Parton distributions of light nuclei within the Poincaré covariant light-front approach

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Sezione di Perugia

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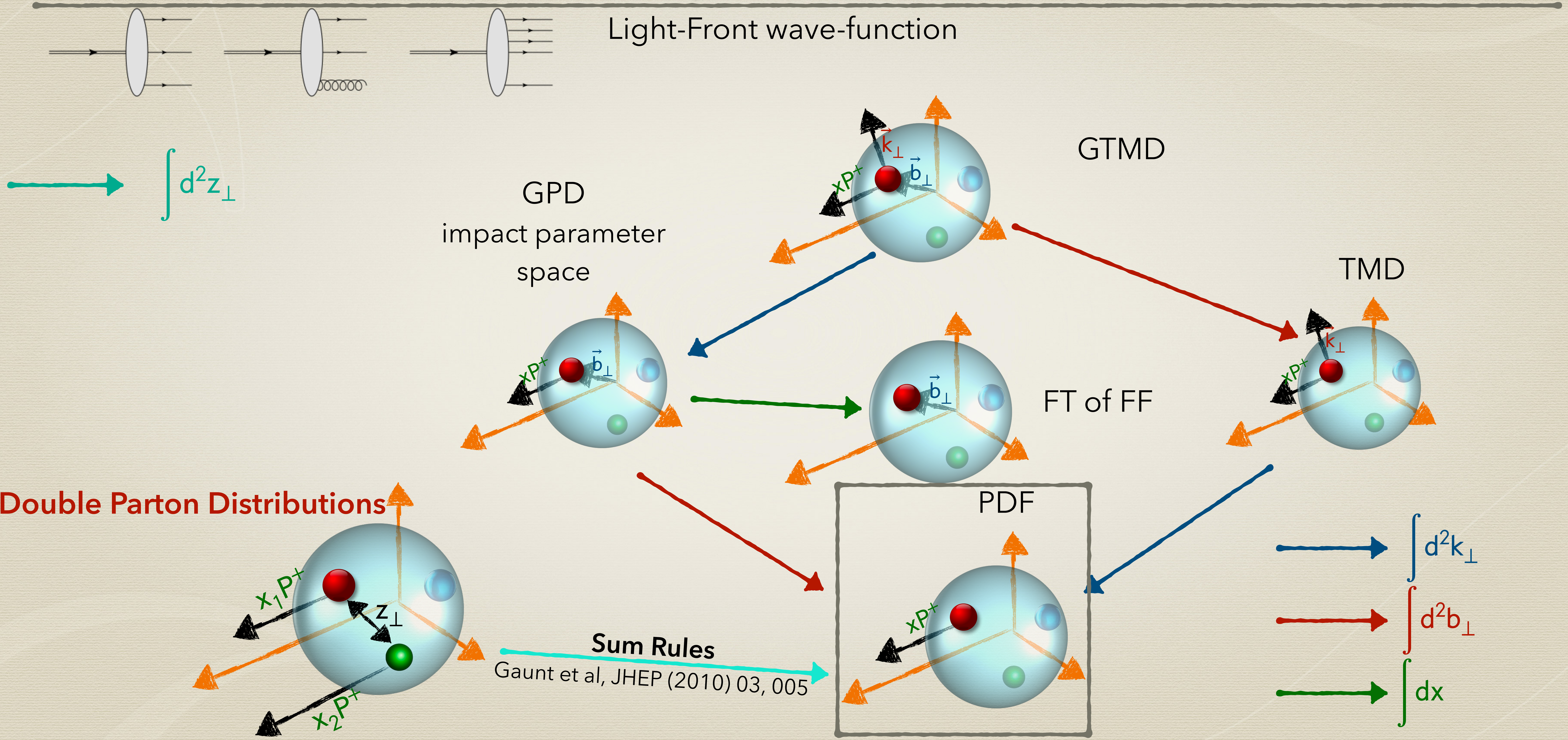
Outline

- The EMC effect
- 3He TMDs (Light Cone Momentum Distributions)
- Polarized ^{3}He Structure functions
- ^{3}He and ^{4}He GPDs
- Coherent J/ψ electro-production on light-nuclei and multiparticle effects
- Double parton scattering on light-nuclei @EIC

Conclusions

Multidimensional picture of hadrons

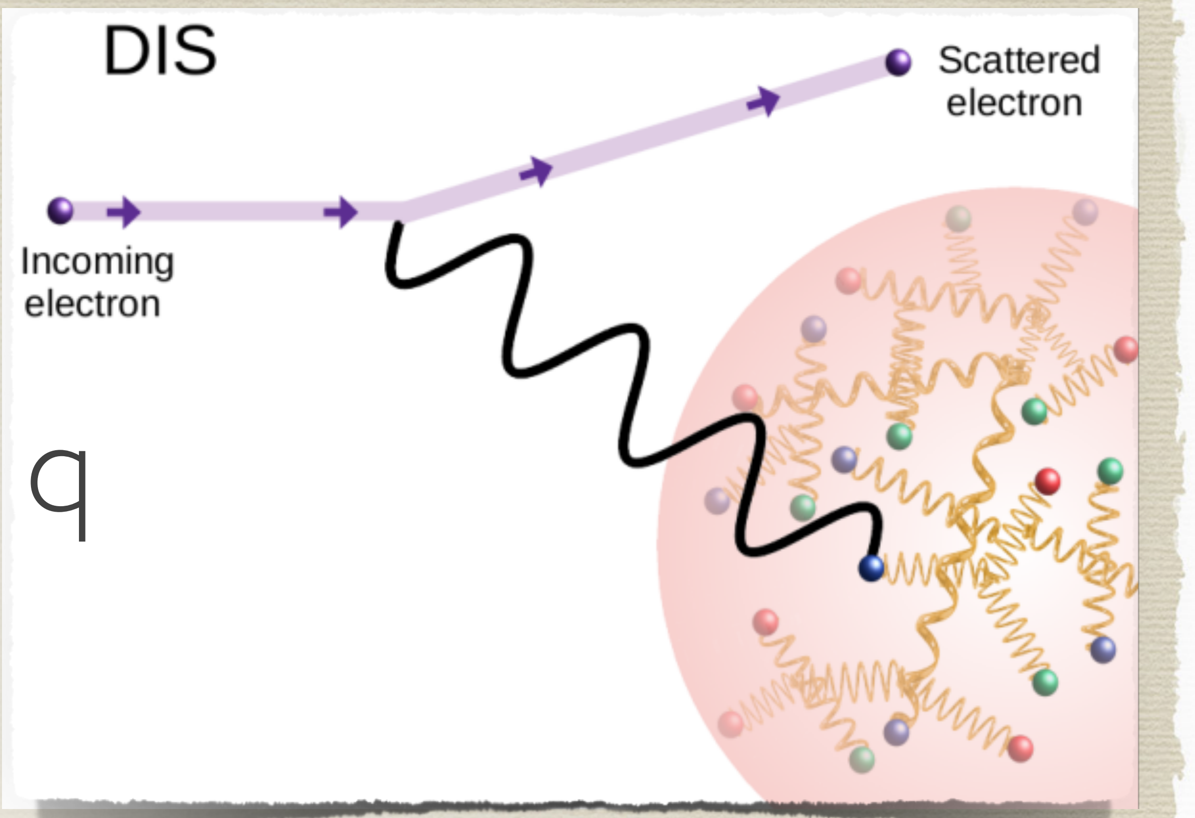


The EMC effect

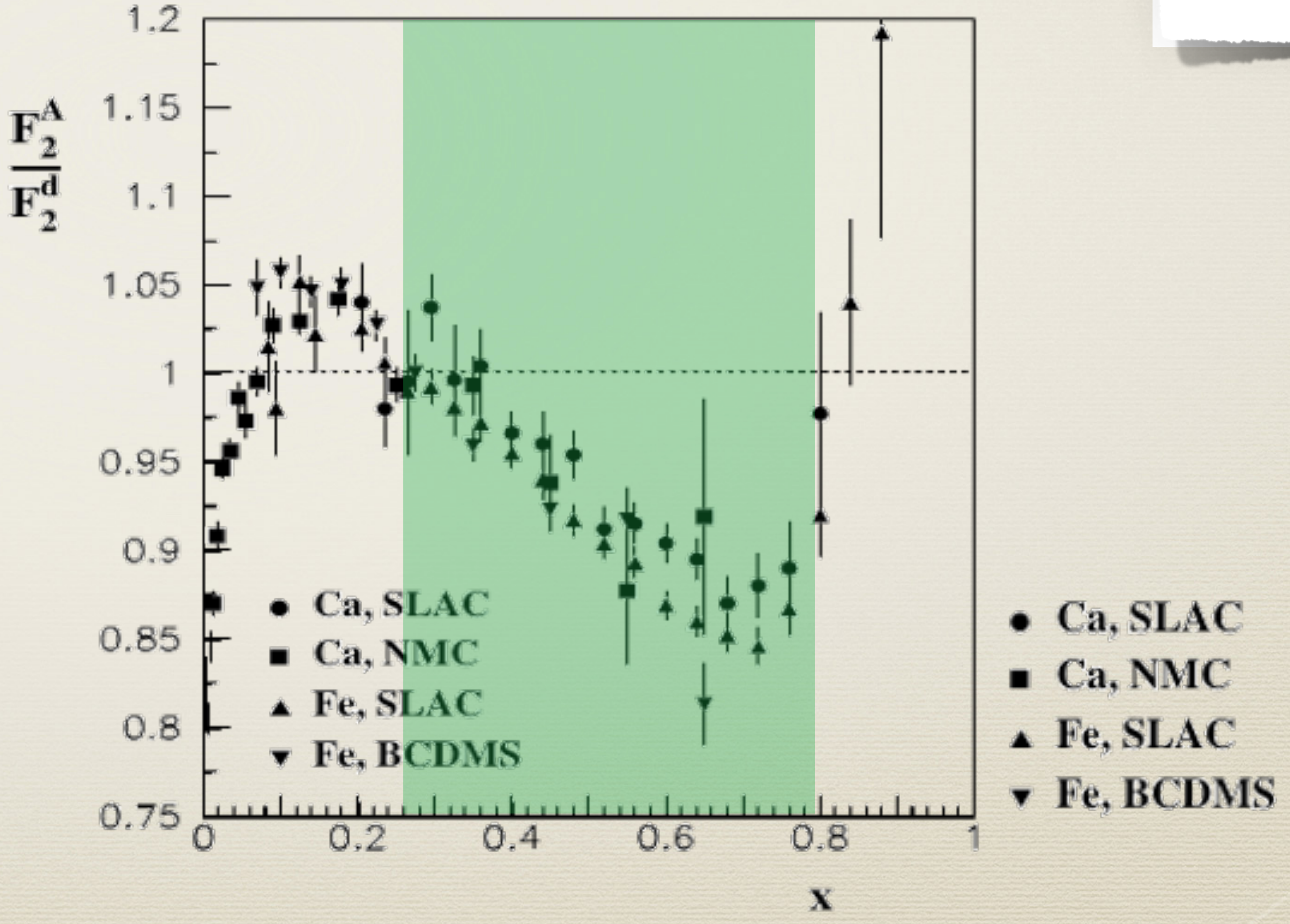
In DIS off a nuclear target with A nucleons:

$$0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$$

$$\frac{d\sigma}{d\Omega dE'} \propto F_2^A(x)$$



0.2 ≤ x ≤ 0.8 "EMC (binding) region":
mainly valence quarks involved



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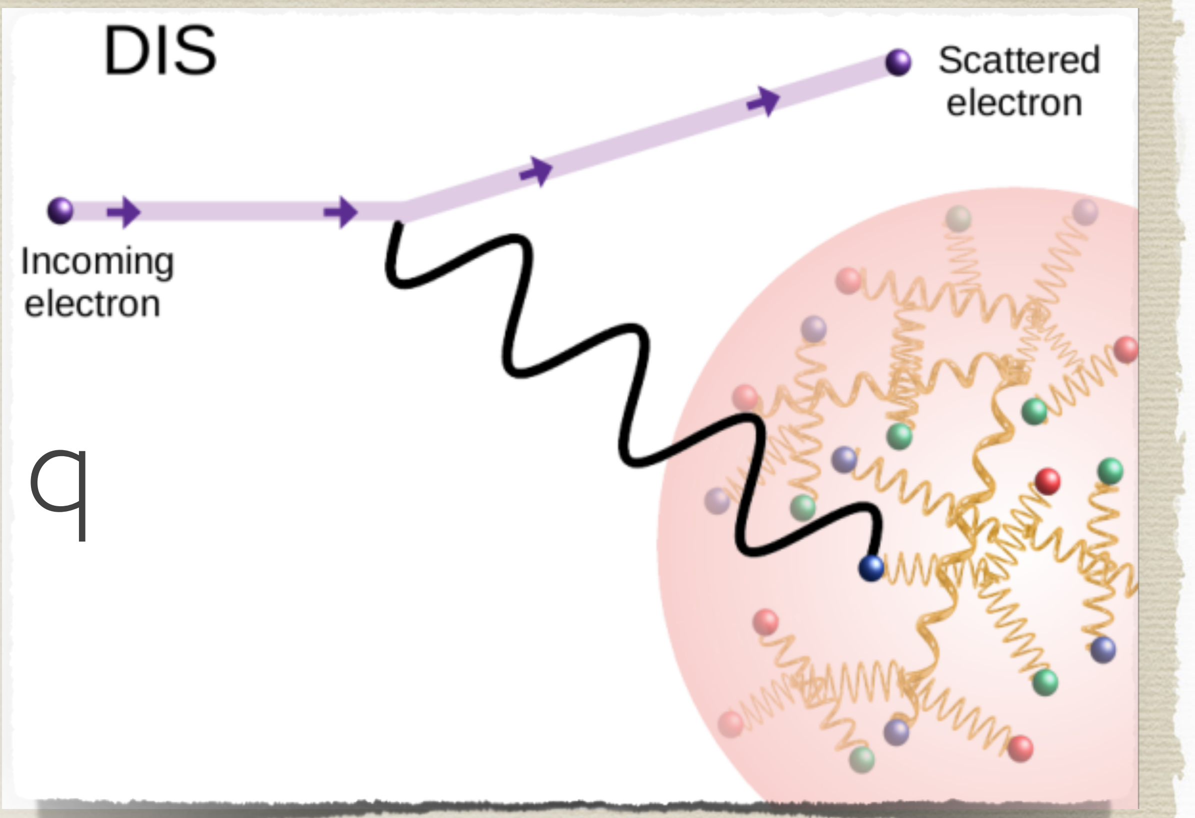

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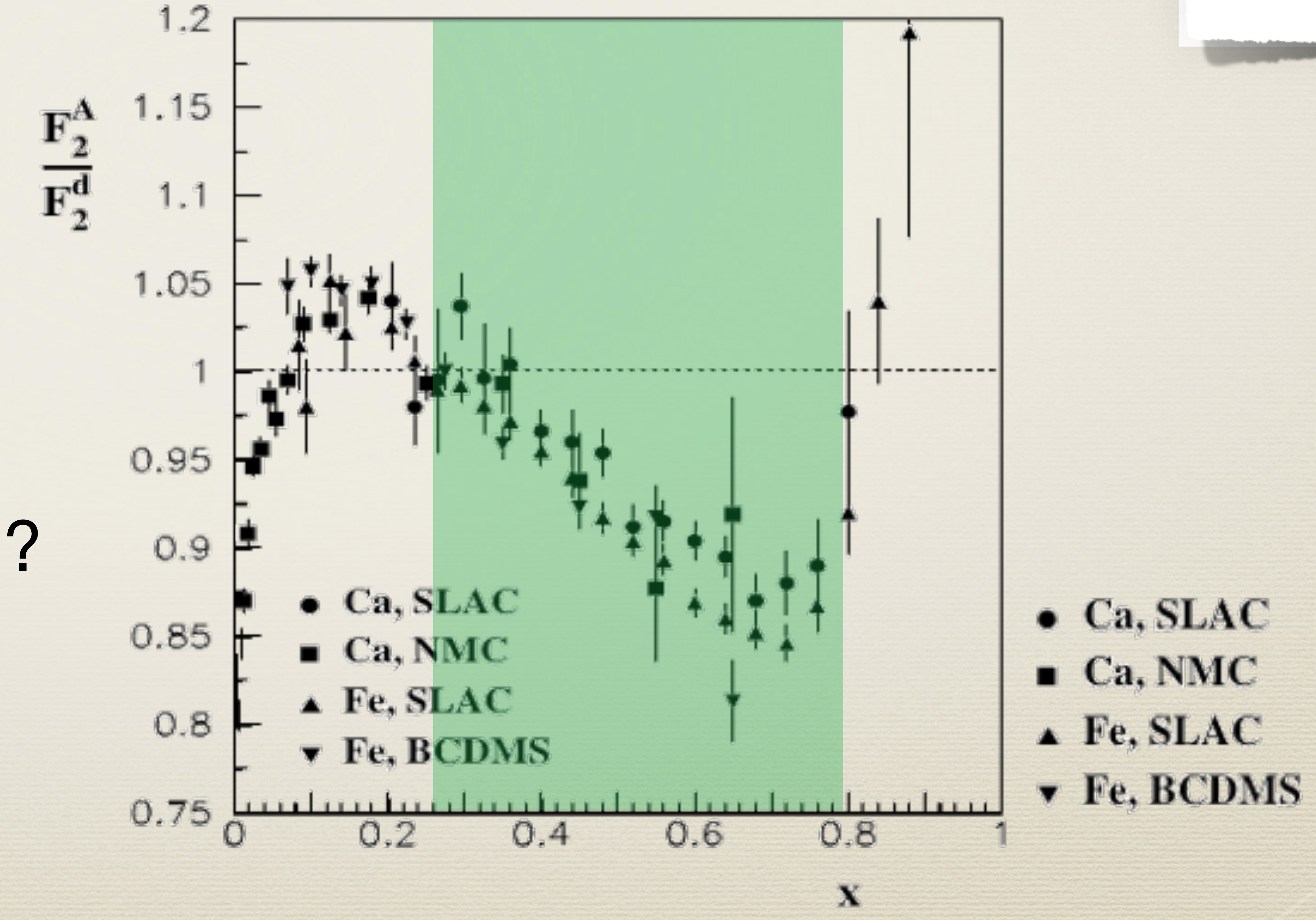
Naive parton model interpretation:

"Valence quarks, in the bound nucleon, are in average slower that in the free nucleon"

Is the bound proton bigger than the free one??



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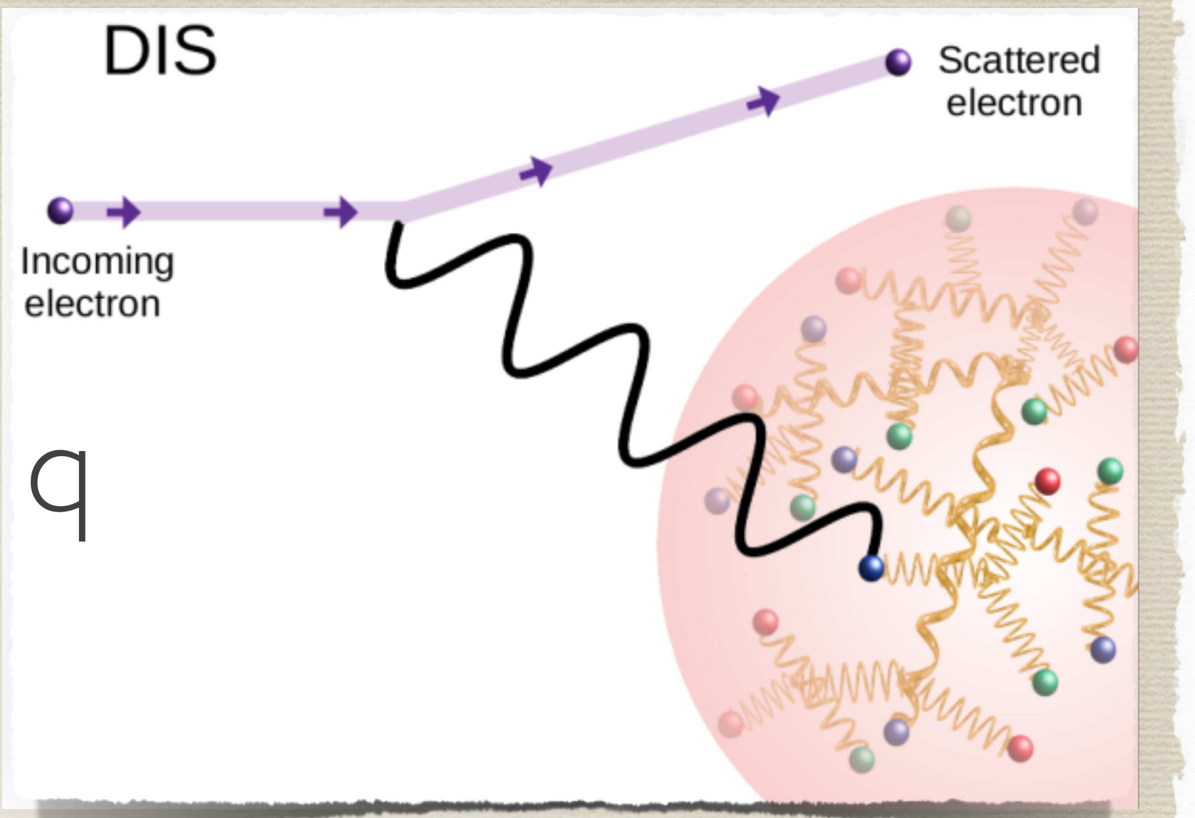

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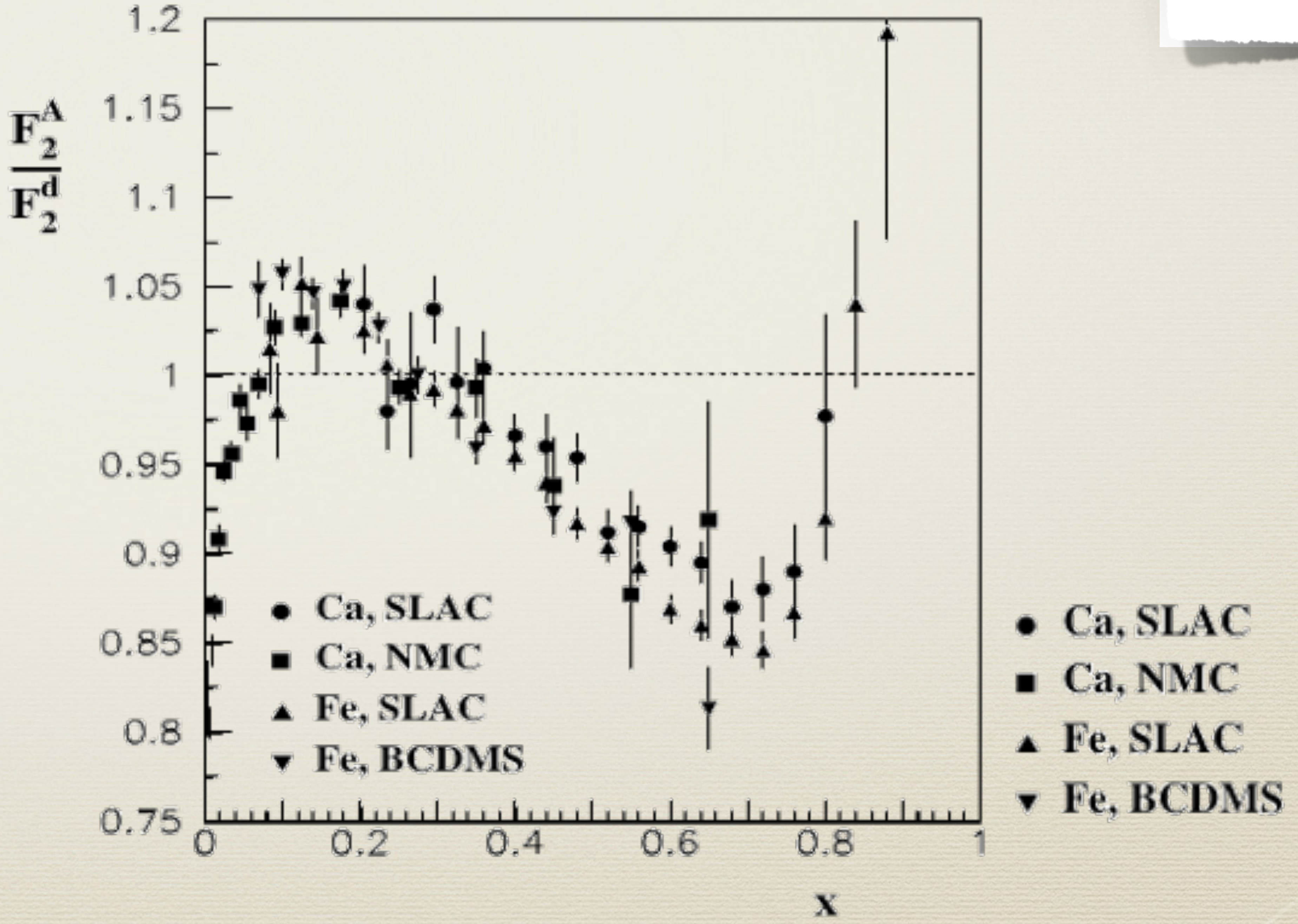
Explanation (exotic) advocated: confinement radius bigger for bound nucleons, quarks in bags with 6, 9,..., 3A
quark, pion cloud effects... Alone or mixed with conventional ones...



The EMC effect

Conventional calculations:

✓ Qualitative agreement

No fulfillment of both particle and momentum sum rules ✗

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No fulfillment of both particle and momentum sum rules ✘

In general, the lack of the **Poincarè covariance** and **macroscopic locality*** generates **biases for the study of genuine QCD effects** (nucleon swelling, exotic quark configurations …)

Macroscopic locality (= **cluster separability** (relevant in nuclear physics)): i.e. observables associated to different space-time regions must commute in the limit of large space like separation (i.e. causally disconnected).

In this way, when a system is separated into disjoint subsystems by a sufficiently large space like separation, then the subsystems behave as independent systems

B.D.Keister and W.N.Polyzou, Adv.Nucl.Phys. 20 (1991), 225-479

LF approach in pills

Poincaré covariance → Find 10 generators:

$P^\mu \rightarrow 4D$ displacements and $M^{\nu\mu} \rightarrow$ Lorentz transformation, that fulfill:

$$[P^\mu, P^\nu] = 0; [M^{\mu\nu}, P^\rho] = -i(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu)$$
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- **7 Kinematical generators** (max n°): i) 3 LF boosts (in instant form they are dynamical!) ; ii) $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$; iii) Rotation around the z-axis
- The LF boosts have a subgroup structure: **trivial separation of intrinsic and global motion, as in the NR case**
- $P^+ \geq 0 \rightarrow$ meaningful Fock expansion, once massless constituents are absent
- **The infinite-monentum frame (IMF) description of DIS is easily included**

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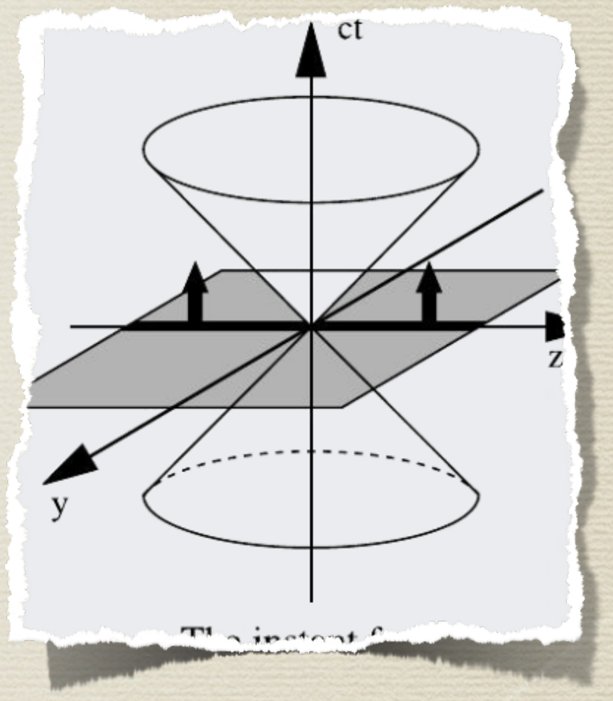

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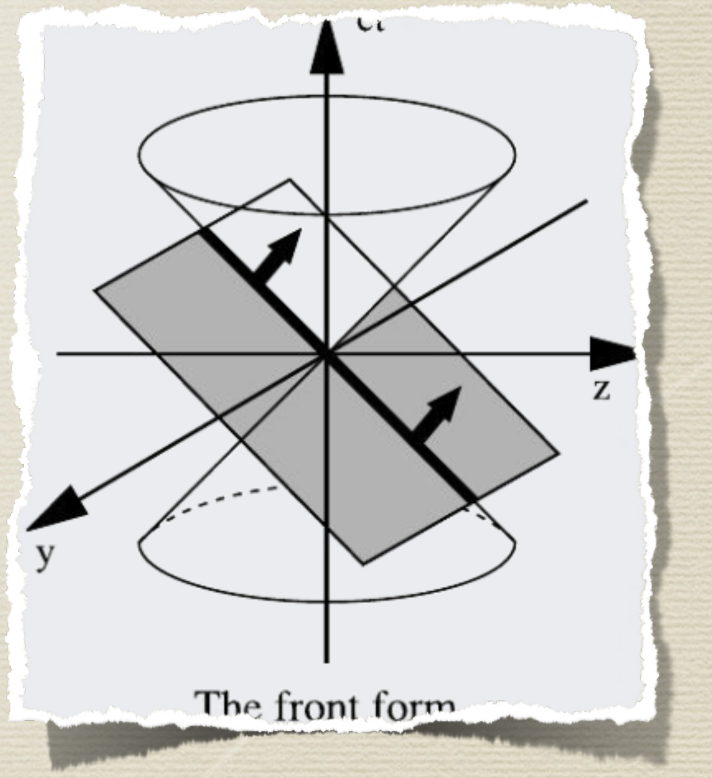

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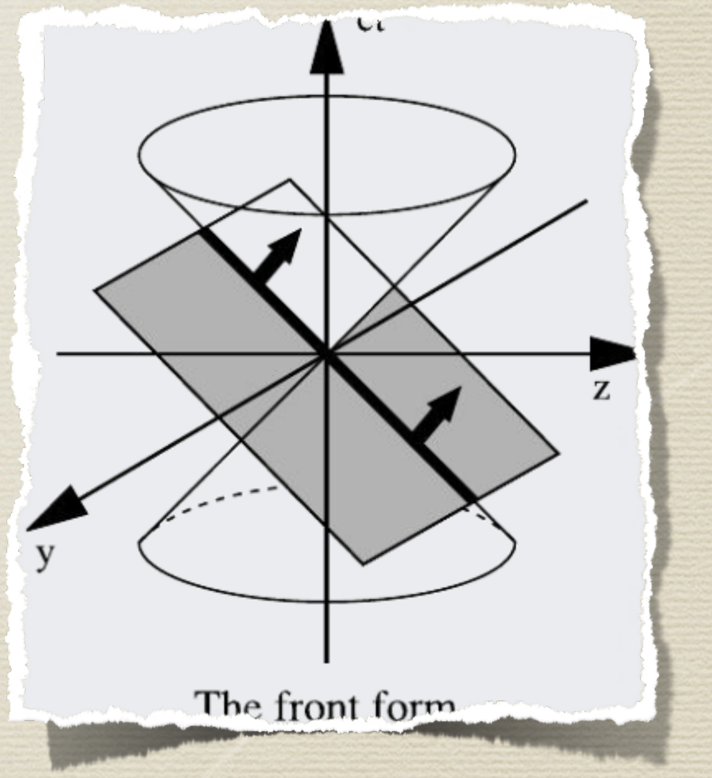

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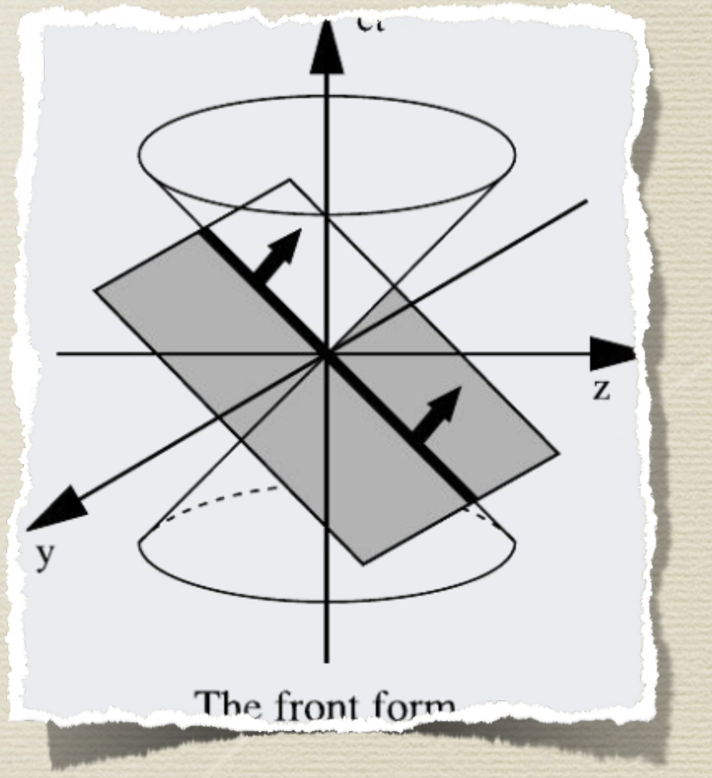

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LF + Bakamjian-Thomas construction

BT properly constructed the 10 Poincaré operators in presence of interactions following this scheme:

i) Only the mass operator M contains the interaction

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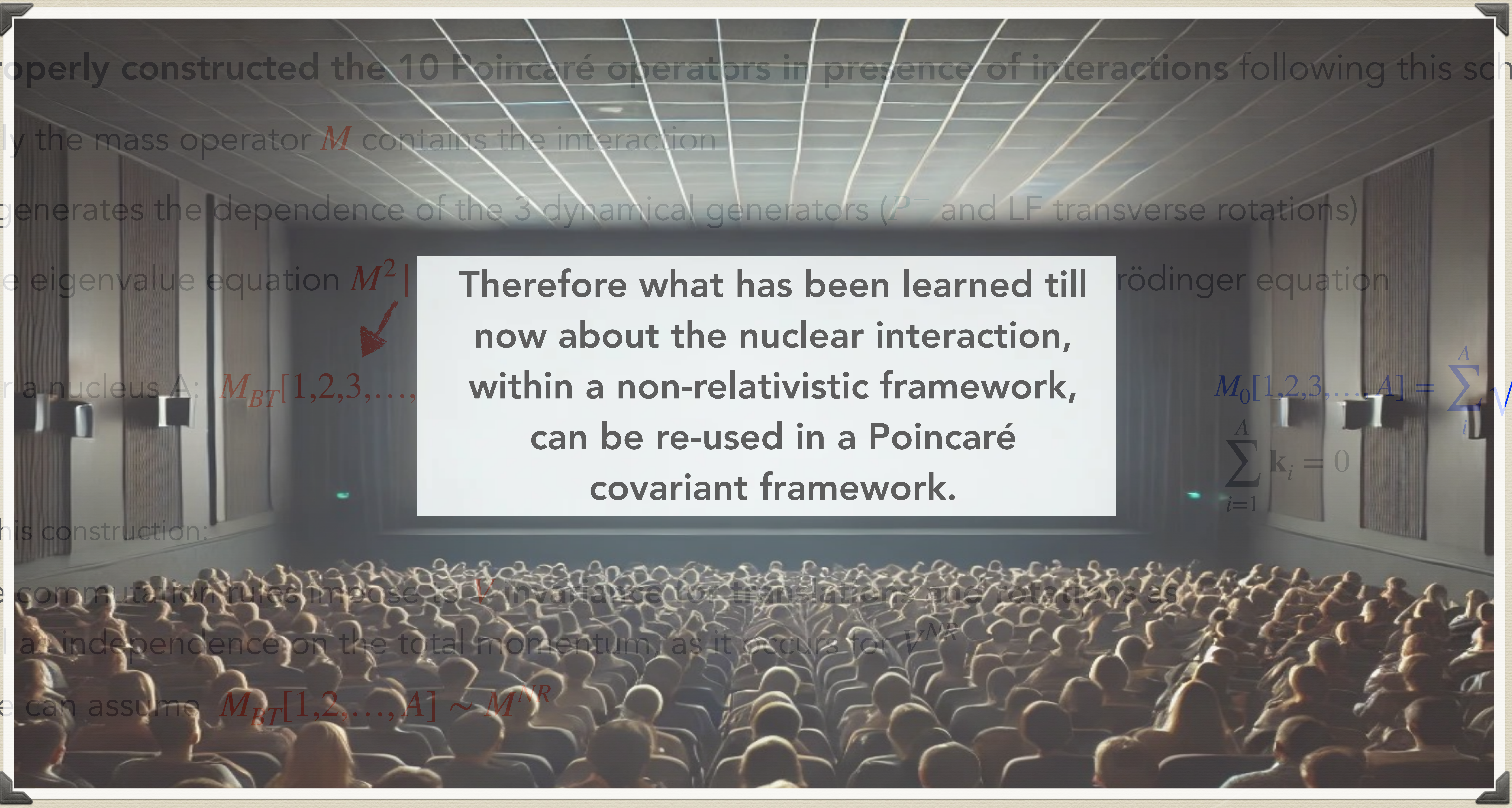

Our approach: LF spectral function I

Since we use an **impulse approximation** assumption, we rely on the **spin-dependent LF spectral function** $P^{\tau}_{\sigma'\sigma}(\tilde{\kappa}, \epsilon, \mathbf{S}, M)$

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The LF spectral function contains the determinant of the Jacobian of the transformation between the intrinsic frames $[1;2,3,\ldots,A-1]$and $[1,2,\ldots,A]$, connected each other by a LF boost

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How do we deal with **LF states**?

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2) Then we can approximate the IF overlap into a NR overlap by using the **NR wave function for the nucleus**, thanks to the BT construction:

$$< tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa | \Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{IF} \sim < tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa | \Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{NR}$$

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$$< tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa | \Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{IF} \; \sim \; < tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa | \Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{NR}$$

Poincaré covariance preserved but using the successful NR phenomenology

Our approach: LF spectral function II

$$P^{N}_{\sigma'\sigma}(\tilde{\kappa},\epsilon,\mathbf{S},M) = \sum_{JJ_z}\sum_{TT_z}\rho(\epsilon)_{LF} < tT;\alpha,\epsilon;JJ_z;\tau\sigma',\tilde{\kappa}\,|\,\Psi_{JM};\mathbf{S},T_AT_{Az} > < \Psi_{JM};\mathbf{S},T_AT_{Az}\,|_{LF}tT;\alpha,\epsilon;JJ_z;\tau\sigma,\tilde{\kappa} >_{LF}$$

How do we deal wit

1) We can express the

$< tT;\alpha,\epsilon;JJ_z;\tau\sigma',\tilde{\kappa}\,|\,\Psi_{JM};$

2) Then we can appro ... the nucleus, thanks to the BT cons

$< tT;\alpha,\epsilon;JJ_z;\tau\sigma'_c,\kappa\,|\,\Psi_{JM}$

Poincarè covariance pre

We used wave functions of $^2H,^3H,^3He,^4He$ calculated through 3 different potentials: **Av18+UIX*** and 2 versions of the **Norfolk χEFT interactions NVIa+3N**** and **NVIb+3N****

*R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, **Phys. Rev. C 51 (1995) 38–51;** R. B. Wiringa et al., **Phys. Rev. Lett. 74 (1995) 4396–4399**

M.Viviani et al., **Phys. Rev. C 107 (1) (2023) 014314; M. Piarulli et al.,**Phys. Rev. Lett. 120 (5) (2018) 052503;** M. Piarulli, S. Pastore, R. B. Wiringa, S. Brusilow, R. Lim,**Phys. Rev. C 107 (1) (2023) 014314**

Nuclear SFs and EMC ratio

To calculate the EMC ratio $R^A_{EMC}(x) = \dfrac{F_2^A(x)}{F_2^d(x)}$ for any nucleus A, we need the nuclear SFs.

Within our approach we have:

$$F_2^A(x) = \sum_N \int_{\xi_{min}}^1 d\xi \; F_2^N\left(\frac{mx}{\xi M_A}\right) \; f_A^N(\xi)$$

*ξ = longitudinal momentum fraction carried by a nucleon in the nucleus

Since our approach **fulfill both macro-locality and Poincaré covariance** the LC momentum distribution satisfies 2 essential sum rules at the same time ():

$A = \int_0^1 d\xi[Zf_1^p(\xi) + (A-Z)f^n(\xi)]$: **Baryon number SR**;

$1 = Z < \xi >_p + (Z-N) < \xi >_n \, ; < \xi >_N = \int_0^1 d\xi \, \xi \, f_1^N(\xi)$: **Momentum SR (MSR)**

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1) in the Bjorken limit we have the LCMD: $f_1^N(\xi) = \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{1}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) \frac{E_s}{1-\xi}$

Unpolarized LF spectral function:
$$P^N(\tilde{\kappa}, \epsilon) = \frac{1}{2j+1} \sum_{\mathcal{M}} P^N_{\sigma\sigma}(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M})$$

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Nuclear SFs and EMC ratio

To calculate the EMC ratio $R^A_{EMC}(x) = \frac{F^A_2(x)}{F^d_2(x)}$ for any nucleus A, we need the nuclear SFs.

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$$F^A_2(x) = \sum_N \int_{\xi_{min}}^1 d\xi \; F^N_2\left(\frac{mx}{\xi M_A}\right) \; f^N_A(\xi)$$

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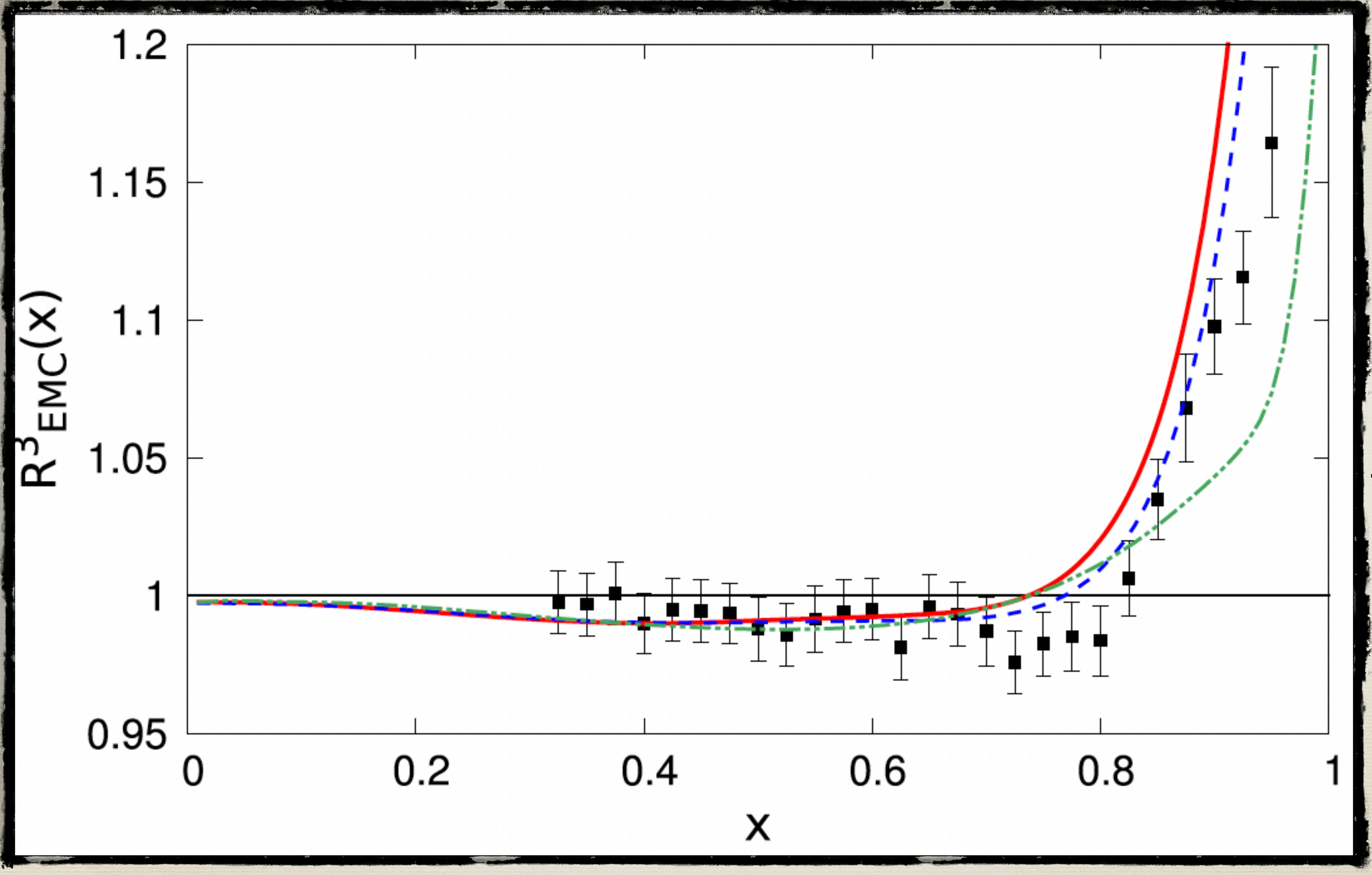
The EMC effect for ^{3}He
E.Pace, M.R. G.Salmè and S.Scopetta, Phys. Lett. B 839(2023) 127810

[1] J. Arrington, et al, **Phys. Rev. C 104 (6) (2021) 065203**

[2] S. A. Kulagin and R. Petti, **Phys. Rev. C 82, 054614 (2010)**



Solid line: Av18/UIX + SMC*
Dashed line:Av18 + SMC*
Dotted-dashed: Av18/UIX +CJ15**

Full squares: JLab data from experiment E03103 [1] **as reanalyzed in** [2]

[B. Adeva, et al., Phys. Lett. B 412 (1997) 414–424.]

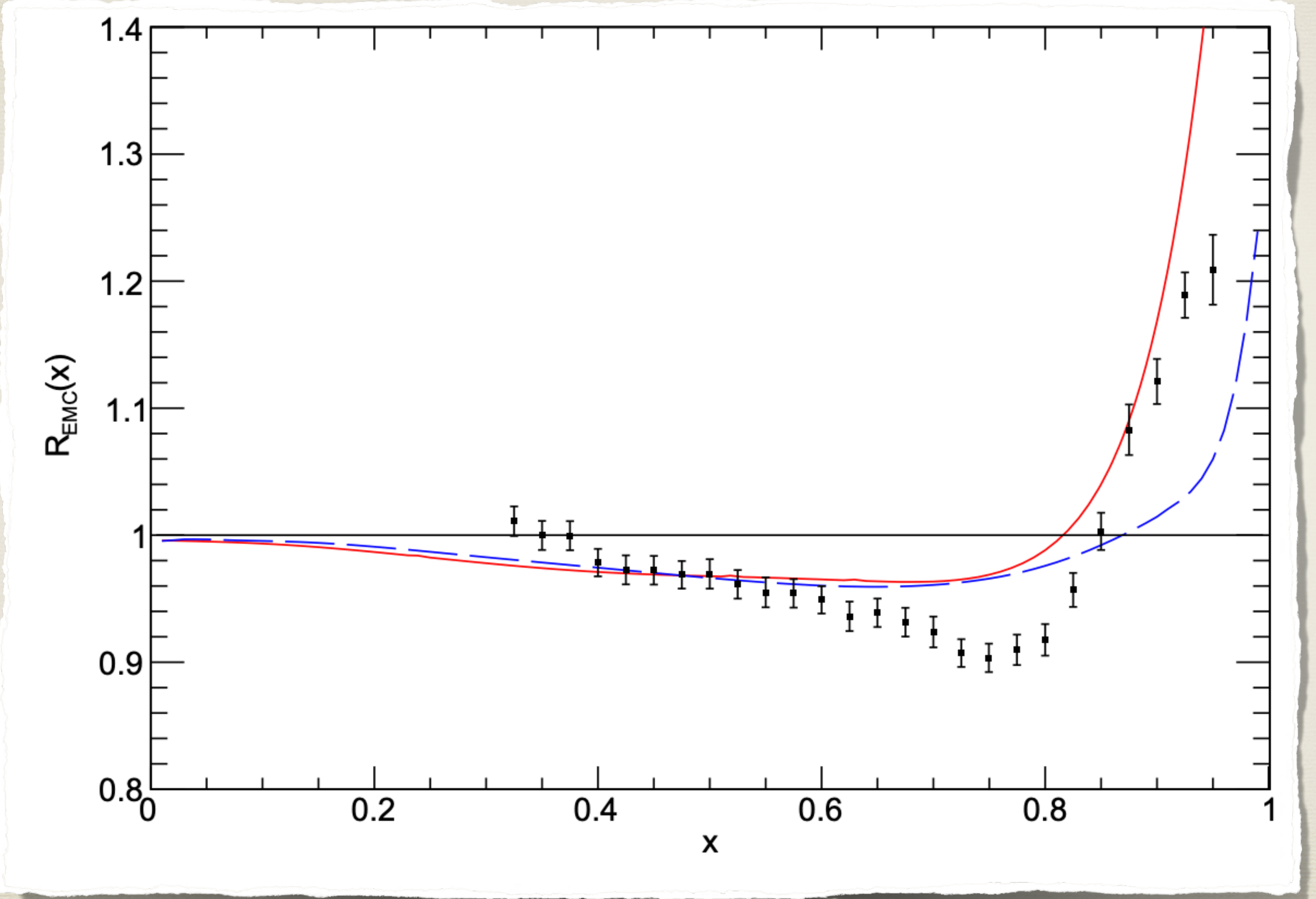
****[A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens, N. Sato, Phys. Rev. D 93 (11) (2016) 114017]**

Small but **solid** effect, comparable to the experimental data

The EMC effect for ^{4}He

F.Fornetti, E.Pace, M.R., G.Salmè, S.Scopetta and M.Viviani, **Phys.Lett.B** 851 (2024) 138587

Full squares: JLab data from experiment E03103



Both lines calculated with Av18/UIX

Solid line: SMC parametrization of F_2^p *

Dashed line: CJ15 +TMC Parametrization of F_2^p**

F_2^n extracted from MARATHON data

****[B. Adeva, et al., Phys. Lett. B 412 (1997) 414–424.]***

****[A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens, N. Sato, Phys. Rev. D 93 (11) (2016) 114017]**

The dependence on the choice of the **free nucleon SFs** is largely under control in the **properly EMC region**

The EMC effect for ^{4}He

F.Fornetti, E.Pace, M.R., G.Salmè, S.Scopetta and M.Viviani, **Phys.Lett.B** 851 (2024) 138587

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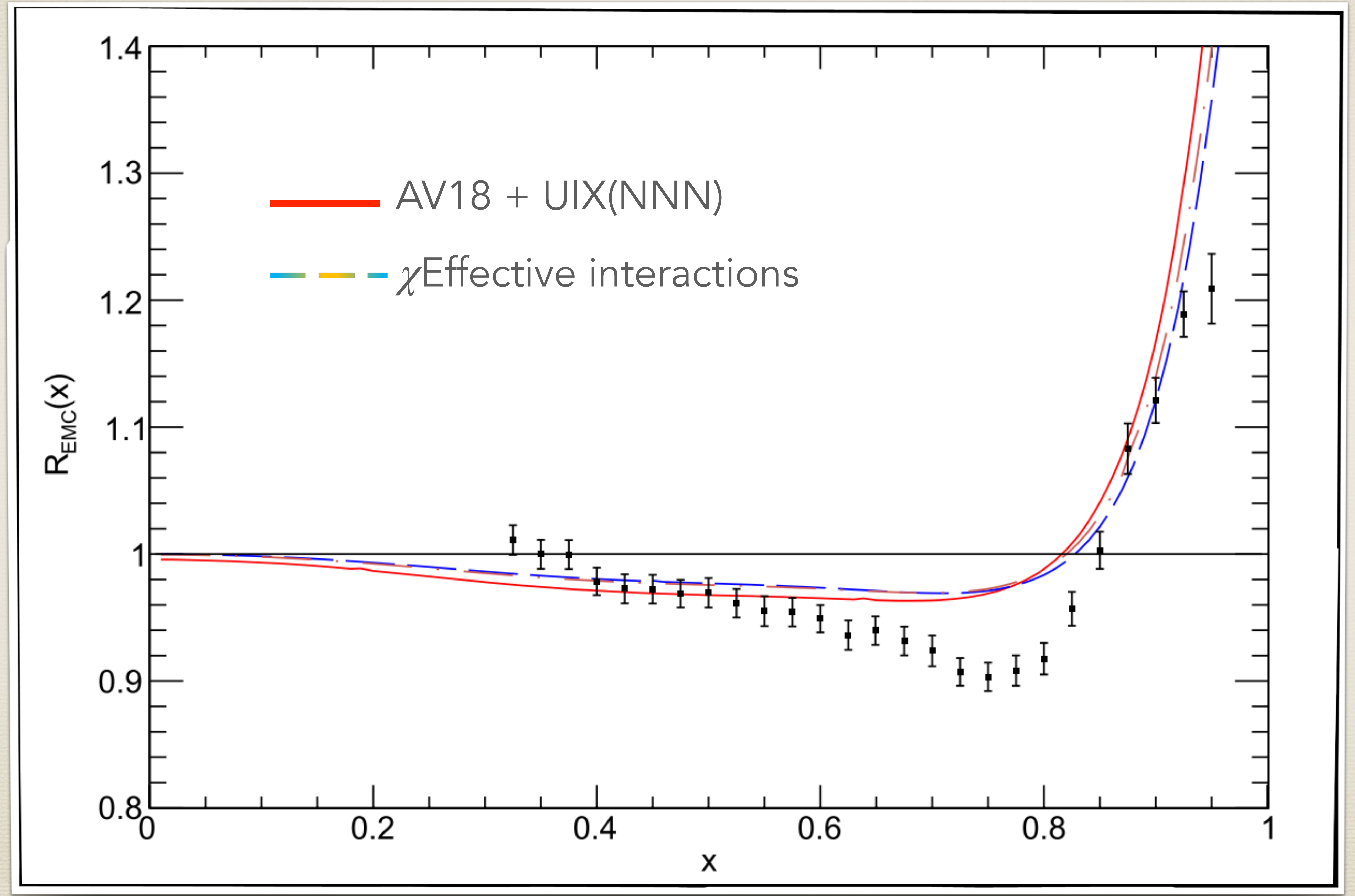

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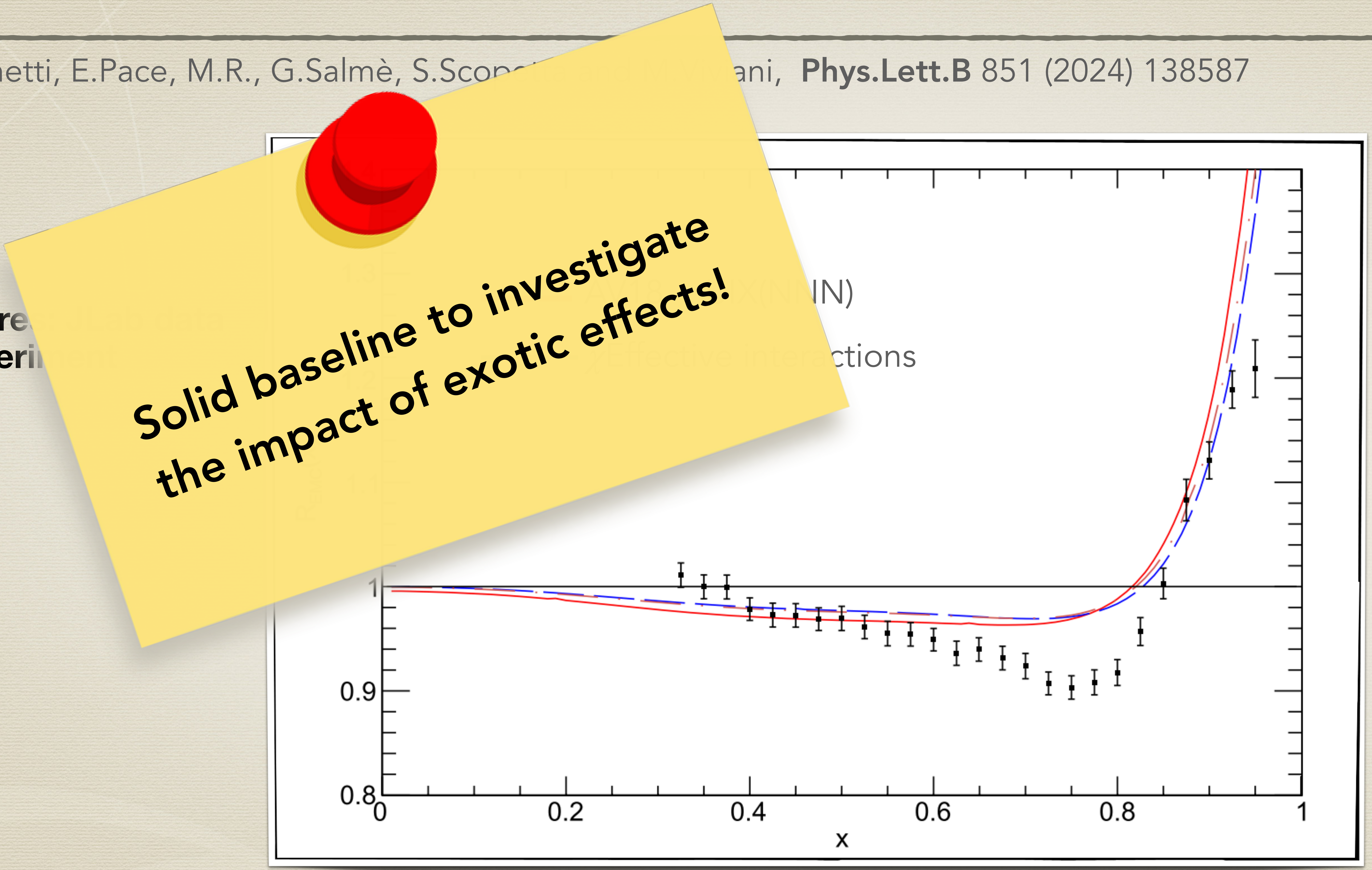
The EMC effect for ^{4}He

F.Fornetti, E.Pace, M.R., G.Salmè, S.Scopa ... ani, **Phys.Lett.B** 851 (2024) 138587

Full square ... from experi... E03103



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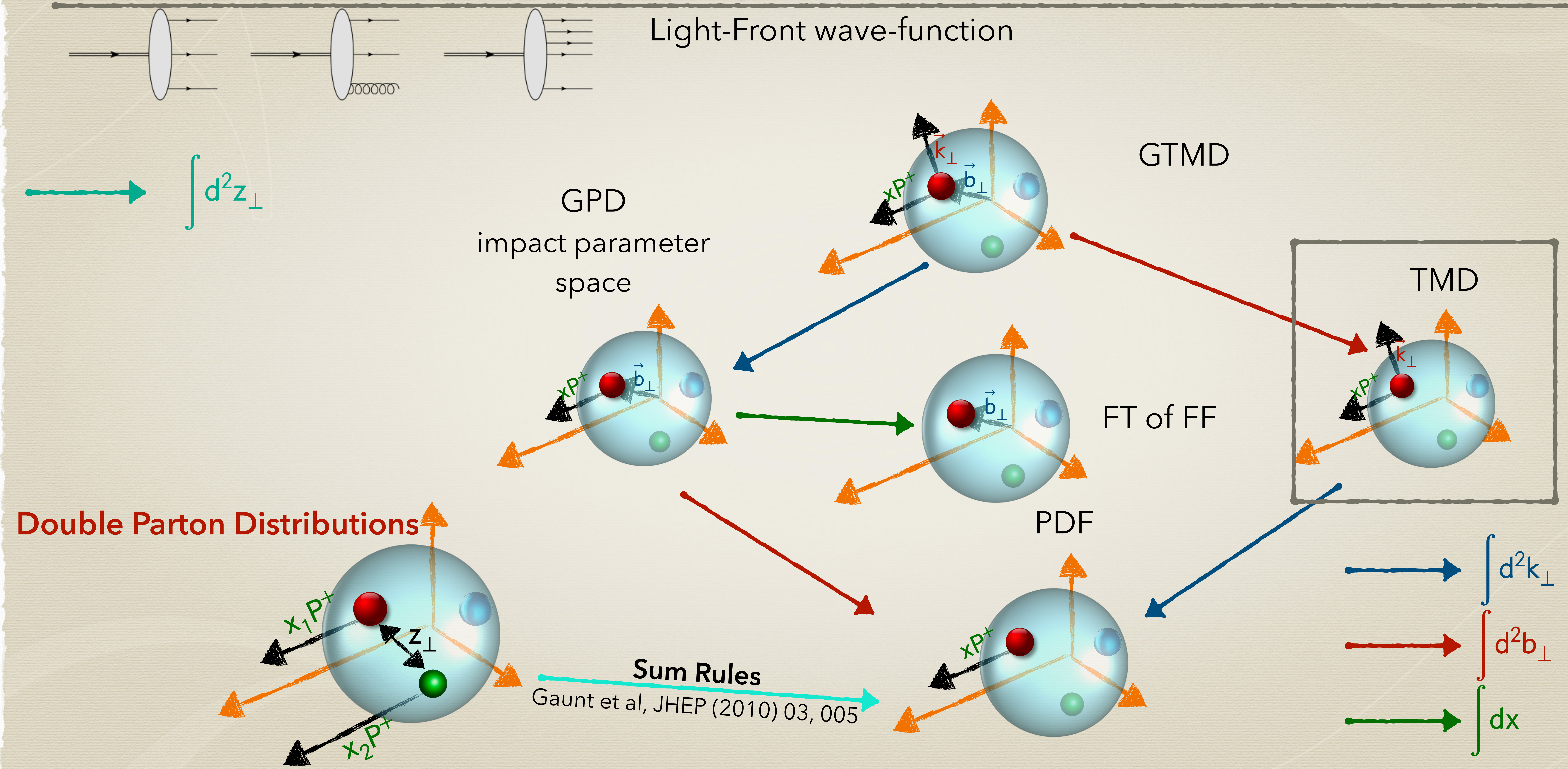
**[B. Adeva, et al., Phys. Lett. B 412 (1997) 414–424.]*

Multidimensional picture of hadrons



^{3}He TMD (LCMDs)

Alessandro,Del Dotto, Pace, Perna, Salm`e, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

Let p be the momentum in the laboratory of an off-mass-shell fermion, with isospin τ, in a bound system of A fermions with total momentum P and spin S. The fermion correlator in terms of the LF coordinates is:
[e.g., **Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002)**]

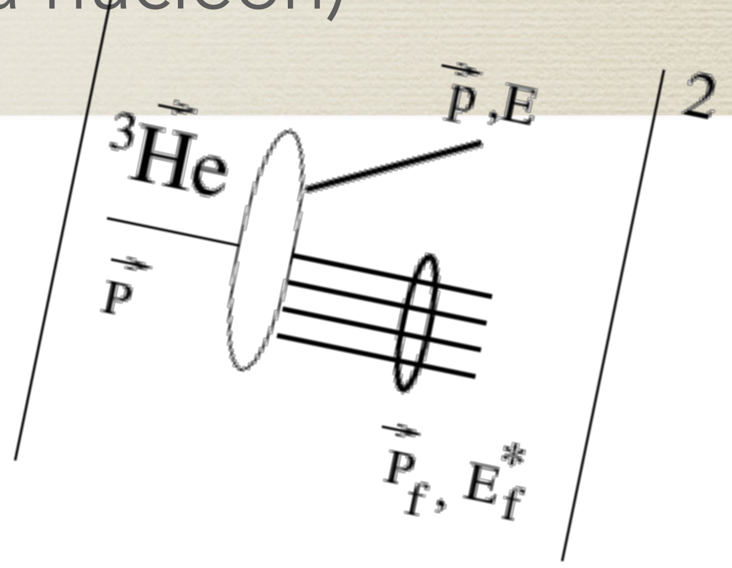
$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \frac{1}{2}\int d\xi^- d\xi^+ d\boldsymbol{\xi}_T \; e^{i\,p\xi} \langle P,S,A|\bar{\psi}^{\tau}_{\beta}(0)\psi^{\tau}_{\alpha}(\xi)|A,S,P\rangle$$

$|A, S, P\rangle =$ the A-particle state

$\psi^{\tau}_{\alpha}(\xi) =$ the particle field (e.g. a nucleon of isospin τ in a nucleus, or in valence approximation a quark in a nucleon)

$$\Phi^{\tau p}(p,P,S) = \frac{(\not p_{on} + m)}{2m}\,\Phi^{\tau}(p,P,S)\,\frac{(\not p_{on} + m)}{2m} = \ldots$$

$$= \frac{2\pi\,(P^+)^2}{(p^+)^2\,4m}\,\frac{E_S}{\mathcal{M}_0[1,(23)]}\sum_{\sigma\sigma'}\{\,u_\alpha(\tilde{\mathbf{p}},\sigma')\,\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\,\bar{u}_\beta(\tilde{\mathbf{p}},\sigma)\,\}$$



^{3}He TMD (LCMDs)

Alessandro, Del Dotto, Pace, Perna, Salm`e, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

The correlation function at the leading twist is given by

$$\Phi(p,P,S) = \frac{1}{2} \not{P} A_1 + \frac{1}{2} \gamma_5 \not{P} \left[A_2 S_z + \frac{1}{M} \widetilde{A}_1 \mathbf{p}_\perp \cdot \mathbf{S}_\perp \right] + \frac{1}{2} \not{P} \gamma_5 \left[A_3 \not{S}_\perp + \widetilde{A}_2 \frac{S_z}{M} \not{p}_\perp + \frac{1}{M^2} \widetilde{A}_3 \mathbf{p}_\perp \cdot \mathbf{S}_\perp \not{p}_\perp \right]$$

The functions A_j, $\tilde{A}_j$ (j = 1,2,3) can be obtained by proper traces of Φ(p,P,S) and Γ matrices. Integrals of A_j, $\tilde{A}_j$ on p+ and p−:
:

$$\mathcal{O}\left[A_j\right] = \frac{1}{2} \int \frac{dp^+ dp^-}{(2\pi)^4} \, \delta[p^+ - xP^+] \, P^+ \left[A_j\right]$$

give the six time reversal even transverse momentum distributions (TMDs)

$$f(x, \mathbf{p}_\perp^2) = \mathcal{O}\left[A_1\right] \qquad \Delta f(x, |\mathbf{p}_\perp|^2) = \mathcal{O}\left[A_2\right] \qquad g_{1T}(x, |\mathbf{p}_\perp|^2) = \mathcal{O}\left[\widetilde{A}_1\right]$$

$$\Delta'_T f(x, |\mathbf{p}_\perp|^2) = \mathcal{O}\left[A_3 + \frac{|\mathbf{p}_\perp|^2}{2M^2} \widetilde{A}_3\right] \qquad h_{1L}^\perp(x, |\mathbf{p}_\perp|^2) = \mathcal{O}\left[\widetilde{A}_2\right] \qquad h_{1T}^\perp(x, |\mathbf{p}_\perp|^2) = \mathcal{O}\left[\widetilde{A}_3\right]$$

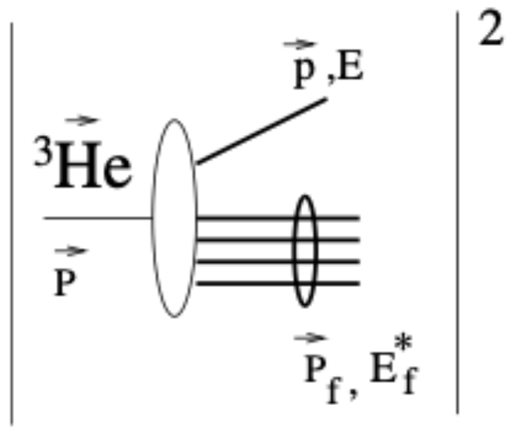
^{3}He TMD (LCMDs)

Alessandro,Del Dotto, Pace, Perna, Salm`e, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

It works for any three-body $J = 1/2$ system in valence approx!

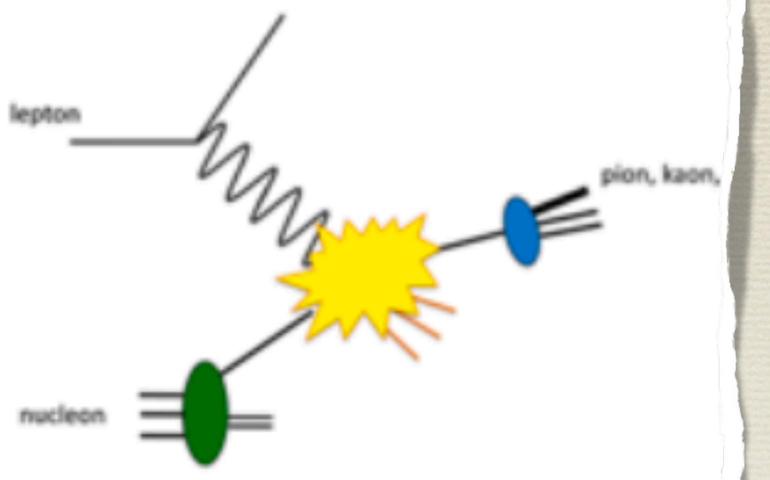
Correspondence:

^{3}He



- p, p, n
- $(e, e'p)$ reactions
- p detection
- PW Impulse Approximation
- spin-dep response functions
- light-cone momentum distributions
- norms, effective polarizations

proton



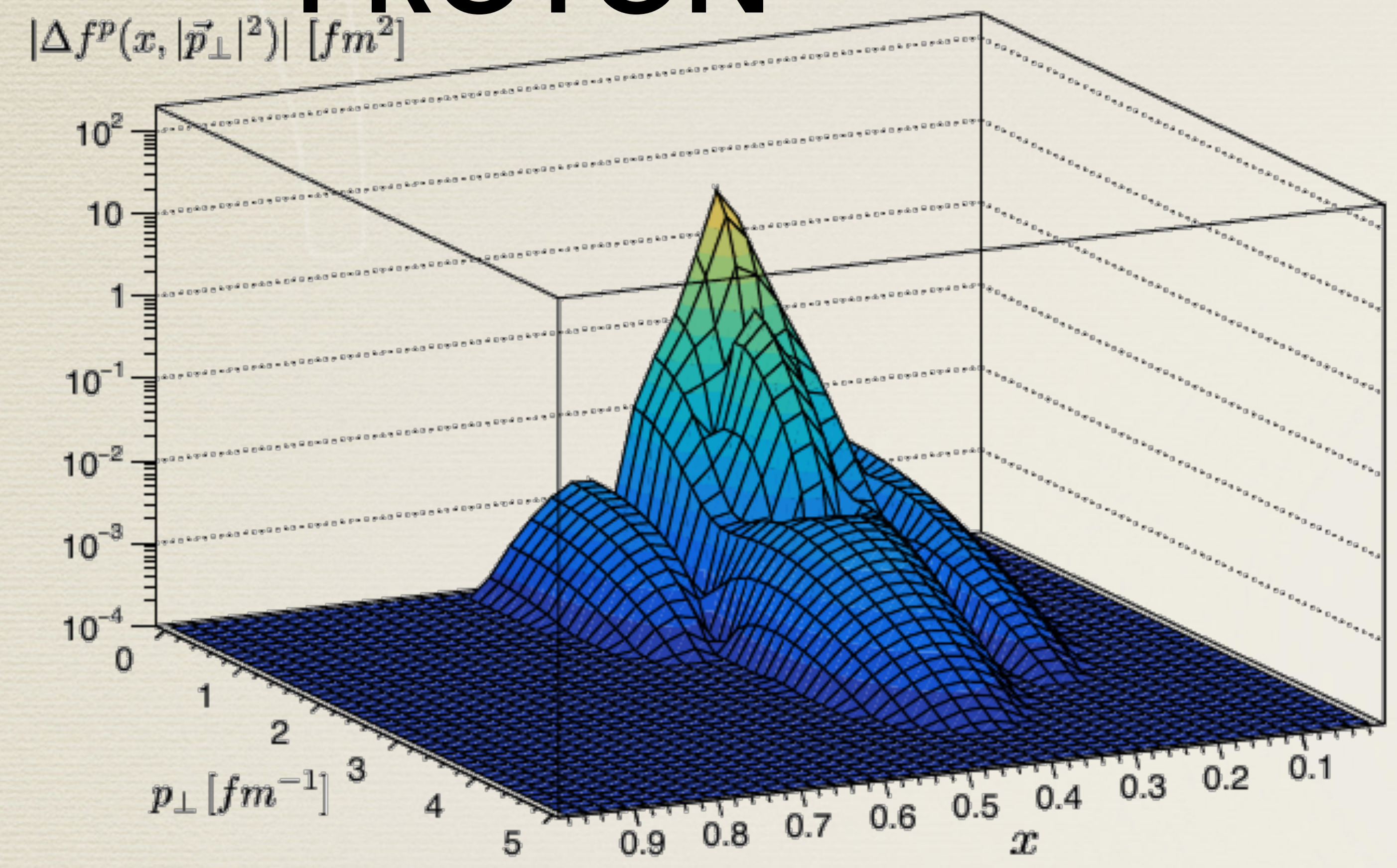
- u_v, u_v, d_v
- SIDIS
- no q_v detection, fragmentation...
- leading twist
- TMDs
- PDFs
- charges (axial, tensor...)

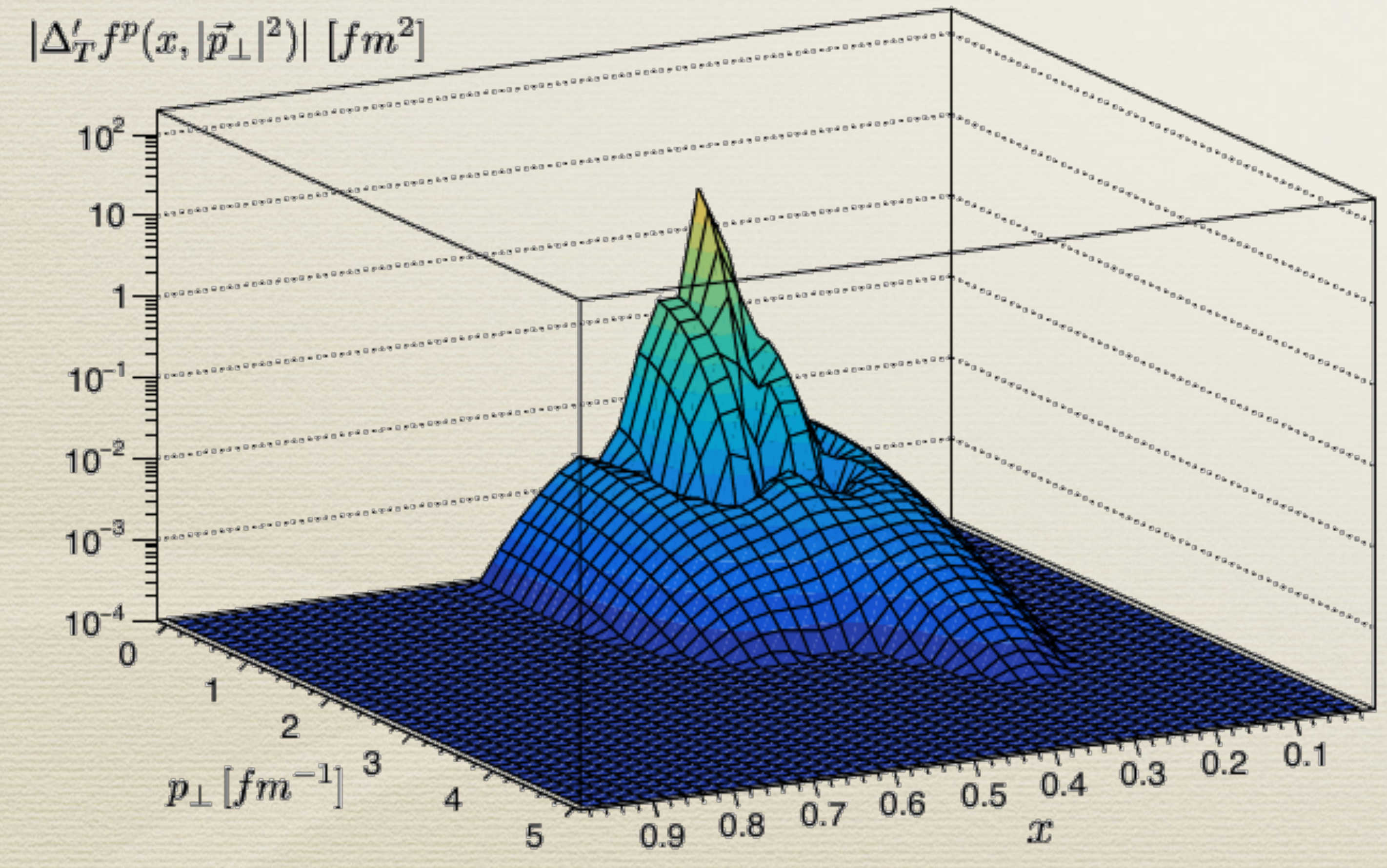
- In the case of ^{3}He the TMDs could be obtained through measurements of appropriate spin asymmetries in $^3\vec{He}(\vec{e}, e'p)$ experiments: in progress!
- We show in the following our calculation for the TMDs of ^{3}He (Alessandro, Del Dotto, Pace, Perna, Salmè, Scopetta, Phys.Rev.C 104 (2021) 6, 065204) , performed using Av18 + UIX wfs (Pisa group, A. Kievsky, M. Viviani et al.)
- Impossible to infer proton properties from ^{3}He, too different dynamics; but a fresh test of LFRHD and of the importance of Relativity in nuclear structure is at hand.

^{3}He TMD (LCMDs)

Alessandro, Del Dotto, Pace, Perna, Salmè, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

PROTON

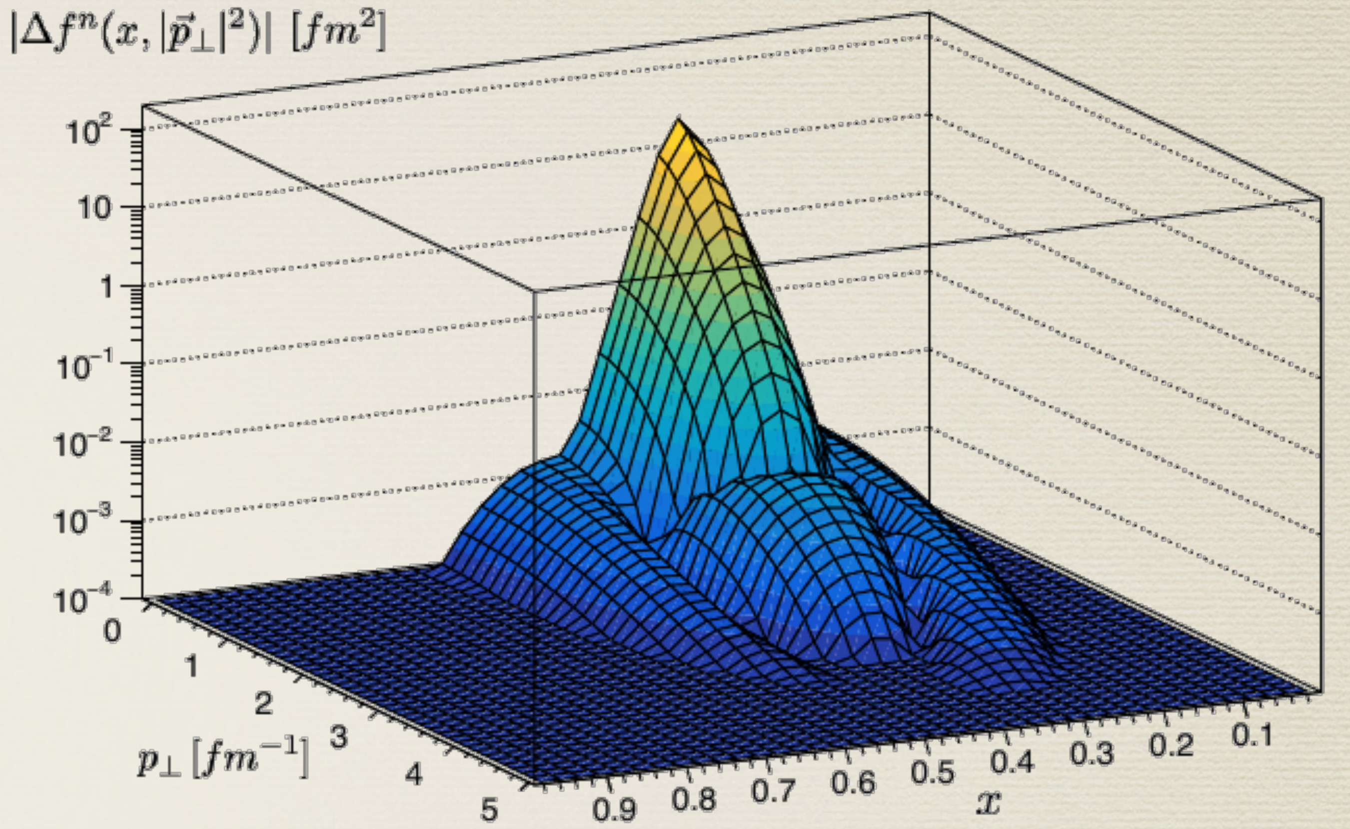


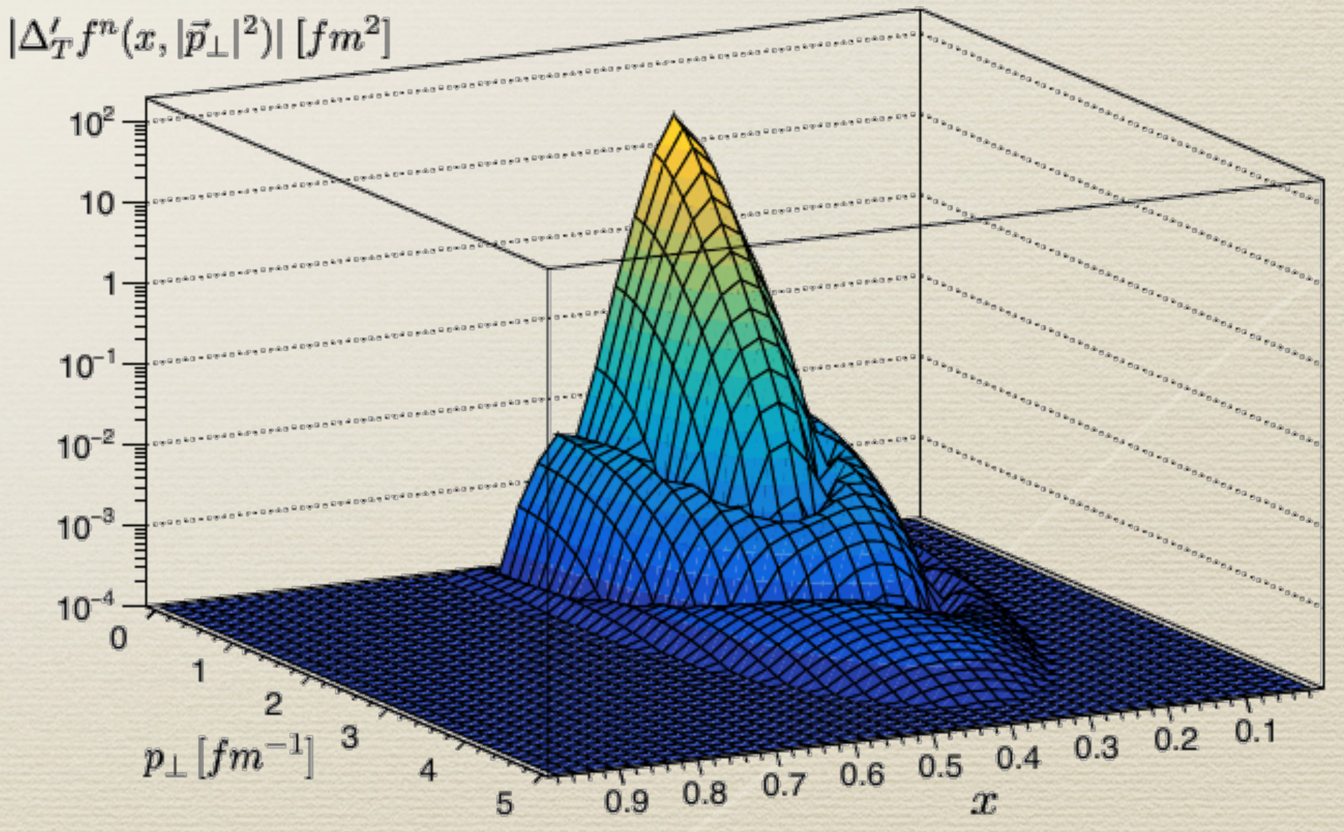


$\Delta f^\tau(x, |\mathbf{p}_\perp|^2)$

$\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$

NEUTRON





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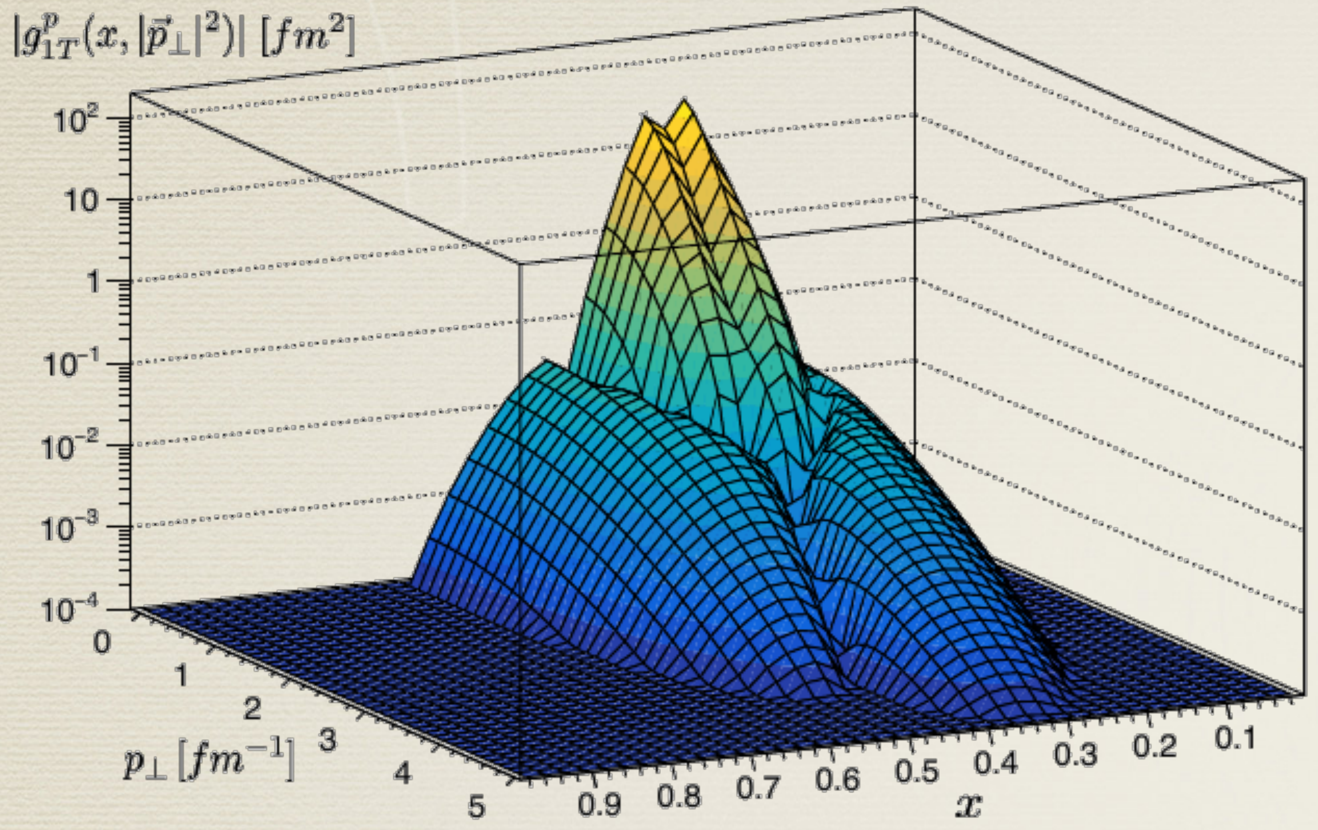
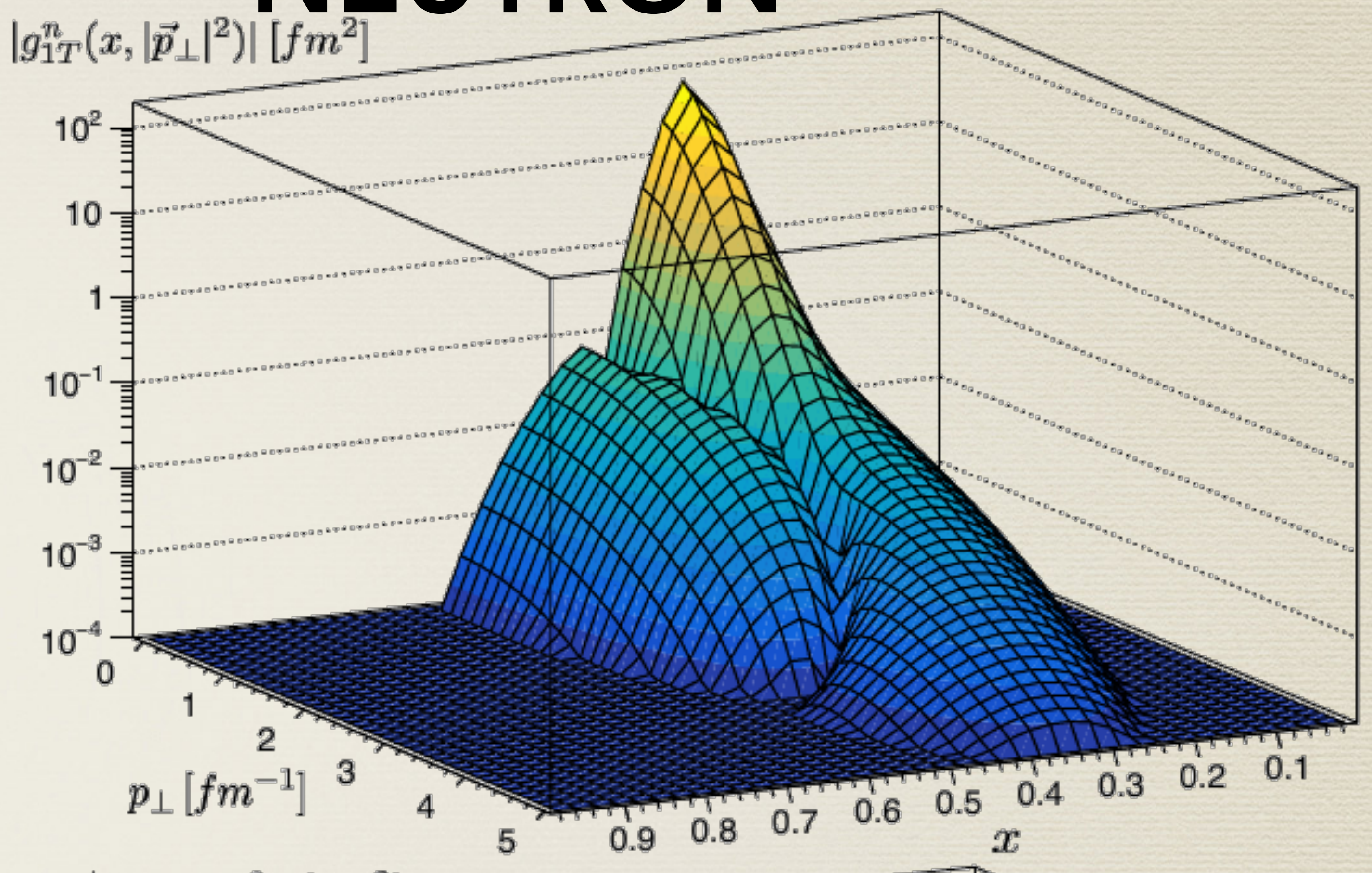
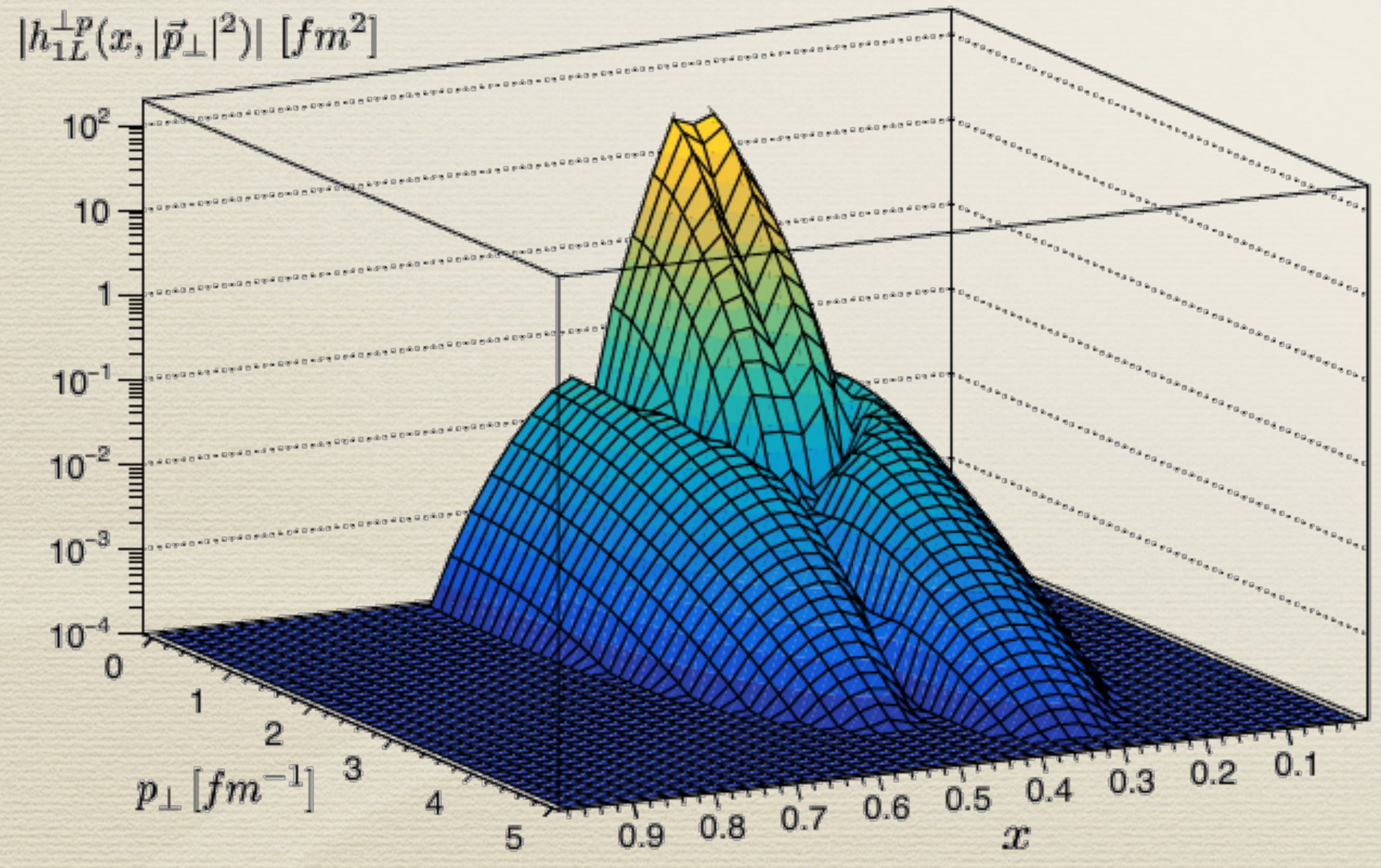
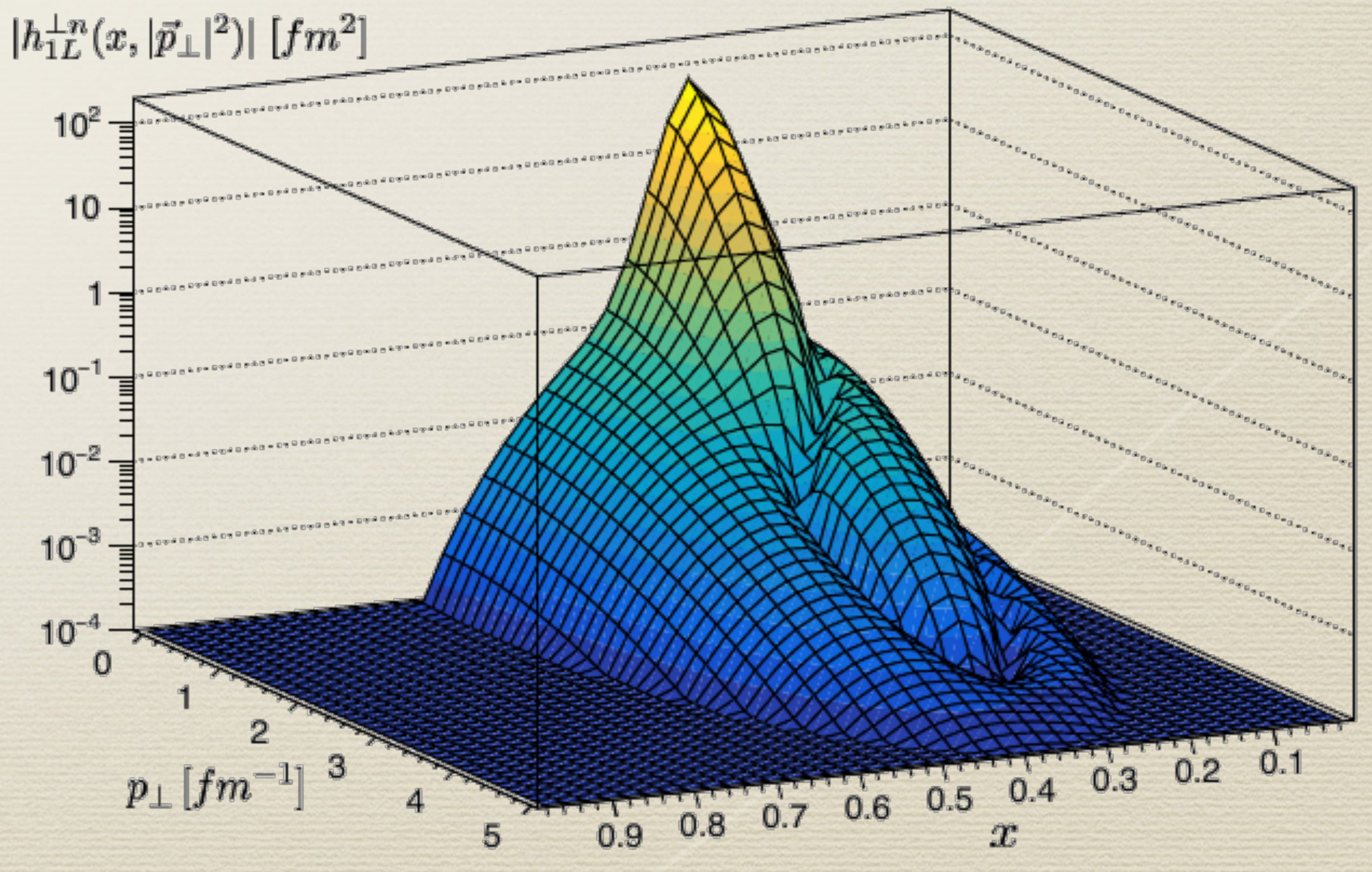


NEUTRON



Absolute value of the nucleon longitudinal-polarization distribution, $g_{1T}^{\tau}(x, |\mathbf{p}_\perp|^2)$, in a transversely polarized ^{3}He.





Absolute value of the nucleon transverse-polarization distribution, $h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2)$ in a longitudinally polarized ^{3}He.

^{3}He spin dependent SFs

F. Fornetti, E. Proietti…, M. R. et al, PRC 110 (2024), 3, L031303

Deep-inelastic scattering (DIS) involving polarized leptons off polarized nuclear targets are the key to accessing unique information on the spin structure inside nucleons and nuclei.
For example to:

1) get the neutron spin-dependent SFs (SSFs)
2) deeply investigate the Bjorken sum rule

Polarized ^{3}He will be used in the future EIC exp. program.

For the process: $\vec{l}(\mathscr{E}) + \vec{A} \to l'(\mathscr{E}') + X$. In particular: k (k′) = 4-momentum of initial (final) electron

$$\frac{d\sigma(+S)}{d\Omega_2 d\nu} - \frac{d\sigma(-S)}{d\Omega_2 d\nu} = 4\frac{\alpha_{em}^2}{Q^4}\, m_e^2 \frac{\mathcal{E}'}{\mathcal{E}} \boxed{L^{a,\mu\nu}} \boxed{W^A_{a,\mu\nu}}$$

$$L^a_{\mu\nu}(h_\ell) = i h_\ell \varepsilon_{\mu\nu\alpha\beta} \frac{k^\alpha\, q^\beta}{2m_e^2},$$

$$W^A_{a,\mu\nu} = i\, \epsilon_{\mu\nu\rho\sigma} q^\rho \left\{ S^\sigma \frac{G_1^A(x,Q^2)}{M_A} + \left[S^\sigma - \frac{S\cdot q}{P_A\cdot q} P_A^\sigma\right] [P_A\cdot q] \frac{G_2^A(x,Q^2)}{M_A^3} \right\}$$

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Antisymmetric Lepton tensor:

$$L^a_{\mu\nu}(h_\ell) = i h_\ell \varepsilon_{\mu\nu\alpha\beta} \frac{k^\alpha \, q^\beta}{2m_e^2} \, ,$$

h_l = helicity of incoming electron

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Antisymmetric hadronic tensor:

$$W^A_{a,\mu\nu} = i\ \epsilon_{\mu\nu\rho\sigma} q^\rho \left\{ S^\sigma\ \frac{G_1^A(x,Q^2)}{M_A} + \left[S^\sigma - \frac{S\cdot q}{P_A\cdot q} P_A^\sigma \right] [P_A\cdot q]\ \frac{G_2^A(x,Q^2)}{M_A^3} \right\}$$

Where G_1^A and G_2^A are nuclear SSFs.

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In the Bjorken limit we use J. R. Ellis and R. L. Jaffe, Phys. Rev. D 9, 1444 (1974)

$$g_1^A(x,Q^2) = P_A\cdot q\, \frac{G_1^A(x,Q^2)}{M_A^2} \qquad g_2^A(x,Q^2) = [P_A\cdot q]^2\, \frac{G_2^A(x,Q^2)}{M_A^4}$$

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^{3}He spin dependent SFs

F. Fornetti, E. Proietti..., M. R. et al, PRC 110 (2024), 3, L031303

For the process: $\vec{l}(\mathscr{E}) + \vec{A} \to l'(\mathscr{E}') + X$. In particular: k (k′) = 4-momentum of initial (final) electron

$$\frac{d\sigma(+S)}{d\Omega_2 d\nu} - \frac{d\sigma(-S)}{d\Omega_2 d\nu} = 4\frac{\alpha_{em}^2}{Q^4}\, m_e^2 \frac{\mathcal{E}'}{\mathcal{E}} \boxed{L^{a,\mu\nu}} \boxed{W^A_{a,\mu\nu}}$$

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Spin-dependent SF

Nucleon hadronic tensor

^{3}He spin dependent SFs

F. Fornetti, E. Proietti…, M. R. et al, PRC 110 (2024), 3, L031303

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In terms of SSFs, as for unpolarized SFs: $$g_j^A(x) = \sum_{N=n,p} \int_{\xi_{min}}^1 d\xi \left\{ g_1^N\left(\frac{x}{\xi}\frac{m}{M_A}\right) l_j^N(\xi) + g_2^N\left(\frac{x}{\xi}\frac{m}{M_A}\right) h_j^N(\xi) \right\},$$

1) $j = 1, 2$
2) $l_j^N(\xi)$, $h_j^N(\xi)$ are the spin-dependent LCMDs evaluated from ^{3}He TMD

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R. Alessandro, A. Del Dotto, E. Pace, G. Perna, G. Salmè, and S. Scopetta, Phys. Rev. C 104, 065204 (2021)

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3) g_j^N free nucleon SSF. While $h_1^N(\xi) = 0$, $l_2^N(\xi) \neq 0 \Rightarrow$ Structure function $g_2^A(x)$ depends also on g_1^N!

^{3}He spin dependent SFs

F. Fornetti, E. Proietti..., M. R. et al, PRC 110 (2024), 3, L031303

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For the numerical calculations we have used:

1) NLO parametrizations of g_1^N of Ref: **M. Gluck et al, Phys. Rev. D 63, 094005 (2001)**
2) Wandzura-Wilczek approximation

$$g_2^N(x) = -g_1^N(x) + \int_x^1 dy\,\frac{g_1^N(y)}{y}$$

Results

F. Fornetti, E. Proietti..., M. R. et al, PRC 110 (2024), 3, L031303

For the process: $\vec{l}(\mathscr{E}) + \vec{A} \to l'(\mathscr{E}') + X.$

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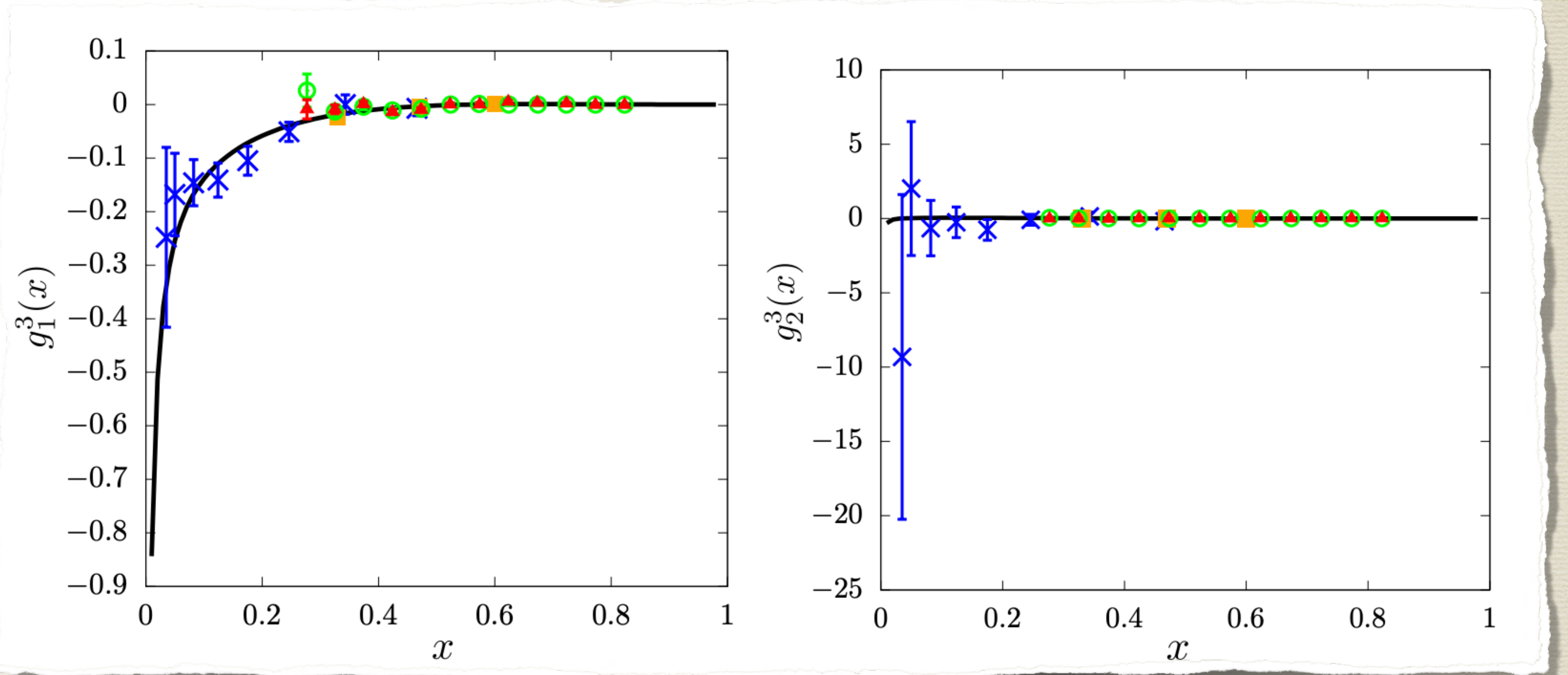
Also in this case there are no free parameters and the ^{3}He w.f. corresponding to the Av18 potential has been used



- Full lines: our calculations
- Experimental analyses:
    - a) crosses: P. L. Anthony et al. (E142), Phys. Rev. D 54, 6620 (1996)
    - b) squares: X. Zheng et al. (JLab Hall A), PRL 92, 012004 (2004)
    - c) empty: D. Flay et al. (Jefferson Lab Hall A), Phys. Rev. D 94, 052003 (2016)

Some recents works…

F. Fornetti, E. Proietti…, M. R. et al, PRC 110 (2024), 3, L031303

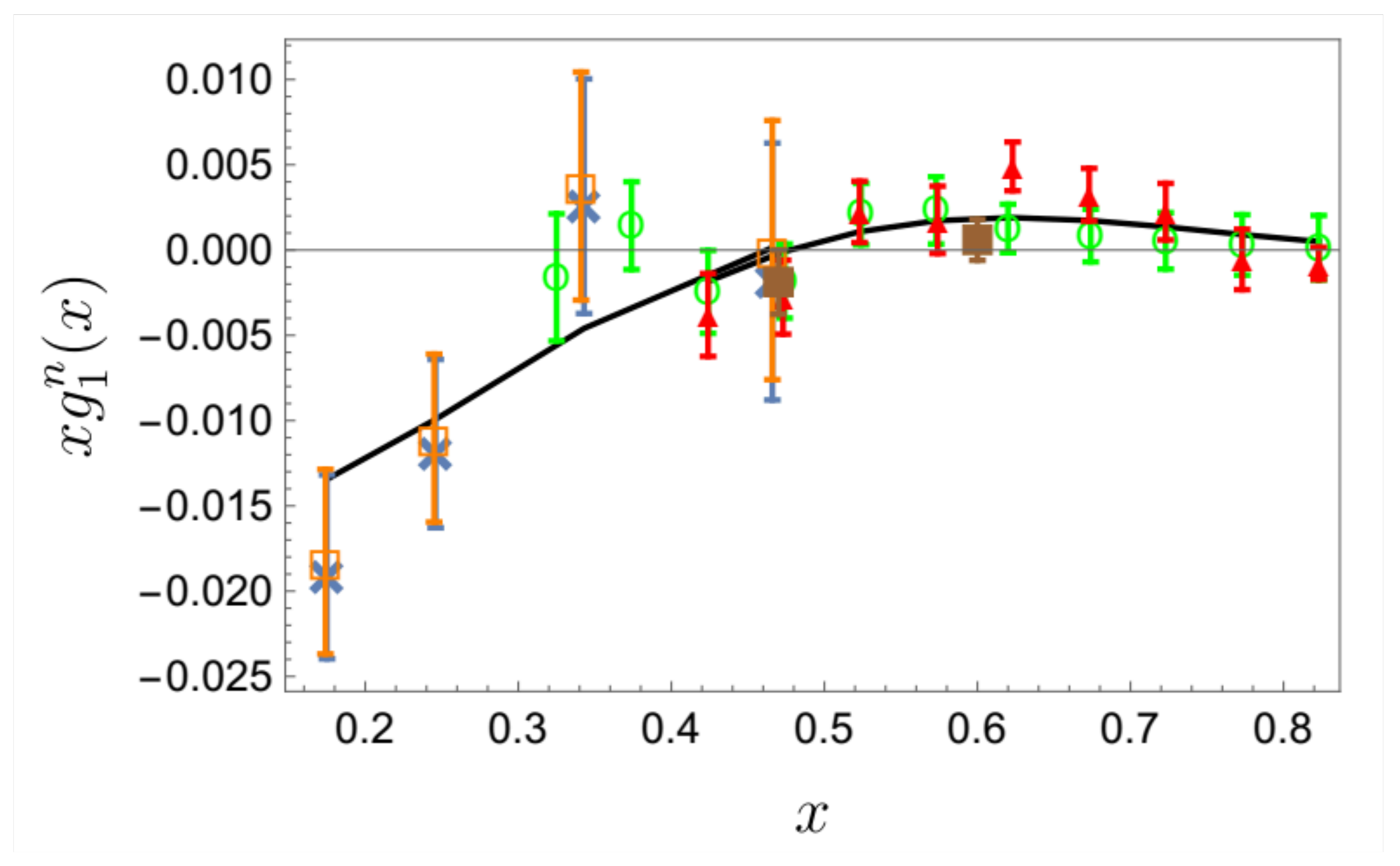
Extraction of the neutron spin dependent structure from ³He data:

$$\bar{g}_j^n(x) = \frac{1}{p_j^n}\left[g_j^3(x) - 2p_j^p g_j^p(x)\right] \quad (j = 1, 2)$$

with the effective polarizations obtained from the ³He w.f.

$$p_1^n \simeq 0.873 \qquad p_2^n \simeq 0.873$$

$$p_1^p \simeq -0.0230 \qquad p_2^p \simeq -0.0245$$

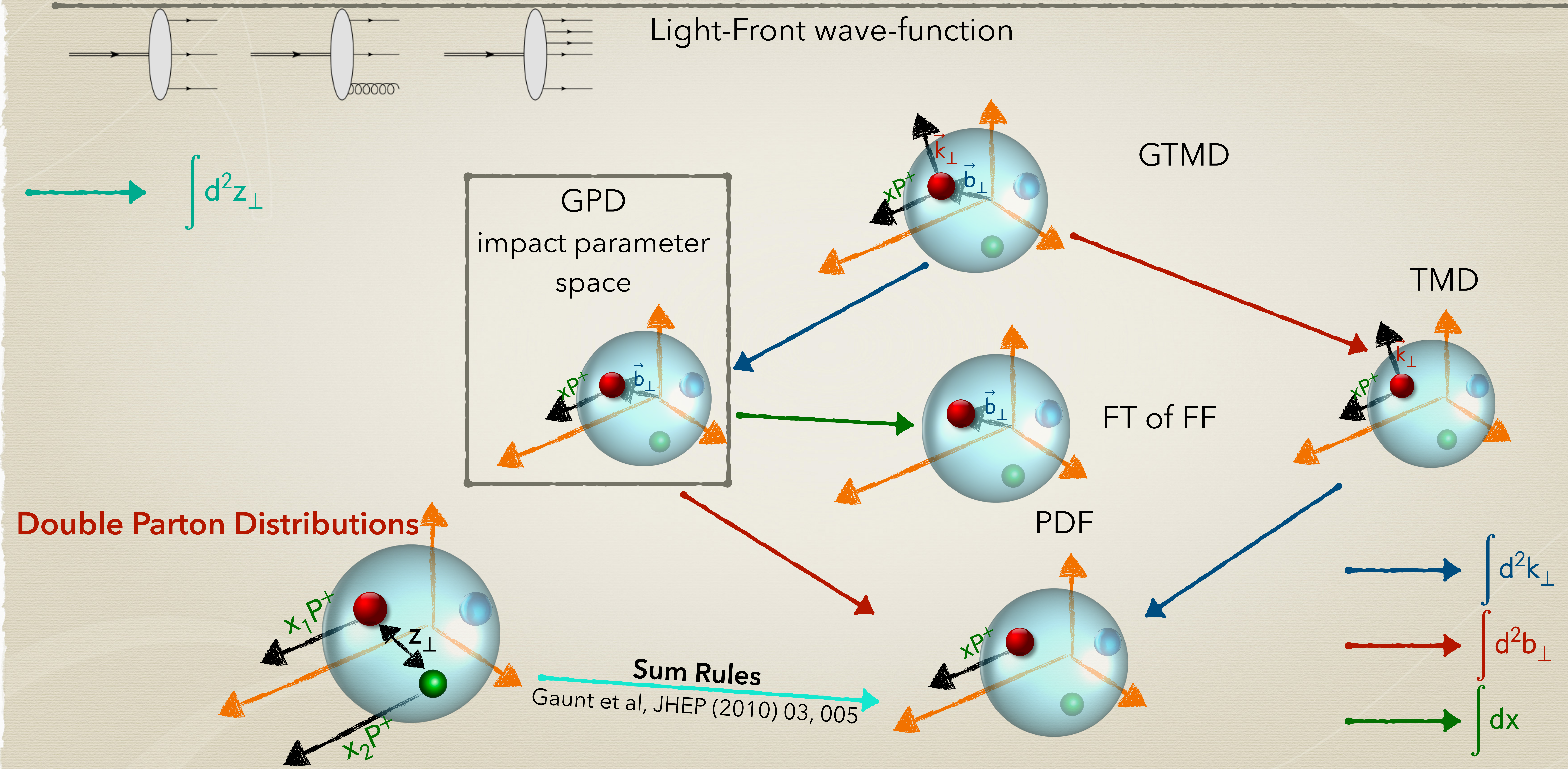


Points: extracted from ³He data using our formula

Line: M. Gluck, et al, Phys. Rev. D 63, 094005 (2001).

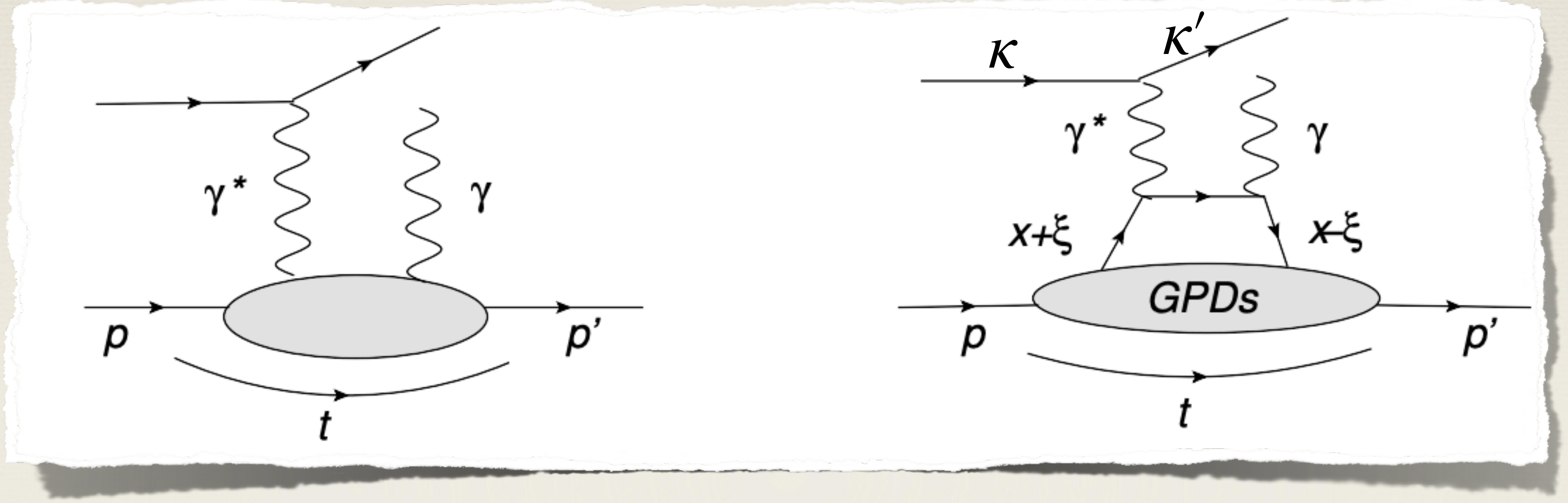
The ³He spin structure is makes this nucleus unique to extract the neutron distributions!

Multidimensional picture of hadrons



Deeply Virtual Compton Scattering

Exclusive electro-production of real photon: access to GPDs:



$$\bar{p}^\mu = \frac{p^\mu + p'^\mu}{2}$$

$$a^\pm = a^0 \pm a_z$$

Light-Cone coordinates

GPDs depend on:

$$\Delta^\mu = (p' - p)^\mu$$

- $t = \Delta^2$
- $x = \frac{\bar{k}^+}{\bar{p}^+}$
- $\xi = \frac{\Delta^+}{2\bar{p}^+}$
- $Q^2 = (\kappa' - \kappa)^2$

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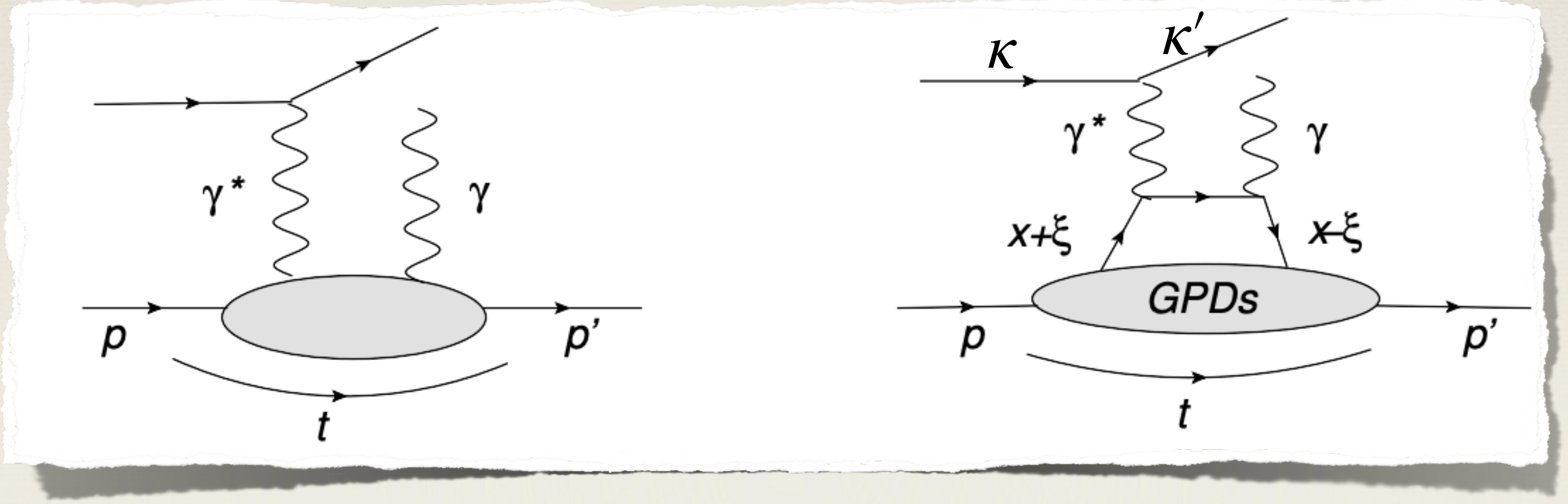

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GPDs are defined from non-local matrix elements

$$F^q_{\lambda,\lambda'}(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{ix\bar{p}^+z^-} \langle p', \lambda' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p, \lambda \rangle |_{z^+=\mathbf{z}_\perp=\mathbf{0}} =$$

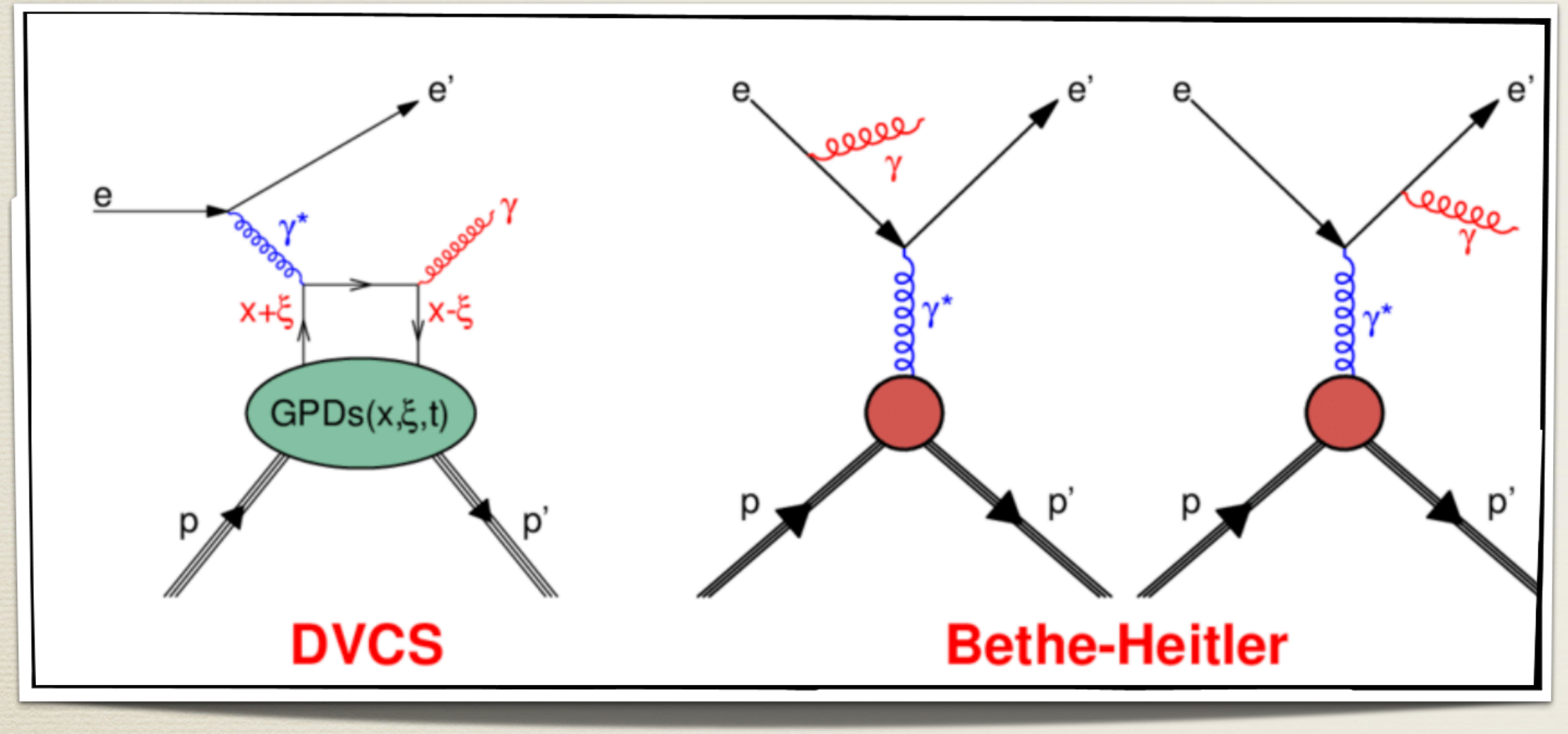
$$\frac{1}{2\bar{p}^+}\left[H_q(x,\xi,t)\bar{u}(p',\lambda')\gamma^+ u(p,\lambda) + E_q(x,\xi,t)\bar{u}(p',\lambda')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p,\lambda)\right]$$

at leading twist and for 1/2 spin target (the scale dependence is omitted)

GPDs meet experiments

There are several GPDs (for quarks and gluons) depending on the spin of the target hence we need several observable from different processes: DVCS, double DVCS, DVMP (double virtual meson production)

We need to take into account the Bethe-Heitler contribution to the final state:



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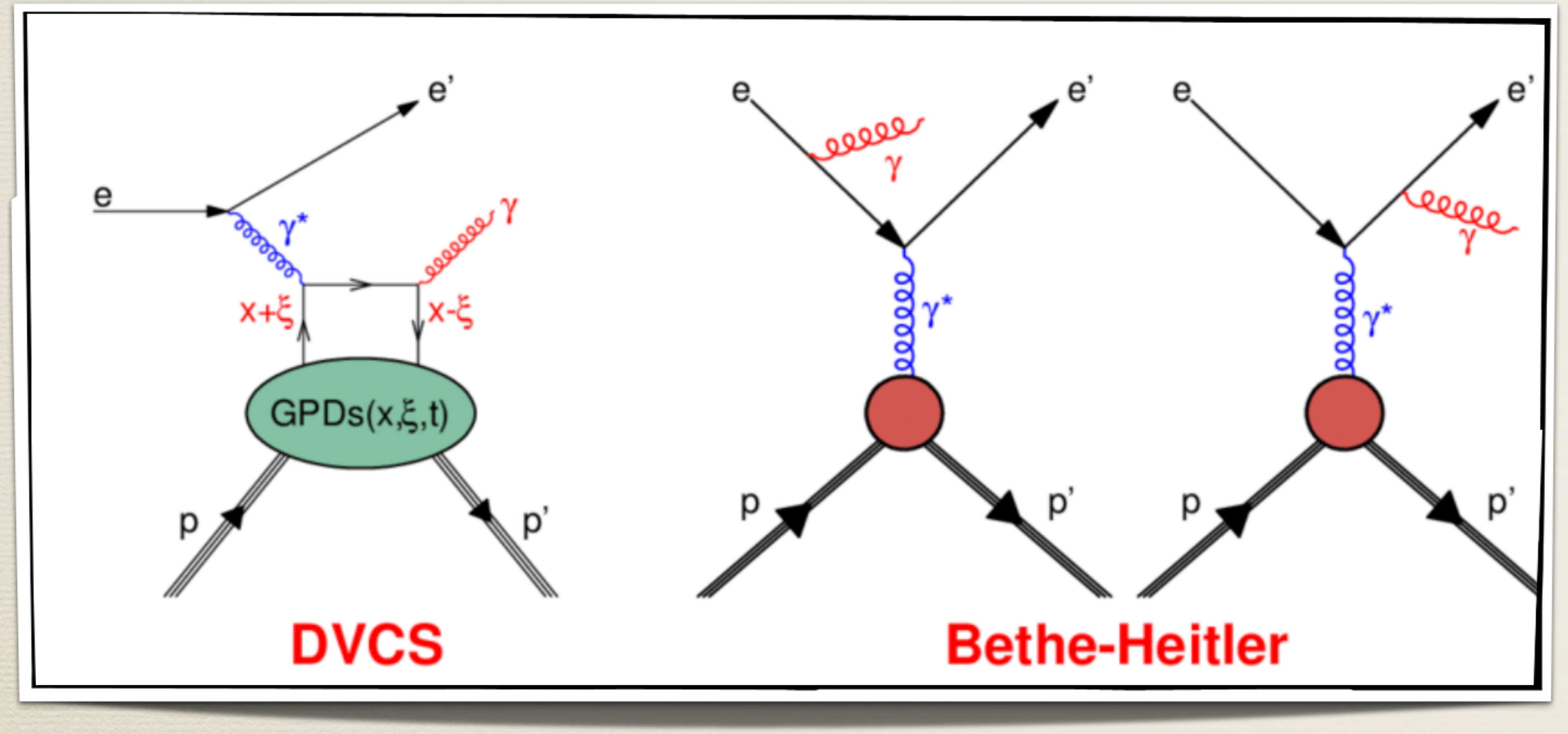

$$\sigma \propto \mathcal{T}^2_{DVCS} + \mathcal{T}^2_{BH} + \mathcal{I}_{DVCS-BH}$$

with

GPD = H, E,...

$\mathcal{T}_{DVCS}$ function of $\mathcal{F}_{q(g)}(\xi, t) = \int_{-1}^{1} \mathrm{dx} \, \frac{\mathrm{F_{q(g)}(x, \xi, t)}}{\mathrm{x} \pm \xi + \mathrm{i}\varepsilon}$

Compton Form Factors (CFFs)

GPDs meet experiments

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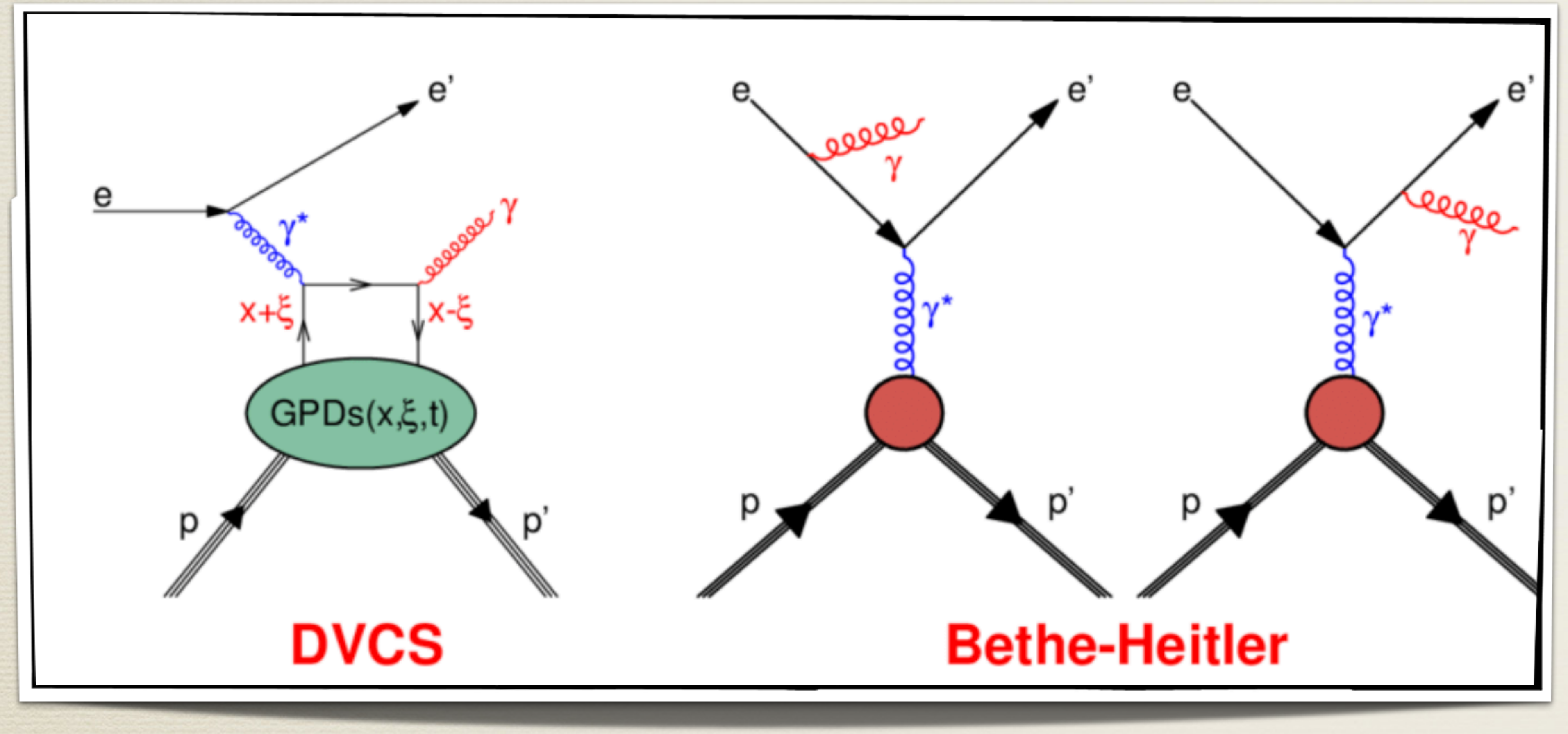

$$\sigma \propto \mathcal{T}^2_{DVCS} + \mathcal{T}^2_{BH} + \mathcal{I}_{DVCS-BH}$$

Asymmetries are fundamental to disentangle the real and imaginary parts of different CFFs.

- **B**eam **C**harge **A**symmetry: $\frac{d\sigma^+ - d\sigma^-}{d\sigma^+ - d\sigma^-} \sim \mathfrak{Re}\mathcal{F}$

- **B**eam **S**pin **A**symmetry: $\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow - d\sigma^\downarrow} \sim \mathfrak{Im}\mathcal{F}$

Why light nuclear targets?

Several reasons. For example:

To access the **neutron** GPDs **Light nuclear targets** play a special role! ^{2}H and ^{3}He are well known and nuclear effects can be properly taken into account thanks to the realistic wave functions available.

To get a complete **flavor decomposition** of GPDs

To study the neutron **spin structure**

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Nuclear tomography!

why is it important?

GPDs as solution to the EMC effect?

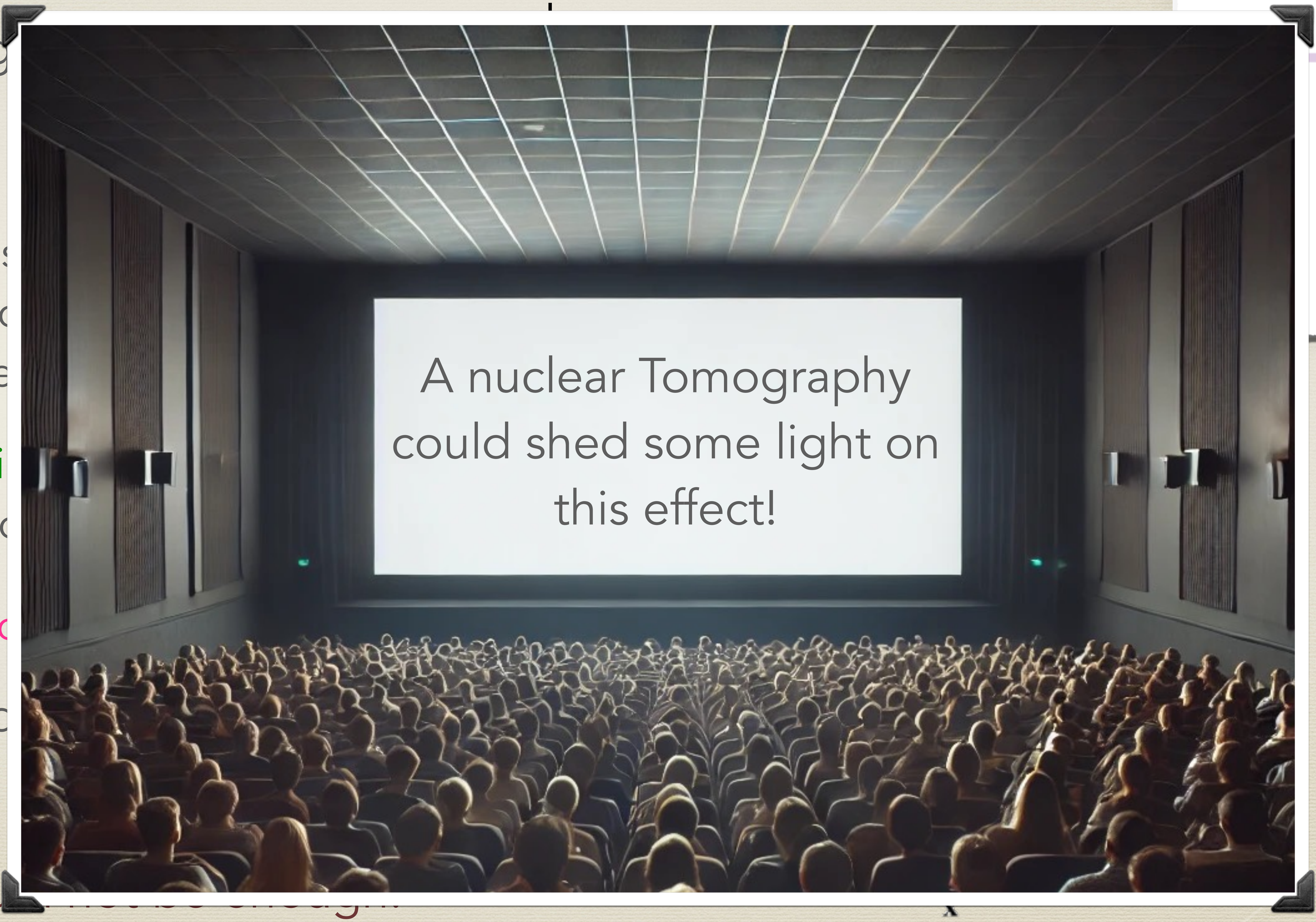
In DIS off a nuclear targ

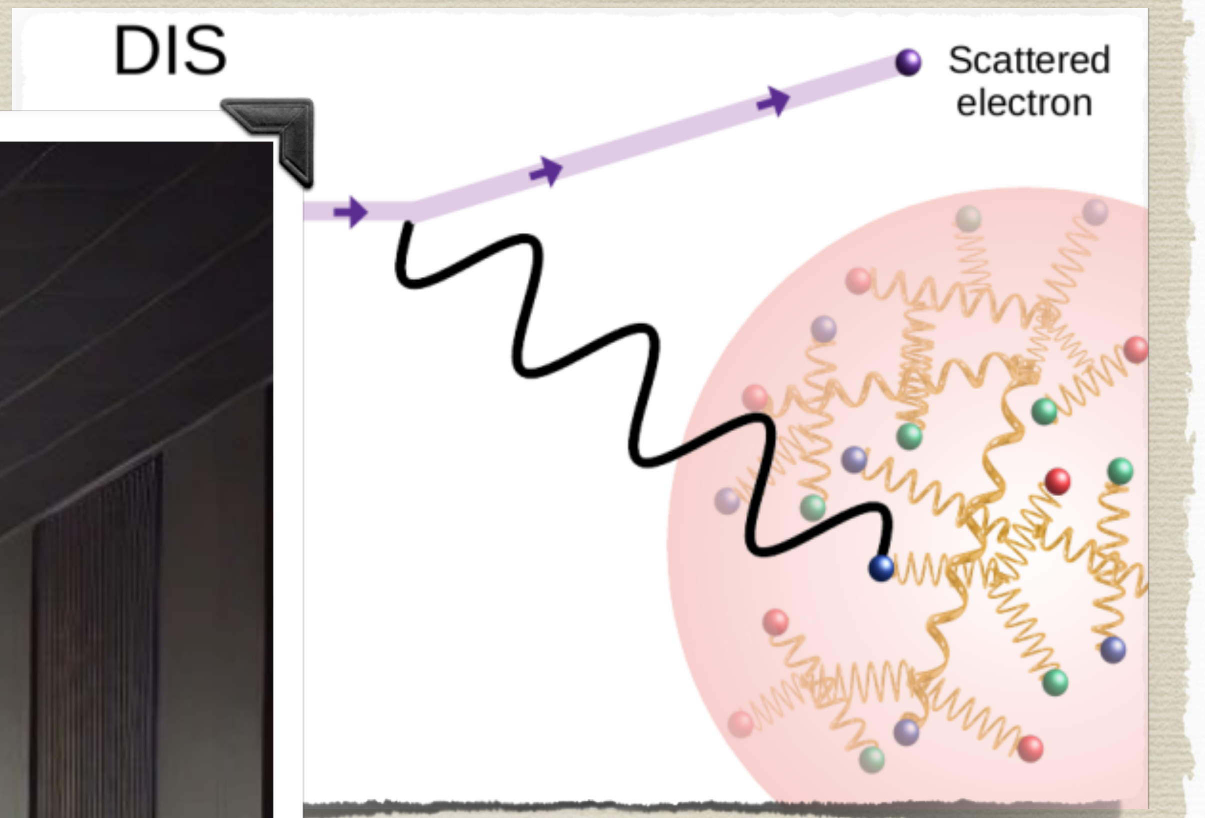
$$0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$$

- $x \leq 0.3$ "Shadowing, anti-s coherence effects, the pho partons belonging to diffe
- $0.2 \leq x \leq 0.8$ "EMC (bindi mainly valence quarks invo
- $0.8 \leq x \leq 1$ "Fermi motio

Small effect! Several moc (**E**veryone's **M**odel is **C**ool)

Collinear information cou

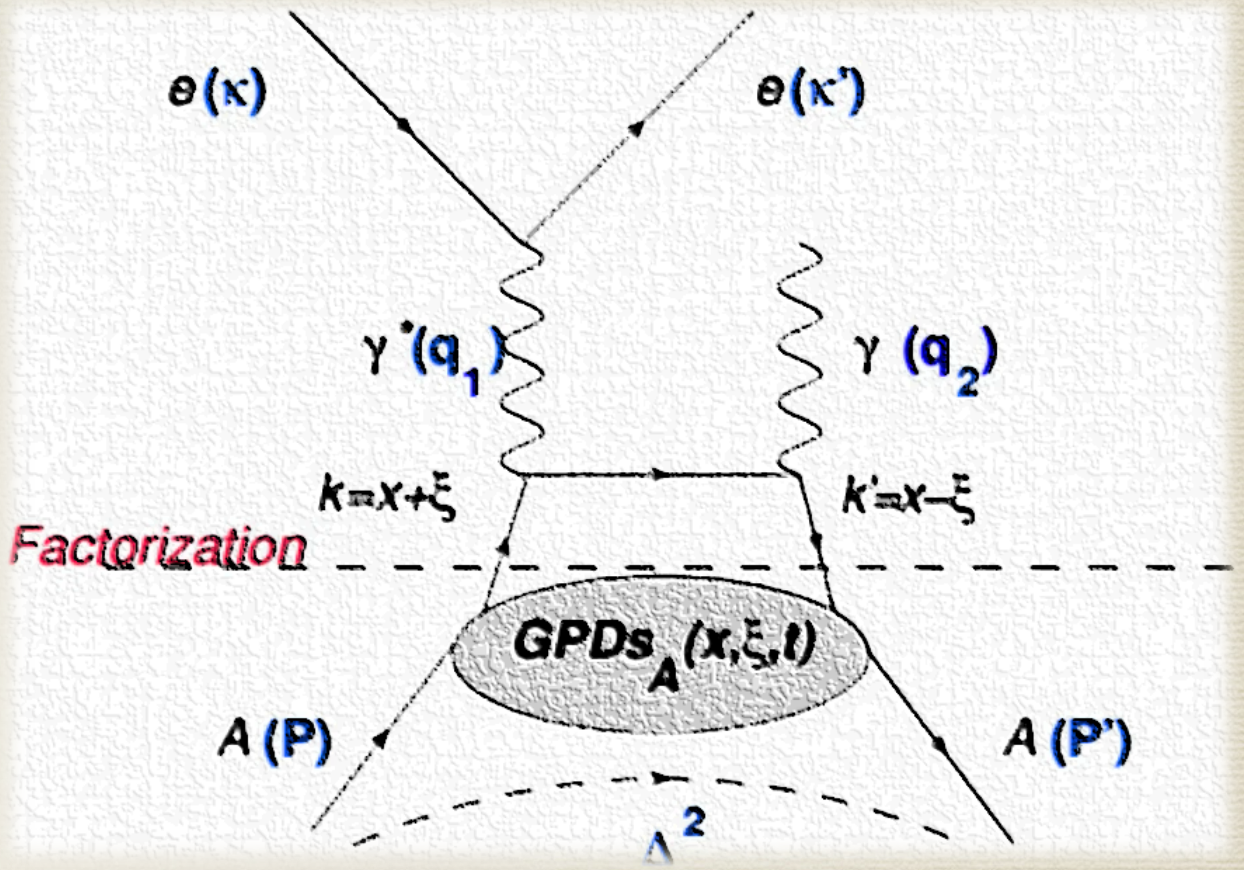


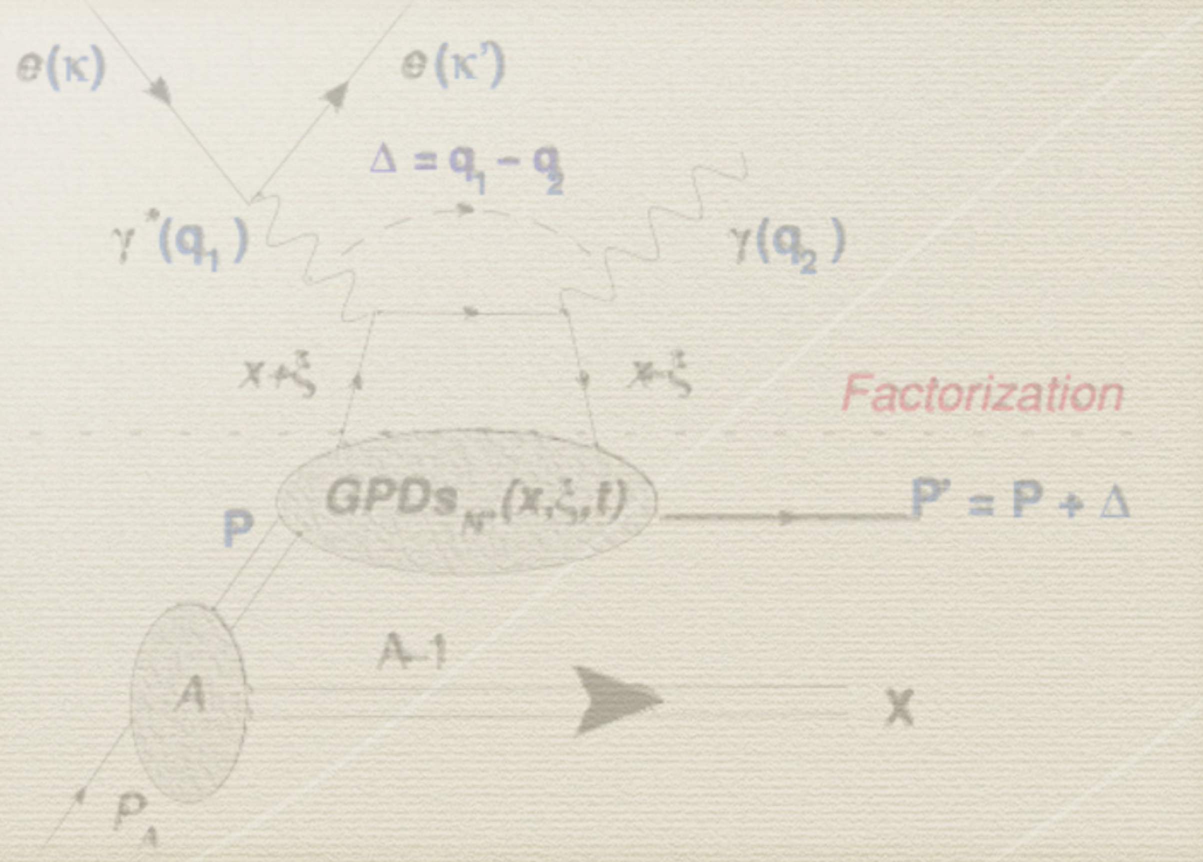


Nuclear DVCS

In the nuclear case we have two channels:

Coherent channel → we access the GPDs of the nucleus
Tomography of the nucleus





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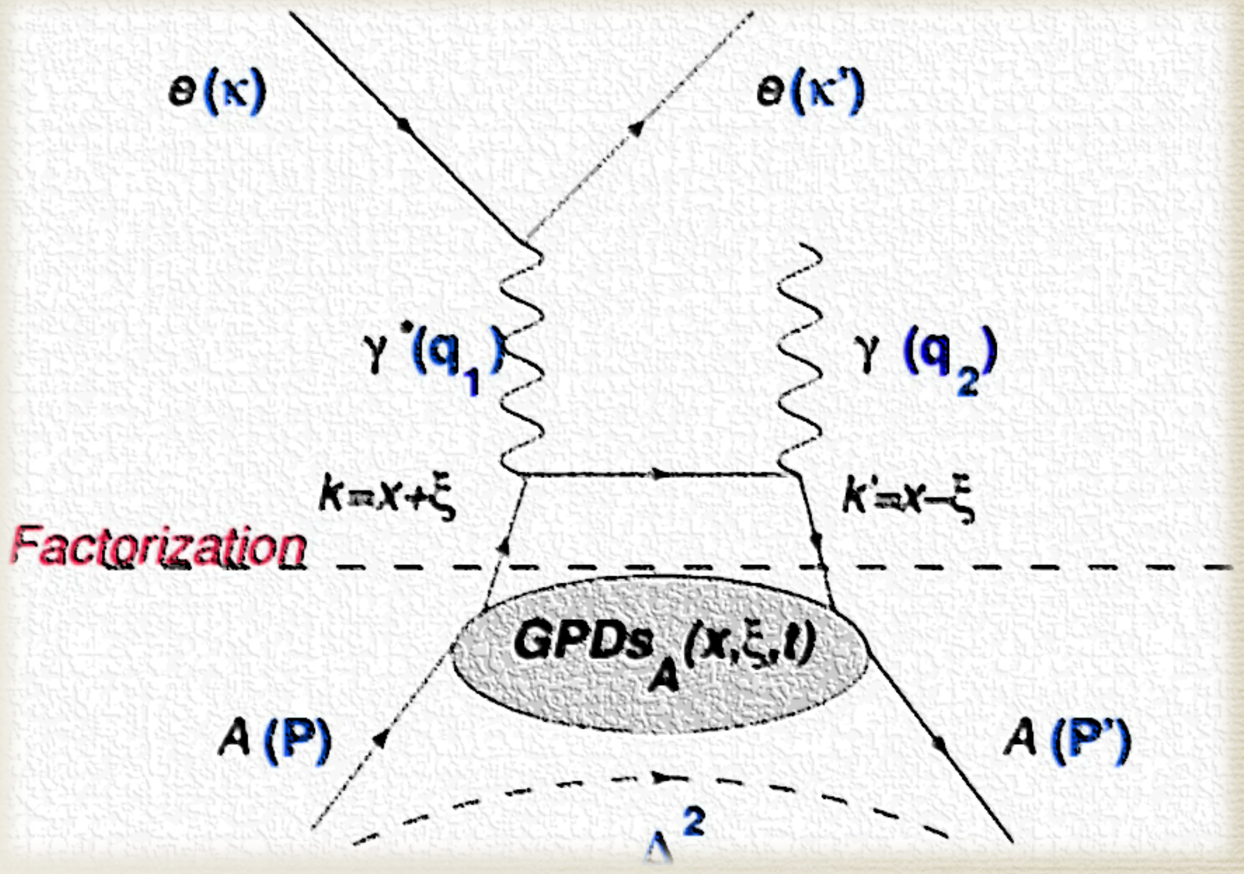
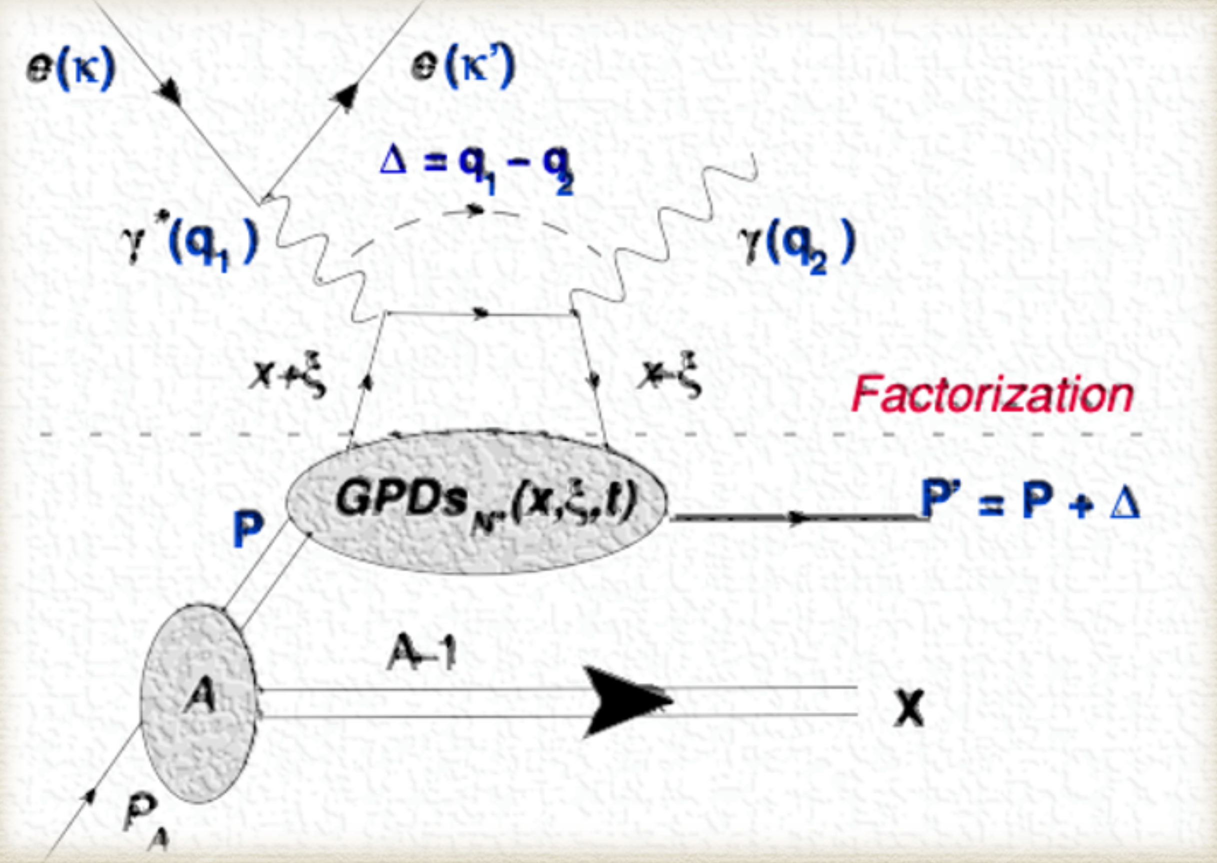


Incoherent channel → we access the GPDs of the bound nucleons
Same distribution of the free one?
Tomography of the bound nucleon



An Impulse Approximation for the coherent case

The leading twist ^{3}He and ^{4}He GPDs:

$$H_q^A(x,\xi,t) \sim \sum_{n=P,N} \int \frac{dz}{z} h^{A,n}(z,\xi,t) H_q^n\left(\frac{x}{z},\frac{\xi}{z},t\right)$$

free nucleon GPD H

we used the e Goloskokov-Kroll model (**EPJA 47 212 (2014)**)

For ^{3}He:
S. Scopetta, PRC 70, 015205 (2004)

For ^{4}He:
S. Fucini et al, PRC 98, 015203 (2018)

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$$h^{A,n}(z,\xi,t) = \int dE d\vec{p}\, P_n^A(\vec{p},\vec{p}+\vec{\Delta},E)\,\delta\left(z-\frac{\bar{p}^+}{\bar{P}^+}\right)$$

Nuclear off-diagonal spectral function

$$P_n^A(\vec{p},\vec{p}+\vec{\Delta},E) \sim \sum_{S,\sigma_n} \rho(E) \langle \vec{P}+\vec{\Delta}\; S | \vec{P}-\vec{p}\; E, \vec{p}+\vec{\Delta}\; \sigma_n \rangle \langle \vec{p}\; \sigma_n, \vec{P}-\vec{p}\; E | \vec{P}\; S \rangle$$

Spin dependent off-diagonal spectral function

$$P^{A,n}_{SS,\sigma_n\sigma_n}(\vec{p},\vec{p}+\vec{\Delta},E)$$

GPDs and binding effects for ^{3}He

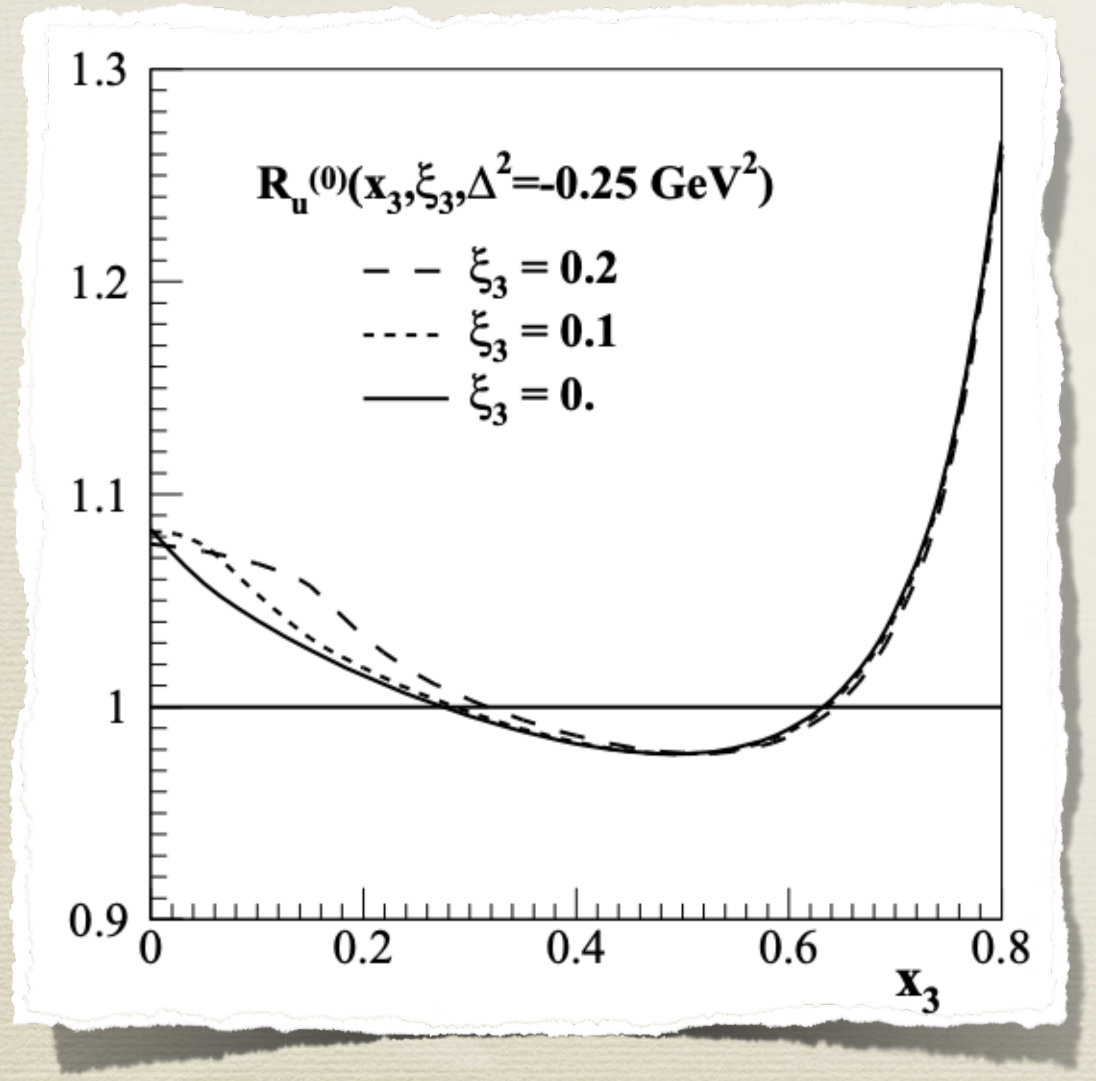
We define an "EMC like" ratio:

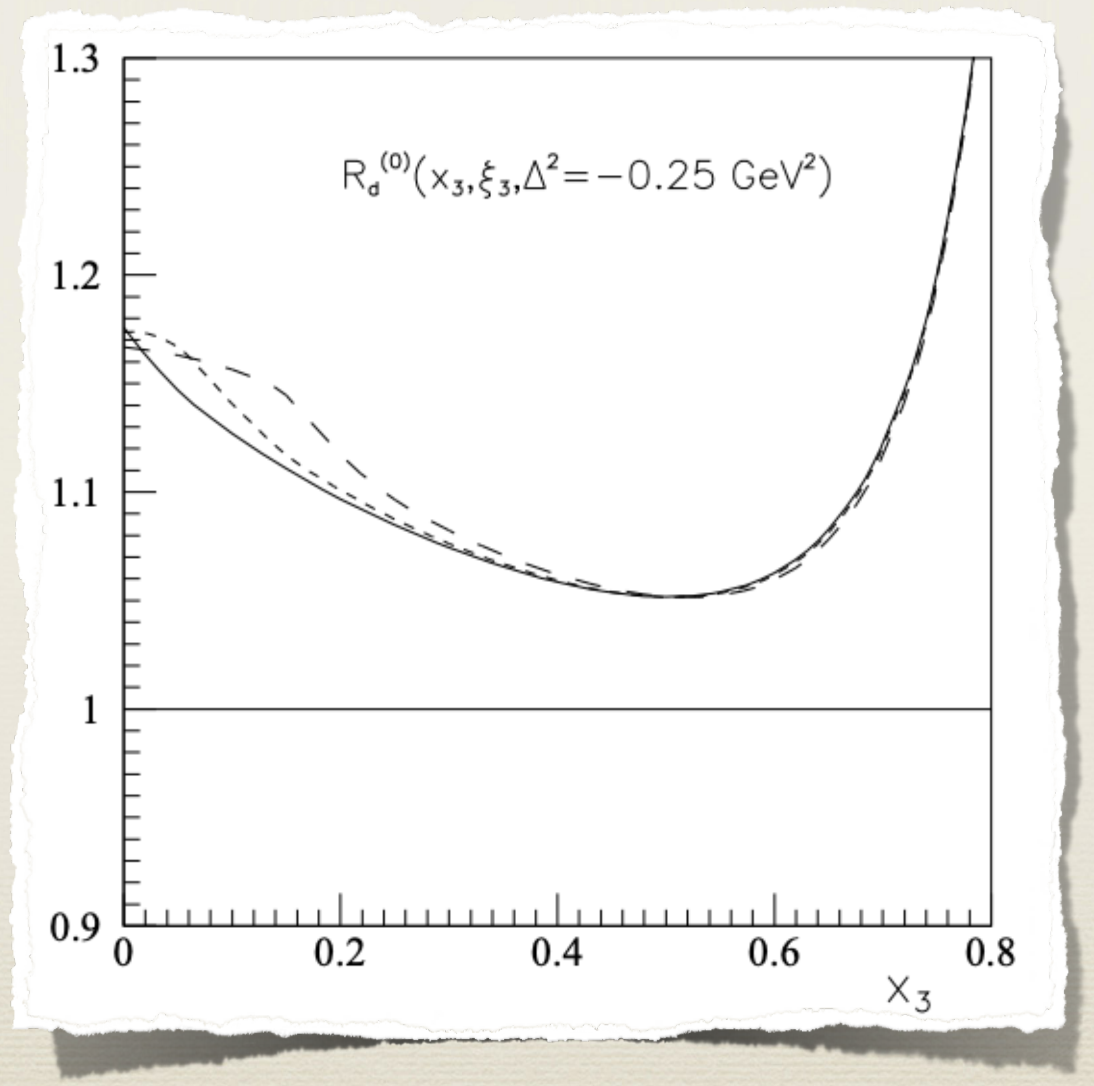
$$R_q^{(0)}(x,\xi,\Delta^2) = \frac{H_q^3(x,\xi,\Delta^2)}{2H_q^{3,p}(x,\xi,\Delta^2) + H_q^{3,n}(x,\xi,\Delta^2)}$$

$$H_q^{3,N}(x,\xi,\Delta^2) = \tilde{H}_q^N(x,\xi)\boxed{F_q^3(\Delta^2)}$$

charge FF

The ratio is equal to 1 if there is no binding!





- Effect grows with ξ and t

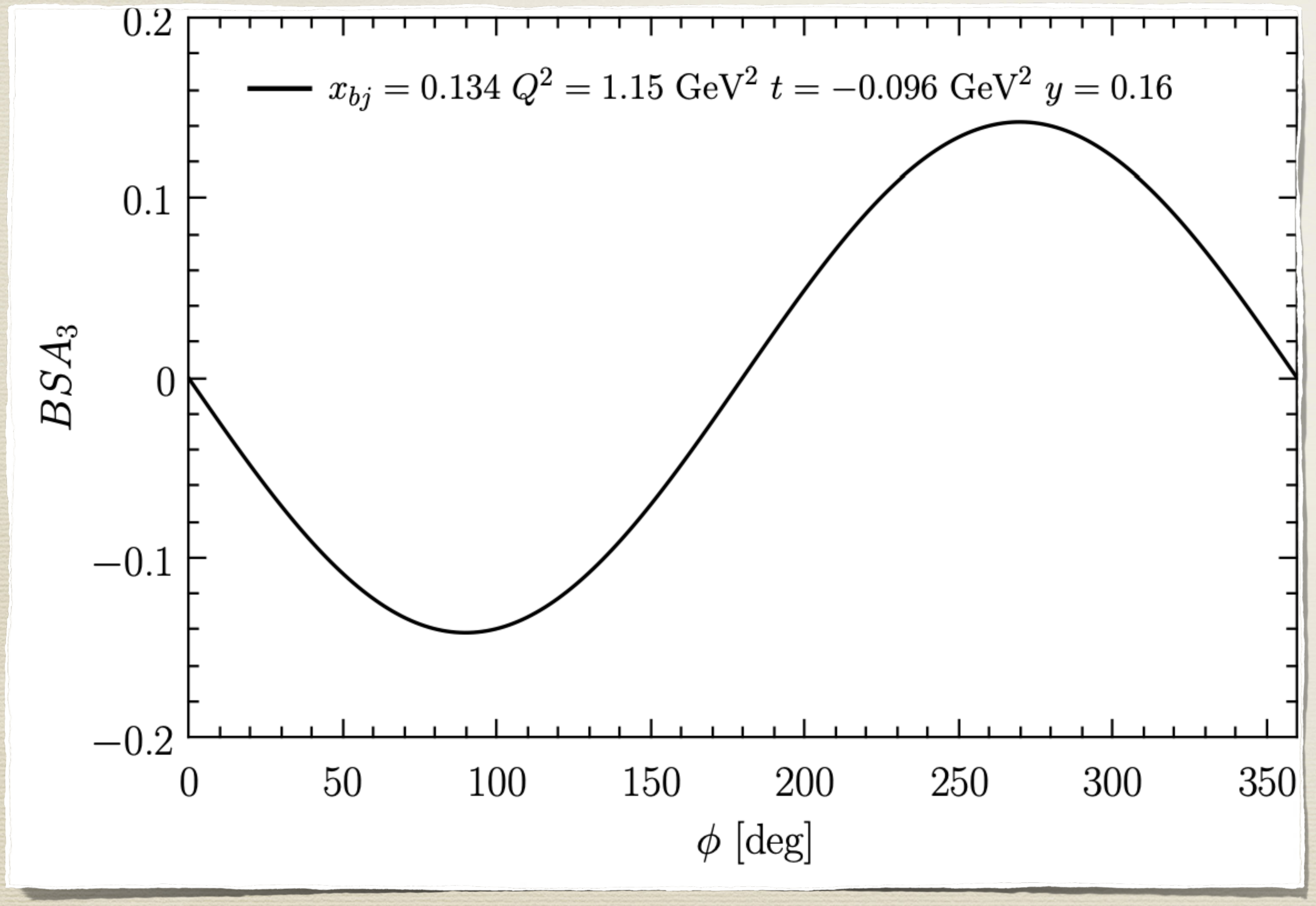
- 5 % to 10 % binding effect between x = 0.4 and 0.7 - much bigger than in the forward case;

Asymmetries for ^{3}He

S. Fucini, M. R. and S. Scopetta, EPJA 57 (2021) 9, 273

The BSA:

$$\mathrm{BSA}_3 \sim \pm \frac{x_{Bj}}{y} \frac{s_1^I}{c_0^{BH}} \sin(\phi)$$



Variables:

- y = lepton energy fraction
- ϕ = the azimuthal angle between the lepton plane and the recoiled nucleus
- c_0^{BH} depends on EM FFs

- s_1^I depends on Imaginary part of CFFs

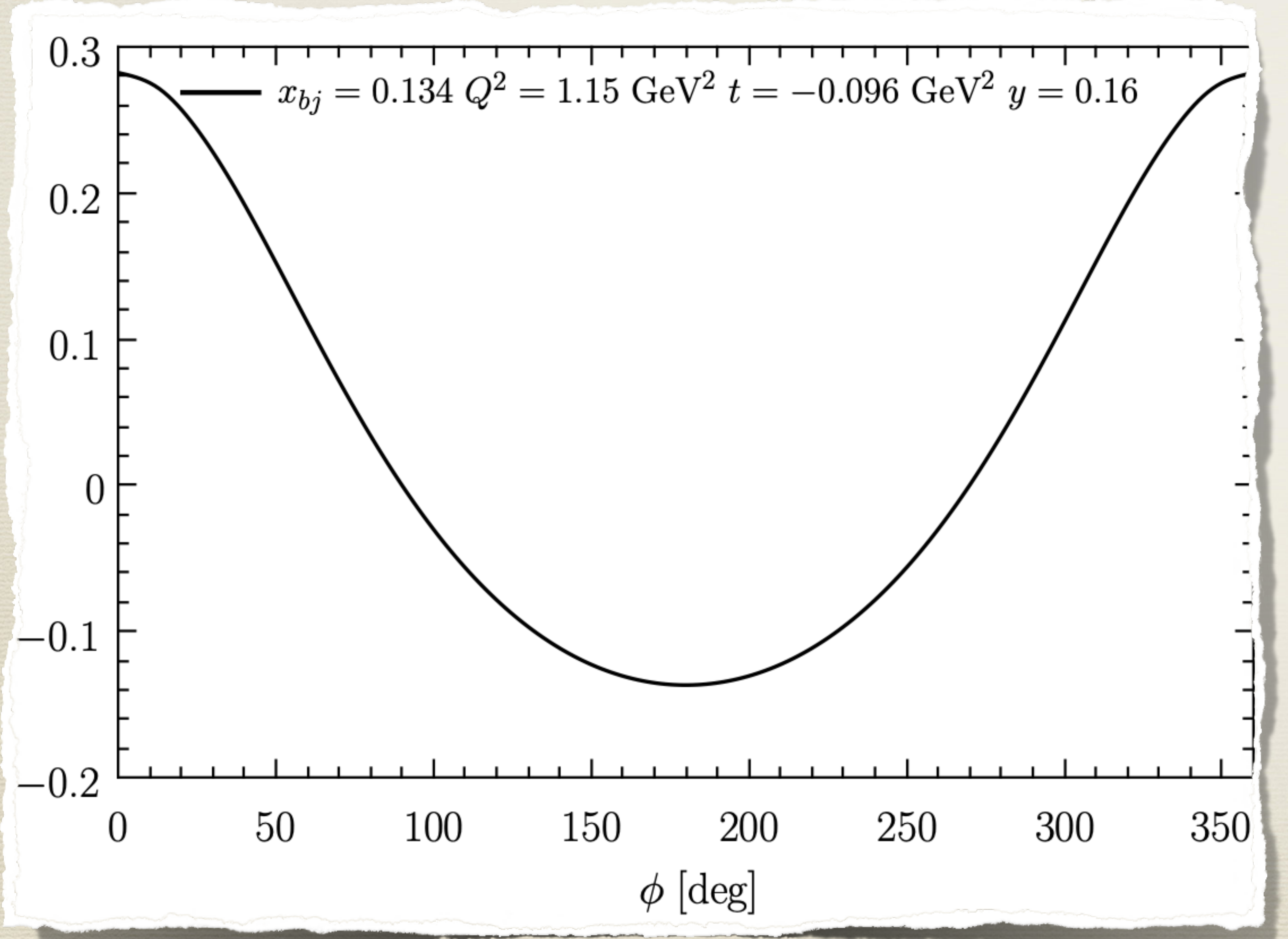
$$\Im m\mathcal{H}_A(\xi, t) = \sum_{q=u,d,s} e_q^2 (H_q^A(\xi, \xi, \Delta^2) - H_q^A(-\xi, \xi, \Delta^2))$$

Asymmetries for ^{3}He

S. Fucini, M. R. and S. Scopetta, EPJA 57 (2021) 9, 273

The BCA:

$$\mathrm{BCA}_3(\phi) = \frac{x_B(1+\varepsilon^2)^2}{y} \frac{c_1^I \cos(\phi)}{c_0^{BH} + c_1^{BH}\cos(\phi)}$$



Variables:

- y = lepton energy fraction
- ϕ = the azimuthal angle between the lepton plane and the recoiled nucleus
- c_0^{BH} and c_1^{BH} depend on EM FFs
- $\epsilon = 2x_{Bj}M_3/Q$
- c_1^I depends on Real part of CFFs

Real part of CFFs are essential quantities!

$$\Re\mathfrak{e}\mathcal{F}(\xi,\mathrm{t}) \propto \mathcal{P}\int_0^1 \mathrm{dx}\, \Im\mathfrak{m}\mathcal{F}(\mathrm{x},\mathrm{t})\left[\frac{1}{\mathrm{x}+\xi} + \frac{1}{\mathrm{x}-\xi}\right] - \delta_\mathrm{t}$$

Asymmetries for ^{3}He

S. Fucini, M. R. and S. Scopetta, EPJA 57 (2021) 9, 273

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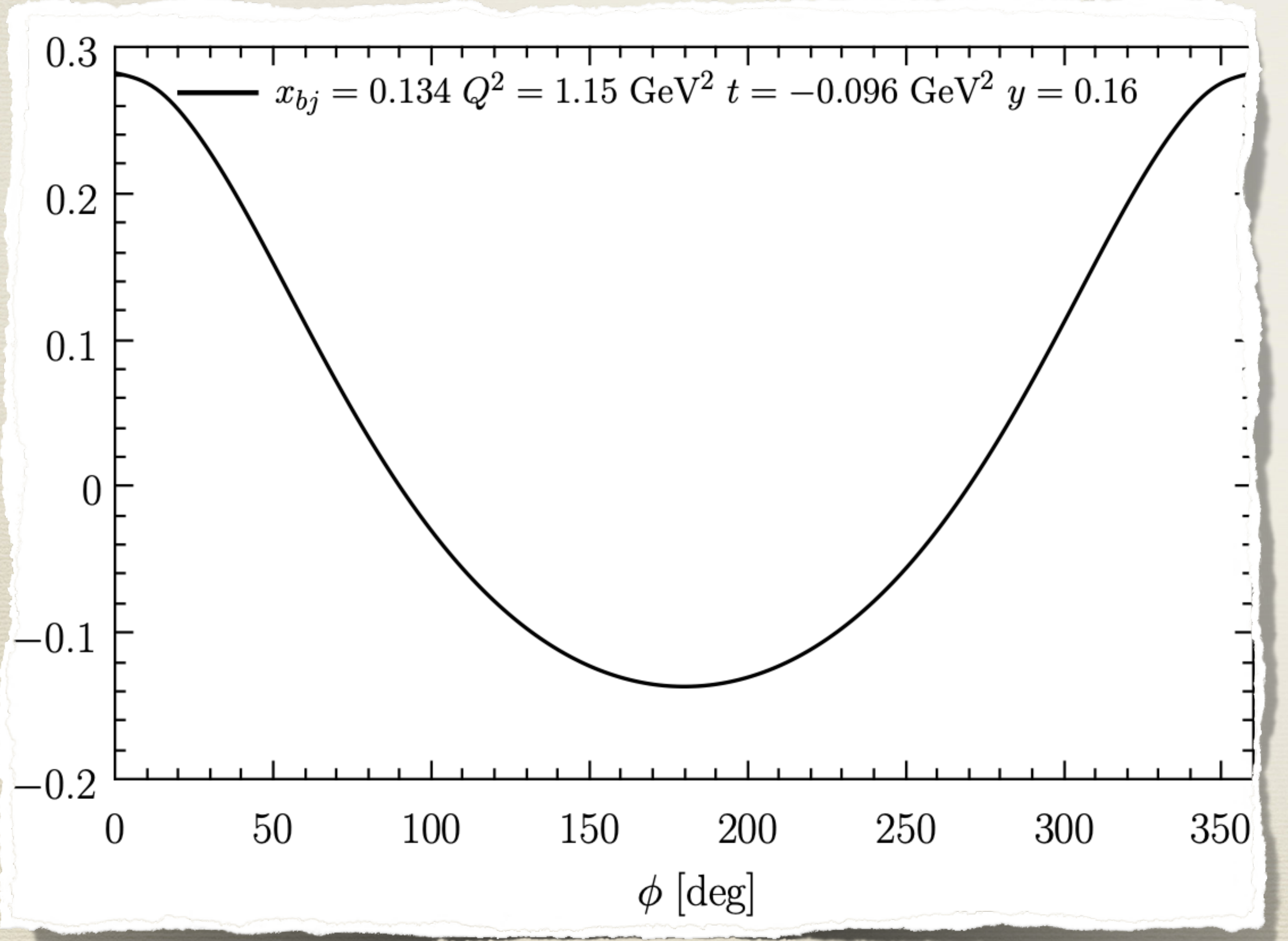

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Real part of CFFs are essential quantities!

$$\mathfrak{Re}\mathcal{F}(\xi,t) \propto \mathcal{P}\int_0^1 dx\, \mathfrak{Im}\mathcal{F}(x,t)\left[\frac{1}{x+\xi} + \frac{1}{x-\xi}\right] - \delta_t$$

This term is related to the *d-term* which encodes information on the mechanical properties of hadrons!

Asymmetries for ^{3}He

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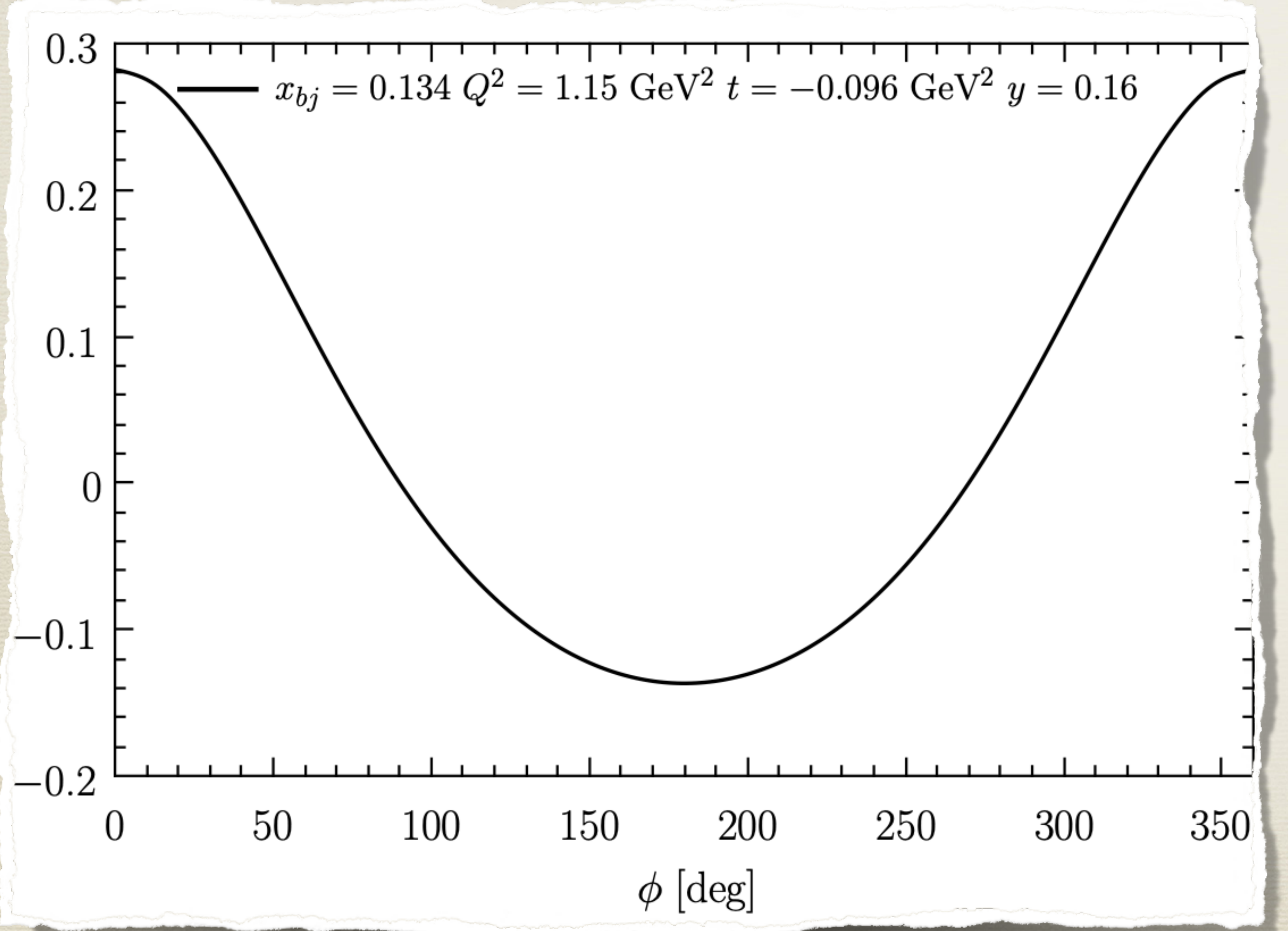

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Real part of CFFs are essential quantities!

BCA BSA

$$\Re\mathfrak{e}\mathcal{F}(\xi,t) \propto \mathcal{P}\int_0^1 dx\, \Im\mathfrak{m}\mathcal{F}(x,t)\left[\frac{1}{x+\xi}+\frac{1}{x-\xi}\right] - \delta_t$$

We need combined informations!

Asymmetries for ^{3}He

S. Fucini, M. R. and S. Scopetta, EPJA 57 (2021) 9, 273

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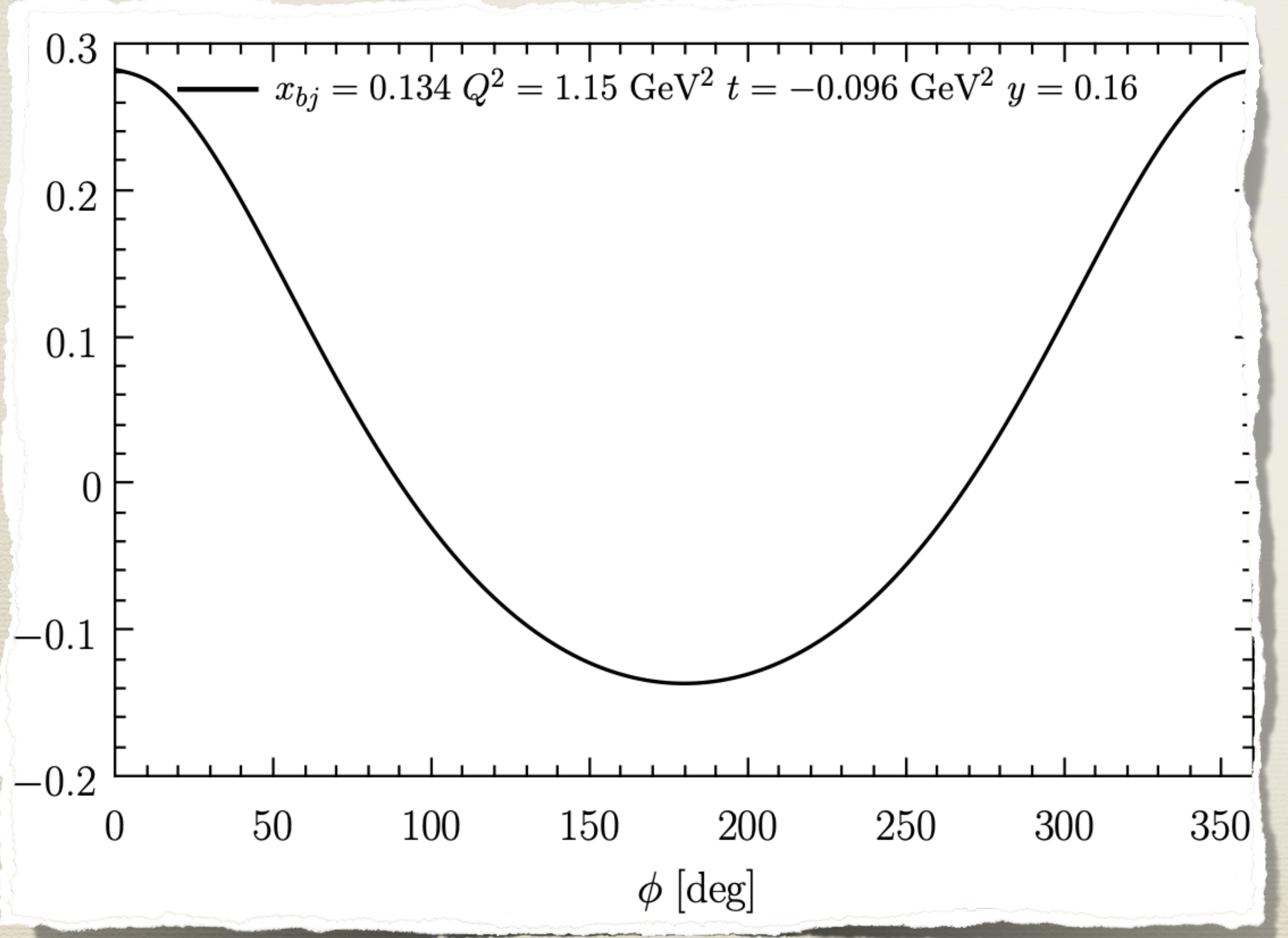

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BCA can be extracted from positron initiated deeply virtual Compton scattering
A. Accardi et al, EPJA 57 (2021) 8,261

Asymmetries for ^{4}He

S. Fucini, S.Scopetta and M. Viviani, PRC 98 (2018) 015203)

S. Fucini, M. R. and S.Scopetta, Few Body Syst. 63 (2021) 9, 273

For ^{4}He (J=0) we have only GPD:

*details on spectral functions in Backup slides

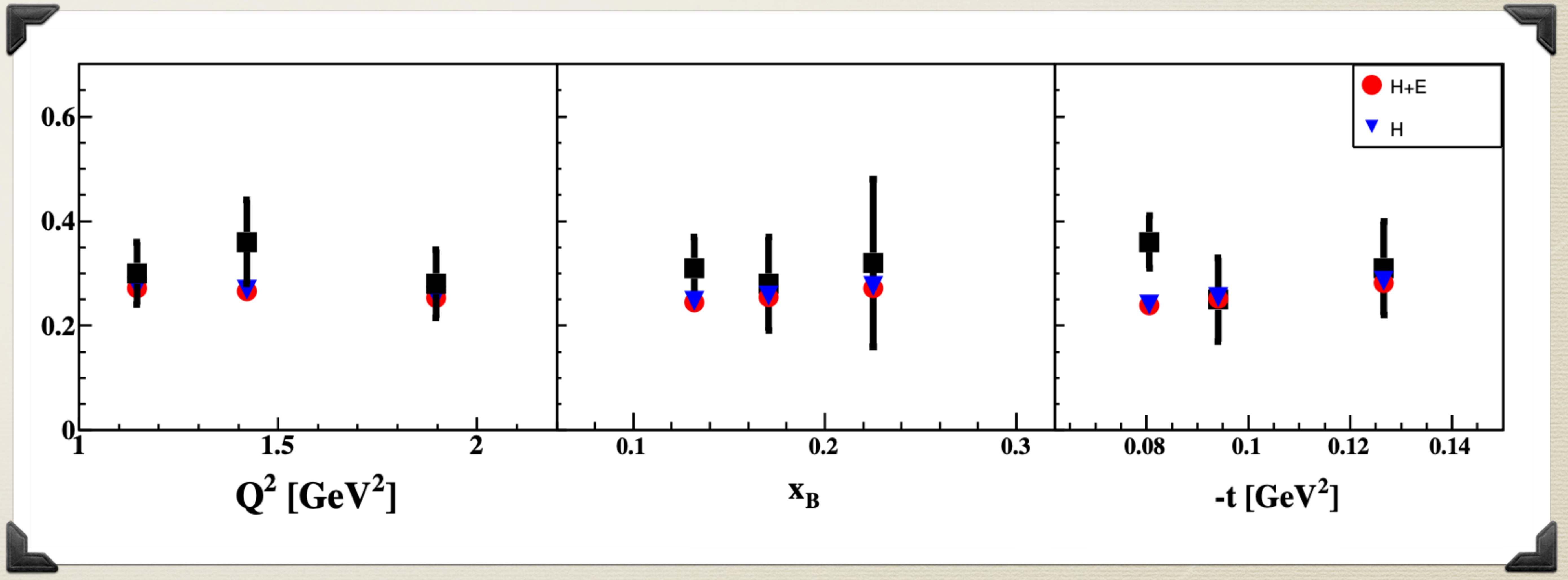
$$H_q^4\left(x,\xi,\Delta^2\right) = \sum_N \int_{|x|}^{1} \frac{dz}{z} h_N^4\left(z,\xi,\Delta^2\right) \quad \mathbf{H_q^N}\left(\frac{x}{\zeta},\frac{\xi}{\zeta},\Delta^2\right)$$

with: $\mathbf{H}_q^N(x,\xi,t) = \sqrt{1-\xi^2}\left[H_q^N(x,\xi,t) - \frac{\xi^2}{1-\xi^2}E_q^N(x,\xi,t)\right]$

We define:

$$\mathrm{BSA}_4 = \frac{\alpha_0(\phi)\Im\mathrm{m}\left(\mathcal{H}_A\right)}{\alpha_1(\phi)+\alpha_2(\phi)\Re\mathrm{e}\left(\mathcal{H}_A\right)+\alpha_3(\phi)\left(\Re\mathrm{e}\left(\mathcal{H}_A\right)^2+\Im\mathrm{m}\left(\mathcal{H}_A\right)^2\right)}$$

- ✔ $H^4(x,\xi,t) = \sum_q e_q^2 H_q^4(x,\xi,t)$
- ✔ $\Im\mathrm{m}\mathcal{H}_A(\xi,t) = H^4(\xi,\xi,t) - H^4(-\xi,\xi,t)$
- ✔ $\Re\mathrm{e}\mathcal{H}_A(\xi,t) = \mathcal{P}\int_{-1}^{1} dx \frac{H^4(x,\xi,t)}{x-\xi+i\varepsilon}$
- ✔ $\alpha_i(\phi)$ **A. V. Belitsky et al., PRD (2009)**



■ JLab data: Hattawy et al, PRL 119 (2017), 202004

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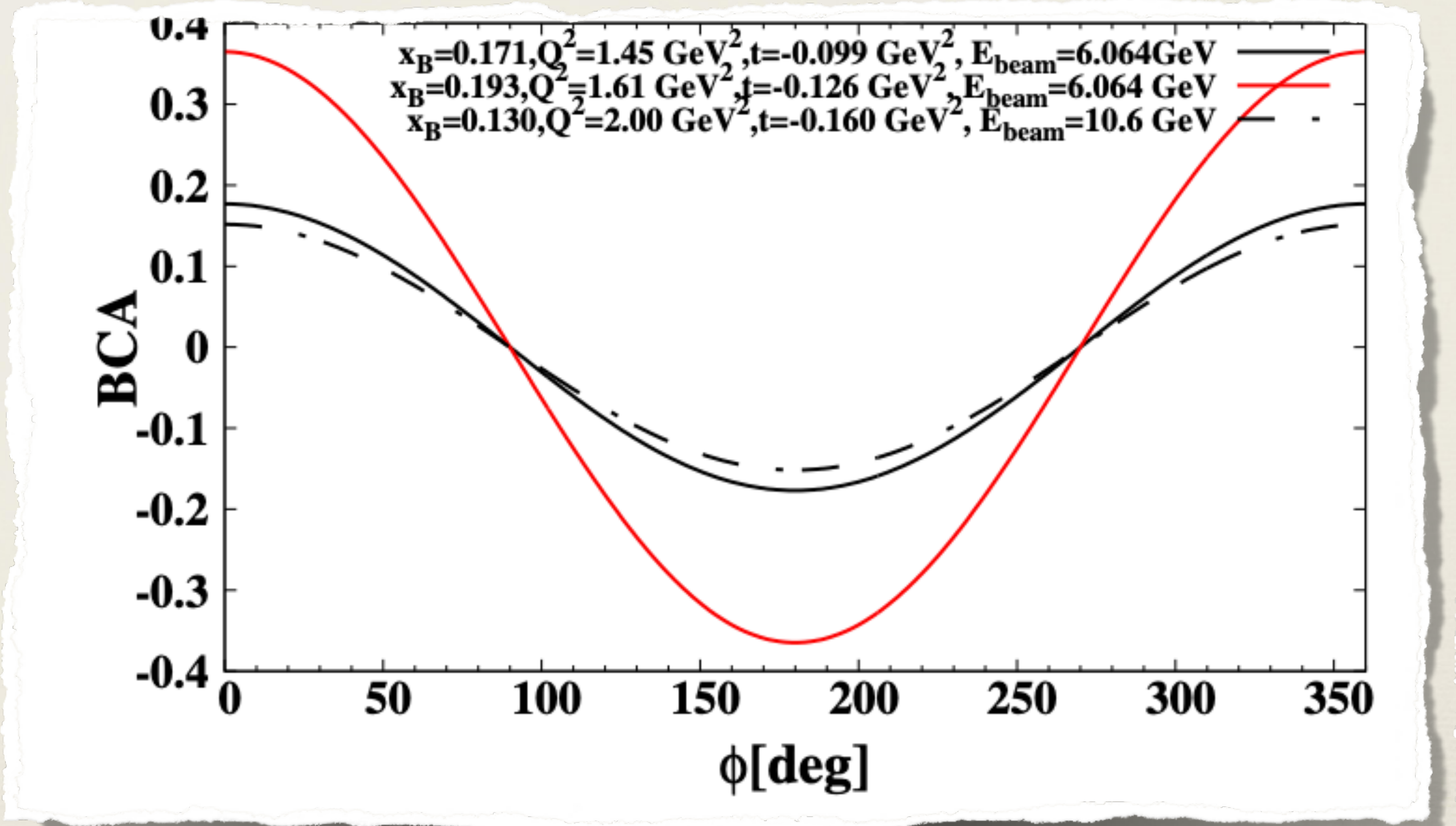
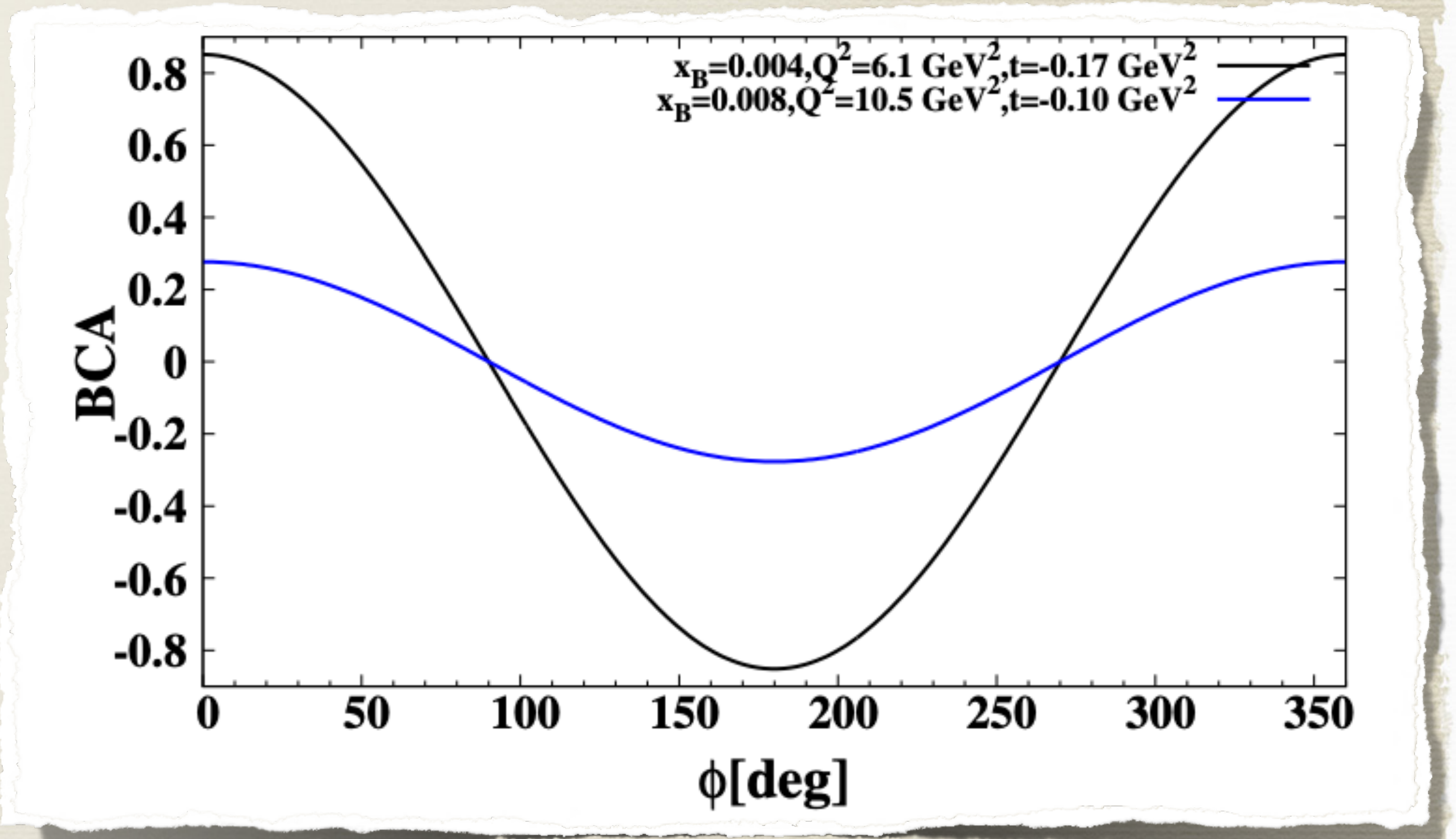
We define:

- ✔ $\mathrm{H}^4(\mathrm{x},\xi,\mathrm{t}) = \sum_\mathrm{q} \mathrm{e}_\mathrm{q}^2 \mathrm{H}_\mathrm{q}^4(\mathrm{x},\xi,\mathrm{t})$
- ✔ $\mathfrak{Im}\mathcal{H}_A(\xi,\mathrm{t}) = \mathrm{H}^4(\xi,\xi,\mathrm{t}) - \mathrm{H}^4(-\xi,\xi,\mathrm{t})$
- ✔ $\mathfrak{Re}\mathcal{H}_A(\xi,\mathrm{t}) = \mathcal{P}\int_{-1}^1 \mathrm{dx}\, \frac{\mathrm{H}^4(\mathrm{x},\xi,\mathrm{t})}{\mathrm{x}-\xi+\mathrm{i}\varepsilon}$
- ✔ $\alpha_\mathrm{i}(\phi)$ **A. V. Belitsky et al., PRD (2009)**

Predictions for BCA asymmetry

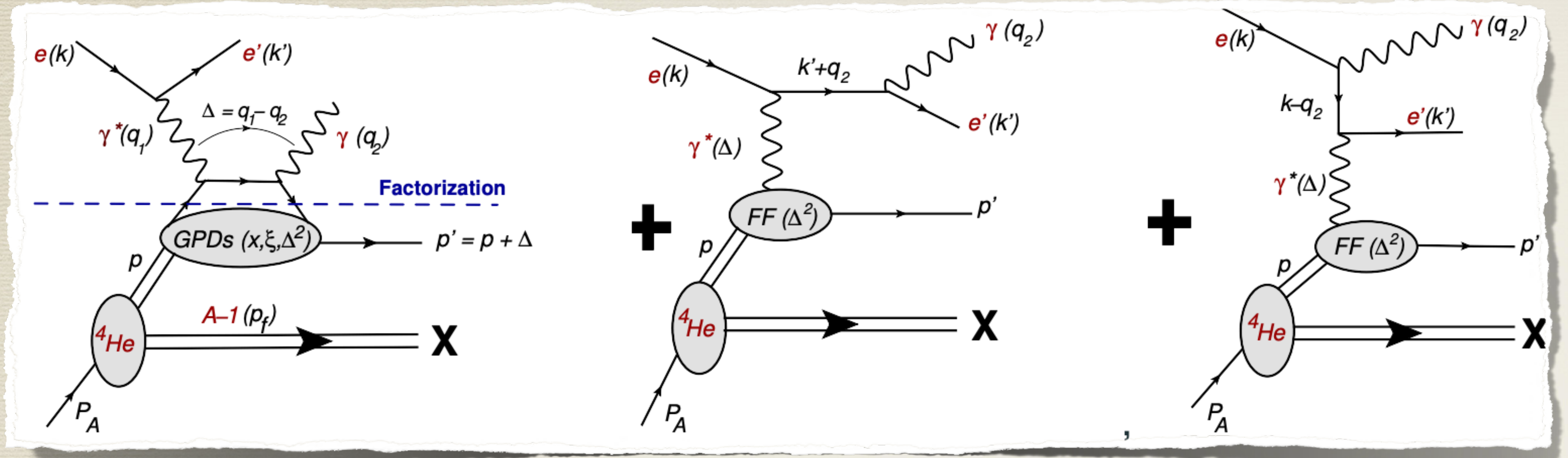




Incoherent DVCS off ^{4}He

S. Fucini, S.Scopetta and M. Viviani, PRC 98 (2018) 015203)

In this case we detect a nucleon:



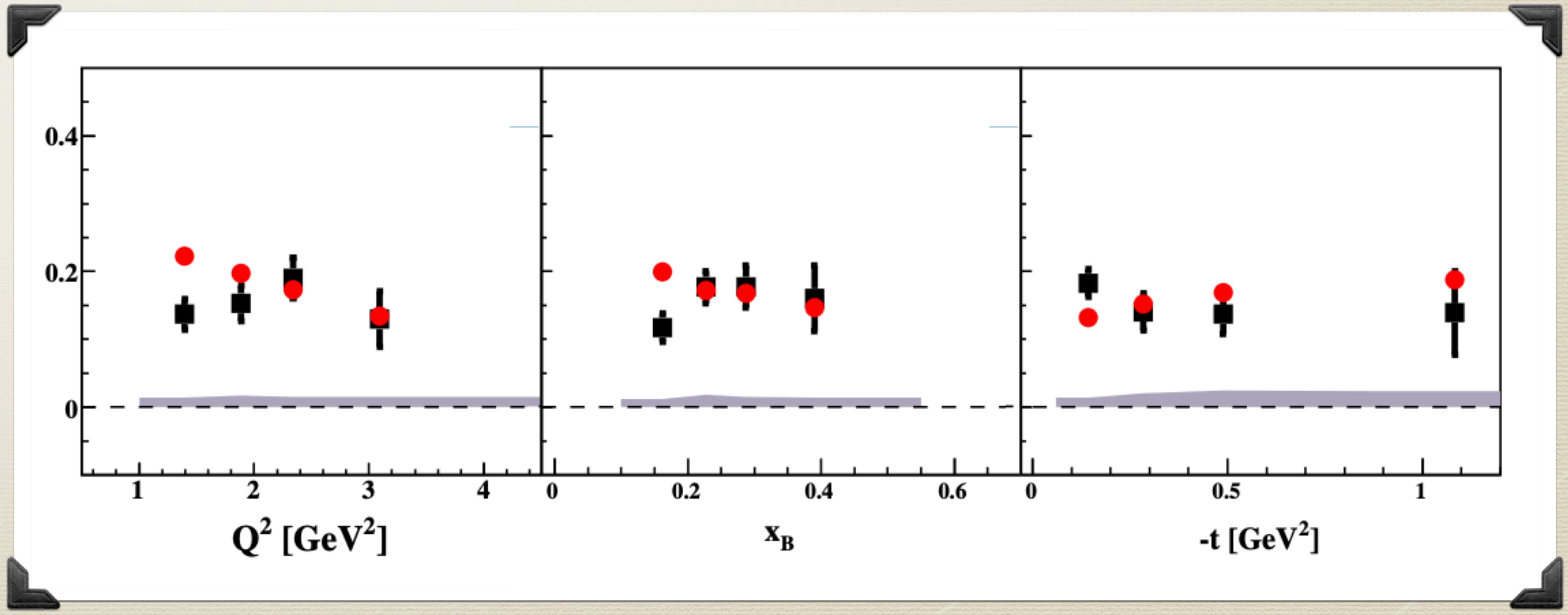
The nucleon is off-shell:

$$p_0 = M_A - \sqrt{M_{A-1}^{*2} + \vec{p}^2} \sim M_N - E - T_{ref} \Rightarrow p^2 \neq m^2$$

In a frame where the target nucleus is at rest, the cross section and its azimuthal dependence are expressed in terms of a convolution formula.

$$BSA_4 = \frac{\int_{exp} dE d\vec{p}\, P^4(\vec{p}, E) g(\vec{p}, E, K) \mathcal{I}(\vec{p}, E, K)}{\int_{exp} dE d\vec{p}\, P^4(\vec{p}, E) g(\vec{p}, E, K) T_{BH}^2(\vec{p}, E, K)}$$

Encode the $\Im m \mathcal{H}^4$ of the bound nucleon



Good agreement in the region of high Q^2

Incoherent DVCS off ^{4}He

S. Fucini, S.Scopetta and M. Viviani, PRC 98 (2018) 015203)

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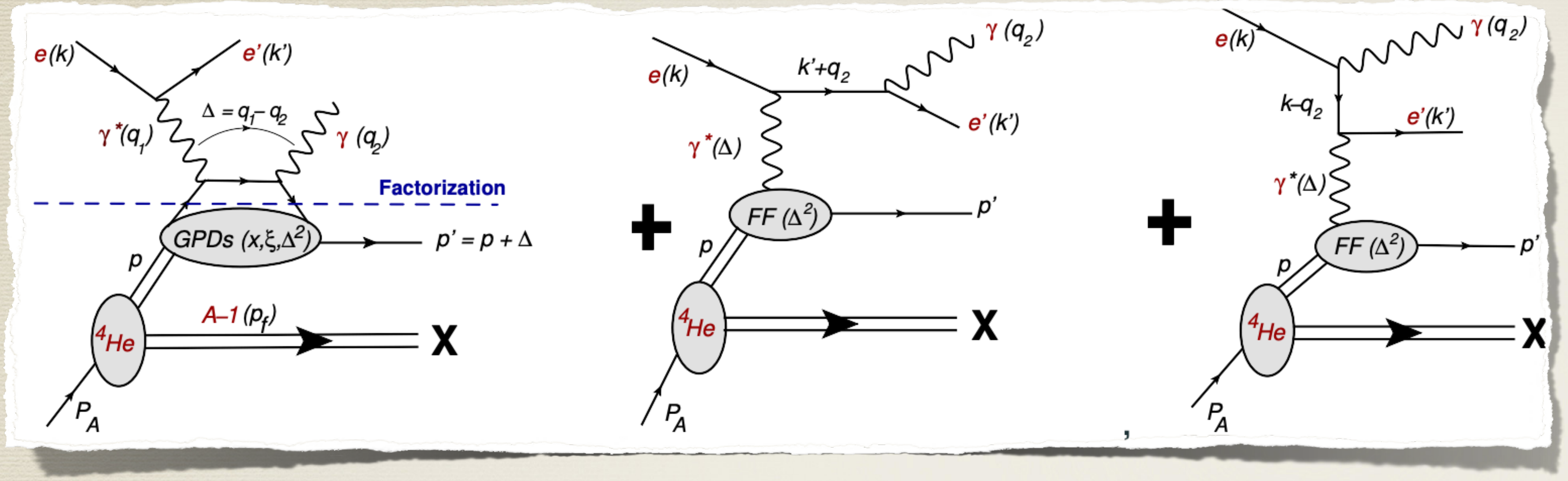

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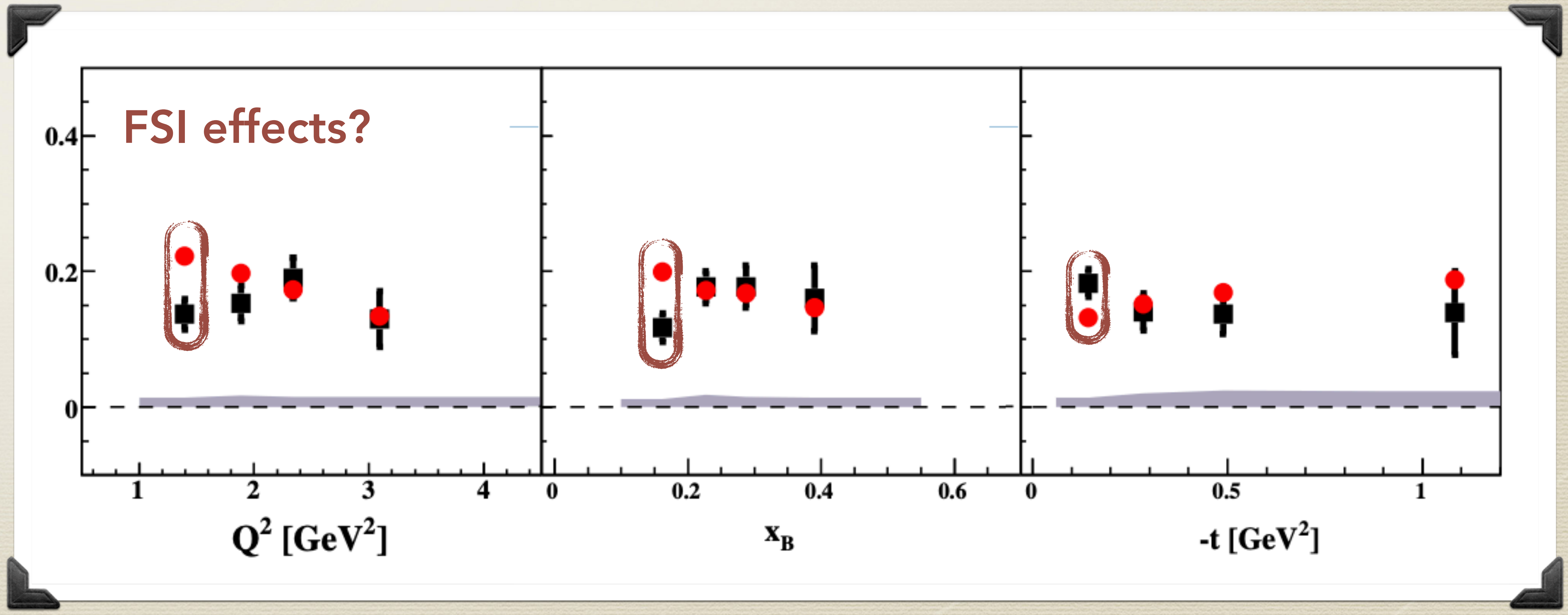
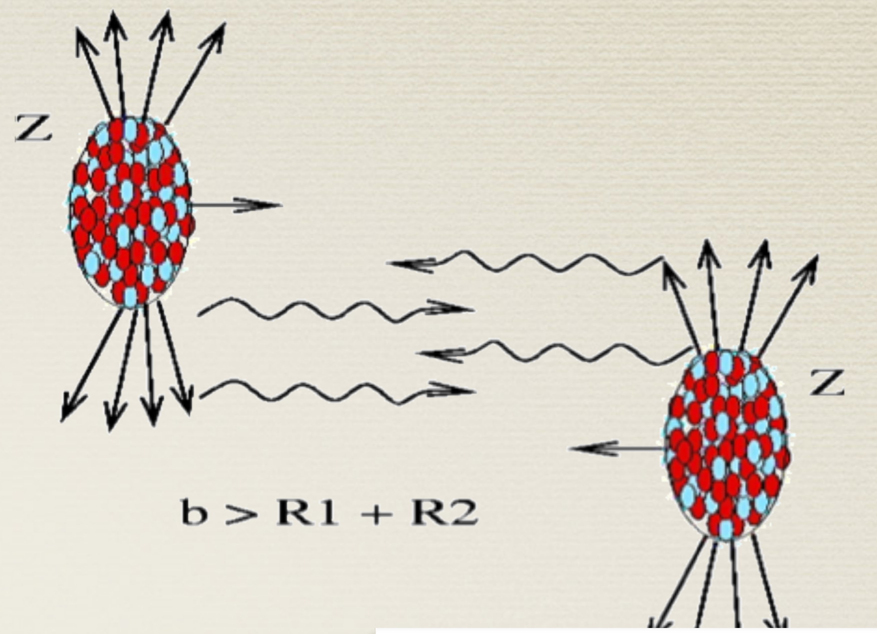


Good agreement in the region of high Q^2

J/ψ electroproduction on light-nuclei

Gluon shadowing in UPC collisions @ LHC

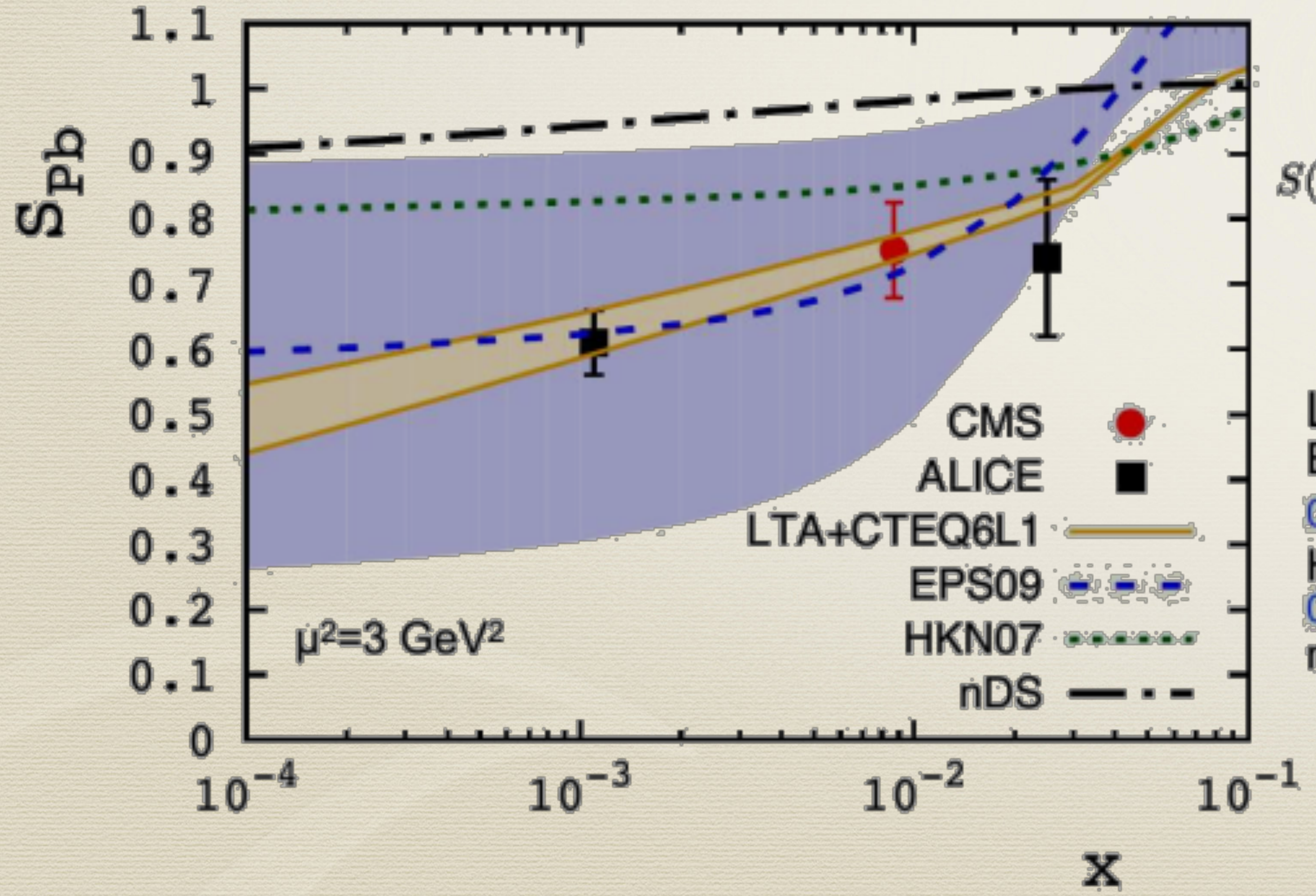


Large (up to 40%) Leading twist (LT) shadowing in:

$\gamma + \text{Pb/Au} \to \rho(J/\Psi) + \text{Pb/Au}$ Explained/predicted

(**Frankfurt, Guzey, Strikman Phys. Rep. 512 (2012) 255**)

Abbas *et al.* [ALICE], EPJ C 73 (2013) 2617; CMS Collab., PLB 772 (2017) 489 ⟶ suppression factor S_{Pb}



$$S(W_{\gamma p}) = \left[\frac{\sigma_{\gamma Pb \to J/\psi Pb}}{\sigma^{IA}_{\gamma Pb \to J/\psi Pb}}\right]^{1/2} = \kappa_{A/N}\frac{G_A(x,\mu^2)}{AG_N(x,\mu^2)}$$

LTA: Guzey, Zhalov JHEP 1310 (2013) 207
EPS09: Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065
HKN07: Hirai, Kumano, Nagai, PRC 76 (2007) 065207
nDS: de Florian, Sassot, PRD 69 (2004) 074028

Introduction. Studies of nuclear shadowing have a long history [1–5]. In quantum mechanics and in the eikonal limit, it is manifested in the total hadron-nucleus cross section being smaller than the sum of individual hadron-nucleon cross sections. In essence, this is due to simultaneous interactions of the projectile with $k \geq 2$ nucleons of the nuclear target, leading to a reduction (shadowing) of the total cross section. In this frame-

J/ψ electroproduction on light-nuclei

- Problem:
  @ EIC/LHC it is challenging to measure coherent scattering at $t \neq 0$ for $A \simeq 200$; Large coherence length: information on interactions with many nucleons, in average

- Solution:
  use the lightest nuclei, especially ^3He and ^4He, to study coherent effects for interactions with exactly 2 nucleons in the range of $0 < -t < 0.5$ GeV2 .

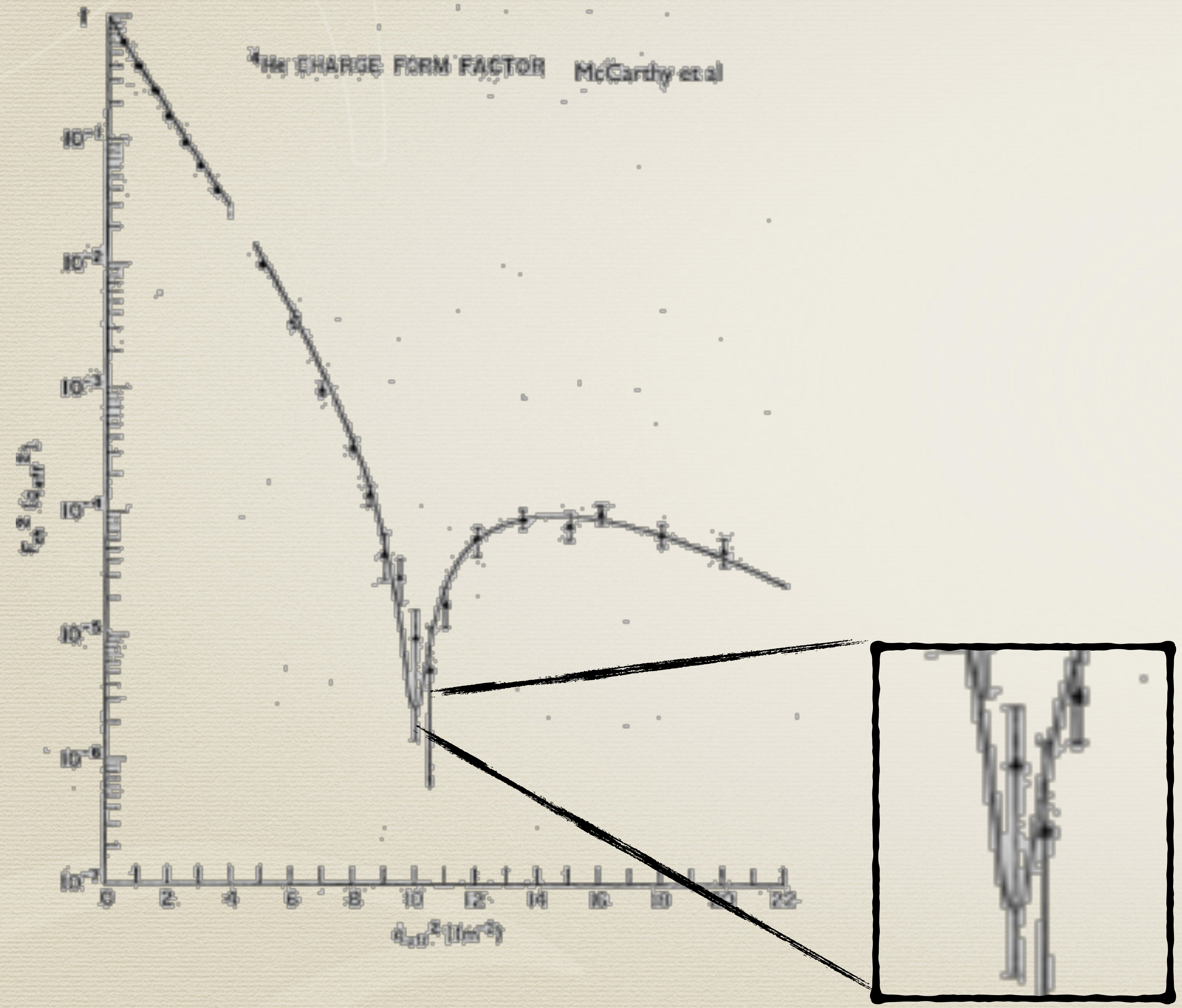
Complementary measurements with light ion beams @ the EIC:

- Scattering off 2 and 3 nucleons can be separately probed
- no excited states -> easy to select coherent events

Here:

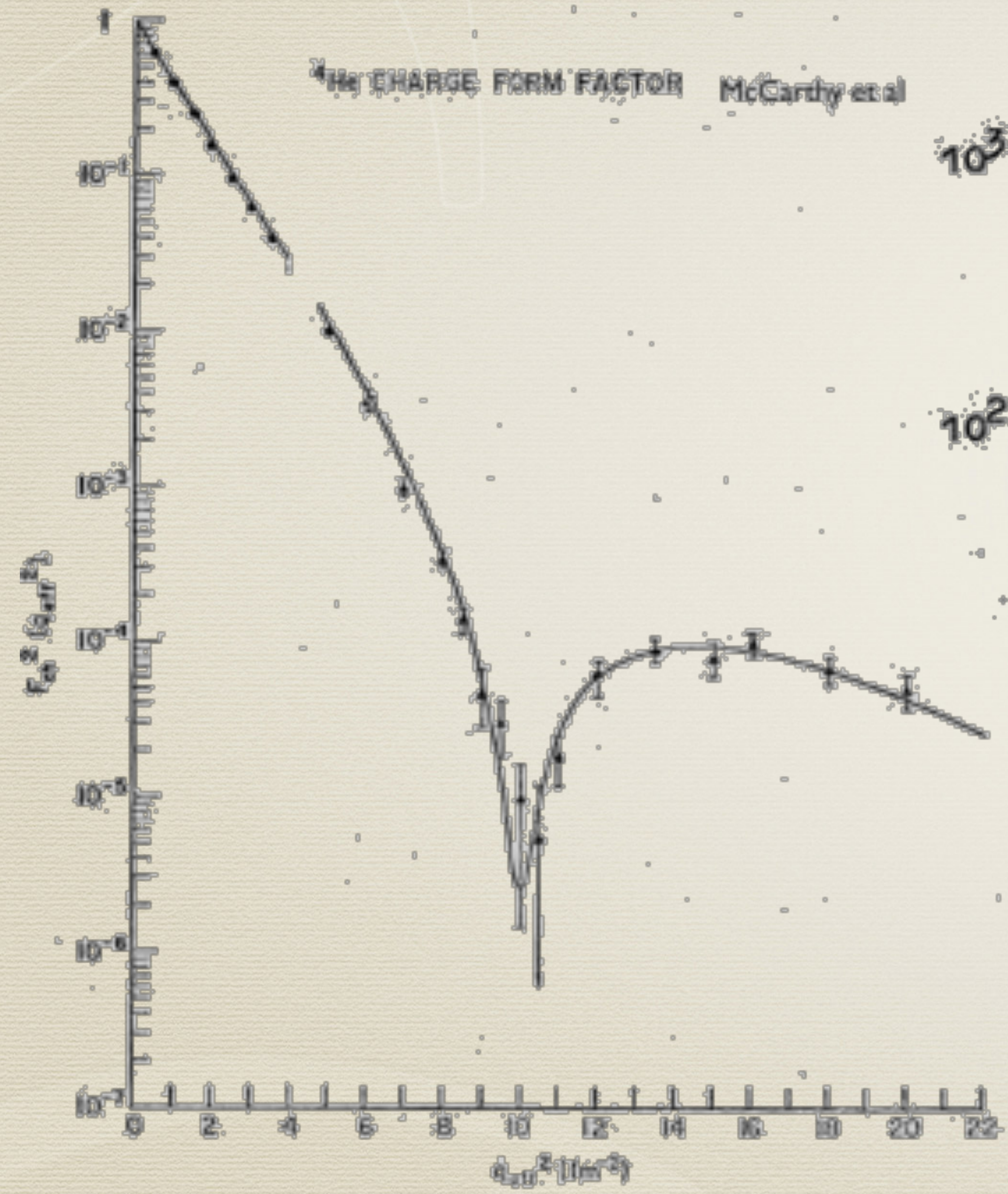
Results on J/Ψ diffractive electro-production off ^3He – ^4He
V. Guzey, M. R., S. Scopetta, M. Strikman and M. Viviani, PRL 129 (2022) 24, 24503

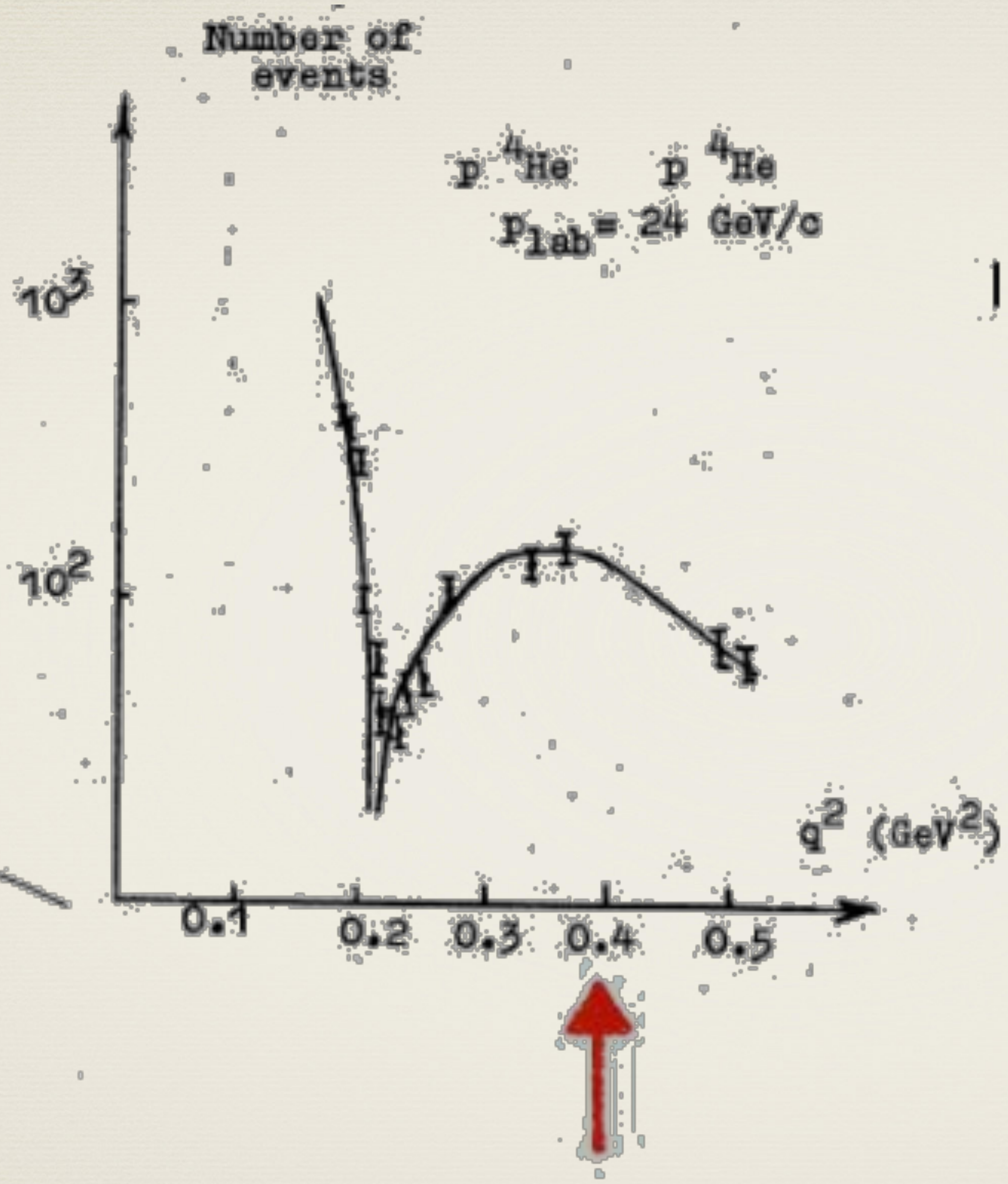
J/ψ electroproduction on light-nuclei



- ^{4}He charge FF, dominated by one-body dynamics (IA) presents the first diffraction minimum at: $-t \simeq 0.4$ GeV^2

J/ψ electroproduction on light-nuclei





no contribution of impulse approximation term

- ^{4}He charge FF, dominated by one-body dynamics (IA) presents the first diffraction minimum at: $-t \simeq 0.4$ GeV^2

- around this value of t, the cross section in p +^{4}He -> p +^{4}He is dominated by effects beyond IA: multinucleon interactions, gluon shadowing for hard processes

J/ψ electroproduction on light-nuclei

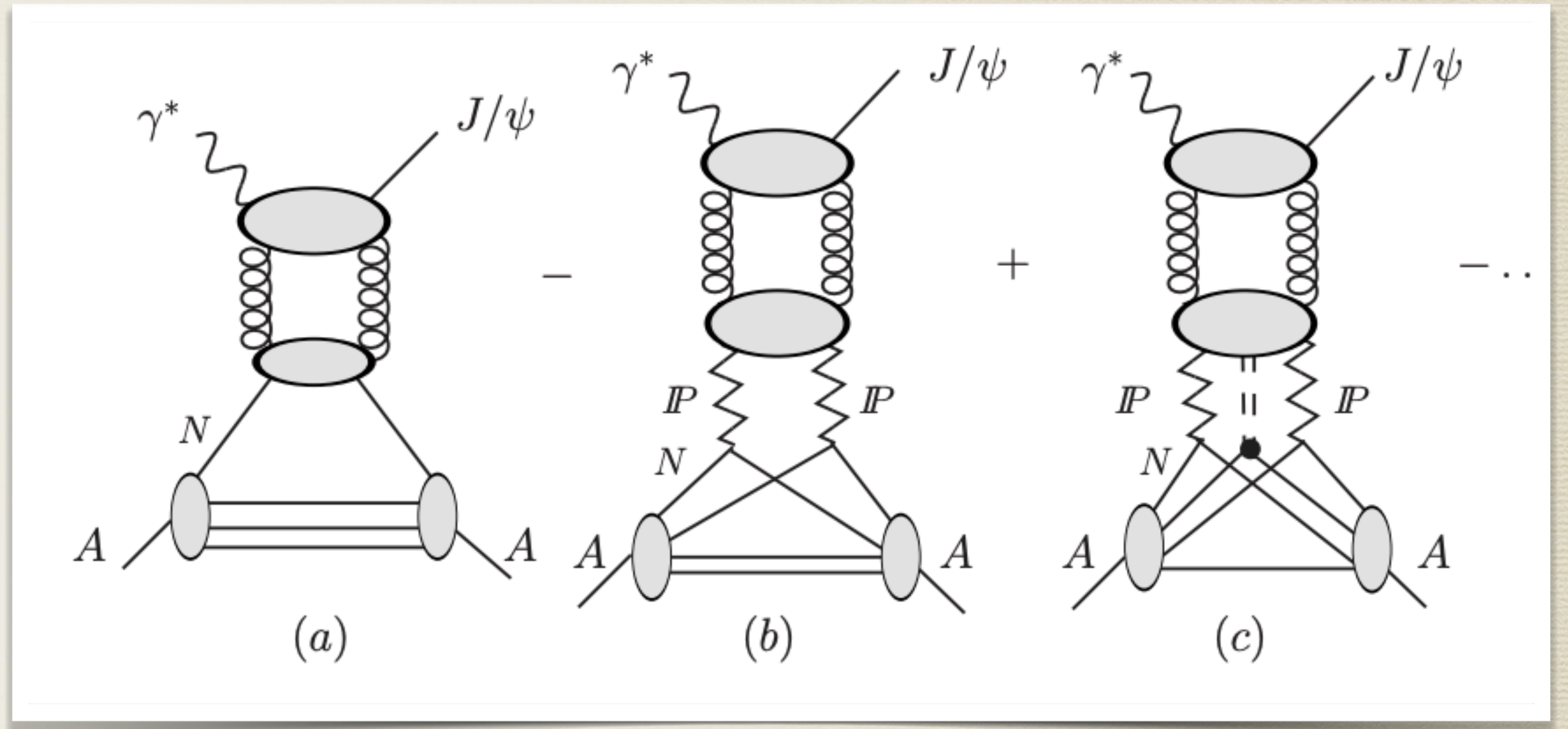
$$\frac{d\sigma_{\gamma^* A\to VA}}{dt} = \frac{d\sigma_{\gamma^* N\to VN}}{dt}(t=0)\left|F_1(t)e^{(B_o/2)t} + \sum_{k=2}^{4} F_k(t)\right|^2$$

$$F_k(q) = \left(\frac{i}{8\pi^2}\right)^{k-1} C_n^k\, A_k \int \prod_{l=1}^{k} d^2 q_l\, f(q_l)\Phi_k(q,q_l)\,\delta\left(\sum_l q_l - q\right) \quad k = 2,3,4$$

$$F_1(q) = 4\Phi_1(q) \quad f(q_l) = scattering\ amplitude\ for\ J/\Psi N \to J/\Psi N$$

$$A_{k>1} = \frac{<\sigma^k>}{<\sigma>}\frac{(1-i\eta)^k}{1-i\eta_0};\ the\ same\ used\ in\ UPC\ studies!$$

LT parton shadowing for J/Ψ coherent production off He (gluon GPDs in He)
(**Frankfurt, Guzey, Strikman Phys. Rep. 512 (2012) 255**)



Parameters:

- B_0
- η (η_0) $=Re(f)/Im(f)$ for $\gamma p \to J/\psi p$ ($J/\psi p \to J/\psi p$)

- moments $<\sigma^i>$ chosen for the specific final state and the specific kinematics
  (Guzey et al. PRC 93 (2016) 055206).

  The model has been tested in J/Ψ photoproduction in Pb-Pb UPCs at the LHC(**V. Guzey and M. Zhalov, JHEP 10, 207 (2013)**)

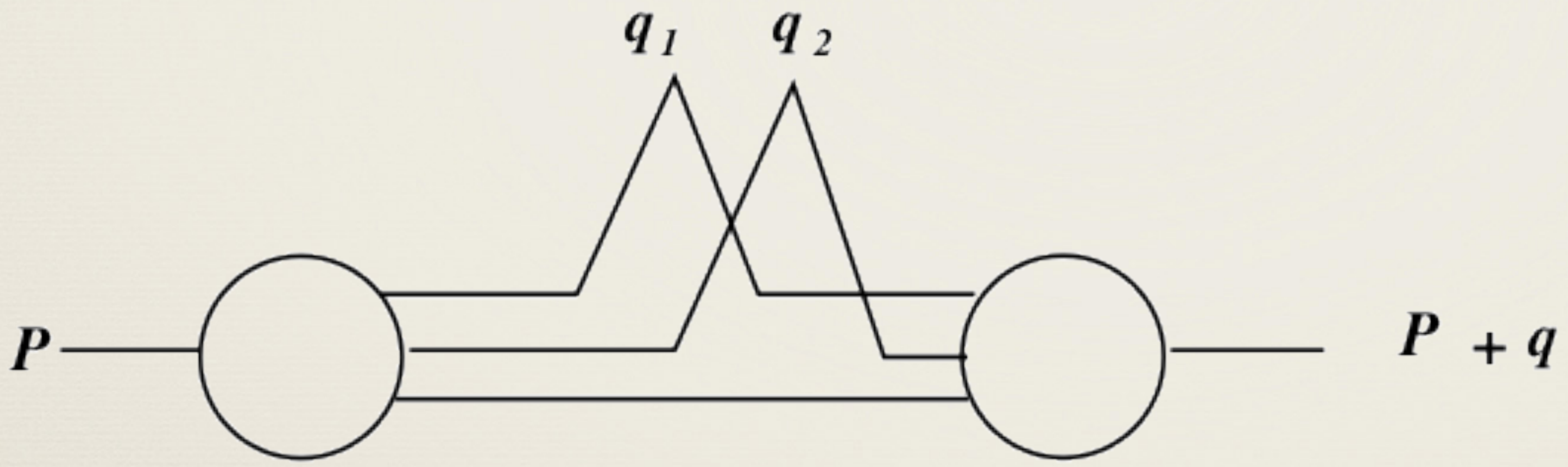
- Φ_k "k-body form factor", is the nuclear input

J/ψ electroproduction on light-nuclei

$$\Phi_k(\vec{q}_1, \dots \vec{q}_k) = \int \prod_{i=N}^{4} \left\{ \frac{d\vec{p}_i}{(2\pi)^3} \right\} \psi^*_{P'}(\vec{p}_1 + \vec{q}_1, \dots \vec{p}_k + \vec{q}_k, \dots, \vec{p}_N) \psi_P(\vec{p}_1, \dots, \vec{p}_k, \dots \vec{p}_N) \delta\left(\sum_{i=1}^{N} \vec{p}_i \right)$$

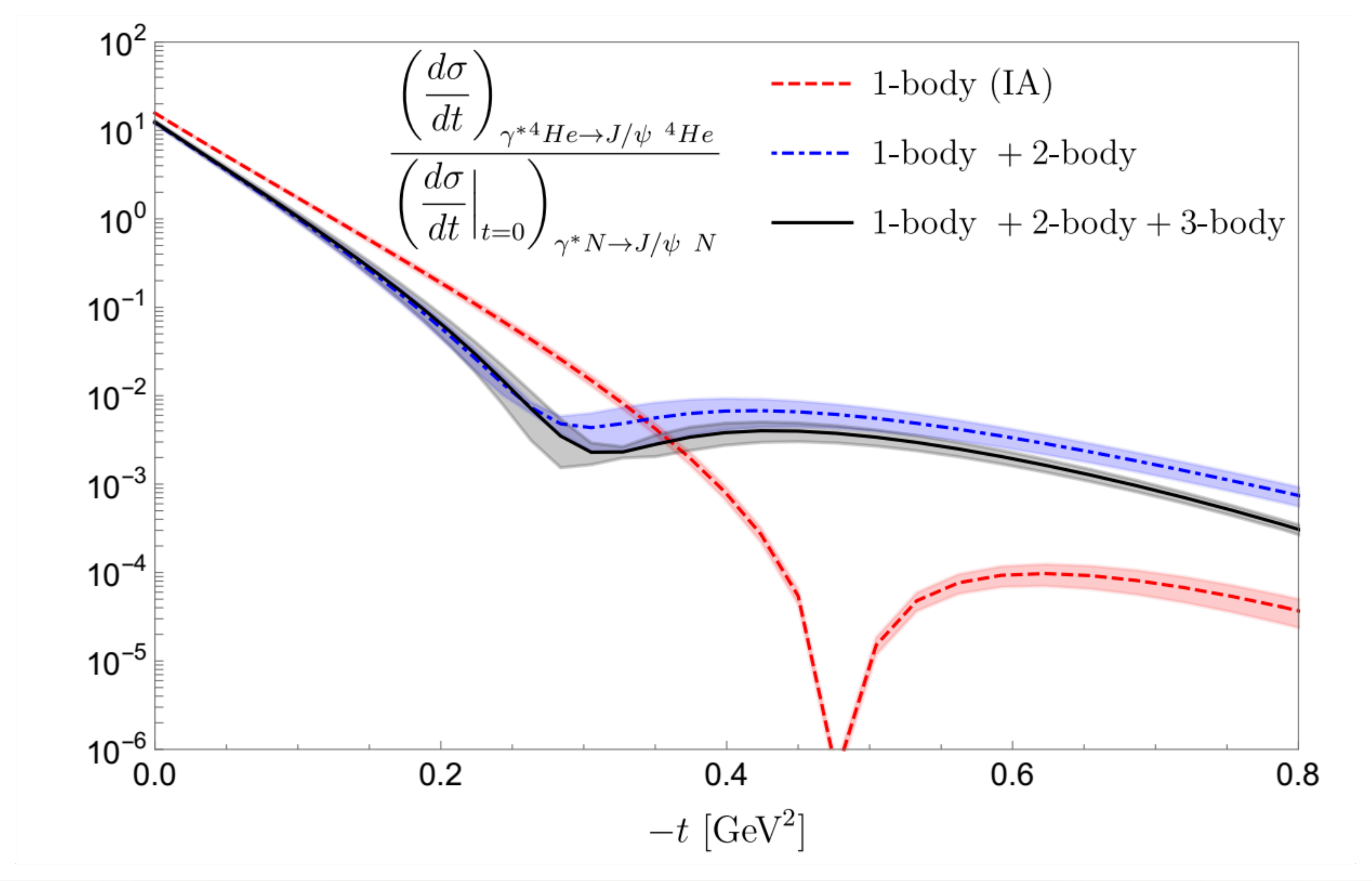
- Φ_1 (IA, very important here), Φ_2 and Φ_3 evaluated using the realistic w. f. obtained by the Pisa group using: *a)* Av18 for ^{3}He *b)* the N4LO chiral potential (D. R. Entem, R. Machleidt, Y. Nosyk, Phys. Rev. C 96, 024004 (2017)) for ^{4}He

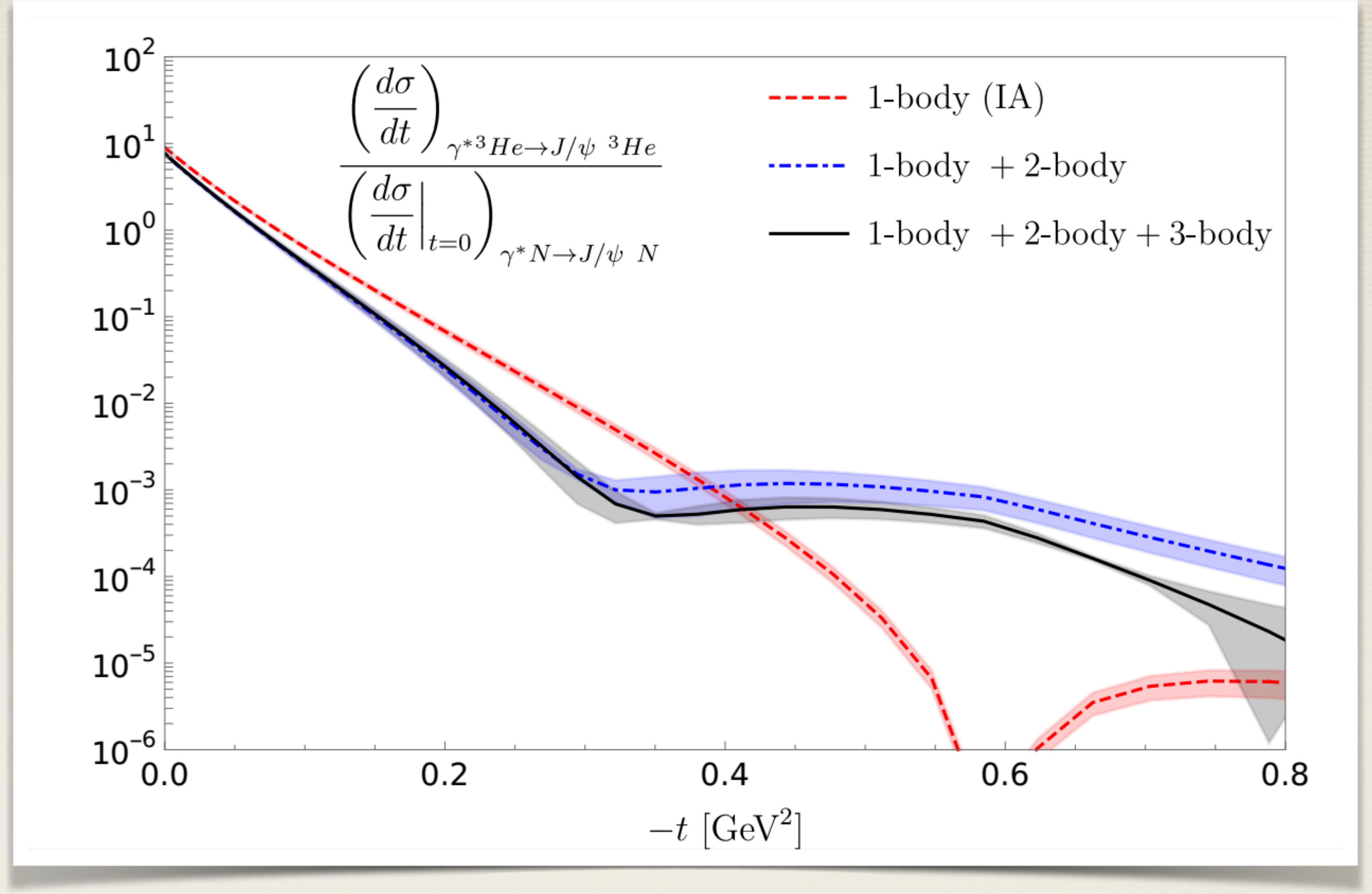
- Example of Φ_2 :



- we remark that $\Phi_2(k_\perp, -k_\perp)$ is the same quantity appearing in the double parton scattering

J/Ψ exclusive production @EIC: xB ≈ 10-3





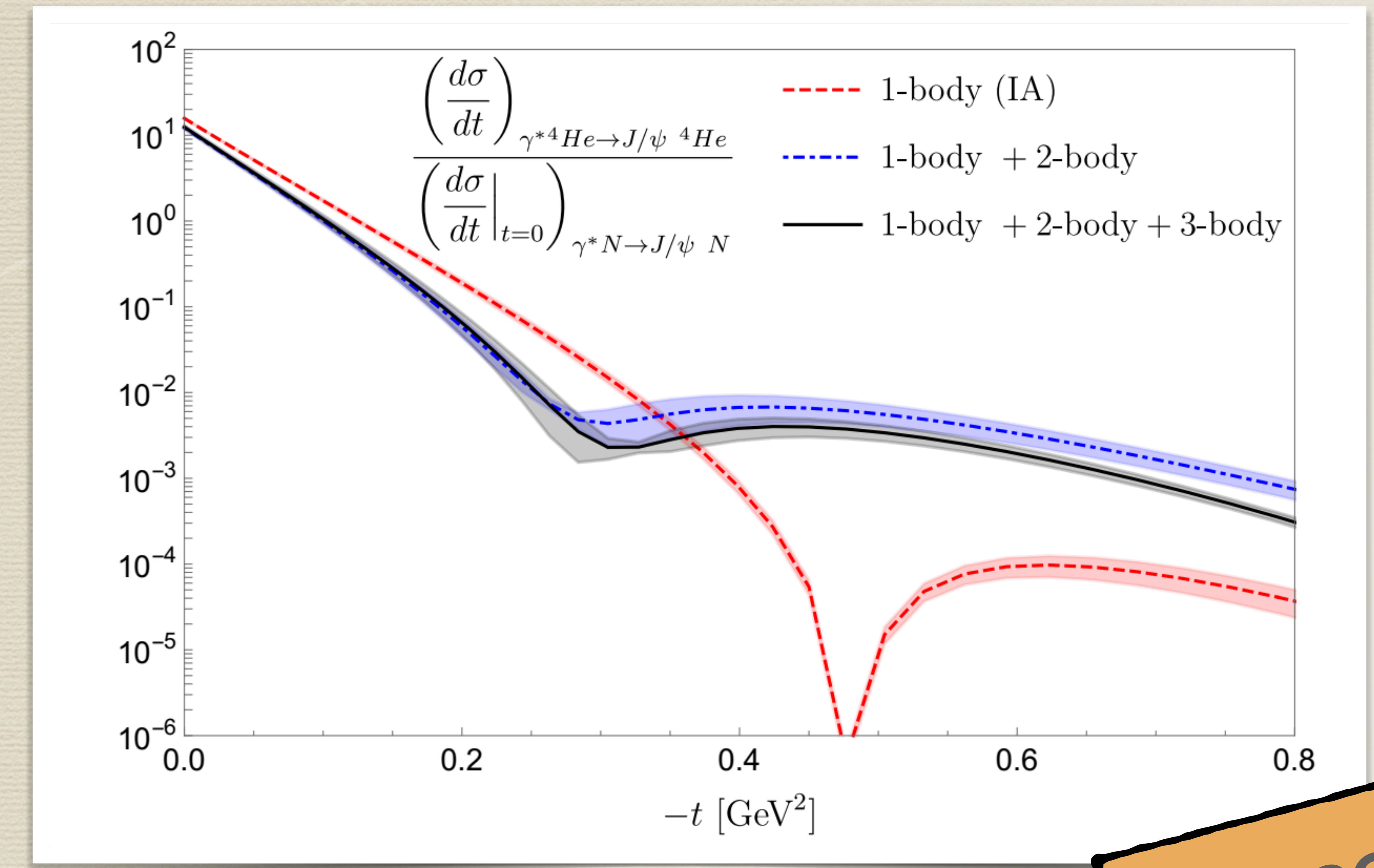
Error bars account:

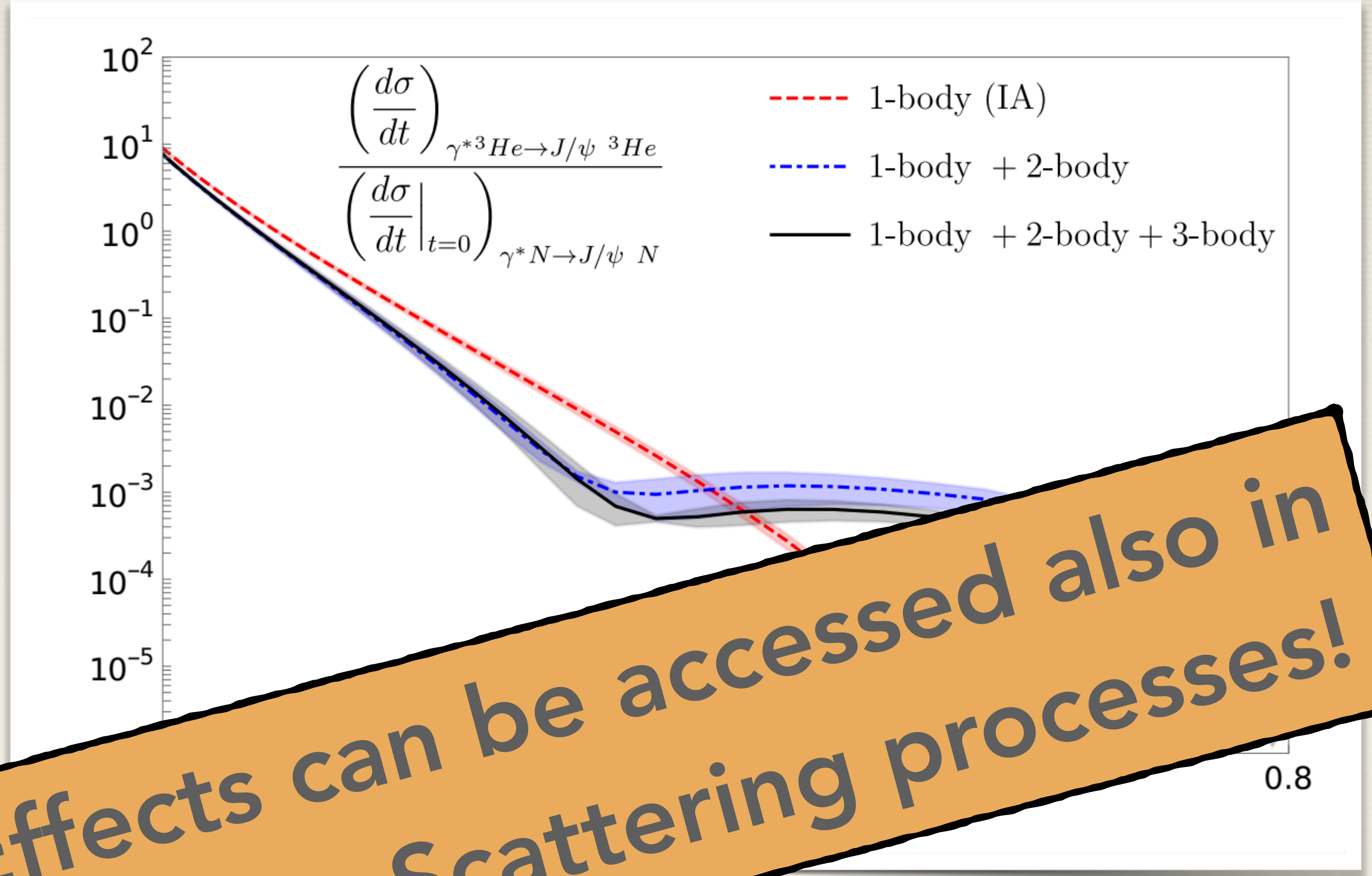
-10% of variation for B_0

-15 of variation in < σ^2 >

- ✔ 1-body + 2-body re-scatterings dominate the cross-sections shift of the minimum due to 2-body dynamics
- ✔ 1-body dynamics under theoretical control: very good chances to disentangle
- ✔ 2-body dynamics (LT gluon shadowing)
- ✔ unique opportunity to access the real part of the scattering amplitudes in a wide range of t
- ✔ The position of the minimum is extremely sensitive to dynamics and the structure!

J/Ψ exclusive production @EIC: xB ≈ 10-3





These Effects can be accessed also in Double Parton Scattering processes!

Error bars account:

-10% of variation for B_0

-15 of variation in < σ^2 >

✔ 1-body + 2-body re-scatterings dominate t… shift of the minimum due to 2-body dynamics

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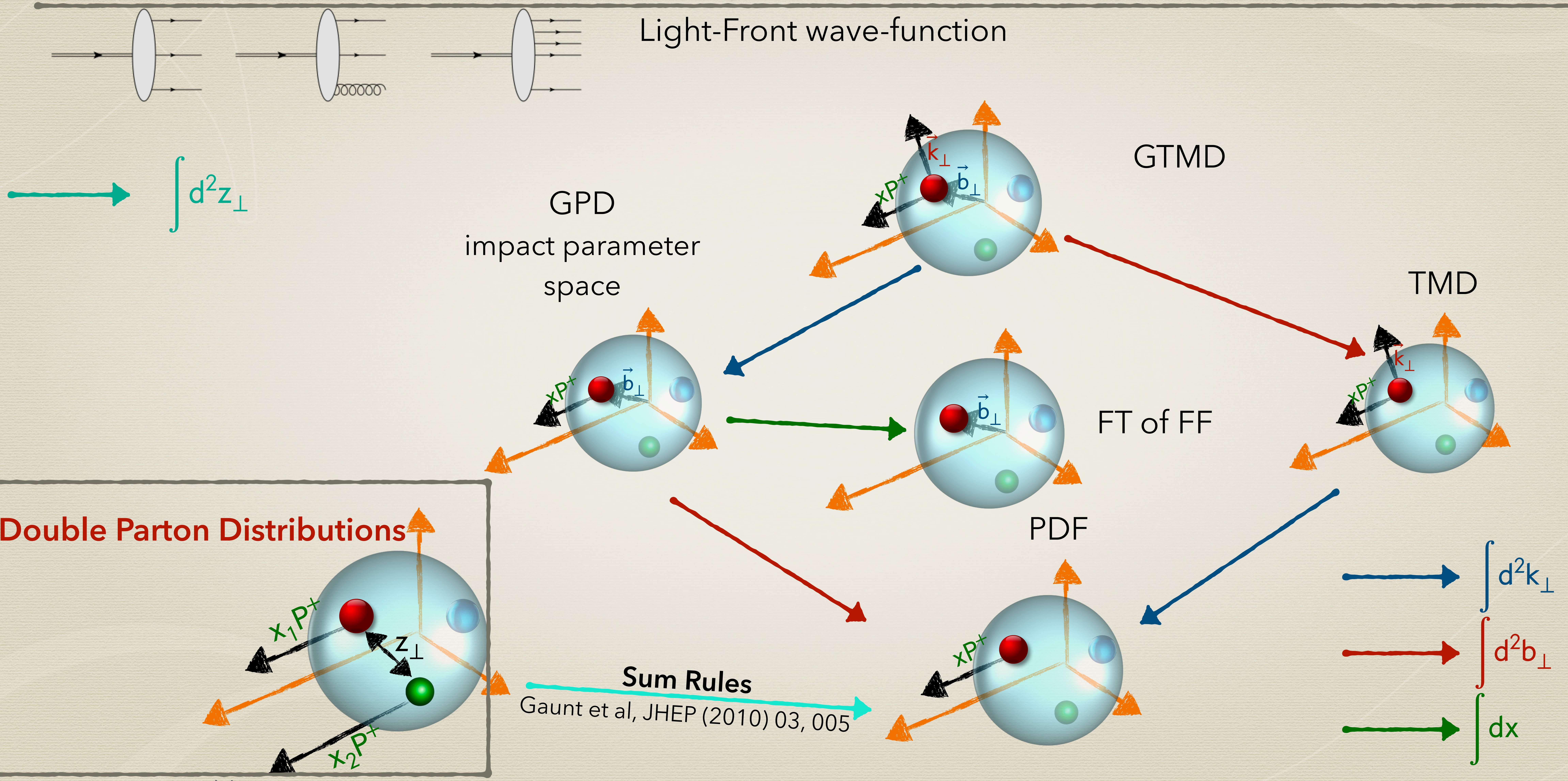
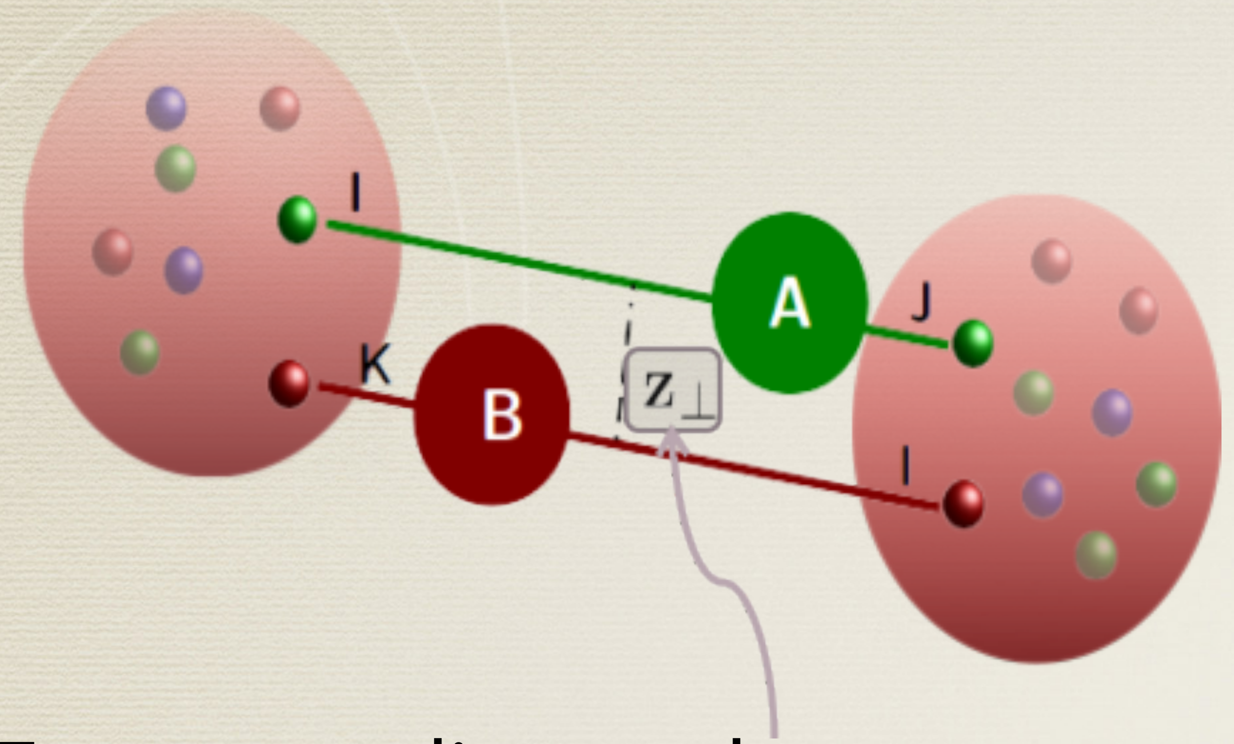
Multidimensional picture of hadrons



Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp, pA and AA cross section @ the LHC:



Transverse distance between two partons

$$d\sigma \propto \int d^2z_\perp \, \underbrace{F_{ij}(x_1, x_2, z_\perp, \mu_A, \mu_B)}F_{kl}(x_3, x_4, z_\perp, \mu_A, \mu_B)$$

Double Parton Distribution (DPD)

N. Paver and D. Treleani, Nuovo Cimento 70A, 215 (1982)
Mekhfi, PRD 32 (1985) 2371
M. Diehl et al, JHEP 03 (2012) 089

$$F_{ij}^{\lambda_1,\lambda_2}(x_1, x_2, \vec{k}_\perp) = (-8\pi P^+)\frac{1}{2}\sum_\lambda \int d\vec{z}_\perp \, e^{i\vec{z}_\perp\cdot\vec{k}_\perp}$$
$$\times \int \left[\prod_l^3 \frac{dz_l^-}{4\pi}\right] e^{ix_1P^+z_1^-/2} e^{ix_2P^+z_2^-/2} e^{-ix_1P^+z_3^-/2}$$
$$\times \langle \lambda, \vec{P} = \vec{0} | \hat{\mathrm{O}}_i^1 \left(z_1^-\frac{\bar{n}}{2}, z_3^-\frac{\bar{n}}{2} + \vec{z}_\perp\right) \hat{\mathrm{O}}_j^2 \left(z_2^-\frac{\bar{n}}{2} + \vec{z}_\perp, 0\right) | \vec{P} = \vec{0}, \lambda\rangle$$

$$\hat{\mathrm{O}}_i^k(z, z') = \bar{q}_i(z)\hat{O}(\lambda_k)q_i(z')$$

$$\hat{O}(\lambda_k) = \frac{\not{\bar{n}}}{2}\frac{1+\lambda_k\gamma_5}{2} .$$

Some Data and Effective Cross Section

$$\sigma_{eff}^{pp} = \frac{m}{2}\frac{\sigma_A^{pp}\sigma_B^{pp}}{\sigma_{DPS}^{pp}}$$

Differential X-section single parton scattering for the process: $pp \longrightarrow A(B) + X$

Differential X-section double parton scattering for the process: $pp \longrightarrow A + B + X$

POCKET FORMULA

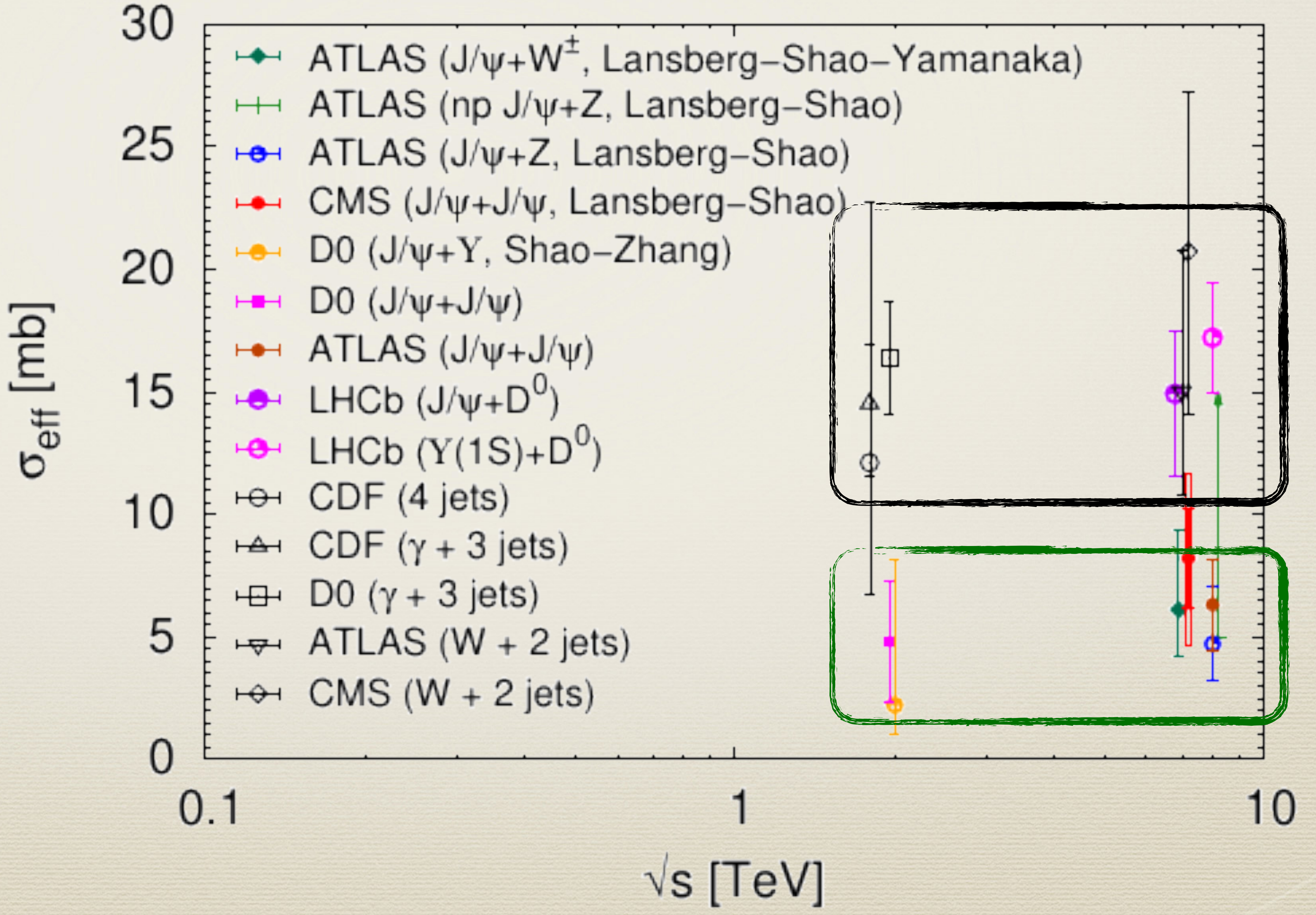
Results for W, Jet productions…

Results for quarkonium productions

1) Process dependent?
2) Sensitive to correlations
3) Sensitive to the inner structure?

predicted by all models!

M.R. et al PLB 752,40 (2016)
M. Traini, M. R. et al, PLB 768, 270 (2017)
M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



First observation of same sign WW via DPS:

$$\sigma_{eff} = 12.2^{+2.9}_{-2.2} \quad \text{mb}$$

[CMS coll.], PRL 131 (2023) 091803

$$\sigma^{DPS} \sim 6.28 \text{ fb}$$

New analysis of same sign WW via DPS:
$\sigma_{eff} = 10.6 \pm 1.8$ mb
[ATLAS coll], arXiv:2505.08313

Some Data and Effective Cross Section

If DPDs factorize in terms of PDFs then

$$\sigma_{\text{eff}}^{-1} = \int d^2 z_\perp \, \tilde{T}(z_\perp)^2 = \int \frac{d^2 k_\perp}{(2\pi)^2} T(k_\perp)^2$$

Effective Form Factor (EFF) =
FT of the probability distribution T

$$T(k_\perp) \propto \int dx_1 dx_2 \, \tilde{F}(x_1, x_2, k_\perp)$$

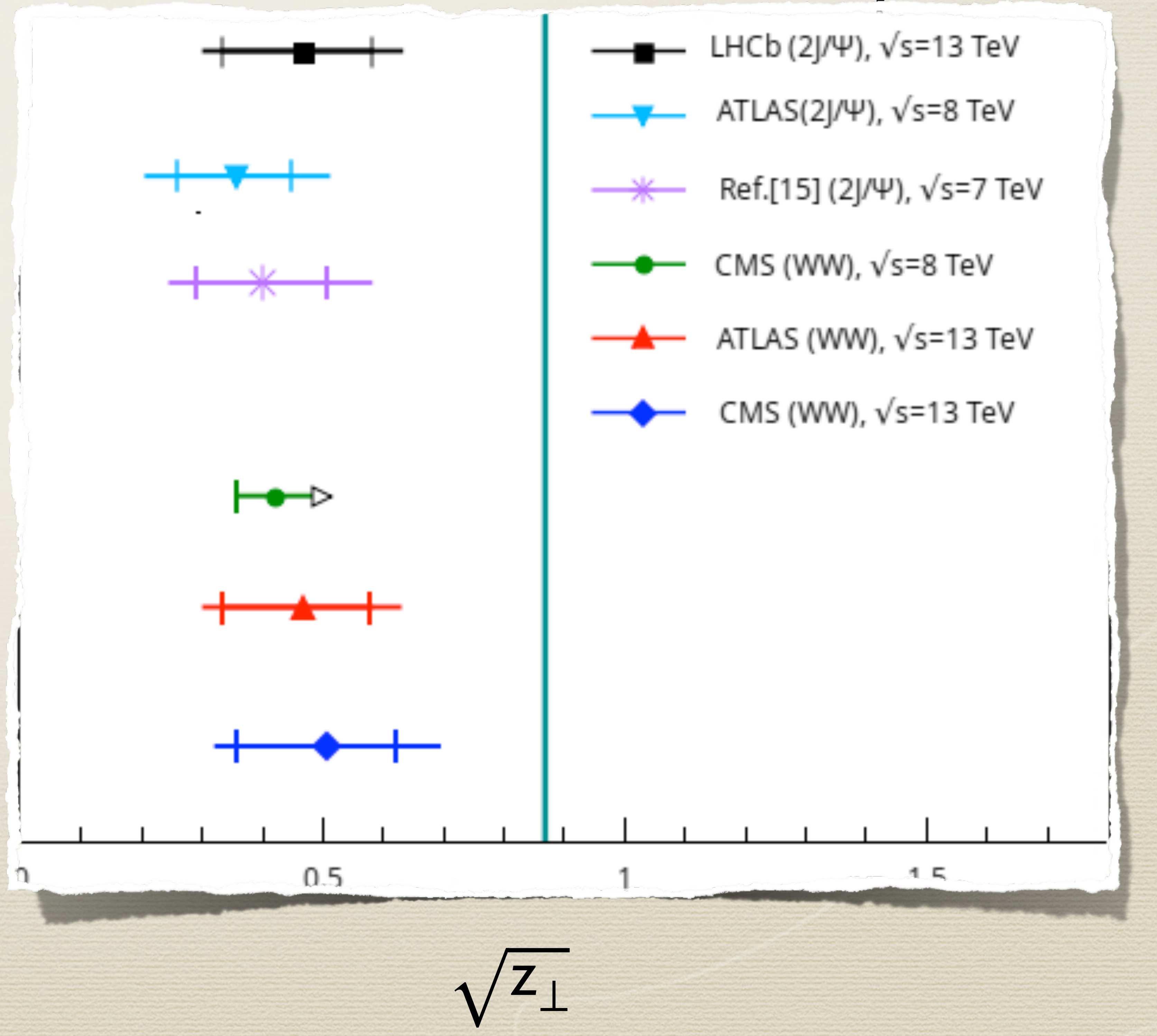
As for the standard FF:

$$\langle z_\perp^2 \rangle \propto \frac{d}{k_\perp dk_\perp} T(k_\perp) \Big|_{k_\perp = 0}$$

From the asymptotic behavior we got the following relation:

$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z_\perp^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$$

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)



Some Data and Effective Cross Section

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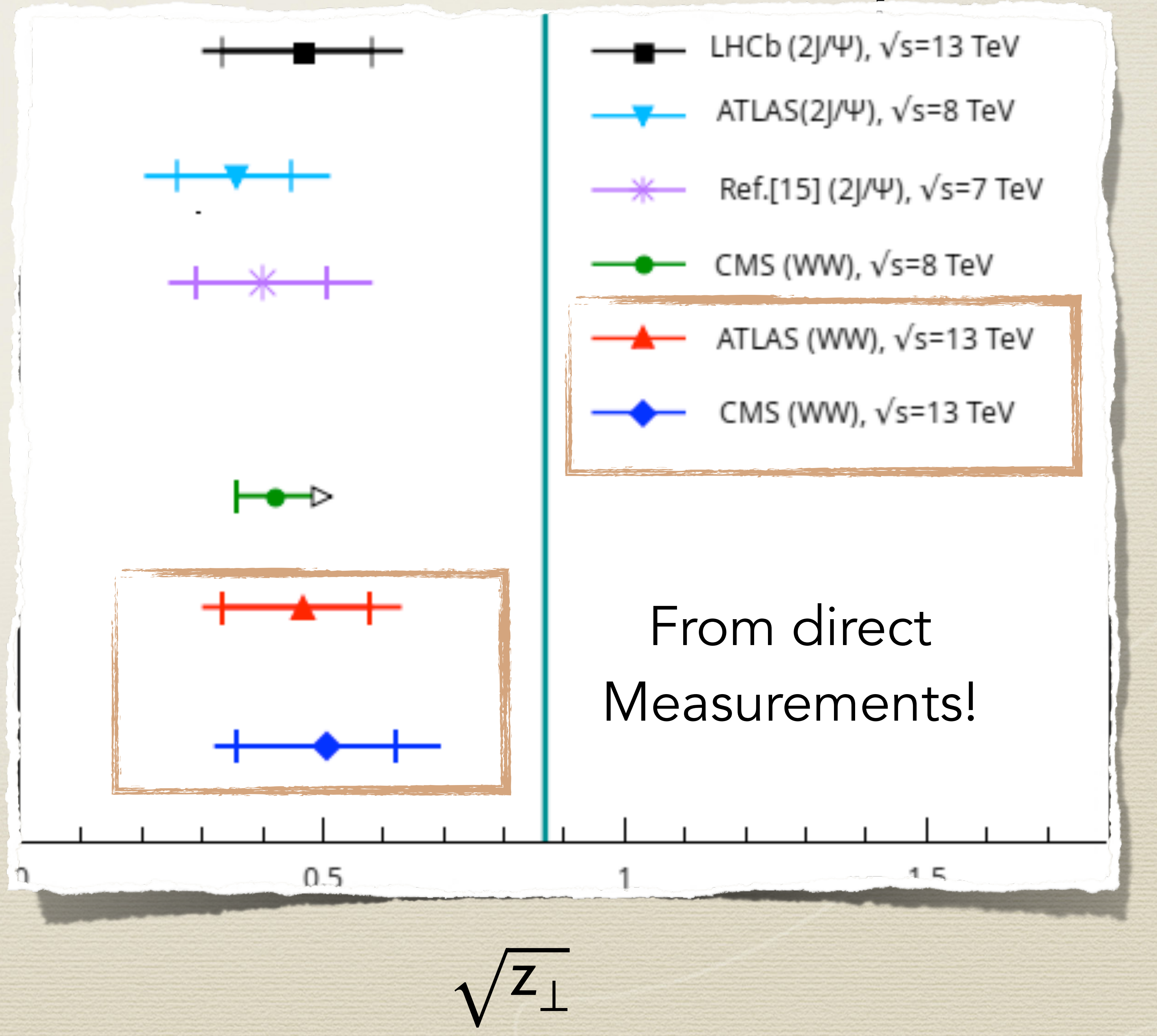
M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)



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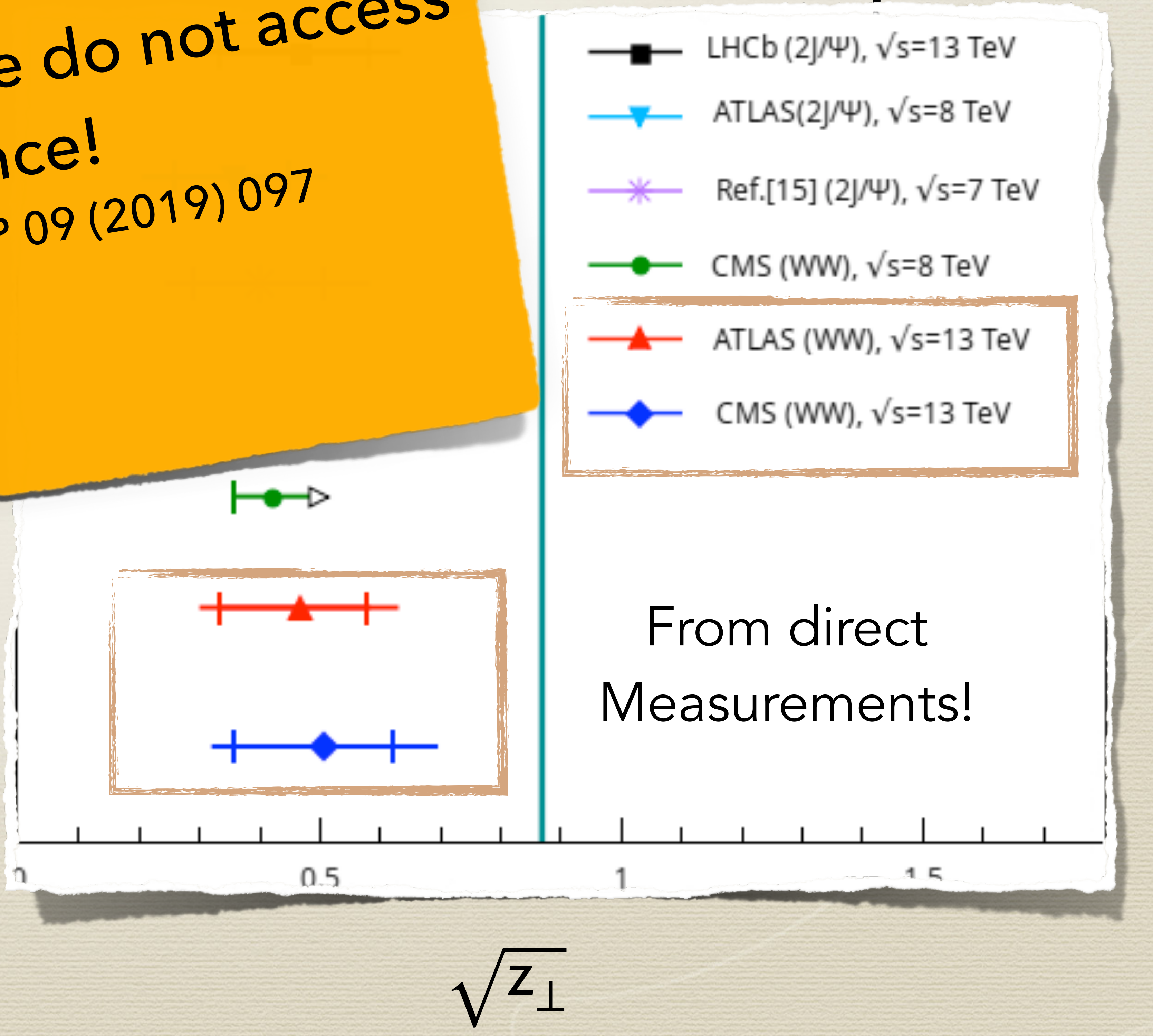
In hadron-hadron collisions we do not access directly the distance!

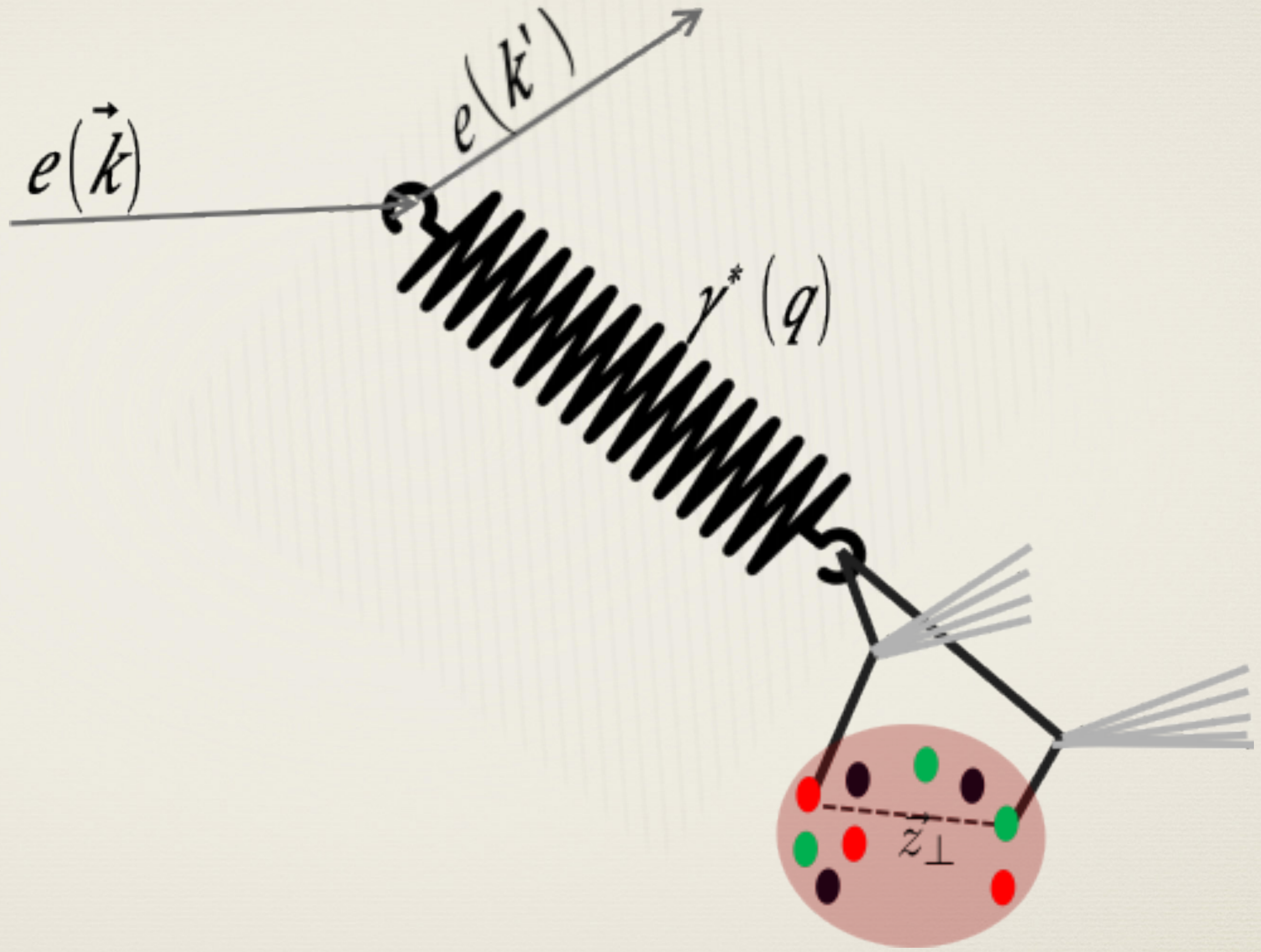
M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097



M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)

DPS in $\gamma - p$ interactions

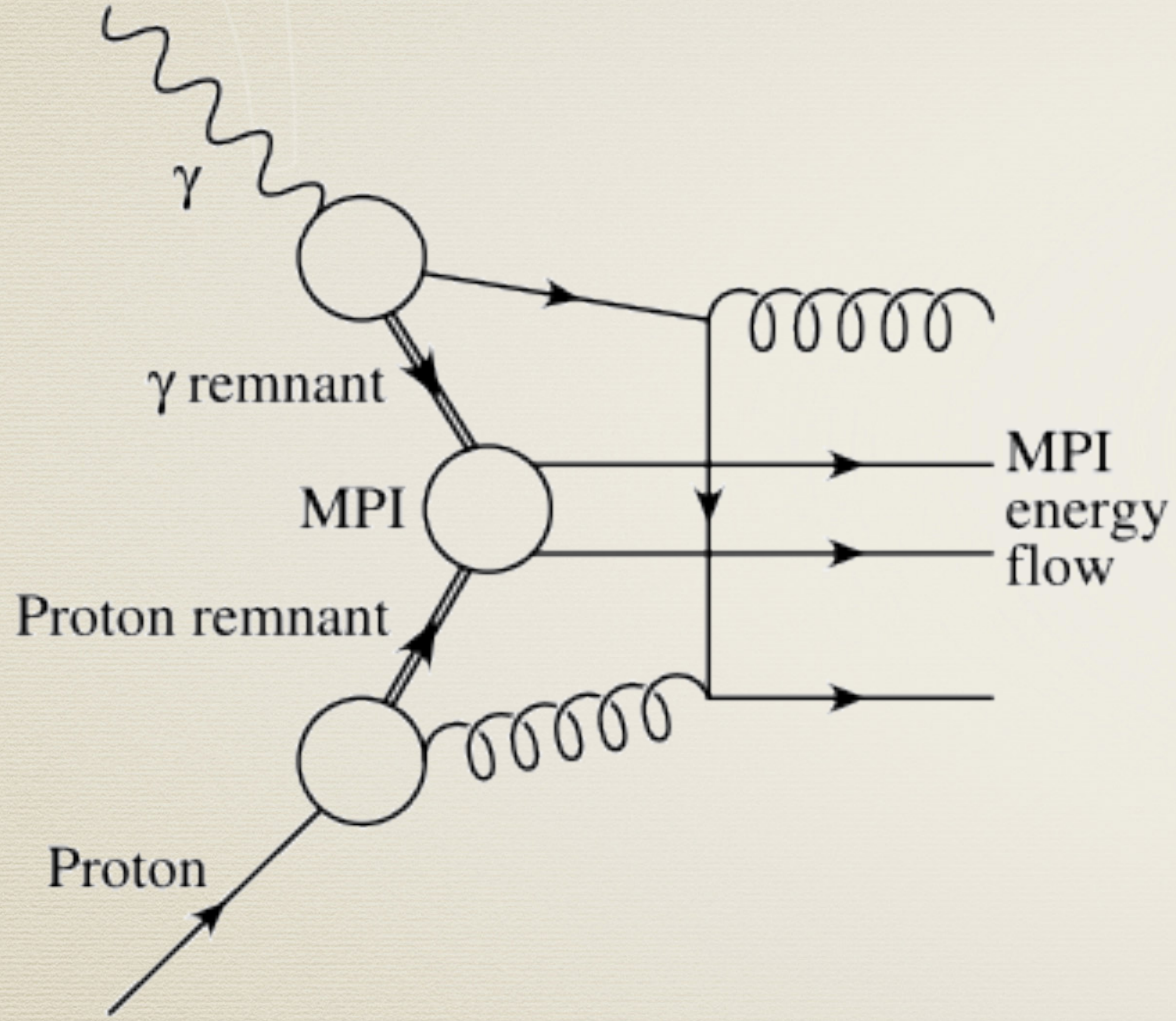
We consider the possibility offered by a DPS process involving a photon FLACTUATING in a quark-antiquark pair interacting with a proton:

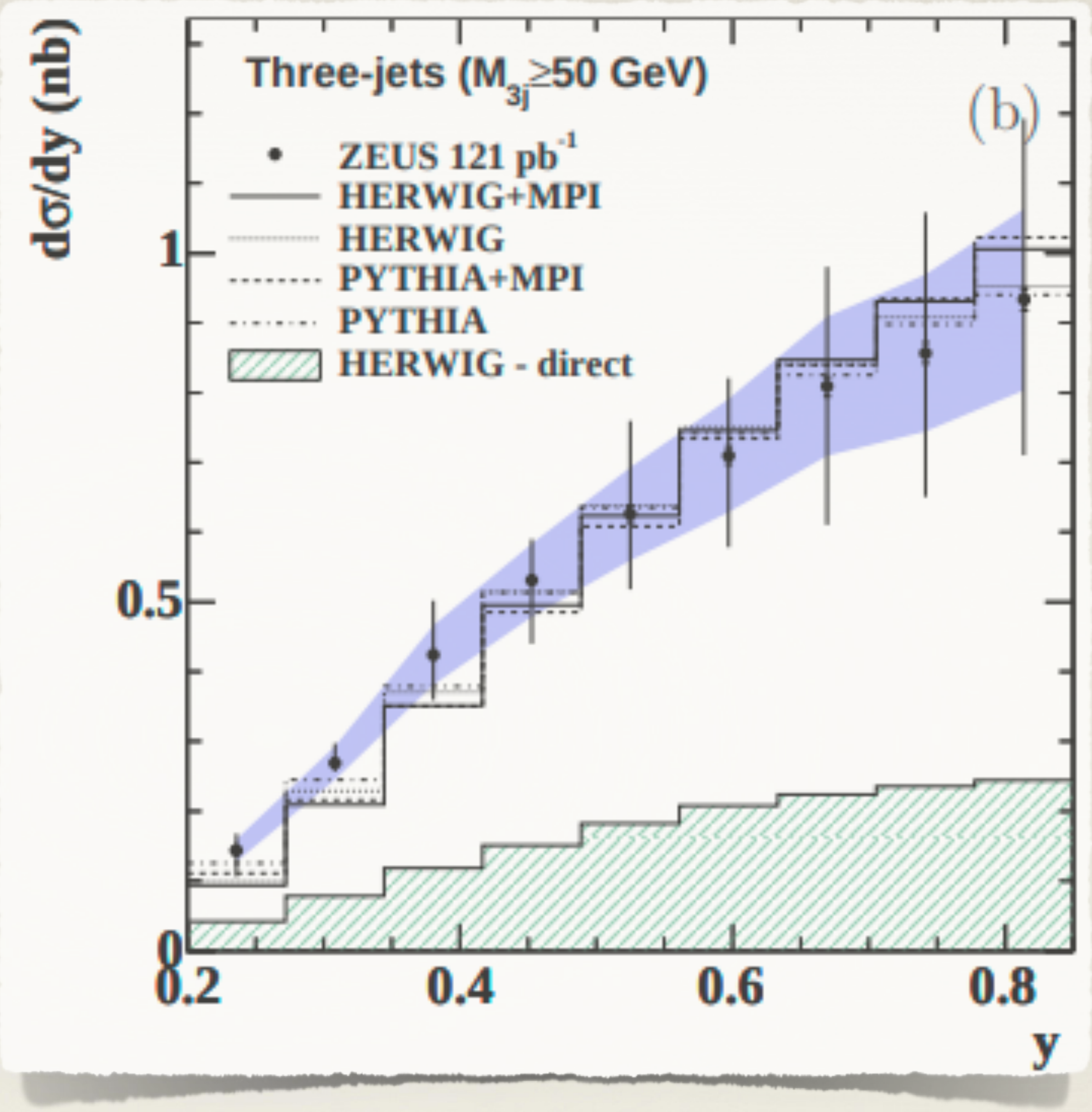


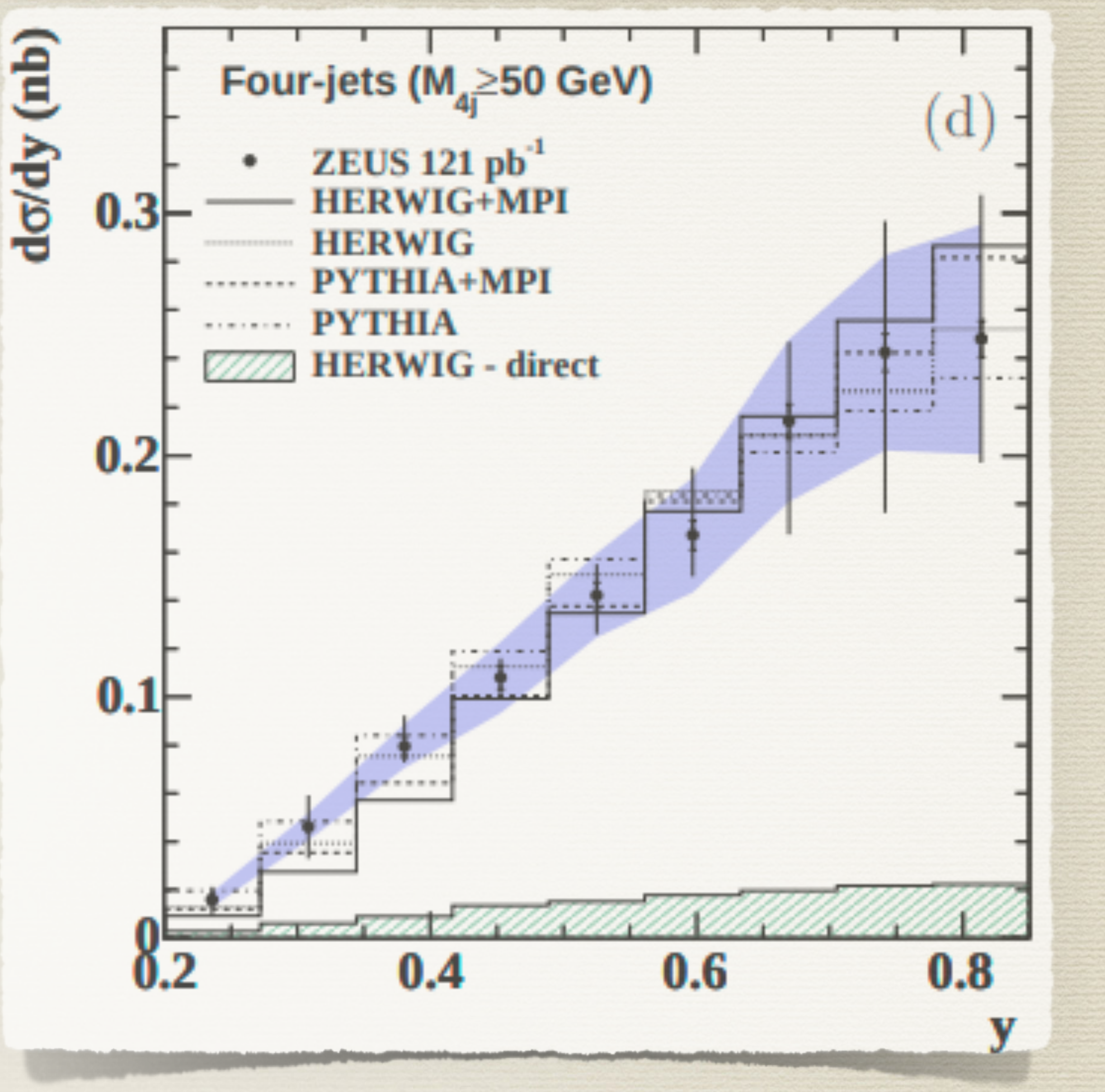
M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

DPS in $\gamma - p$ interactions

Already at HERA the importance of MPI for the **3,4 jets photo-production** has been addressed:





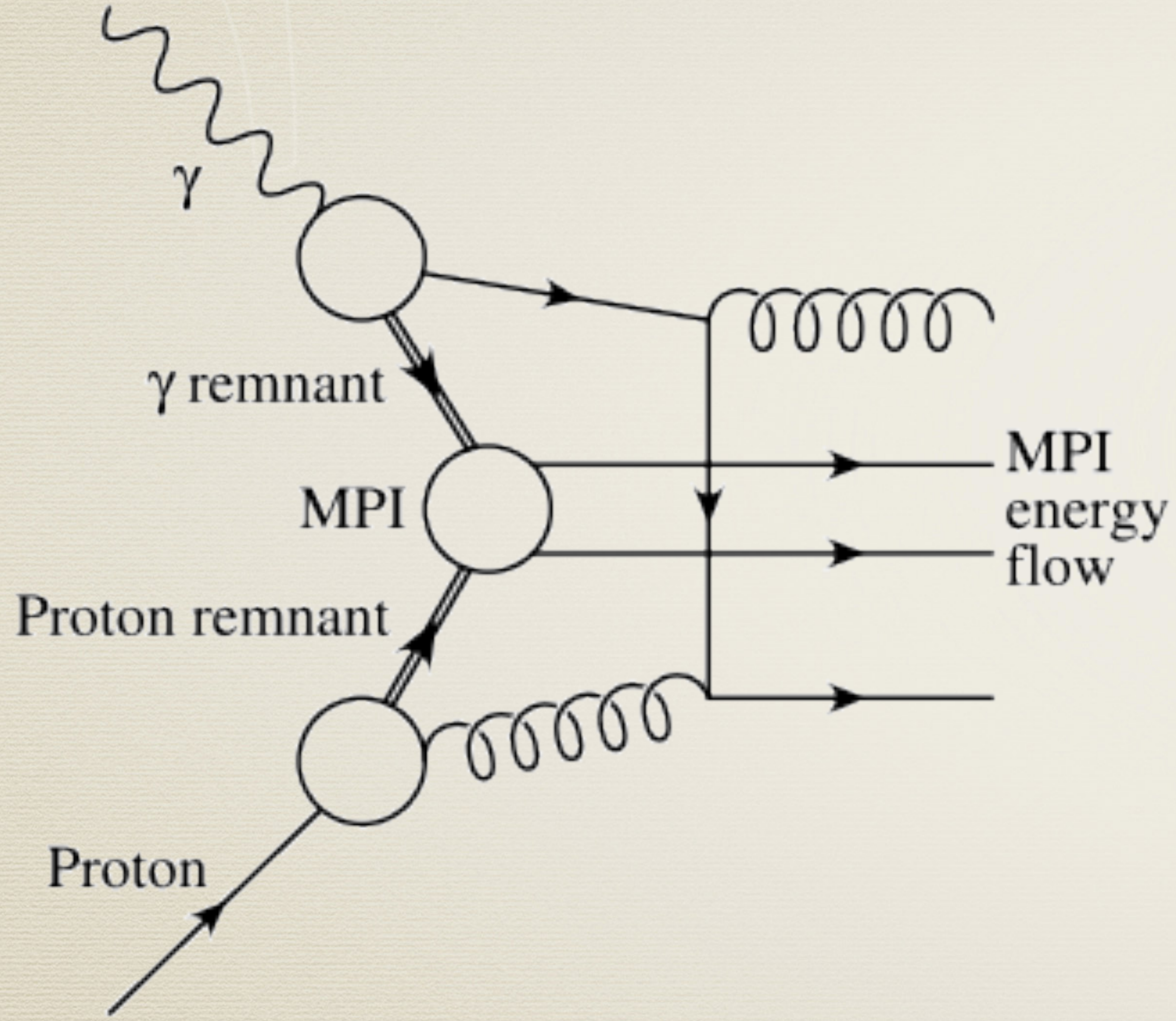


J. R. Forshaw et al, Z phys. C 72, 637

S. Chekanov et al [ZEUS coll.], Nucl. Phys B 792,1 (2008)

DPS in $\gamma - p$ interactions

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photo-production at HERA (**S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)**)



For this first investigation, we make use of the POCKET FORMULA:

Flux Factor
P. Nason et al,PLB319

$$d\sigma^{4j}_{DPS} = \frac{1}{2}\sum_{ab,cd}\int dy\, dQ^2 \frac{f_{\gamma/e}(y,Q^2)}{\sigma_{eff}^{\gamma p}(Q^2)} \times$$

$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}^{2j}_{ab}(x_{p_a}, x_{\gamma_b}) \quad \text{SPS}$$

$$*$$

$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}^{2j}_{cd}(x_{p_c}, x_{\gamma_d}) \quad \text{SPS}$$

Proton PDF
(J. Pumplin et al. JHEP 07, 012 (2002)

Photon PDF
(M. Gluck et al. PRD46, 1973 (1992)

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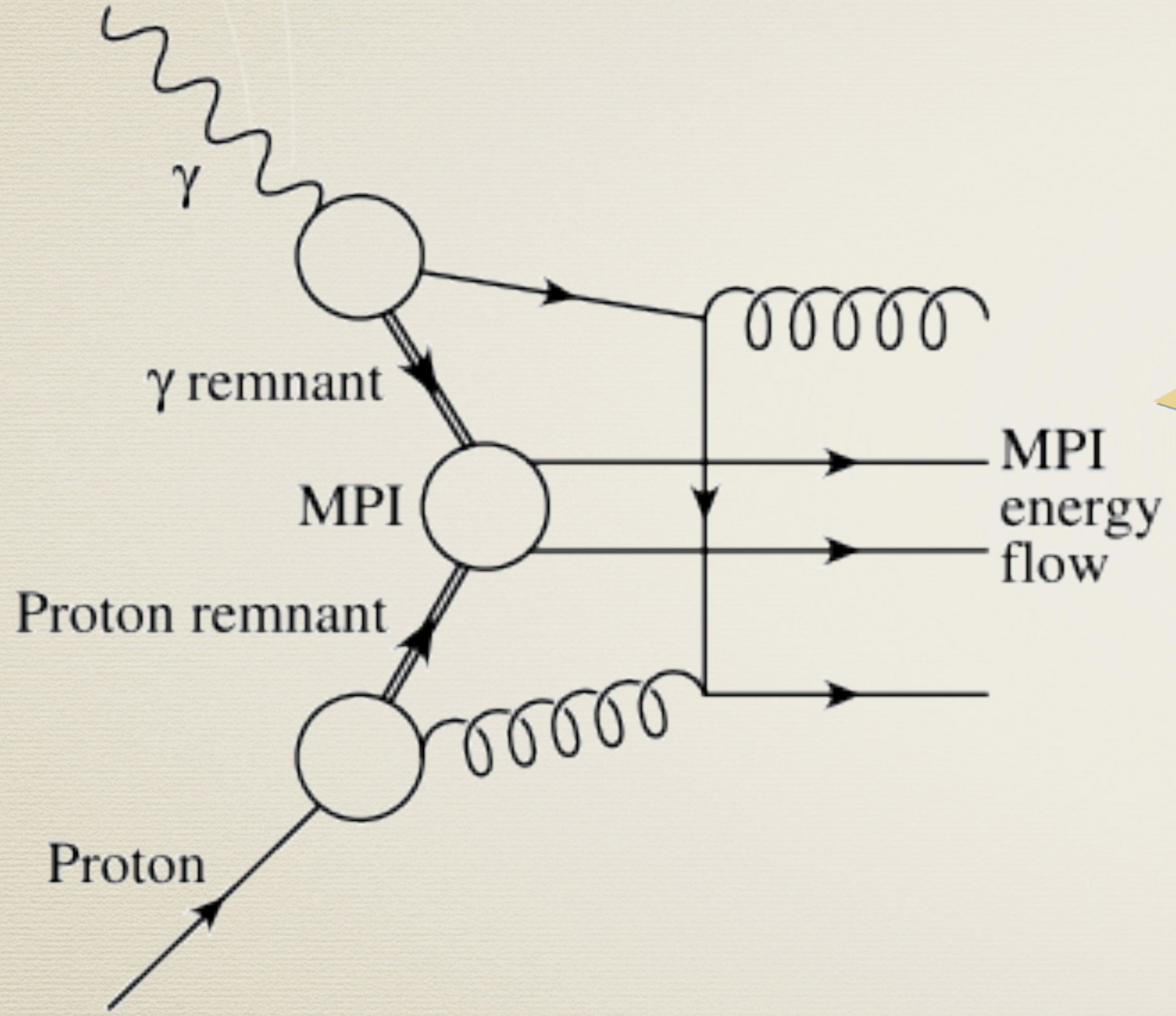


(J. Pumplin et al.

For this first investigation, we make use of the POCKET FORMULA:

Flux Factor
P. Nason et al,PLB319

$$f_{\gamma/e}(y, Q^2) \times \sigma_{eff}^{\gamma p}(Q^2)$$

$(x_{\gamma_b}) d\hat{\sigma}^{2j}_{ab}(x_{p_a}, x_{\gamma_b})$ — SPS

*

$\gamma(x_{\gamma_d}) d\hat{\sigma}^{2j}_{cd}(x_{p_c}, x_{\gamma_d})$ — SPS

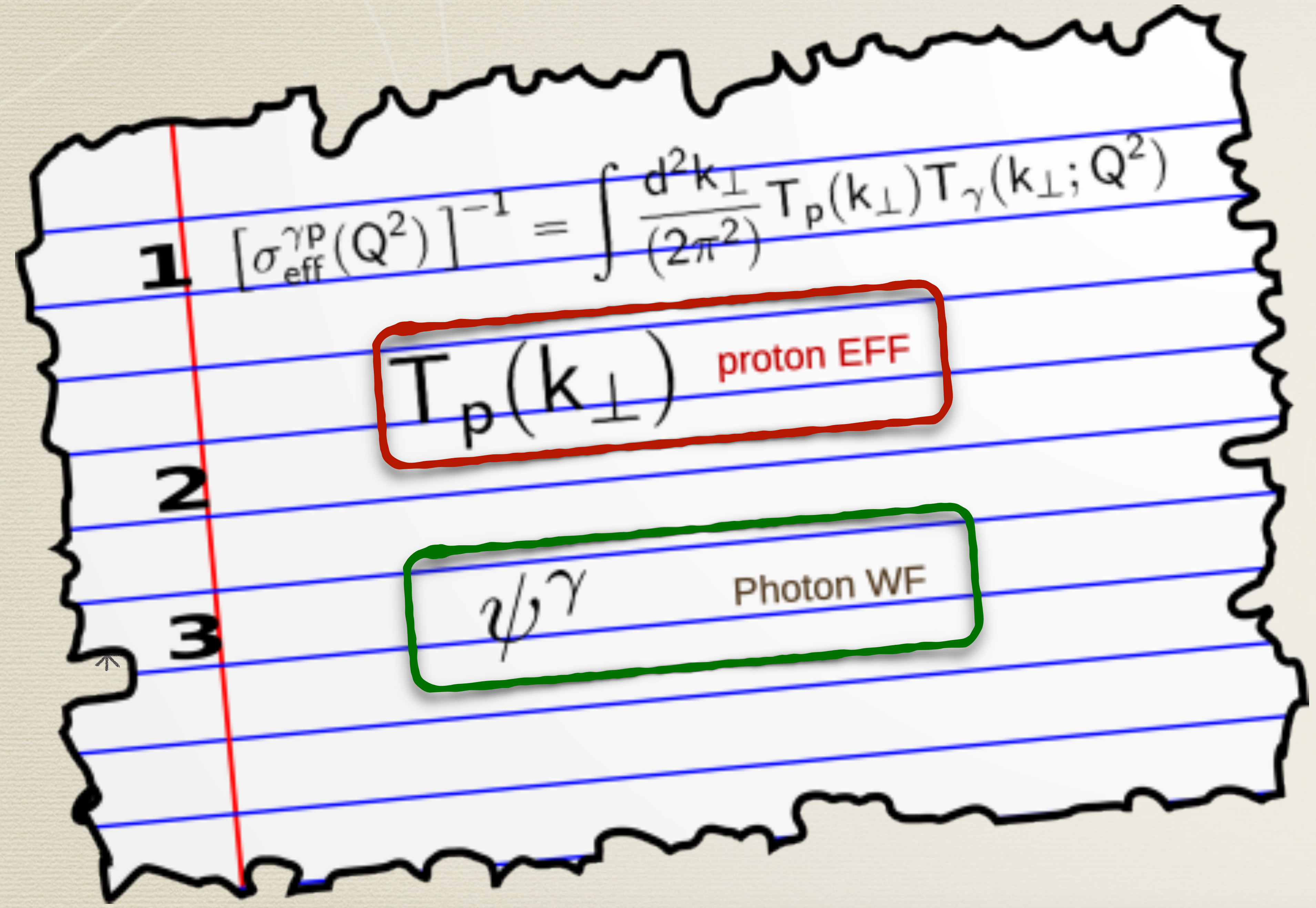
Photon PDF
(**M. Gluck et al. PRD46, 1973 (1992)**)

The main quantity we have to evaluate is:

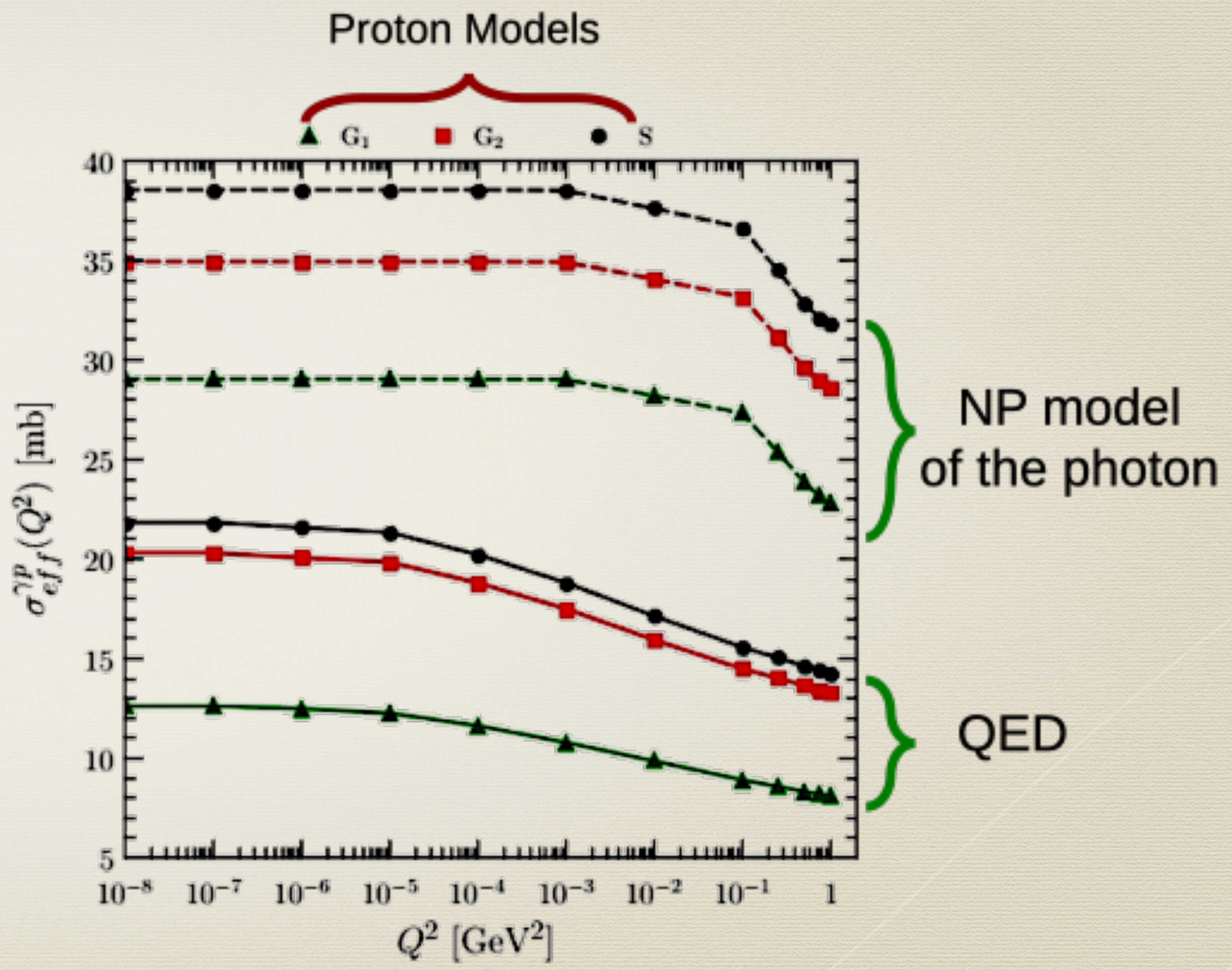
$$\sigma_{eff}^{\gamma p}(Q^2)$$

The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501



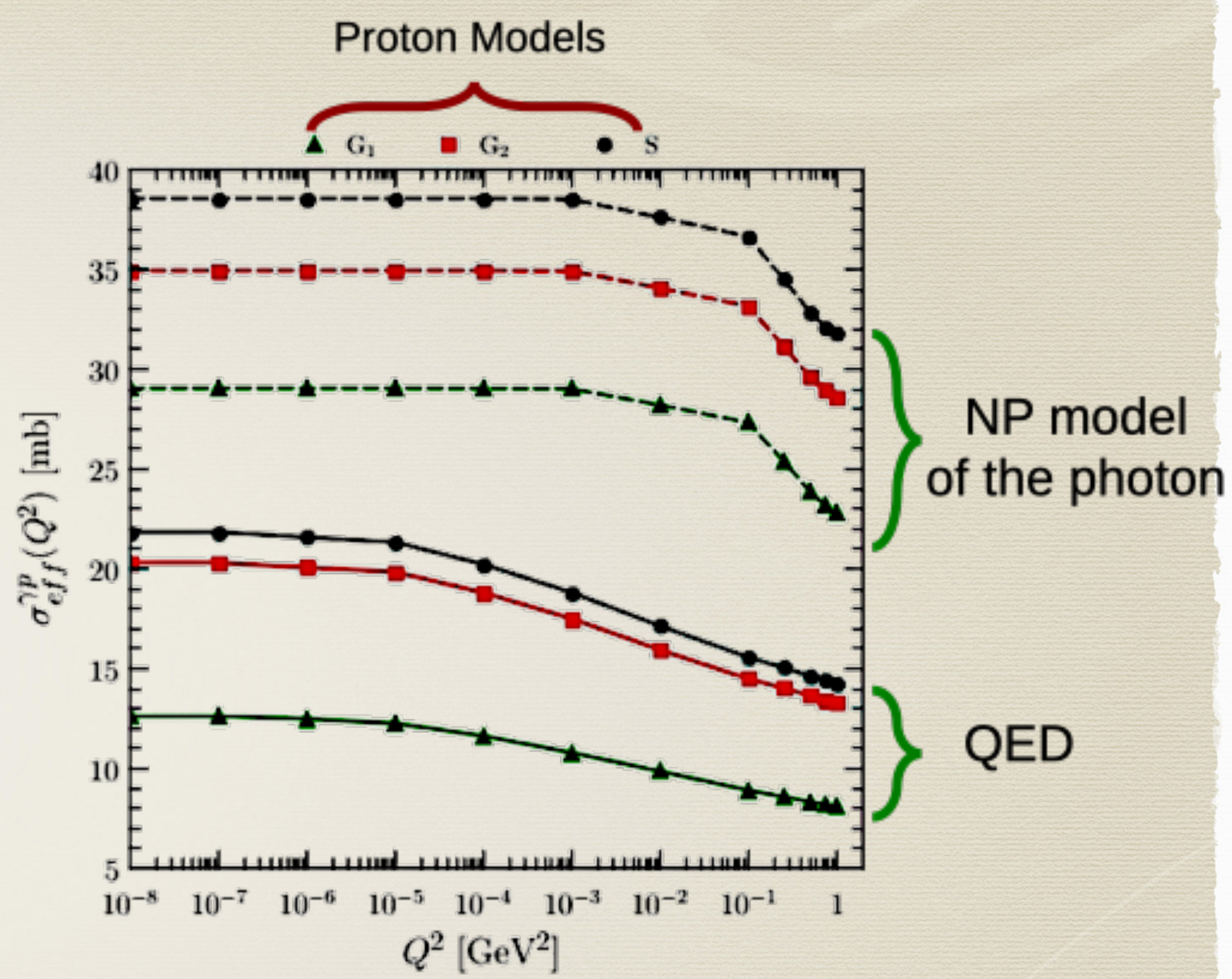
The effective cross-section depends on the photon virtuality! (NEW)



The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$d\sigma^{4j}_{DPS} = \frac{1}{2}\sum_{ab,cd}\int dy\, dQ^2\, \frac{f_{\gamma/e}(y,Q^2)}{\sigma^{\gamma p}_{eff}(Q^2)} \times$$
$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}^{2j}_{ab}(x_{p_a}, x_{\gamma_b})$$
$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}^{2j}_{cd}(x_{p_c}, x_{\gamma_d})$$



KINEMATICS:

$E_T^{jet} > 6$ GeV

$|\eta_{jet}| < 2.4$

$Q^2 < 1$ GeV^2

$0.2 \leqslant y \leqslant 0.85$

The ZEUS collaboration quoted an integrated total 4-jet cross section of 136 pb
S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

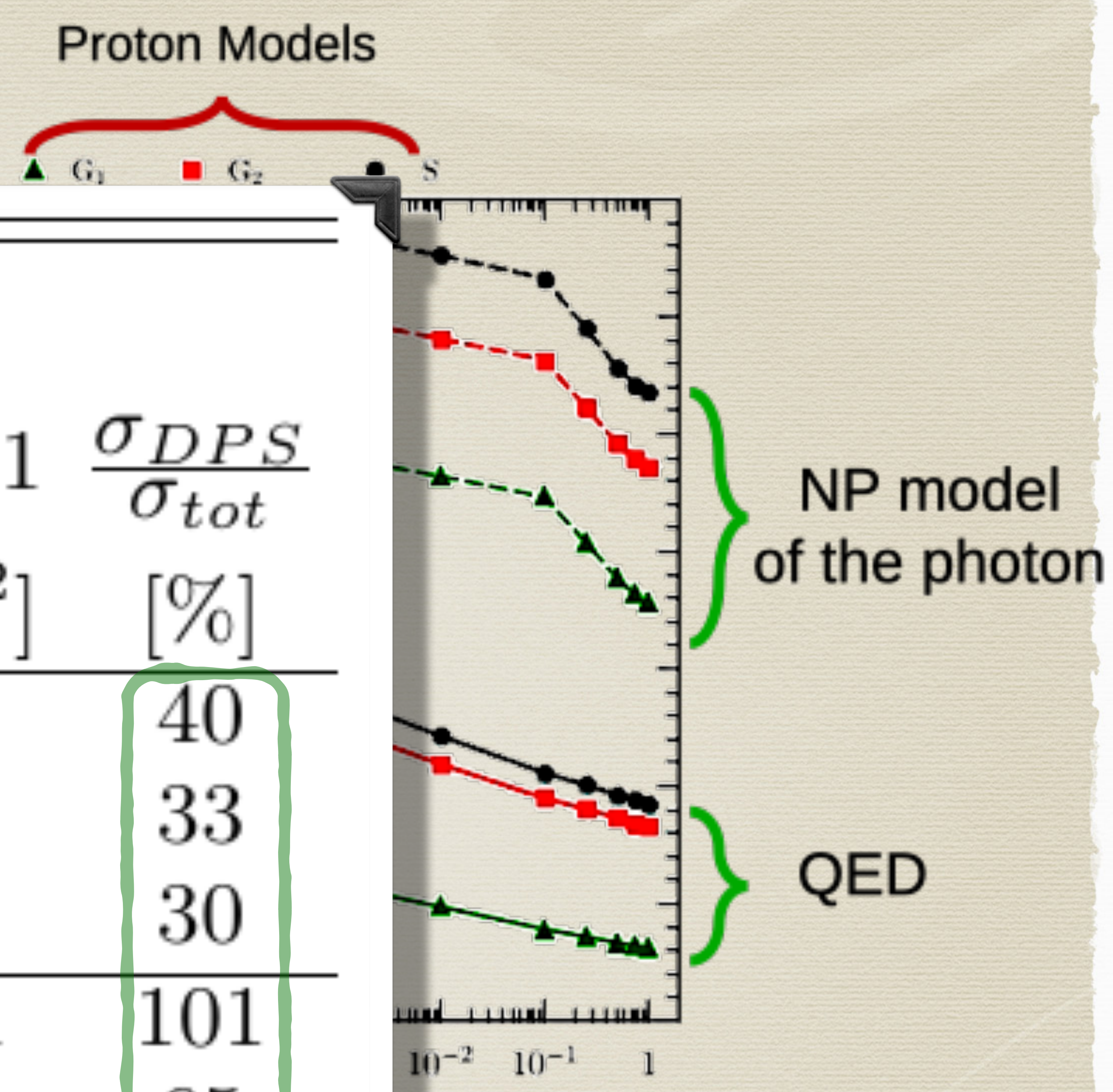
The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$d\sigma^{4j}_{DPS} = \frac{1}{2}\sum_{ab,cd}\int \ldots$$

$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(\ldots$$

$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(\ldots$$



Photon \ Proton		σ_{DPS} [pb] $Q^2 \leq 10^{-2}$ [GeV^2]	$10^{-2} \leq Q^2 \leq 1$ [GeV^2]	$Q^2 \leq 1$ [GeV^2]	$\frac{\sigma_{DPS}}{\sigma_{tot}}$ [%]
NP Model	G_1	35.1	18.6	53.7	40
	G_2	29.1	15.2	44.3	33
	S	26.4	13.7	40.1	30
QED	G_1	87.8	54.3	142.1	101
	G_2	54.3	33.4	87.7	65
	S	50.5	31.1	81.6	60

KINEMATICS:

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S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

A key to the proton structure

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The effective cross section can be also written in terms of probability distribution:

$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2)\right]^{-1} = \int d^2z_\perp \, \tilde{F}_2^{p}(z_\perp)\tilde{F}_2^{\gamma}(z_\perp;Q^2)$$

$^*\tilde{F}_2^{A}(z_\perp) =$ prob. distr. of finding two partons at given transverse distance

We can expand the distribution related to the photon:

$$\tilde{F}_2^{\gamma}(z_\perp;Q^2) = \sum_n C_n(Q^2) z_\perp^n$$

Coefficients determined in a given approach describing the photon structure

$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2)\right]^{-1} = \sum_n C_n(Q^2)\langle z_\perp^n \rangle_p$$

Mean value of the transverse distance between two partons in the PROTON

If we could measure $\sigma_{\text{eff}}^{\gamma p}(Q^2)$ we could access NEW INFORMATION ON THE PROTON STRUCTURE

DPS in pA collisions

For DPS in pA and AA collisions the following references were missing:

1)Same-sign WW production in proton-nucleus collisions at the LHC as a signal for double parton scattering
D. d'E. & A. Snigirev, PLB 718 (2013) 1395-1400

2)Enhanced J/ΨJ/\PsiJ/Ψ-pair production from double parton scatterings in nucleus-nucleus collisions at the Large Hadron Collider
D. d'E. & A. Snigirev, PLB 727 (2013) 157-162

3)Pair production of quarkonia and electroweak bosons from double-parton scatterings in nuclear collisions at the LHC
D. d'E. & A. Snigirev, Nucl. Phys. A 931 (2014) 303-308

and for TPS:

Triple-parton scatterings in proton–nucleus collisions at high energies
D. d'E. & A. Snigirev, EPJC 78 (2018) 5, 359

DPS in pA collisions

$$F_{a_1 a_2}(x_1, x_2, \mathbf{y}_\perp) = 2p^+ \int \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} dy^- e^{i(x_1 z_1^- + x_2 z_2^-)p^+}$$
$$\times \langle A | \mathcal{O}_{a_2}(0, z_2) \mathcal{O}_{a_1}(y, z_1) | A \rangle$$

In this case we have two mechanisms that contribute:

DPS in pA collisions

$$F_{a_1a_2}(x_1, x_2, \mathbf{y}_\perp) = 2p^+ \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi} dy^- e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle A|\mathcal{O}_{a_2}(0, z_2)\mathcal{O}_{a_1}(y, z_1)|A\rangle$$

In this case we have two mechanisms that contribute:

B. Blok et al, EPJC (2013) 73:2422

DPS 1: The two partons belong to the SAME nucleon in the nucleus!

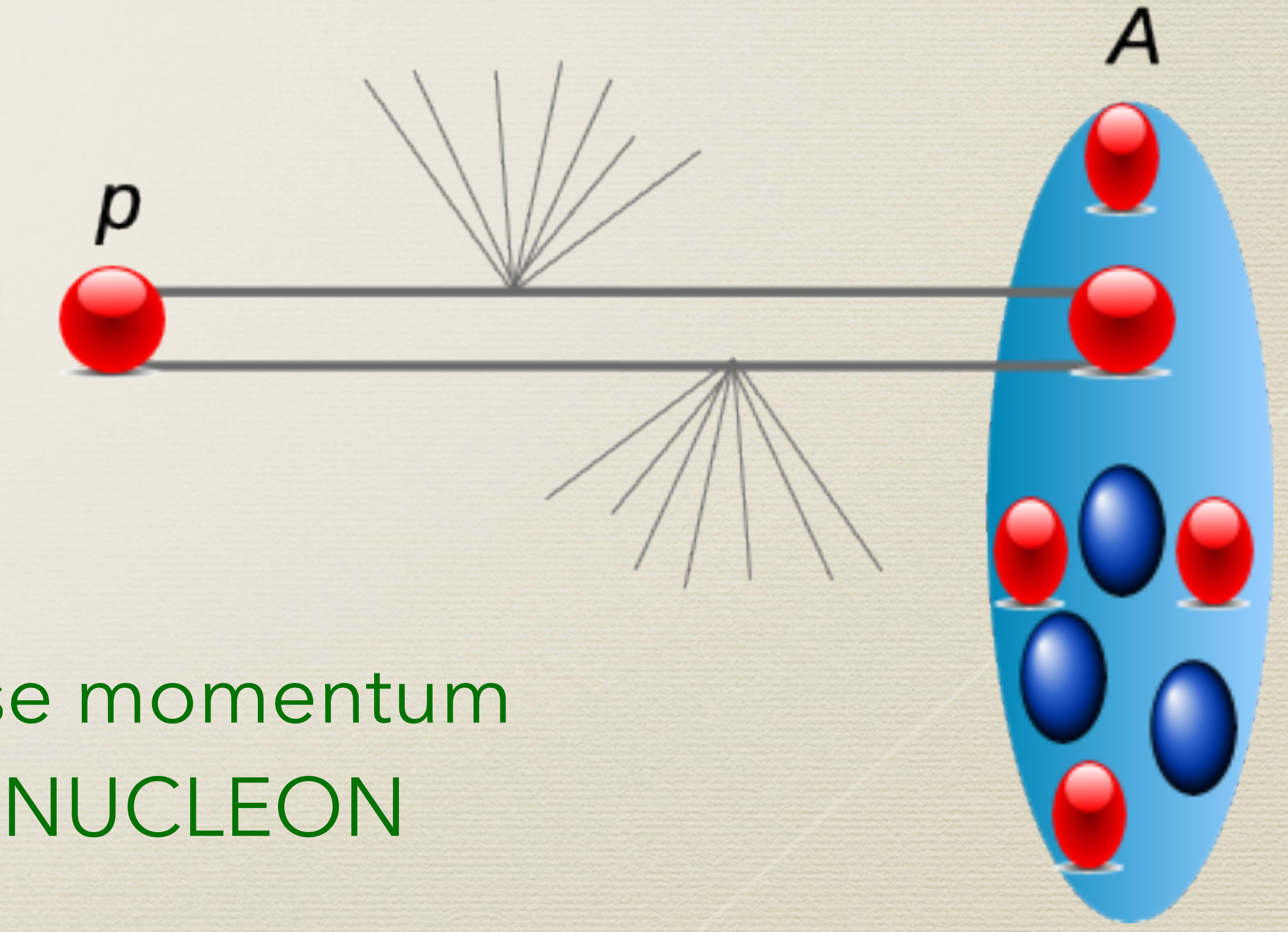
$$\tilde{F}^1_{a_1a_2}(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^N_{a_1a_2}\left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp\right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2p_{t,N}$$

DPD of te nucleon inside the nucleus

Momentum fraction carried by a NUCLEON

Light-Cone Momentum Distribution

Transverse momentum of the NUCLEON



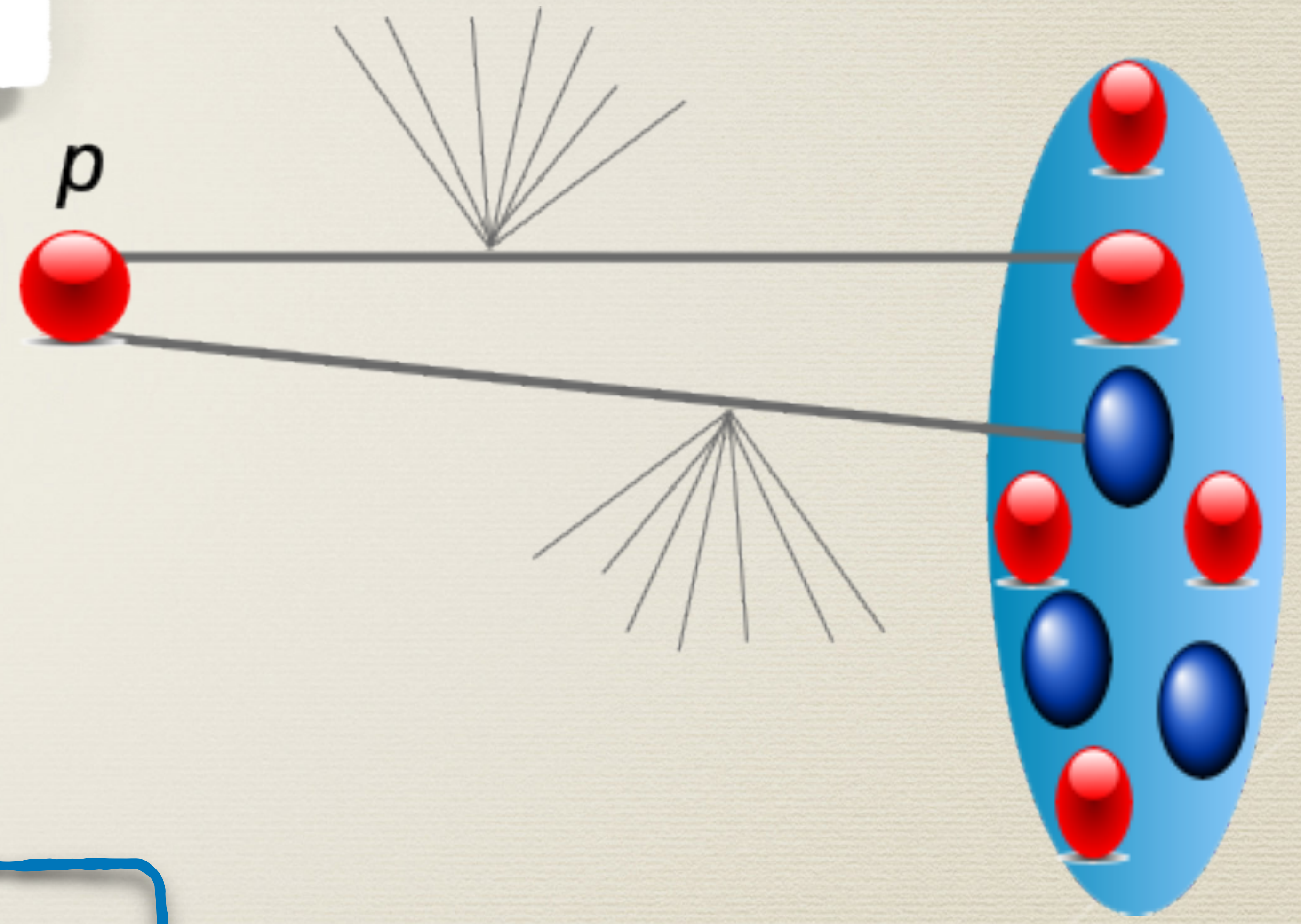
DPS in pA collisions

$$F_{a_1a_2}(x_1, x_2, \mathbf{y}_\perp) = 2p^+ \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi} dy^- e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle A | \mathcal{O}_{a_2}(0, z_2)\mathcal{O}_{a_1}(y, z_1) | A \rangle$$

In this case we have two mechanisms that contribute:

B. Blok et al, EPJC (2013) 73:2422

DPS 2: The two partons belong to the DIFFERENT nucleons in the nucleus!

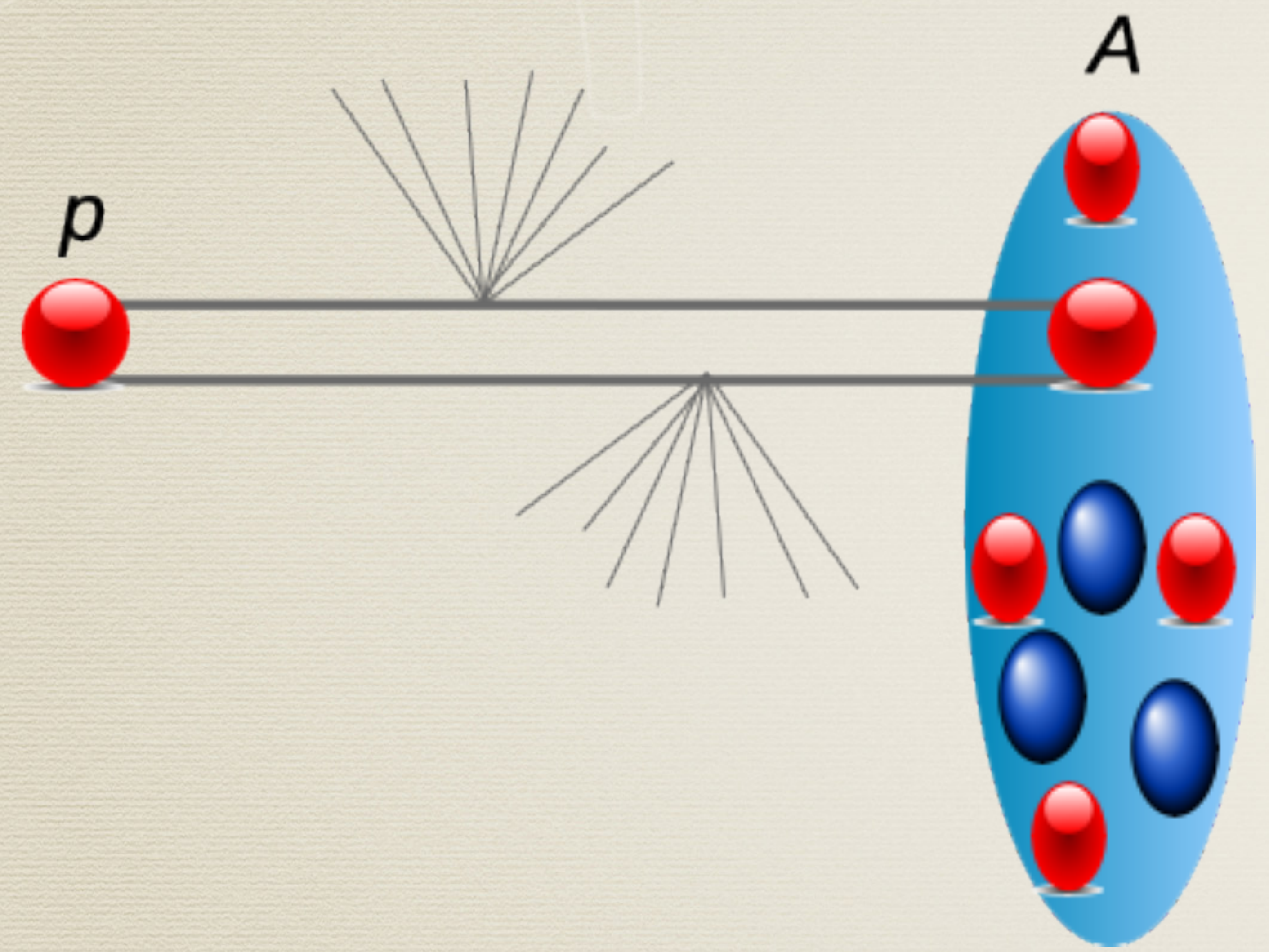


$$\tilde{F}^2_{a_1a_2}(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1\xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, p_{t1}, p_{t2}, \ldots) \times \psi_A\left(\xi_1, \xi_2, p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp, \ldots\right) G_{a_1}^{N_1}\left(x_1/\xi_1, |\vec{k}_\perp|\right) G_{a_2}^{N_2}\left(x_2/\xi_2, |\vec{k}_\perp|\right)$$

Nucleus wf

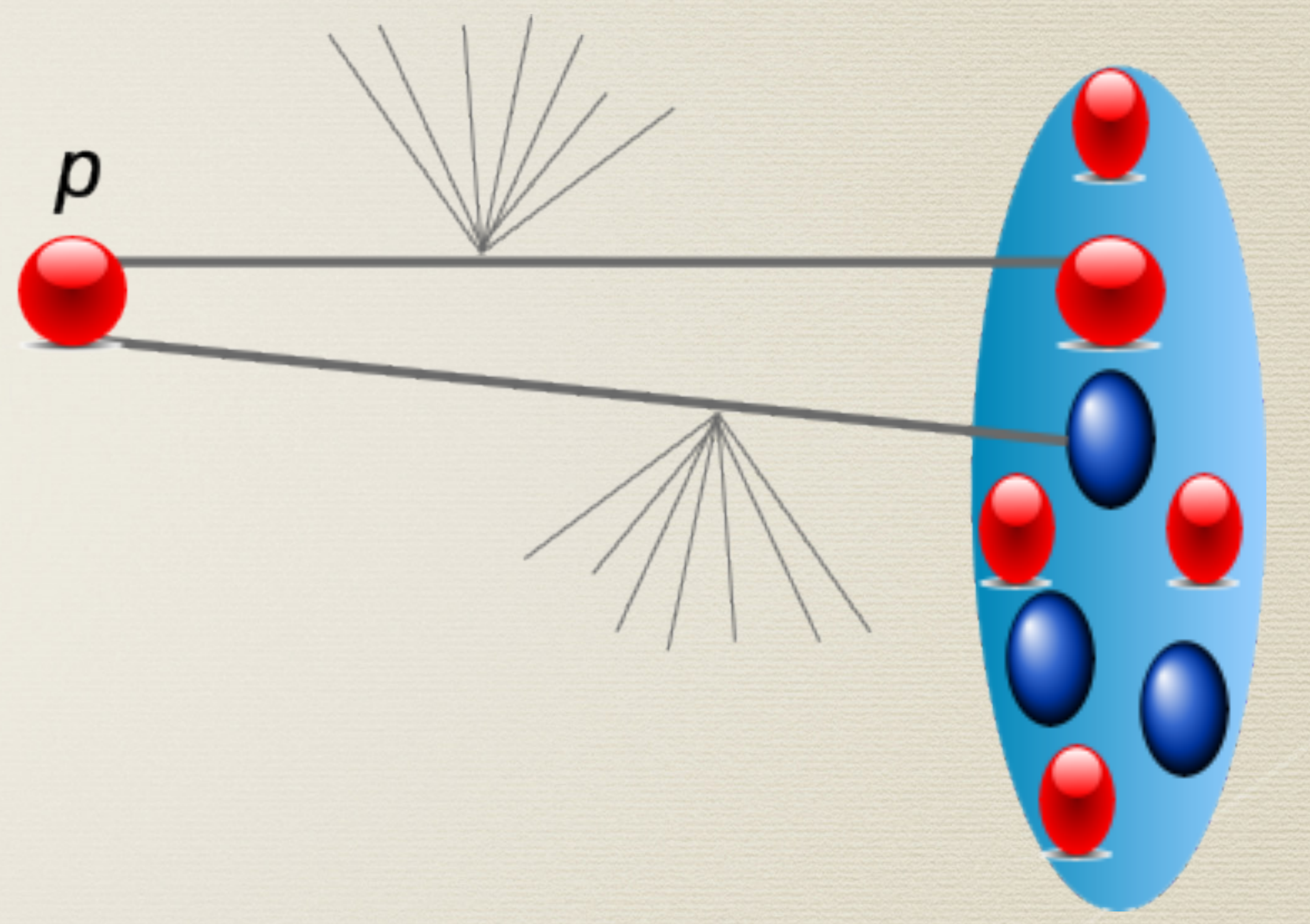
Nucleon GPD

DPS in pA collisions



$$\sigma_{DPS2} \sim A^{1/3}\sigma_{DPS1}$$

$$\sigma_{DPS1} \sim A\sigma^{pp}_{DPS}$$



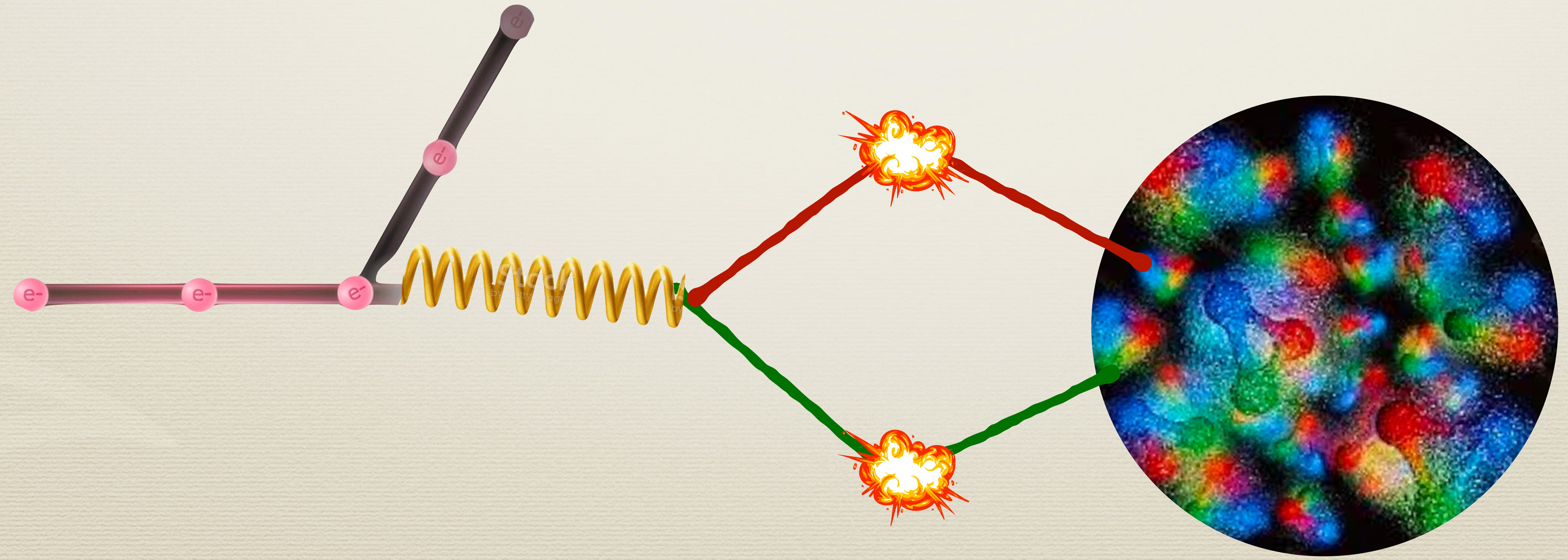
DPS in γA collisions with light nuclei?

In p-Pb collisions there are some difficulties (personal view):

1) both cross-sections (DPS1 and DPS2) depends on proton DPD (still almost unknown) therefore both mechanisms are very important → could be difficult to extract some information on the proton DPD

2) for heavy nuclei is difficult to perform calculation with wave-function obtained from realistic potentials

POSSIBLE SOLUTION?



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2) for heavy nuclei is difficult to perform calculation with wave-function obtained from realistic potentials

POSSIBLE SOLUTION?

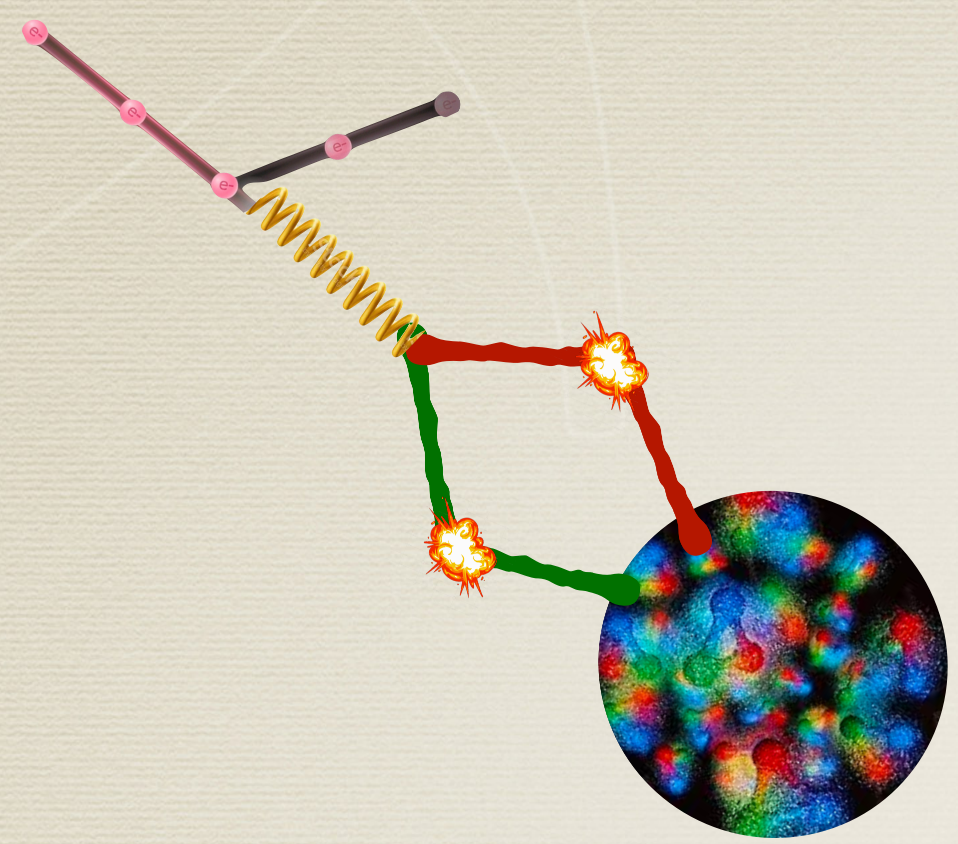
1) In γA the DPS2 will not contain any DPD of the proton → this mechanism can now be viewed as a background that can be evaluated if we properly treat the photon (as previously discussed) and the Nuclear geometry

2) For light nuclei these calculations can be done starting from realistic wave-function (Av18 or chiral potential)!

Could we access the DPD of bound nucleons? Double EMC effect?

DPS1 in γA collisions with light nuclei

For example in DPS1:



$$\tilde{F}^{1}_{a_1 a_2}(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^{N}_{a_1 a_2}\left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp\right) \rho^{N}_{A}(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$

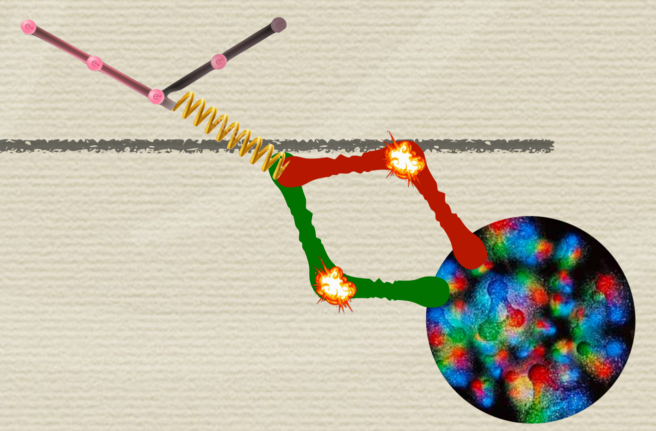
The nuclear light-cone distribution can be evaluated with realistic wave-function (from Av18 +UIV potential) in a fully relativistic and Poincaré covariant approach for:

1) H^2 in **E. Pace and G. Salmé, TNPI2000 (2001), arXiv:nucl-th/0106004**

2) He^3 in e.g. **A. Del Dotto et al, PRC 95, 014001 (2017), M.R. et al, PLB 839 (2023), 137810**

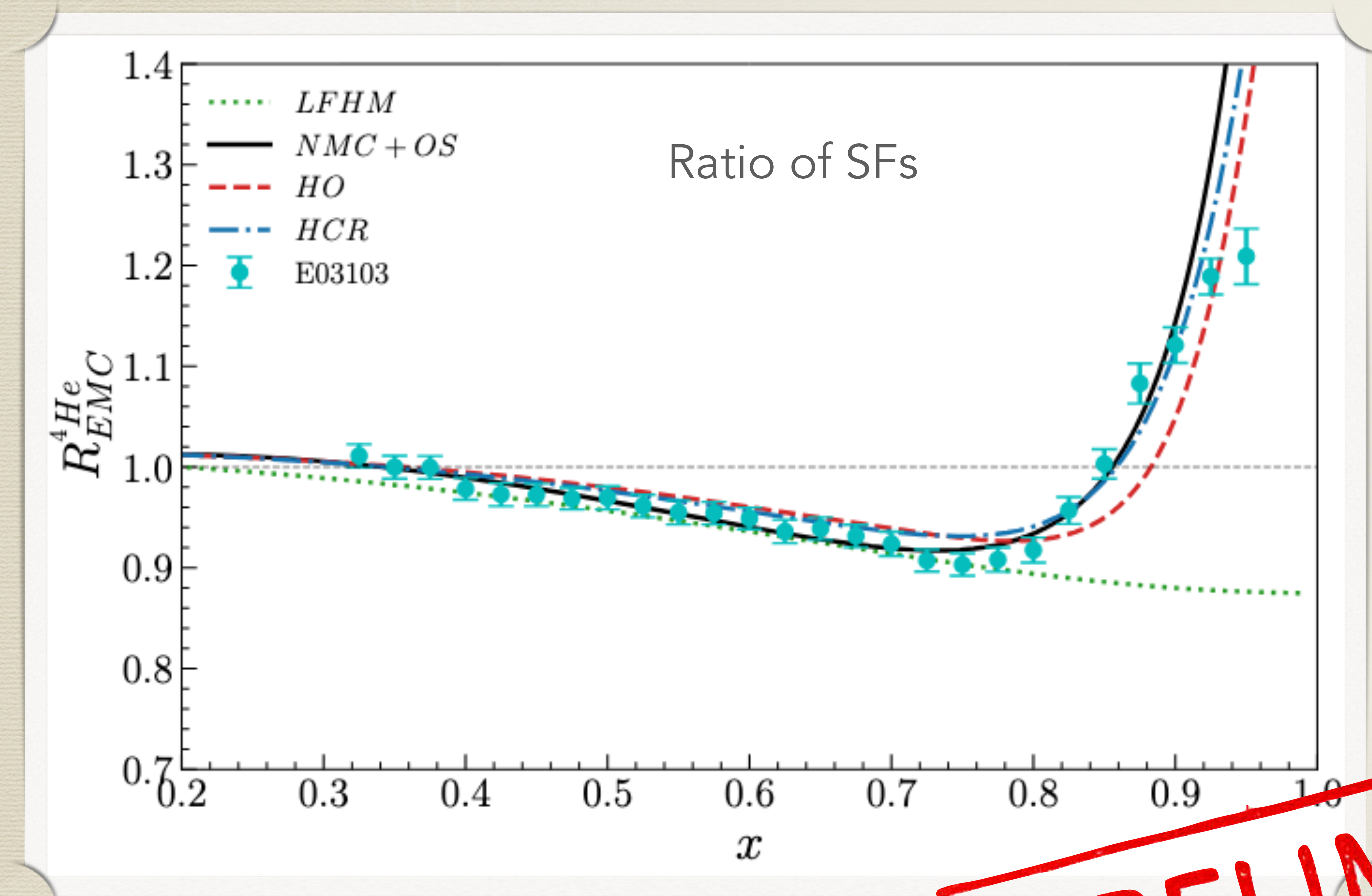
3) ^4He from **F. Fornetti, E. Pace, M. R., G. Salmé, S. Scopetta and M. Viviani, PLB 851 (2024), 138587**

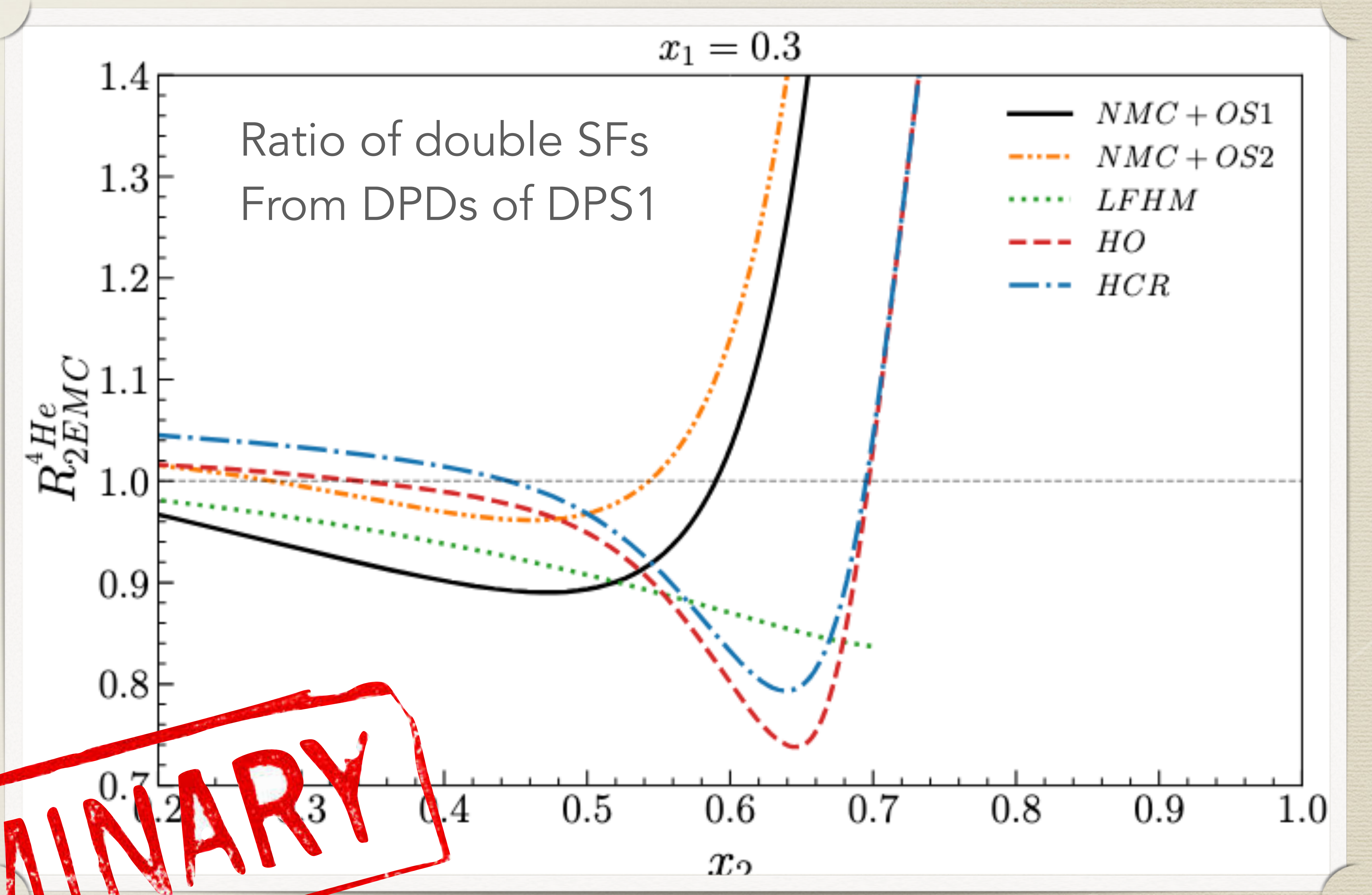
DPS1 in γA collisions with light nuclei



For example in DPS1: $\tilde{F}^{1}_{a_1 a_2}(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^{N}_{a_1 a_2}\left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp\right) \rho^{N}_{A}(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$

Possible solution to the EMC effect. In fact:





DPS1 in γA collisions with light nuclei

For example in DPS1: $\tilde{F}^{1}_{a_1 a_2}(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^{N}_{a_1 a_2}\left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp\right) \rho^{N}_{A}(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$

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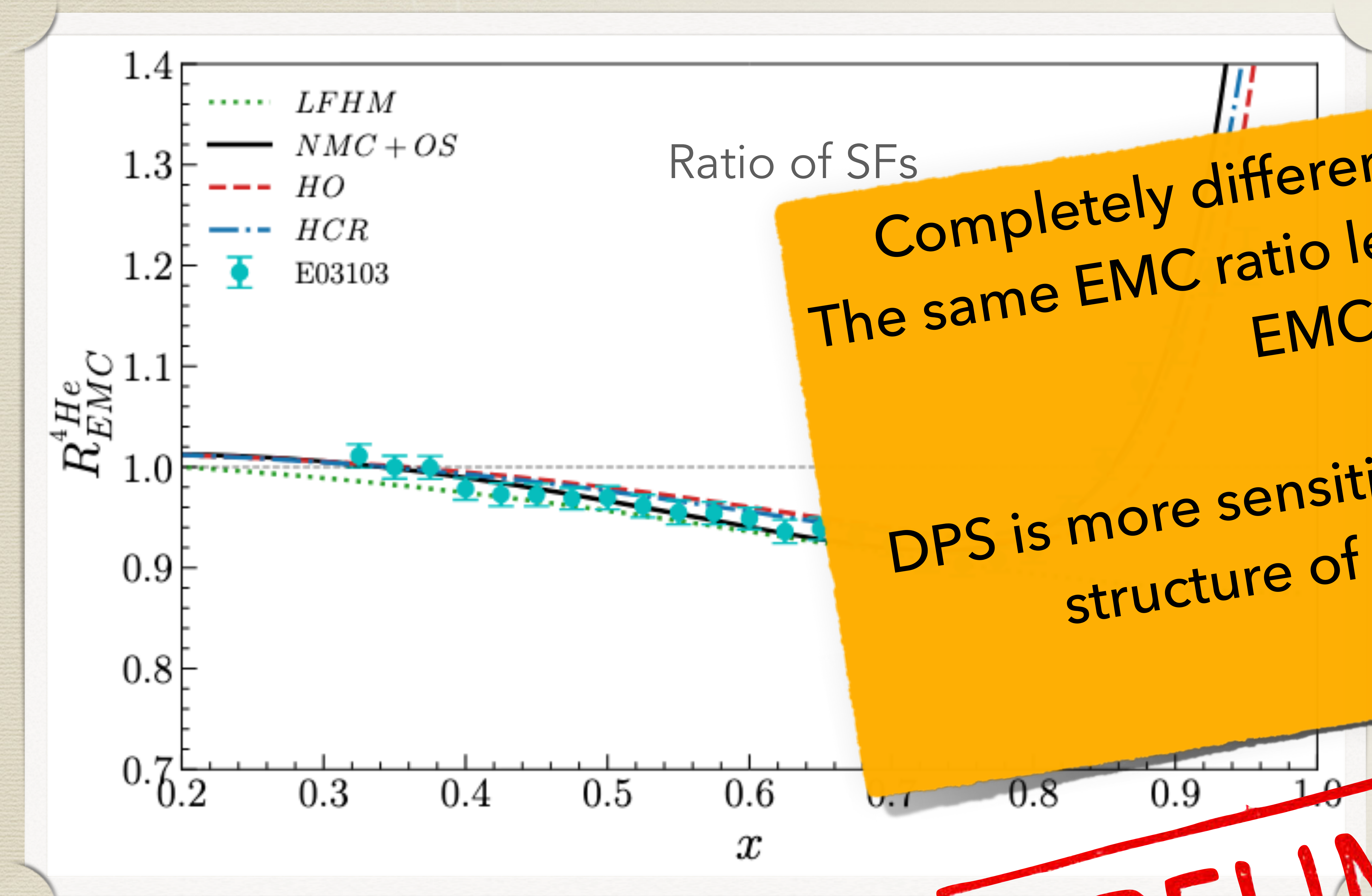
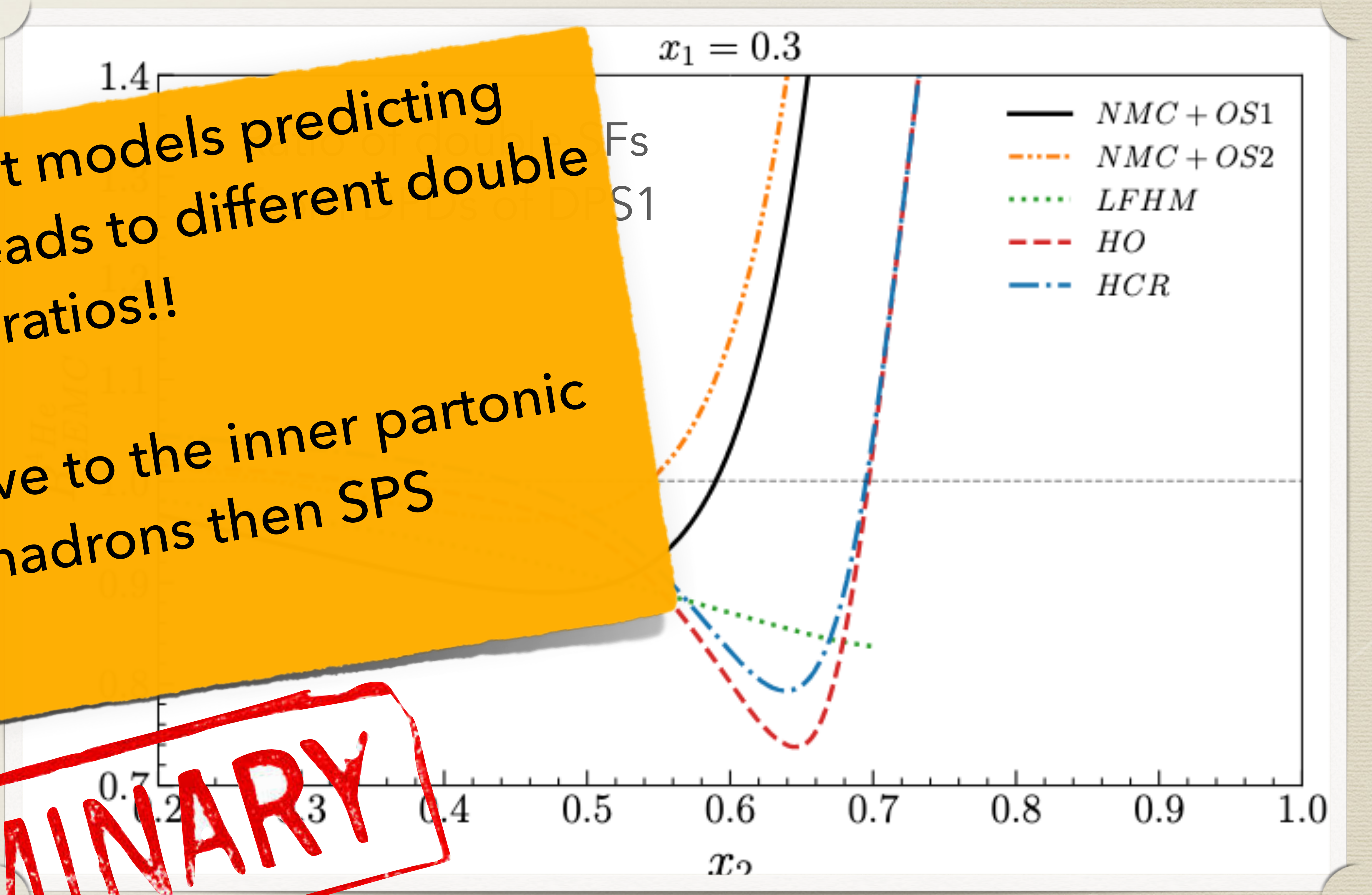




Completely different models predicting the same EMC ratio leads to different double EMC ratios!!

DPS is more sensitive to the inner partonic structure of hadrons then SPS

PRELIMINARY

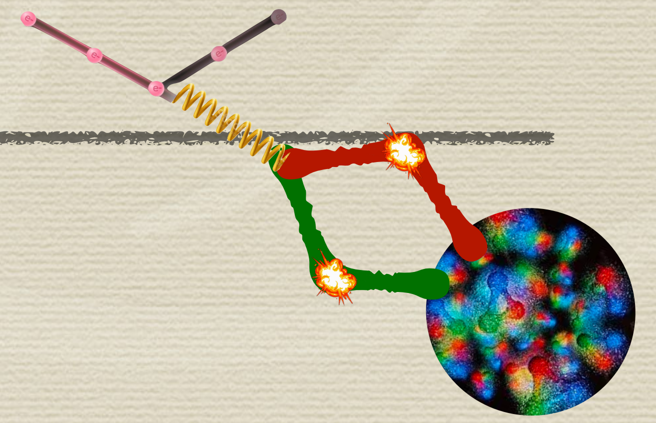
DPS2 in γA collisions with light nuclei

For example in DPS2:

$$D_{ij}^{A,2}(x_1, x_2, \mathbf{k}_\perp) = A(A-1) \sum_{\tau_1,\tau_2=n,p} \sum_{\lambda_1,\lambda_2} \sum_{\lambda_1',\lambda_2'} \int d\xi_1 \, \frac{\bar{\xi}}{\xi_1} \int d\xi_2 \, \frac{\bar{\xi}}{\xi_2} \, \rho^A_{\tau_1\tau_2}(\xi_1, \xi_2, \mathbf{k}_\perp, \lambda_1, \lambda_2, \lambda_1', \lambda_2')$$

$$\times \, \Phi^{\tau_1,i}_{\lambda_1,\lambda_1'}\left(x_1 \frac{\bar{\xi}}{\xi_1}, 0, \mathbf{k}_\perp\right) \Phi^{\tau_2,j}_{\lambda_2,\lambda_2'}\left(x_2 \frac{\bar{\xi}}{\xi_2}, 0, -\mathbf{k}_\perp\right) .$$

DPS2 in γA collisions with light nuclei



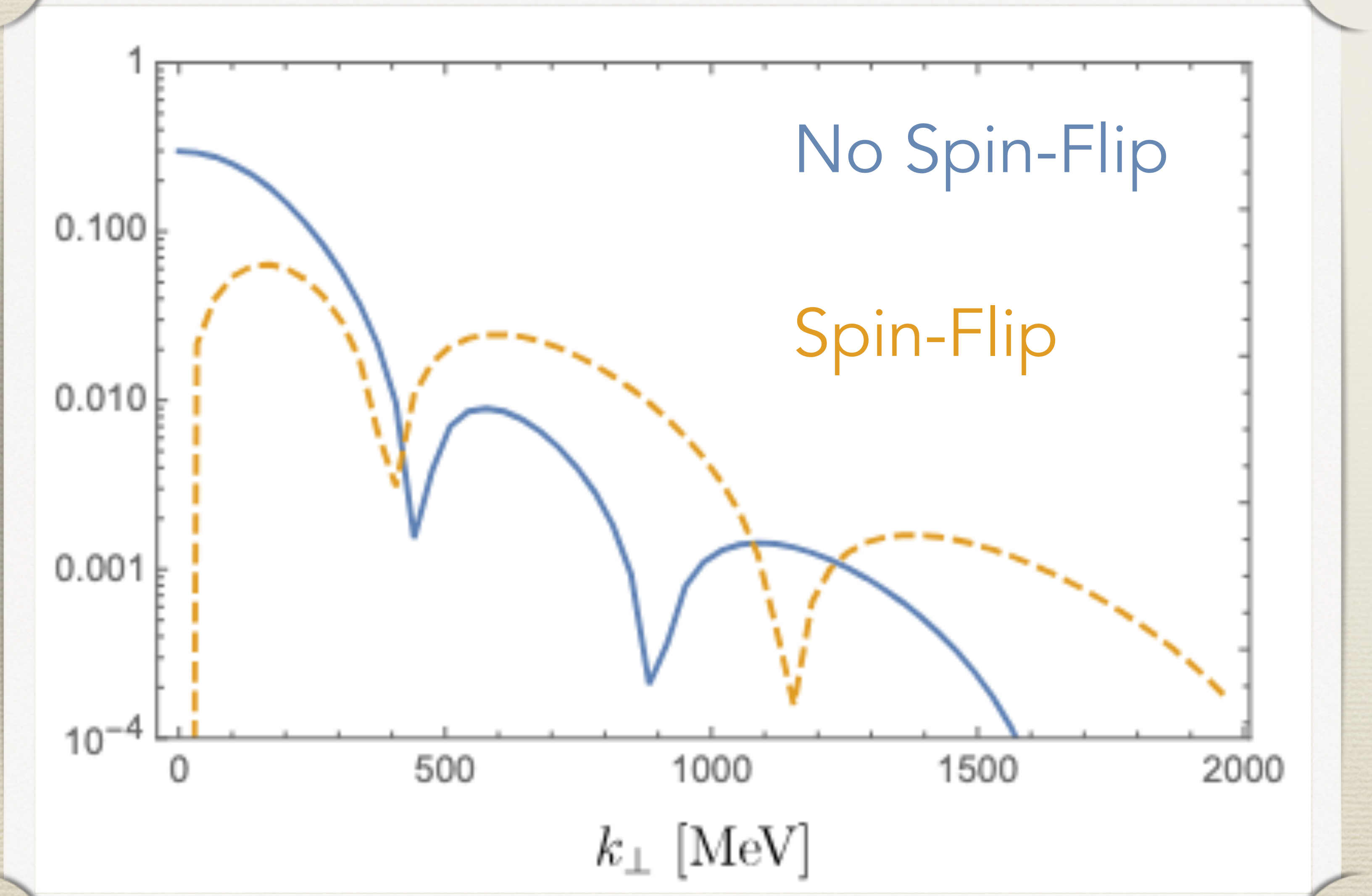
For example in DPS2:

$$D_{ij}^{A,2}(x_1,x_2,\mathbf{k}_\perp) = A(A-1)\sum_{\tau_1,\tau_2=n,p}\sum_{\lambda_1,\lambda_2}\sum_{\lambda_1',\lambda_2'}\int d\xi_1\,\frac{\xi}{\xi_1}\int d\xi_2\,\frac{\xi}{\xi_2}\,\rho^A_{\tau_1\tau_2}(\xi_1,\xi_2,\mathbf{k}_\perp,\lambda_1,\lambda_2,\lambda_1',\lambda_2')$$
$$\times\,\Phi^{\tau_1,i}_{\lambda_1,\lambda_1'}\left(x_1\frac{\bar\xi}{\xi_1},0,\mathbf{k}_\perp\right)\Phi^{\tau_2,j}_{\lambda_2,\lambda_2'}\left(x_2\frac{\bar\xi}{\xi_2},0,-\mathbf{k}_\perp\right).$$

Off-forward
LC momentum distribution

Standard LC correlator parametrized by GPDs

We address the possible role of nucleon spin-flip effects for the first time!



We have:

1) the Off-forward LCMDs which depends of the deuteron obtained within the Av18 Potential + LF approach to properly fulfill the Poincaré covariance

2) the role of spin effects could be important to make Realistic predictions

DPS in γA collisions with light nuclei

Before closing let us mention that the integral over ξ_1 and ξ_2 yields the nuclear two-body form factor:

$$F^{double}_{A,\tau_1,\tau_2}(\mathbf{k}_\perp) = \int \frac{d\xi_1}{\xi_1} \int \frac{d\xi_2}{\xi_2}\; \bar{\xi}^2 \bar{\rho}^A_{\tau_1,\tau_2}(\xi_1, \xi_2, \mathbf{k}_\perp)$$

Nuclear 2-body form factor

Calculated for ^{3}He and ^{4}He in:

V. Guzey, M.R., S. Scopetta, M. Strikman and M. Viviani et al, *"Coherent J/Ψ electroproduction on He4 and He3 at the EIC: probing Nuclear shadowing one nucleon at a time"*, PRL 129 (2022) 24, 242503

DPS in γA collisions with light nuclei

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Nuclear

WE HAVE A LINK BETWEEN
2 DIFFERENT PROCESSES!

Calculated for ^{3}He and ^{4}He in:

V. Guzey, M.R., S. Scopetta, M. Strikman a... ...*ctroproduction on He4 and He3 at the EIC: probing Nuclear shadow*... ...9 (2022) 24, 242503

DPS in γA collisions with light nuclei

Finally:

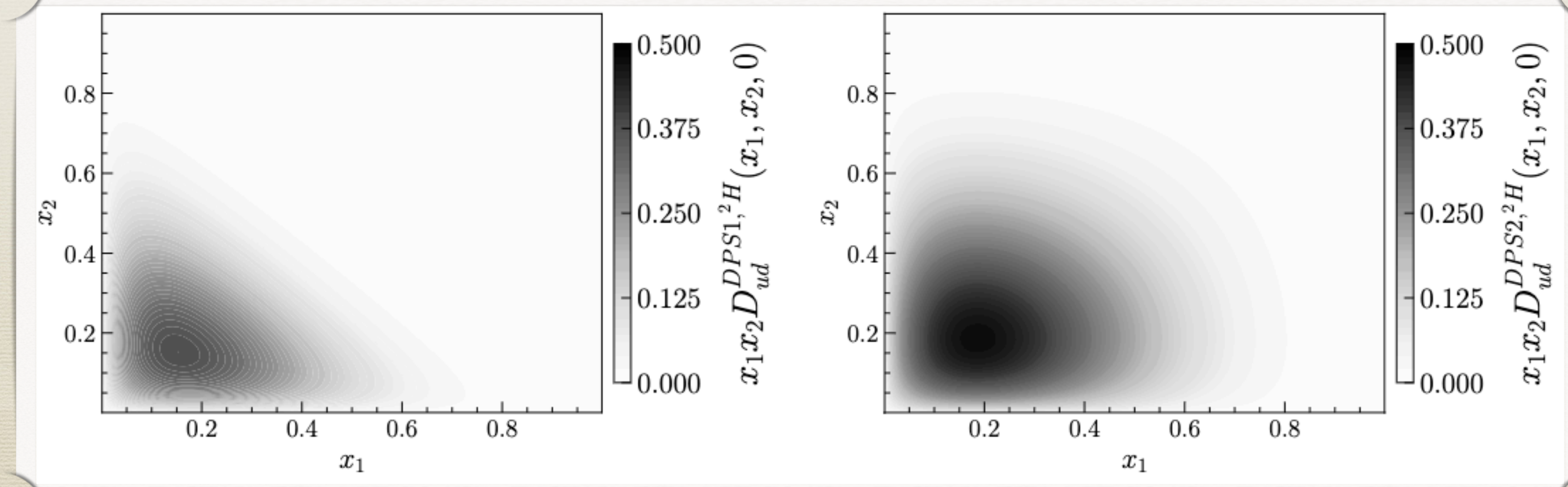
$$D_{ij}^{A,1}(x_1,x_2,\mathbf{k}_\perp) = \int d^2y_\perp \; e^{-i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \tilde{D}_{ij}^{A,1}(x_1,x_2,\mathbf{y}_\perp) = A \sum_{\tau=n,p} \int d\xi \; \frac{\bar{\xi}^2}{\xi^2} \; \rho_\tau^A(\xi) \; D_{ij}^\tau \left(x_1 \frac{\bar{\xi}}{\xi}, x_2 \frac{\bar{\xi}}{\xi}, \mathbf{k}_\perp \right)$$

$$D_{ij}^{A,2}(x_1,x_2,\mathbf{k}_\perp) = A(A-1) \sum_{\tau_1,\tau_2=n,p} \sum_{\lambda_1,\lambda_2} \sum_{\lambda_1',\lambda_2'} \int d\xi_1 \; \frac{\xi}{\xi_1} \int d\xi_2 \; \frac{\xi}{\xi_2} \; \rho_{\tau_1\tau_2}^A(\xi_1,\xi_2,\mathbf{k}_\perp,\lambda_1,\lambda_2,\lambda_1',\lambda_2')$$
$$\times \, \Phi_{\lambda_1,\lambda_1'}^{\tau_1,i} \left(x_1 \frac{\bar{\xi}}{\xi_1}, 0, \mathbf{k}_\perp \right) \Phi_{\lambda_2,\lambda_2'}^{\tau_2,j} \left(x_2 \frac{\bar{\xi}}{\xi_2}, 0, -\mathbf{k}_\perp \right) \; .$$

1) For DPS1 we used the product of PDFs as phenomenological nucleon DPDs (standard strategy)

2) For DPS2 we used the Goloskokv-Kroll model for the nucleon GPDs

Full deuteron DPDs at $k_\perp = 0$:



Conclusions

- **EMC of light-nuclei within a Poincaré covariant LF approach**
  - ☑ We developed a rigorous formalism for the calculation of nuclear SFs, TMD LCMDs, spin-dependent SFs and DPDs involving only nucleonic DOF with the conventional nuclear physics
  - ☑ For ^{3}He we obtain results in agreement with experimental data for the EMC effect.
  - ☑ For the deviations from experimental data could be ascribed to genuine QCD effects: **our results provide a reliable baseline to study exotic phenomena**
  - ☑ The approach has been successfully applied to the calculation of spin-dependent SFs
- **NR calculations**
  - ☑ Calculation of the ^{4}He GPDs which are in good agreement with data (both for coherent and incoherent) channels
  - ☑ Calculation of the ^{3}He GPDs and predictions for asymmetries for the positron beam JLab upgrade
  - ☑ Calculation of the J/ψ electro-production of ^{3}He and ^{4}He including effects beyond IA
- **To do next**
  - ☐ Application of the approach to calculate the EMC effect of heavier nuclei (^{6}Li starting project)
  - ☐ Calculate the Double Parton Scattering cross-section of light-nuclei

An Impulse Approximation for the coherent case

The leading twist ³He GPD E:

M. R, S.Scopetta PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)

$$\underbrace{H_q^A(x,\xi,t) + E_q^A(x,\xi,t)}_{\tilde{G}_q^A(x,\xi,t)} \sim \sum_{n=P,N} \int \frac{dz}{z} \bar{h}^{A,n}(z,\xi,t) \left[\underbrace{H_q^n\left(\frac{x}{z},\frac{\xi}{z},t\right) + E_q^n\left(\frac{x}{z},\frac{\xi}{z},t\right)}_{\tilde{G}_q^n\left(\frac{x}{z},\frac{\xi}{z},t\right)} \right]$$

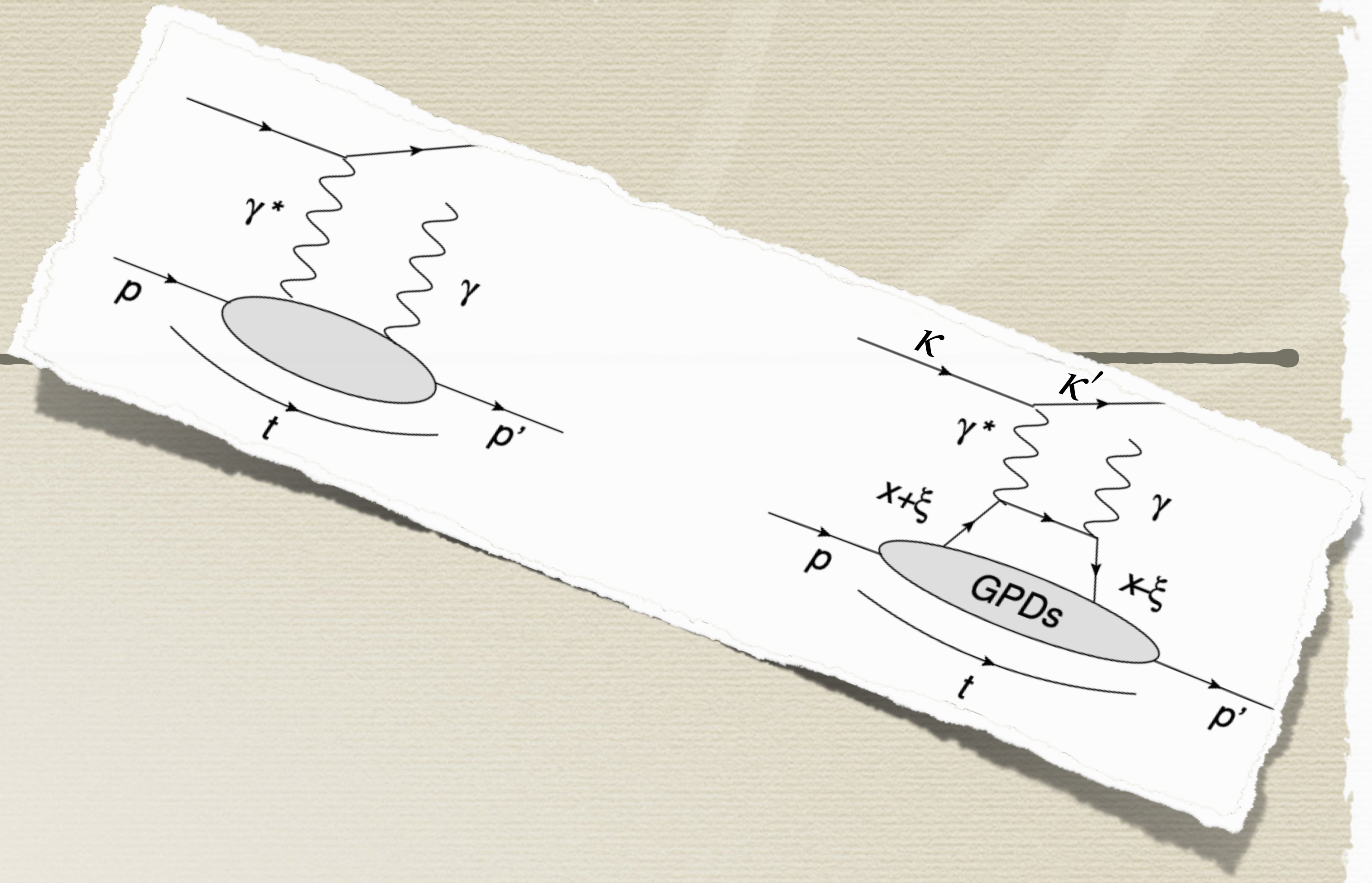
The integral gives the ³He magnetic FF

$$\tilde{H}_q^A(x,\xi,t) \sim \sum_{n=P,N} \int \frac{dz}{z} \bar{h}^{A,n}(z,\xi,t) \left[\tilde{H}_q^n\left(\frac{x}{z},\frac{\xi}{z},t\right) \right]$$

$$\bar{h}^{A,n}(z,\xi,t) = \int dE d\vec{p} \; [P^{A,n}_{+-,+-}(\vec{p},\vec{p}+\vec{\Delta},E) - P^{A,n}_{+-,-+}(\vec{p},\vec{p}+\vec{\Delta},E)] \delta\left(z - \frac{\bar{p}^+}{\bar{P}^+}\right)$$

the spectral functions have been evaluated by means of a realistic treatment based on Av18 wave functions (w.f. from **A. Kievsky et al NPA 577, 511 (1994), A. Kievsky et. al, PRC 56, 64 (1997)**).

GPDs properties



- Forward limit: $\Delta^\mu = 0$

$$H(x,\xi,t) \xrightarrow[\Delta^\mu \to 0]{} f(x)$$

f(x) = Parton Distribution Function (PDF)

- First moment: relations between GPDs and form factors

$$\int_{-1}^{1} dx\; H_q(x,\xi,t) = F_1^q(t) \qquad \int_{-1}^{1} dx\; E_q(x,\xi,t) = F_2^q(t)$$

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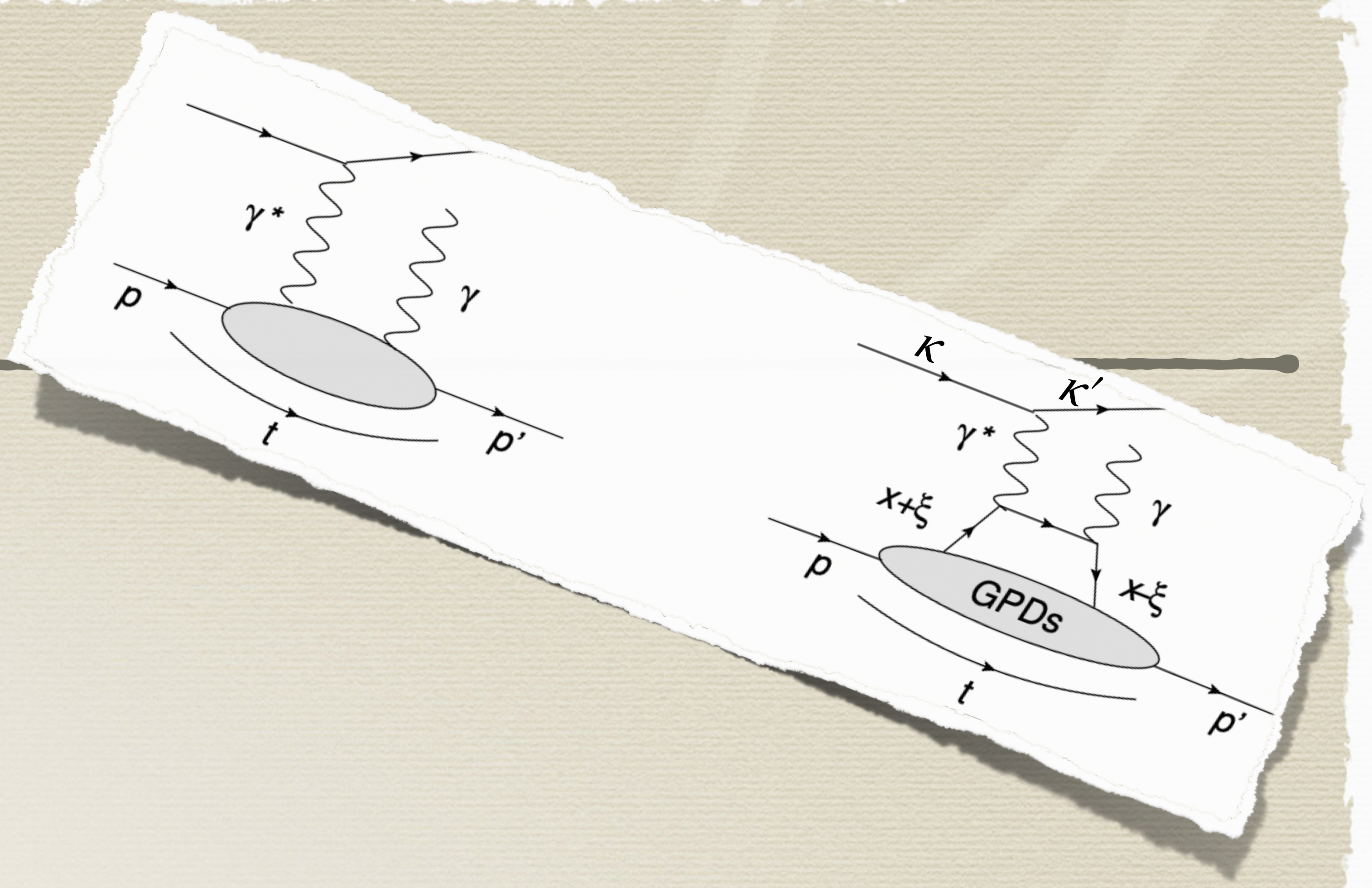

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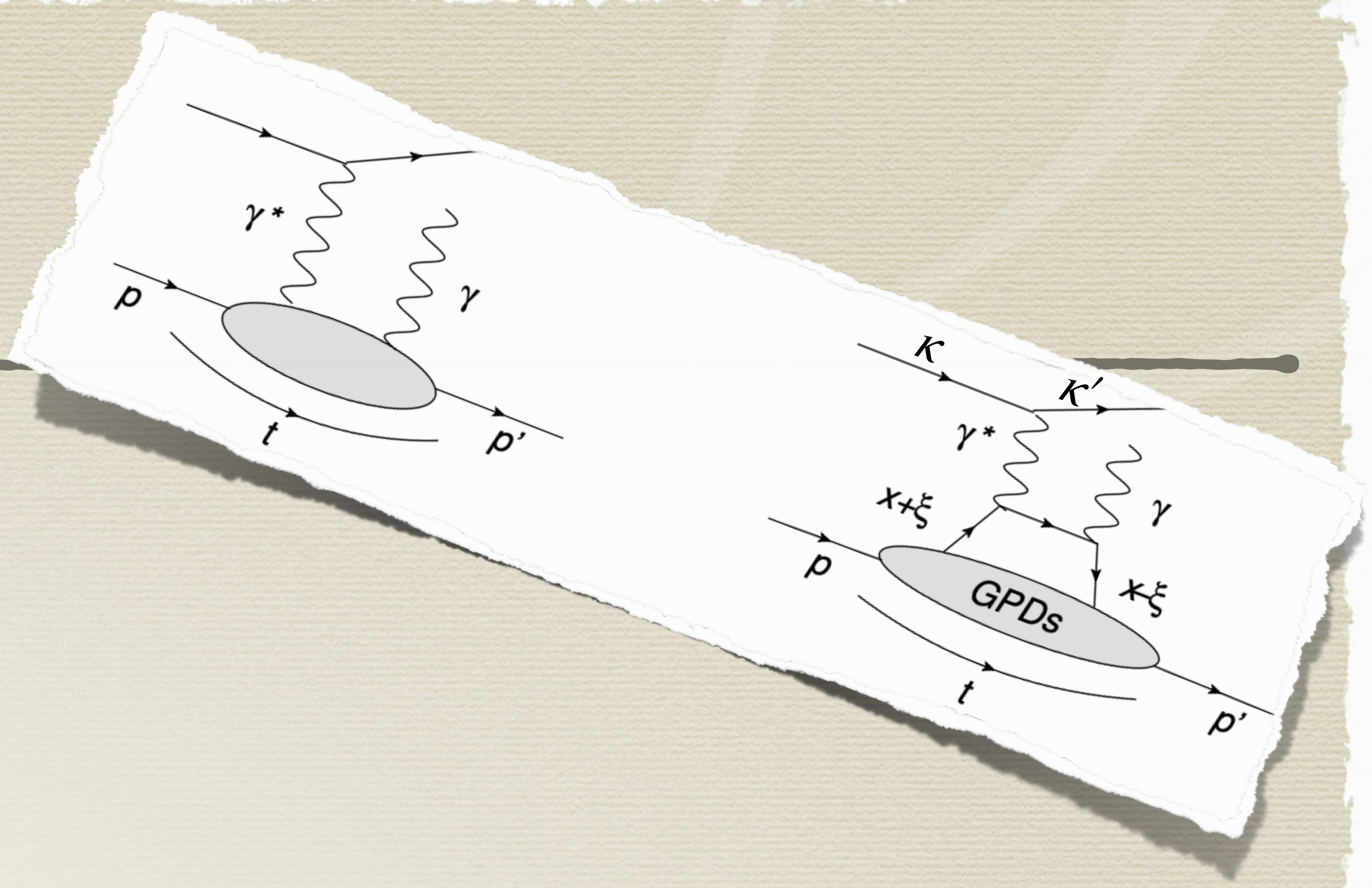
ξ-independence is a consequence of Lorentz invariance

GPDs properties



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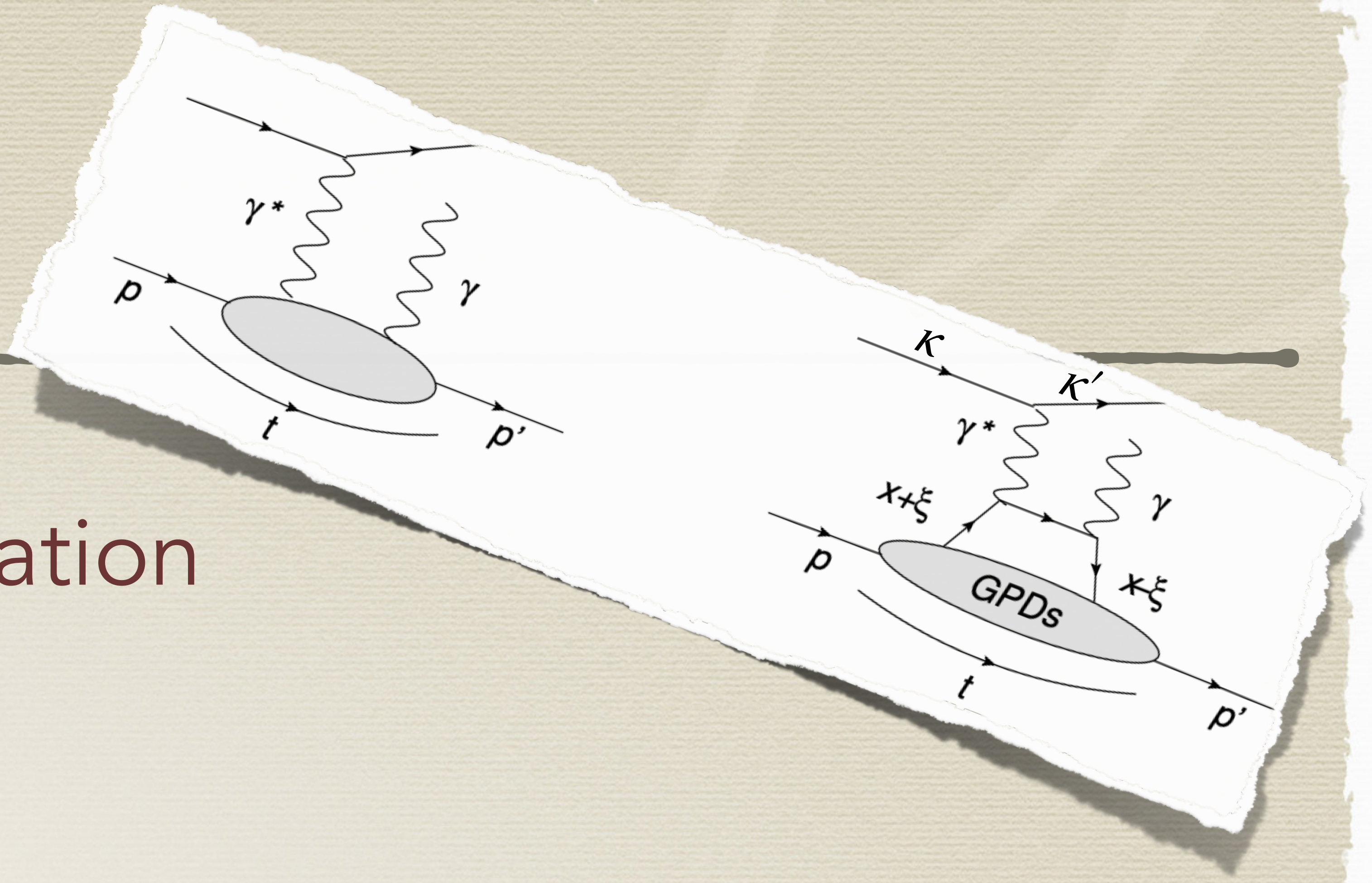
- Lorentz invariance implies **polynomiality**:

$$\int_{-1}^{1} dx\, x^n H^q(x,\xi,t) = \sum_{\substack{i=0 \\ \text{even}}}^{n} (2\xi)^i A^q_{n+1,i}(t) + \text{mod}(n,2)\,(2\xi)^{n+1} C^q_{n+1}(t),$$

$$\int_{-1}^{1} dx\, x^n E^q(x,\xi,t) = \sum_{\substack{i=0 \\ \text{even}}}^{n} (2\xi)^i B^q_{n+1,i}(t) - \text{mod}(n,2)\,(2\xi)^{n+1} C^q_{n+1}(t).$$

GPDs properties



- The Fourier Transform of GPDs at $\xi = 0$ have a probabilistic interpretation

$$\rho_q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} H_q(x, 0, t) + \ldots$$

Hadron tomography

- Moments of GPDs $\int dx\ x^n \mathrm{GPDs}$ are related to **gravitational form factors**

Mechanical properties of hadrons

- Ji's sum rule:

$$\langle J_q \rangle = \int_{-1}^{1} dx\ x \left[H_q(x, \xi, 0) + E(x, \xi, 0) \right]$$

Solution to the proton spin crisis?

Our approach: Reference frames

In order to implement **macro-locality**, it is crucial to distinguish between different frames:

- The Lab frame, where $\tilde{P} = (M_{BT}, \mathbf{0}_\perp)$
- The intrinsic LF frame of the whole system, $[1,2,\ldots,A]$, where $\tilde{P} = (M_0[1,2,\ldots,A], \mathbf{0}_\perp)$ with $k_i^+ = \xi_i M_0[1,2,\ldots,A]$ and $M_0[1,2,\ldots,A] = \sum_{i=1}^{A} \sqrt{m^2 + \mathbf{k}_i^2}$
- The intrinsic LF frame of the cluster $[1;2,3,\ldots,(A-1)]$ where $\tilde{P} = (\mathscr{M}_0[1;2,3,\ldots,A-1]), \mathbf{0}_\perp)$ with $k^+ = \xi \mathscr{M}_0[1;2,3,\ldots,A-1]$ and $\mathscr{M}_0[1;2,3,\ldots,A-1] = \sqrt{m^2+\kappa^2} + \sqrt{M_s^2+\kappa^2}$

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While $\mathbf{p}_\perp^{LAB} = \mathbf{k}_{1\perp} = \kappa_\perp$

$M_s = (A-1)m + \epsilon$ is the mass of the fully interacting spectator system

^{3}He TMDs

Alessandro,Del Dotto, Pace, Perna, Salm`e, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

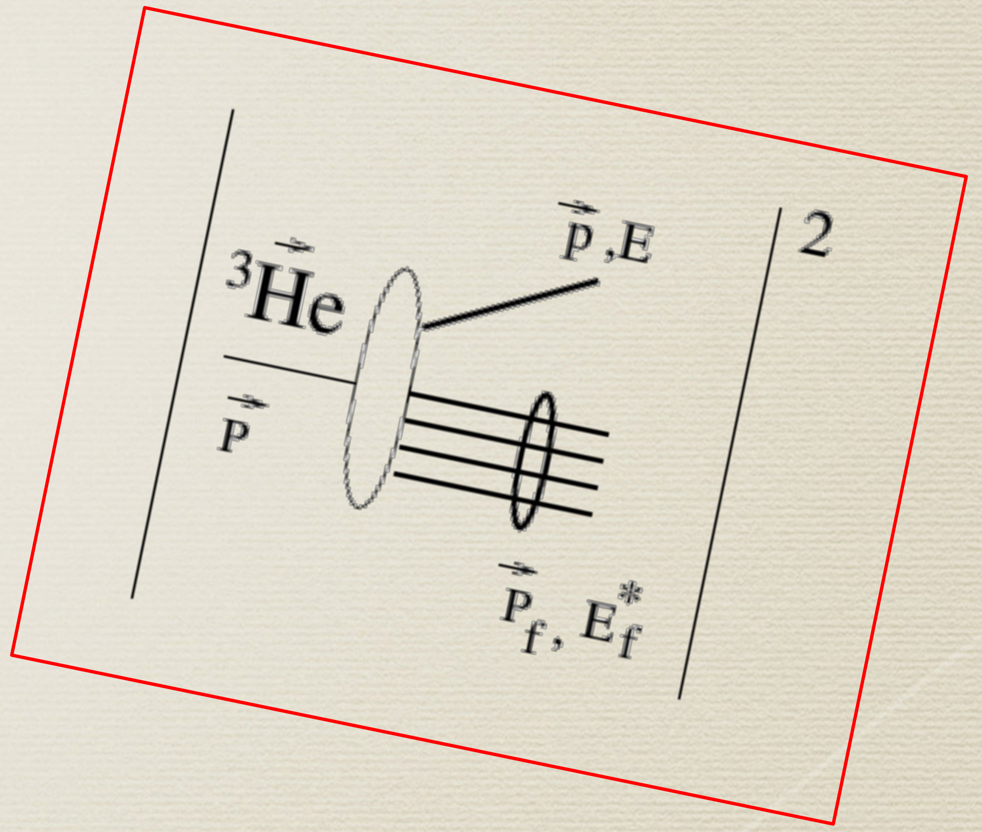
From a theoretical point of view, we need:

a description of the nuclear dynamics which retains as many general properties as possible...

... leading to realistic procedures to *extract* the Nucleon (neutron) structure

In the presented approach the key quantity is the nuclear *SPECTRAL FUNCTION*
(Nucleon Green's function in the medium)

$$P_{\sigma'\sigma}(k,E) = -\frac{1}{\pi}\Im m\left\{\langle\Psi_{gr}|a^{\dagger}_{\mathbf{k},\sigma'}\,\frac{1}{E-H+i\epsilon}\,a_{\mathbf{k},\sigma}|\Psi_{gr}\rangle\right\}$$



Quite familiar in nuclear Physics;in hadron physics one introduces the LC correlator:

$$\Phi^{\tau}(x,y) = \langle\Psi_{gr}|\bar{\psi}_{\tau}(x)\mathcal{W}(\hat{n}\cdot A)\psi_{\tau}(y)|\Psi_{gr}\rangle$$

Our point: in valence approximation, one can relate $P_{\sigma'\sigma}(k,E)$(given in a Poincaré covariant framework) and $\Phi^{\tau}(x,y)$

^{3}He TMDs

Alessandro,Del Dotto, Pace, Perna, Salm`e, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

- In Instant form (initial hyperplane t=0), one can couple spins and orbital angular momenta via Clebsch-Gordan (CG) coefficients. In this form the three rotation generators are independent of the the interaction.

- To embed the CG machinery in the LFHD one needs unitary operators, the so-called Melosh rotations that relate the LF spin wave function and the canonical one. For a particle of spin (1/2) with LF momentum $\tilde{\mathbf{k}} \equiv \{k^+, \vec{k}_\perp\}$

$$|\mathbf{k}; \frac{1}{2}, \sigma\rangle_c = \sum_{\sigma'} \underbrace{D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))}_{\text{Wigner rotation for the J=1/2 case}} |\tilde{\mathbf{k}}; \frac{1}{2}, \sigma'\rangle_{LF}$$

- $R_M(\tilde{\mathbf{k}})$ is the Melosh rotation connecting the intrinsic LF and canonical frames, reached through different boosts from a given frame where the particle is moving

$$D^{1/2}[R_M(\tilde{\mathbf{k}})]_{\sigma'\sigma} = \chi^\dagger_{\sigma'} \frac{m + k^+ - \imath\boldsymbol{\sigma} \cdot (\hat{z} \times \mathbf{k}_\perp)}{\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}} \chi_\sigma = {}_{LF}\langle \tilde{\mathbf{k}}; s\sigma' | \mathbf{k}; s\sigma\rangle_c$$

N.B. If $|\mathbf{k}_\perp| << k^+, m \longrightarrow D_{\sigma'\sigma} \simeq I_{\sigma'\sigma}$

^{3}He TMDs

Alessandro,Del Dotto, Pace, Perna, Salm`e, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

The Spectral Function: probability distribution to find inside a bound system a particle with a given $\tilde{\kappa}$ when the rest of the system has energy ϵ , with a polarization vector **S**:

$$\mathcal{P}^{\tau}_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \rho(\epsilon) \sum_{JJ_z\alpha} \sum_{Tt} {}_{LF}\langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\boldsymbol{\kappa}} | \Psi_{\mathcal{M}}; ST_z\rangle \langle ST_z; \Psi_{\mathcal{M}} | \tilde{\boldsymbol{\kappa}}, \sigma\tau; JJ_z; \epsilon, \alpha; Tt\rangle_{LF}$$

$$|\Psi_{\mathcal{M}}; ST_z\rangle = \sum_m |\Psi_m; S_z T_z\rangle \; D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha, \beta, \gamma)$$

Euler angles of rotation from the z-axis to the polarization vector S

$|\Psi_m; S_z T_z\rangle = |j, j_z; \epsilon^3; \frac{1}{2} T_z\rangle$ three-body bound eigenstate of $M_{BT}(123) \sim M^{NR}$

$|\tilde{\boldsymbol{\kappa}}, \sigma\tau; JJ_z; \epsilon, \alpha; T\tau\rangle_{LF}$ tensor product of a plane wave for particle 1 with LF momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the $[1 + (23)]$ cluster times the fully interacting state of the (23) pair of energy eigenvalue ϵ. It has eigenvalue

$$\mathcal{M}_0(1, 23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} + E_S \quad E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2} \qquad M_S = 2\sqrt{m^2 + m\epsilon}$$

and fulfills the macroscopic locality (Keister, Polyzou, Adv. N. P. 20, 225 (1991)).

$$\tilde{\boldsymbol{\kappa}} = (\kappa^+ = \xi \; \mathcal{M}_0(1, 23), \mathbf{k}_\perp = \boldsymbol{\kappa}_\perp)$$

^{3}He TMDs

Alessandro, Del Dotto, Pace, Perna, Salm`e, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

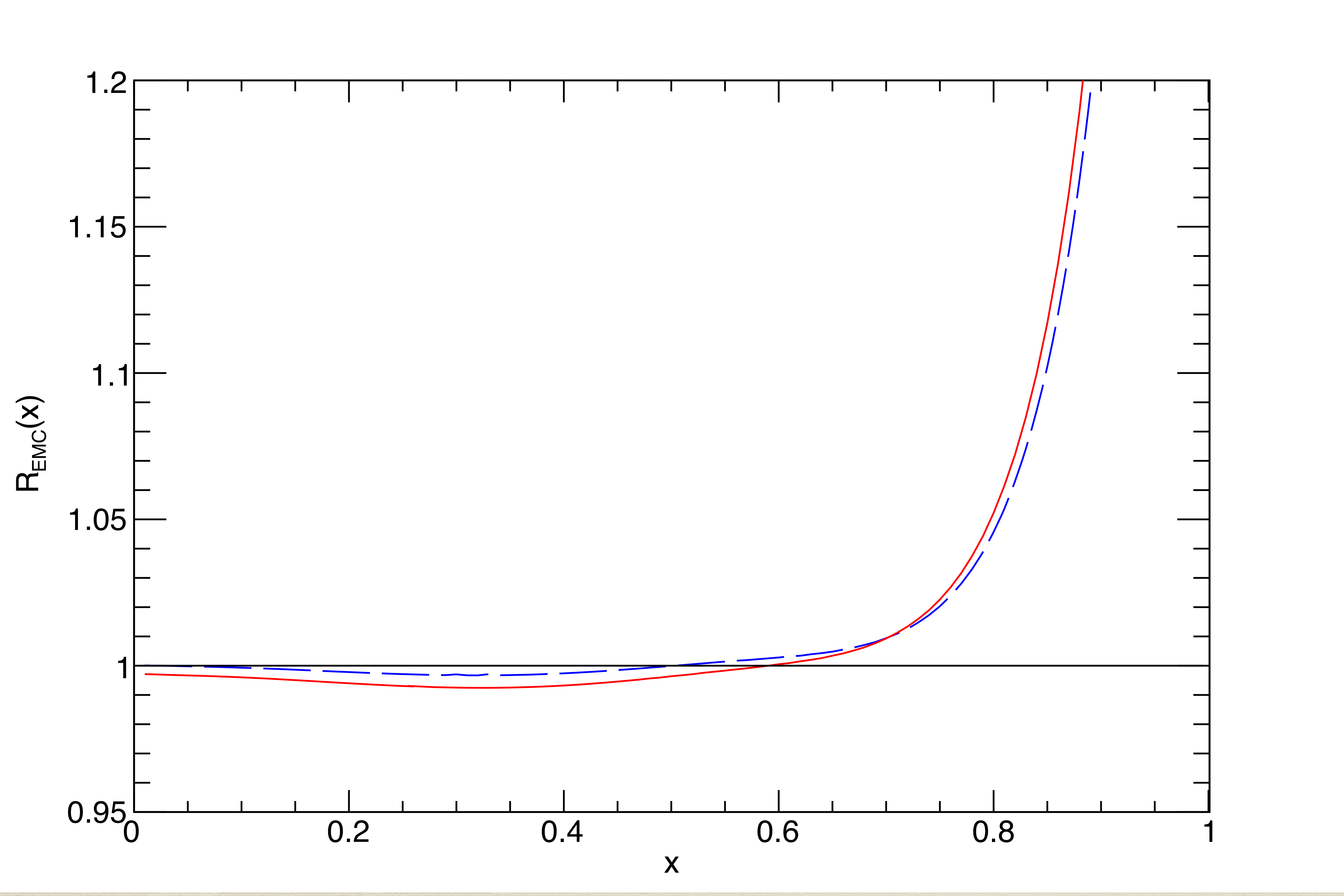
The LF overlaps for ^{3}He SF in terms of the IF ones are

$$\overbrace{\langle T\tau;\alpha,\epsilon;JJ_z;\tau_1\sigma,\tilde{\boldsymbol{\kappa}}}^{<\tilde{\boldsymbol{\kappa}}|\times 2N\ state}|\overbrace{j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{LF}}^{3N\ bound\ state} = \sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}\sum_{\sigma'_1} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'_1}$$

$$\sqrt{(2\pi)^3\,2E(\mathbf{k})}\sqrt{\frac{\kappa^+E_{23}}{k^+E_S}}\sum_{\sigma''_2,\sigma''_3}\sum_{\sigma'_2,\sigma'_3}\mathcal{D}_{\sigma''_2,\sigma'_2}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\,\mathcal{D}_{\sigma''_3,\sigma'_3}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)$$

$${}_{IF}\langle T,\tau;\alpha,\epsilon;JJ_z|\mathbf{k}_{23},\sigma''_2,\sigma''_3;\tau_2,\tau_3\rangle\,\langle\sigma'_3,\sigma'_2,\sigma'_1;\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{IF}$$

Backup slides



Results similar to ^3He and ^4He

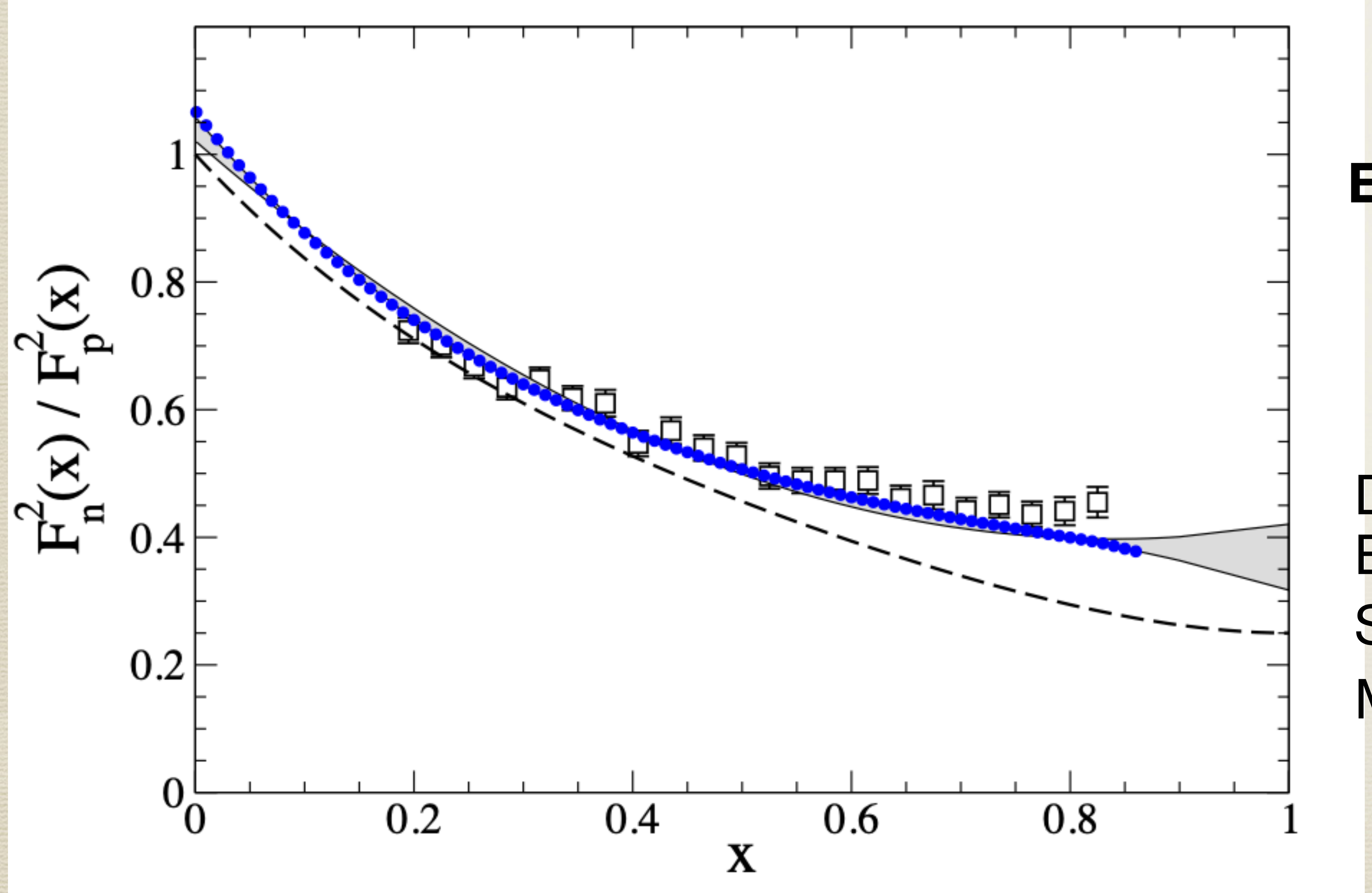
Solid line: Av18/UIX; Dashed-line: NVIb/UIX

Backup slides

MARATHON coll. : experimental data of the super-ratio $R^{ht}(x) = F_2^{^3He}(x)/F_2^{3H}(x)$

^3He: 2p + n; ^3H: n + 2p

Is possible to extract the ratio $F_2^n(x)/F_2^p(x)$ through the super-ratio



E.Pace, M.Rinaldi, G.Salmè and S.Scopetta Phys. Lett. B 839(2023) 127810

Dashed line: ratio from SMC collaboration
Empty squares: MARATHON extraction
Solid line: cubic and conic extractions from F_2^p SMC parametrization, fitted to MARATHON data

Backup slides

* E.Pace, M.Rinaldi, G.Salmè and S. Scopetta, **Phys. Scr. 95, 064008 (2020)**

$$W_A^{s,\mu\nu} = \sum_N \sum_\sigma \not{\sum} d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{2(2\pi)^3 \kappa^+} \frac{1}{\xi} P^N(\tilde{\kappa}, \epsilon) \; w_{N,\sigma}^{s,\mu\nu}(p,q)$$

hadronic tensor of the nucleon

Unpolarized LF spectral function:

$$P^N(\tilde{\kappa}, \epsilon) = \frac{1}{2j+1} \sum_{\mathcal{M}} P^N_{\sigma\sigma}(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M})$$

$W_A^{s,\mu\nu}$ is parametrized by the SFs $F_2^A(x)$ and $F_1^A(x)$:

$$F_2^A(x) = -\frac{1}{2} x g_{\mu\nu} W_A^{s,\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{d\kappa^+}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) F_2^N(z)$$

Free nucleon SF

Where $x = \dfrac{Q^2}{2P_A \cdot q}$ and $\xi = \dfrac{\kappa^+}{\mathcal{M}_0[1;2,3,\ldots,A-1]}$ with $z = \dfrac{Q^2}{2p \cdot q} = \dfrac{p}{P_A^+}\dfrac{x}{\xi}$

Backup slides

* A. Del Dotto, E.Pace, G. Salmè and S.Scopetta, **Phys. Rev. C 95,014001 (2017)**

$$F_2^A(x) = -\frac{1}{2} x g_{\mu\nu} W_A^{s,\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{d\kappa^+}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) F_2^N(z)$$

In the Bjorken limit $\int d\epsilon \int d\kappa^+ = \int d\kappa^+ \int d\epsilon$ so we can use the **light-cone momentum distribution** (LCMD) instead of the **LF spectral function** *

LCMD: $f_1^N(\xi) = \sum \int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{1}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) \frac{E_s}{1-\xi} = \int d\mathbf{k}_\perp n^n(\xi, \mathbf{k}_\perp)$

LF momentum distribution:

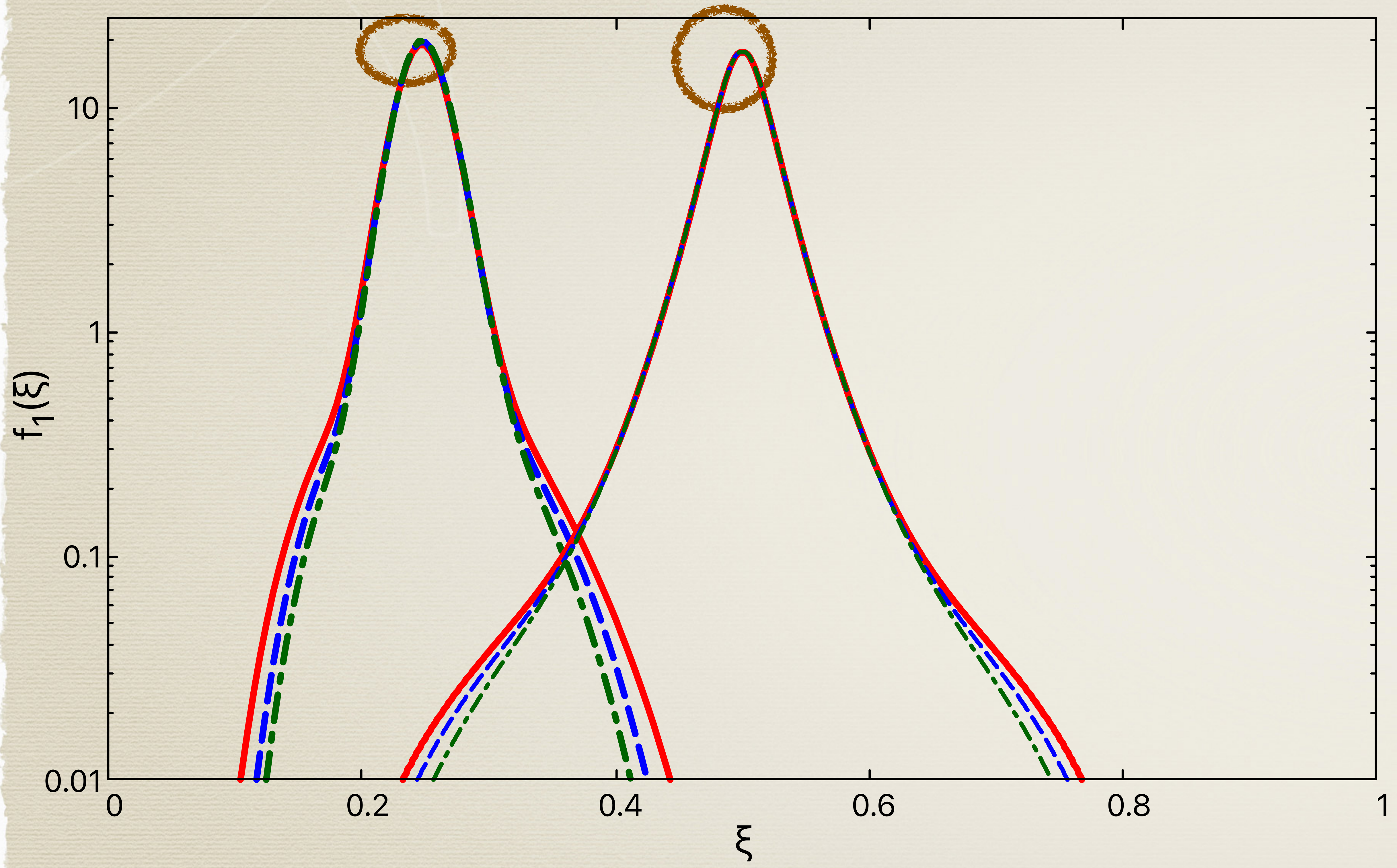
$$n^N(\xi, \mathbf{k}_\perp) = \frac{1}{2\pi} \int \prod_{i=2}^{A-1} [d\mathbf{k}_i] \; \left| \frac{\partial k_z}{\partial \xi} \right| \; \mathcal{N}^N(\mathbf{k}, \mathbf{k}_2, \ldots, \mathbf{k}_{A-1})$$

Squared nuclear wave function. **Thanks to the BT construction, one is allowed to use the NR one**

Determinant of the Jacobian matrix. **LF boost: effect of a Poincaré covariance approach**

Backup slides

The distributions are peaked at 1/A with an accuracy of 1/1000:



F.F, E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani, ***Phys.Lett.B* 851 (2024)**

- The tails of the distributions are generated by the **short range correlations (SRC)** induced by the potentials (i.e the high-momentum content of the 1-body momentum distribution)
- The tails of the LC momentum distribution calculated by the Av18/UIX potential is larger than the ones obtained by the χEFT potentials for both ^4He and deuteron

LC momentum distribution for ^4He (peaked at 0.25) and deuteron (peaked at

Backup slides

For the **polarized DIS** we need to calculate the **antysimmetric** part of the **hadronic tensor**:

$$W_A^{a,\mu\nu} = \sum_N \sum_\sigma \oint d\epsilon \int \frac{d\kappa d\kappa^+}{2(2\pi)^3 \kappa^+} \frac{1}{\xi} P_\sigma^N(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M}) \; w_{N,\sigma}^{a,\mu\nu}(p,q)$$

hadronic tensor of the nucleon

Spin-dependent LF spectral function

$W_A^{a,\mu\nu}$ is parametrized by the the **spin-dependent SFs (SSFs)** $g_1^A(x, Q^2)$ and $g_2^A(x, Q^2)$

As for the unpolarized case, in the **Bjorken limit** we can write a **convolution formula** for the **SSFs**:

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi \left[g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi) \right], j = 1,2$$

Backup slides

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi \left[g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi) \right], j = 1,2$$

The spin-dependent **LCMD** $l_j^N(\xi)$ and $h_j^N(\xi)$ are related to the **transverse momentum-**

We used the **TMDs** for ^3He calculated with the **Av18** potential in Ref. **[1]**

GRSV parametrization [2] for the $g_1^N(x)$ SSF

$g_2^N(x)$ extracted by $g_1^N(x)$ with the **Wandzura-Wilczek** formula [3]:

$$g_2^N(x) = -g_1^N(x) + \int_x^1 dy \frac{g_1^N(y)}{y}$$

[1] R.Alessandro, A.Del Dotto, E.Pace, G.Perna, G.Salmè and S.Scopetta. **Phys.Rev.C 104(2021) 6,065204**

[2] M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, **Phys. Rev. D 63, 094005 (2001)**

[3] S. Wandzura and F. Wilczek, **Phys. Lett. B 72, 195 (1977)**

Backup slides

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi \left[g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi) \right]$$

One can **approximate** this equation using that $l_j^N(\xi), h_j^N(\xi)$ are **peaked** around $\xi \simeq 1/A$ and so

$$g_j^{\bar{n}}(x) = \frac{1}{p_j^n} \left[g_j^{^3He}(x) - 2 p_j^p g_j^p(x) \right]$$

Where the **effective polarization** p_j^N are **integral** of the **TMDs** $\Delta f(\xi, k_\perp)$ **and** $\Delta_T' f(\xi, k_\perp)$*

$$p_1^N = \int_0^1 d\xi \int d\mathbf{k}_\perp \Delta f(\xi, k_\perp) \text{ and } p_2^N = \int_0^1 d\xi \int d\mathbf{k}_\perp \Delta_T' f(\xi, k_\perp)$$

We compared our extraction of the **neutron SSFS** with the one of the **GRSV parametrization** and with the **NR extraction**, obtained through the effective polarizations calculated from a NR

Neutron GPDs from ^{3}He

M. R, S.Scopetta PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)

Let us consider $\tilde{G} = H + E$

$$\tilde{G}_q^3(x,\xi,t) \sim \sum_{n=P,N} \int \frac{dz}{z} \bar{h}^{3,n}(z,\xi,t) \left[\tilde{G}_q^n\left(\frac{x}{z},\frac{\xi}{z},t\right)\right]$$

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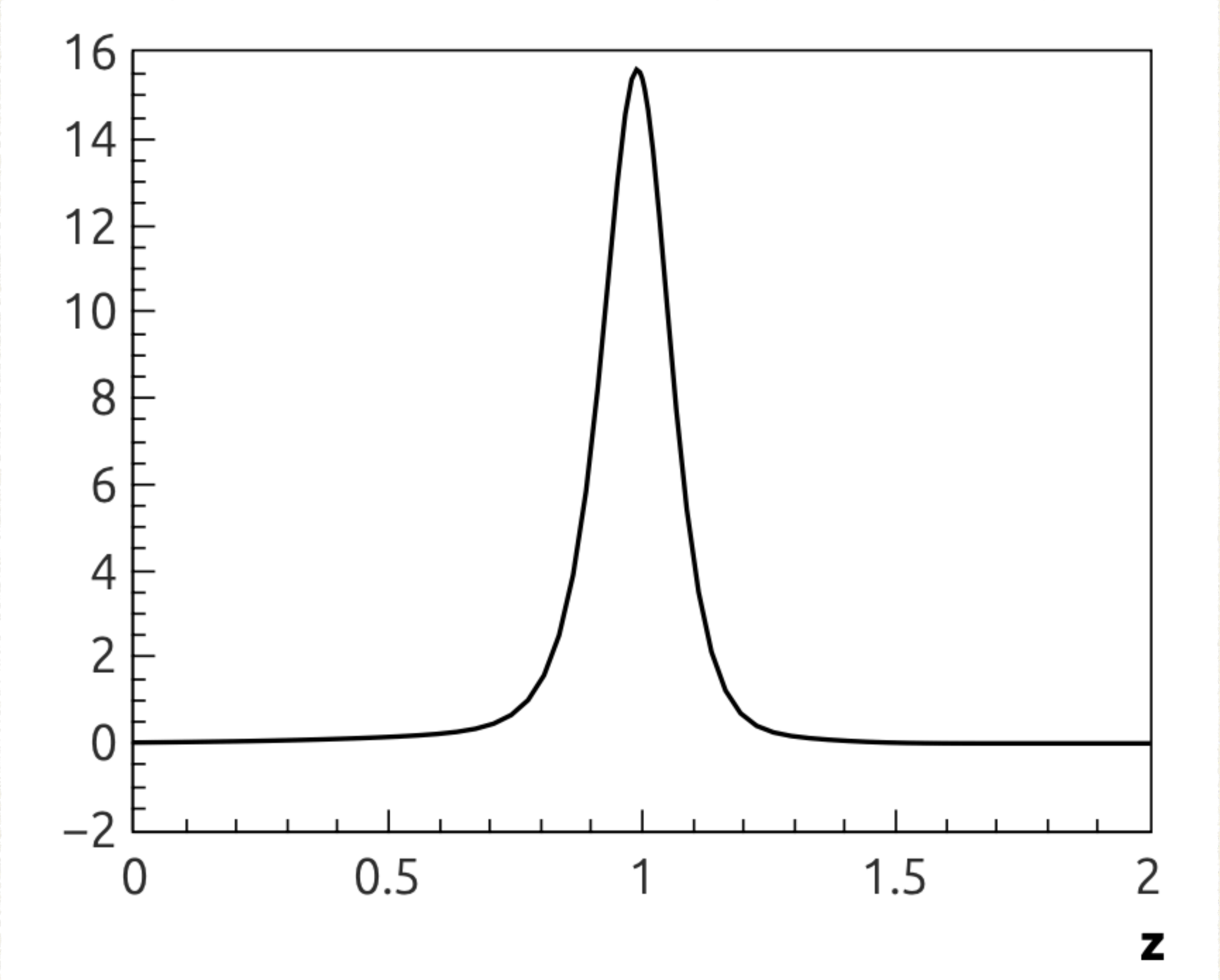

Peaked around z ~ 1

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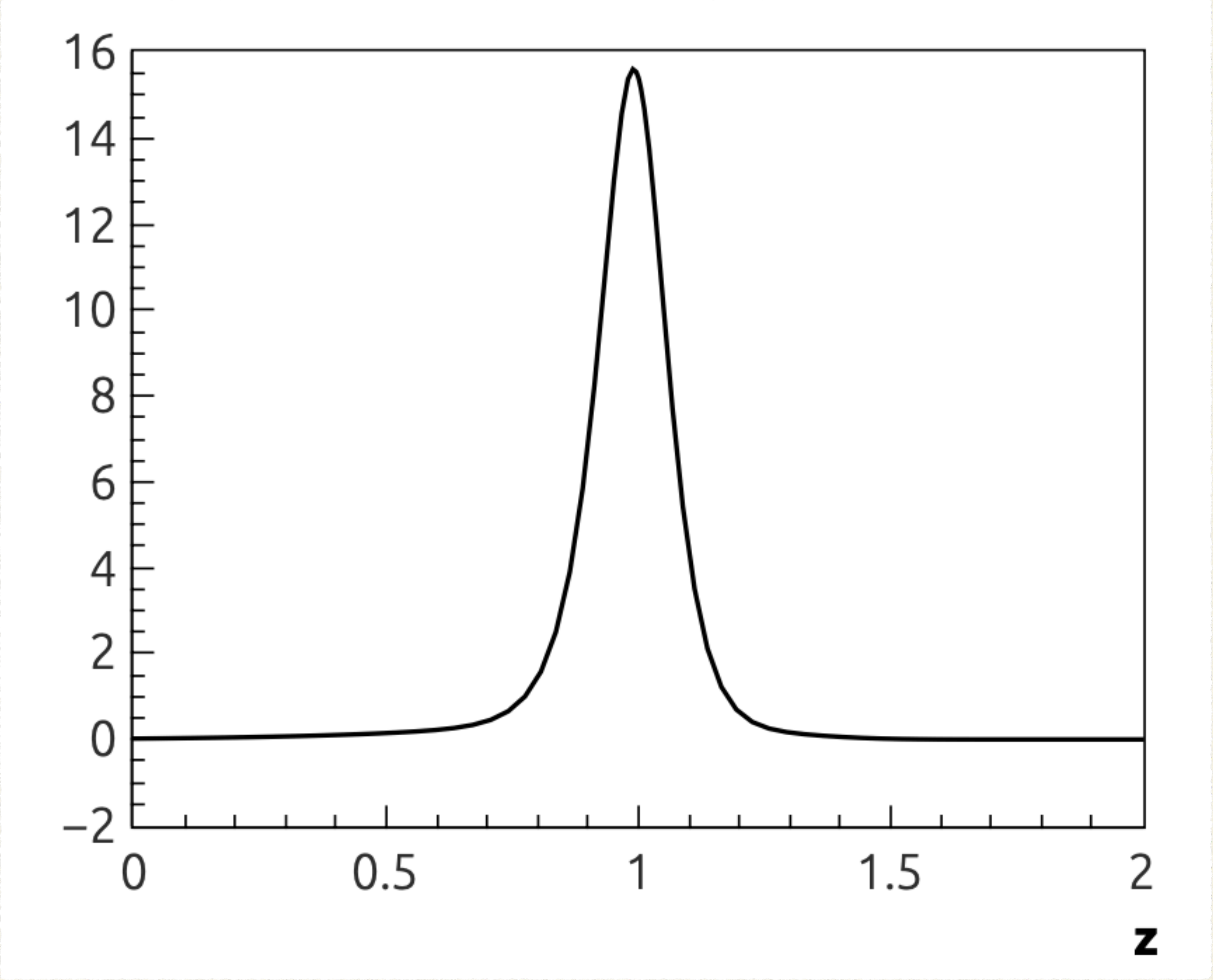

Peaked around z ∼ 1, hence:

$$\tilde{G}_q^3(x,\xi,t) \sim G_{point}^{3,p}(t)\tilde{G}_q^p(x,\xi,t) + G_{point}^{3,n}(t)\tilde{G}_q^n(x,\xi,t)$$

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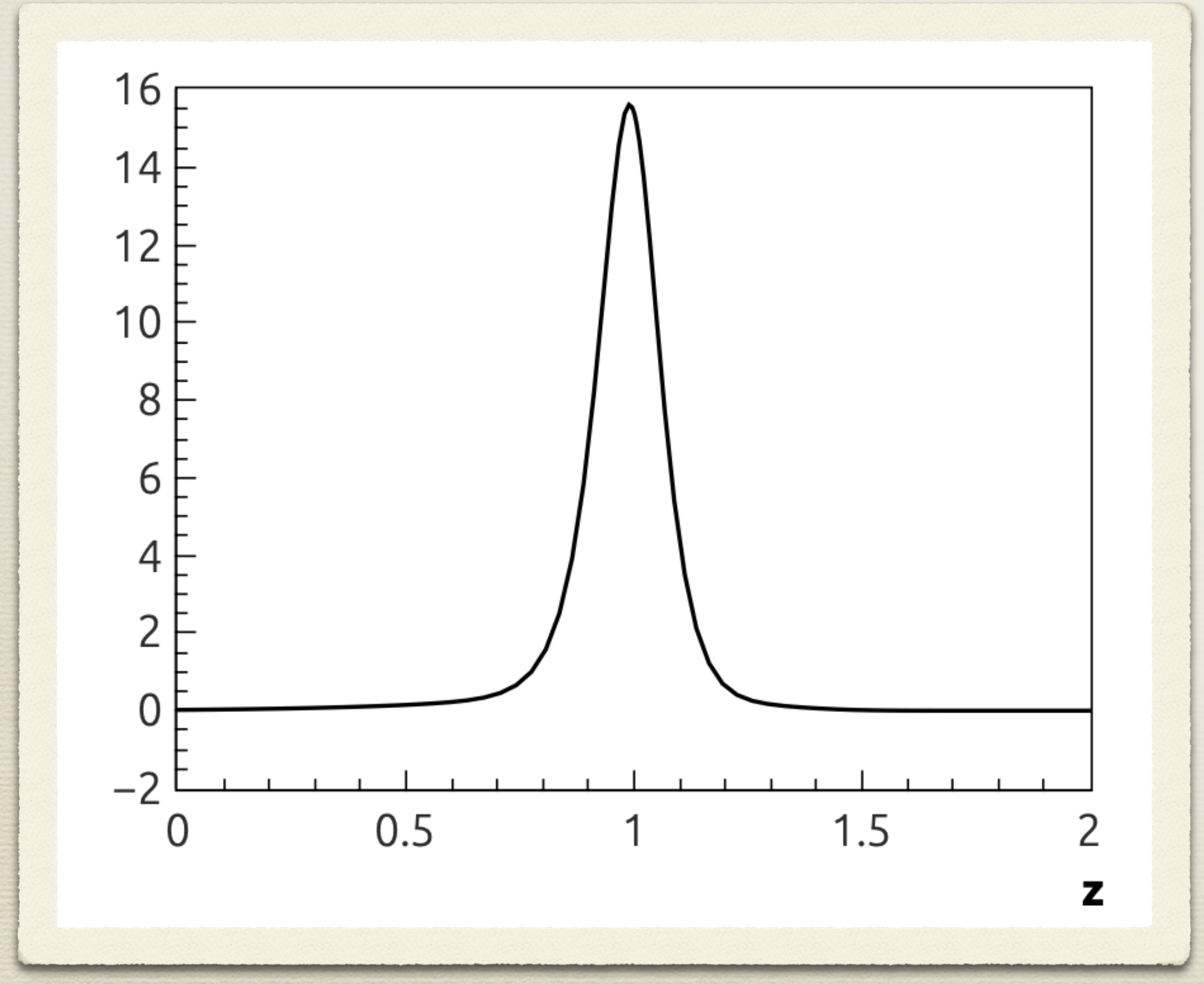

Peaked around $z \sim 1$, hence:

$$\tilde{G}^3_q(x,\xi,t) \sim G^{3,p}_{point}(t)\tilde{G}^p_q(x,\xi,t) + G^{3,n}_{point}(t)\tilde{G}^n_q(x,\xi,t)$$

where:

$$G^{3,n}_{point}(t) = \int dE d\vec{p}\, \left[P^{A,n}_{+-,+-}(\vec{p},\vec{p}+\vec{\Delta},E) - P^{A,n}_{+-,-+}(\vec{p},\vec{p}+\vec{\Delta},E)\right]$$

realistic calculations with AV18 w.f. available!

Neutron GPDs from ^{3}He

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We can extract the neutron GPDs from ^{3}He data:

$$\tilde{G}_q^3(x,\xi,t) \sim G_{point}^{3,p}(t)\tilde{G}_q^p(x,\xi,t) + G_{point}^{3,n}(t)\tilde{G}_q^n(x,\xi,t) \quad \Longrightarrow \quad \tilde{G}_q^n(x,\xi,t) = \frac{\tilde{G}_q^3(x,\xi,t) - G_{point}^{3,p}(t)\tilde{G}_q^p(x,\xi,t)}{G_{point}^{3,n}(t)}$$

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We test the extraction procedure:

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2) we evaluate the ^{3}He GPDs and the point-like FFs

Neutron GPDs from ³He

M. R, S.Scopetta PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)

We can extract the neutron GPDs from ³He data:

$$\tilde{G}_q^3(x,\xi,t) \sim G^{3,p}_{point}(t)\tilde{G}_q^p(x,\xi,t) + G^{3,n}_{point}(t)\tilde{G}_q^n(x,\xi,t) \quad \Rightarrow \quad \tilde{G}_q^n(x,\xi,t) = \frac{\tilde{G}_q^3(x,\xi,t) - G^{3,p}_{point}(t)\tilde{G}_q^p(x,\xi,t)}{G^{3,n}_{point}(t)}$$

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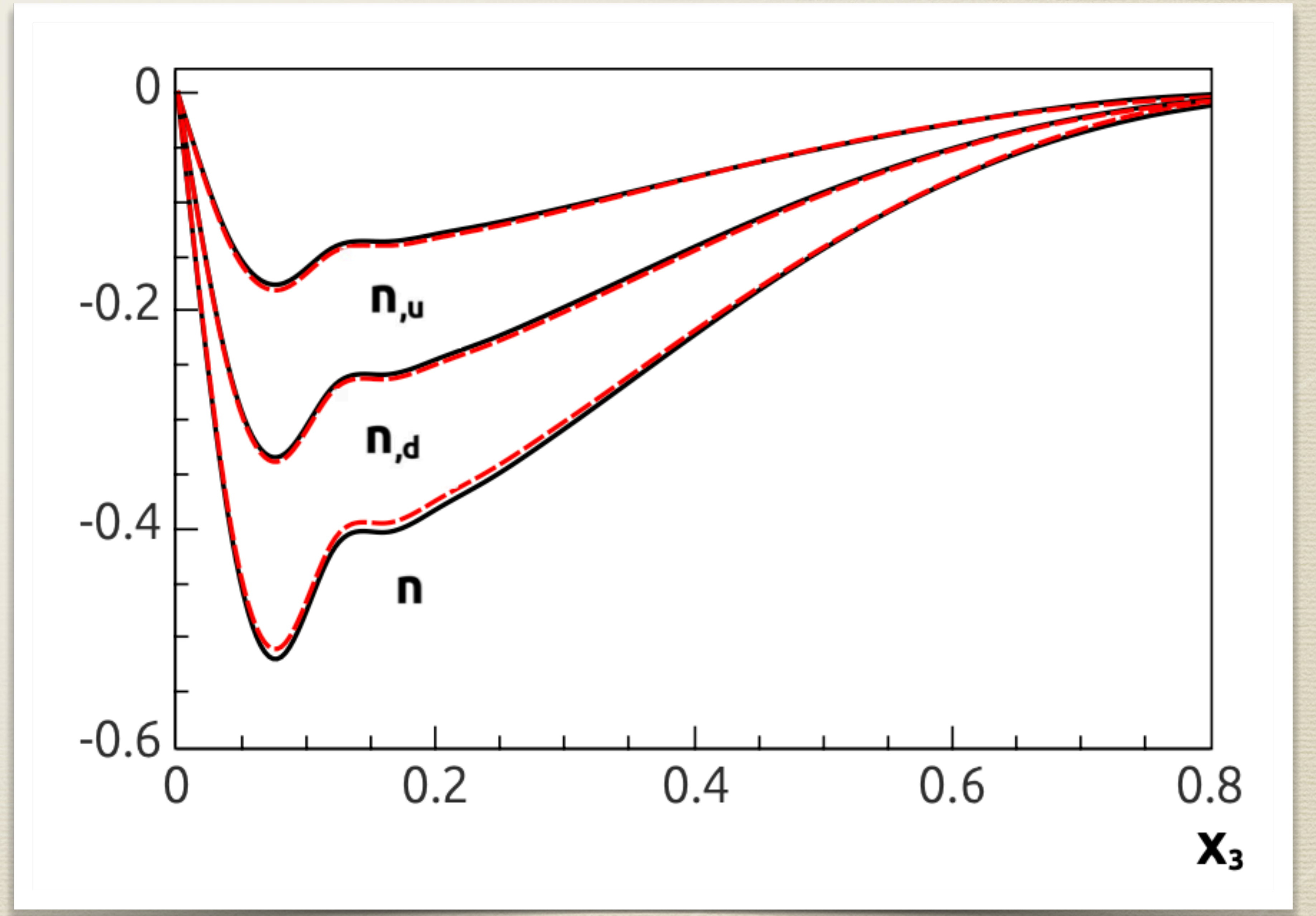
3) we compare the neutron GPD extracted (fill line) with that of the model (dashed)



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Therefore, if get:

- data for $\tilde{G}_q^3(x,\xi,t)$

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Therefore, if get:

- data for $\tilde{G}_q^3(x,\xi,t)$
- data for $\tilde{G}_q^p(x,\xi,t)$
- realistic calculations for $G_{point}^{3,n}(t)$ (**we have them**)

Neutron GPDs from ^{3}He

M. R, S.Scopetta PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)

We can extract the neutron GPDs from ^{3}He data:

$\tilde{G}^3_q(x,\xi,t) \sim G^{3,p}_{point}(t)\tilde{G}^p_q(x,\xi,t) + G^{3,n}_{point}$ … $\xi, t) = \dfrac{\tilde{G}^3_q(x,\xi,t) - G^{3,p}_{point}(t)\tilde{G}^p_q(x,\xi,t)}{G^{3,n}_{point}(t)}$

Therefore, if get:

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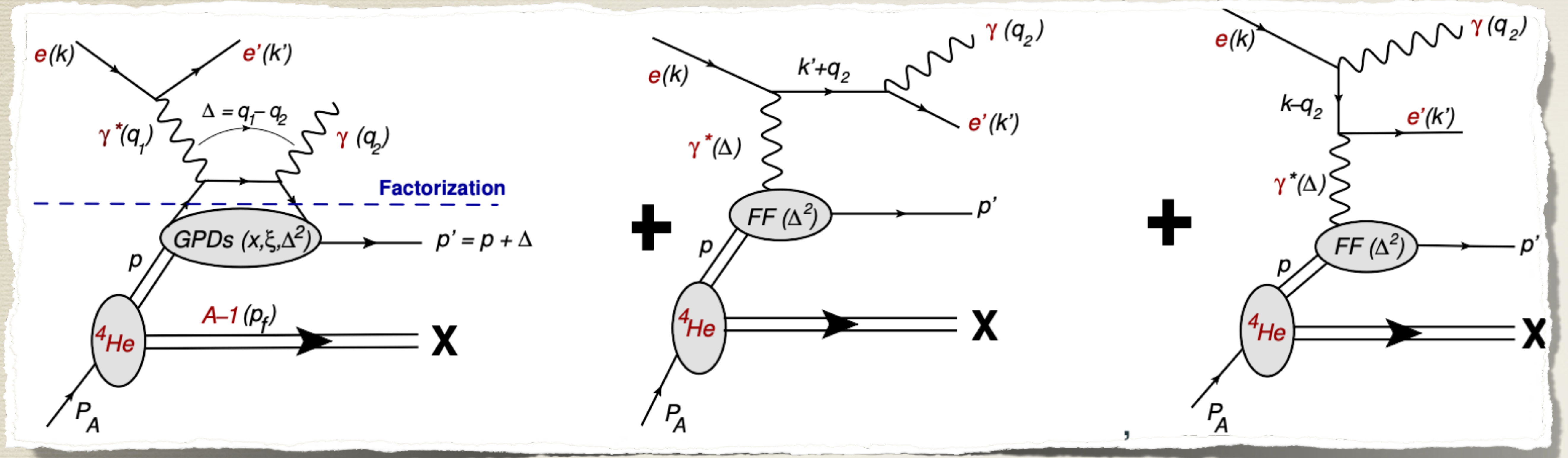
- realistic calculations for G…

Incoherent DVCS off ^{4}He

S. Fucini, S.Scopetta and M. Viviani, PRC 98 (2018) 015203)

In this case we detect a nucleon:



The nucleon is off-shell:

$$p_0 = M_A - \sqrt{M_{A-1}^{*2} + \vec{p}^2} \sim M_N - E - T_{ref} \Rightarrow p^2 \neq m^2$$

In a frame where the target nucleus is at rest, the cross section and its azimuthal dependence are expressed in terms of a convolution formula.

$$BSA_4 = \frac{\int_{exp} dEd\vec{p}\, P^4(\vec{p}, E) g(\vec{p}, E, K) \mathcal{I}(\vec{p}, E, K)}{\int_{exp} dEd\vec{p}\, P^4(\vec{p}, E) g(\vec{p}, E, K) T_{BH}^2(\vec{p}, E, K)}$$

diagonal nuclear spectral function

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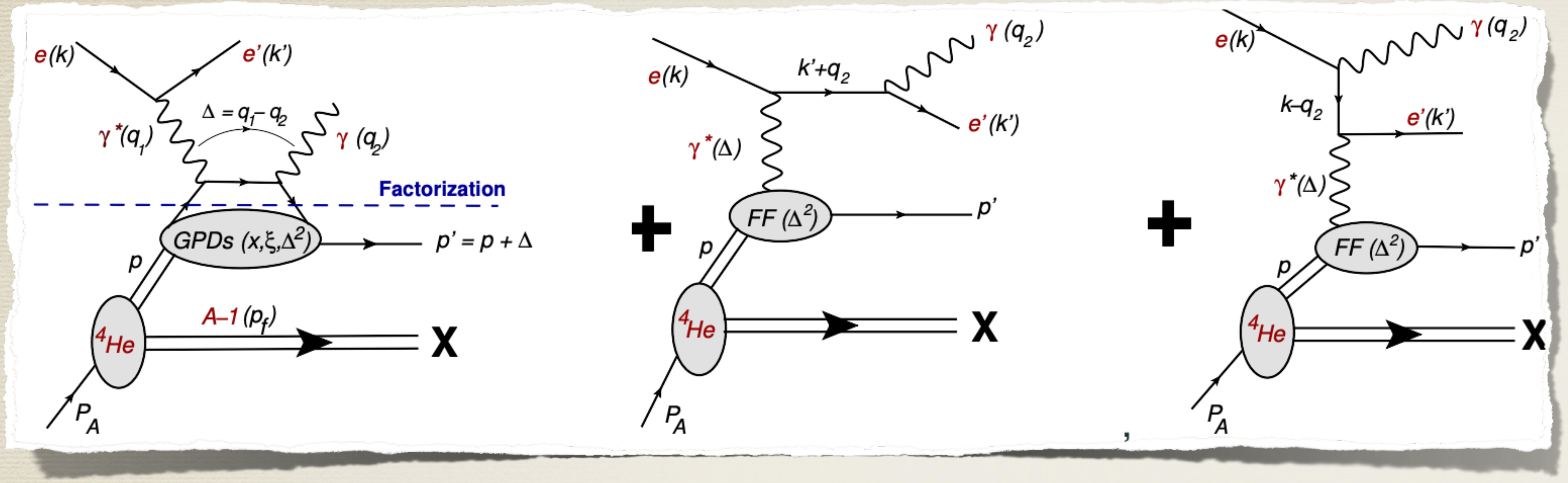

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diagonal nuclear spectral function

terms from LIPS

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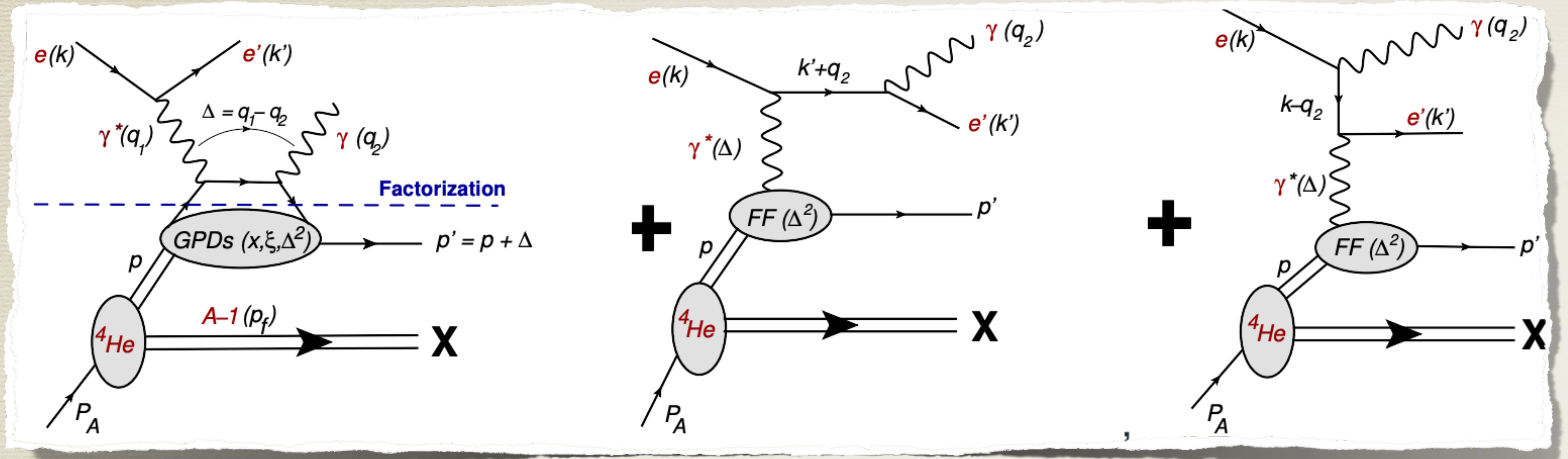

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diagonal nuclear spectral function — terms from LIPS — Nucleon FFs

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S. Fucini, S.Scopetta and M. Viviani, PRC 98 (2018) 015203)

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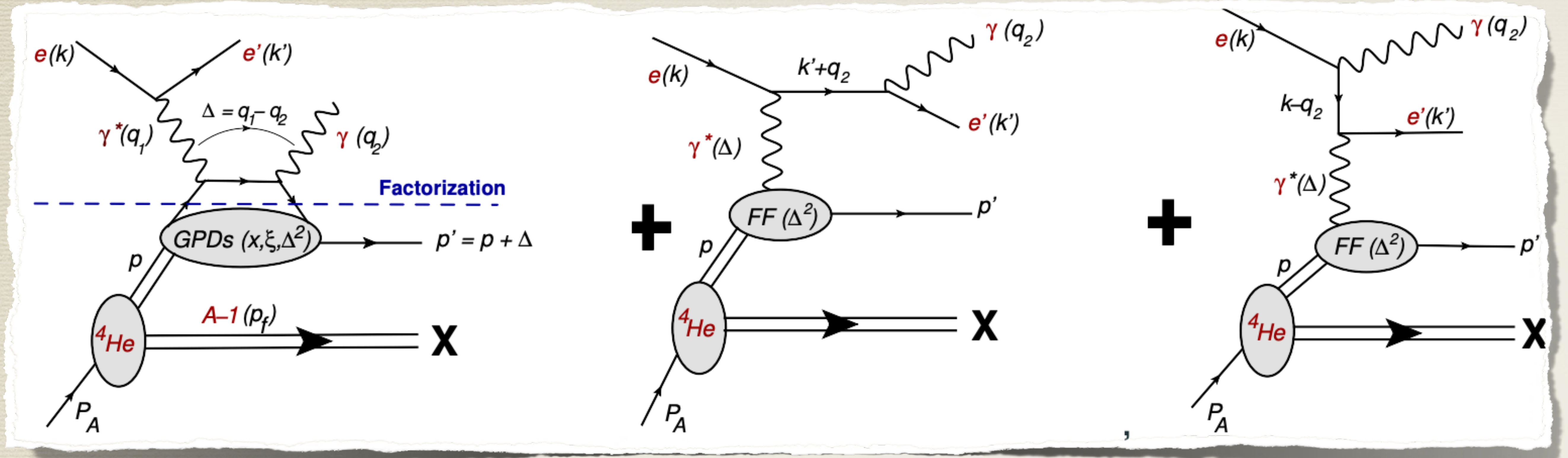

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Encode the $\Im m \mathcal{H}^4$ of the bound nucleon

- $K = \{x_B = \frac{Q^2}{M\nu}, Q^2, \phi, \Delta^2\}$ fixes the proper range of integration
- $g(\vec{p}, E, K)$ arises from the integration of LIPS and includes also the flux factor

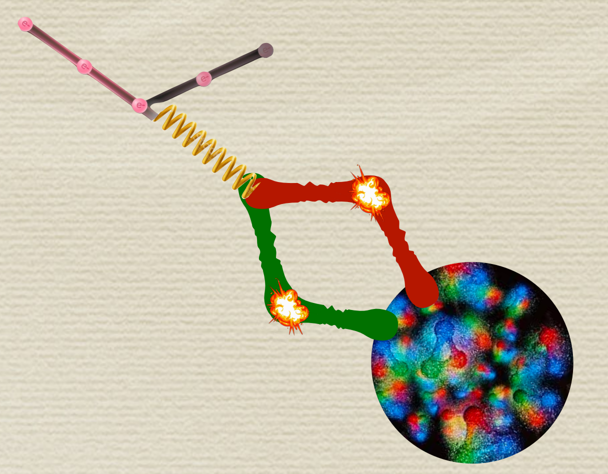
diagonal nuclear spectral function

terms from LIPS

Nucleon FFs

DPS in γA collisions with light nuclei?

For example in DPS1: $$\tilde{F}^{1}_{a_1 a_2}(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^{N}_{a_1 a_2}\left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp\right) \rho^{N}_{A}(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$



Let us check sum rules:

$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 F_{i_1 i_2}(x_1, x_2, k_\perp = 0) = \begin{cases} N_{i_1} N_{i_2} & \text{for } i_1 \neq i_2 \\ \left(N_{i_1} - 1\right) N_{i_2} & \text{for } i_1 = i_2 \end{cases}$$

Gaunt's sum rules

J. R. Gaunt and W. J. Stirling, JHEP 03, 005 (2010)

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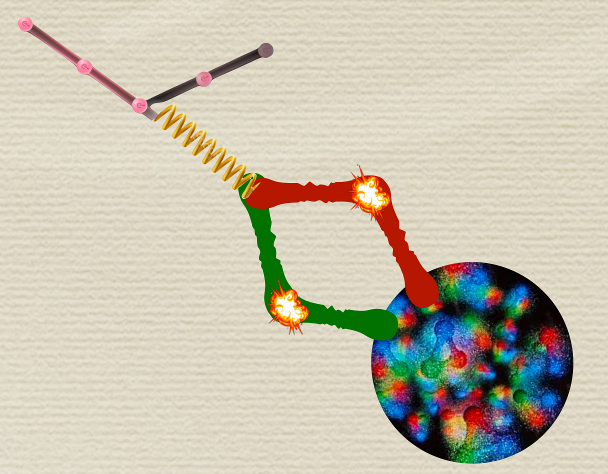

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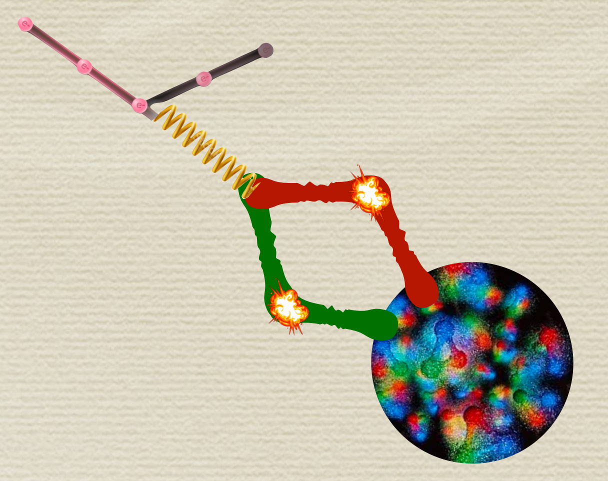
However for the nuclear case one needs also the DPS2

Thus we can introduce approximated partial sum rules (APSR)

DPS in γA collisions with light nuclei?



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$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 F_{i_1i_2}(x_1, x_2, k_\perp = 0) = \begin{cases} N_{i_1}N_{i_2} & \text{for } i_1 \neq i_2 \\ \left(N_{i_1} - 1\right) N_{i_2} & \text{for } i_1 = i_2 \end{cases}$$

Gaunt's sum rules

J. R. Gaunt and W. J. Stirling, JHEP 03, 005 (2010)

APSR: Since $f^A_n(\xi) = \int d^2p_{t,N}\, \rho^N_A(\xi, p_{t,N})$ is peaked around 1/A

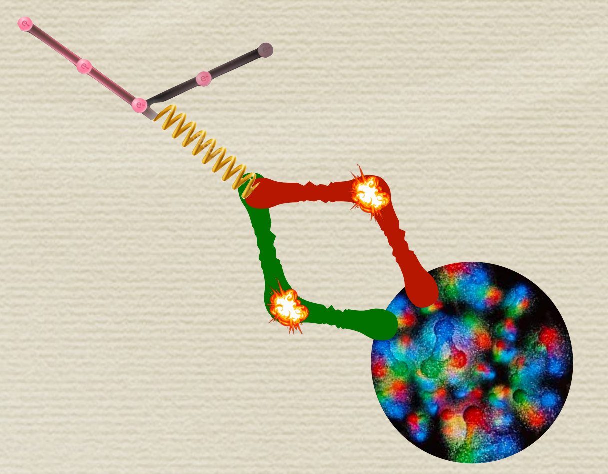
$$\int_0^A dx_1 \int_0^{A-x_1} dx_2 \tilde{F}^{A,1}_{i_1i_2}(x_1, x_2, 0) \sim \sum_{n=N,P} \int d\xi f^A_n(\xi) \begin{cases} \left(N^n_{i_1} - 1\right) N^n_{i_2} & i_1 = i_2 \\ N^n_{i_1}N^n_{i_2} & i_1 \neq i_2 \end{cases}$$

Normalized to 1

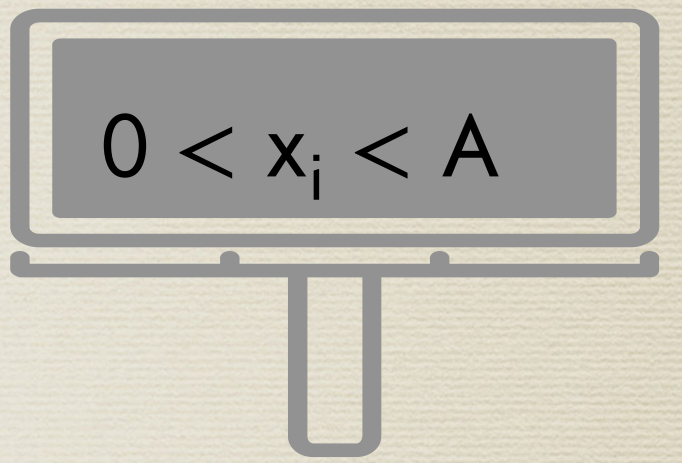
Gaunt's sum rules for the nucleon DPD: numbers of quarks with given flavor i in the nucleon n

DPS in γA collisions with light nuclei?

For example in DPS1: $$\tilde{F}^1_{a_1a_2}(x_1,x_2,k_\perp) = \sum_{N=p,n}\int \frac{1}{\xi}\tilde{F}^N_{a_1a_2}\left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right)\rho^N_A(\xi,p_{t,N})\frac{d\xi}{\xi}d^2p_{t,N}$$



- Only for valence quarks. We used a relativistic constituent quark model for the nucleon DPD
- verified approximated partial sum rules (numerically)



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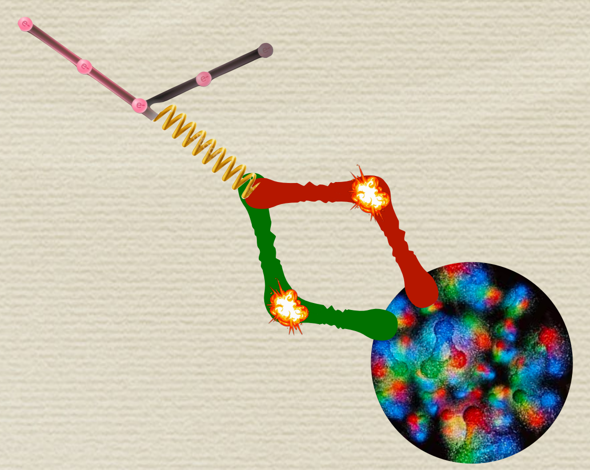

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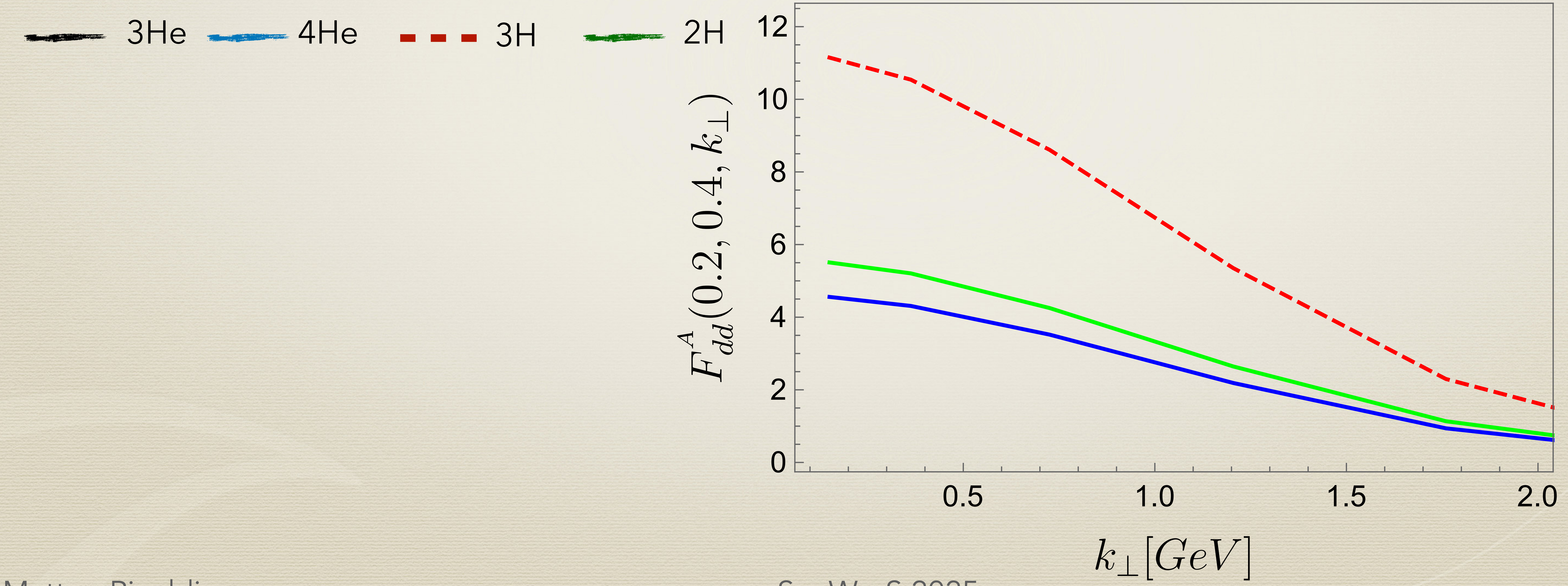
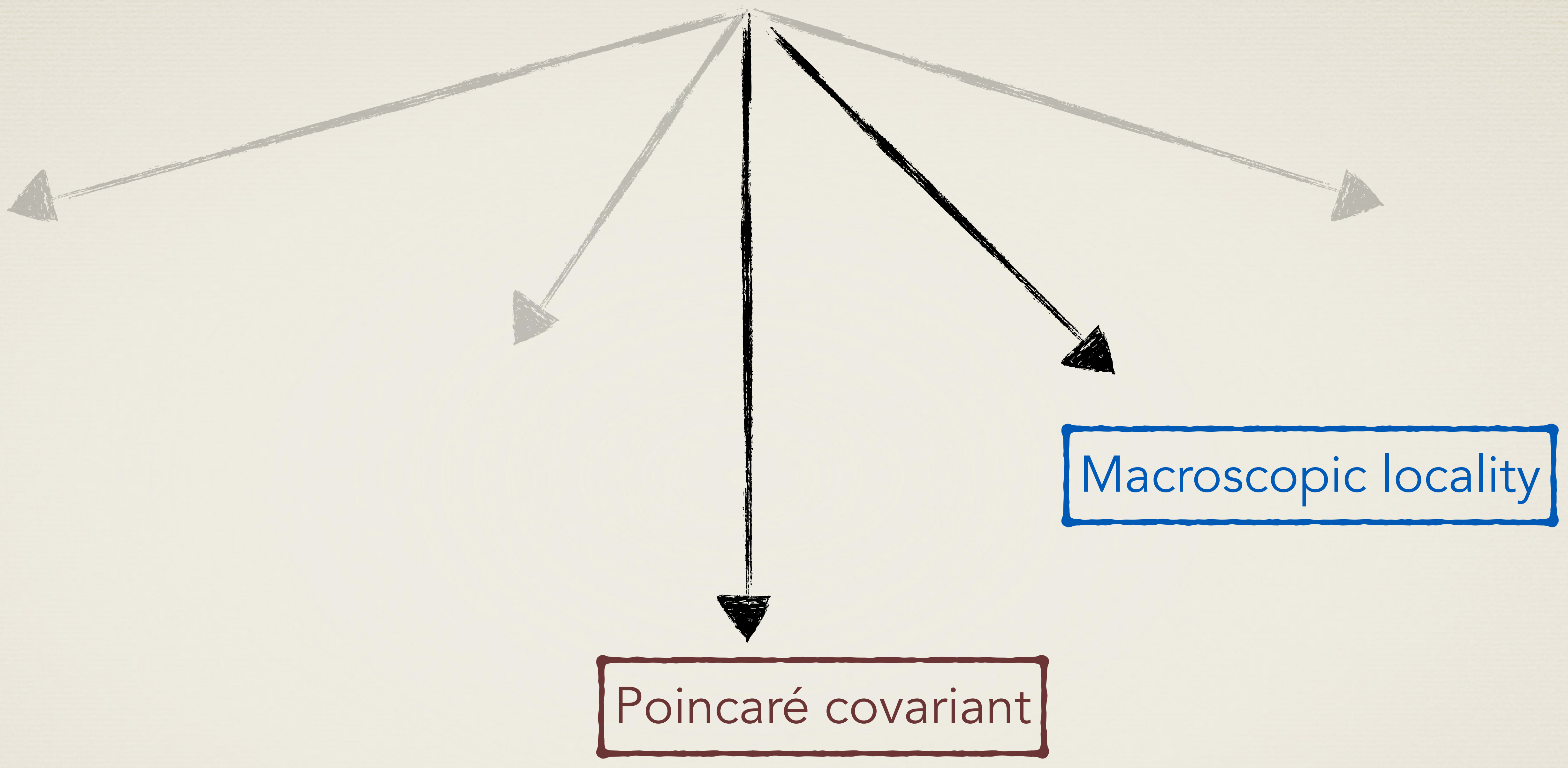


The EMC effect

Our approach (Light-Front + Bakamjian-Thomas)



B.D.Keister and W.N.Polyzou, Adv.Nucl.Phys. 20 (1991), 225-479
P. A. M. Dirac, Rev. Mod. Phys. 21 (1949) 392–399

B. Bakamjian, L. H. Thomas, Phys. Rev. 92 (1953) 1300–1310

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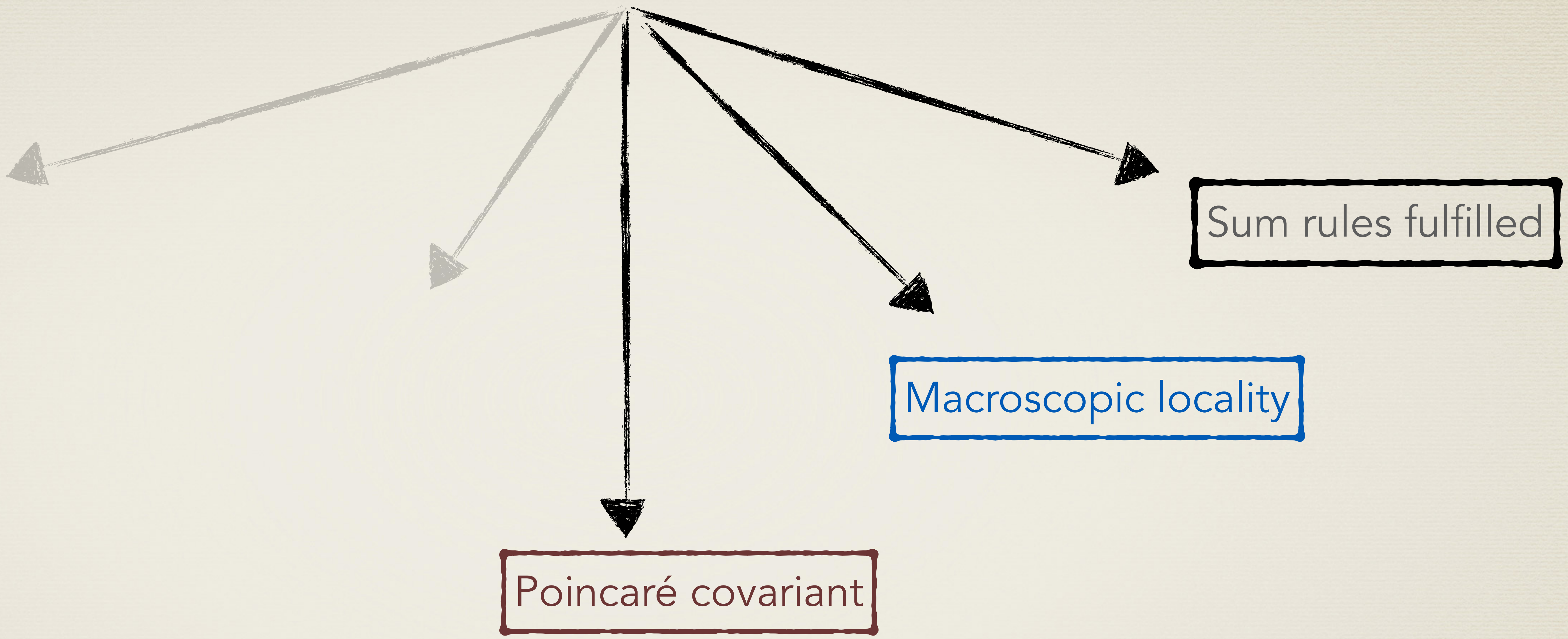

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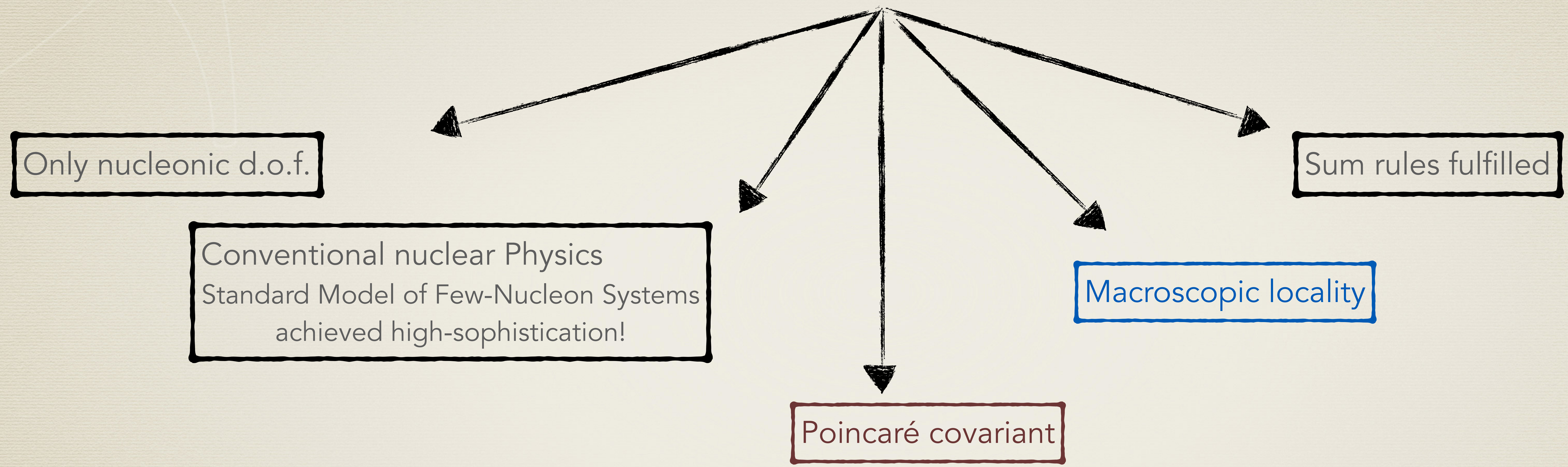

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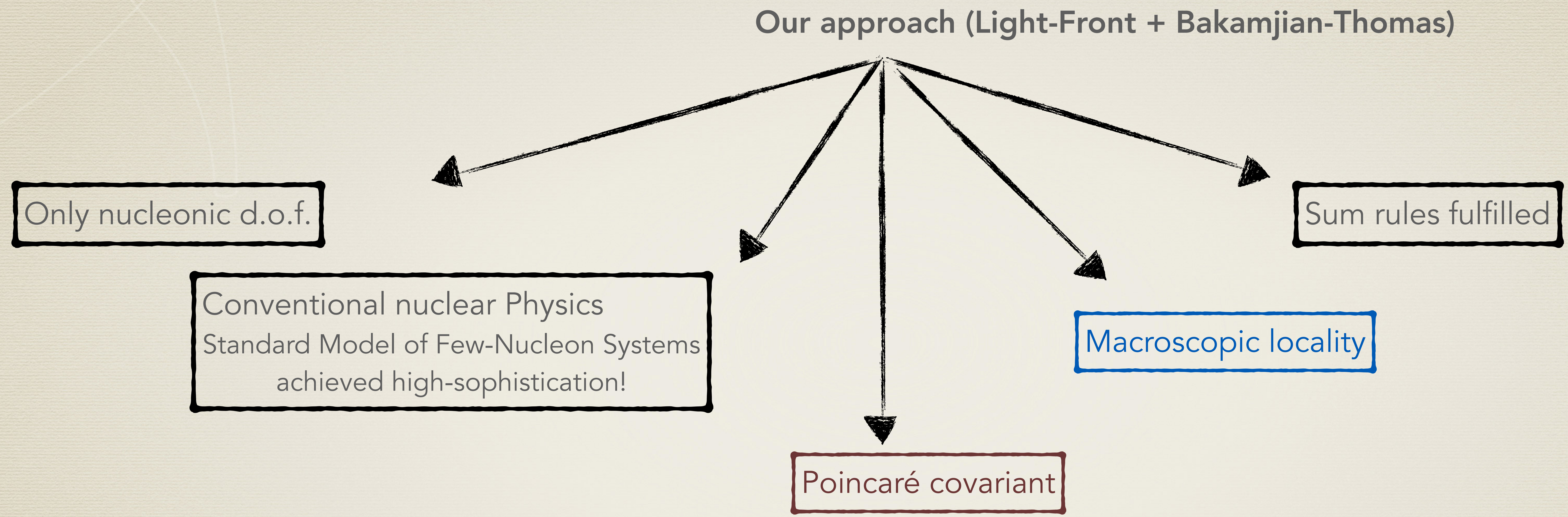


- We provide a **reliable baseline for the calculation of the nuclear SFs** where only the well known nuclear part is considered
- This relativistic treatment is needed for the kinematics of the **JLab12, JLab22 and EIC**

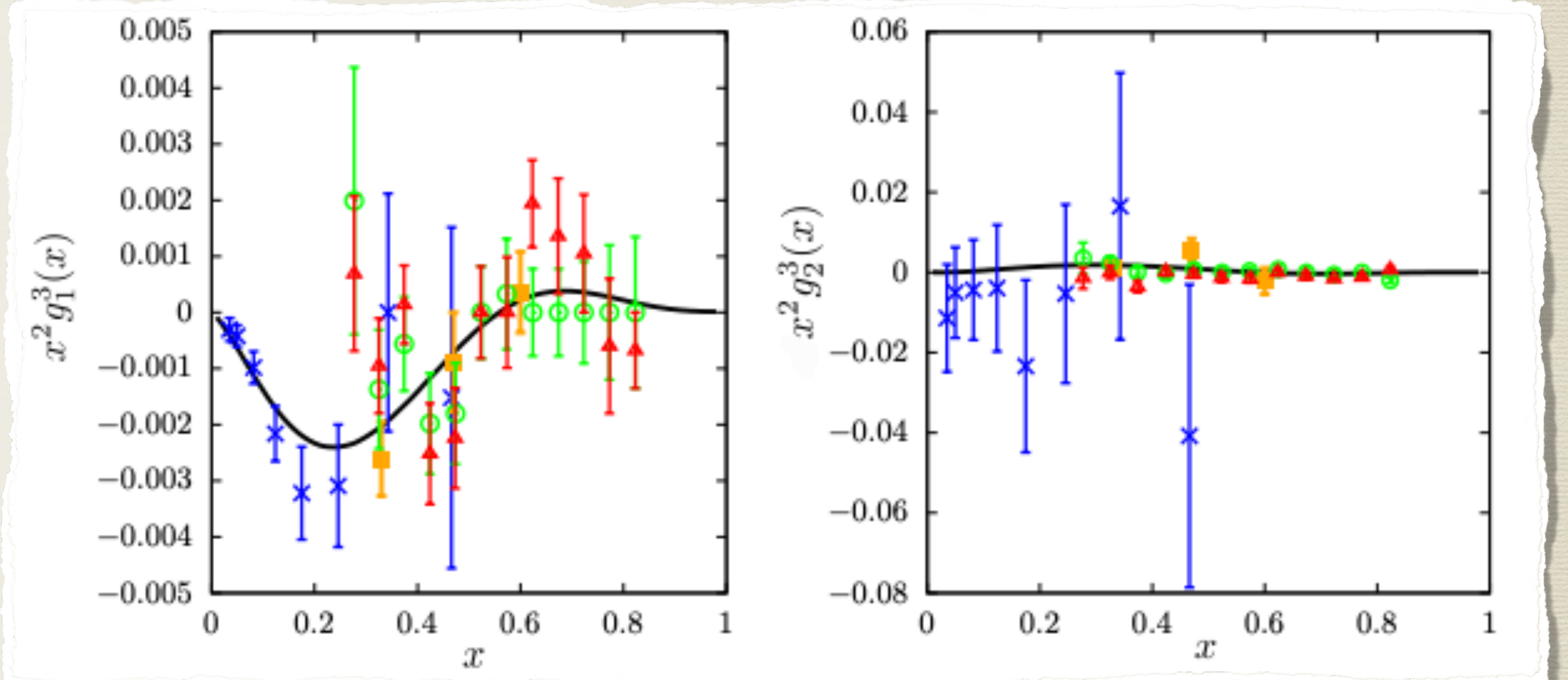
Results

F. Fornetti, E. Proietti..., M. R. et al, PRC 110 (2024), 3, L031303

For the process: $\vec{l}(\mathscr{E}) + \vec{A} \to l'(\mathscr{E}') + X.$

$$g_j^A(x) = \sum_{N=n,p} \int_{\xi_{min}}^{1} d\xi \left\{ g_1^N\left(\frac{x}{\xi}\frac{m}{M_A}\right) l_j^N(\xi) + g_2^N\left(\frac{x}{\xi}\frac{m}{M_A}\right) h_j^N(\xi) \right\},$$

Also in this case there are no free parameters and the ^3He w.f. corresponding to the Av18 potential has been used

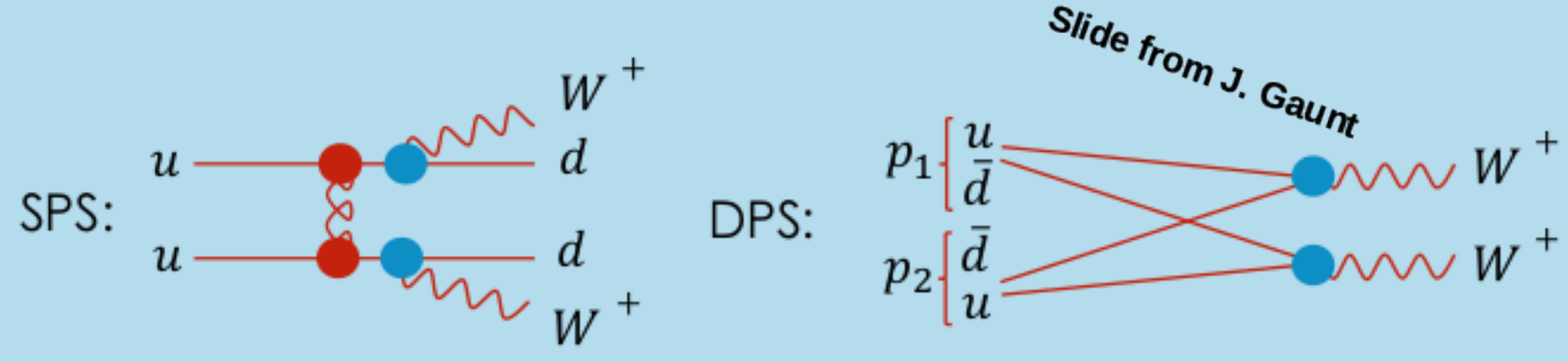


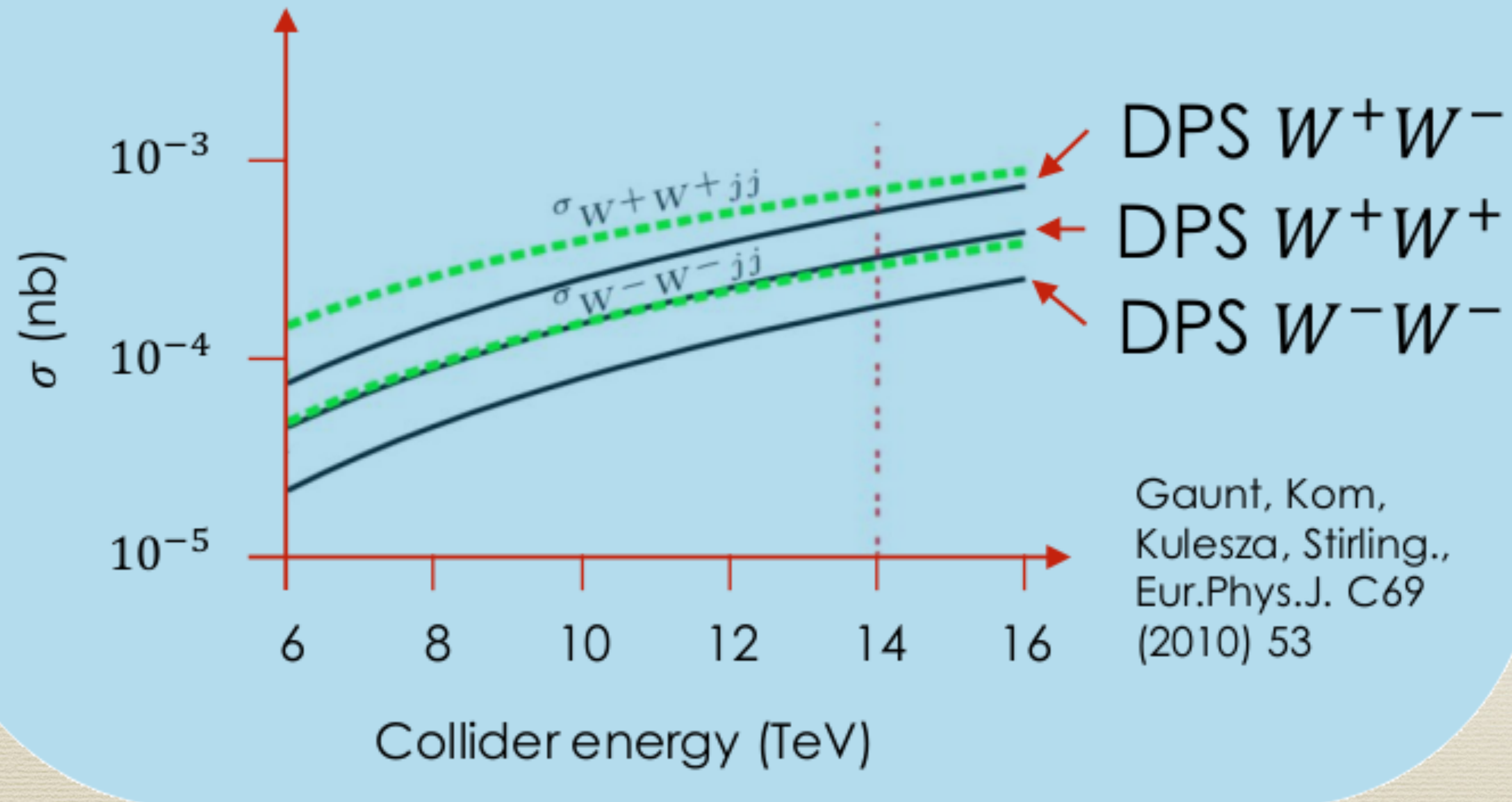
- Full lines: our calculations
- Experimental analyses:
  a) crosses: P. L. Anthony et al. (E142), Phys. Rev. D 54, 6620 (1996)
  b) squares: X. Zheng et al. (JLab Hall A), PRL 92, 012004 (2004)
  c) empty: D. Flay et al. (Jefferson Lab Hall A), Phys. Rev. D 94, 052003 (2016)

Where and Why DPS?

DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:

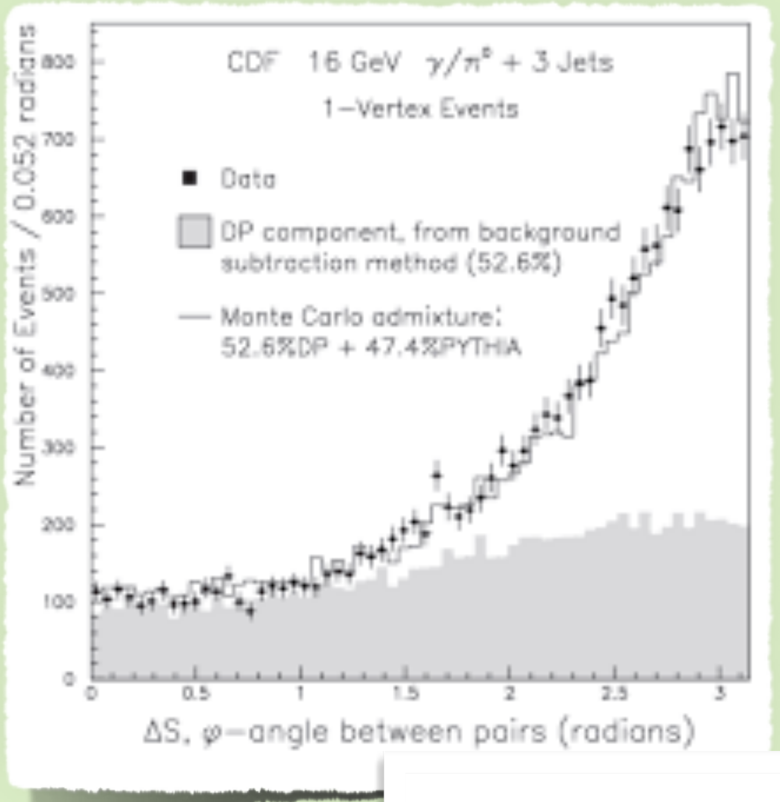
Slide from J. Gaunt





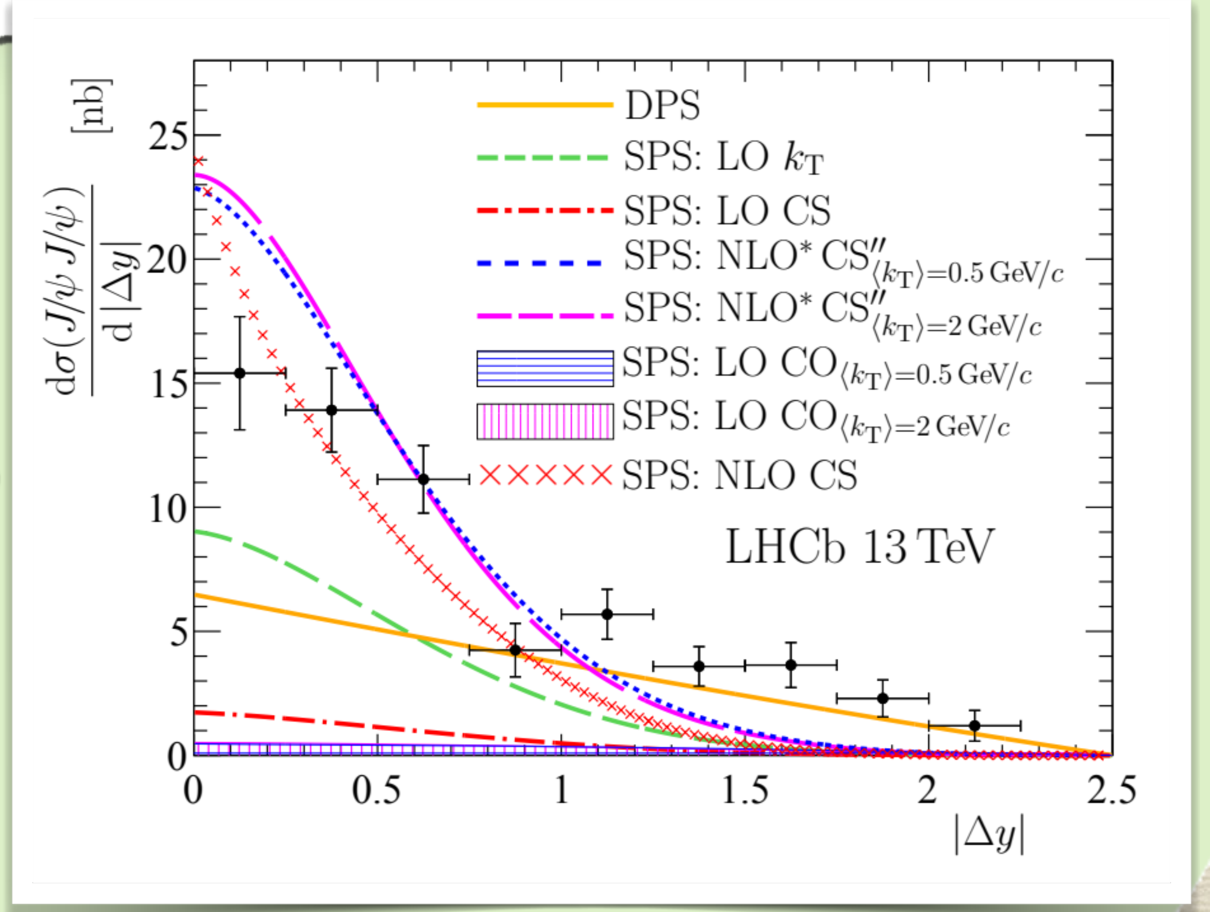
Gaunt, Kom, Kulesza, Stirling., Eur.Phys.J. C69 (2010) 53

...or in certain phase space regions

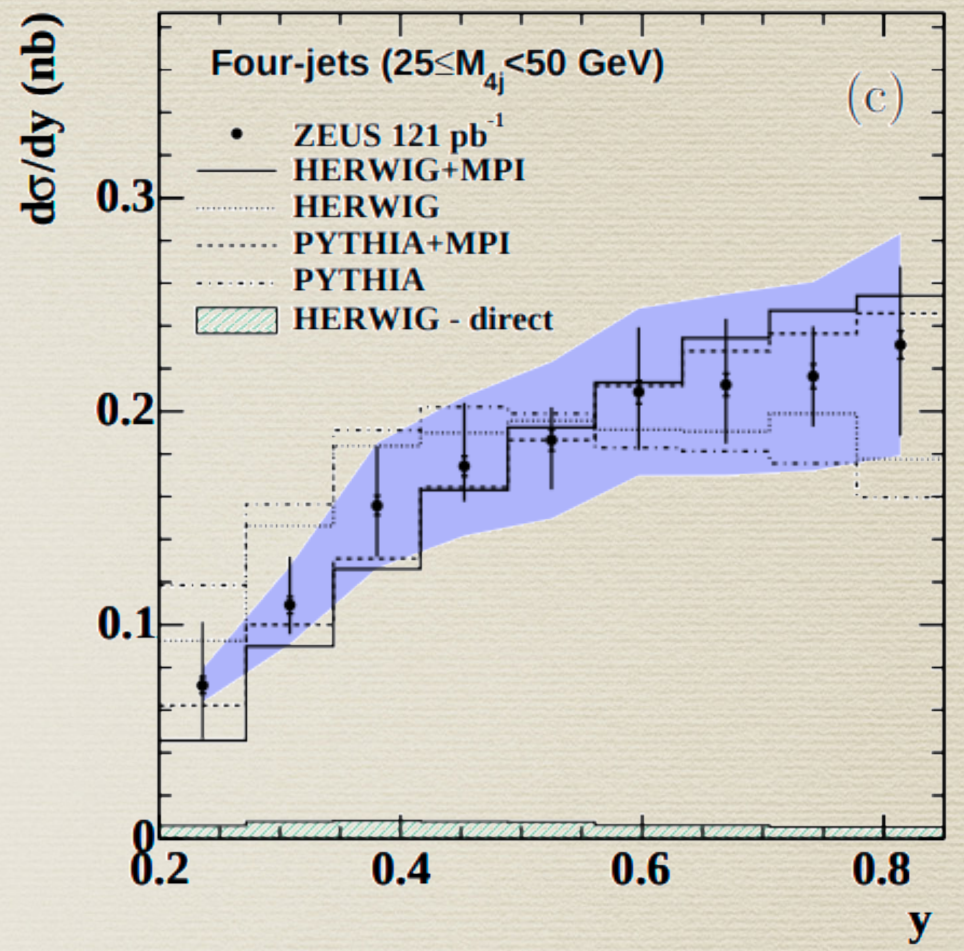


CDF, $\gamma + 3j$, Phys.Rev. D56 (1997) 3811-3832

LHCb, double J/ψ, JHEP 06, 047, (2017)



in ep Colliders?



HERA data, ZEUS coll, Nucl. Phys. B 729, 1 (2008)

Access to:
- double parton correlations
- the transverse distance distribution of partons!!

all **UNKNOWN**

Nuclear SFs and EMC ratio

To calculate the EMC ratio $R^A_{EMC}(x) = \dfrac{F_2^A(x)}{F_2^d(x)}$ for any nucleus A, we need the nuclear SFs.

Within our approach we have:

$$F_2^A(x) = \sum_N \int_{\xi_{min}}^1 d\xi \; F_2^N\left(\frac{mx}{\xi M_A}\right) \; f_A^N(\xi)$$

*ξ = longitudinal momentum fraction carried by a nucleon in the nucleus

1) in the Bjorken limit we have the LCMD: $f_1^N(\xi) = \sum\!\!\!\!\!\!\int d\epsilon \int \dfrac{d\kappa_\perp}{(2\pi)^3} \dfrac{1}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) \dfrac{E_s}{1-\xi}$

Unpolarized LF spectral function:

$$P^N(\tilde{\kappa}, \epsilon) = \frac{1}{2j+1} \sum_{\mathcal{M}} P^N_{\sigma\sigma}(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M})$$

2) Procedure:

a) we choose a **parametrization** for $F_2^p(x)$

b) we use the **MARATHON data (**MARATHON Coll., **Phys. Rev. Lett 128 (2022) 13,132003)** for the parametrization of the ratio $\dfrac{F_2^n}{F_2^p}$ to get F_2^n

^{4}He CFFs

S. Fucini, S.Scopetta and M. Viviani, PRC 98 (2018) 015203)

For ^{4}He (J=0) we have only GPD:

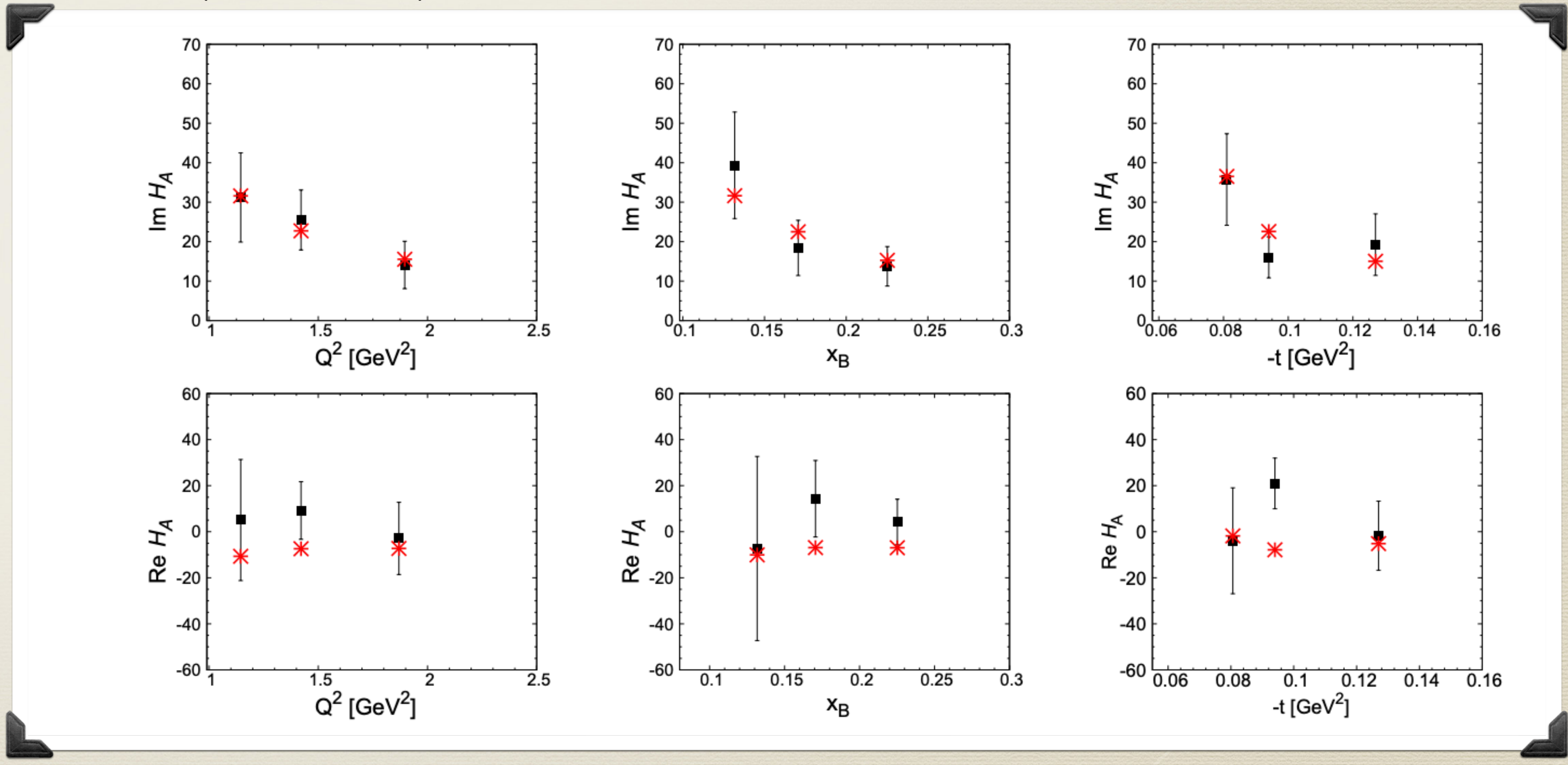
*details on spectral functions in Backup slides

$$H_q^4\left(x,\xi,\Delta^2\right) = \sum_N \int_{|x|}^1 \frac{dz}{z} h_N^4\left(z,\xi,\Delta^2\right) \quad \mathbf{H_q^N}\left(\frac{x}{\zeta},\frac{\xi}{\zeta},\Delta^2\right)$$

with: $\mathbf{H_q^N}(x,\xi,t) = \sqrt{1-\xi^2}\left[H_q^N(x,\xi,t) - \frac{\xi^2}{1-\xi^2}E_q^N(x,\xi,t)\right]$

We define:

- ✓ $H^4(x,\xi,t) = \sum_q e_q^2 H_q^4(x,\xi,t)$
- ✓ $\Im m \mathcal{H}_A(\xi,t) = H^4(\xi,\xi,t) - H^4(-\xi,\xi,t)$
- ✓ $\Re e \mathcal{H}_A(\xi,t) = \mathcal{P}\int_{-1}^1 dx \frac{H^4(x,\xi,t)}{x-\xi+i\varepsilon}$
- ✓ $\alpha_i(\phi)$ **A. V. Belitsky et al., PRD (2009)**



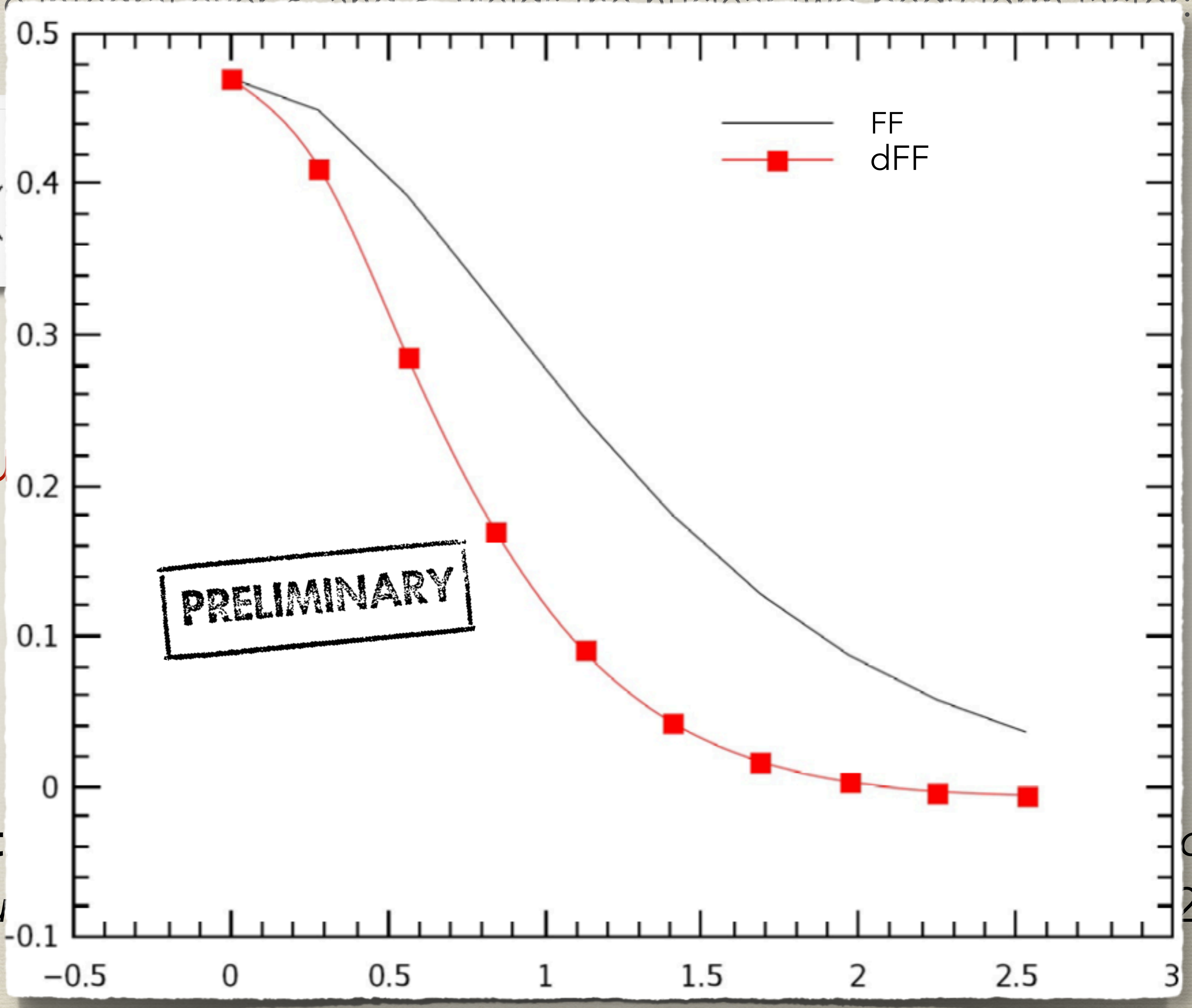
■ JLab data: Hattawy et al, PRL 119 (2017), 202004

DPS in γA collisions with light nuclei

Before closing let us mention that th…

$F^{double}_{A,\tau_1,\tau_2}($…

Nu…



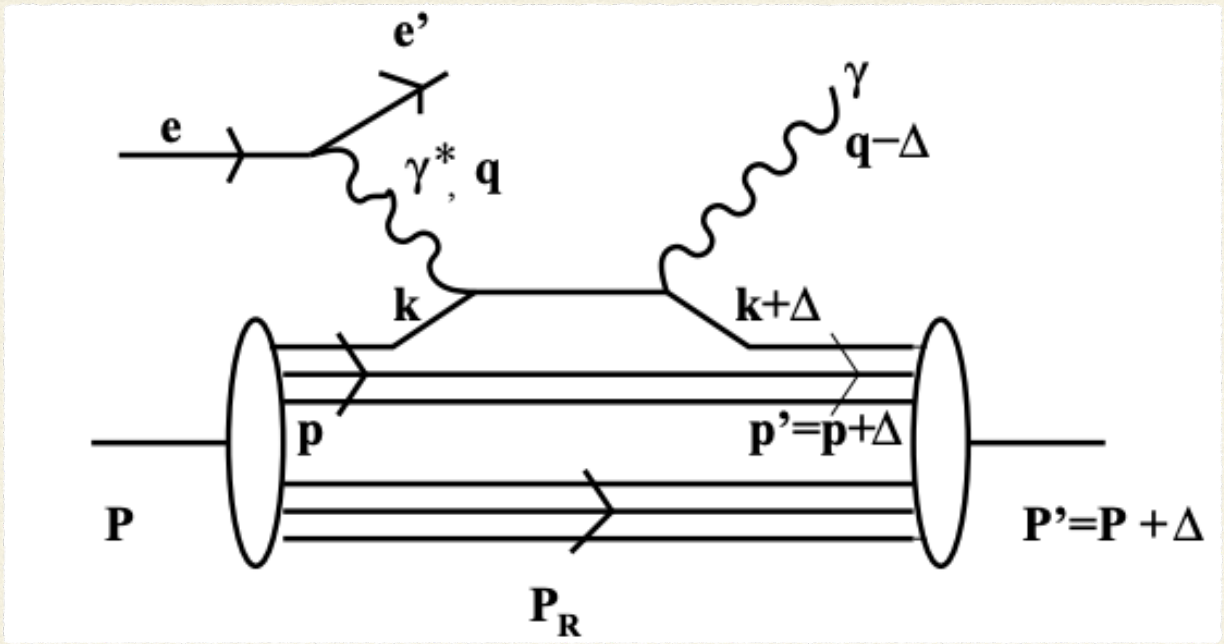
Calculated for ^{3}He and ^{4}He in:

V. Guzey, M.R., S. Scopetta, M. St… …duction on
He4 and He3 at the EIC: probing Nu… …2) 24, 242503

An Impulse Approximation for the coherent case

- We consider only nucleonic d.o.f.

- Nucleons are kinematically off-shell:



$$p_0 = M_A - \sqrt{M^{*2}_{A-1} + \vec{p}^2} \simeq M - E - T_{rec} \longrightarrow \mathbf{p^2 \neq M^2}$$

Here the **Removal Energy** $E = |E_A| - |E_{A-1}| - E^*$

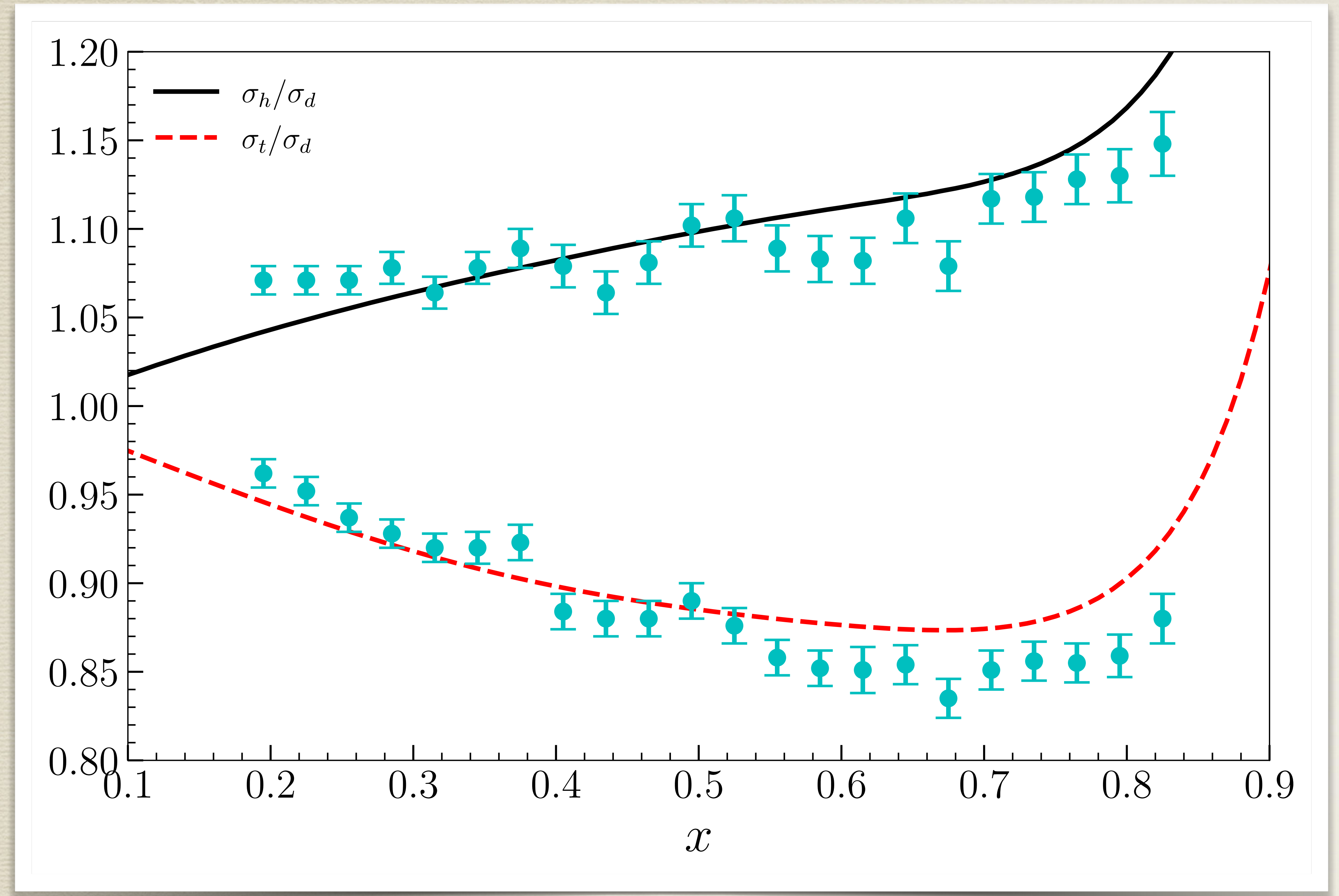
- The photon interacts with a nucleon but then we measure the nucleus!

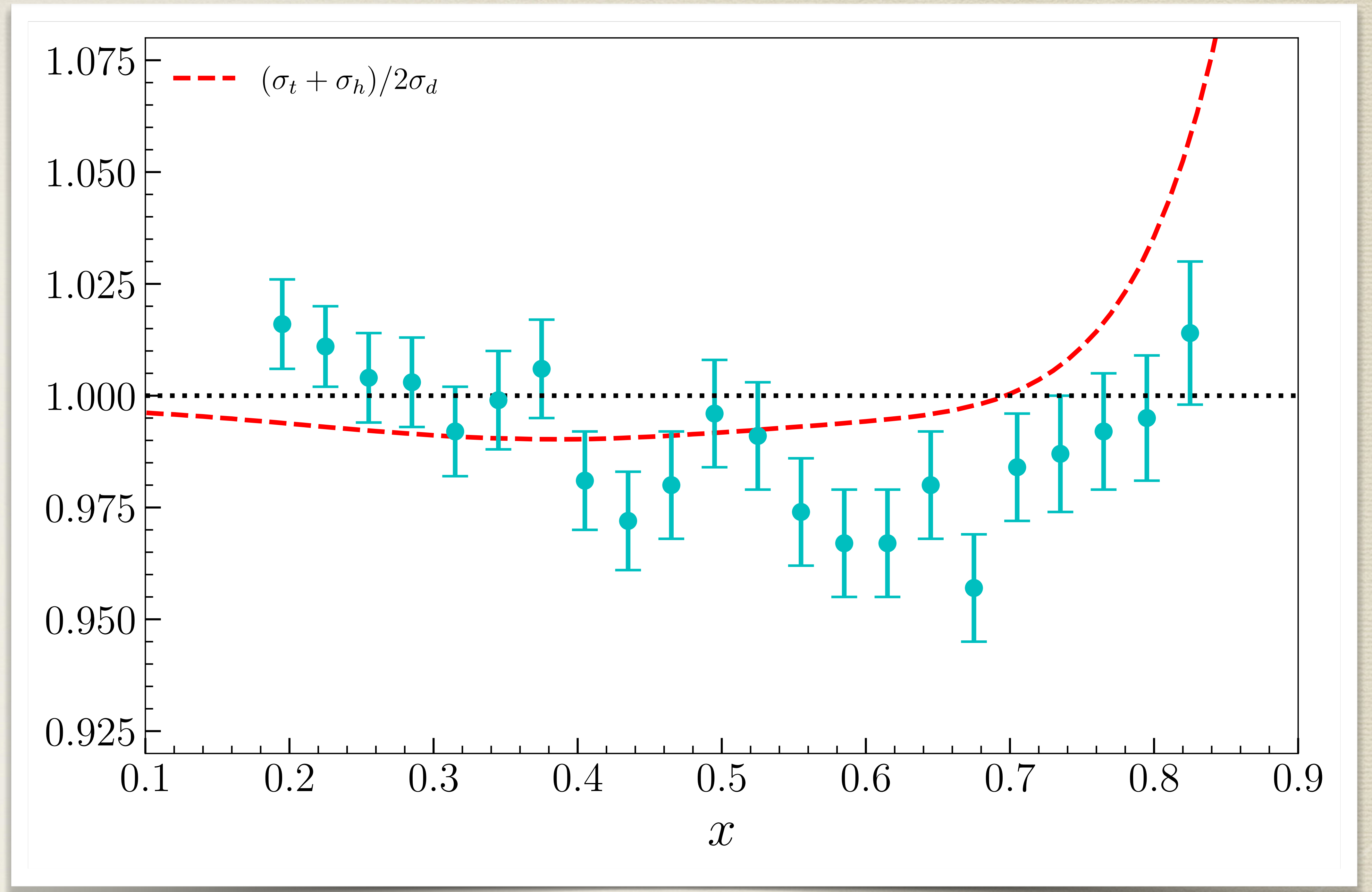
$$GPD^A \sim \sum_{n=P,N} h^{A,n}_{GPDs} \otimes GPD^n$$

Free nucleon GPD

Nuclear Light-Cone momentum distribution

Recent (ongoing) calculations





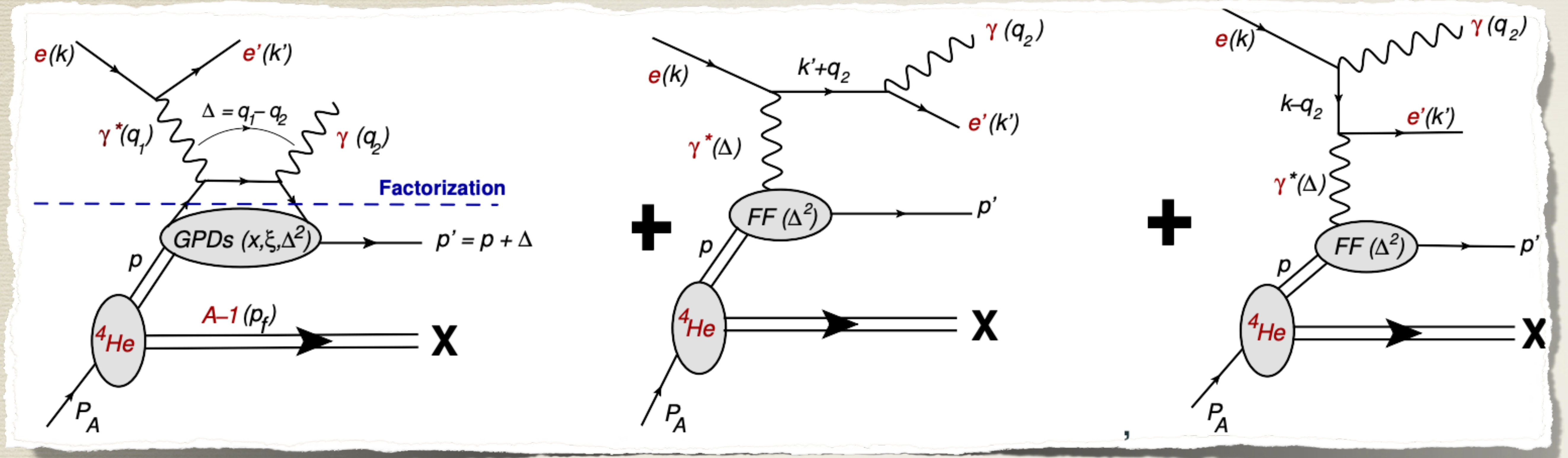
Data from:

D. Abrams, H. Albataineh, B.~S. Aljawrneh,…,et al,
``The EMC Effect of Tritium and Helium-3 from the JLab MARATHON Experiment,''
[arXiv:2410.12099 [nucl-ex]].

Incoherent DVCS off ^{4}He

S. Fucini, S.Scopetta and M. Viviani, PRC 98 (2018) 015203)

In this case we detect a nucleon:



The nucleon is off-shell:

$$p_0 = M_A - \sqrt{M_{A-1}^{*2} + \vec{p}^{\,2}} \sim M_N - E - T_{ref} \Rightarrow p^2 \neq m^2$$

In a frame where the target nucleus is at rest, the cross section and its azimuthal dependence are expressed in terms of a convolution formula.

- the **diagonal** spectral function $P^{^4He}$ of the inner nucleons

$$d\sigma^{\pm}_{Incoh} = \int_{exp} dE d\vec{p} \frac{p \cdot k}{p_0 |\vec{k}|} P^{^4He}(\vec{p}, E) \, d\sigma^{\pm}_{b}(\vec{p}, E, K)$$

- the DVCS cross section off a bound proton