Joint thrust and TMD resummation in SIDIS

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Content

Content

Goals

We present the framework for obtaining precise predictions for the transverse momentum of hadrons with respect to the thrust axis in ep collisions. To this end, we

- Use factorization theorems within Soft Collinear Effective Theory for $e^+e^- \to hX$ processes.
- Give a complete definition of the soft matrix elements.
- Make use of the δ-regulator to calculate and renormalize the soft functions.
- Check the consistency relation between functions.
- Extend the formalism to SIDIS.
- Plot the cross-section for SIDIS at NNLO.

Content

Thrust

For e^+e^- thrust is defined like

$$T = 1 - \tau = \max_{\hat{t}} \frac{\sum_j |\vec{p}_j \cdot \hat{t}|}{\sum_j |\vec{p}_j|}$$

pencil-like → spherical

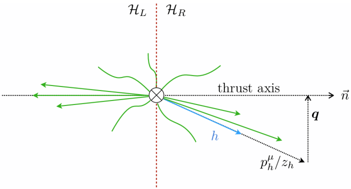
Figure 1: τ from 0 to $1/2$

Thrust axis

We choose coordinates such that $\hat{t} = (0,0,1)$ and in terms of light-cone coordinates

$$\tau \equiv 1 - T = \frac{1}{Q}\sum_j \min\{p_j^+, p_j^-\}$$

We define the transverse momentum with respect to the thrust axis $q_T^2 = \boldsymbol{p}_h^2/z_h^2$ where $\boldsymbol{p}_h$ is the hadron momentum and $z_h = 2E_h/Q$.



$$\boldsymbol{p}_h = -\sum_{i \in \text{hemi. with } i \neq h} \boldsymbol{p}_i$$

Kinematic Regions

The factorization in each kinematic region was written by [Y. Makris, F. Ringer, and W. J. Waalewijn, JHEP 02, 070] for

$$\sqrt{\tau} \gtrsim q_T/Q \gtrsim \tau$$

They identify three kinematic regions:

- Region 1: $\sqrt{\tau} \gg q_T/Q \sim \tau$
- Region 2: $\sqrt{\tau} \gg q_T/Q \gg \tau$
- Region 3: $\sqrt{\tau} \sim q_T/Q \gg \tau$

Region 1: $\sqrt{\tau} \gg q_T/Q \sim \tau$

We have

$$p_n^\mu \sim (\lambda_n^2, 1, \lambda_n)Q\,, \quad p_{\bar{n}}^\mu \sim (1, \lambda_{\bar{n}}^2, \lambda_{\bar{n}})Q\,, \quad p_s^\mu \sim (\lambda_s, \lambda_s, \lambda_s)Q$$

Therefore, if the hadron goes in the direction n

$$\tau \sim \lambda_n^2 + \lambda_{\bar{n}}^2 + \lambda_s\,, \quad q_T/Q \sim \lambda_n + \lambda_s$$

For the region $q_T/Q \sim \tau$

$$\lambda_{\bar{n}} \sim \sqrt{\tau}\,, \quad \lambda_n \sim \lambda_s \sim q_T/Q \sim \tau$$

Cross section for Region 1

For the kinematic region $\sqrt{\tau} \gg q_T/Q \sim \tau$

Cross section

$$\frac{d\sigma_1}{dz_h d\boldsymbol{q} d\tau} = \sum_j \hat{\sigma}_j^0(Q) \int_{-\infty}^{\infty} \frac{d\boldsymbol{b}}{(2\pi)^2} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{du}{2\pi i} e^{i\boldsymbol{b}\cdot\boldsymbol{q}+u\tau} H(Q,\mu)$$

$$\times J\left(\frac{u}{Q^2},\mu\right) S_{\text{hemi}}\left(\boldsymbol{b},\frac{u}{Q},\mu,\zeta\right) D_{1,j\to h}(z_h,\boldsymbol{b},\mu,\zeta)\left[1+\mathcal{O}\left(\tau,\frac{q_T^2}{\tau Q^2}\right)\right]$$

- Hard function $H(Q,\mu)$
- Jet function $J(\frac{u}{Q^2},\mu)$
- TMD Fragmentation function $D_{1,j\to h}(z_h,\boldsymbol{b},\mu,\zeta)$
- Soft function $S_{\text{hemi}}(\boldsymbol{b},\frac{u}{Q},\mu,\zeta)$

Region 2: $\sqrt{\tau} \gg q_T/Q \gg \tau$

We had

$$\tau \sim \lambda_n^2 + \lambda_{\bar{n}}^2 + \lambda_s\,, \quad q_T/Q \sim \lambda_n + \lambda_s$$

The assumption $\sqrt{\tau} \gg q_T/Q \gg \tau$ implies

$$\lambda_{\bar{n}} \sim \sqrt{\tau}\,, \quad \lambda_n \sim q_T/Q\,, \quad \lambda_s \sim \tau$$

This does not lead to a consistent factorization theorem. Because τ and q_T/Q are no longer correlated there is an additional collinear-soft mode, whose power counting is fixed by the fact that it contributes to both τ and q_T [M. Procura, W.J. Waalewijn and L. Zeune, JHEP 02 (2015) 117]

$$p_{cs}^{\mu} \sim (\tau Q, q_T^2/(\tau Q), q_T)$$

Cross section for Region 2

For the kinematic region $\sqrt{\tau} \gg q_T/Q \gg \tau$

Cross section

$$\frac{d\sigma_2}{dz_h d\boldsymbol{q} d\tau} = \sum_j \hat{\sigma}_j^0(Q) \int_{-\infty}^{\infty} \frac{d\boldsymbol{b}}{(2\pi)^2} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{du}{2\pi i} e^{i\boldsymbol{b}\cdot\boldsymbol{q}+u\tau} H(Q,\mu) J(\frac{u}{Q^2},\mu)$$

$$\times S_{\rm thr}(\frac{u}{Q},\mu) C(\boldsymbol{b},\frac{u}{Q},\mu,\zeta) D_{1,j\to h}(z_h,\boldsymbol{b},\mu,\zeta)\left[1+\mathcal{O}(\frac{q_T^2}{\tau Q^2},\frac{\tau^2 Q^2}{q_T^2})\right]$$

The new ingredients for this region are

- Thrust soft function $S_{\rm thr}(\frac{u}{Q},\mu)$
- Collinear Soft function $C(\boldsymbol{b},\frac{u}{Q},\mu,\zeta)$

Region 3: $\sqrt{\tau} \sim q_T/Q \gg \tau$

The assumption $\sqrt{\tau} \sim q_T/Q$ implies

$$\lambda_{\bar{n}} \sim \lambda_n \sim \sqrt{\tau} \sim q_T/Q\,, \quad \lambda_s \sim \tau$$

Cross section

$$\frac{\mathrm{d}\sigma_3}{\mathrm{d}z_h \mathrm{d}\boldsymbol{q} \mathrm{d}\tau} = \sum_j \hat{\sigma}_j^0(Q) \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\mathrm{d}u}{2\pi i} e^{u\tau} H(Q,\mu) J(\frac{u}{Q^2},\mu)$$
$$\times S_{\rm thr}(\frac{u}{Q},\mu) \mathscr{G}_{j\to h}(\frac{u}{Q^2}, z_h, \boldsymbol{q}, \mu) \Big[1 + \mathscr{O}(\tau, \frac{q_T^2}{\tau Q^2}, \frac{\tau^2 Q^2}{q_T^2})\Big]$$

The new ingredients for this region is the generalized fragmenting jet function (FJF) $\mathscr{G}_{j\to h}(\frac{u}{Q^2}, z_h, \boldsymbol{q}, \mu)$. From this we conclude that region 3 is not interesting for constraining nonperturbative TMD physics.

Content

Soft functions

The $S_{\rm thr}$, S, and C are defined by the following matrix elements [M. Procura, W.J. Waalewijn and L. Zeune, JHEP 02 (2015) 117]

$$S_{\rm thr} = \frac{1}{N_c}\left\langle 0 \middle| \mathrm{Tr}\left[\overline{\mathbf{T}}\left(S_n^\dagger(0)S_{\bar{n}}(0)\right)\delta\left(k^+ - \boldsymbol{P}_1^+ - \boldsymbol{P}_2^-\right)\mathbf{T}\left(S_{\bar{n}}^\dagger(0)S_n(0)\right)\right]\middle| 0\right\rangle$$

$$S = \frac{1}{N_c}\left\langle 0 \middle| \mathrm{Tr}\left[\overline{\mathbf{T}}\left(S_n^\dagger(0)S_{\bar{n}}(0)\right)\delta\left(k^+ - \boldsymbol{P}_1^+ - \boldsymbol{P}_2^-\right)\delta^2(\vec{k}_\perp - \vec{\boldsymbol{P}}_\perp)\mathbf{T}\left(S_{\bar{n}}^\dagger(0)S_n(0)\right)\right]\middle| 0\right\rangle$$

$$C = \frac{1}{N_c}\left\langle 0 \middle| \mathrm{Tr}\left[\overline{\mathbf{T}}\left(X_n^\dagger(0)V_n(0)\right)\delta\left(k^+ - \boldsymbol{P}^+\right)\delta^2(\vec{k}_\perp - \vec{\boldsymbol{P}}_\perp)\mathbf{T}\left(V_n(0)^\dagger X_n(0)\right)\right]\middle| 0\right\rangle$$

where $(\bar{\mathrm{T}})$ T is the (anti)time ordering operator and the operator $\boldsymbol{P}_1$ ($\boldsymbol{P}_2$) gives the momentum of the soft radiation going into the hemisphere defined by $p_i^+ < p_i^-$ ($p_i^+ > p_i^-$).

Hemisphere-Thrust-TMD Soft function

$S_{\rm hemi}$ was not defined before, we find

$$S_{\rm hemi} = \frac{1}{N_c} \left\langle 0 \middle| \mathrm{Tr} \left[\overline{\mathbf{T}} \left(S_n^\dagger(0) S_{\bar{n}}(0) \right) \delta\left(k^+ - \boldsymbol{P}_1^+ - \boldsymbol{P}_2^-\right) \delta^2(\vec{k}_\perp - \vec{\boldsymbol{P}}_{2\perp}) \mathbf{T} \left(S_{\bar{n}}^\dagger(0) S_n(0) \right) \right] \middle| 0 \right\rangle$$

The soft radiation goes into the hemisphere of the identified hadron only.

δ-Regulator

The factorization of the cross-section introduces rapidity divergences, which we regulate with the δ-regulator [M. G. Echevarria, I. Scimemi, and A. Vladimirov, Phys. Rev. D 93, 054004 (2016)]. It consists on the following modification of the Wilson lines

$$S_n(0) = \boldsymbol{P}\exp\left[ig\int_{-\infty}^{0} ds\; n\cdot A_s(sn)\right] \to \boldsymbol{P}\exp\left[ig\int_{-\infty}^{0} ds\; n\cdot A_s(sn)e^{\delta^+ s}\right]$$

$$S_{\bar{n}}(0) = \boldsymbol{P}\exp\left[-ig\int_{0}^{\infty} ds\; \bar{n}\cdot A_s(s\bar{n})\right] \to \boldsymbol{P}\exp\left[-ig\int_{0}^{\infty} ds\; \bar{n}\cdot A_s(s\bar{n})e^{-\delta^- s}\right]$$

At the level of Feynman diagrams in momentum space this translates to

$$\frac{1}{(k^+ - i0)} \to \frac{1}{(k^+ - i\delta^+)}$$

The gauge properties are recovered in the limit $\delta \to 0^+$.

One loop calculation of $S_{\rm hemi}$

At one loop the calculation is similar to that of S [M. Procura, W.J. Waalewijn and L. Zeune, JHEP 02 (2015) 117]. Considering only the transverse momentum of the radiation that goes into the hemisphere defined by $p_i^+ > p_i^-$ translates to

$$\delta^2(\vec{\ell}_\perp - \vec{k}_\perp) \longrightarrow \delta^2(\vec{\ell}_\perp - \vec{k}_\perp)\theta(\ell^+ - \ell^-) + \theta(\ell^- - \ell^+)$$

The calculation reduces to integrals of the type

$$\frac{2g^2 C_F Q}{(2\pi)^{3-2\varepsilon}} \left(\frac{e^{\gamma_E}\mu^2}{4\pi}\right)^{\varepsilon} \int \mathrm{d}^d\ell \int_0^\infty d\tau \frac{e^{-u\tau}\theta\left(\ell_0\right)\delta\left(\ell^2\right)}{(\ell^+ + \mathrm{i}Q\delta^+)(\ell^- - \mathrm{i}Q\delta^-)}$$
$$\times\left[e^{-\mathrm{i}\ell\cdot\boldsymbol{b}}\delta\left(\ell^- - Q\tau\right)\theta(\ell^+ - \ell^-) + \delta\left(\ell^+ - Q\tau\right)\theta\left(\ell^- - \ell^+\right)\right]$$

Hemisphere soft function

The final result for the bare hemisphere soft function is

$$S^{(1)}_{\rm hemi}(u, \boldsymbol{b}, \mu, \delta^2) = -\frac{4a_S C_F}{\varepsilon^2} - \frac{2a_S C_F}{\varepsilon} \ln\left(\frac{e^{2\gamma_E}\mu^2 u^2}{Q^2}\right) - \frac{2a_S C_F}{\varepsilon} \ln\left(\frac{\mu^2}{Q^2\delta^2}\right)$$
$$+4a_S C_F\Big[-\frac{1}{2}\ln\left(B^2 e^{2\gamma_E}\mu^2\right)\ln\left(\frac{\mu^2}{Q^2\delta^2}\right) + \frac{1}{4}\ln^2(B^2 e^{2\gamma_E}\mu^2)$$
$$+\frac{B^2 Q^2 F}{u^2} - \frac{1}{4}\ln^2\left(\frac{B^2 Q^2}{u^2}\right) - \frac{1}{4}\ln^2\left(\frac{\mu^2 u^2 e^{2\gamma_E}}{Q^2}\right) - \frac{\pi^2}{4}\Big]$$

where $F = {}_4F_3[\frac{3}{2}, 1, 1, 1; 2, 2, 2; -\frac{4B^2Q^2}{u^2}]$. Our definition gives the expected result at one loop [Y. Makris, F. Ringer, and W. J. Waalewijn, JHEP 02, 070].

One-loop relation between soft functions

$$S^{(1)}_{\rm hemi} = \frac{1}{2}S^{(1)} + \frac{1}{2}S^{(1)}_{\rm thr}$$

Renormalized soft functions

Since in the cross-section the TMD appears unsubtracted, we still have to perform the subtraction of the TMD soft function [M. G. Echevarria, I. Scimemi, and A. Vladimirov, Phys. Rev. D 93, 054004 (2016)]. Then, the one loop results agree with the literature

$$\mathscr{T}_{\text{hemi}}(u,\boldsymbol{b},\mu,\zeta) = \frac{S_{\text{hemi}}(u,\boldsymbol{b},\mu,\delta^2)}{\sqrt{S_{DY}(\boldsymbol{b},\mu,\delta^2\zeta/Q^2)}} = 1 + 4a_S C_F\Big[\frac{B^2Q^2F}{u^2} - \frac{1}{2}\ln\left(B^2e^{2\gamma_E}\mu^2\right)\ln\left(\frac{\zeta}{Q^2}\right) - \frac{1}{4}\ln^2\left(\frac{B^2Q^2}{u^2}\right) - \frac{1}{4}\ln^2\left(\frac{Q^2}{\mu^2u^2e^{2\gamma_E}}\right) - \frac{7\pi^2}{24}\Big]$$

$$\hat{C}(u,\boldsymbol{b},\mu,\zeta) = \frac{C(u,\boldsymbol{b},\mu,\delta^2)}{\sqrt{S_{DY}(\boldsymbol{b},\mu,\delta^2\zeta/Q^2)}}$$

$$= 1 + 2a_S C_F\Big[\ln\left(\frac{\mu^2u^2e^{2\gamma_E}}{\zeta}\right)\ln\left(B^2e^{2\gamma_E}\mu^2\right) - \frac{1}{2}\ln^2(\mu^2B^2e^{2\gamma_E}) - \frac{\pi^2}{12}\Big]$$

Content

SIDIS

We can extend this formalism to SIDIS. In the Breit frame, the virtual photon momentum is given by

$$q^\mu = \frac{Q}{2}(\bar{n}^\mu - n^\mu) = Q(0,0,0,-1)$$

where $n^\mu = (1,0,0,1)$ and $\bar{n}^\mu = (1,0,0,-1)$. Up to mass corrections, the proton momentum can be written as

$$P^\mu \simeq Q/(2x)n^\mu = Q/(2x)(1,0,0,1)$$

At Born level, the struck quark back-scatters against the photon with momentum

$$p_q^\mu = \xi P^\mu + q^\mu \simeq (Q/2)\bar{n}^\mu$$

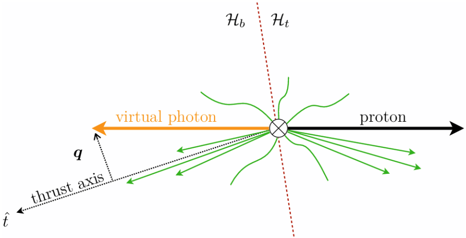
it fragments and produces a jet-like structure which points close to the opposite of the beam direction.

Thrust

The thrust axis $\hat{t}$ and event shape we use are defined by

$$\tau = \min_{\hat{t}} \frac{1}{Q} \sum_i \min\{n \cdot p_i, n_t \cdot p_i\}$$

The transverse momentum $\boldsymbol{q}$ of the photon with respect to the thrust axis, is directly related to the total transverse momentum $\boldsymbol{p_t}$ of radiation in hemisphere $\mathscr{H}_t$ through $\boldsymbol{q} = -2\boldsymbol{p_t} = 2(\boldsymbol{p}_{c,b} + \boldsymbol{p}_{s,b})$



SIDIS

This measurement is related to the transverse momentum of a hadron in e^+e^- collisions by crossing the outgoing hadron to an incoming proton, thereby

$$D_{1,j\to h}(z_h, \boldsymbol{b}, \mu, \zeta) \to F_{1,j}(x, \boldsymbol{b}, \mu, \zeta)$$

And exchanging an outgoing Wilson line for an incoming one in the global soft functions.
Furthermore, the kinematic regions and modes are the same as those discussed for the e^+e^- case.

Cross sections for SIDIS

Region 1: $\sqrt{\tau} \gg q_T/Q \sim \tau$

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\boldsymbol{p}_t\mathrm{d}\tau} = \sum_j \sigma_{0,j}(x,Q) \int_{-\infty}^{\infty} \frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\mathrm{d}u}{2\pi i} e^{i\boldsymbol{b}\cdot\boldsymbol{p}_t + u\tau} H(Q,\mu)$$

$$\times J\left(\frac{u}{Q^2},\mu\right) S_{\mathrm{hemi}}\left(\boldsymbol{b},\frac{u}{Q},\mu,\zeta\right) F_{1,j}(x,\boldsymbol{b},\mu,\zeta) \left[1 + \mathcal{O}\left(\tau, \frac{q_T^2}{\tau Q^2}\right)\right]$$

Region 3: $\sqrt{\tau} \sim q_T/Q \gg \tau$

$$\frac{\mathrm{d}\sigma_3}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\boldsymbol{p}_t\mathrm{d}\tau} = \sum_j \sigma_{0,j}(x,Q) \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\mathrm{d}u}{2\pi i} e^{u\tau} H(Q,\mu)$$

$$\times J\left(\frac{u}{Q^2},\mu\right) S_{\mathrm{thr}}\left(\frac{u}{Q},\mu\right) \mathscr{B}_j\left(\frac{u}{Q^2},x,\boldsymbol{p}_t,\mu\right) \left[1 + \mathcal{O}\left(\tau, \frac{q_T^2}{\tau Q^2}, \frac{\tau^2 Q^2}{q_T^2}\right)\right]$$

Cross section for Region 2

In what follows, we will focus on each of the functions appearing in Region 2

Region 2: $\sqrt{\tau} \gg q_T/Q \gg \tau$

$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\boldsymbol{p}_t\mathrm{d}\tau} = \sum_j \sigma_{0,j}(x,Q) \int_{-\infty}^{\infty} \frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\mathrm{d}u}{2\pi i} e^{i\boldsymbol{b}\cdot\boldsymbol{p}_t+u\tau} H(Q,\mu)$$
$$\times J(\frac{u}{Q^2},\mu) S_{\mathrm{thr}}(\frac{u}{Q},\mu) C(\boldsymbol{b},\frac{u}{Q},\mu,\zeta) F_{1,j}(x,\boldsymbol{b},\mu,\zeta)\Big[1+\mathscr{O}(\frac{q_T^2}{\tau Q^2},\frac{\tau^2 Q^2}{q_T^2})\Big]$$

Content

Evolution for Region 2

The large logarithms in q_T/Q and τ are resummed by evaluating the ingredients at their natural scale and evolving them to a common scale [see Patricia Gutiérrez's talk] using the RGEs.

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} S_i(\mu) = \gamma_{S_i}(\mu) S_i(\mu),$$

The natural renormalization scales of the ingredients in the factorization are given by

$$\mu_H \sim Q, \qquad \mu_J \sim \sqrt{\tau} Q, \qquad \mu_F \sim \mu_C \sim 1/b_T, \qquad \mu_{S_{\mathrm{thr}}} \sim \tau Q$$
$$\zeta_C \sim 1/(b_T^2 \tau Q)$$

The final scales of the evolution are binded to the hard scale of factorization

$$\mu_f^2 = Q^2 \qquad \zeta_f = Q^2$$

Evolution

The RGEs for the hard, jet and thrust soft functions are given by:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} H(Q^2,\mu) = \left[2\Gamma_{\mathrm{cusp}}(a_s)\ln\left(\frac{Q^2}{\mu^2}\right) + 2\gamma^H(a_s)\right] H(Q^2,\mu)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} J(\ln\frac{sQ^2}{\mu^2},\mu) = \left[-2\Gamma_{\mathrm{cusp}}(a_s)\ln\left(\frac{sQ^2}{\mu^2}\right) - 2\gamma^J(a_s)\right] J(\ln\frac{sQ^2}{\mu^2},\mu)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} S_{\mathrm{thr}}(\ln\frac{sQ}{\mu},\mu) = \left[4\Gamma_{\mathrm{cusp}}(a_s)\ln\left(\frac{sQ}{\mu}\right) - 2\gamma^S(a_s)\right] S_{\mathrm{thr}}(\ln\frac{sQ}{\mu},\mu)$$

Evolution for Region 2

For the hard, jet and thrust soft, the solutions of the RGEs are:

$$H(Q^2,\mu) = H(Q^2,\mu_H)\exp\left[4S(\mu_H,\mu) - 2A_H(\mu_H,\mu)\right]\left(\frac{Q^2}{\mu_H^2}\right)^{-2A_\Gamma(\mu_H,\mu)}$$

$$J(\ln\frac{sQ^2}{\mu^2},\mu) = J(\ln\frac{sQ^2}{\mu_J^2},\mu_J)\exp\left[-4S(\mu_J,\mu) + 2A_J(\mu_J,\mu)\right]\left(\frac{sQ^2}{\mu_J^2}\right)^{2A_\Gamma(\mu_J,\mu)}$$

$$S_{\rm thr}(\ln\frac{sQ}{\mu},\mu) = S_{\rm thr}(\ln\frac{sQ}{\mu_S},\mu_S)\exp\left[4S(\mu_S,\mu) + 2A_S(\mu_S,\mu)\right]\left(\frac{sQ}{\mu_S}\right)^{-4A_\Gamma(\mu_S,\mu)}$$

Evolution for Region 2

The necessary ingredients are

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} \mathrm{d}\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{\mathrm{d}\alpha'}{\beta(\alpha')}$$

$$A_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} \mathrm{d}\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)}$$

$$A_H(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} \mathrm{d}\alpha \frac{\gamma^H(\alpha)}{\beta(\alpha)}$$

$A_J(\nu,\mu)$ and $A_S(\nu,\mu)$ are obtained from A_H by substituting γ^J, γ^S for γ^H respectively. $S(\nu,\mu)$ and $A_\Gamma(\nu,\mu)$ can be found at [T. Becher, M. Neubert and B. Pecjak, JHEP 0701:076,2007]. The anomalous dimensions at NNLO can be found at [T. Becher and M.D. Schwartz, JHEP 0807:034,2008].

Double scale evolution

For the TMD as well as for C, there are two different scales and therefore, two RGEs

$$\mu^2 \frac{d}{d\mu^2} F = \frac{\gamma_F}{2} F$$

$$\zeta \frac{d}{d\zeta} F = -\mathscr{D}(\boldsymbol{b}, \mu) F$$

Since the anomalous dimensions of all the other functions in the cross-section are known to NNLO order, we can obtain the anomalous dimension of the collinear soft function from the consistency relation:

$$2\gamma^H - 2\gamma^J - 2\gamma^S - \gamma^V - \gamma^C = 0$$

Double scale evolution

This allows us to obtain the evolution of the collinear soft function, given by the solution of the above eqs

$$C(u,b,\mu_f,\zeta_f) = R_C[u,b,(\mu_C,\zeta_C) \to (\mu_f,\zeta_f)]C(u,b)$$

where the general form of the evolution factor is

$$R_C[u,b,(\mu_C,\zeta_C) \to (\mu_f,\zeta_f)] = \exp\left[\int_P (\gamma_C(\mu,\zeta)\frac{\mathrm{d}\mu}{\mu} - \mathscr{D}(u,b)\frac{\mathrm{d}\zeta}{\zeta})\right]$$

where (μ_C,ζ_C) and (μ_f,ζ_f) refer respectively to an initial and final set of scales.

Double scale evolution

The initial scale is chosen with ζ-prescription. In particular we work with the special null-evolution line, a line through which the distribution does not evolve and that passes through the saddle point. That is, a solution of

$$\gamma_C(u,\mu,\zeta_\mu(b)) = 2\mathscr{D}(b,\mu)\frac{\mathrm{d}\ln\zeta_\mu(b)}{\mathrm{d}\ln\mu^2}$$

$$\gamma_C(u,\mu_C,\zeta_C) = 0$$
$$\mathscr{D}(\boldsymbol{b},\mu_C) = 0$$

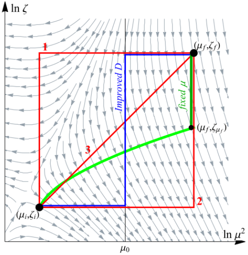


Figure 2: Null Evolution curve

Double scale evolution

Under these conditions we have

$$C(u,b,\mu_f,\zeta_f) = R_C[u,b,(\mu_C,\zeta_C) \to (\mu_f,\zeta_f)]\,C(u,b)$$

where

$$R_C[u,b,(\mu_C,\zeta_C) \to (\mu_f,\zeta_f)] = \left(\frac{\zeta_f}{\zeta_{\mu_f}(\mu_C,\zeta_C)}\right)^{-\mathscr{D}(b,\mu_f)}$$

$$C(u,b,\mu_C,\zeta_C) = C(u,b), \qquad \mu_C \sim 1/b_T, \qquad \zeta_C \sim 1/(b_T^2 \tau Q)$$

The explicit form of $\zeta_{\mu_f}(\mu,\zeta)$ and $\mathscr{D}(b,\mu)$ can be found at [I. Scimemi and A. Vladimirov, 1803.11089]. Therefore, we have all the ingredients present in the cross-section up to order NNLO (except the finite part of the collinear soft function).

Non perturbative model

The last step is to consider a model to take into account the non-perturbative contribution from the collinear soft function. There is a large freedom in the definition of the non perturbative models. The main criterion for their construction is to have the maximum flexibility with the smallest number of free parameters. We opted for the following form

$$\hat{C}_{NP}(u,b) = \frac{1}{1+u(\lambda_1 + \lambda_2 b + \lambda_3 b^2)}$$

Plots

PRELIMINARY!!: Finally, for SIDIS at NNLO for an energy of 100 GeV we have

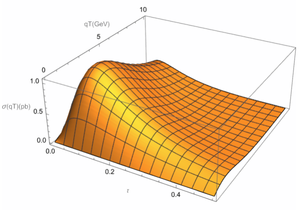


Figure 3: $\lambda_1 = \lambda_2 = \lambda_3 = 0.1$

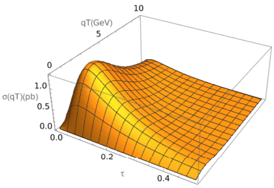


Figure 4: $\lambda_1 = \lambda_3 = 0.1, \lambda_2 = 0$

Results

Summary:

- The factorization given in previous works lacks one soft function operator definition.
- New definition of the soft function S_{hemi}.
- Explicit calculation of S_{hemi}, S, S_{thr} and C at one loop order with the δ-regulator.
- Checked the relation between S_{hemi} and the other known soft functions.
- Resummation of the collinear soft function using ζ-prescription.
- Preliminary predictions for the cross-section for SIDIS at NNLO using the TMD PDF and evolution kernel from previous fits.
- TO DO: comparison with Pythia and e^+e^- case.