Particle Production from Gravitational Inhomogeneities

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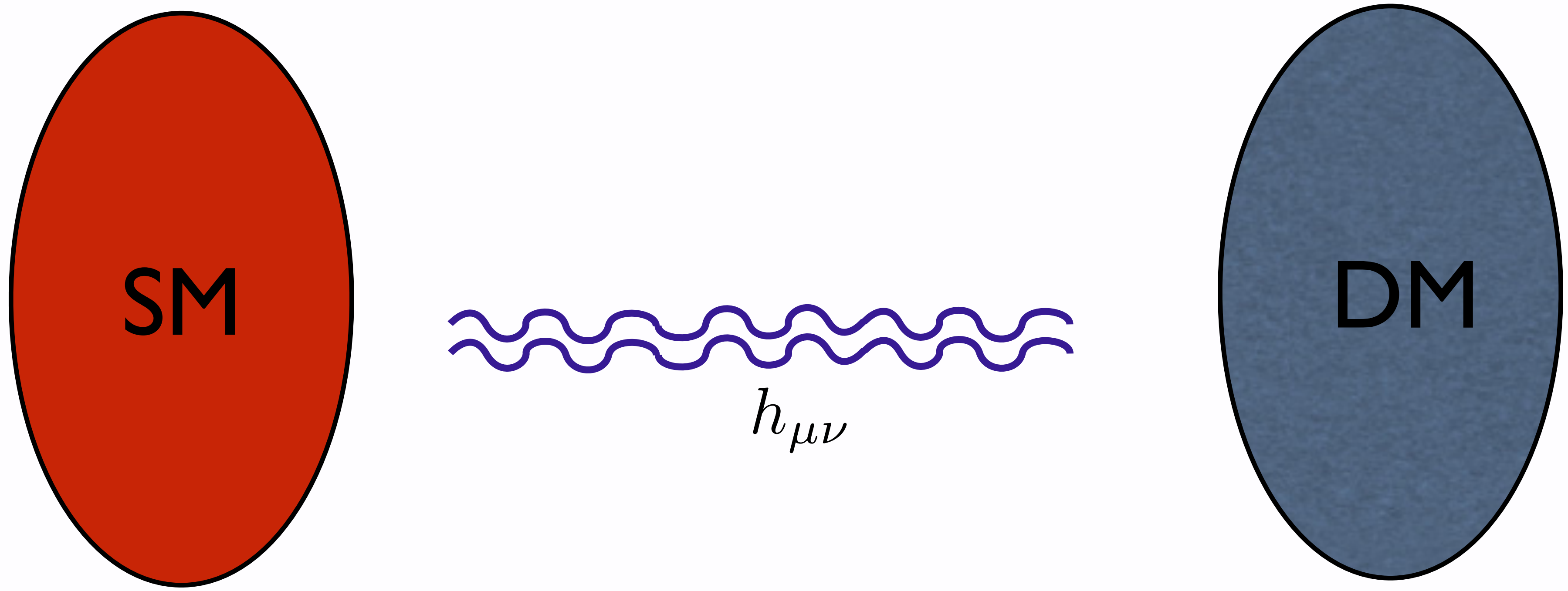
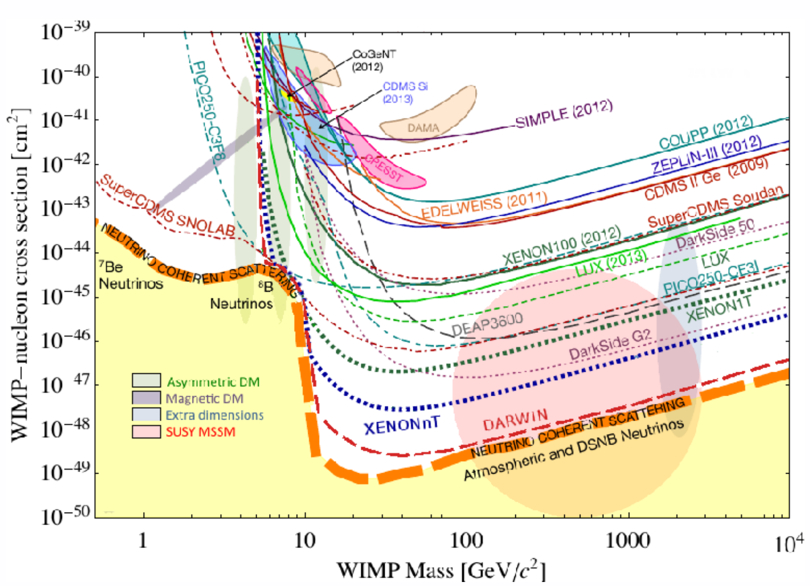
with Garani and Tesi

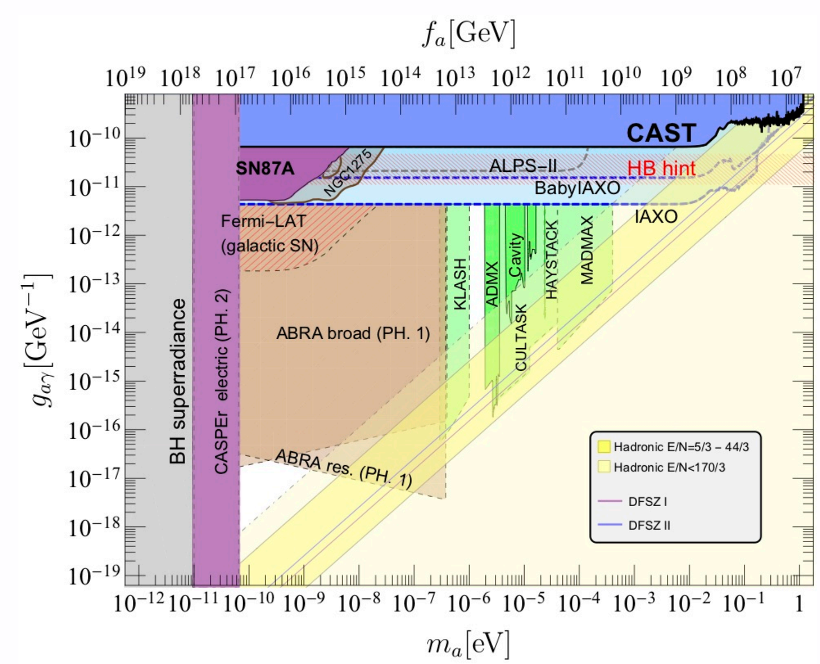
Firenze - 19 March 2025

A fundamental question is how Dark Matter couples to the SM



It would be great if DM had other interactions with SM:





What if DM interacts only gravitationally?

Apparently we live in an expanding universe:

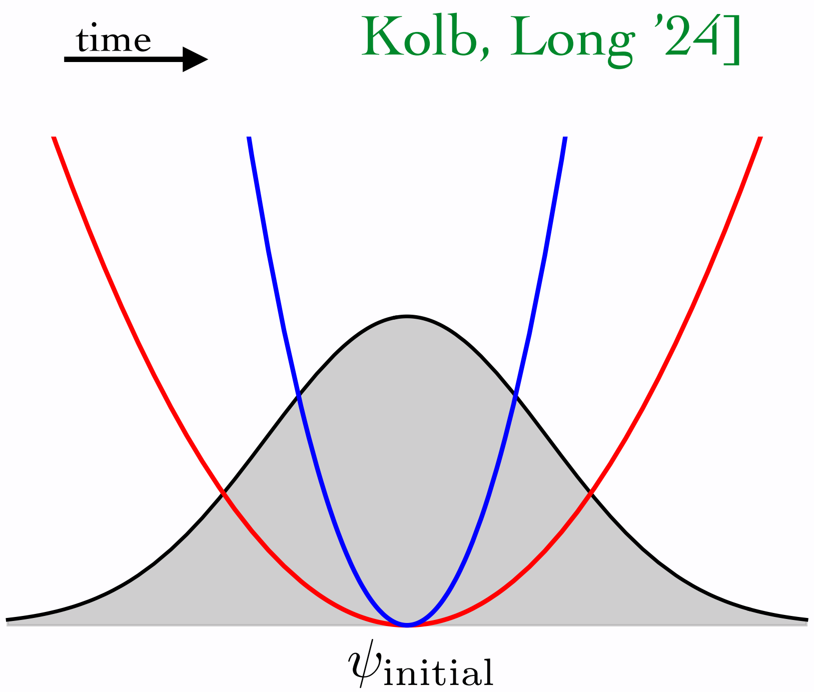
$$ds^2 = a(\tau)^2(d\tau^2 - d\vec{x}^2)$$

In a time dependent background particles are produced due to the non-adiabatic evolution of the vacuum.

[Schroedinger '39,
Ford '87,
Kolb, Riotto, Giudice '90s
Kolb, Long '24]

$$v''_{\vec{k}}(\tau) + \omega^2_{\vec{k}}(\tau) v_{\vec{k}}(\tau) = 0$$

$$\omega^2_{\vec{k}}(\tau) = |\vec{k}|^2 + M^2 a^2(\tau) - \frac{a''(\tau)}{a(\tau)}(1 - 6\xi)$$

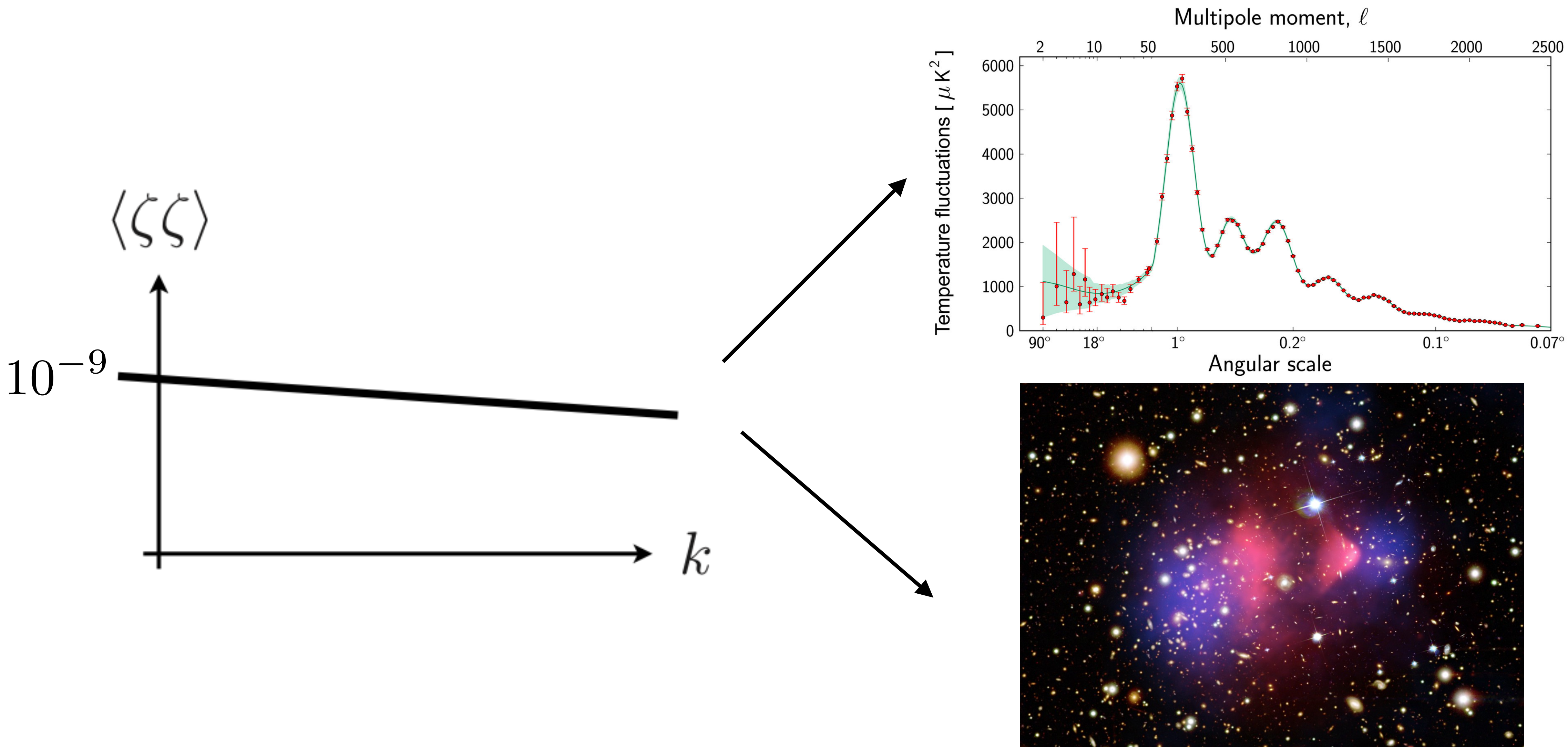


As for the time dependent harmonic oscillator the initial vacuum is interpreted as an excited state at late times.

A phase of quasi de-Sitter — inflation — appears necessary to explain the initial conditions of our universe,

$$a(\tau) \approx -\frac{1}{H_I \tau}$$

Production of the inflaton generates the seeds of perturbations that eventually give rise to all that we see:



Could Dark Matter be produced by the expansion of the universe?

Cosmological particle production is associated to the breaking of Weyl invariance ($g_{\mu\nu}(x) \rightarrow \Omega(x)^2 g_{\mu\nu}(x)$):

- Kinetic term:

$$L = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \qquad\qquad \Omega_{\rm DM} \sim \sqrt{\frac{M}{10^{-5}{\rm eV}}} \left(\frac{H_I}{10^{14} GeV} \right)^2$$

- Mass term:

$$L = \bar{\Psi}(i\gamma^\mu D_\mu - M)\Psi \qquad\qquad \Omega_{\rm DM} \sim \left(\frac{M}{10^9 \ {\rm GeV}} \right)^{5/2}$$

Massless fermions and in general conformally coupled theories are not produced in a homogeneous FLRW background

$$ds^2 = a(\tau)^2 (d\tau^2 - d\vec{x}^2) \longrightarrow \Box \chi = 0$$

With inhomogeneities the background is not Weyl flat and particle production takes place even in Weyl invariant theories:

$$ds^2 = a^2(\tau)[\eta_{\mu\nu} + h_{\mu\nu}(\tau, \vec{x})]dx^\mu dx^\nu$$

To first order:

$$S_{\text{int}} = \frac{1}{2}\int d^4x h_{\mu\nu}(x)T^{\mu\nu}(x) \longrightarrow \Box\varphi = \frac{1}{2}h_{\mu\nu}\frac{\delta T^{\mu\nu}}{\delta\varphi}$$

Recall Schwinger pair production

$$\langle \text{out}|\text{in}\rangle = e^{i\Gamma} \longrightarrow |\langle \text{out}|\text{in}\rangle|^2 = e^{-2\text{Im}\Gamma_{1\text{PI}}}$$

The number of events is:

$$N \approx 2\text{Im}[\Gamma_{1\text{PI}}]$$

Contrary to Schwinger pair production it is very easy to compute $\Gamma_{1\text{PI}}$

$$\Gamma_{1PI} = \frac{1}{4}\int \frac{d^4q}{(2\pi)^4} h^{\mu\nu}(-q)\langle T_{\mu\nu}(q)T_{\rho\sigma}(-q)\rangle h^{\rho\sigma}(q)$$

For conformally coupled particles <TT> is completely fixed!

$$\langle T_{\mu\nu}(q) T_{\rho\sigma}(q')\rangle = (2\pi)^4 \delta^4(q+q') \frac{c_J}{7680\pi^2} \Pi_{\mu\nu\rho\sigma}(q) \log(-q^2)$$

$$\Pi_{\mu\nu\rho\sigma}(q) \equiv (2\pi_{\mu\nu}\pi_{\rho\sigma} - 3\pi_{\mu\rho}\pi_{\nu\sigma} - 3\pi_{\mu\sigma}\pi_{\nu\rho}) \ , \qquad \pi_{\mu\nu} \equiv \eta_{\mu\nu} q^2 - q_\mu q_\nu$$

The total number of particles is:

$$N_{\text{particles}} = 4 \mathrm{Im}\Gamma_{1PI} = \frac{c_J}{15360\pi} \int \frac{d^4 q}{(2\pi)^4} \theta(q^2) h^{\mu\nu}(q) \Pi_{\mu\nu\rho\sigma}(q) h^{\rho\sigma}(-q)$$

There is one last candy... the theory is Weyl invariant so the answer must contain the Weyl tensor!

$$N_{\text{particles}} = \frac{c_J}{1280\pi} \int \frac{d^4 q}{(2\pi)^4} \theta(q^2) W^{\mu\nu\rho\sigma}(q) W_{\mu\nu\rho\sigma}(-q)$$

In cosmology we are often interested in stochastic backgrounds.

scalar perturbations:

$$ds^2 = a^2 d\tau^2[1 + 2\Psi(\tau, \vec{x})] - a^2[1 - 2\Psi(\tau, \vec{x})]d\vec{x}^2 \,.$$

$$\langle \Psi_{\vec{q}}(\tau)\Psi^*_{\vec{q}'}(\tau')\rangle = (2\pi)^3\delta^3(\vec{q} - \vec{q'})\frac{2\pi^2}{q^3}\Delta_\Psi(q, \tau, \tau')$$

$$\frac{d(na^3)}{dq_0} = \frac{c_J}{240\pi^2}\int d(\log q)\,\theta(q_0^2 - q^2)q^4\Delta_\Psi(q, q_0, -q_0)$$

tensor perturbations:

$$\frac{d(na^3)}{dq_0} = \frac{c_J}{640\pi^2}\int d(\log q)\,\theta(q_0^2 - q^2)(q_0^2 - q^2)^2\Delta_h(q, q_0, -q_0)$$

Stochastic Dark Matter

This mechanism allows to produce cosmologically dark sectors when the mass scale is negligible:

- massless fermions and gauge fields
- interacting CFTs

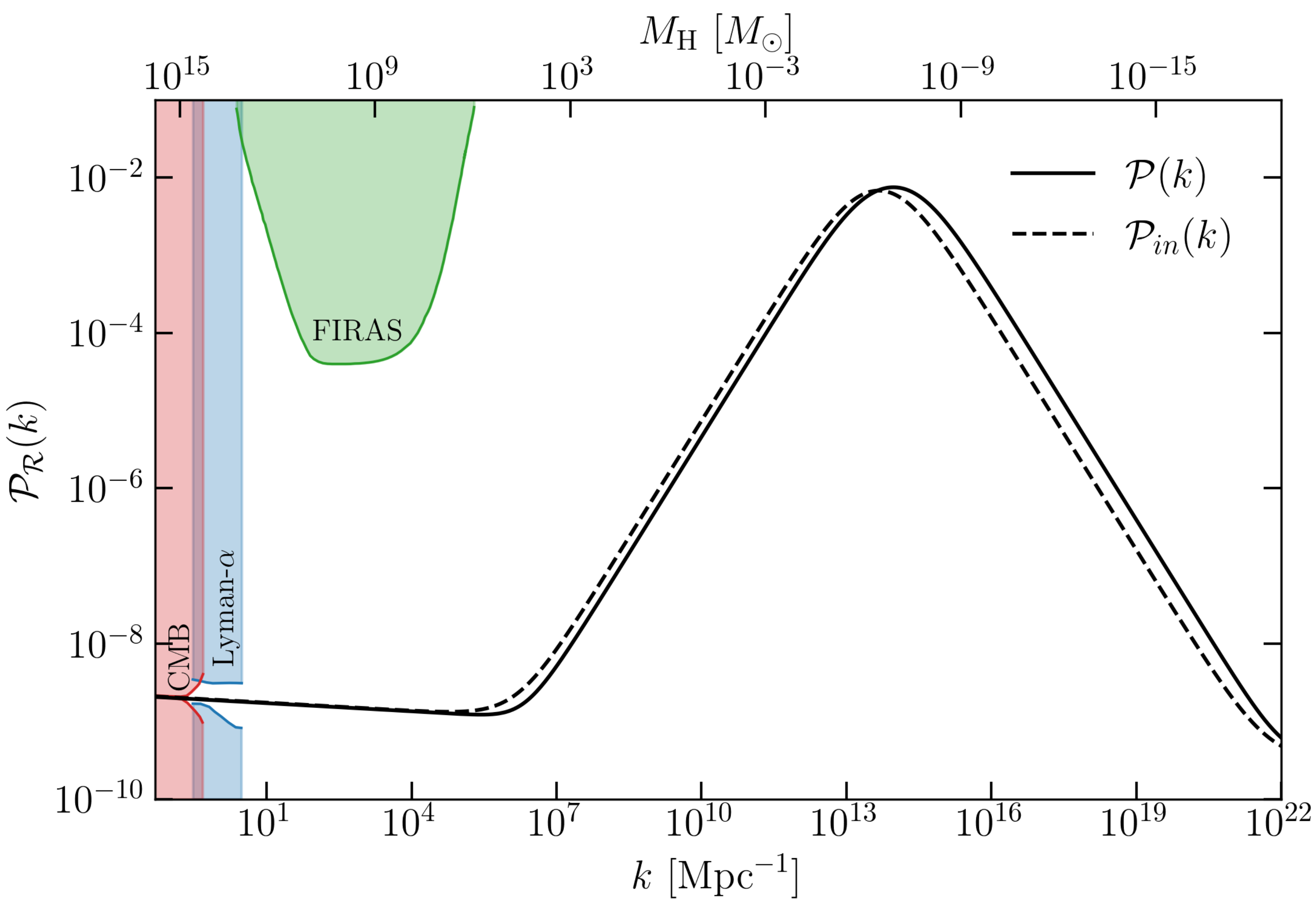
If these sectors are gapped by mass or confinement they will contribute to DM.

The background could be produced by inflation, first order phase transitions or any other violent event in the universe.

- Inflationary production:

[Garani, MR, Tesi '24]

Inflation produces curvature perturbations that depend on the evolution of the inflaton. At large scales the perturbations are small but they could be large at smaller scales,



$$n_{\rm DM} \approx 10^{-4}\, c_J \times \Delta_\zeta(q_{\rm peak}) q_{\rm peak}^3$$

Abundance is dominated by modes that exit horizon towards the end of inflation

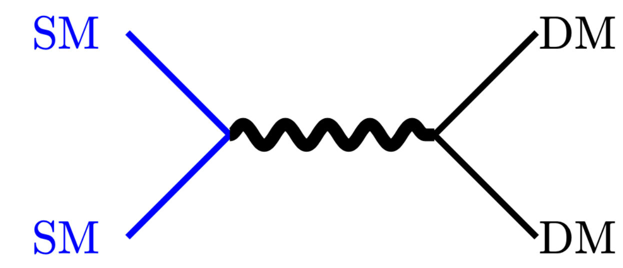
$$\Omega_{\rm DM} \approx c_J \frac{\Delta_\zeta(q_{\rm max})}{10^{-2}} \frac{M}{10^7\,{\rm GeV}}$$

Other contributions can easily be smaller,

- Gravitational freeze-in:

[Garny-Sandora-Sloth '15
MR-Tesi-Tillim '20]

$$\Omega_{\rm DM}|_{\rm GFI} \approx 10^{-5} \frac{M k_R^3}{3M_{\rm Pl}^2 H_0^2} \times c_J$$



- Time dependent background:

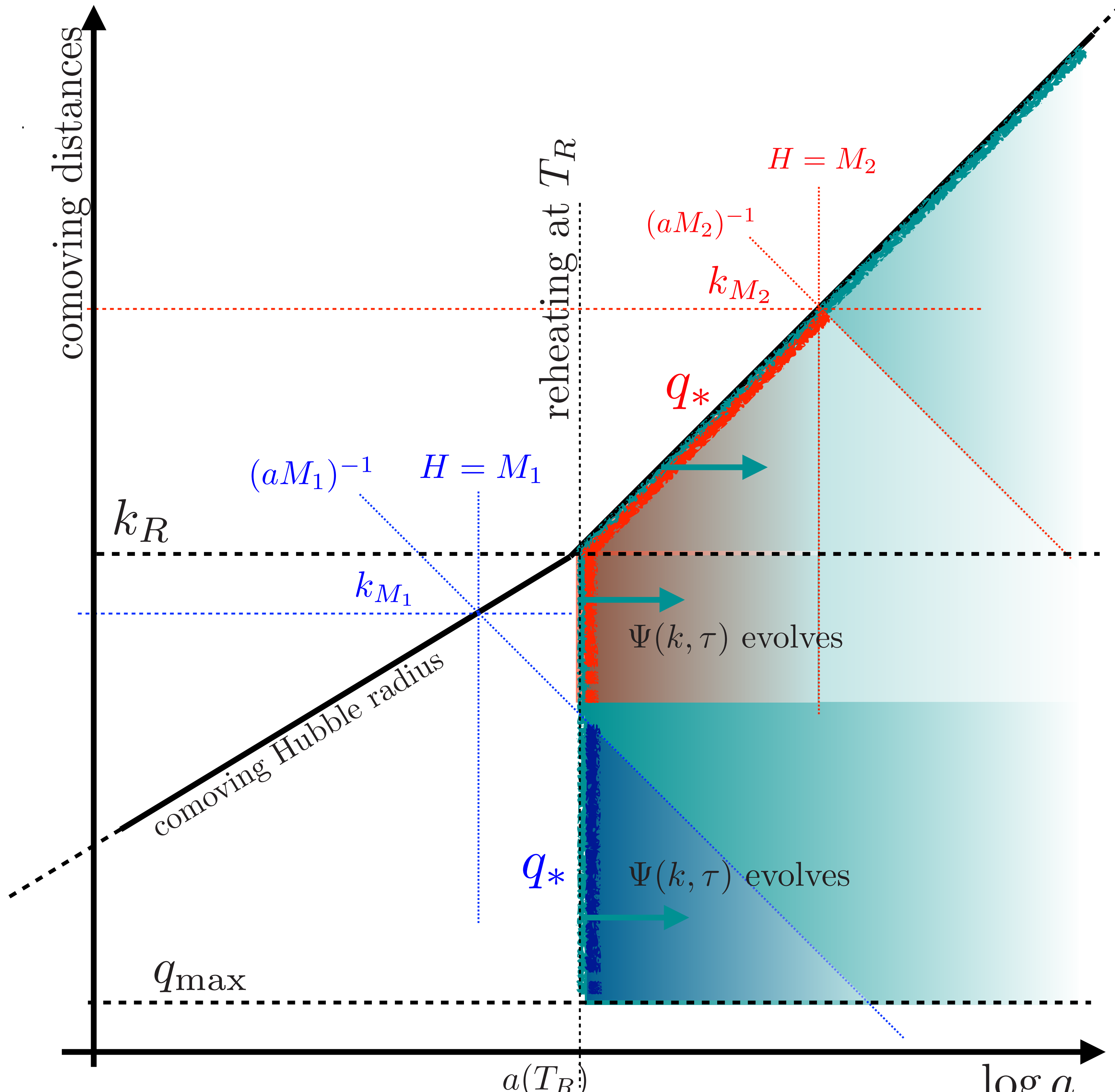
$$\Omega_{\rm DM}|_{\rm GPP} \approx 10^{-2} \frac{M\,k_M^3}{3M_{\rm Pl}^2 H_0^2} \lesssim \left(\frac{M}{10^9\,{\rm GeV}}\right)^{5/2}$$

SUMMARY

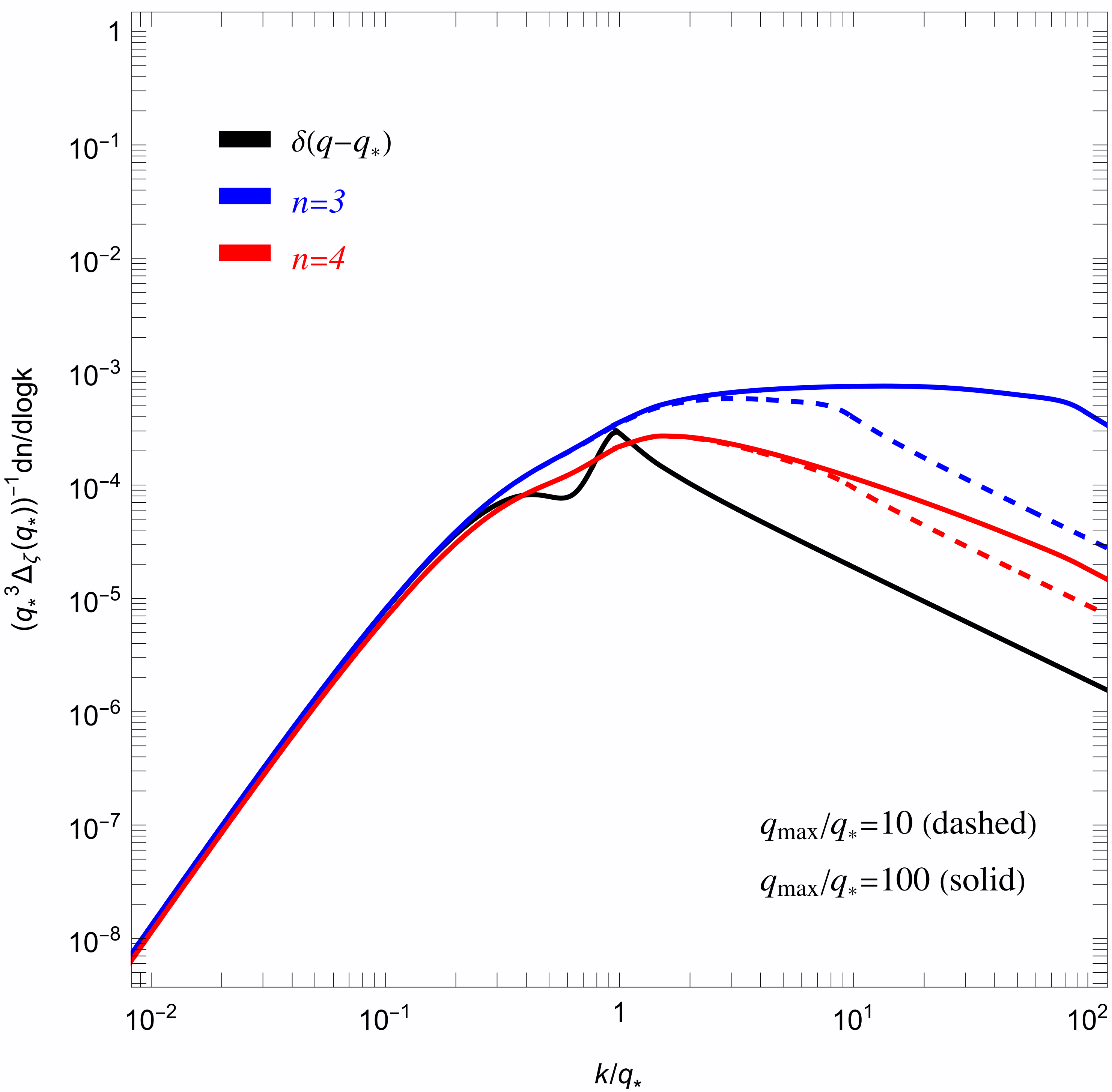
- Inhomogeneities allow for a new mechanism of particle production that works even for conformally coupled particles such as fermions and gauge fields.

- Inclusive quantities are simply determined by the central charge.

- This mechanism can be applied to the production of dark sectors that can host (heavy) dark matter. The formalism can be also applied to production from astrophysical objects or other defects.



comoving distances
reheating at T_R
$H = M_2$
$(aM_2)^{-1}$
k_{M_2}
q_*
$(aM_1)^{-1}$
$H = M_1$
k_R
k_{M_1}
$\Psi(k,\tau)$ evolves
comoving Hubble radius
q_*
$\Psi(k,\tau)$ evolves
$q_{\rm max}$
$a(T_R)$
$\log a$



$\delta(q-q_*)$
$n=3$
$n=4$
$q_{\rm max}/q_*=10$ (dashed)
$q_{\rm max}/q_*=100$ (solid)
$(q_*^3\Delta_\zeta(q_*))^{-1}$dn/dlogk
k/q_*
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{1}
10^{2}