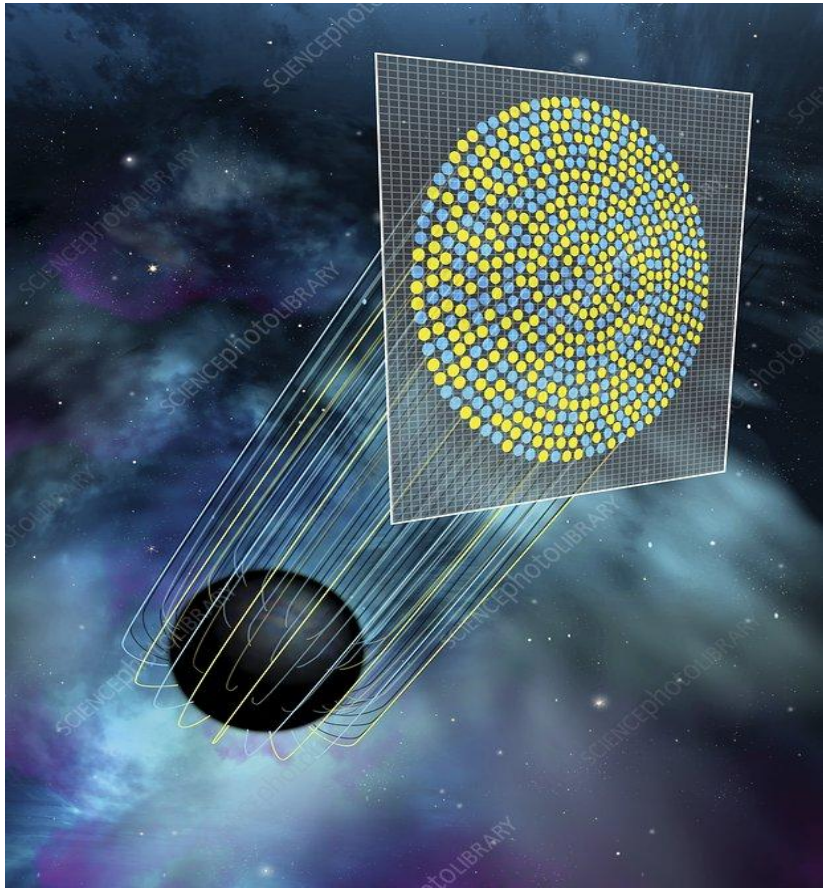
Bridging Holography in Maximally Symmetric Space-times

Lorenzo IACOBACCI
The Galileo Galilei Institute for Theoretical Physics

loreiacobacci@gmail.com

Quantum Gravity is Holographic



«The situation can be compared with a hologram of a three-dimensional image on a two-dimensional surface.»

(T'Hooft, «Dimensional reduction in quantum gravity», Conf. Proc. **C** 930308 (1993), 284-296))

«In a certain sense, the world is two-dimensional and not three-dimensional as previously supposed.»

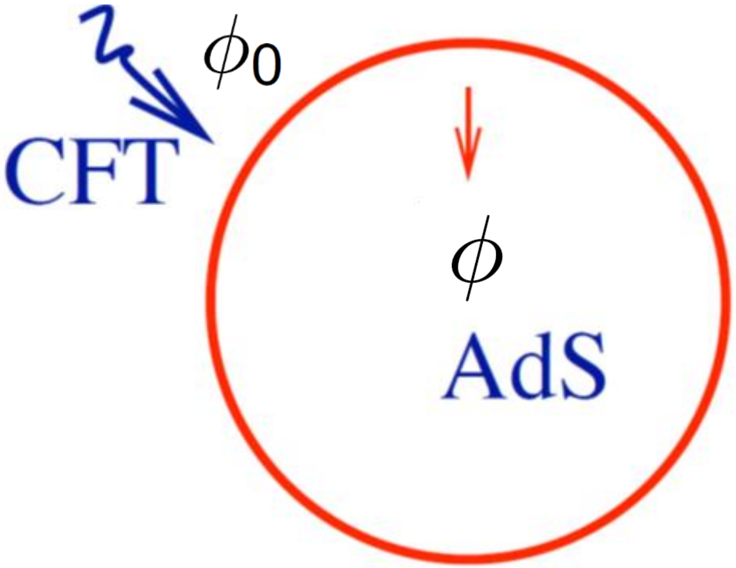
(Susskind, «The World as a Hologram», J. Math. Phys. **36** (1995), 6377-6396))

WHAT IS THE THEORY ON THE BOUNDARY OF THE UNIVERSE?

The AdS/CFT Correspondence

- A Conformal field theory in D-dimensions is dual to a quantum theory of gravity in (D+1)-dim. (Witten, «ANTI DE SITTER SPACE AND HOLOGRAPHY», Adv. Theor. Math.Phys. **2**, 253 (1998))

$$\left\langle \exp \int_{\mathbf{S}^d} \phi_0 \mathcal{O} \right\rangle_{CFT} = Z_S(\phi_0)$$



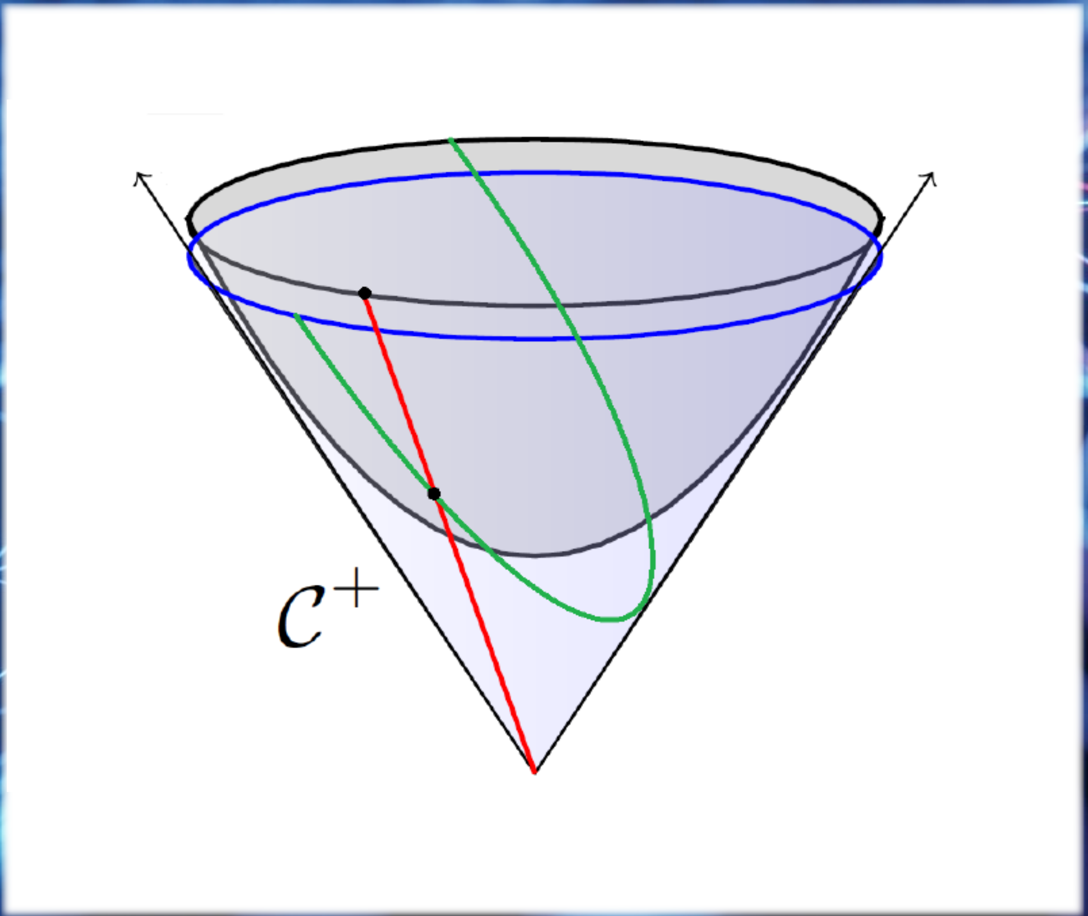
- The right-hand side is the partition function of the bulk theory of gravity (string theory, SUGRA).

- The left hand side is the partition function of a CFT in a fixed background.

- In the semiclassical approximation:

$$Z_S \simeq e^{-S(\phi, A_\mu, \dots)} \longrightarrow \langle O(x_1) \cdots O(x_n) \rangle = (-1)^{n+1} \frac{\delta^3 S_{\text{onshell}}}{\delta\phi_{(0)}(x_1) \cdots \delta\phi_{(0)}(x_n)} \Big|_{\phi_{(0)}}$$

EAdS Conformal Compactification



- H_{d+1} (in grey) is the EAdS unitary slice in (d+2)-dimensional Minkowski.

$$\hat{p}(y,\vec{\omega}) = (2y)^{-1}(1+y^2+|\vec{\omega}|^2, 1-y^2-|\vec{\omega}|^2, 2\vec{\omega})$$

$$\mathrm{d}s^2_{H_{d+1}} = y^{-2}[\mathrm{d}y^2 + \mathrm{d}\vec{\omega}\cdot\mathrm{d}\vec{\omega}]$$

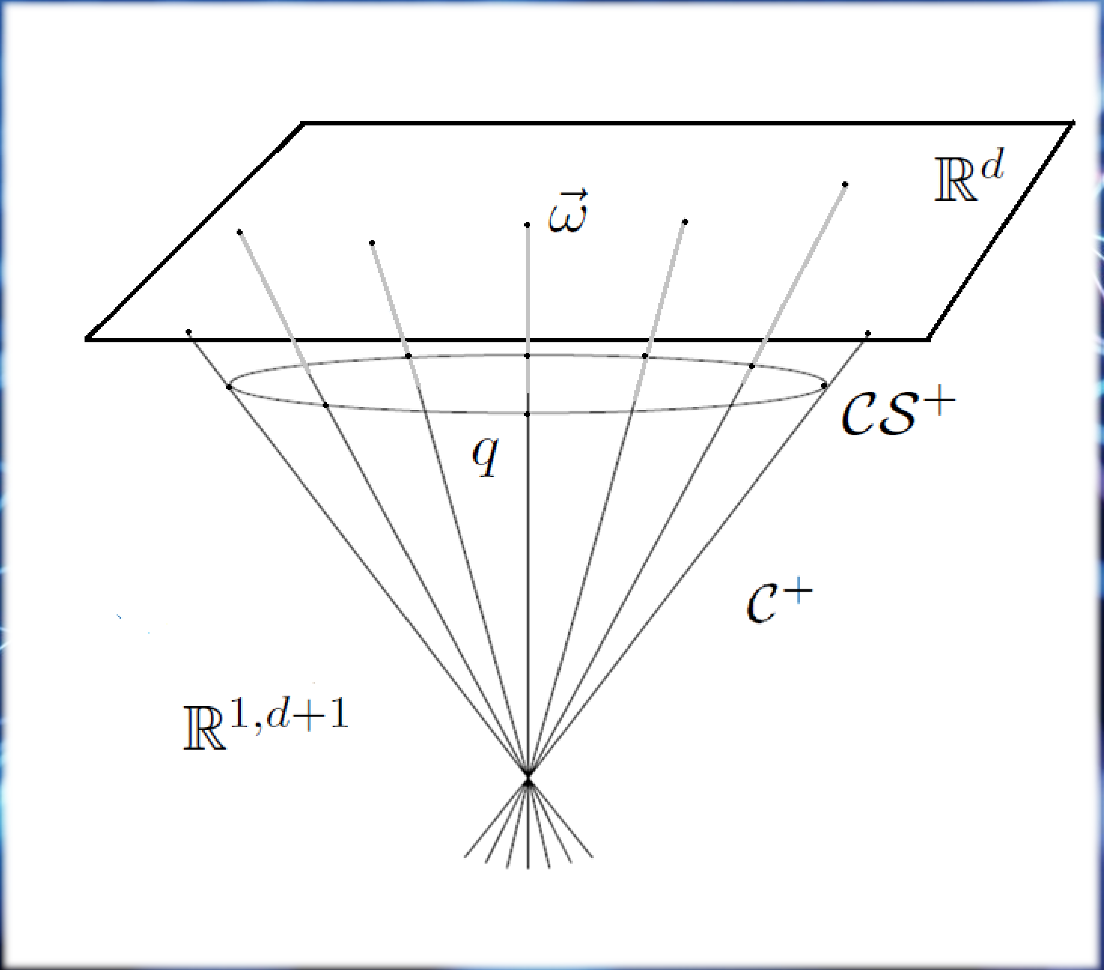
- *CS* is the boundary circle (in blue) of H_{d+1} at infinity.

$$\mathrm{d}s^2_{CS} = \lim_{y\to 0} y^2 \mathrm{d}s^2_{H_{d+1}} = \mathrm{d}\vec{\omega}\cdot\mathrm{d}\vec{\omega}$$

$$q(\vec{\omega}) = \lim_{y\to 0} y\hat{p} = (1+|\vec{\omega}|^2, 1-|\vec{\omega}|^2, \vec{\omega})$$

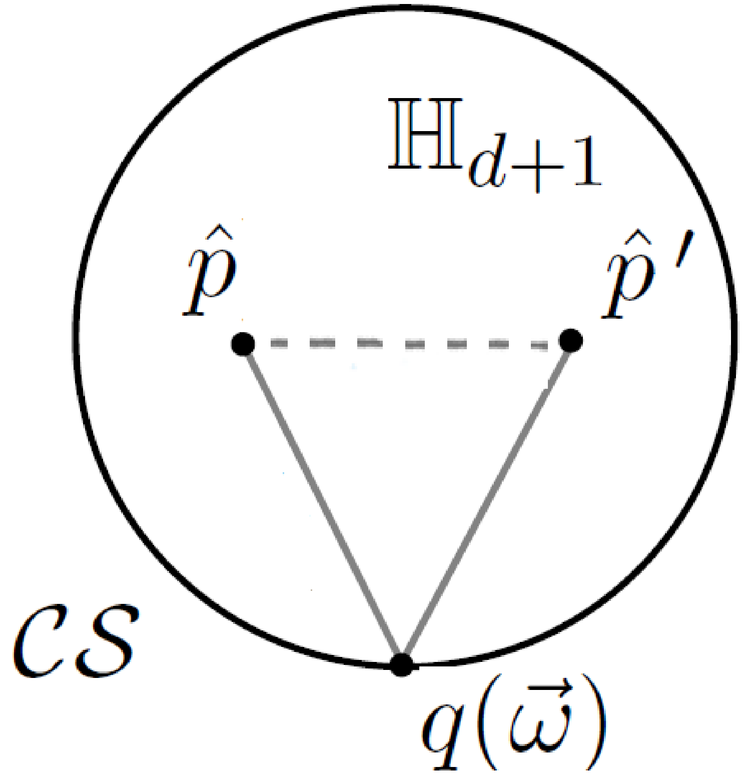
- *CS* conformally compactifies H_{d+1}.

Celestial Sphere as Projective Space



- The celestial sphere *CS* is the set of the light-rays of Minkowski space-time;
- *CS* can be visualized as the sphere that each light-ray intersects in one point at infinity;
- Points q ∈ *CS* are mapped in R^d by a projective map;
- q(ω) transforms under the action of SO(1, d+1) as a scalar conformal primary operator of conformal weight 1.

SO(1, d+1) Symmetries



The group SO(1,d+1) acts as:

1. The Lorentz group of $R^{1,d+1}$;
2. The Isometry group of H_{d+1}.
3. The Conformal group of R^d;

Two types of Propagators:

= Bulk-to-Boundary Propagator

= Bulk-to-Bulk Propagator

Scalar Conformal Primary Basis

- The massive scalar conformal primary wavefunctions are given by [Pasteski, Shao 2017] (*Δ=d/2+iv, v>0*)

$$\phi^{\pm}_{\Delta}(X, q(\vec{\omega})) = \int_{\mathbb{H}_{d+1}} [\mathrm{d}\hat{p}]\, K^{\mathrm{AdS}}_{\Delta}(\hat{p}; q(\vec{\omega}))\, \mathrm{e}^{\pm i m \hat{p}\cdot X},$$

where $[d\hat{p}]$ is the SO(1,d+1)-invariant measure, p=m$\hat{p}$ and

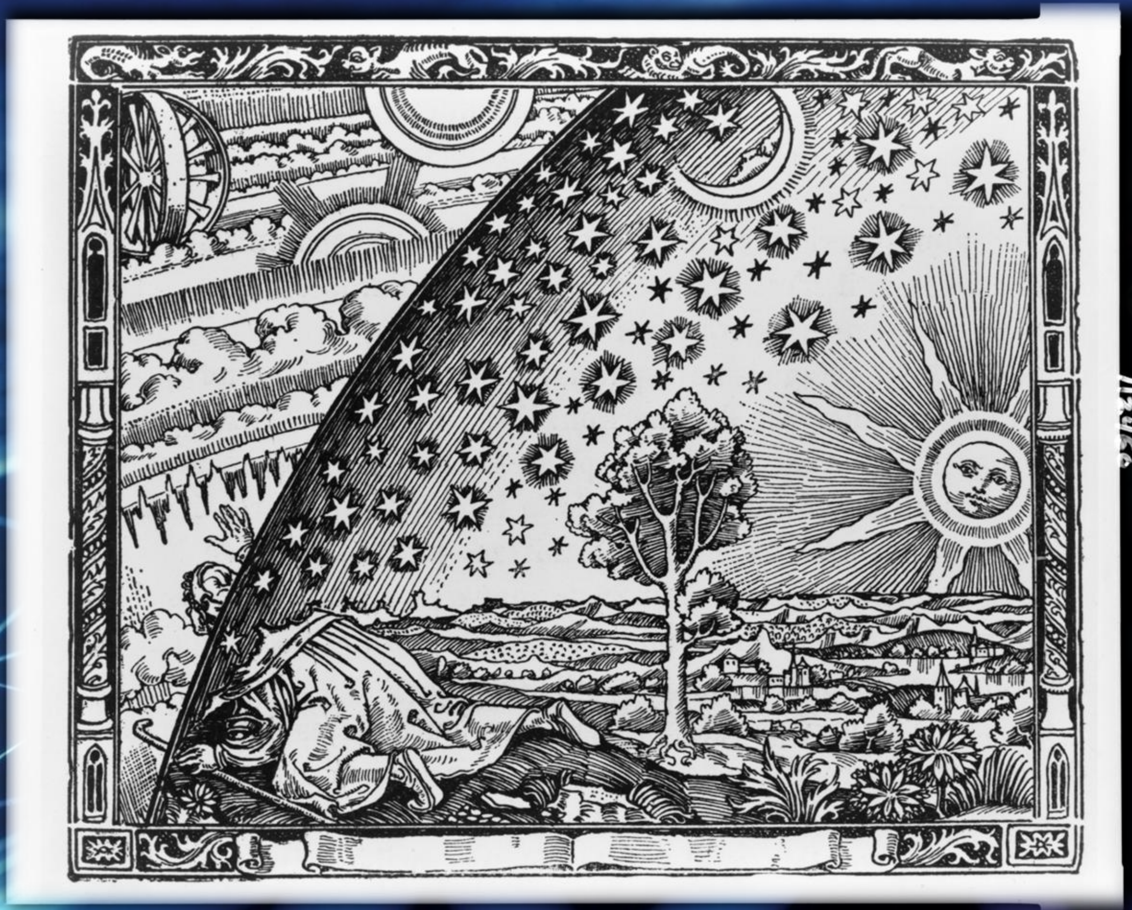
$$K^{\mathrm{AdS}}_{\Delta}(\hat{p}; q) = \frac{C^{\mathrm{AdS}}_{\Delta}}{(-2\hat{p}\cdot q)^{\Delta}},$$ is the scalar bulk-to-boundary propagator.

- The massless scalar conformal primary wavefunctions take the form of Mellin transform of plane waves

$$\varphi^{\pm}_{\Delta}(X; \vec{\omega}) = \int_0^{+\infty} \frac{\mathrm{d}y}{y}\, y^{\Delta} \mathrm{e}^{\pm i y q(\vec{\omega})\cdot X} = \frac{(\mp i)^{\Delta}\Gamma(\Delta)}{(-2q(\vec{\omega})\cdot X \mp i\epsilon)^{\Delta}}$$

Δ=d/2+iv, with *v* real number.

Celestial Holography



- Celestial Holography is a candidate for being a holographic theory in flat space-time;
- It postulates a duality between a QFT in Minkowski and a CFT on the CS;
- It is formulated in a new class of bases known as «Conformal Primary Bases»;
- In this new description, SO(1,d+1)-covariance is manifest;
- The boundary theory is an Euclidean CFT defined on a co-dimension 2 surface;
- We will refer to the boundary theory as a Celestial CFT (CCFT).

Massive Celestial Amplitudes

- The massive Celestial amplitudes are related with the momentum amplitudes by ([Pasterski, Shao, Strominger 2016] & [Pasterski, Shao 2017])

$$\widetilde{\mathcal{A}}(\Delta_k, \vec{\omega}_k) = \prod_{i=1}^{n} \int_{H_{d+1}^{+}} [\mathrm{d}\hat{p}]\, K_{\Delta_i}^{\mathrm{AdS}}(\hat{p}_i, \vec{\omega}_i)\, \mathcal{A}(m_k \hat{p}_k)$$

- They transform covariantly as d-dimensional CFT correlation functions.

- Pasterki, Shao and Strominger used this formula to compute the contact 3-point function setting $m_1 = m_2 = m,\ m_3 = 2m(1+\epsilon)$ and $d = 2,$ at first order in ε.

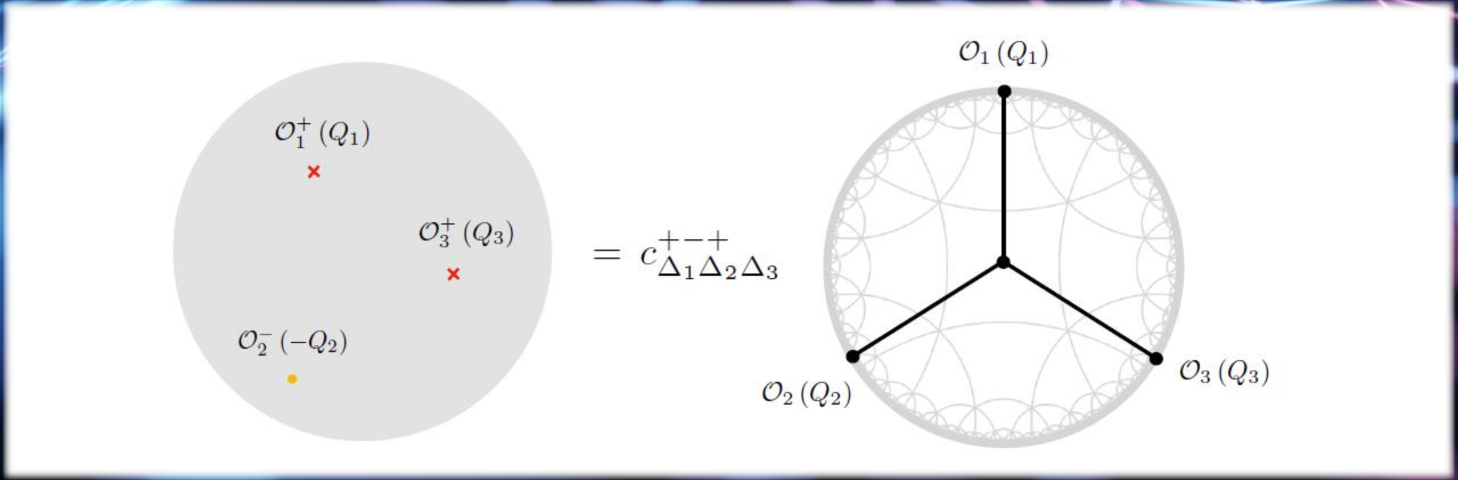
- They found that the Celestial 3-point contact amplitude is proportional to the 3-point contact diagram on EAdS, at leading order in ε.

Celestial Contact Amplitudes

Computing contact amplitudes in Celestial holography, we found the proportionality law

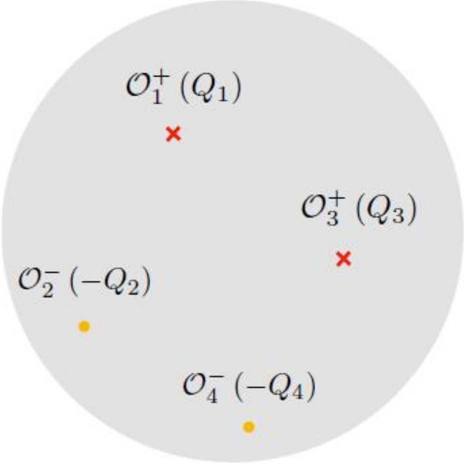
$$\tilde{\mathcal{A}}^{\text{contact}}_{\Delta_1\ldots\Delta_n} = c^{\pm_1\ldots\pm_n}_{\Delta_1\ldots\Delta_n} \times \underbrace{\int_{\mathcal{H}^+_{d+1}} \mathrm{d}\hat{X}_{\text{AdS}}\, K^{\text{AdS}}_{\Delta_1}(\hat{X}_{\text{AdS}};q_1)\ldots K^{\text{AdS}}_{\Delta_n}(\hat{X}_{\text{AdS}};q_n)\,.}_{^{(\text{AdS})}\tilde{\mathcal{A}}^{\text{c}}_{\Delta_1\ldots\Delta_n}(q_1,\ldots,q_n)}$$

The n-point Celestial contact amplitude is proportional to the corresponding EAdS contact amplitude by a coefficient that depends on the masses and the conformal weights of the fields.

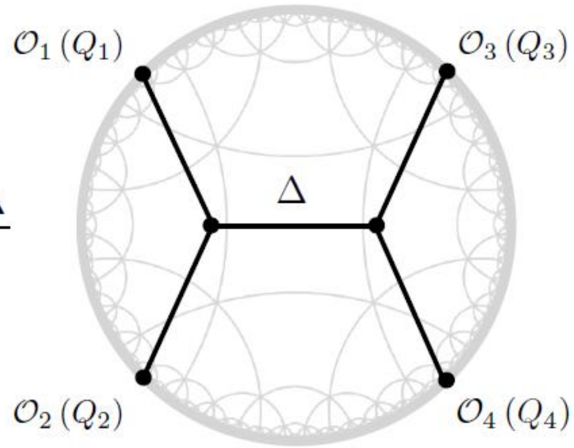


Celestial Correlators from EAdS Witten Diagrams

Each contribution to massive Celestial correlators can be recast in terms of a linear combination of contributions of corresponding massive Witten correlators in EAdS.

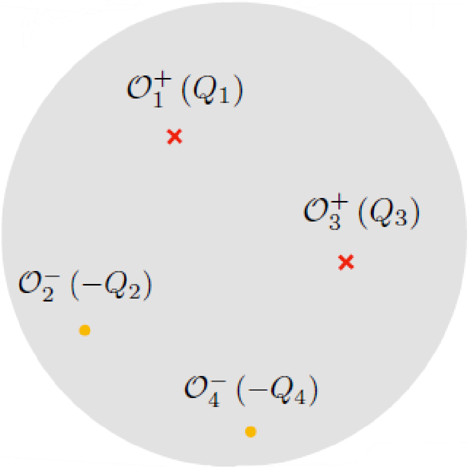


$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i} \sum_{\pm} \frac{c^{+-\pm}_{\Delta_1\Delta_2\Delta}\, c^{+-\mp}_{\Delta_3\Delta_4\Delta}}{c_\Delta}$$

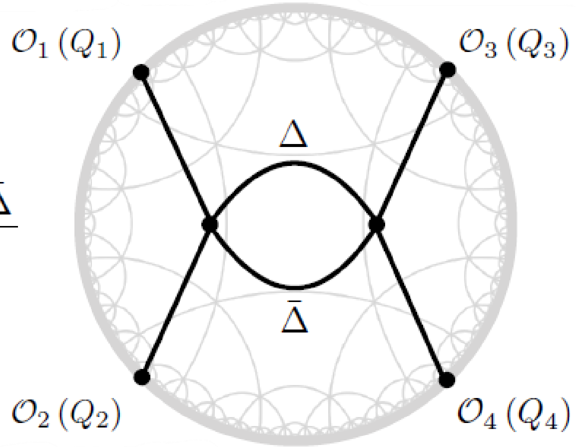


Celestial Correlators from EAdS Witten Diagrams

Each contribution to massive Celestial correlators can be recast in terms of a linear combination of contributions of corresponding massive Witten correlators in EAdS.

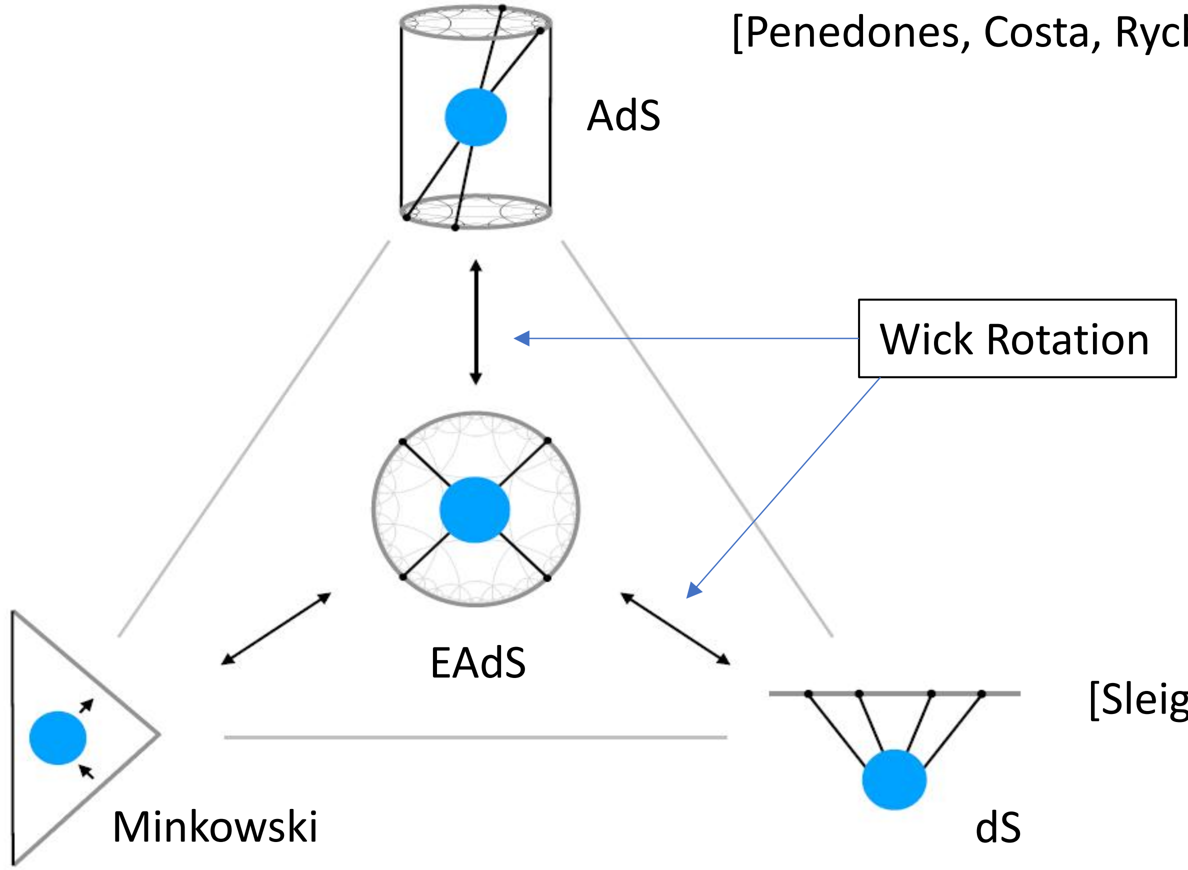


$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i}\,\frac{d\bar{\Delta}}{2\pi i} \sum_{\pm\hat{\pm}} \frac{c^{+-\pm\hat{\pm}}_{\Delta_1\Delta_2\Delta\bar{\Delta}}\, c^{+-\mp\hat{\mp}}_{\Delta_3\Delta_4\Delta\bar{\Delta}}}{c_\Delta c_{\bar{\Delta}}}$$



The Holographic Triangle

Observables at infinity in different maximally symmetric spaces can all be recast in terms of boundary observables in EAdS.



[Penedones, Costa, Rychkov, Paulos, Duffin, etc…]

[Iacobacci, Sleight, Taronna, arXiv:2208.01629 & 2401.16591]

[Sleight, Taronna, arXiv:2109.0272]

Conclusions

- Each contribution to massive Celestial correlators can be recast in terms of a linear combination of contributions of corresponding massive Witten correlators in EAdS;

- Euclidean AdS plays a central role in bridging holographic theories on maximally symmetric spaces;

- We can leverage the network of connections unveiled in this presentation to transfer well-defined concepts from flat space to (A)dS, such as the S-matrix;

- EAdS correlators emerge as fundamental building blocks in the construction of a holographic framework for asymptotically flat space-time.