# Cooperative Shielding in Long Range Systems


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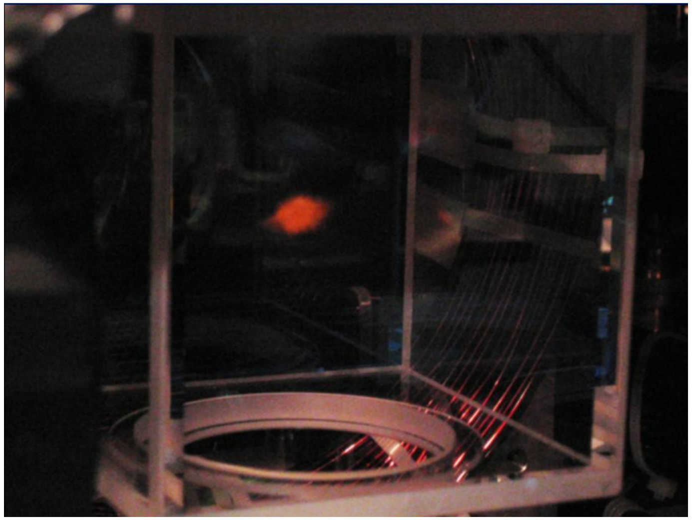
# Experimental Relevance of long range

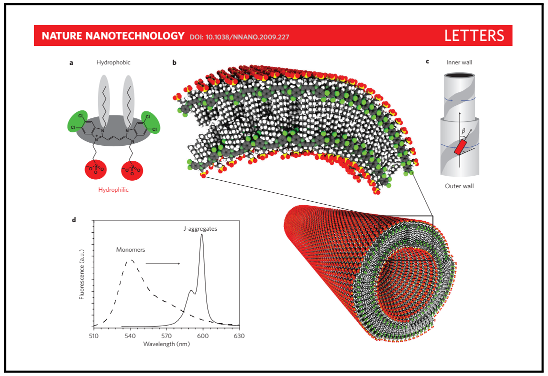
Long-range interacting systems: $1/r^{\alpha}$ **with** $\alpha < d$**. Gravity, EM.**

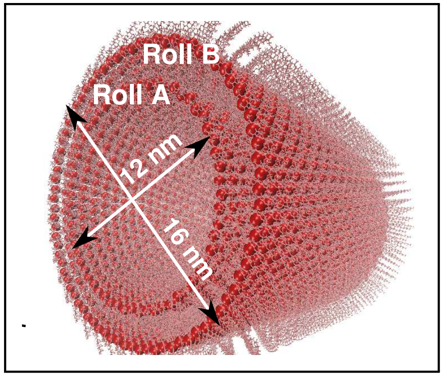
## Cold Atomic Clouds:

Robin Kaiser (CNRS, France)



## Biological Systems.




## CAVITY PHYSICS

J. Feist and F. J. Garcia-Vidal

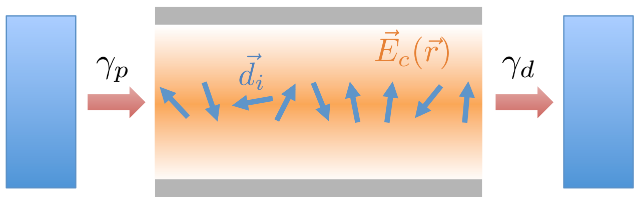
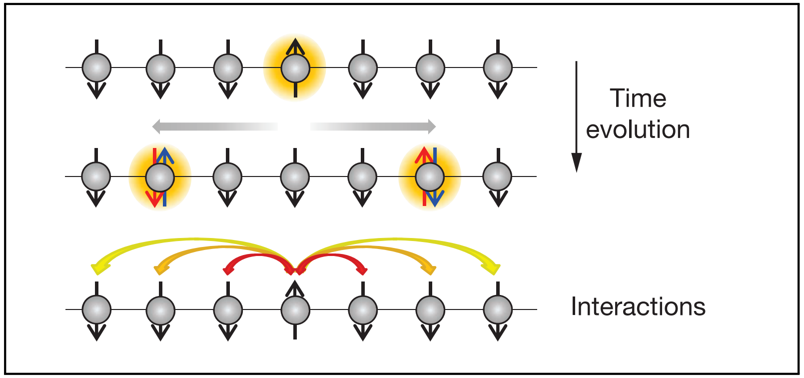



FIG. 1. Sketch of the model system. A 1D chain of (possibly disordered) quantum emitters with dipole moments $\vec{d}_i$ inside a cavity with cavity mode $\vec{E}_c(\vec{r})$. Excitons are pumped into the system from the left reservoir with rate $\gamma_p$. The exciton current is measured by the excitons reaching the sink reservoir on the right,

## Ion Traps: tuning interaction range



# Long-Range Interaction: information spreading

Ion Traps experiment

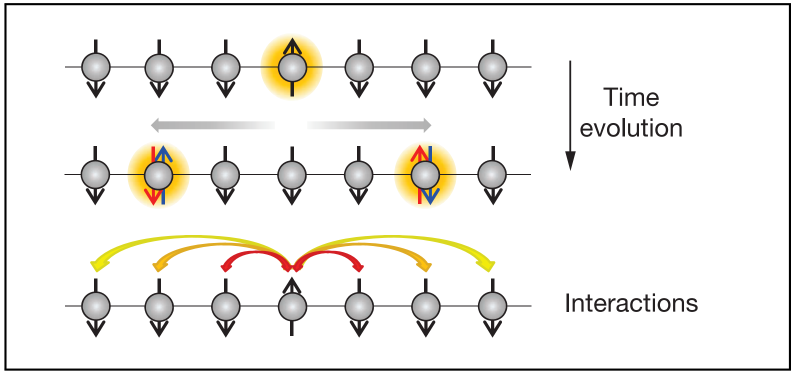
1d Many Body Hamiltonian:

$$H = B\sum_k \sigma_k^z + J\sum_{i<j}\frac{\sigma_i^x\sigma_j^x}{|i-j|^\alpha}$$

with $0 \leq \alpha \leq 3$ .

Breaking of Lieb-Robinson bounds in Ion Trap

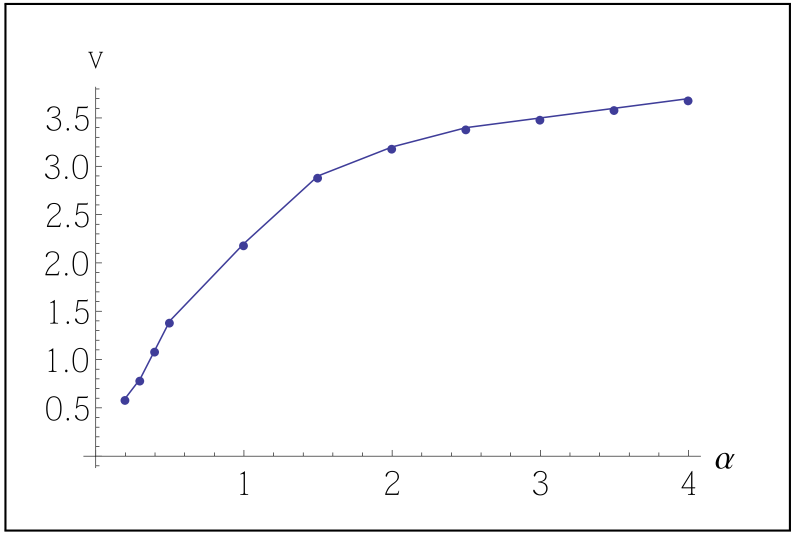
Richerme et al., Nature Letter **511**, 198 (2014); P. Jurcevic et. al., Nature, **511**, 202 (2014).




Theoretical work:

Suppression of the velocity of spreading with the increase of the interaction range $\alpha$.

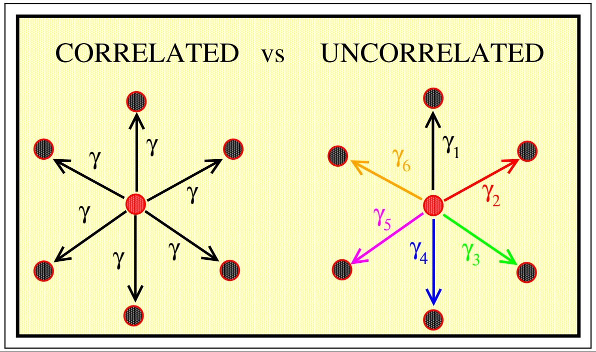
M. Kastner, New J. Phys. **17**, 063021 (2015)




Cooperative Shielding can help to explain such contradictory features

# A COMMON MECHANISM TO COLLECTIVE PHENOMENA

## Correlation Induced Localization



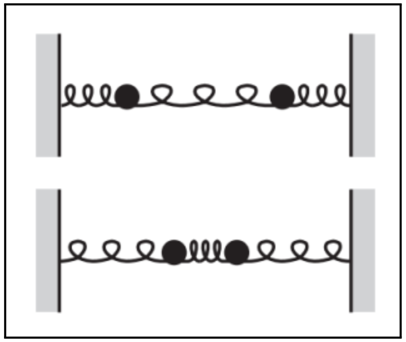

- X. Deng, V.E. Kravtsov, G.V. Shlyapnikov, and L. Santos Phys.Rev. Lett. 120, 110602 (2018).
- Rahul M. Nandkishore and S.L. Sondhi Phys. Rev. X 7, 041021 (2017).
- J. T. Cantin, T. Xu, and R. V. Krems Phys. Rev. B 98, 014204 (2018).
-P. A. Nosov, I. M. Khaymovich, and V. E. Kravtsov Phys. Rev. B 99, 104203 (2019)
-A. Lerose, B. Zunkovic, A. Silva and A. Gambassi, Phys. Rev. B 99, 121112(R) (2019)
-F. Liu, R. Lundgren, P. Titum, G. Pagano, J. Zhang, C. Monroe, and A. V. Gorshkov, Phys. Rev. Lett. 122, 150601 (2019).



**Fano Rev Mod Phys: “superconductivity, plasmons, nuclear deformations…cmmon to these phenomena is the role of a dense spectrum of states…The seemingly weak interactions among these states often condensates into a single eigenvalue separated from the rest of the spectrum by an enegy gap.”**

# WHAT ARE COOPERATIVE EFFECTS?

- Cooperative effect are based on collective correlated motion.



- Collective motion does not imply cooperative effects: example: normal modes of linearly coupled oscillators.
- $H = H_0 + V$
- $V$ : coupling to an external environment or intrinsic coupling. $V$ selects states with large couplng.
- **A common mechanism of collective phenomena** U. Fano, Rev. Mod. Phys. 64, 313 (1992).
  **Real and imaginary energy gaps**: a comparison between single excitation Superradiance and Superconductivity and robustness to disorder, Eur. Phys. J. B (2019) 92: 144.

# Cooperative Shielding in many-body.

Experimentally accessible 1d spin $1/2$ Hamiltonian:

$$H = H_0 + V, \tag{1}$$
$$H_0 = B\sum_{n=1}^{L} \sigma_n^z$$
$$V = \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x.$$

- $\alpha < 1$: long range. $\alpha > 1$: short range.



# Cooperative Shielding in many-body.

Experimentally accessible spin $1/2$ Hamiltonian:

$$H = H_0 + V, \tag{2}$$
$$H_0 = \sum_{n=1}^{L-1} J_z \sigma_n^z \sigma_{n+1}^z,$$
$$V = \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x.$$

- $\alpha < 1$: long range. $\alpha > 1$: short range.

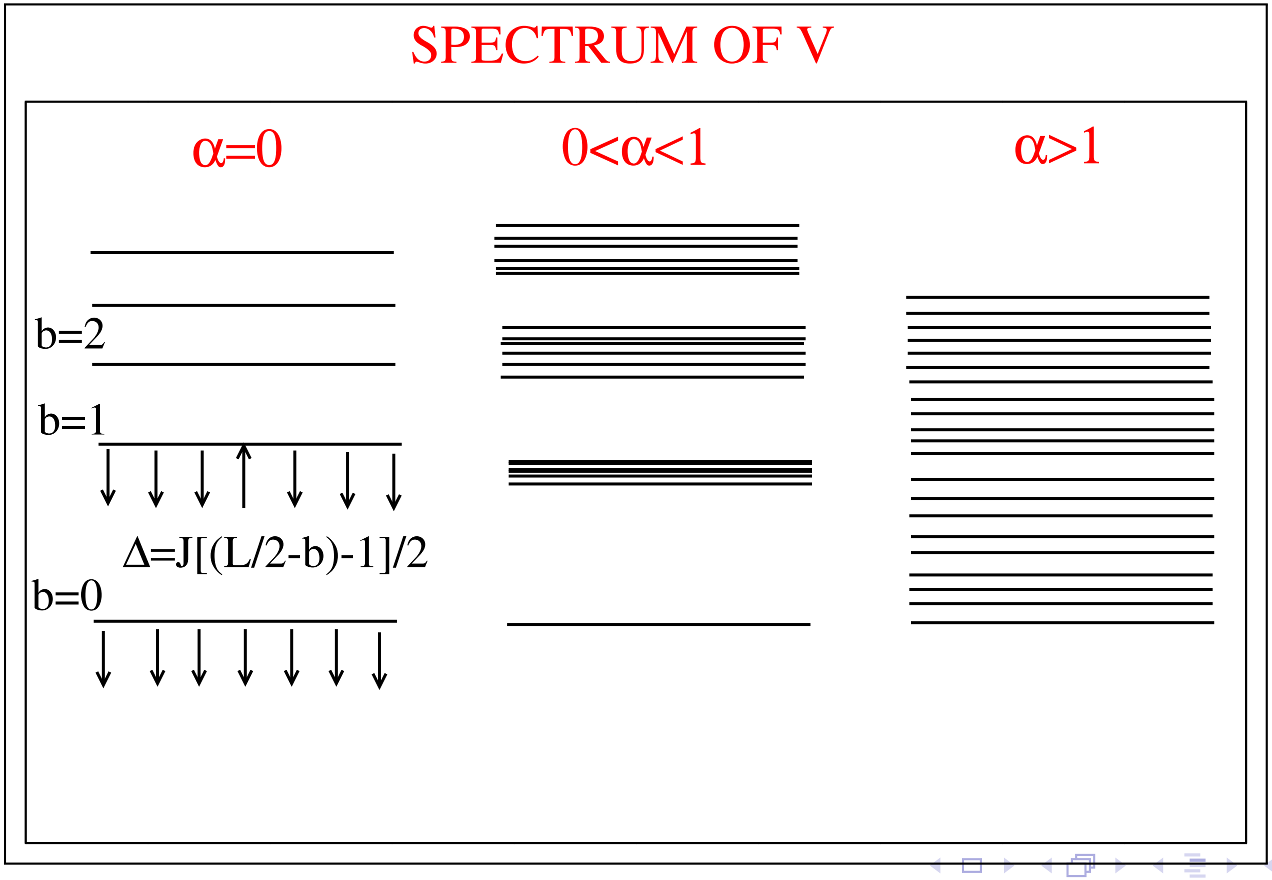
**NN+ LONG RANGE**

# Spectrum of V

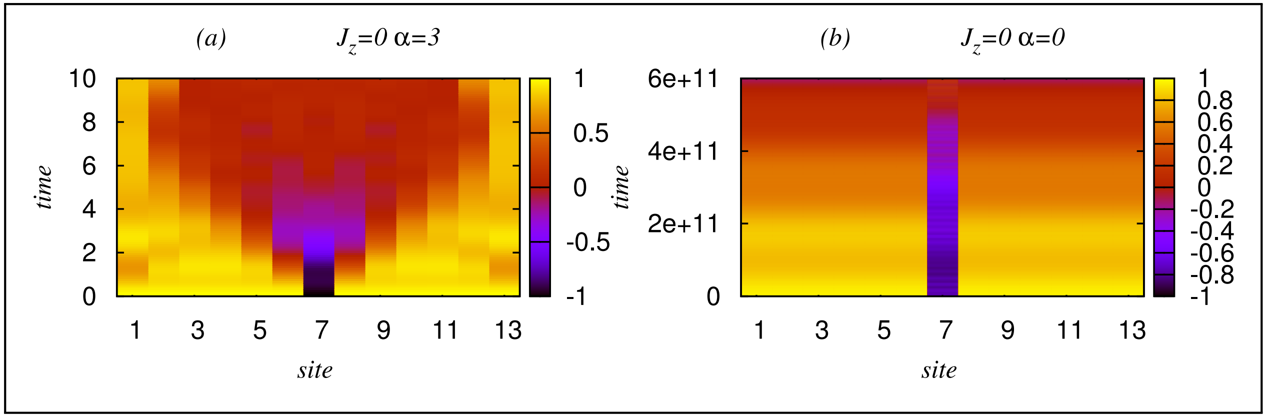
The case $\alpha = 0$ :

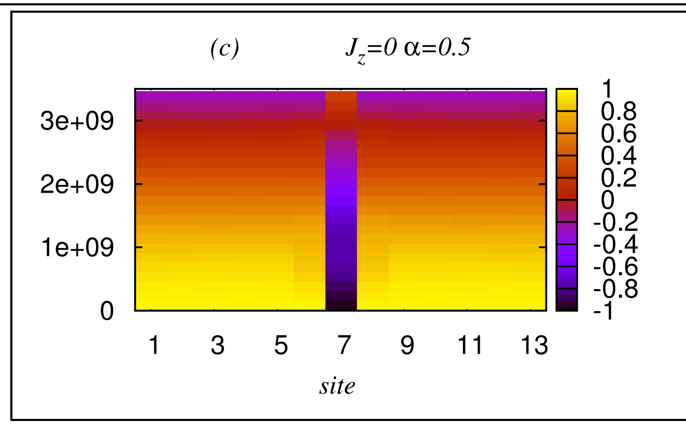
$$V = J\sum_{n<m} \sigma_n^x \sigma_m^x = \frac{JM_x^2}{2} - \frac{JL}{2} \quad \text{where} \quad M_x = \sum_n \sigma_n^x$$

$$V_b = J(L/2-b)^2/2 - JL/2, \quad \text{where} \quad b = 0, 1, \ldots L/2$$



# Light-cones



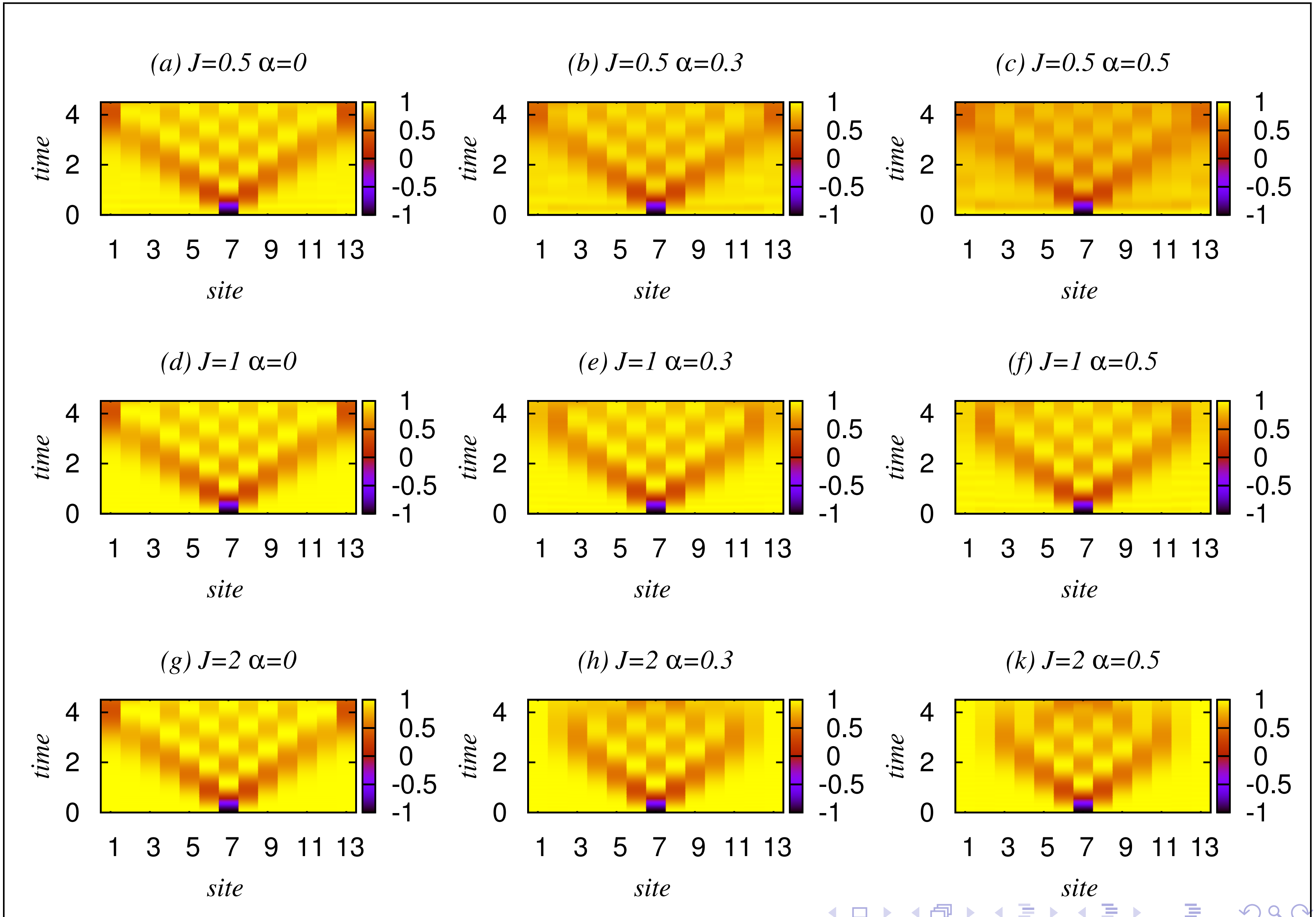




Initial State:

$$|\psi_0\rangle = |\uparrow, \uparrow, .., \downarrow, .., \uparrow, \uparrow\rangle_X$$

*a*) $B = 0.5, \alpha = 3$ light-cone;
*b*) $B = 0.5, \alpha = 0$ localization without disorder;
*c*) $B = 0.5, \alpha = 0.5$

# Shielding


(a) J=0.5 α=0
(b) J=0.5 α=0.3
(c) J=0.5 α=0.5
(d) J=1 α=0
(e) J=1 α=0.3
(f) J=1 α=0.5
(g) J=2 α=0
(h) J=2 α=0.3
(k) J=2 α=0.5
time
site
0
2
4
1 3 5 7 9 11 13
1
0.5
0
-0.5
-1

# CONCLUSIONS

- **Cooperative Shielding: existence of subspaces in long range interacting systems where the dynamics can be described by emergent short range Hamiltonians.**
- **Role of cooperativity in many body interacting systems can lead to new discoveries!**

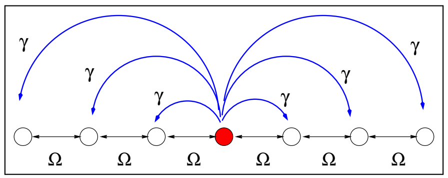
**THANK YOU!**

## Cooperative Shielding. Single excitation transport.

- 1d Anderson model with long range hopping:

$$H = D + H_{\rm NN} + V_{\rm LR} = \sum_i \epsilon_i^0 |i\rangle\langle i| - \Omega \sum_{\langle i,j \rangle} (|j\rangle\langle i| + |i\rangle\langle j|) - \gamma \sum_{i \neq j} \frac{|i\rangle\langle j|}{r_{i,j}^{\alpha}}$$

- $\epsilon_j^0$: are random energies $[-W/2, +W/2]$; $r_{i,j} = |i - j|$; long range for $\alpha < 1$. $\alpha = 0$: all to all.
- $\Omega > 0, \gamma > 0$: the tunnelling transition amplitude.



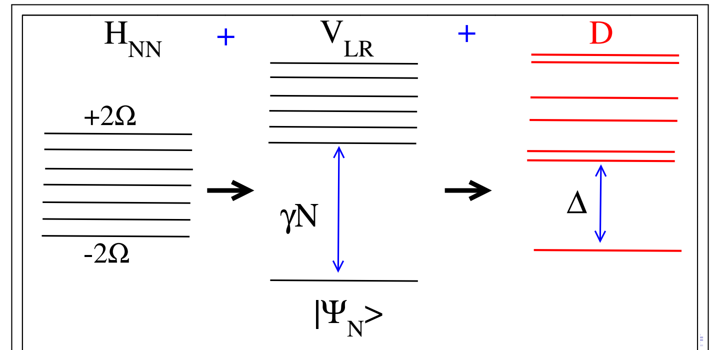

G.L.C., R. Kaiser, and F. Borgonovi, PRB **94**, 144206 (2016).



## Spectrum and Energy Gap: Does shielding survive disorder?

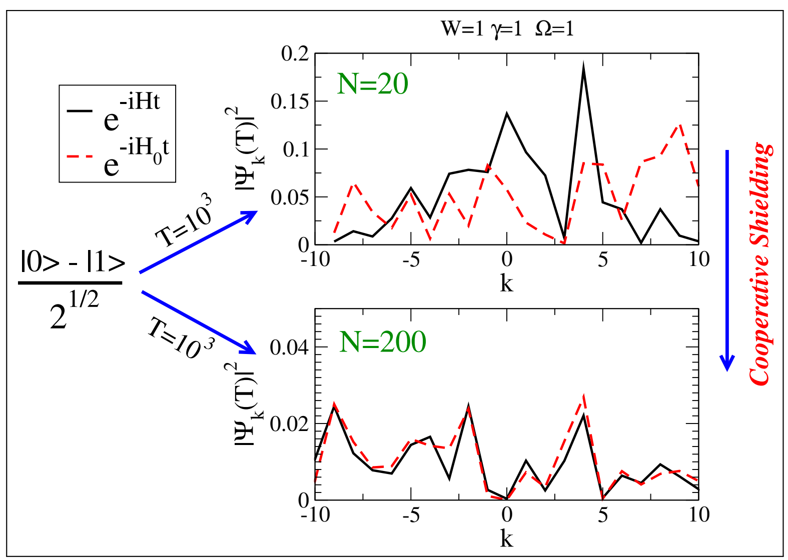
$$H = H_{NN} + V_{LR} + D$$

$$H = -\Omega \sum_i (|i\rangle\langle i+1| + h.c.) - \gamma \sum_{i \neq j} |i\rangle\langle j| + \sum_i \epsilon_i^0 |i\rangle\langle i|$$

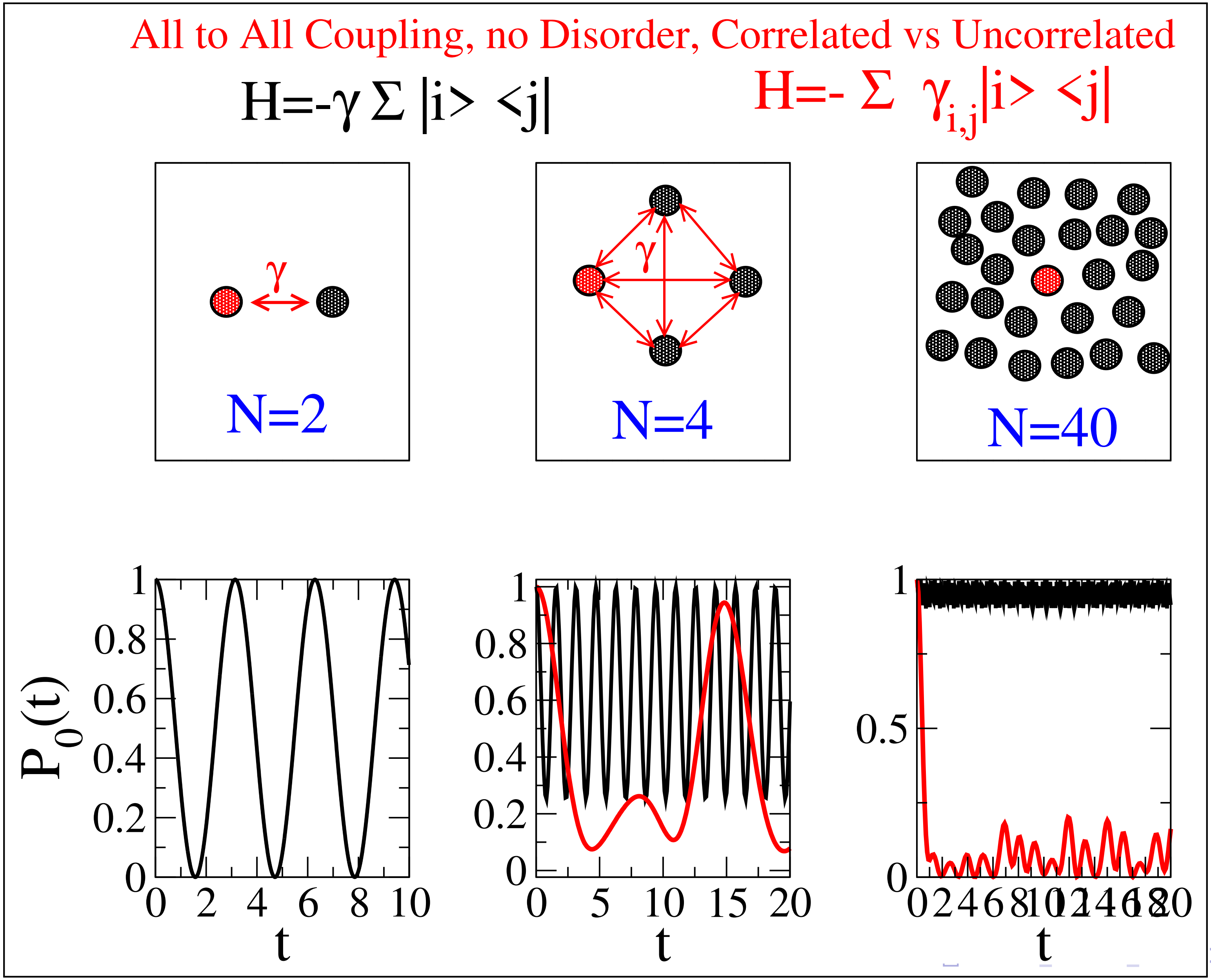





## Cooperative Shielding



# Suppression of Long Range interaction



# The Shielding effect

- Let us consider a system:

$$H = H_0 + V, \qquad \text{with} \qquad [H_0, V] = 0$$

with $V$ highly degenerate $V|v_k\rangle = v|v_k\rangle$

- $|\psi_0\rangle = \sum_{k=1}^{g} c_k|v_k\rangle$
- $V$ contributes only with global phase

$$|\psi(t)\rangle = e^{iHt}|\psi_0\rangle = e^{ivt}e^{iH_0t}|\psi_0\rangle$$

We have shielding from $V$!!. $H_0$: emerging Hamiltonian.

- What if $[H_0, V] \neq 0$?
- What if spectrum of $V$ is not degenerate? What is the connection with long range? Is this a cooperative effects? What is the emergent Hamiltonian?