Holography and Confining Gauge Theories

Federico Castellani

Università degli Studi di Firenze

- "Resonance contributions to nucleon spin structure in holographic QCD", F.Bigazzi and F.C., JHEP 04 (2024) 037;
- "Nucleon electric and magnetic polarizabilities in holographic QCD", F.C., Phys.Rev.D 110 (2024) 6, 066001;
- "Hagedorn temperature in holography: world-sheet and effective approaches", F.Bigazzi, T.Canneti, A.L.Cotrone, W. Mück, F.C., JHEP 09 (2024) 193;
- "Holography for Confined and Deformed Theories: TsT-Generated Solutions in type IIB Supergravity", C.Nunez and F.C., JHEP12(2024)155;
- "TsT-Generated Solutions in Type IIB Supergravity from Twisted Compactification of $AdS_5 \times T^{1,1}$", F.C., JHEP03(2025)xxx;
- "Vacuum configuration of winding superstrings from non-standard semiclassical quantization", T. Canneti, W. Mück, and F.C., arXiv:2501.14532 [hep-th];

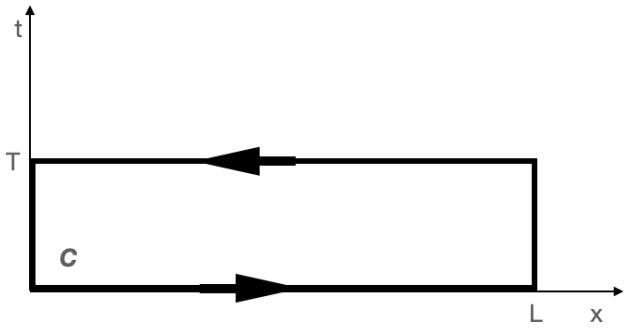
Holographic Correspondence

[Maldacena '97, Witten '98, Gubser et al '98]

- Conjecture: duality between certain quantum field theories and gravity theories in higher dimensions;
- "Weak/strong duality";
- Countless checks of its validity;
- Applied in various physics research areas;

Confinement in Holography

- In a gauge theory, the rectangular Wilson loop is related to the potential between a non-dynamical quark-anti-quark pair.
- Its expectation value gives indications for confinement/deconfinement phases;
- $V(L) \sim L$, the theory is confining;

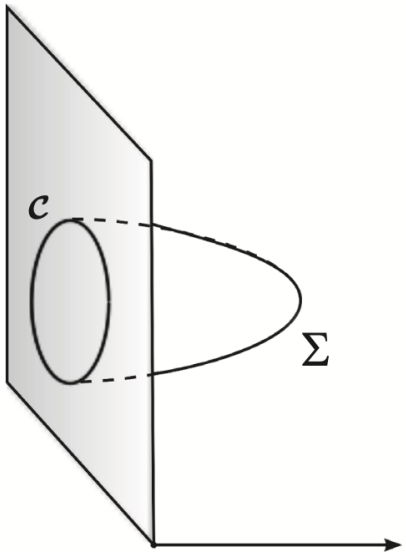


$$\langle W_C \rangle = \mathrm{Tr}\, P \exp\left(-i \oint_C A\right) \sim \exp\left(-TV(L)\right)$$

Confinement in Holography

- Holographically, $\langle W_C \rangle$ is computed through the on-shell Nambu-Goto action of a string exploring the bulk dual geometry;
- The worldsheet Σ approaches the contour C at the boundary:

$$\langle W_C \rangle = \exp\left(-S_{NG}^{on-shell}[\Sigma]\right) .$$



- One way to build up confining holographic models is via wrapped D_p branes constructions with shrinking q-cycle Σ_q;
- Infinite tower of massive Kaluza-Klein (KK) modes;
- In the supergravity approximation, no hierarchy between the masses of the KK modes and the QFT's strong coupling scale;
- It is challenging to disentangle the effects of these modes from the low-energy dynamics;

Twisted Compactified $\mathcal{N} = 4$ SYM [Kumar, Stuardo '24]

- 4D $\mathcal{N} = 4$ Super Yang Mills theory compactified on a circle ϕ of radius R with SUSY breaking b.c.;
- Background gauge field $\mathcal{A}_\mu = Q\,\delta_\mu^\phi$ for the diagonal combination of the three $U(1)$ generators of the maximal abelian subgroup of R-symnmetry $SO(6)$;
- Gaugino λ_4 can now have a zero mode precisely when

$$Q = \frac{1}{3R}\,,$$

  and it can be identified as the gaugino for the 3D gauge multiplet.
- 3D theory preserves 4 supercharges and consists of a massless vector multiplet (A_μ, λ_4) and the KK tower of three massive chiral multiplets $(Y_{\tilde{a}}, \lambda_{\tilde{a}})$.

Holographic Description [Anabalon, Ross '21, Nunez et al. '24]

- SUSY Soliton: $\mathcal{N}=1$ SUSY preserving deformation of $AdS_5 \times S^5$

$$ds^2 = \frac{r^2}{\ell^2}\left(dx_{2,1}^2 + f(r)d\phi^2\right) + \frac{dr^2}{\frac{r^2}{\ell^2}f(r)} + \ell^2 \sum_{i=1}^{3}\left[d\mu_i^2 + \mu_i^2\left(d\phi_i + Q^3\ell^4\zeta(r)d\phi\right)^2\right],$$

$$F_5 = G_5 + \star G_5,$$

$$G_5 = -\frac{4}{\ell}\mathrm{Vol}(\mathcal{M}_5) + Q^3\ell^4 \sum_{i=1}^{3} d(\mu_i^2) \wedge \left(d\phi_i + Q^3\ell^4\zeta(r)d\phi\right) \wedge dt \wedge dx_1 \wedge dx_2$$

where

$$f(r) = 1 - \frac{Q^6\ell^{12}}{r^6}, \qquad \zeta(r) = \frac{1}{r^2} - \frac{1}{Q^2\ell^4}, \qquad \ell = (4\pi g_s N)^{1/4},$$

$$\mu_1 = \sin\theta\sin\varphi, \quad \mu_2 = \sin\theta\cos\varphi, \quad \mu_3 = \cos\theta, \quad Q = \frac{1}{3R}.$$

TsT transfomation

- Compare an observable calculated in the seed background with the same observable in a "deformed" background.
- Deformation obtained by performing a so-called TsT transformation (T-duality, shift of coordinates, and T-duality) on the seed geometry. These transformations typically involve $U(1) \times U(1)$ global symmetry.
- Lunin and Maldacena proposed the TsT transformation as a way of producing backgrounds dual to marginal deformations of SCFTs.
- The KK-modes are typically charged under R-symmetries of the QFT, associated with internal isometries of the dual background.

TsT transfomation [Nunez, Castellani '24]

- The marginal deformed background reads

$$ds^2 = \frac{r^2}{\ell^2}\left(dx_{2,1}^2 + f(r)d\phi^2\right) + \frac{dr^2}{\frac{r^2}{\ell^2}f(r)} + \ell^2 d\tilde{\Omega}_5^2\,,$$

$$d\tilde{\Omega}_5^2 = \sum_{i=1}^{3}\left[d\mu_i^2 + \mu_i^2 G\left(d\phi_i + Q^3\ell^4\zeta(r)d\phi\right)^2\right] + \gamma^2\mu_1^2\mu_2^2\mu_3^2 G\left(\sum_{i=1}^{3} d\phi_i\right)^2,$$

$$F_5 = G_5 + \star G_5\,, \quad F_7 = G_5 \wedge B\,, \quad F_3 = -\star F_7\,,$$

$$B = \gamma G\left(\mu_1^2\mu_2^2 d\phi_1 d\phi_2 + \mu_2^2\mu_3^2 d\phi_2 d\phi_3 + \mu_3^2\mu_1^2 d\phi_3 d\phi_1\right)\,, \quad e^{2\Phi} = G\,,$$

  where

$$G^{-1} = 1 + \gamma^2\left(\mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_3^2\mu_1^2\right)\,.$$

- In the dual QFT, Lagrangian deformed by introducing an alternative field product

$$\Phi_A\Phi_B \longrightarrow \Phi_A \star \Phi_B = e^{i\pi\gamma\left(Q^1_{\Phi_A}Q^2_{\Phi_B} - Q^2_{\Phi_A}Q^1_{\Phi_B}\right)}\Phi_A\Phi_B\,, \gamma \in \mathbb{R}\,.$$

KK modes and TsT [Nunez, Castellani '24, Castellani '25]

- The Lagrangian terms involving the vector multiplet (A_μ, λ_4) are invariant under the deformation;
- After the compactification, the zero modes belong only to the spectrum of the latter fields;
- If an observable of the deformed theory depends on the γ parameter, then it receives contributions by the massive KK modes;
- We can discern between observables that depend on the IR theory only or also in its UV completion;

Observables: Wilson loops and confinement properties

- The probe string does not extend inside the deformed five-sphere

$$t = \tau\,, \qquad x^1 = \sigma\,, \qquad r = r(\sigma)\,, \qquad \sigma \in [-L/2, L/2]\ .$$

- No pull-back on the worldsheet of the B field and induced metric independent on the γ-parameter

$$\mathrm{d}s^2_{\mathrm{ind}} = r^2\left[-\mathrm{d}\tau^2 + \left(1 + \frac{r'(\sigma)^2}{r^4 f(r)}\right)\mathrm{d}\sigma^2\right]\,, \quad r'(\sigma) = \partial_\sigma r(\sigma)\,,$$

$$S_{NG} \longrightarrow S_{NG} = \frac{T}{2\pi}\int_{-L/2}^{L/2}\mathrm{d}\sigma\sqrt{r^4 + \frac{r'(\sigma)^2}{f(r)}}\,.$$

- $\langle W_C\rangle$ is blind respect with the marginal transformation.
- Confining properties of the theory depend only on the IR physics.

Summary

- $\mathcal{N} = 1$ twisted compactification of 4D $\mathcal{N} = 4$ $SU(N)$ SYM on $\mathbb{R}^{2,1} \times S^1$.
- Holographic dual: SUSY soliton on $\mathrm{AdS}_5 \times S^5$. This solution contains a cigar geometry, which fibers the S^5.
- TsT transformation of the SUSY soliton corresponding to a marginal deformation in the dual field theory.
- We presented an algorithm to discern between observables of seed theory that depend on the IR theory only or also in its UV completion.

Thank you!