SIDIS Kinematic regions

and their role in giving the correct theory interpretation to experimental measurements

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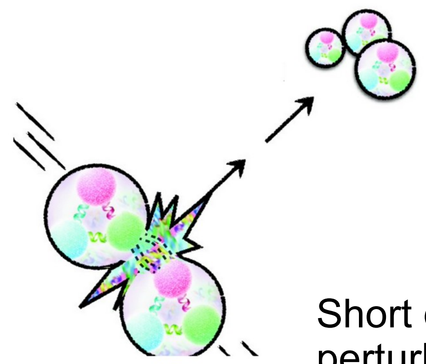
QCD and Factorization

QCD

- Non perturbative regime (non computable but universal terms)
- Perturbative regime (computable but process dependent terms)



- The interplay between **perturbative** and **non-perturbative** regimes is currently one of the most challenging aspects in phenomenology, which is presently explored in many different dedicated experiments (existing or being planned for the near future).
- **Factorization** allows to separate the perturbative content of an observable from its non-perturbative content. At large Q and small m, the non-perturbative contributions are separated out from anything that can be computed by using perturbative techniques, and identified with universal quantities (structure functions).
- **Factorization** restores the predictive power of QCD

Factorization: region classification

J. Collins, Foundations of perturbative QCD (Cambridge University Press, 2011)

Particles are classified according to how they propagate in space, i.e. according to their virtuality.

Momentum regions:

HARD
$k \sim (Q,\, Q,\, Q)$

Short-distance contributions $\longrightarrow$ Large virtuality

Long-distance contributions $\longrightarrow$ Low virtuality

SOFT
$k \sim (\lambda_S,\, \lambda_S,\, \lambda_S)$

$\lambda_S = \lambda^2/Q$

COLLINEAR
$k \sim (Q,\, \lambda^2/Q,\, \lambda)$

Factorization: region classification

General structure of a generic factorization theorem:

$$\mathcal{O} = H \times \boxed{S \times \prod_j C_j} + p.s.$$

Power suppressed terms

IR-safe hard contribution

Soft and collinear contributions, accounting for non-perturbative effects → PDFs and TMDs

- Each term is equipped with proper subtractions.
- The soft factor S encodes the *correlation* among the various collinear parts.
- While **H can be computed in pQCD, S and C have to be determined using non perturbative methods**. For instance they can be modeled and extracted from experimental data, or computed in lattice QCD

FACTORIZATION: Collinear vs TMD

- Factorization: **identify, isolate, separate perturbative from non-perturbative contributions**

- This is intimately related to a clear identification of **different kinematic regions**, where different theoretical schemes must be applied

COLLINEAR

$q_T \gtrsim Q$

There is *enough* transverse momentum to produce jets at wide angles in the final state.

The low transverse momenta of the struck parton, of the fragmenting parton and of soft radiation are totally negligible

TMD

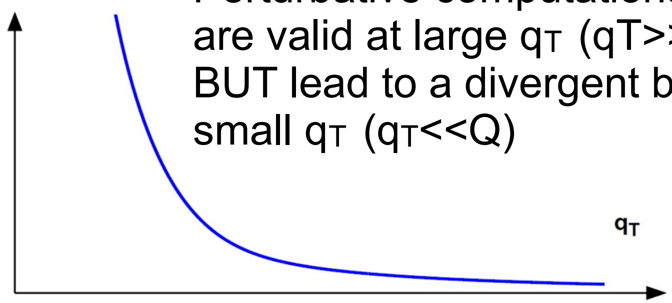
$q_T \ll Q$

The production of hard jets with high transverse momentum is strongly suppressed

The low transverse momenta of the struck parton, of the fragmenting parton and of soft radiation are *relevant*

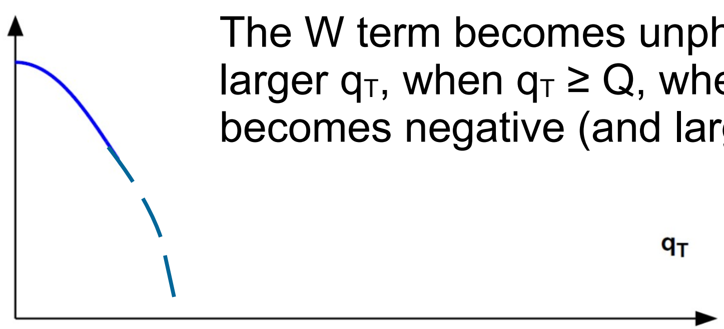
Factorization: collinear vs TMD

Perturbative computations to fixed order are valid at large q_T ($qT>>Q$),
BUT lead to a divergent behavior at small q_T ($q_T<<Q$)



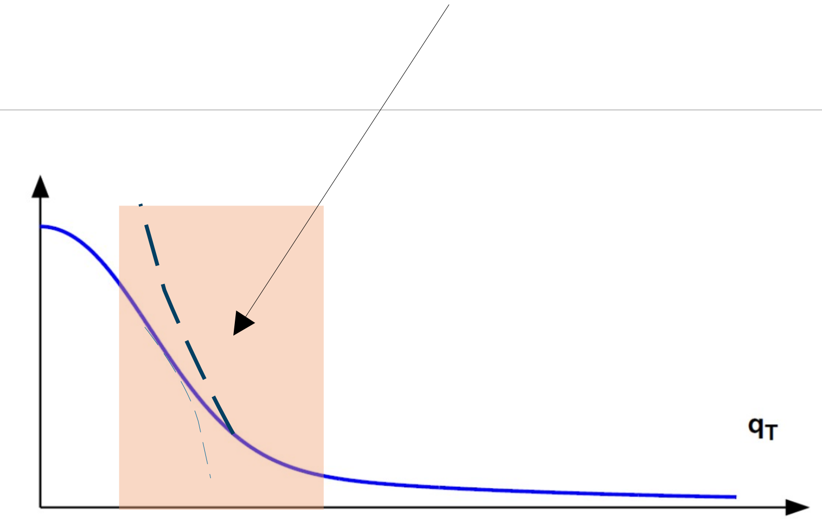
q_T >> Q
collinear region
pQCD

The W term becomes unphysical at larger q_T, when $q_T \geq Q$, where it becomes negative (and large).



q_T << Q
TMD region
non-pQCD

Transition from perturbative to non-perturbative regimes



Central region is where TMD and collinear cross sections are expected to be matched

The Y term corrects for the misbehavior of W as q_T gets larger, providing a consistent (and positive) q_T differential cross section.

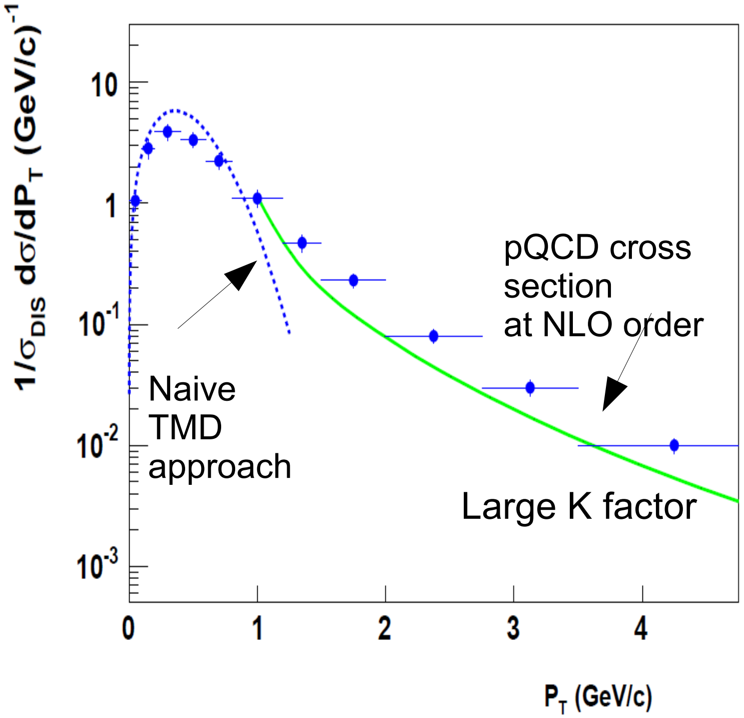
Factorization in SIDIS

Factorization regions

- Fixed order pQCD calculations describe the SIDIS cross section at large q_T
  → COLLINEAR factorization
- The cross section at small q_T is dominated by non perturbative contributions
  → TMD factorization (complemented with non perturbative modeling)
- The intermediate region is a “matching region”

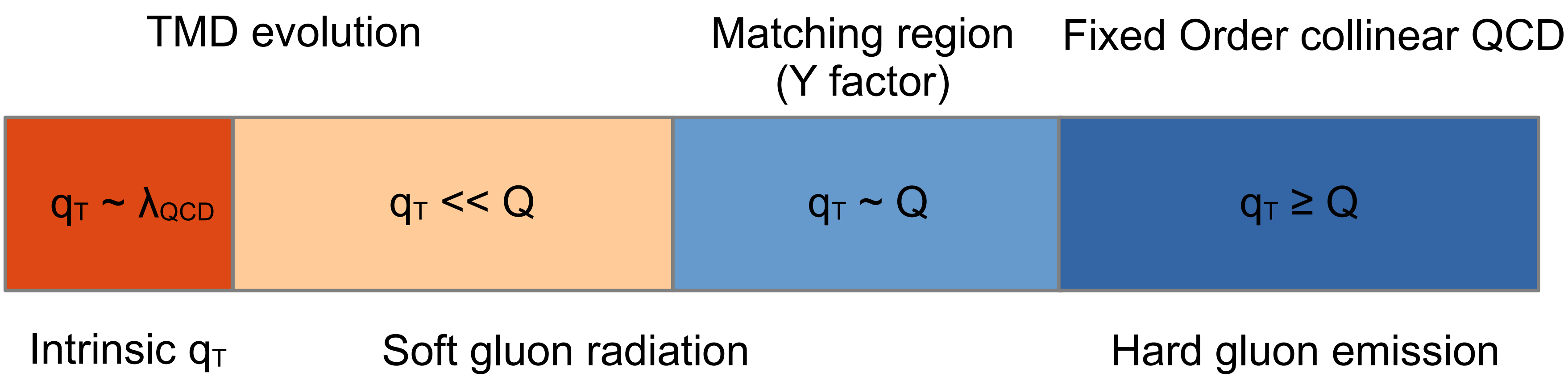
Anselmino, Boglione, Prokudin, Turk, Eur.Phys.J. A31 (2007) 373-381

ZEUS Collaboration (M. Derrick), Z. Phys. C 70, 1 (1996)



TMD vs Collinear regions

- For this scheme to work, distinct kinematic regions have to be identified
- They should be large enough and well separated



Warning: factorization identifies 2 regions, at the 2 ends of the q_T spectrum!

Issues arise in the **intermediate region** (transition from pert. to non-pert. regimes)

How do we know where the boundaries of the kinematic regions are?

Perturbative vs non-perturbative contributions

- Study evolution in the transverse coordinate space, b_T (F.T. space of k_T)
- Freeze all perturbative scales when reaching the non perturbative region

b* prescription

$$b_T \longrightarrow b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \longrightarrow \boldsymbol{b}_*(b_\mathrm{T}) = \begin{cases} \boldsymbol{b}_\mathrm{T} & b_T \ll b_\mathrm{max} \\ \boldsymbol{b}_\mathrm{max} & b_T \gg b_\mathrm{max} \end{cases}$$

- Define a non-perturbative function for large b_T:

$$\text{TMD} = \tilde{f}_{j/h}(x, b_\mathrm{T}; \mu_{Q_0}, Q_0^2) = \underbrace{\tilde{f}_{j/h}(x, b_*; \mu_{Q_0}, Q_0^2)}_{\text{Perturbative (small } b_T)} \underbrace{\left(\frac{\tilde{f}_{j/h}(x, b_\mathrm{T}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/h}(x, b_*; \mu_{Q_0}, Q_0^2)} \right)}_{\text{Non perturbative (large } b_T)}$$

- Model the (unknown) non perturbative function
- Let the fit determine the value of the free parameters of the non-pert. model

Perturbative vs non-perturbative contributions

- **ISSUE 1**: the b* prescription isolates the perturbative part of the TMD and restricts it to low values of b_T (i.e. to large values of k_T)

  BUT in this prescription **nothing prevents infiltrations of the non-perturbative model into the perturbative region**, at small values of b_T.

- **ISSUE 2**: in practical implementations, the function $b^*(b_T)$ can become another non perturbative model, and the value of b_{MAX} a model parameter.

- **ISSUE 3**: TMDs should satisfy an integral relation which links them to their collinear counterparts

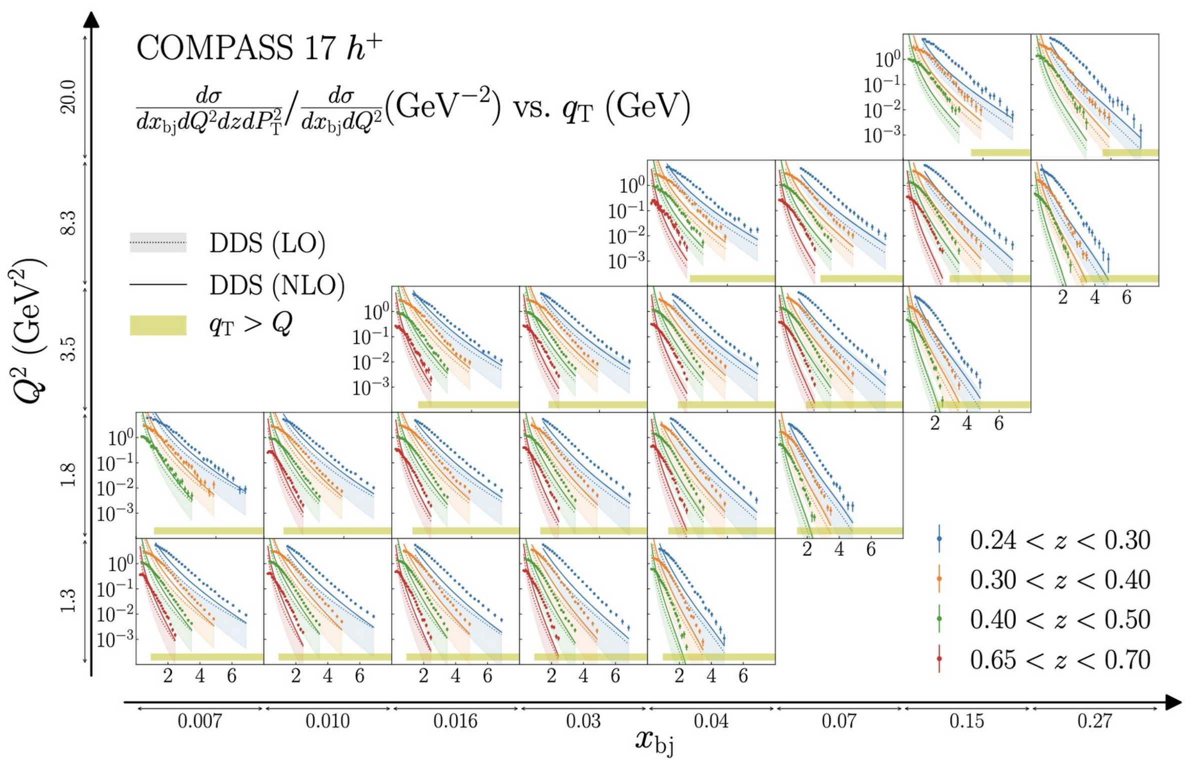
$$f(x) = \int d^2\mathbf{k_T}\, f(x, \mathbf{k_T})$$

- **ISSUE 4**: discrepancies observed in the size of the collinear large q_T tail cross sections

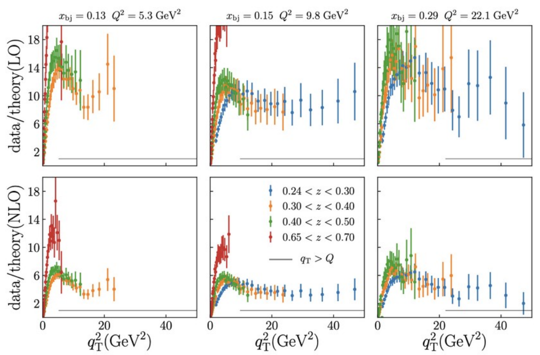
  ...

Collinear cross sections at large q_T

★ At high q_T the collinear formalism should be valid, but large discrepancies are observed



Discrepancy is about one order of magnitude



Gonzalez-Hernandez, Rogers, Sato, Wang, Phys.Rev.D 98 (2018) 11, 114005

Perturbative vs non-perturbative contributions

- These complications are ultimately connected to the fact that the usual b* organization prescribes the existence of only 2 very sharply defined contributions: 1 that involves entirely perturbative b_T dependence and 1 that is entirely non-perturbative.

- A more realistic view is that there are 3 types of b_T-dependence:

1) A totally non-perturbative behavior as $b_T \to \infty$
2) A very reliable collinear factorization as $b_T \to 1/Q_0$
3) An intermediate transition region around $b_T \sim 1/Q_0$ where b_T-dependence is reasonably well-described by perturbative collinear factorization, but is not as isolated from non-perturbative effects as region 2.

Ideally, the **intermediate region** should be described by a **physically motivated model** that interpolates between the TMD and the collinear regions

Hadron Structure Oriented approach (HSO)

Aslan, Boglione, Gonzalez-Hernandez,Rainaldi, Rogers, Simonelli, Phys.Rev.D 110 (2024) 7, 074016

- Construct a smooth and continuous parametrization of the TMD that directly interpolates between perturbative and non-perturbative regimes
- Impose the integral relation on this parametrization

$$f(x) = \int_{reg} d^2\mathbf{k_T}\, f(x, \mathbf{k_T}; \mu_Q, \mu_Q^2) + \Delta + p.s.$$

- Avoid intrusion of the non-perturbative model in the kinematical region where the behaviour of the TMD should be dominated by perturbative physics
- Make sure that the TMD converges fast enough to its perturbative tail at large k_T
- Ensure a reliable **physical interpretation of the extracted TMD**

The role of experimental data
(JLab12 and JLab22)

- Do experimental data help us to assess energy and transverse momentum ranges which are crucial to improve the current understanding of QCD in terms of factorization theorems?
- Do experimental data shed light on the hadronization mechanism?
- Do experimental data allow a reliable separation between TMD and collinear regimes?
- How does QCD manifest itself in the “intermediate/matching region”?

Affinity

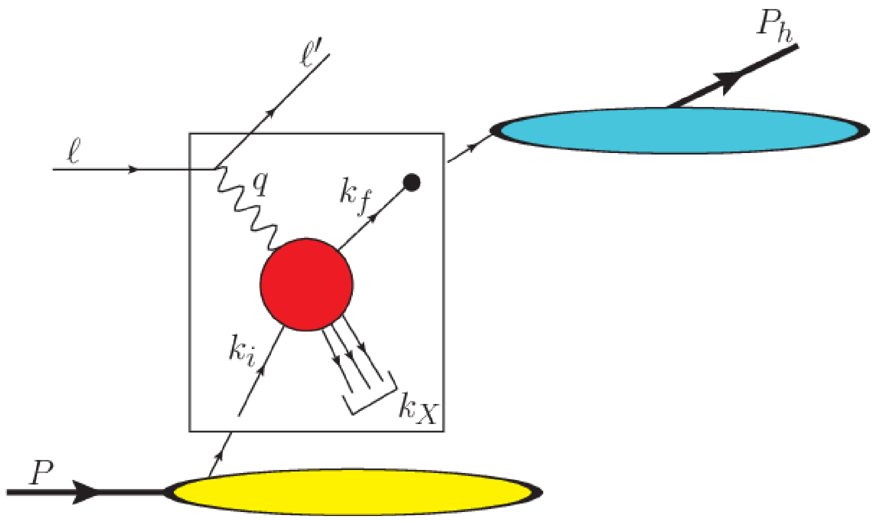
Boglione,Kelleher, Prokudin, Vossen, Yushkevych, in preparation

What is "affinity" ?

- Affinity is a phenomenological tool based on momentum region indicators to guide the analysis and interpretation of SIDIS measurements.
- The new tool, referred to as "affinity", is devised to help visualize and quantify the proximity of any experimental kinematic bin to a particular hadron production region, such as that associated with transverse momentum dependent factorization.
- Bin centers are located in the points corresponding to the bin averaged values of x_{Bj} and Q^2, and in each of these bins various values of z_h and q_T/Q can be measured.
- In each bin of fixed z and q_T /Q, the affinity is indicated by a dot with size proportional to the number of events in that specific bin.
- The affinity is color coded according to the scheme on the right of the panels: red (and smaller) symbols correspond to low TMD affinity, while dark blue (and larger) symbols correspond to high TMD affinity

Momentum region indicators

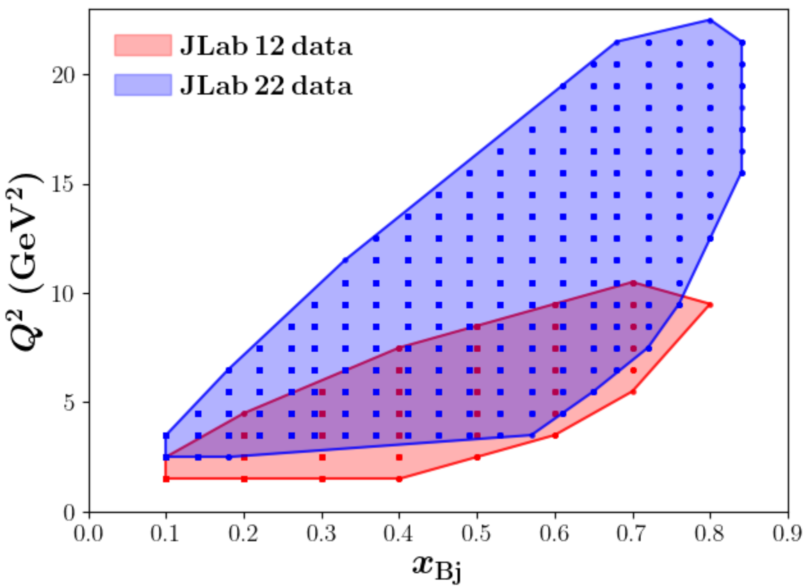
	TMD Region	COLLINEAR Region
$R_0 = \max\left(\left\lvert\frac{k_i^2}{Q^2}\right\rvert, \left\lvert\frac{k_f^2}{Q^2}\right\rvert, \left\lvert\frac{\delta k_T^2}{Q^2}\right\rvert\right)$	Small	Small
$R_1 = \frac{P_h \cdot k_f}{P_h \cdot k_i}$	Small	Small
$R_2 = \frac{\lvert k^2 \rvert}{Q^2}$	Small	Large

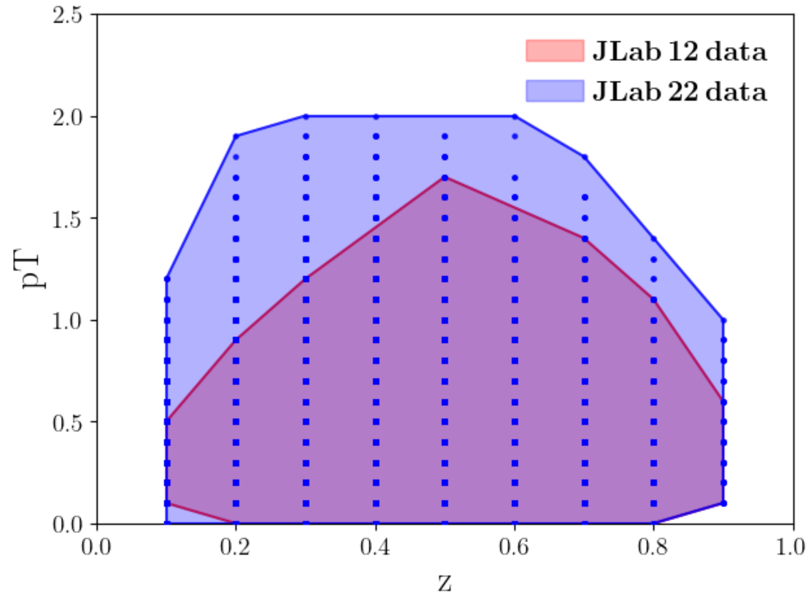


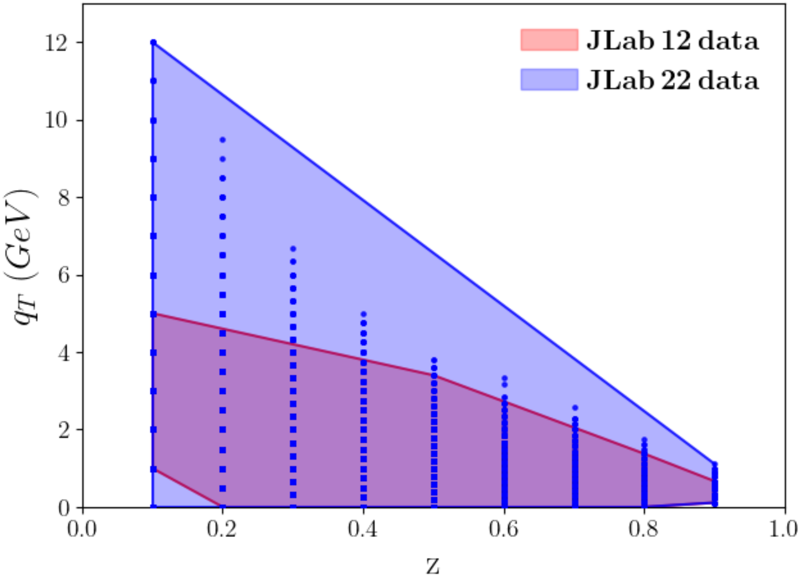
$$k = k_f - q$$

$$k_X = k_i + q - k_f$$

Phase space – JLab12 and JLab22







Data set: 100k events

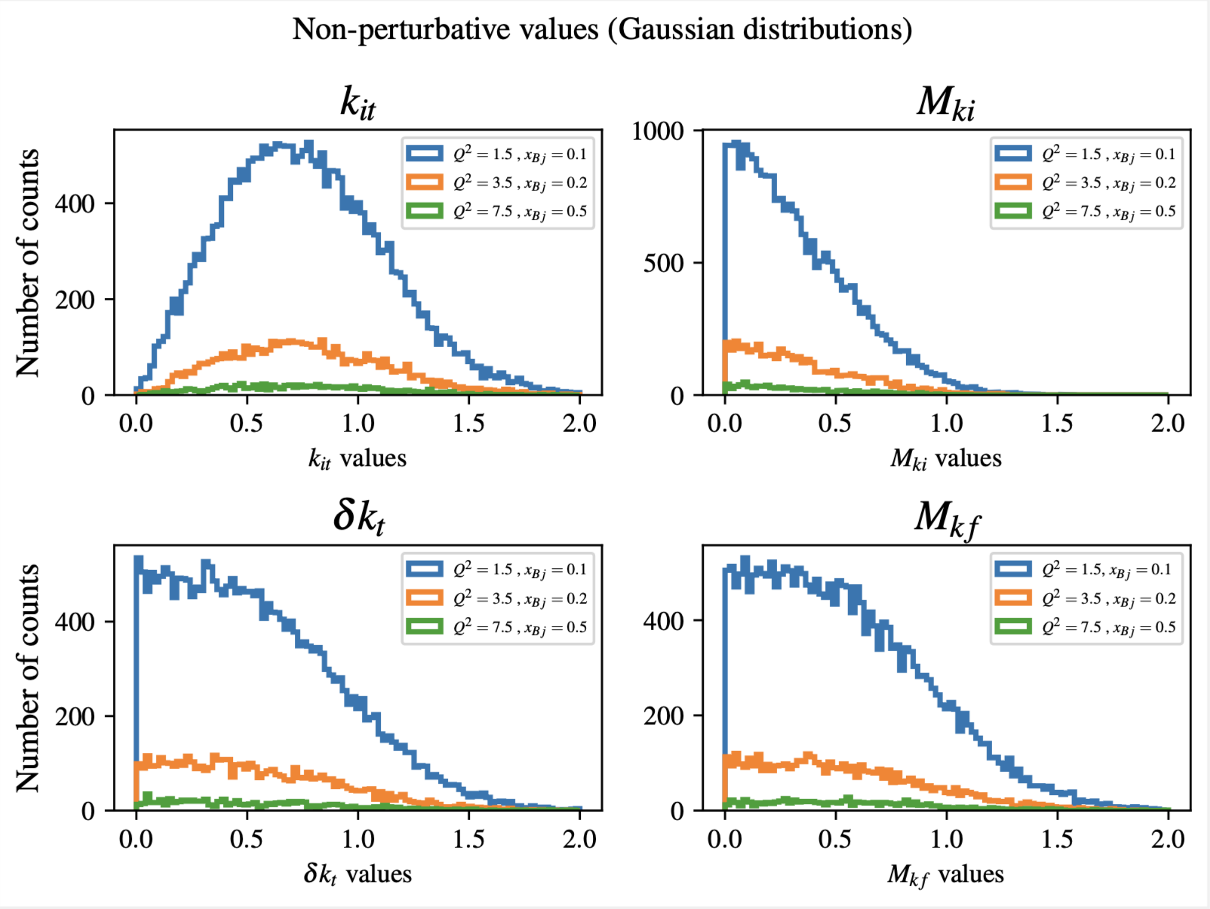
Each event is identified by:

- a set of hadronic variables **(x, Q^2, z, p_T)** - measured
- a set of partonic variables **(k_i, M_i, M_f, δk_T)** - unknown

Reconstruction of hadronic variables

Two possible strategies:

1. partonic variables are assumed to follow **Gaussian distributions** (same for all values of hadronic variables)

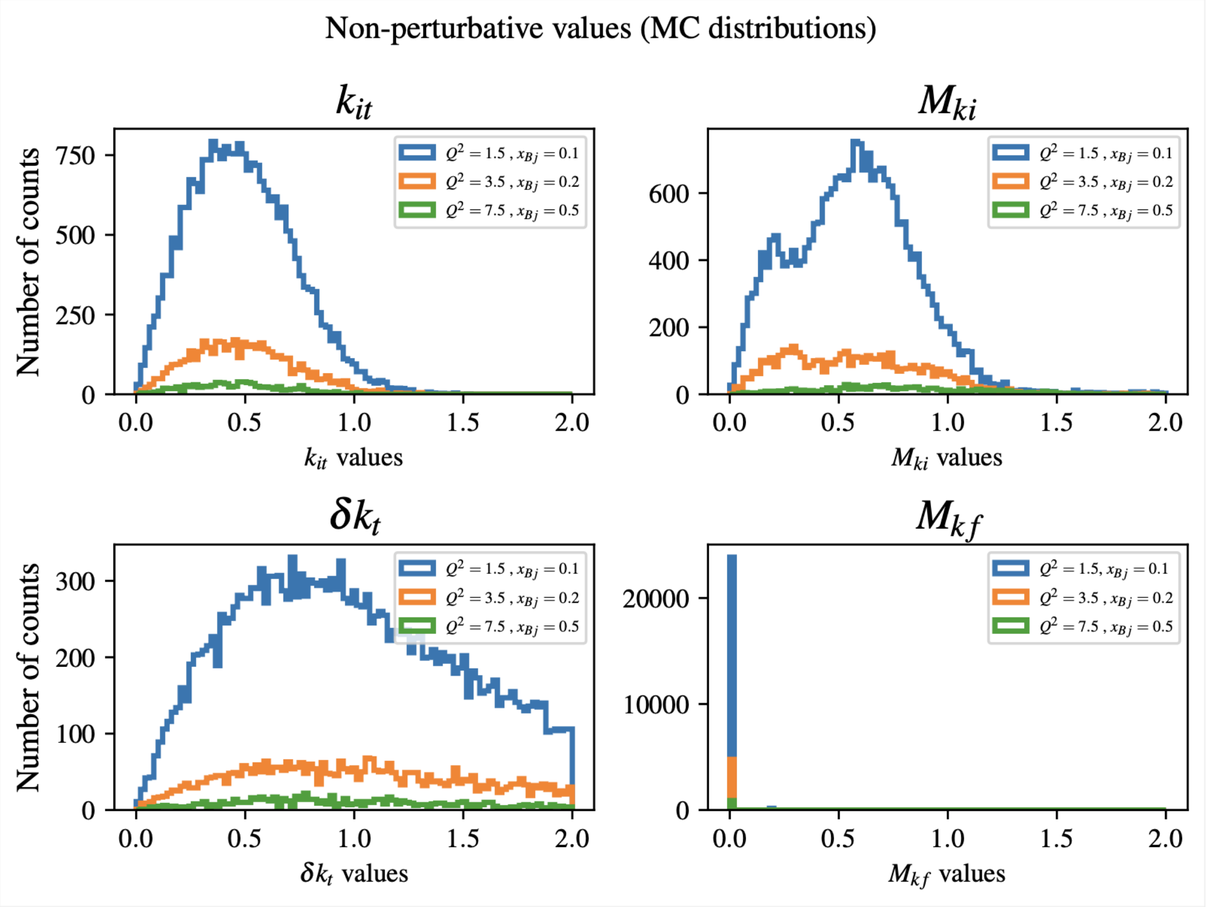


Boglione, Diefenthaler, Dolan, Gamberg, Melnitchouk, Pitonyak, Prokudin, Sato, Scalyer, JHEP 04 (2022) 084

Reconstruction of hadronic variables

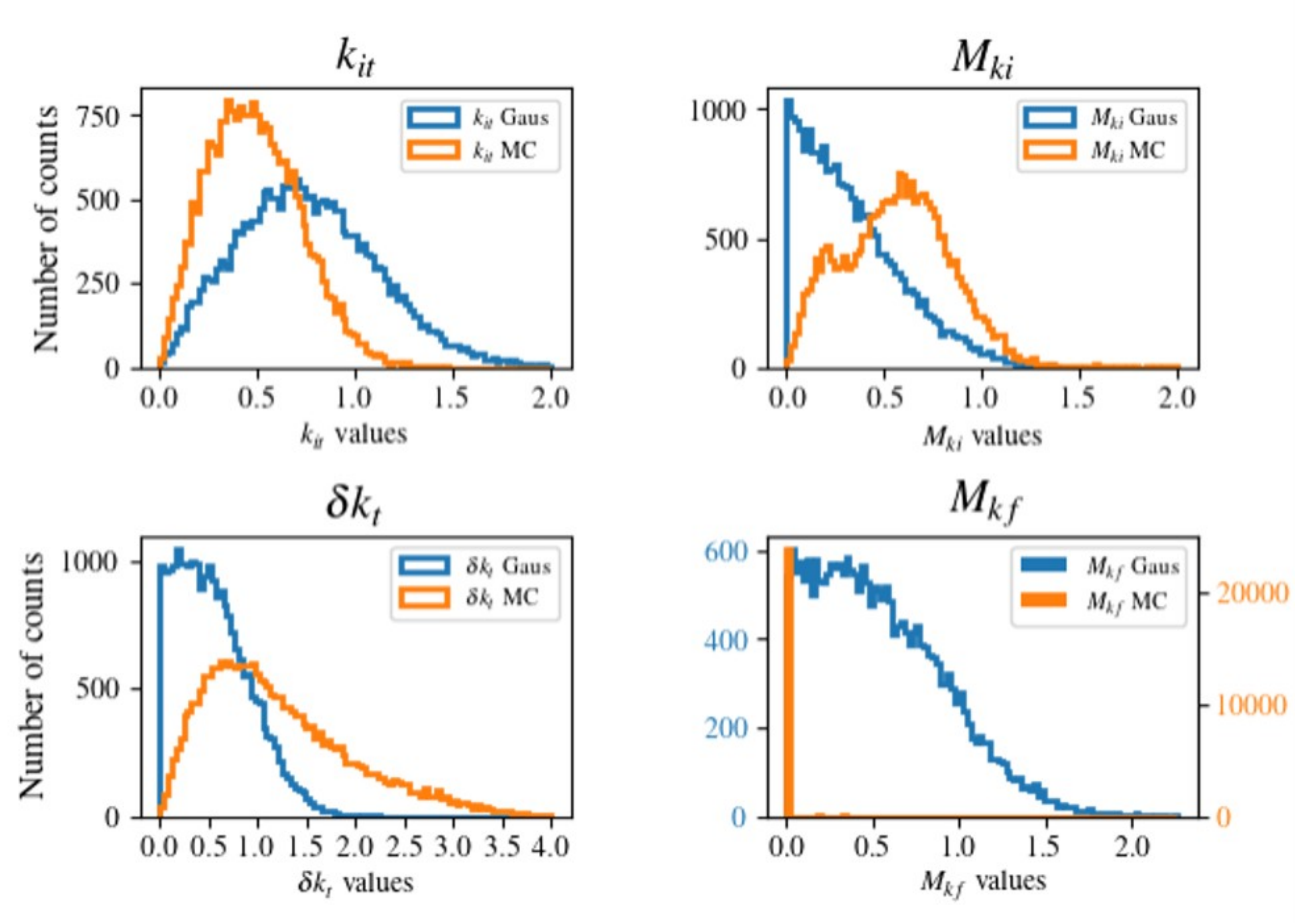
Two possible strategies:

2. partonic variables distributions are reconstructed, event-by-event, from a **MC generator** (correlations preserved)

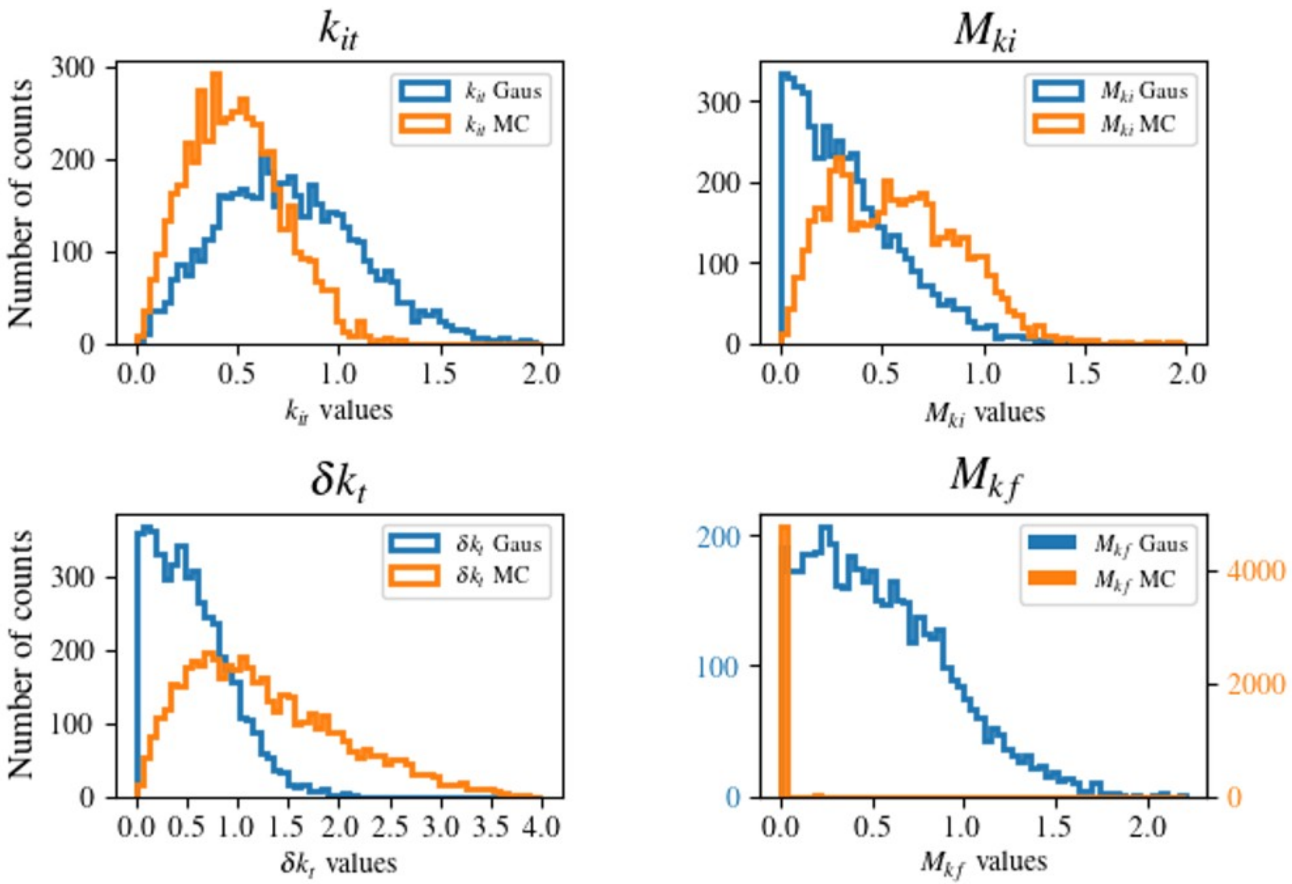


Boglione, Kelleher, Prokudin, Vossen Yushkevych, in preparation

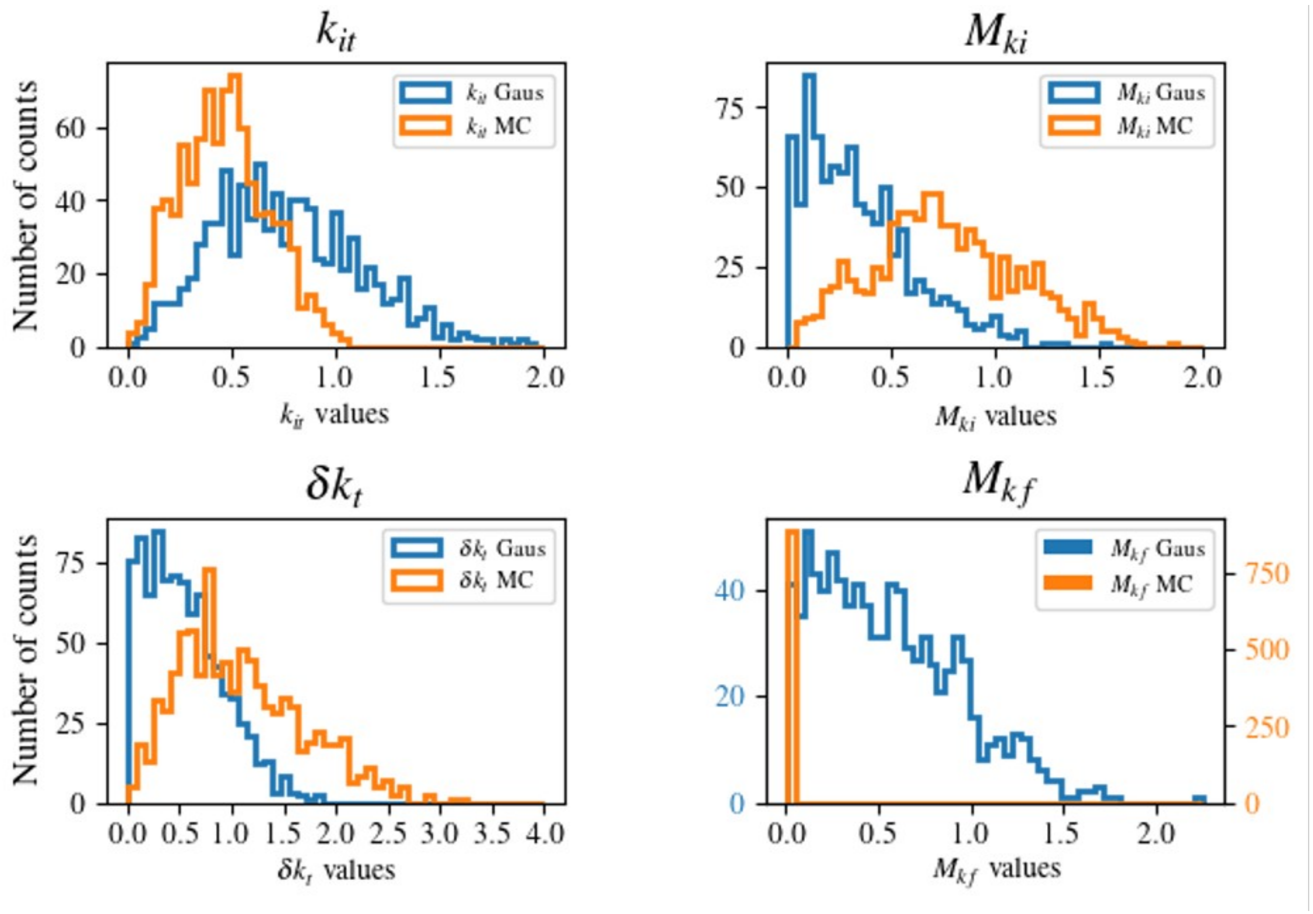
Non-perturbative parameters for [Q^2=1.5, x_{Bj}=0.1]



Non-perturbative parameters for $[Q^2=3.5,\ x_{Bj}=0.3]$

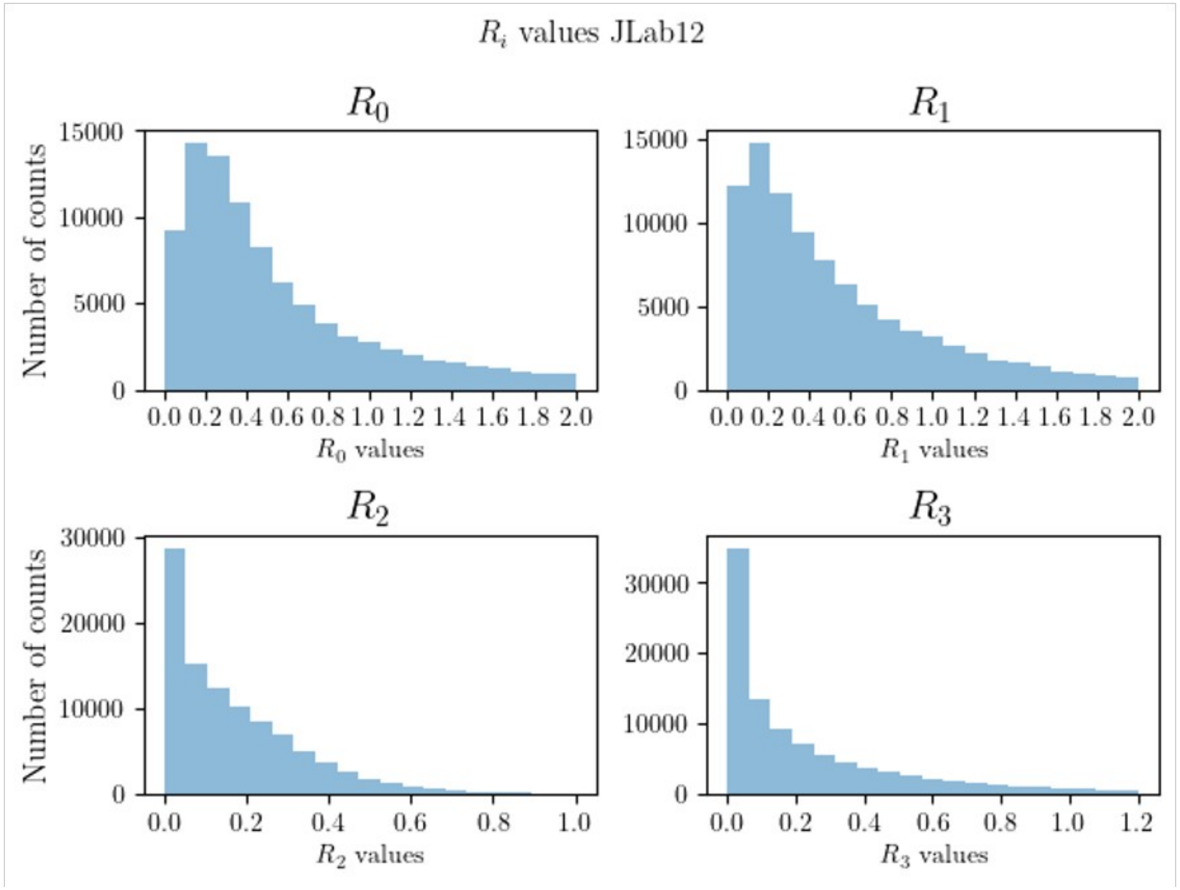


Non-perturbative parameters for $[Q^2=7.5,\ x_{Bj}=0.5]$



Region indicators

Region indicators are computed for each event:



$$R_0 = \max\left(\left|\frac{k_i^2}{Q^2}\right|, \left|\frac{k_f^2}{Q^2}\right|, \left|\frac{\delta k_T^2}{Q^2}\right|\right)$$

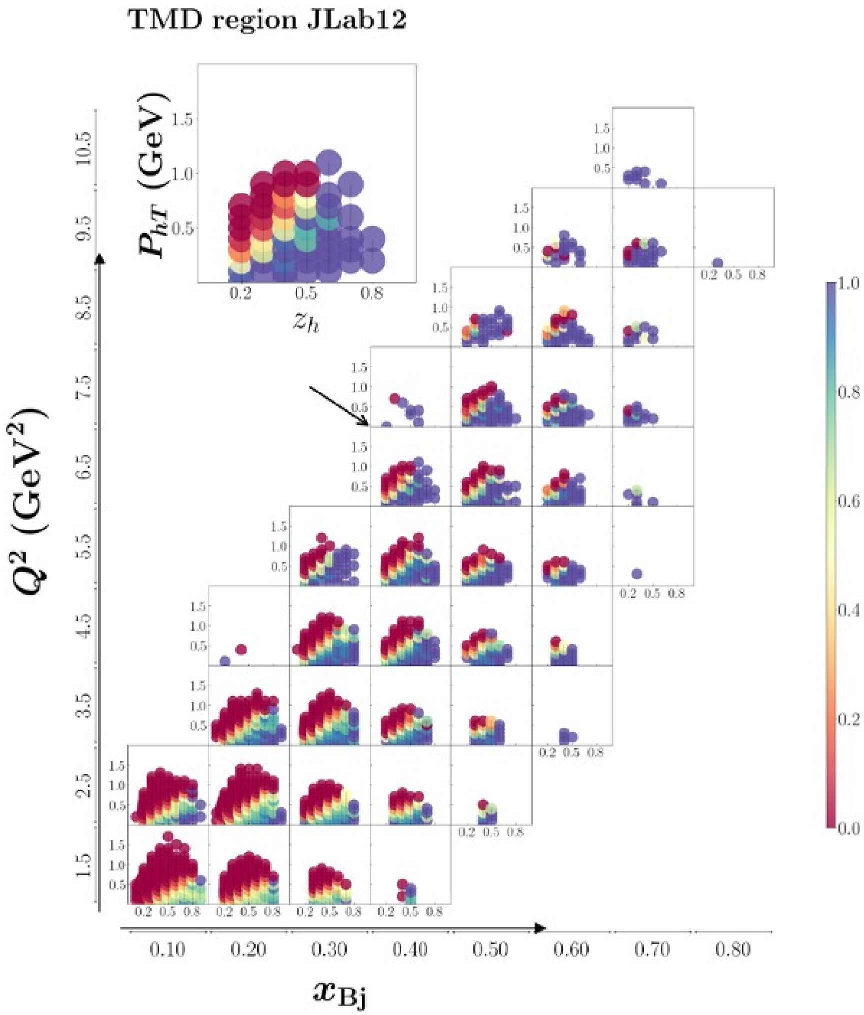
$$R_1 = \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

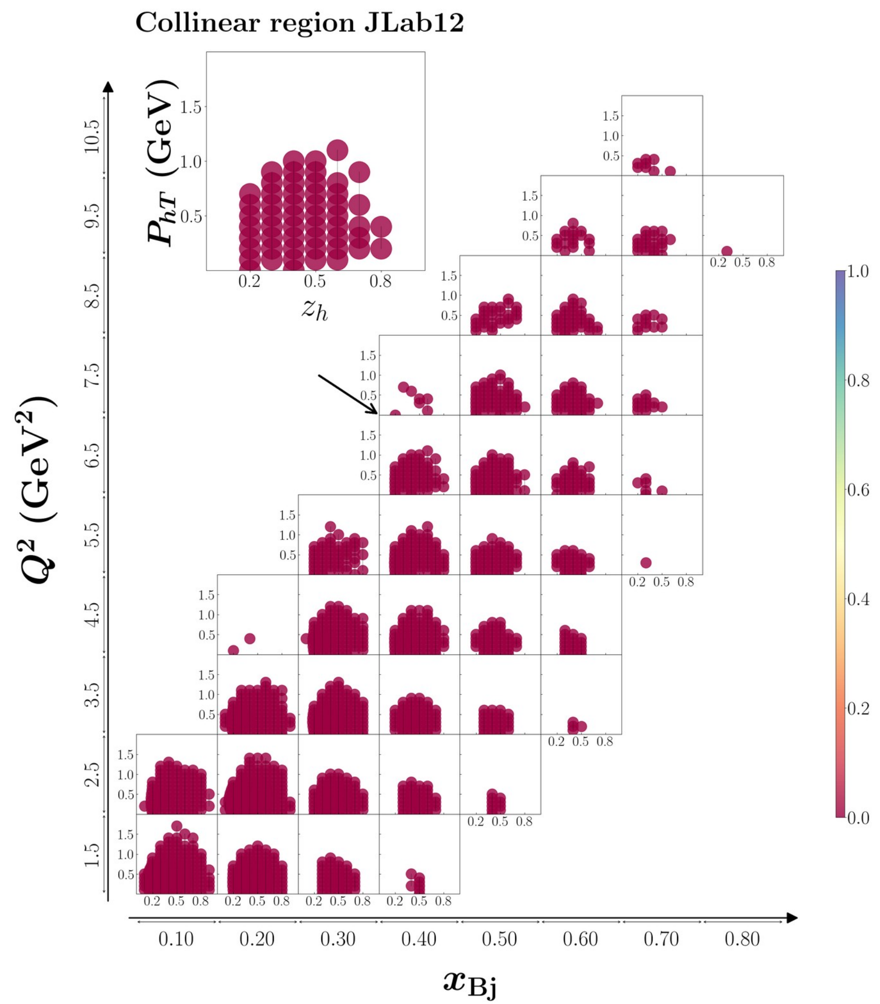
$$R_2 = \frac{|k^2|}{Q^2}$$

$$R_3 \equiv \frac{|k_X^2|}{Q^2}$$

TMD-Affinity@JLab12
(MC event-by-event method)

Boglione, Kelleher, Prokudin, Vossen Yushkevych, in preparation

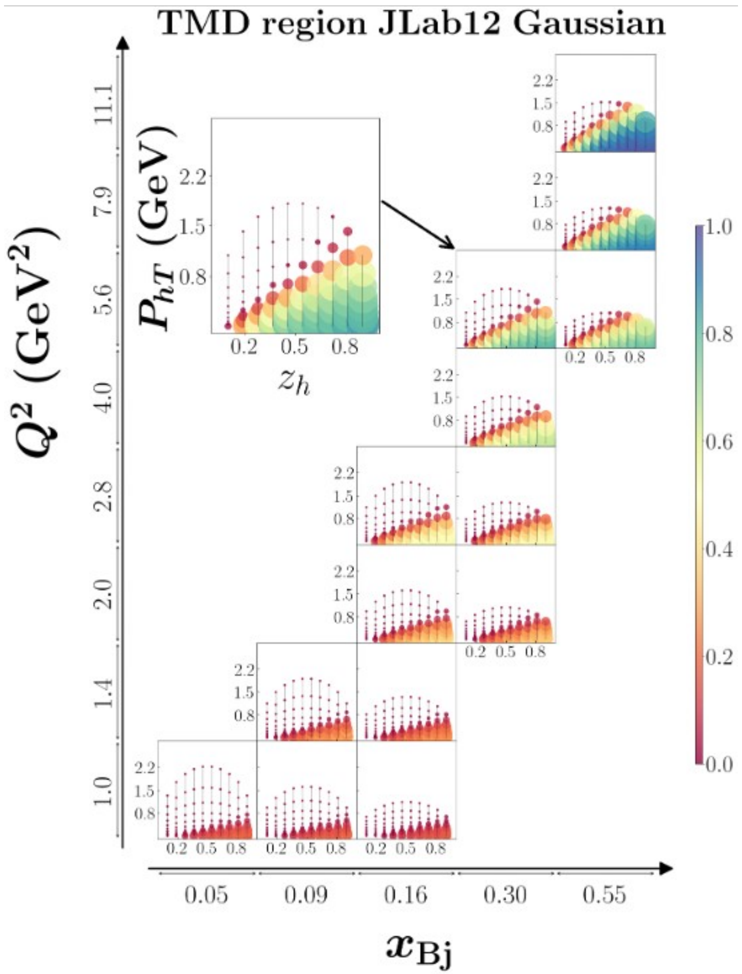




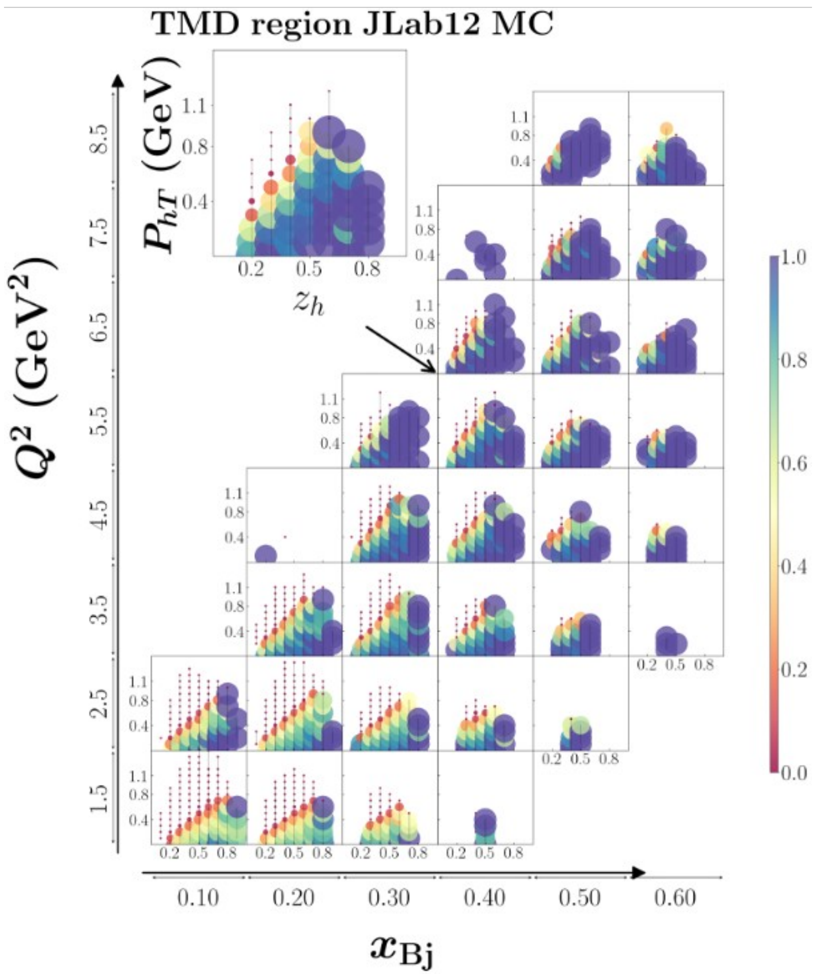
blob sizes are based on number of events, blob colors are based on affinity

TMD-Affinity@JLab12
(Gaussian distributions vs. event-by-event MC generated distributions)

Boglione, Diefenthaler, Dolan, Gamberg, Melnitchouk,Pitonyak, Prokudin, Sato, Scalyer, JHEP 04 (2022) 084

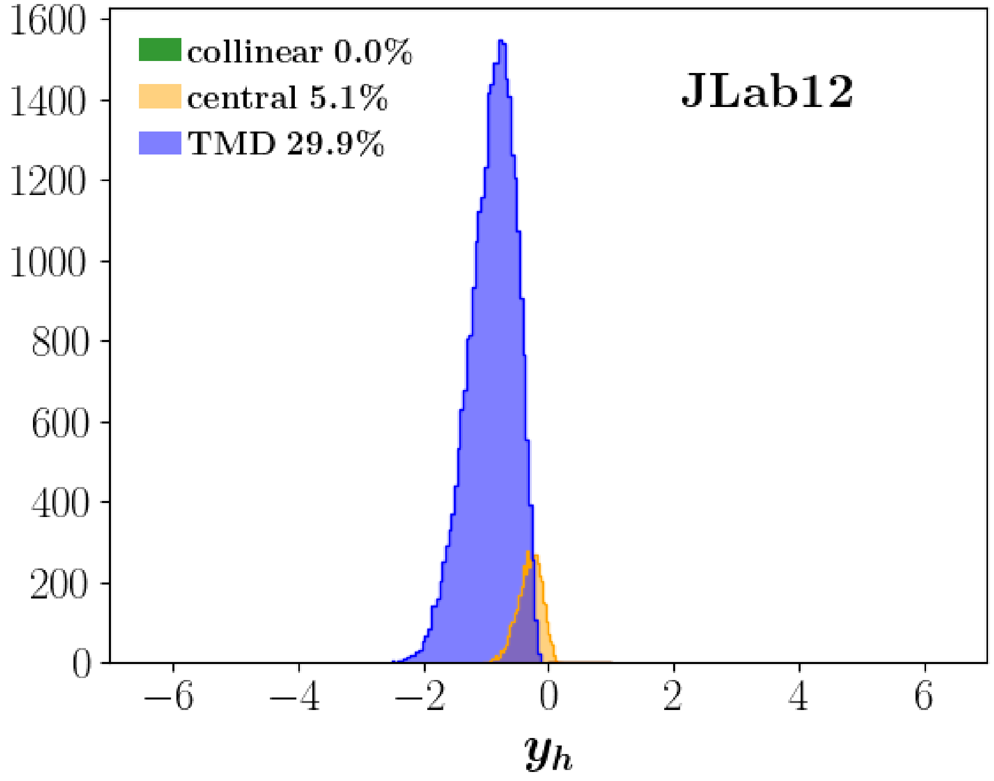


Boglione, Kelleher, Prokudin, Vossen, Yushkevych, in preparation



blob sizes as well as blob colors are based on affinity / binning is different

JLab12 affinity to TMD and collinear regions



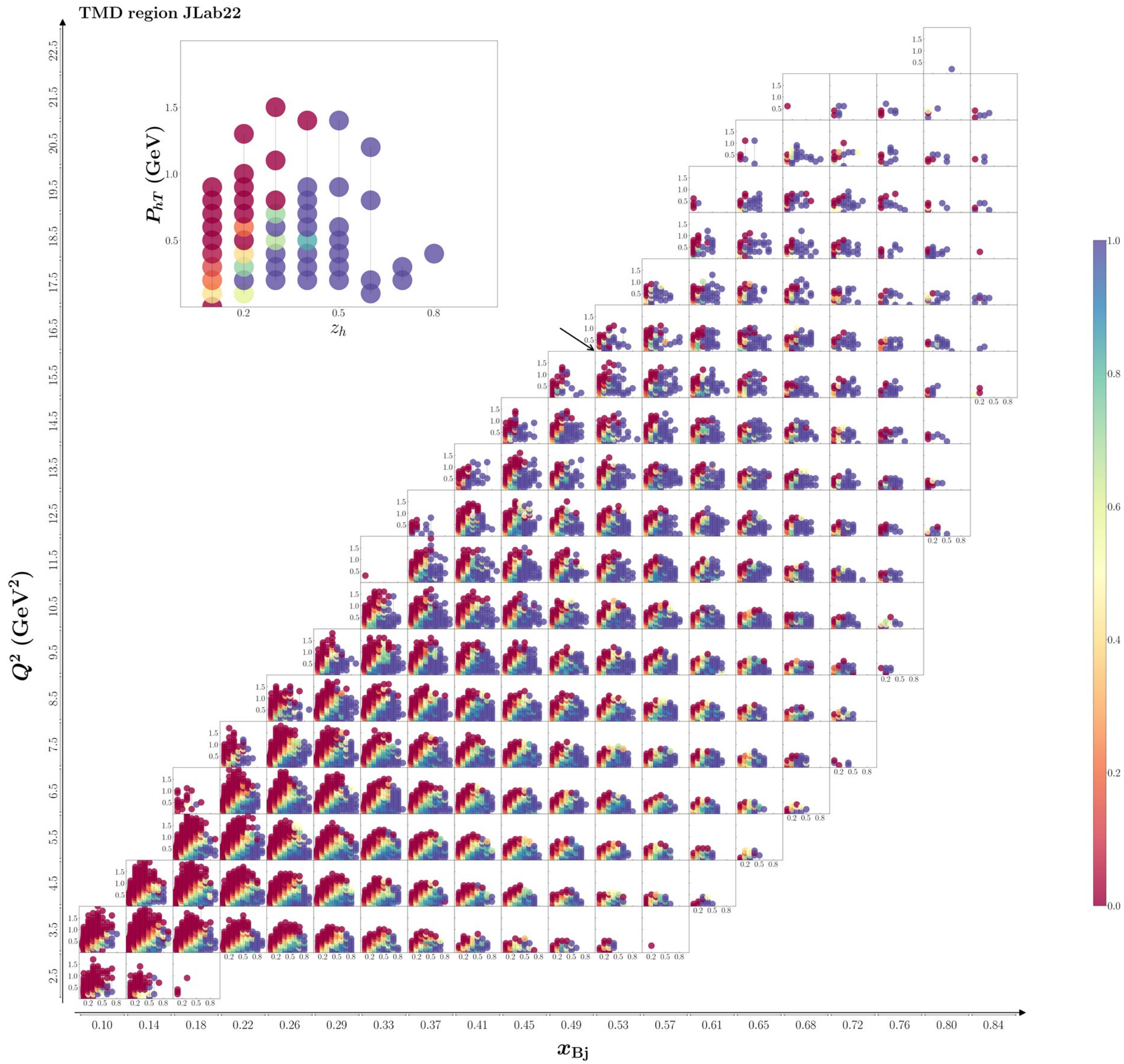
Phase space in rapidity y_h of produced hadron, with for TMD, collinear, central and target regions indicated.

The legends show the percentage of all bins with corresponding affinity above 5%

As expected Jlab12 phase space covers mostly the TMD region

JLAb22

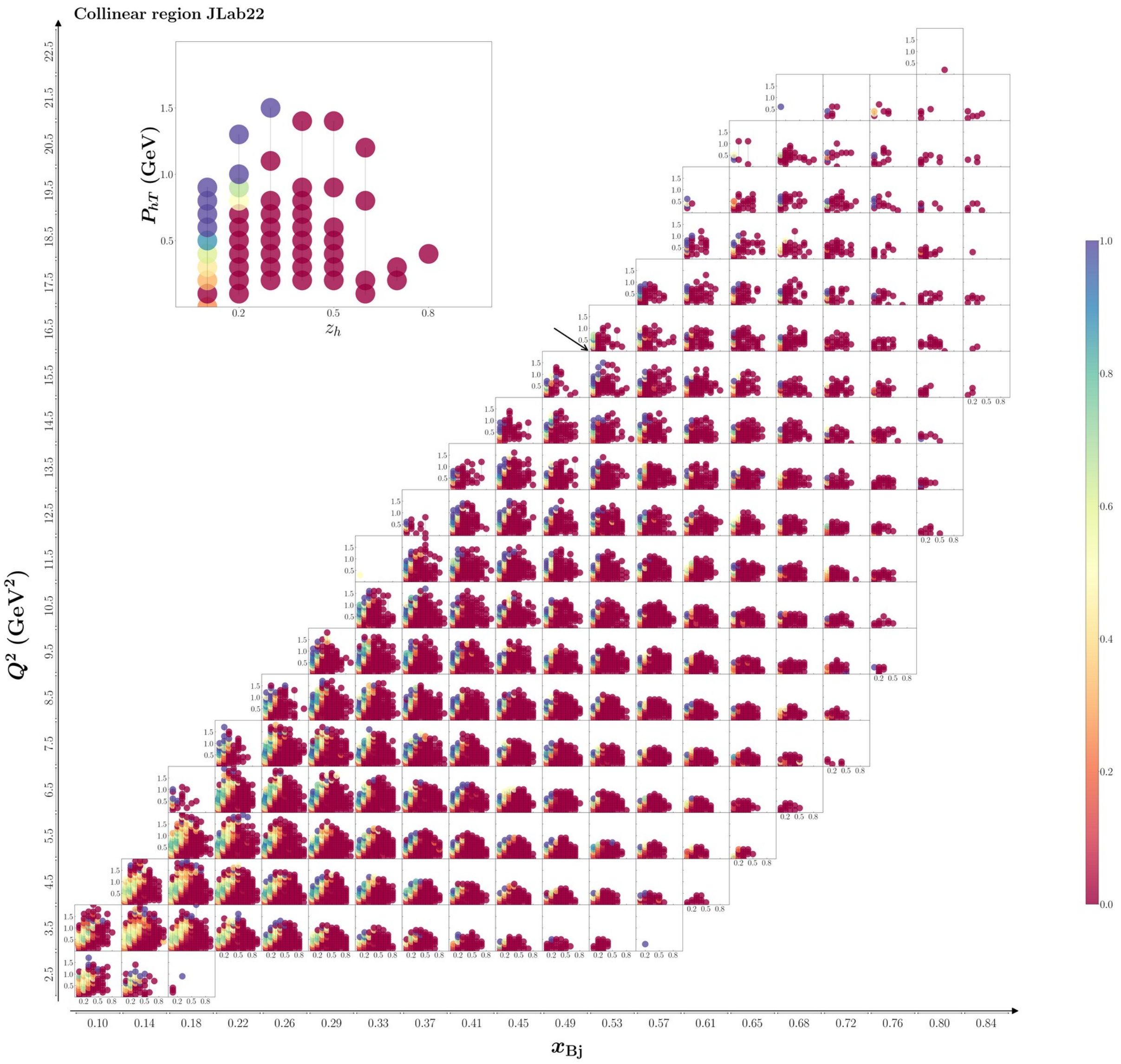
TMD-Affinity@JLab22



- Very high statistics and fine binning will improve the 3D maps of hadron structure
- **TMD region** will be accessible (large Q, large z, **small P_T**)
- **Collinear region** will be accessible (large Q, moderate z, **large P_T**)
- A unique feature of Jlab22 is that it will offer an unprecedented insight onto the matching **region,** which cannot be explored in any other SIDIS experiment

Boglione, Prokudin, Yushkevych, in preparation

Collinear-Affinity@JLab22



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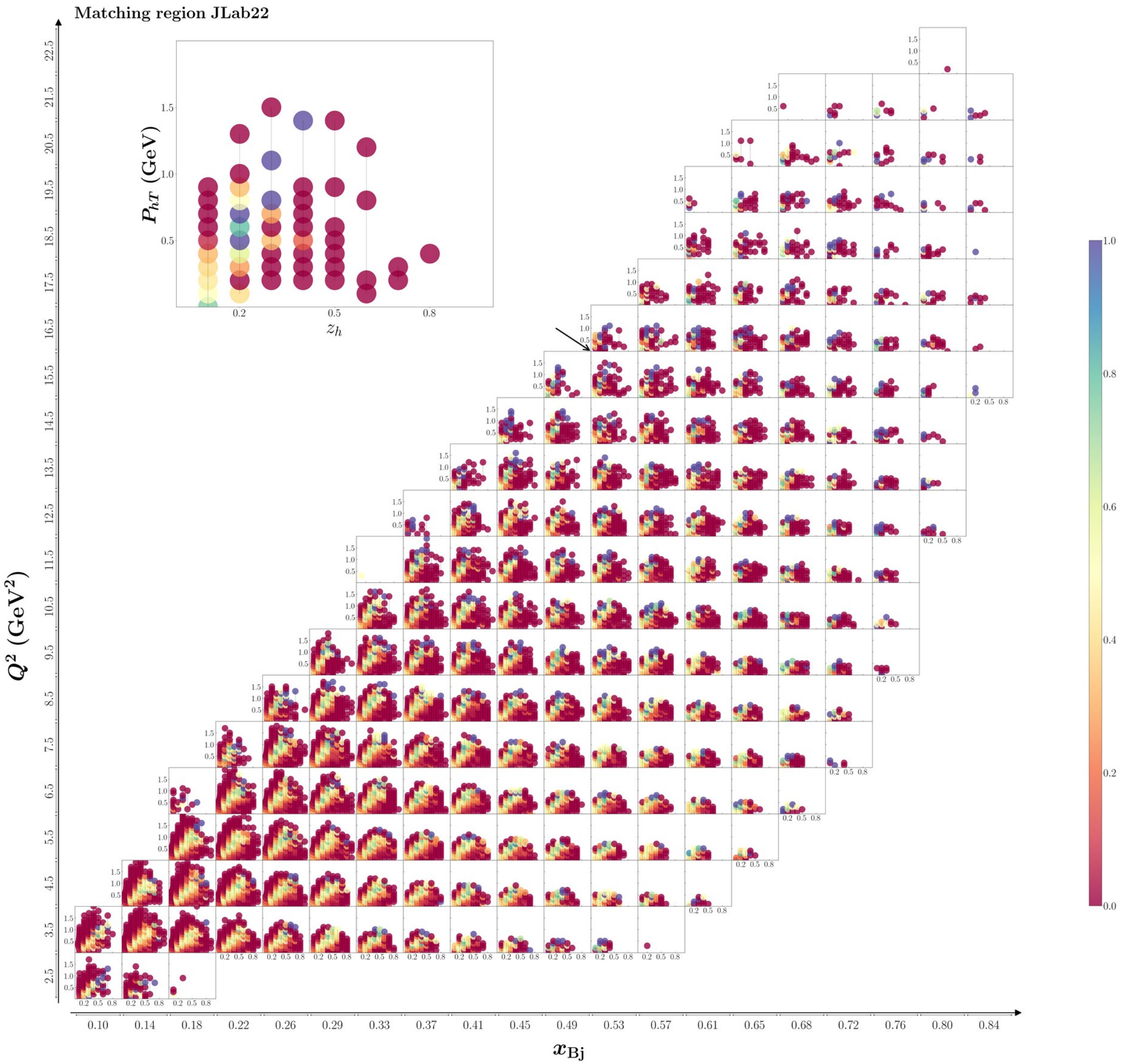
Boglione, Prokudin, Yushkevych, in preparation

Central-Affinity@JLab22



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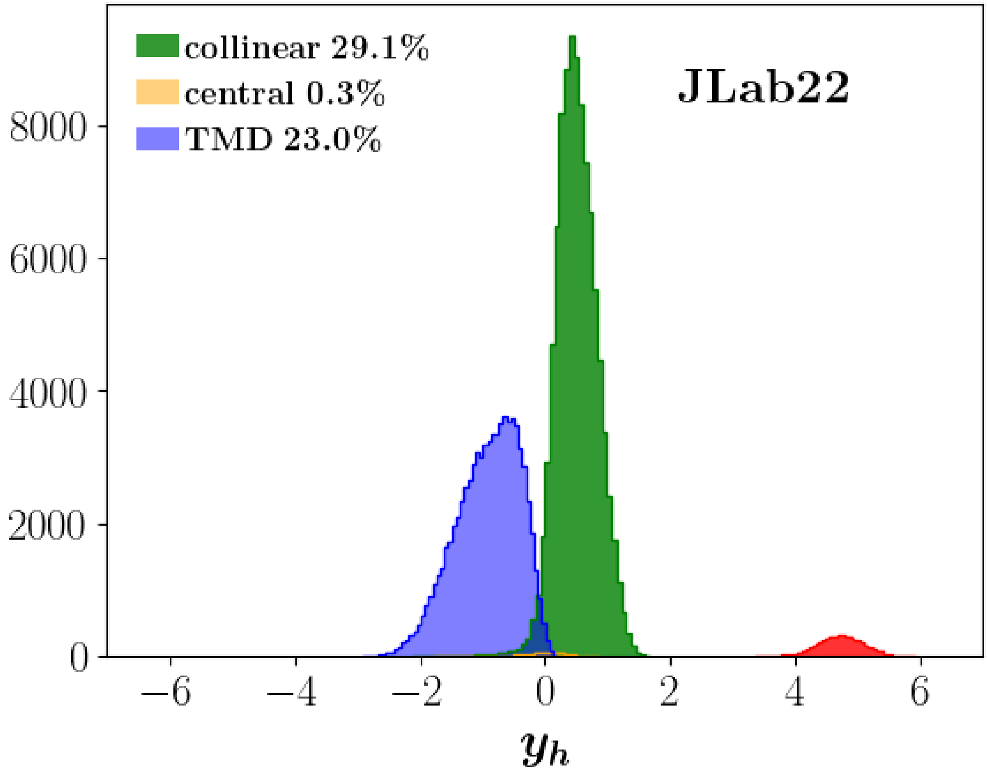
Boglione, Prokudin, Vossen, Yushkevych, in preparation

JLab22 affinity to TMD and collinear regions



Phase space in rapidity y_h of produced hadron, with for TMD, collinear, central and target regions indicated.

The legends show the percentage of all bins with corresponding affinity above 5%

At JLab22 TMD and collinear regions are well separated

Conclusions

- Phenomenological studies of TMD factorization have entered a high precision hera. Many aspects of the interplay between perturbative and non-perturbative contributions are now better understood

- Some issues remain open and need further investigation, especially as far as the phenomenological application of factorization theorems is concerned:
  - Hard to avoid non-perturbative models leeks into perturbative regions (→ HSO)
  - Hard to work in b_T space where we loose phenomenological intuition
  - F.T. involves integration of an oscillating function over b_T up to infinity: upon integration one loses track of “small b_T“ and “large b_T”.
  - ...

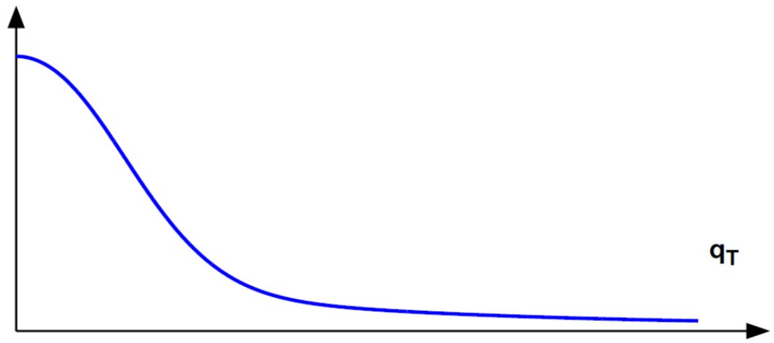
- Simultaneous fits of SIDIS, Drell-Yan and e+e- annihilation data are very valuable, BUT they should be performed within consistent and solid frameworks, to allow for a reliable interpretation of the fit outcomes.

Conclusions

- A thorough assessment of SIDIS kinematic regions can have a considerable impact on our understanding of TMD physics
- These studies will help us to assess energy and transverse momentum ranges which are crucial to improve the current understanding of QCD in terms of factorization theorems, and shed light on the hadronization mechanism
- Affinity will allow a reliable separation between TMD and collinear regimes, and possibly give access to the matching region

Back up

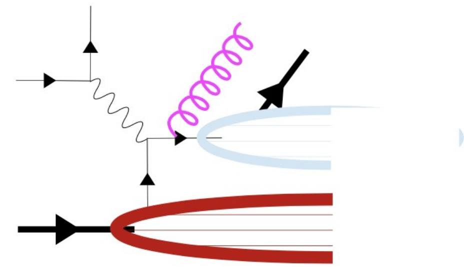
Factorization



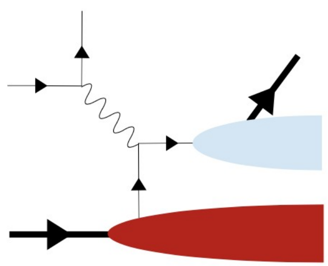
q_T<<Q
TMD region

q_T~Q
Matching region

q_T>>Q
pQCD



Hard gluon radiation



$$\frac{d\sigma^{\rm NLO}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\rm ASY}}{dx\,dy\,dz\,dq_T^2} + Y$$

$$\frac{d\sigma^{\rm total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{\rm DIS}\int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}W^{\rm SIDIS}(x,z,b_T,Q) + Y^{\rm SIDIS}(x,z,q_T,Q)$$

Resummation / TMD evolution

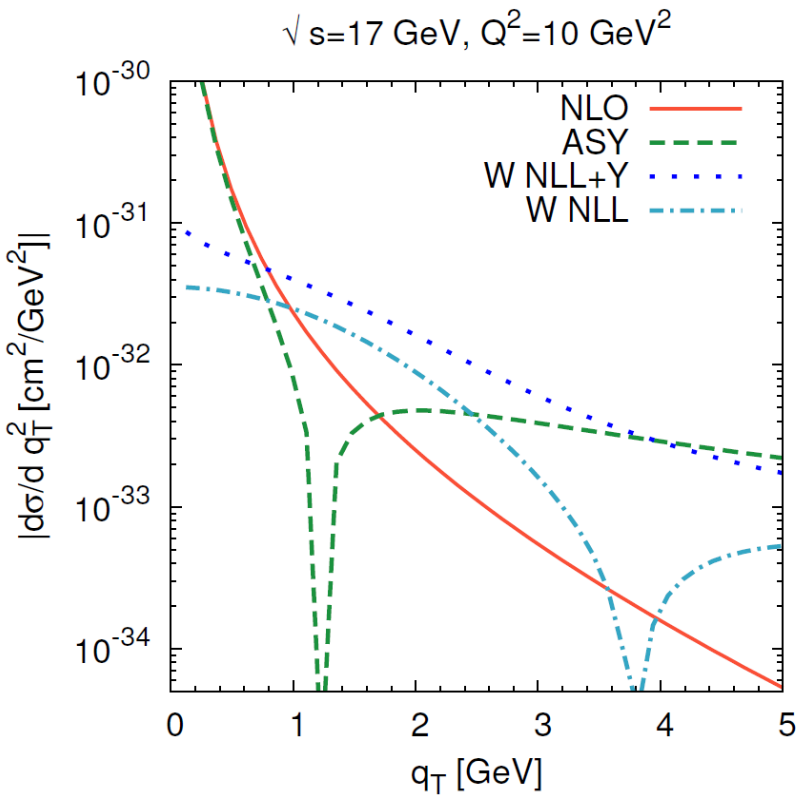
$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j \overbrace{e_j^2 W_j(x_1,x_2,b_T,Q)}^{\text{Resummed term}} + \overbrace{Y(x_1,x_2,q_T,Q)}^{\text{Finite term}}$$

$Y = \sigma^{FO} - \sigma^{ASY}$

- The W term is designed to work well at low and moderate q_T, when $q_T \ll Q$. (Notice that W is devised to work down to $q_T \sim 0$, however collinear-factorization works up to $q_T > M$; therefore, TMD-factorization and collinear-factorization can be simultaneously applied only when $q_T \gg M$).
- The W term becomes unphysical at larger q_T, when $q_T \geq Q$, where it becomes negative (and large).
- The Y term corrects for the misbehavior of W as q_T gets larger, providing a consistent (and positive) q_T differential cross section.
- The Y term should provide an effective smooth transition to large q_T, where fixed order perturbative calculations are expected to work.

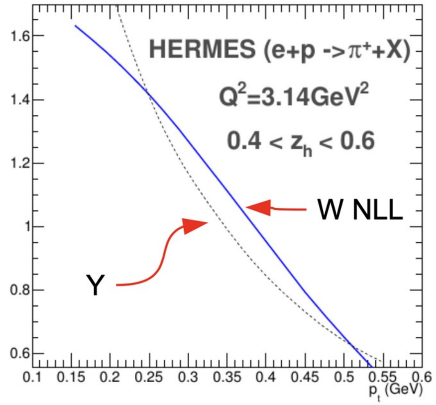
SIDIS - Y factor

Boglione, Gonzalez, Melis, Prokudin, JHEP 02 (2015) 095



$$\frac{d\sigma^{\rm NLO}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\rm ASY}}{dx\,dy\,dz\,dq_T^2} + Y$$

- The Y factor should not be neglected
- The Y factor is very large (as large as the cross section itself) even at low q_T



Sun et al arXiv:1406.3073

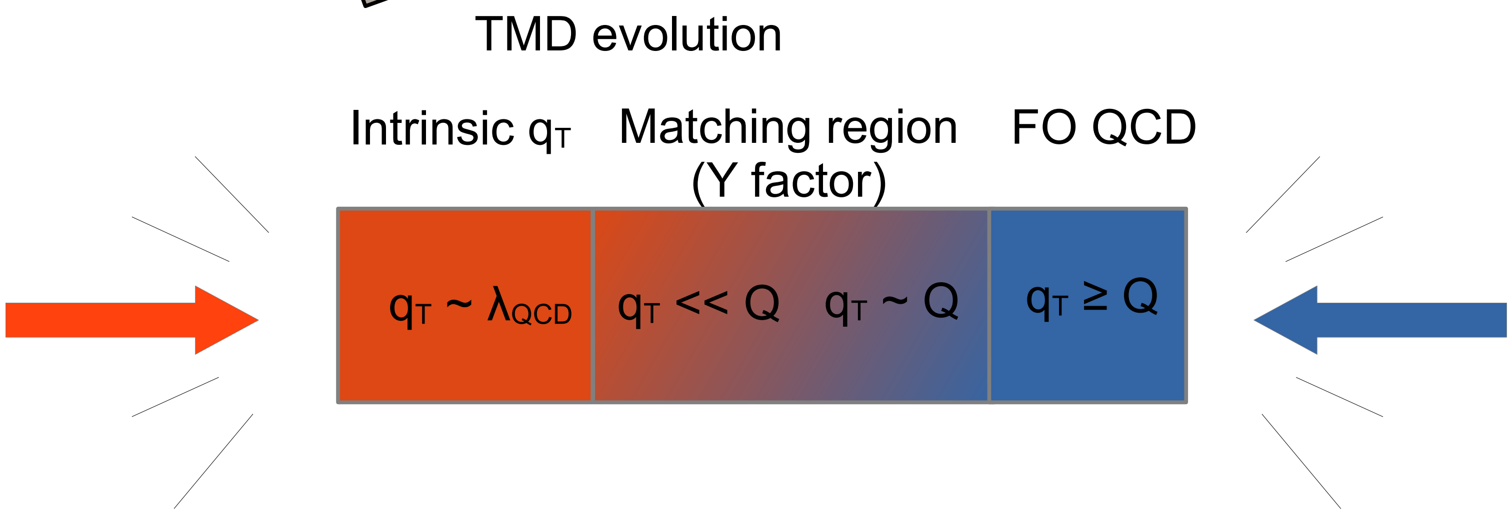
- However, it could be affected by large theoretical uncertainties

$\sigma^{ASY} = Q^2/q_T^2\ [A\ \ \mathrm{Ln}(Q^2/q_T^2) + B + C]$

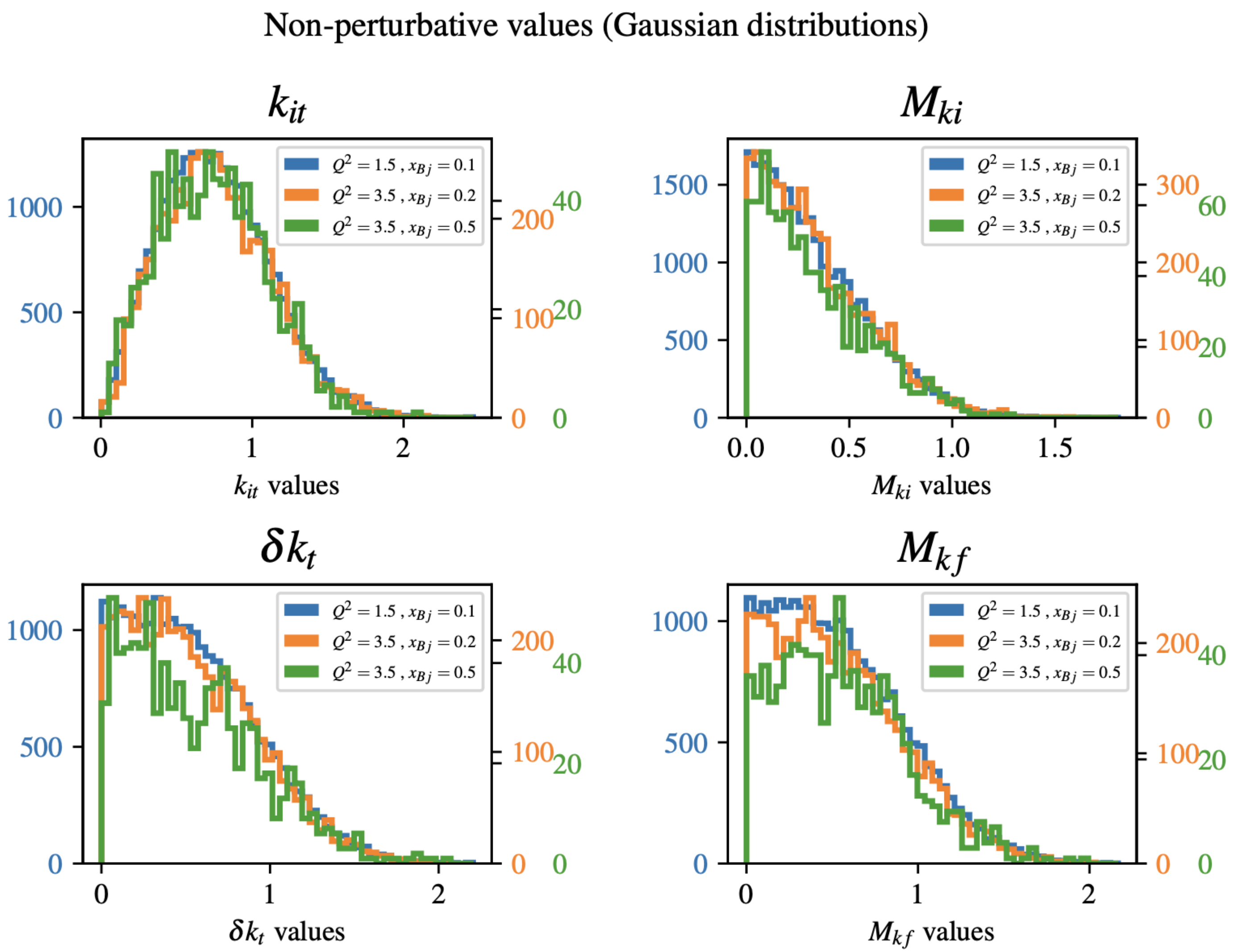
TMD regions

- For this scheme to work, 4 distinct kinematic r… …ave to be identified
- They should be large enough a… …d

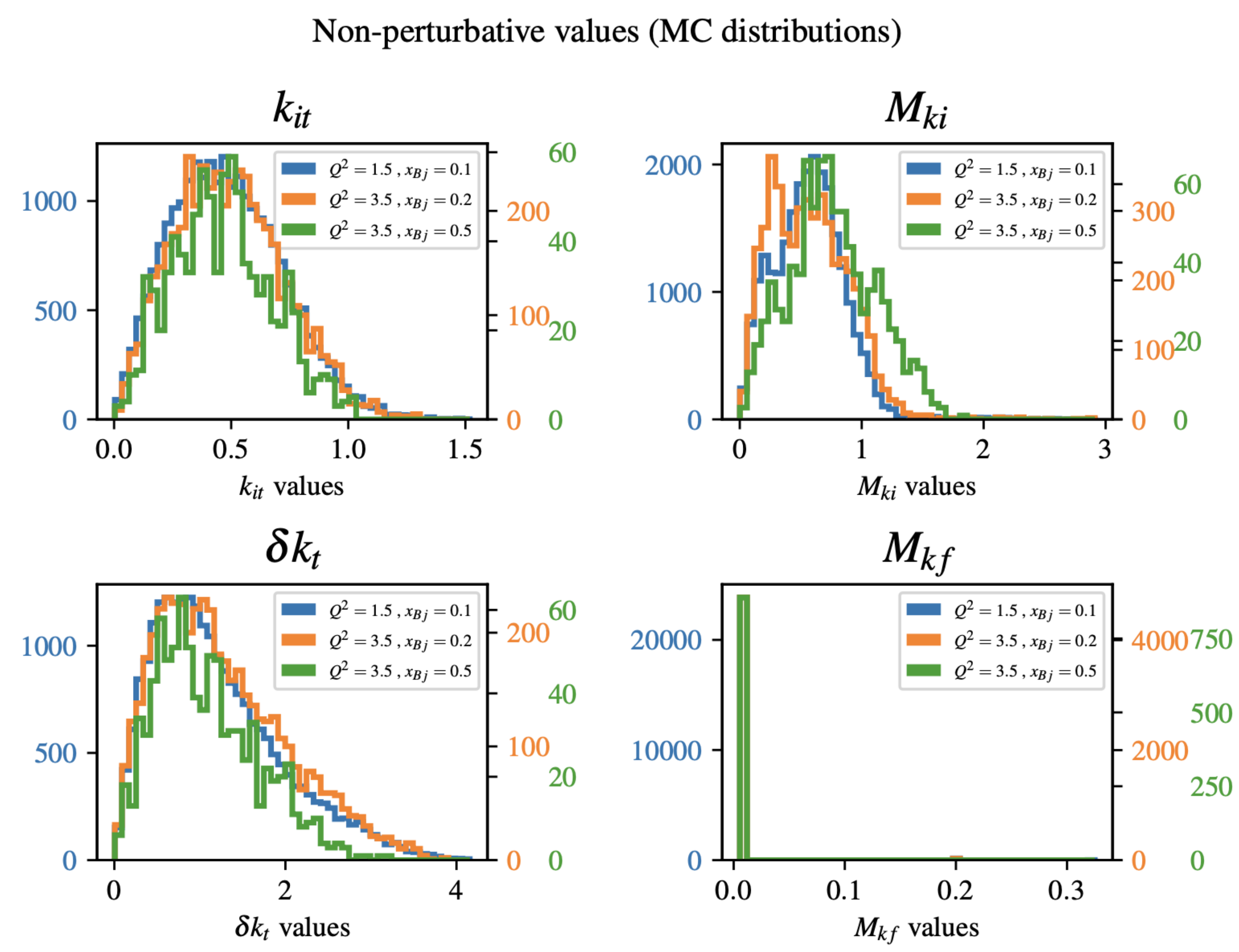
Does not work in SIDIS



Non-perturbative parameters
Gaussian distributions

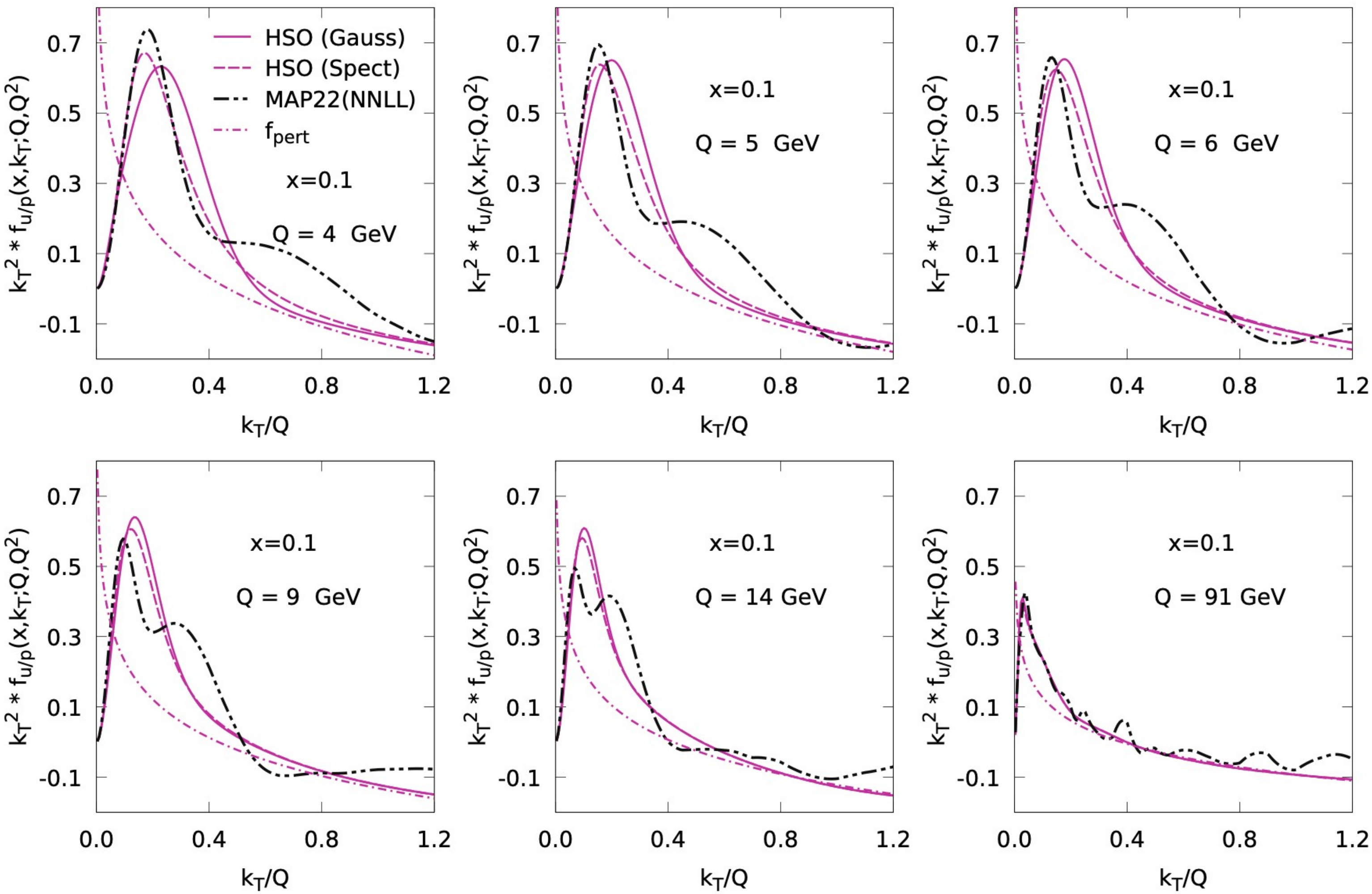


Non perturbative parameters - MC



Hadron Structure Oriented approach (HSO)

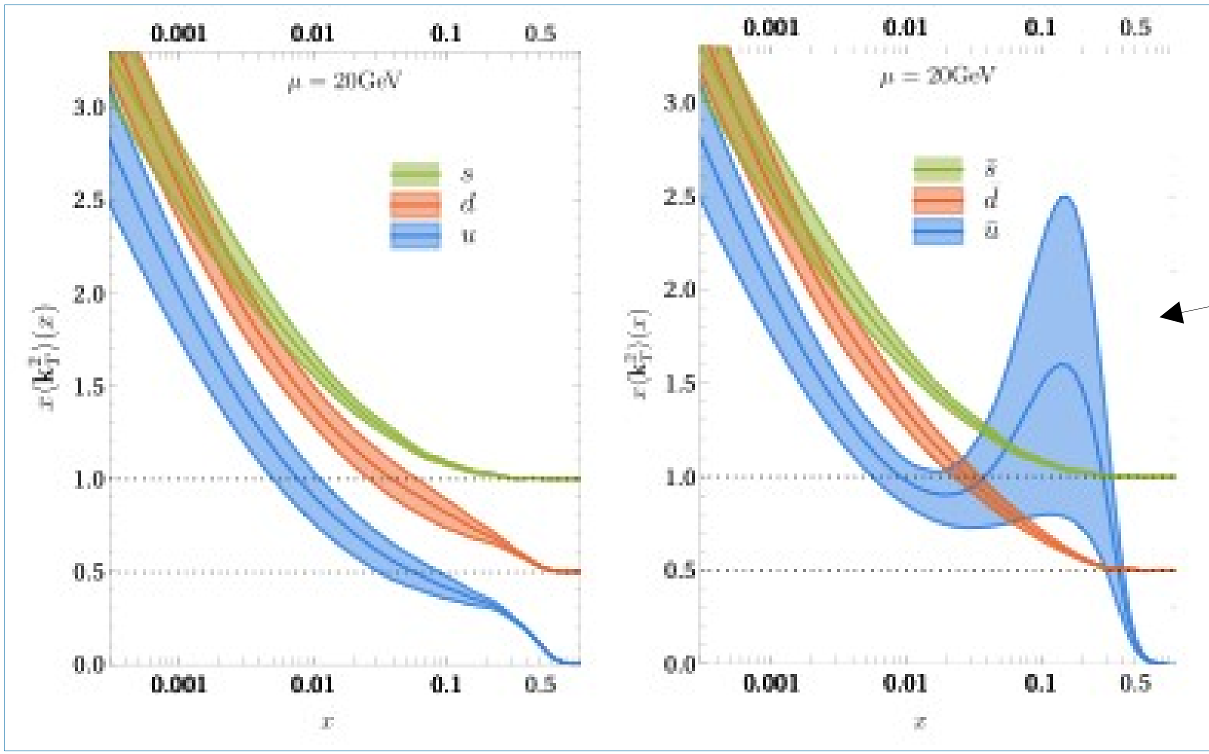
Aslan, Boglione, Gonzalez-Hernandez,Rainaldi, Rogers, Simonelli, Phys.Rev.D 110 (2024) 7, 074016



Numerical Instabilities?

Possible effects of numerical instabilities

Valentin Moos, Ignazio Scimemi, Alexey Vladimirov, Pia Zurita



Unphysical behaviour due to numerical instabilities

Hadron Structure Oriented approach (HSO)

Aslan, Boglione, Gonzalez-Hernandez,Rainaldi, Rogers, Simonelli, Phys.Rev.D 110 (2024) 7, 074016

… and you postdict this