NLO Corrections and Factorization for Transverse Single-Spin Asymmetries

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Contents



Introduction



$$A_{UT} \equiv \frac{\sigma(\vec{s}_T) - \sigma(-\vec{s}_T)}{\sigma(\vec{s}_T) + \sigma(-\vec{s}_T)} \sim 40\%$$

in $pp^{\uparrow} \rightarrow (h, \gamma, jet, l, J/\psi, ...)X$

Klem, R. D., et al., 1976, <u>Phys. Rev. Lett. 36, 929</u> Dragoset, W. H., et al., 1978, <u>Phys. Rev. D 18, 3939</u> Antille, J., et al., 1980, <u>Phys. Lett. 94B, 523</u> Apokin, V. D., et al., 1990, <u>Phys. Lett. B 243, 461</u> Saroff, S., et al., 1990, <u>Phys. Rev. Lett. 64, 995</u>

Simpler Similar Processes?

 $lp^{\uparrow} \rightarrow lX$ (DIS)

N. Christ, T. D. Lee, Phys. Rev. 143, 1310 (1966)

Vanishes for 1-photon exchange Suppression with α_{em} for 2-photon exchange

 $lp^{\uparrow} \rightarrow h(or \ jet)X$

This talk! arXiv: 2503.16119 arXiv: 2503.16097

 $lp^{\uparrow} \rightarrow l' \gamma X$

"_YSIDIS" W.S. Albaltan, A. Prokudin, M. Schlegel, Phys. Lett. B 804, 135367 (2020) M. Harris, J. Marsh, D. Pitonyak, A. Prokudin, J. Putnam, D. Rein, M. Schlegel arXiv 2505.02711 or single-inclusive:

 $lp^{\uparrow} \rightarrow \gamma X$

Goals

- Show that collinear twist-3 factorization holds at the one-loop level
 - For a **"truly single-inclusive"** process (no separation into leptonic/hadronic part; no use of structure functions)
- Learn about quark-gluon (gluon-gluon) correlations inside the target nucleon
 - Encoded in the correlation functions F_{FT}^q , G_{FT}^q
- Build towards $pp^{\uparrow} \rightarrow (h, \gamma, jet, l, J/\psi, ...)X$

Collinear Twist-3 Factorization Approach

Intrinsic

higher twist terms of familiar quark-quark correlator

→Vanishes for this process

Kinematical

Effects due to transverse motion of the partons, non-zero k_T

 \rightarrow first k_T -moment of TMDs

 $f_{1T}^{\perp(1),q}(x) = +\pi F_{FT}^{q}(x,x)$

Dynamical

Quark-gluon-quark correlation functions $F_{FT}^{q}(x, x'), G_{FT}^{q}(x, x')$

(... and tri-gluon correlation functions)

The notation in the following slides follows the conventions from K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak, M. Schlegel, Phys. Rev. D 93, 054024 (2016)

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Dynamical Twist-3 Distribution Functions



Connection with the first k_T moment of the Sivers function

 $\pi F_{FT}^{q}(x,x) = f_{1T}^{\perp(1),q}(x)$

D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. B 667 (2003)



Overview of Partonic Channels



Overview of Partonic Channels



Overview of Partonic Channels



Key Features of the Calculation

$$LO \text{ for } e(l)p^{\uparrow}(P) \to h(P_h)X:$$

$$E_h \frac{d\sigma_{LO}}{d^3 P_h}(S) = \sigma_0(S) \int_{v_0}^{v_1} dv \int_{x_0}^{1} \frac{dw}{w} \hat{\sigma}_{LO}(v,w) \sum_q e_q^2 \left[\left(1 - x \frac{d}{dx} \right) F_{FT}^q(x,x) D_1^q(z) \right] \Big|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}} + h_1^q \otimes \Im [\widehat{H}_{FU}^q]$$

General schematic form of the dynamical part at NLO:

$$\int \frac{dz}{z^2} \int dx \int \frac{dx'}{x'-x} \Big[\hat{\sigma}^1(x,x',z) F_{FT}^q(x,x') + \hat{\sigma}^5(x,x',z) G_{FT}^q(x,x') \Big] D_1^q(z) + c.c.$$

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Twist-3 fragmentation, not

Key Features of the Dynamical Part



Key Features of the Dynamical Part



Key Features of the Dynamical Part

But Careful!
$$\tilde{x} = \frac{2x \, l \cdot r}{\hat{s} - 2k \cdot r} \rightarrow F_{FT}^q(x, \tilde{x}), G_{FT}^q(x, \tilde{x}) \propto r$$

Keep $i\eta$ in the propagator, perform the phase space integral first, then take the imaginary part
Extract imaginary part via the general formula
 $\log[y \pm i\eta] \xrightarrow{\eta \to 0^+} \log[|y|] \pm \Theta(-y)i\pi$
An integral over the region $0 \le x' \le x$ remains!

"Integral Contribution"

Dynamical Twist-3 Distribution Functions



Spin Asymmetries

Final Result

 $E_{h}\frac{d\sigma_{NLO}^{lN^{\uparrow} \to hX}}{d^{3}P_{h}}(S) = \underbrace{E_{h}\frac{d\sigma_{NLO}^{qg \to q}}{d^{3}P_{h}}(S)}_{q^{3}P_{h}} + E_{h}\frac{d\sigma_{NLO}^{qg \to q}}{d^{3}P_{h}}(S) + E_{h}\frac{d\sigma_{NLO}^{qq \to q}}{d^{3}P_{h}}(S) + E_{h}\frac{d\sigma_{NLO}^{qg \to q}}{d^{3}P_{h}}(S) + E_{h}\frac{d\sigma_{NLO}^{qg \to q}}{d^{3}P_{h}}(S) + E_{h}\frac{d\sigma_{NLO}^{qg \to q}}{d^{3}P_{h}}(S)$ $E_h \frac{d\sigma_{NLO}^{qg \to q}}{d^3 P_h}(S) = E_h \frac{d\sigma_{Int}^{qg \to q}}{d^3 P_h}(S) + E_h \frac{d\sigma_{SGP}^{qg \to q}}{d^3 P_h}(S) + \underbrace{E_h \frac{d\sigma_{HP}^{qg \to q}}{d^3 P_h}(S)}_{d^3 P_h}(S) + E_h \frac{d\sigma_{SFP}^{qg \to q}}{d^3 P_h}(S)$ $E_h \frac{\mathrm{d}\sigma_{\mathrm{HP}}^{qg \to q}}{\mathrm{d}^3 \mathbf{P_h}} = \sigma_0(S) \frac{\alpha_s(\mu)}{2\pi} \int_{v_0}^{v_1} \mathrm{d}v \int_{x_0}^1 \frac{\mathrm{d}w}{w} \sum e_q^2 D_1^q \left(\frac{1-v_1}{1-v}, \mu\right) \times$ $\hat{\sigma}_{\mathrm{HP},F}^{qg \to q,1}(v,w,\chi_{\mu}) F^{q}(\frac{x_{0}}{w},x_{0},\mu)$ $-\hat{\sigma}_{\mathrm{HP},\partial_{1}F}^{qg \to q,1}(v,w,\chi_{\mu}) x_{0} \left(\partial_{1}F^{q}\right)\left(\frac{x_{0}}{w},x_{0},\mu\right) - \hat{\sigma}_{\mathrm{HP},\partial_{2}F}^{qg \to q,1}(v,w,\chi_{\mu}) x_{0} \left(\partial_{2}F^{q}\right)\left(\frac{x_{0}}{w},x_{0},\mu\right)$ $+\hat{\sigma}_{\mathrm{HP},\partial_{\tau}F}^{qg \to q,1}(v,w) x_0^2 (\partial_1^2 F^q)(\frac{x_0}{w},x_0,\mu) + \hat{\sigma}_{\mathrm{HP},\partial_1\partial_{\tau}F}^{qg \to q,1}(v,w) x_0^2 (\partial_1\partial_2 F^q)(\frac{x_0}{w},x_0,\mu)$ $+\hat{\sigma}_{\text{HP}\,G}^{qg \rightarrow q,5}(v,w,\chi_{\mu}) G^{q}(\frac{x_{0}}{w},x_{0},\mu)$ $-\hat{\sigma}_{\mathrm{HP},\partial_1G}^{qg \to q,5}(v,w) x_0 \left(\partial_1 G^q\right)(\frac{x_0}{w}, x_0, \mu) - \hat{\sigma}_{\mathrm{HP},\partial_2G}^{qg \to q,5}(v,w,\chi_{\mu}) x_0 \left(\partial_2 G^q\right)(\frac{x_0}{w}, x_0, \mu)$ $\left. \left. + \hat{\sigma}^{qg \rightarrow q,5}_{\operatorname{HP},\partial_1^2 G}(v,w) \, x_0^2 \, (\partial_1^2 G^q)(\tfrac{x_0}{w},x_0,\mu) + \hat{\sigma}^{qg \rightarrow q,5}_{\operatorname{HP},\partial_1\partial_2 G}(v,w) \, x_0^2 \, (\partial_1\partial_2 G^q)(\tfrac{x_0}{w},x_0,\mu) \right| .$ $\sigma_0(S) \equiv \frac{8\pi\alpha_{\rm em}^2}{\alpha_{\rm em}^2} \frac{M\epsilon^{lPP_hS}}{m^2}$ NLO Corrections and Factorization for Transverse Single-

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Spin Asymmetries

$$x = r \cos\left(\varphi + \frac{\pi}{4}\right), x' = r \sin\left(\varphi + \frac{\pi}{4}\right)$$

Periodicity $F_{FT}^q(r, \varphi + 2\pi) = F_{FT}^q(r, \varphi)$ can be used to set up fourier series!

$$\begin{split} F_{FT}^{q}(r,\varphi) \\ &= \frac{1}{2\pi} f_{1T}^{\perp(1),q+\bar{q}} \left(\frac{r}{\sqrt{2}}\right) \left(1 + \sum_{n} a_{2n}^{q} [\cos(2n\varphi) - 1]\right) \\ &+ \frac{1}{2\pi} f_{1T}^{\perp(1),q-\bar{q}} \left(\frac{r}{\sqrt{2}}\right) \left(\cos(\varphi) + \sum_{n} a_{2n+1}^{q} [\cos((2n+1)\varphi) - \cos(\varphi)]\right) \end{split}$$

Truncate the series, find realistic choices for $(a_2^q, a_4^q, a_6^q; a_3^q, a_5^q, a_7^q)$

Constraint from lattice QCD:

$$\int_{-1}^{1} dx \int_{-1}^{1} dx' F_{FT}^{q}(x, x') = -d_{2}^{q}$$



 $d_2^u = -0.00365(25), d_2^d = 0$ J. M. Bickerton, Ph.D. thesis, Adelaide U. (2020)

Scenario 1 ("Realistic" Off-Diagonal)

$$F_{FT}^{u}: a^{u} = \left(2.5308, -\frac{2}{3}, -\frac{2}{3}; -\frac{1}{3}, -1, -\frac{1}{3}\right)$$

-0.6

$$F_{FT}^d$$
: $a^d = \left(-0.3429, \frac{2}{3}, \frac{2}{3}; \frac{1}{3}, 1, \frac{1}{3}\right)$





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Combined Plot of the Asymmetries for π^+ , EIC



Combined Plot of the Asymmetries for π^{\pm} , HERMES $\sqrt{s} = 7.25 \ GeV, \mu = \langle P_T \rangle = 1.15 \ GeV$



Data from www.desy.de based on A. Airapetian et al. Phys. Lett. B 728, 183 (2014)

Summary/Outlook

Summary:

- Twist-3 formalism: plethora of different contributions + distribution (and fragmentation) functions
- Collinear twist-3 factorization holds at 1-loop level!
- There is potential for Future experiments to distinguish the different model scenarios

Outlook:

- Include model for the tri-gluon correlation functions in the numerical predictions
- Generate plots vs. transverse momentum $P_{h,T} \rightarrow$ include evolution (Honeycomb/Snowflake S. Rodini, L. Rossi, A. Vladimirov <u>arXiv 2405.01162</u>)
- Include twist-3 fragmentation effects
- $pp^{\uparrow} \rightarrow (h, \gamma, jet, l, J/\psi, ...)X$