MAP meeting 16 dec. 2024

Number density interpretation of dihadron fragmentation functions

D. Pitonyak,[1] C. Cocuzza,[2] A. Metz,[2] A. Prokudin,[3,4] and N. Sato[4]

[1] *Department of Physics, Lebanon Valley College, Annville, Pennsylvania 17003, USA*
[2] *Department of Physics, SERC, Temple University, Philadelphia, Pennsylvania 19122, USA*
[3] *Division of Science, Penn State University Berks, Reading, Pennsylvania 19610, USA*
[4] *Jefferson Lab, Newport News, Virginia 23606, USA*

We present a new quantum field-theoretic definition of fully unintegrated dihadron fragmentation functions (DiFFs) as well as a generalized version for n-hadron fragmentation functions. We demonstrate that this definition allows certain sum rules to be satisfied, making it consistent with a number density interpretation. Moreover, we show how our corresponding so-called extended DiFFs that enter existing phenomenological studies are number densities and also derive their evolution equations. Within this new framework, DiFFs extracted from experimental measurements will have a clear physical meaning.

P.R.L. **132** (2024) 011902, arXiv:2305.11995

QCD factorization with multihadron fragmentation functions

T. C. Rogers [*]
Department of Physics, Old Dominion University, Norfolk, VA 23529, USA and
Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

M. Radici [†]
INFN - Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy

A. Courtoy [‡]
Instituto de Física, Universidad Nacional Autónoma de México,
Apartado Postal 20-364, 01000 Ciudad de México, Mexico

T. Rainaldi [§]
Department of Physics, Old Dominion University, Norfolk, VA 23529, USA

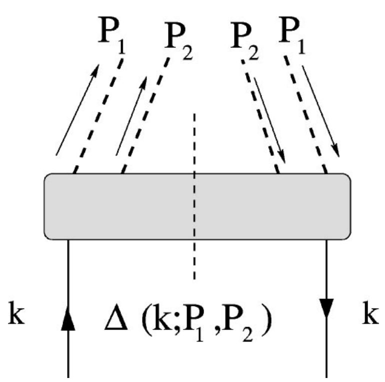
to be submitted to arXiv today

JAM statement:

almost 25 years ago in the pioneering paper of Bianconi, Boffi, Jakob, and Radici (BBJR) [20]. This work has been the basis for all subsequent dihadron-related research for observables sensitive to the relative transverse momentum of the two hadrons [21–25, 27, 31, 32, 35–39, 41–48]. Unfortunately, the BBJR definition does not allow the uDiFFs, nor the so-called extended DiFFs (extDiFFs) that are the focus of existing phenomenological analyses, to retain a number density interpretation in a parton model framework.

The main purpose of this Letter is to disseminate a new definition of uDiFFs that corrects this issue. We justify its number density interpretation by explicitly proving certain sum rules. We also show our corresponding extDiFFs are number densities and derive their evolution equations. Given the existing electron-positron

BBJR: P.R. D**62** (2000) 034008, hep-ph/9907475



quark-quark correlator

$$\Delta_{ij}(k;P_1,P_2)=\sum\!\!\!\!\!\!\!\int_X \int \frac{d^4x}{(2\pi)^4}\, e^{ik\cdot x}\langle 0|\psi_i(x)a_2^\dagger(P_2)a_1^\dagger(P_1)|X\rangle\langle X|a_1(P_1)a_2(P_2)\bar\psi_j(0)|0\rangle$$

$$\Delta^{[\Gamma]}=\frac{1}{4z_h}\int dk^+\int dk^-\,\delta\!\left(k^- - \frac{P_h^-}{z_h}\right)\mathrm{Tr}[\Delta\Gamma].$$

$P_h=P_1+P_2$

$R=(P_1-P_2)/2$

$z_h=P_h^-/k^-=z_1+z_2$

$\zeta=(z_1-z_2)/z_h$

unpolarized DiFF

$$\Delta^{[\gamma^-]}(z_h,\zeta,\vec k_T^2,\vec R_T^2,\vec k_T\cdot\vec R_T)\equiv D_1(z_h,\zeta,\vec k_T^2,\vec R_T^2,\vec k_T\cdot\vec R_T)$$

new definition

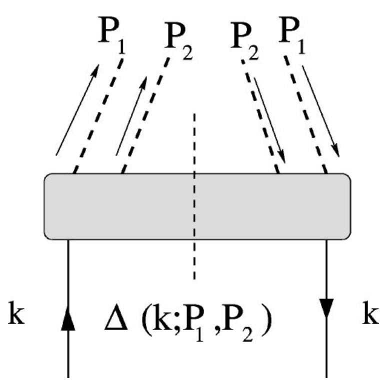
$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1h_2/q}(z_1,z_2,\vec P_{1\perp},\vec P_{2\perp})\gamma^-\right]=D_1^{h_1h_2/q}(z_1,z_2,\vec P_{1\perp}^2,\vec P_{2\perp}^2,\vec P_{1\perp}\cdot\vec P_{2\perp})$$

JAM statement:

almost 25 years ago in the pioneering paper of Bianconi, Boffi, Jakob, and Radici (BBJR) [20]. This work has been the basis for all subsequent dihadron-related research for observables sensitive to the relative transverse momentum of the two hadrons [21–25, 27, 31, 32, 35–39, 41–48]. Unfortunately, the BBJR definition does not allow the uDiFFs, nor the so-called extended DiFFs (extDiFFs) that are the focus of existing phenomenological analyses, to retain a number density interpretation in a parton model framework.

The main purpose of this Letter is to disseminate a new definition of uDiFFs that corrects this issue. We justify its number density interpretation by explicitly proving certain sum rules. We also show our corresponding extDiFFs are number densities and derive their evolution equations. Given the existing electron-positron

BBJR: P.R. D**62** (2000) 034008, hep-ph/9907475



quark-quark correlator $\Delta_{ij}(k;P_1,P_2)=\sum_X \int \frac{d^4}{(2\pi)^4} e^{ik\cdot} \langle 0|\psi_i(\ )a_2^\dagger(P_2)a_1^\dagger(P_1)|X\rangle\langle X|a_1(P_1)a_2(P_2)\bar{\psi}_j(0)|0\rangle$

$$\Delta^{[\Gamma]} = \frac{1}{4z_h}\int dk^+ \int dk^- \delta\left(k^- - \frac{P_h^-}{z_h}\right)\mathrm{Tr}[\Delta\Gamma].$$

$P_h = P_1 + P_2$

$R = (P_1 - P_2)/2$

$z_h = P_h^-/k^- = z_1 + z_2$

$\zeta = (z_1 - z_2)/z_h$

unpolarized DiFF $\Delta^{[\gamma^-]}(z_h,\zeta,\vec{k}_T^2,\vec{R}_T^2,\vec{k}_T\cdot\vec{R}_T) \equiv D_1(z_h,\zeta,\vec{k}_T^2,\vec{R}_T^2,\vec{k}_T\cdot\vec{R}_T)$

new definition $\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\right] = D_1^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$

sum rule $\int d\mathcal{PS}\, D_1^{h_1h_2/i}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = \langle\mathcal{N}(\mathcal{N}-1)\rangle$

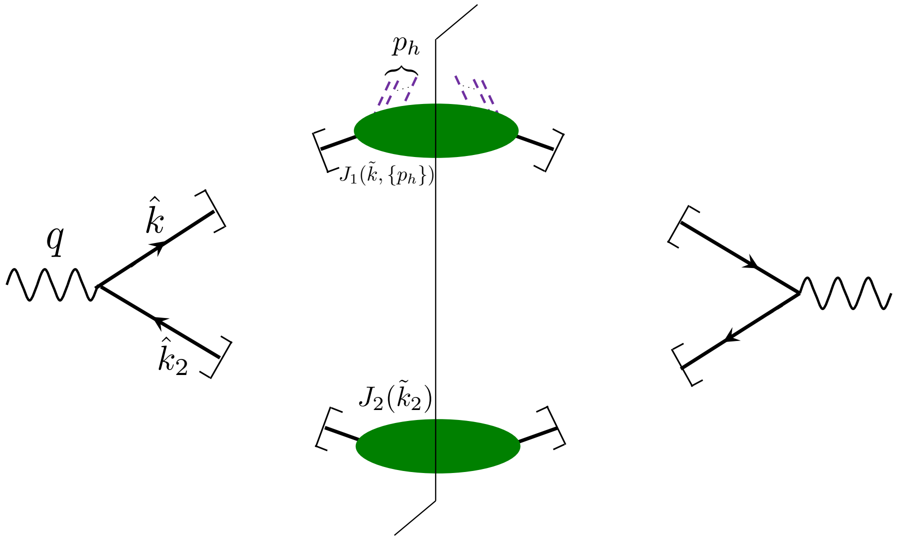
total # of hadron pairs

$\int d\mathcal{PS} = \sum_{h_1}\sum_{h_2}\int_0^1 dz_2 \int_0^{1-z_2} dz_1 \int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}$

recover number density interpretation

generalizable to n-hadron case: $\frac{1}{4(16\pi^3)^{n-1} z_1\cdots z_n}\mathrm{Tr}\Big[\ \ldots..$

BBJR definition of correlator compatible with factorization theorem

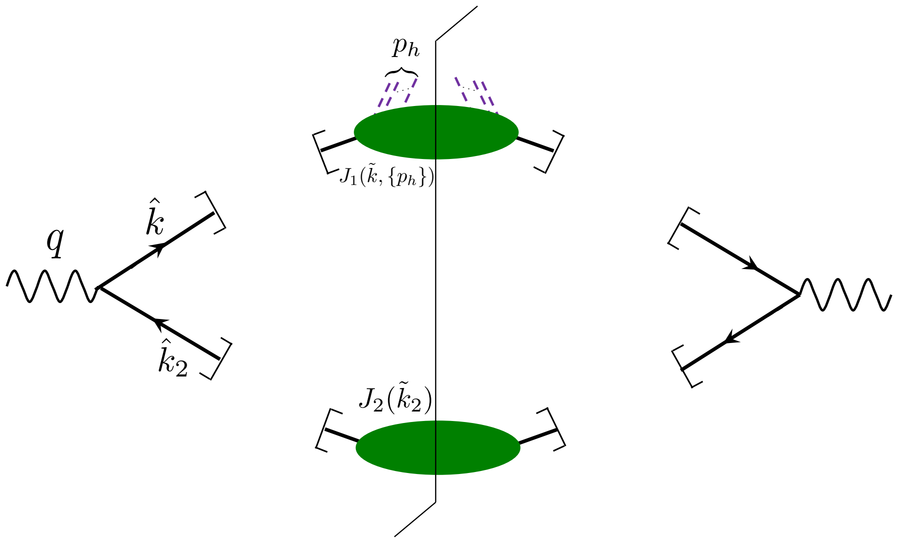


$$2E_{p_{h_1}}(2\pi)^3 2E_{p_{h_2}}(2\pi)^3 \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_{h_1}\,\mathrm{d}^3\boldsymbol{p}_{h_2}} = \int_z^1 \frac{\mathrm{d}\xi}{\xi^2}\left(2E_{\hat{k}}(2\pi)^3\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\hat{\boldsymbol{k}}}\right) d(\xi,\{p_h\}) + \mathrm{p.s.}$$

$$\xi = \xi_1 + \xi_2$$

$$\xi_i = \frac{p^+_{h_i}}{k^+} = \frac{p^+_{h_i}}{p^+_h}\,\frac{p^+_h}{k^+} = \frac{z_i}{z}\xi$$

BBJR definition of correlator compatible with factorization theorem



$$2E_{p_{h_1}}(2\pi)^3 2E_{p_{h_2}}(2\pi)^3 \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_{h_1}\ \mathrm{d}^3\boldsymbol{p}_{h_2}} = \int_z^1 \frac{\mathrm{d}\xi}{\xi^2}\left(2E_{\hat{k}}(2\pi)^3 \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\hat{\boldsymbol{k}}}\right) d(\xi, \{p_h\}) + \text{p.s.}$$

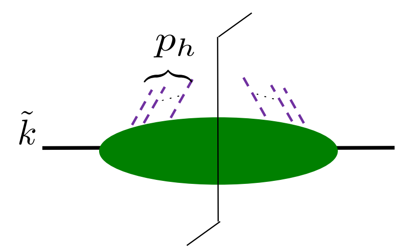
$$\xi = \xi_1 + \xi_2$$

$$\xi_i = \frac{p_{h_i}^+}{k^+} = \frac{p_{h_i}^+}{p_h^+}\frac{p_h^+}{k^+} = \frac{z_i}{z}\xi$$

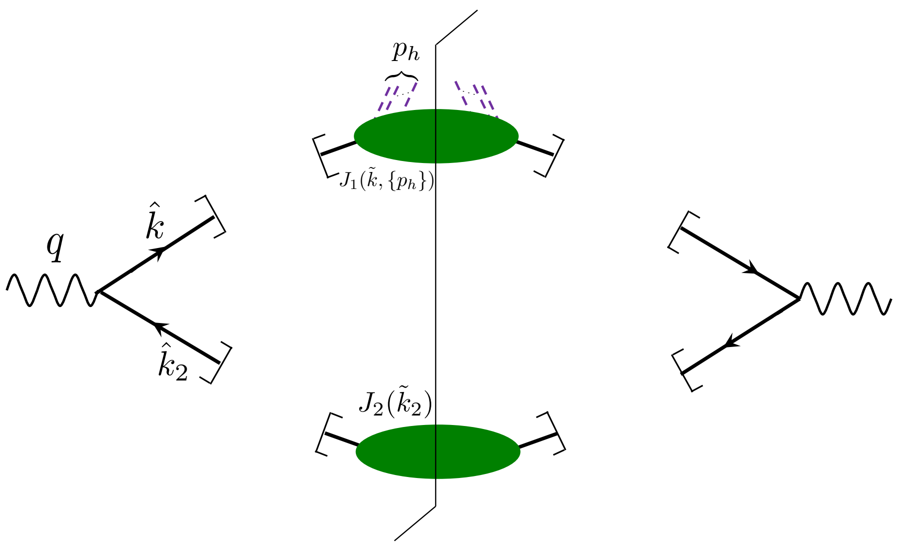
$$D_1(\xi, \zeta, \boldsymbol{k}_T, M_h^2) = \frac{1}{4\xi}\int \mathrm{d}k^+ \, \mathrm{Tr}\left[\Delta(k, P_h, R)\,\gamma^-\right]|_{k^-=P_h^-/\xi}$$

$$= \frac{1}{4\xi}\int \frac{\mathrm{d}k^+}{(2\pi)^4}\,\mathrm{Tr}\left[J_1(k, \{p_h\})\,\gamma^-\right]|_{k^-=P_h^-/\xi}$$

$$= \frac{1}{4\xi}\int \frac{\mathrm{d}k_H^-}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma^+ \quad \right]_{k_H^+=p_{h,H}^+/\xi}$$



BBJR definition of correlator compatible with factorization theorem



$$2E_{p_{h_1}}(2\pi)^3 2E_{p_{h_2}}(2\pi)^3 \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_{h_1}\,\mathrm{d}^3\boldsymbol{p}_{h_2}} = \int_z^1 \frac{\mathrm{d}\xi}{\xi^2}\left(2E_{\hat{k}}(2\pi)^3\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\hat{\boldsymbol{k}}}\right) d(\xi,\{p_h\}) + \text{p.s.}$$

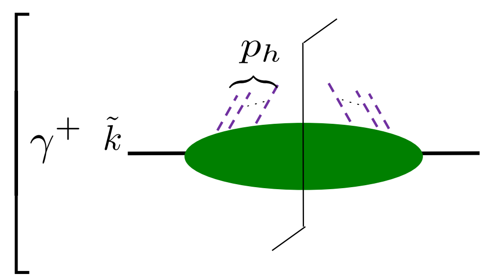
$$\xi = \xi_1 + \xi_2$$

$$\xi_i = \frac{p_{h_i}^+}{k^+} = \frac{p_{h_i}^+}{p_h^+}\frac{p_h^+}{k^+} = \frac{z_i}{z}\xi$$

$$D_1(\xi,\zeta,\boldsymbol{k}_T,M_h^2) = \frac{1}{4\xi}\int \mathrm{d}k^+\,\mathrm{Tr}\left[\Delta(k,P_h,R)\,\gamma^-\right]|_{k^-=P_h^-/\xi}$$

$$= \frac{1}{4\xi}\int \frac{\mathrm{d}k^+}{(2\pi)^4}\,\mathrm{Tr}\left[J_1(k,\{p_h\})\,\gamma^-\right]|_{k^-=P_h^-/\xi}$$

$$= \frac{1}{4\xi}\int \frac{\mathrm{d}k_H^-}{(2\pi)^4}\mathrm{Tr}\Big[\;\cdots\;\Big]_{k_H^+=p_{h,H}^+/\xi}$$



same factor as in single-hadron fragmentation: factorization works the same irrespective of the kind of final hadronic state (1,2,..n hadrons) provided that $M_h^2 \ll Q^2$ and z is fixed

$$-\zeta_{\max} \le \zeta = \sqrt{1 - \frac{(m_{h_1}+m_{h_2})^2}{M_h^2}} \le \zeta_{\max} \Rightarrow \zeta = \frac{z_1 - z_2}{z}$$

limited

critique of JAM:

1. z_1, z_2, are external kinematical variables, not to be confused with parton momentum fractions ξ_1, ξ_2:

$$z_i = \frac{2p_{h_i}\cdot q}{q^2} \xrightarrow[\substack{\text{only in parton model approx.}\\ \xi \to z \text{ and } \xi_i \to z_i}]{} \xi_i = \frac{p_{h_i}^+}{k^+} = \frac{p_{h_i}^+}{p_h^+}\,\frac{p_h^+}{k^+} = \frac{z_i}{z}\xi$$

parton momentum fractions ξ_i must be used in definition of correlators

critique of JAM:

1. z_1, z_2, are external kinematical variables, not to be confused with parton momentum fractions ξ_1, ξ_2:

$$z_i = \frac{2p_{h_i}\cdot q}{q^2} \xrightarrow[\substack{\text{only in parton model approx.}\\ \xi \to z \text{ and } \xi_i \to z_i}]{} \xi_i = \frac{p^+_{h_i}}{k^+} = \frac{p^+_{h_i}}{p^+_h}\frac{p^+_h}{k^+} = \frac{z_i}{z}\xi$$

parton momentum fractions ξ_i must be used in definition of correlators

2. if we use JAM definition of correlator with parton momentum fractions

$$\frac{1}{64\pi^3\xi_1\xi_2}\mathrm{Tr}[\ldots]$$

then the general formula for factorized e+e- cross section becomes

$$2E_{p_{h_1}}(2\pi)^3 2E_{p_{h_2}}(2\pi)^3 \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_{h_1}\,\mathrm{d}^3\boldsymbol{p}_{h_2}} = \int_z^1 \frac{\mathrm{d}\xi}{\xi^2}\left(2E_{\hat{k}}(2\pi)^3\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\hat{\boldsymbol{k}}}\right) d(\xi,\{p_h\}) + \text{p.s.}$$

$$2E_{p_{h_1}}(2\pi)^3 2E_{p_{h_2}}(2\pi)^3 \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_{h_1}\,\mathrm{d}^3\boldsymbol{p}_{h_2}} = \int_z^1 \frac{\mathrm{d}\xi}{\xi^2}\left(2E_{\hat{k}}(2\pi)^3\xi\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\hat{\boldsymbol{k}}}\right) d_{\mathrm{mod}}(\xi,\{p_h\}) + \text{p.s.}$$

critique of JAM:

1. z_1, z_2, are external kinematical variables, not to be confused with parton momentum fractions ξ_1, ξ_2:

$$z_i = \frac{2p_{h_i} \cdot q}{q^2} \xrightarrow[\substack{\text{only in parton model approx.} \\ \xi \to z \text{ and } \xi_i \to z_i}]{} \xi_i = \frac{p_{h_i}^+}{k^+} = \frac{p_{h_i}^+}{p_h^+}\frac{p_h^+}{k^+} = \frac{z_i}{z}\xi$$

parton momentum fractions ξ_i must be used in definition of correlators

2. if we use JAM definition of correlator with parton momentum fractions

$$\frac{1}{64\pi^3 \xi_1 \xi_2}\mathrm{Tr}[\ldots]$$

then the general formula for factorized e+e- cross section becomes

$$2E_{p_{h_1}}(2\pi)^3 2E_{p_{h_2}}(2\pi)^3 \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_{h_1}\,\mathrm{d}^3\boldsymbol{p}_{h_2}} = \int_z^1 \frac{\mathrm{d}\xi}{\xi^2}\left(2E_{\hat{k}}(2\pi)^3\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\hat{\boldsymbol{k}}}\right) d(\xi, \{p_h\}) + \mathrm{p.s.}$$

$$2E_{p_{h_1}}(2\pi)^3 2E_{p_{h_2}}(2\pi)^3 \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_{h_1}\,\mathrm{d}^3\boldsymbol{p}_{h_2}} = \int_z^1 \frac{\mathrm{d}\xi}{\xi^2}\left(2E_{\hat{k}}(2\pi)^3\xi\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\hat{\boldsymbol{k}}}\right) d_{\mathrm{mod}}(\xi, \{p_h\}) + \mathrm{p.s.}$$

- hard part not independent on details of nonperturbative final state
- changes evolution kernel

critique :
(continued)

3. generalization to n hadrons $\frac{1}{4(16\pi^3)^{n-1}\xi_1\xi_2\ldots\xi_n}\mathrm{Tr}[\ldots]$

$$2E_{p_{h_1}}(2\pi)^3 2E_{p_{h_2}}(2\pi)^3 \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_{h_1}\,\mathrm{d}^3\boldsymbol{p}_{h_2}} = \int_z^1 \frac{\mathrm{d}\xi}{\xi^2}\left(2E_{\hat{k}}(2\pi)^3 \overset{\xi^{n-1}}{\cancel{\xi}} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\hat{\boldsymbol{k}}}\right) d_{\mathrm{mod}}(\xi,\{p_h\}) + \mathrm{p.s.}$$

worsen breaking of factorization..

4. if we have really to interpret JAM formula as $\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}[\ldots]$

kinematic factors

then operator definition would depend on process because z_1, z_2 depend on q => breaking universality
(and still factorization formula would be broken by a 1/ξ term…)