FEA-Based Vibrational Analysis of the Inner Barrel Layers (L0–L1) in the SVT Detector

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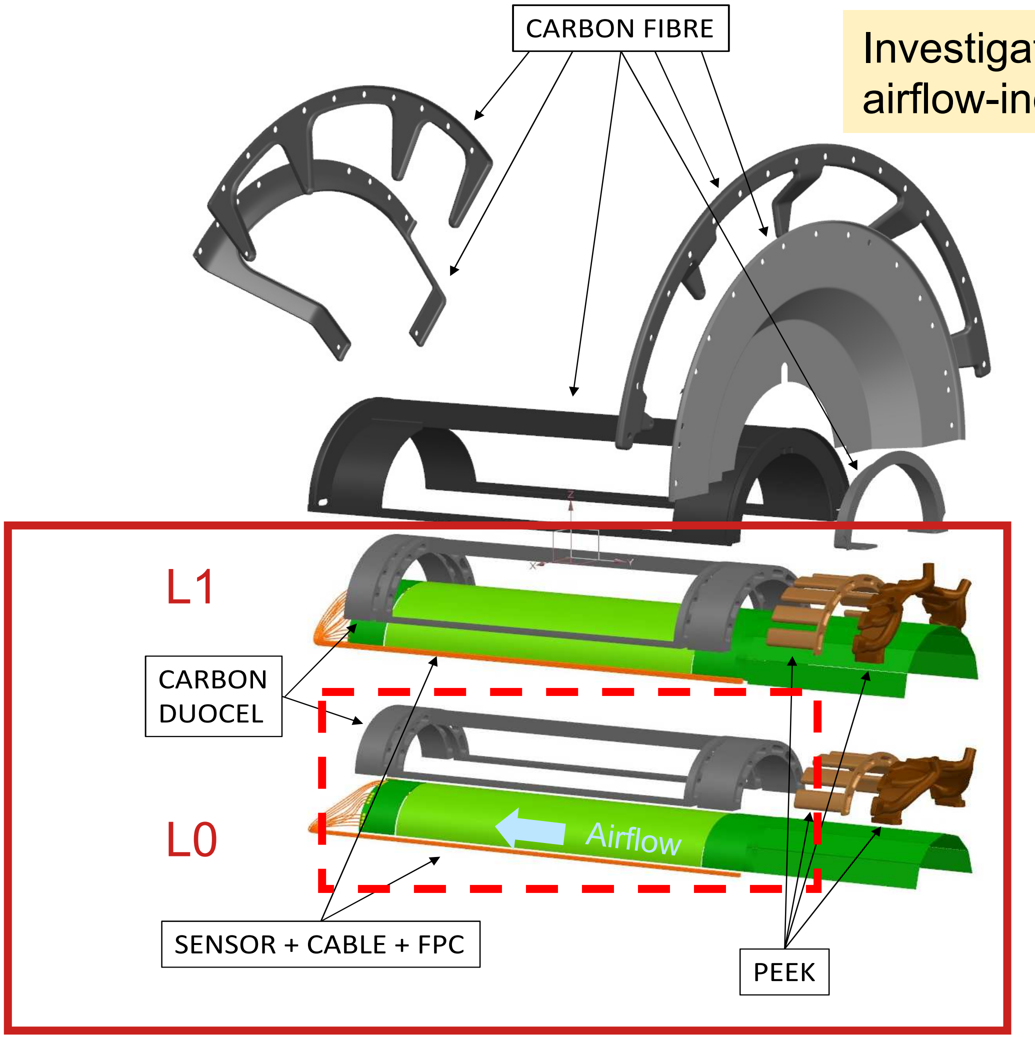
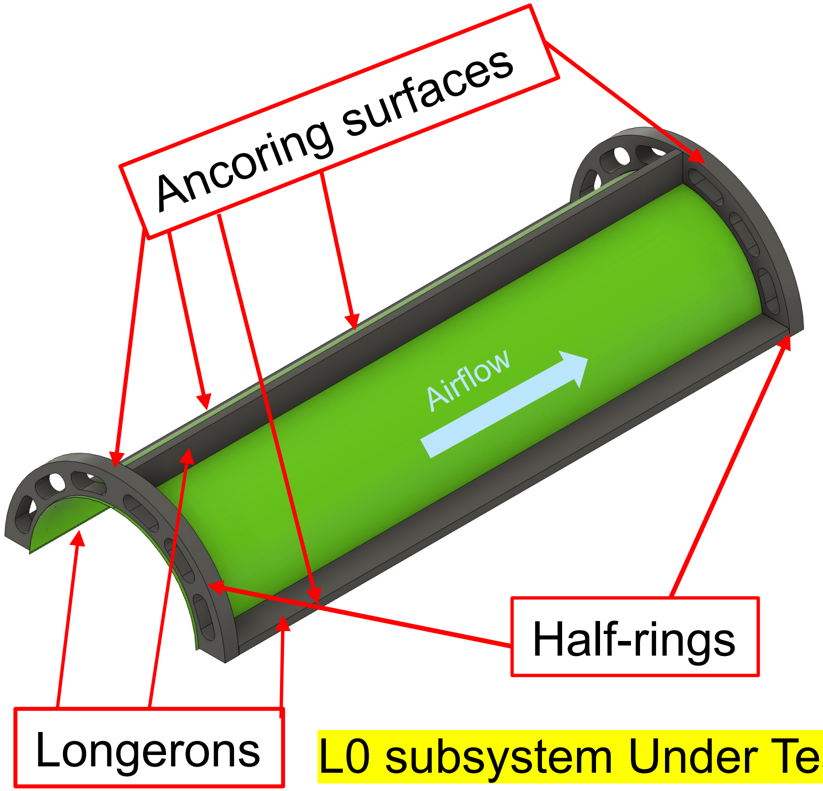
Outline:

- Structural modeling of the L0–L1 SVT assembly (geometry and materials)
- Representation of the silicon layers as circular cylindrical shells with simply supported or clamped edges
- Validation of finite element model (FEM) results
- Realistic modal analysis of the L0–L1 SVT assembly
- PSD-based random vibration testing
- Conclusions and prospects

Structural modelling of the L0-L1 SVT assembly

Investigation of SVT vibrations due to multiple sources: airflow-induced, environment & thermal induced vibrations

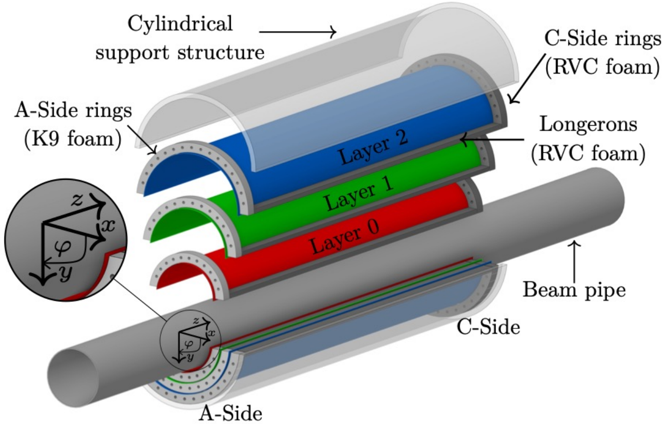


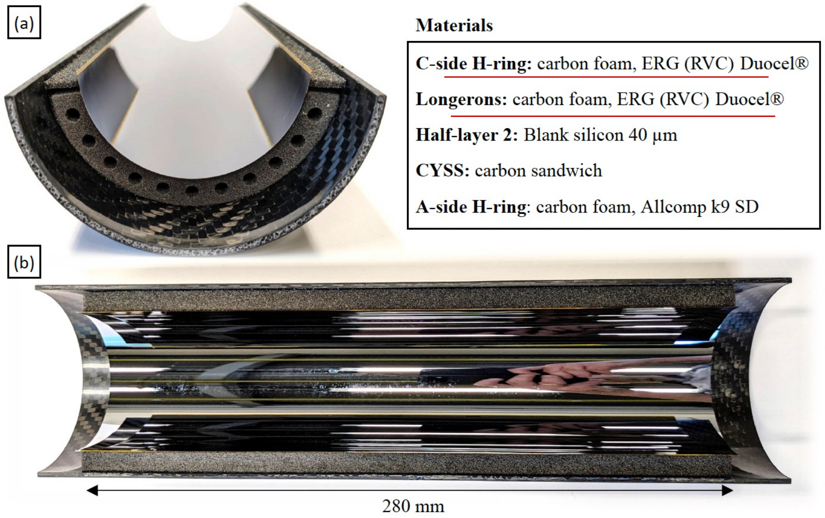


L0 subsystem Under Test

Materials & detector assembly

Alice ITS3

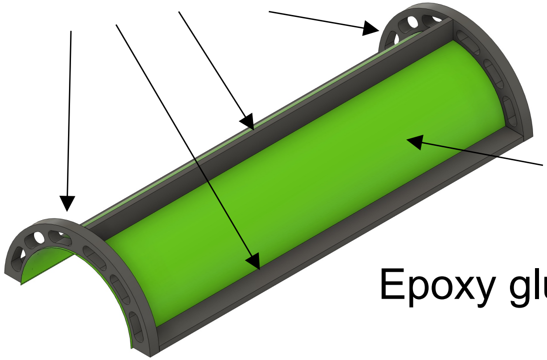


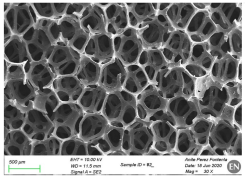


Courteously from Alice TDR021 for ITS3

Shared challenge with ALICE- ITS3: minimum material budget achieved by a thin carbon cylindrical exoskeleton for the support of the three half-la

Radius/Longitudinal/ contact angle	
R_{0iC}	38.15 [mm]
R_{0iG}	38.05 [mm]
R_{0iSi}	38.00 [mm]
L_{Si}	251.3 [mm]
L_{TOT}	264.00 [mm]
α_c	3.65°





Towards the mechanical characterisation of SVT

1- Potential failure and evaluate the short-term/long-term reliability of SVT
2- Position stability of the sensor over time.

$$\mathrm{RMS_{tot}} = \sqrt{\mathrm{RMS}^2_{\mathrm{air\ flow}} + \mathrm{RMS}^2_{\mathrm{noise}} + \mathrm{RMS}^2_{\Delta T}} < 1\ \mu\mathrm{m}$$

Aero-elasticity | Elasticity | Thermo-elasticity

With ref. TDR021 for ITS3

Short-term displacement is induced by air flow of the cooling system, the seismic (<1 Hz) and cultural noise (>1 Hz), and thermoelastic expansion caused by short-term temperature fluctuation.

Simulation FEM Workflow supporting experimental activity:

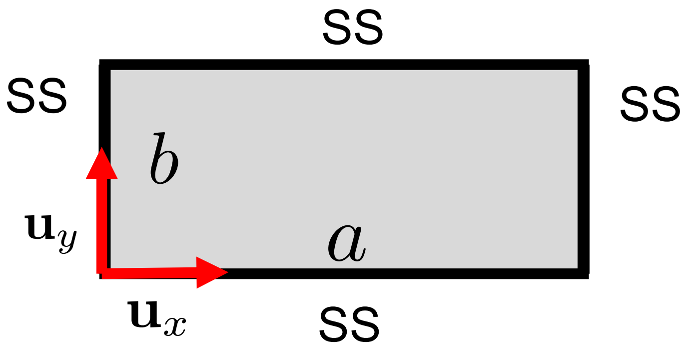
- LES Simulations – Incompressible Navier-Stokes fluid
Evaluate static and fluctuating pressure fields on L0–L1 layers to estimate aerodynamic loads.

- Harmonic Simulations – Transfer Function – PSD random vibrations
Assess how external acoustic excitations are transmitted to the L0–L1 layers.

- Transient Thermo-Structural Analysis
Analyse displacement evolution on L0–L1 layers due to thermal gradients with materials with different Coefficients of Thermal Expansion (CTEs).

Modelling of Si layers: circular cylindrical shells

Simply-supported plate

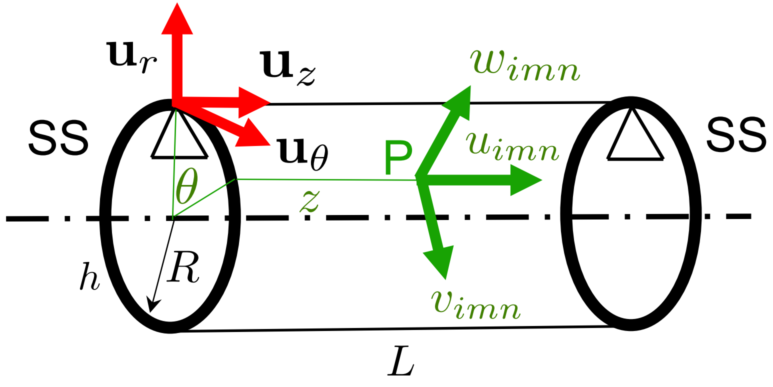


Natural frequencies & mode shape

$$D\nabla^2 w + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \qquad \omega_{mn} = ((\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2)\sqrt{\frac{D}{\rho h}}$$

$$w(x,y,t) = \sum_m \sum_n C_{mn} \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \cos(\omega_{mn} t)$$

Simply-supported cylindrical shell



$$u_z(0,\theta,t) = 0$$
$$u_\theta(0,\theta,t) = 0$$
$$M_{zz}(0,\theta,t) = 0$$
$$N_{zz}(0,\theta,t) = 0$$

$$u_r(L,\theta,t) = 0$$
$$u_\theta(L,\theta,t) = 0$$
$$M_{zz}(L,\theta,t) = 0$$
$$N_{zz}(L,\theta,t) = 0$$

For every (m.n) we have three frequencies i=1,2,3

Mode shape function for the cylindrical shell

$$u_{imn}(z,\theta,t) = A_{imn} \cos\frac{m\pi z}{L} \cos(n\theta - \phi) \cos(\omega_{imn} t)$$
$$v_{imn}(z,\theta,t) = B_{imn} \sin\frac{m\pi z}{L} \sin(n\theta - \phi) \cos(\omega_{imn} t)$$
$$w_{imn}(z,\theta,t) = C_{imn} \sin\frac{m\pi z}{L} \cos(n\theta - \phi) \cos(\omega_{imn} t)$$

m - longitudinal index
n – radial index

Ref. Vibrations of Shells and Plates
Werner Soedel

Cylindrical shell simply supported at the edges – approximation D-M-V

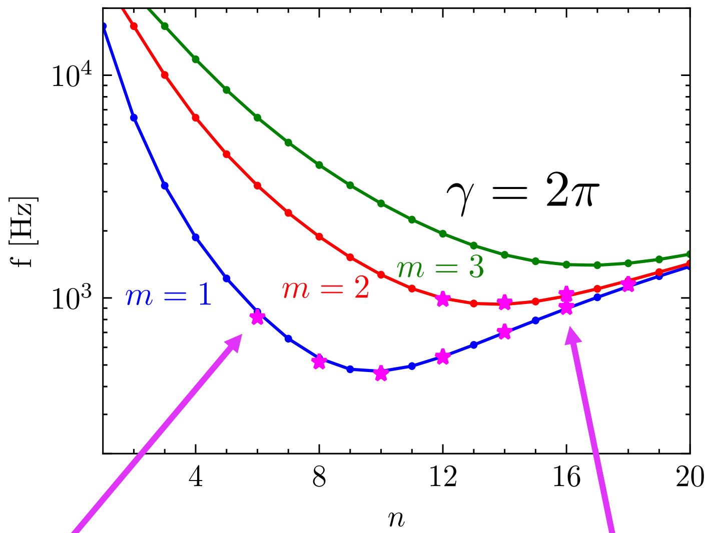
To understand the vibrational behaviours of the cylinder shell, we consider the Donnell–Mushtari–Vlasov approximation (shallow shell equations). Closed form is useful for mode identification following the approach of D–M–V with the guess function:

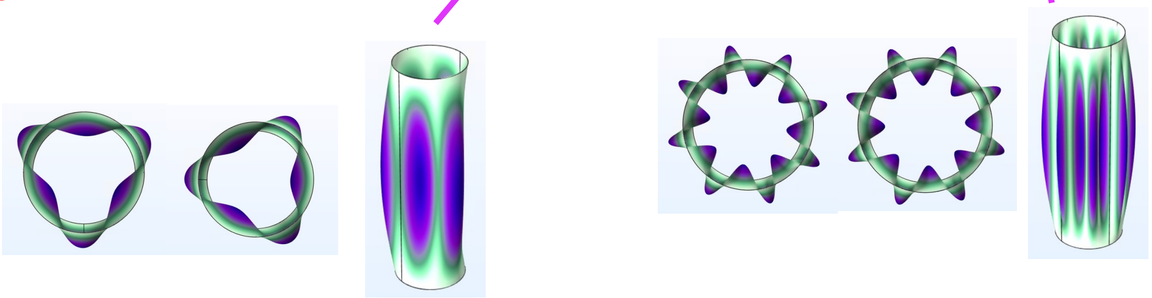
$$\tilde{w}_{mn}(z,\theta) = \sin\frac{m\pi z}{L}\cos\left(\frac{n(\theta-\phi)}{2}\right)$$

$$\omega_{mn} = \frac{1}{R}\sqrt{\frac{(m\pi R/L)^4}{[(m\pi R/L)^2+(n/2)^2]^2} + \frac{(h/R)^2}{12(1-\nu^2)}\left[\left(\frac{m\pi R}{L}\right)^2+\left(\frac{n}{2}\right)^2\right]^2}\sqrt{\frac{Y}{\rho}}$$

Membrane stiffness   Bending stiffness

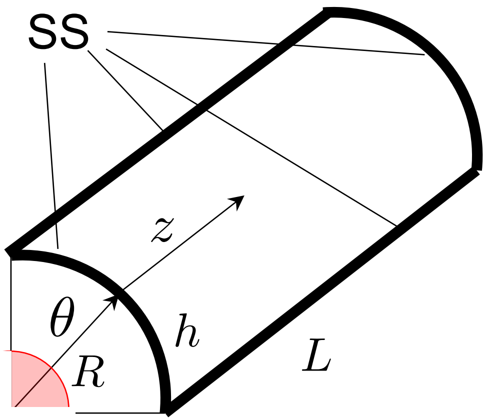
Analytical vs FEM





Ref. Vibrations of Shells and Plates - Werner Soedel

Cylindrical panel simply supported at all edges 1



$\gamma = \pi/2$

General solution

$$u_{imn}(z,\theta,t) = A_{imn} \cos\frac{m\pi z}{L} \sin(\frac{n\pi\theta}{\gamma}) \cos(\omega_{imn}t)$$

$$v_{imn}(z,\theta,t) = B_{imn} \sin\frac{m\pi z}{L} \cos(\frac{n\pi\theta}{\gamma}) \cos(\omega_{imn}t)$$

$$w_{imn}(z,\theta,t) = C_{imn} \sin\frac{m\pi z}{L} \sin(\frac{n\pi\theta}{\gamma}) \cos(\omega_{imn}t)$$

Donnell–Mushtari–Vlasov approximation with guess function:

$$\tilde{w}_{mn}(z,\theta) = \sin\frac{m\pi z}{L} \sin(\frac{n\pi\theta}{\gamma})$$

$$\omega_{mn} = \frac{1}{R}\sqrt{\frac{(m\pi R/L)^4}{[(m\pi R/L)^2 + (n\pi/\gamma)^2]^2} + \frac{(h/R)^2}{12(1-\nu^2)}[(\frac{m\pi R}{L})^2 + (\frac{n\pi}{\gamma})^2]^2}\sqrt{\frac{Y}{\rho}}$$

Membrane stiffness

Bending stiffness

Two additional BCs

$$u_r(z,0,t) = 0 \qquad u_r(z,\gamma,t) = 0$$

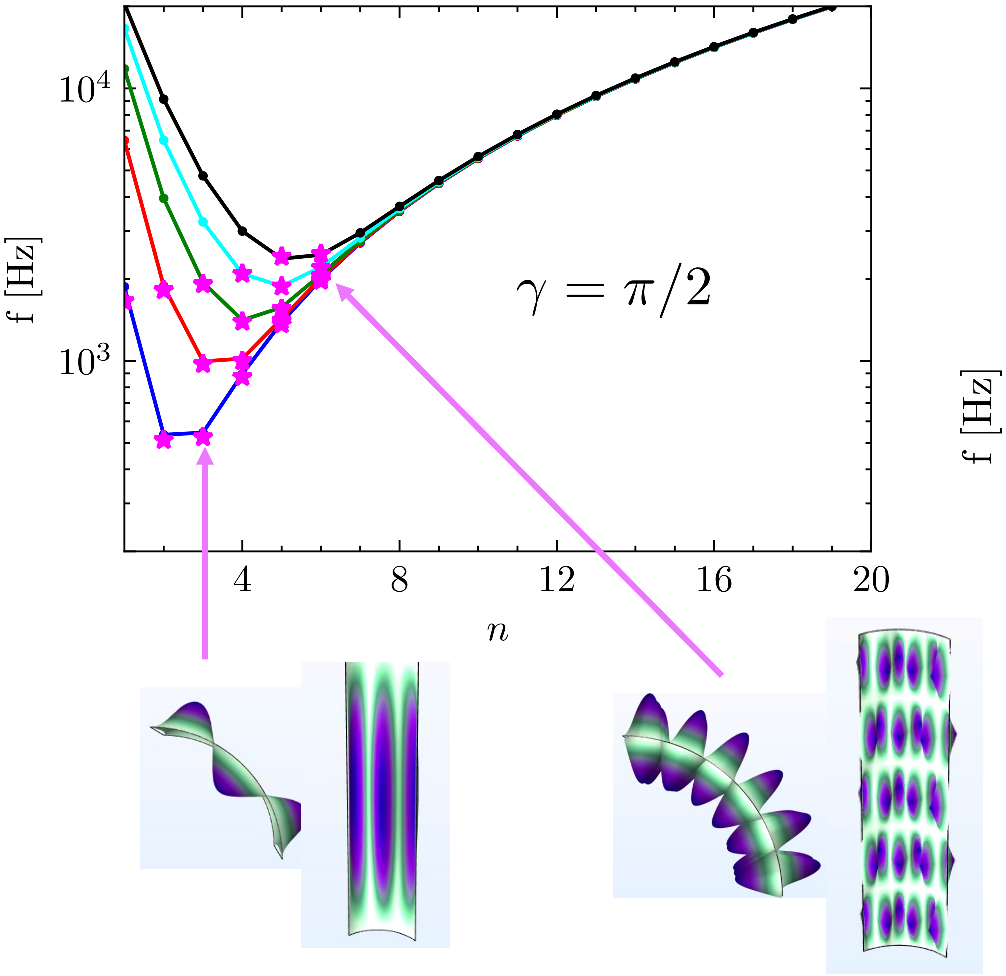
$$u_z(z,0,t) = 0 \qquad u_z(z,\gamma,t) = 0$$

$$M_{\theta\theta}(z,0,t) = 0 \qquad M_{\theta\theta}(z,\gamma,t) = 0$$

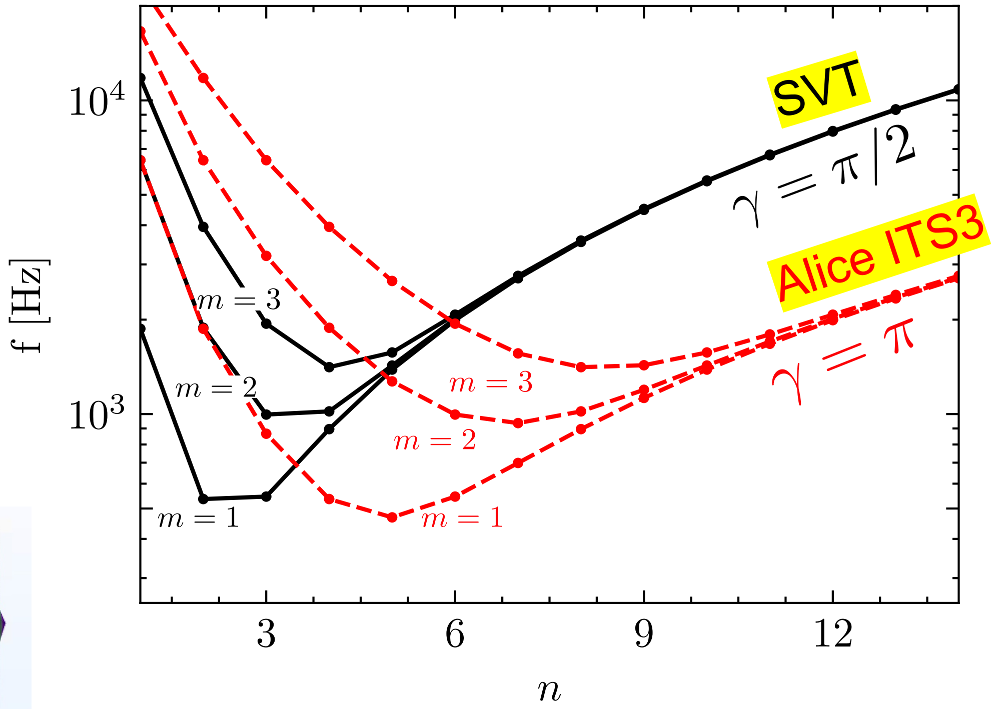
$$N_{\theta\theta}(z,0,t) = 0 \qquad N_{\theta\theta}(z,\gamma,t) = 0$$

Cylindrical panel simply supported at all edges 2

Analitical vs FEM

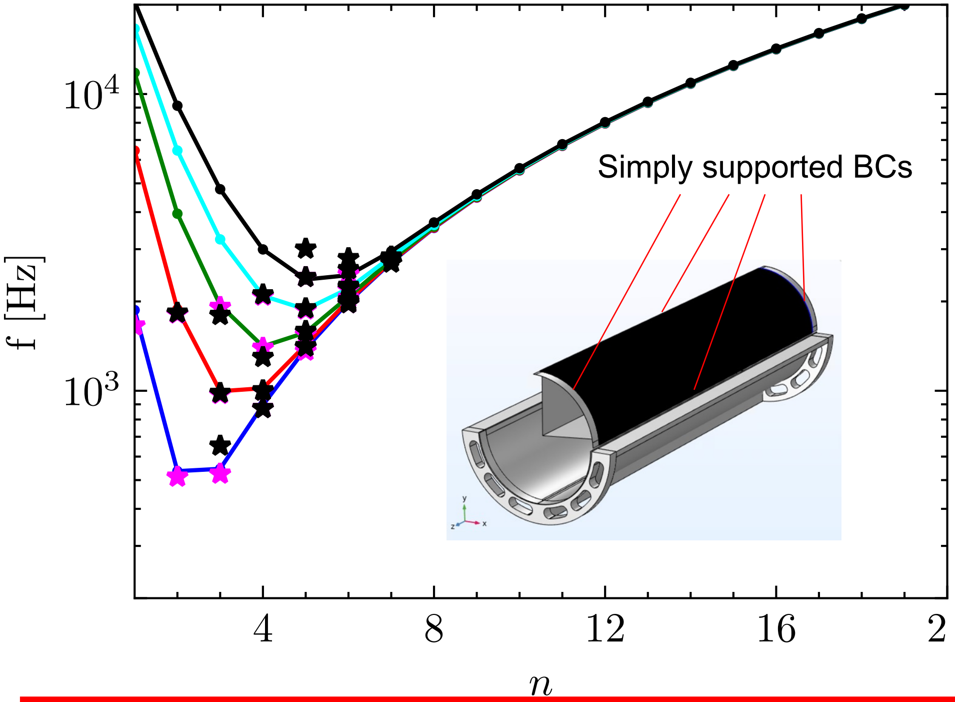


Comparison between two half layers

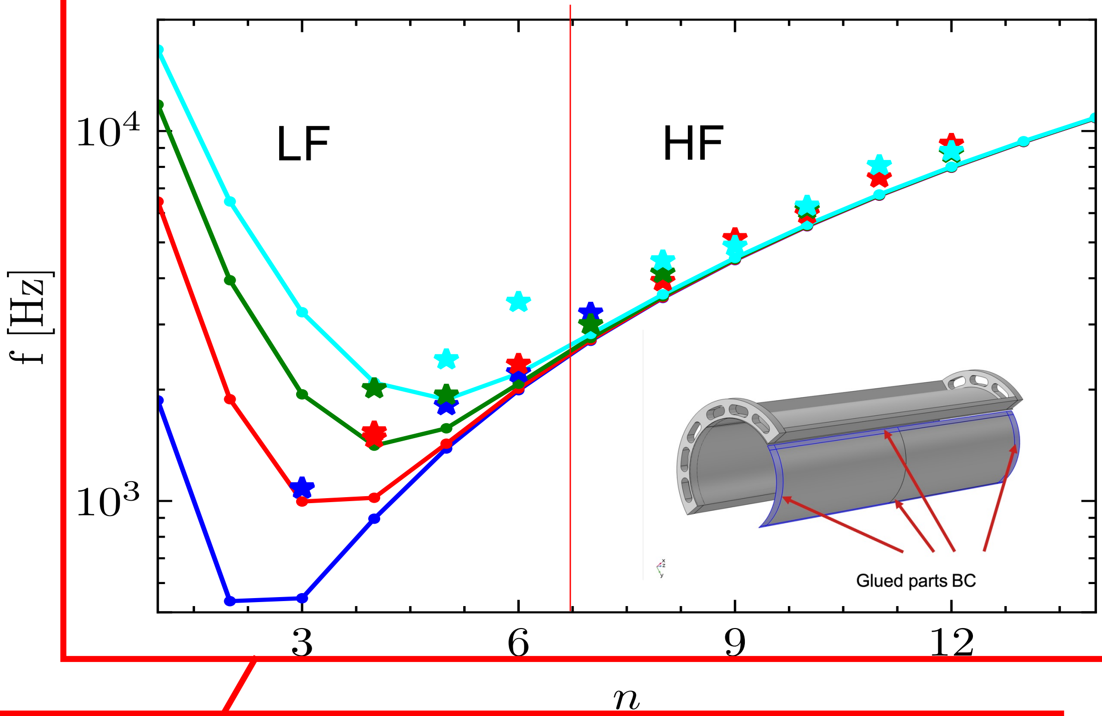


Modal analysis with two BCs of the SVT-L0 model

Simply- Supported FEM analysis

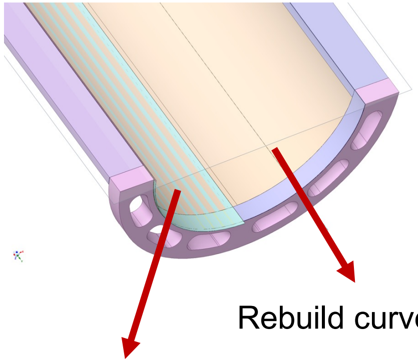


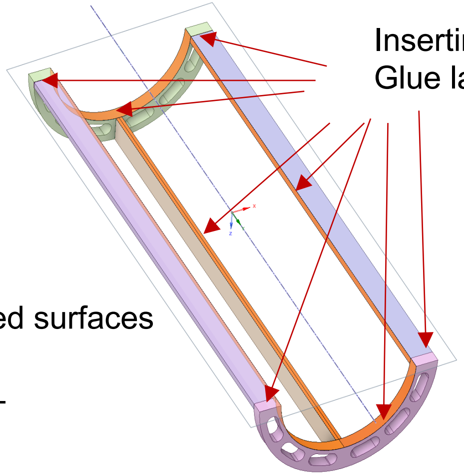
Built-in edges



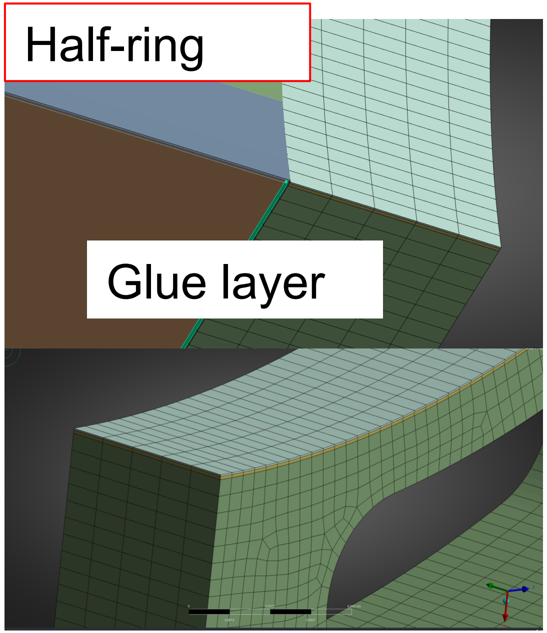
$1079.25[Hz]\ (m=1, n=3)$ $1268.01[Hz]\ (m=1, n=4)$ $1550.11[Hz]\ (m=2, n=4)$ $1936.62[Hz]\ (m=3, n=5)$

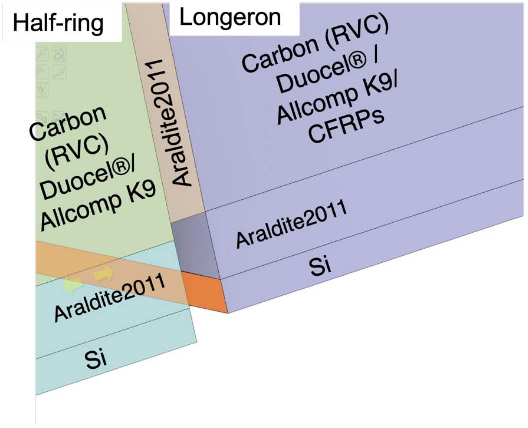
CAD modelling & mesh strategy of the SVT-L0

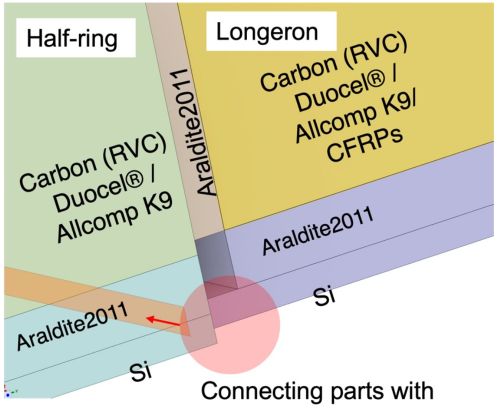




Ansys model

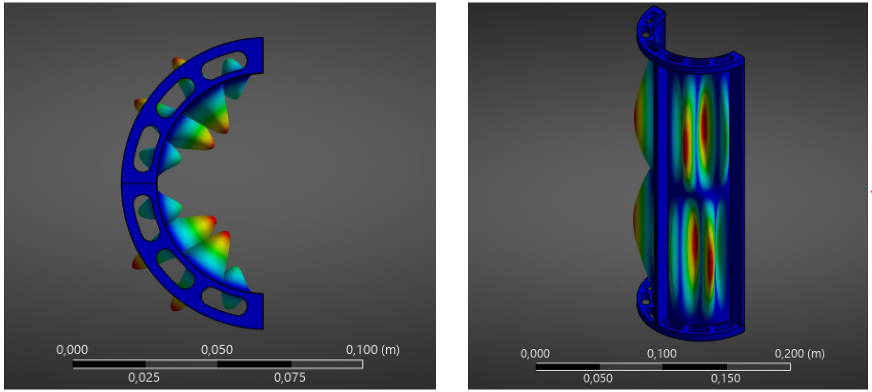




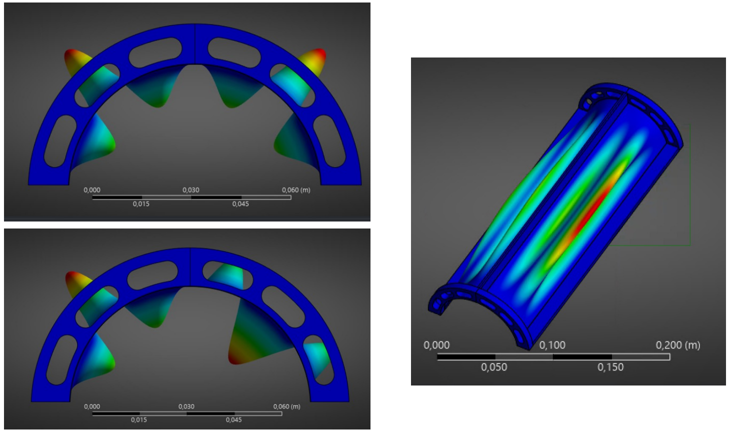


Mesh sweeping to obtain a regular and a good mesh quality.

Fully constrained half-layer SVT-L0 model

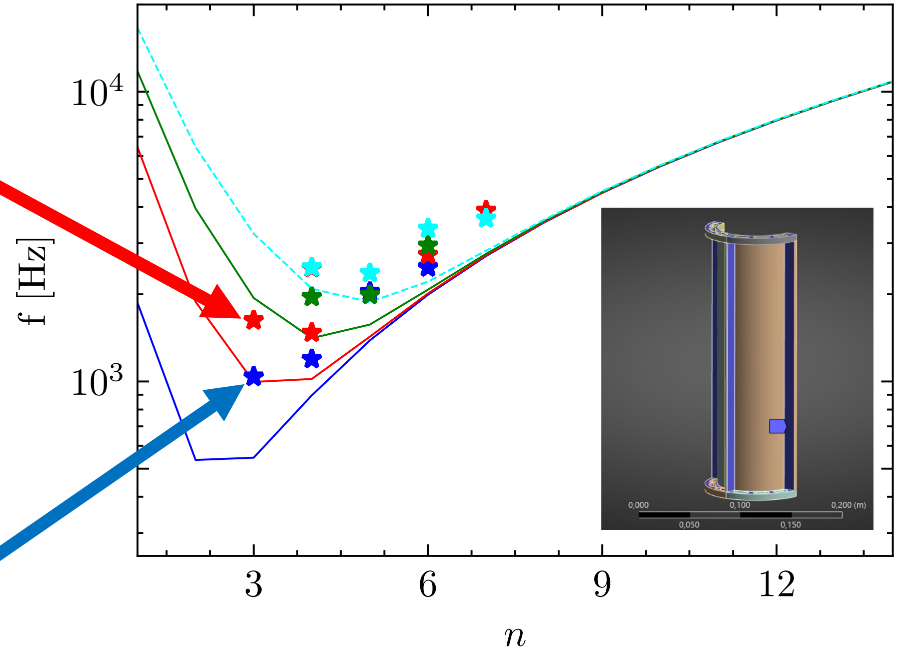


$1624.00[Hz](m = 2, n = 4)$



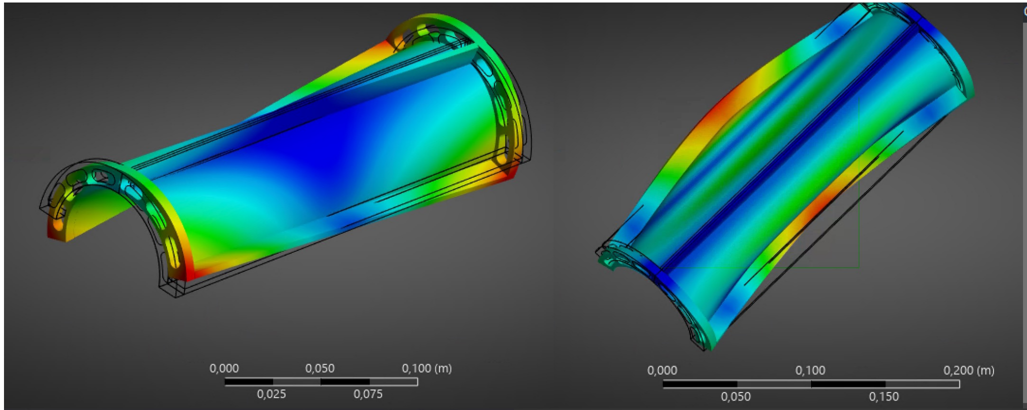
$1036.5[Hz](m = 1, n = 3)$

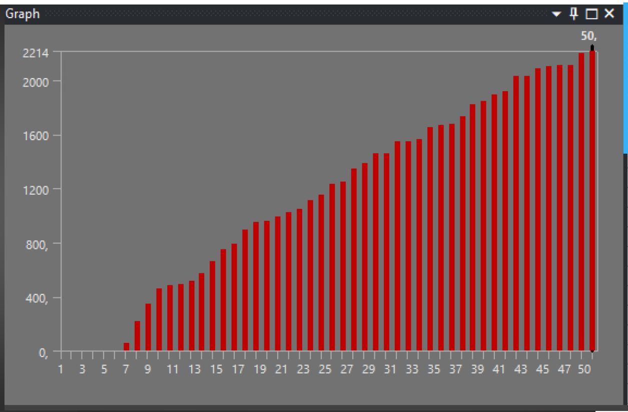
Analytical SS – Ansys FEM Build-in edeges



At low frequencies, increasing the number of radial nodes (*n*) raises the mode frequency. For higher *n*, frequencies become less sensitive to longitudinal boundary conditions, similar to a supported case. Additional mode shapes (not shown) contribute similarly at higher frequencies.

Unconstrained half-layer SVT-L0 model – mode couplings



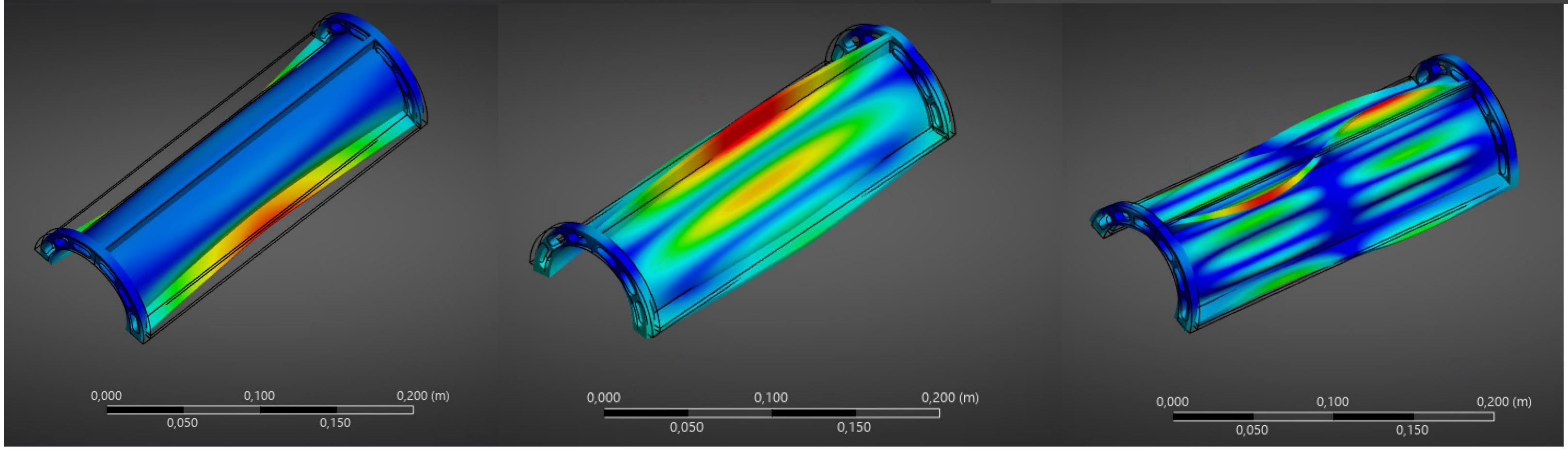


Tabular Data

	Mode	Frequency [Hz]
1	1,	0,
2	2,	0,
3	3,	0,
4	4,	0,
5	5,	0,
6	6,	0,47594
7	7,	55,736
8	8,	220,43
9	9,	342,34
10	10,	456,09
11	11,	486,16
12	12,	490,84
13	13,	517,87
14	14,	571,85
15	15,	660,63
16	16,	746,27
17	17,	786,94
18	18,	891,72
19	19,	947,63

Rigid motions

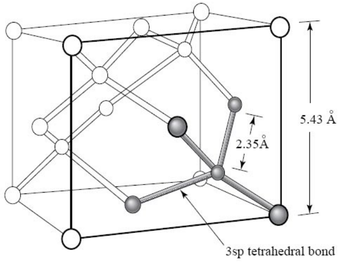
Several modes in the 1 kHz bandwidth



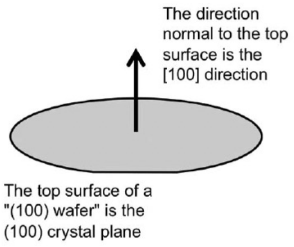
Carbon foam structures (longerons and half rings) have modes very close to the vibration of the silicon shell.

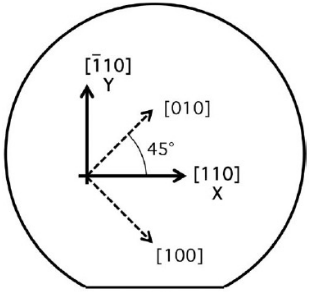
Silicon anisotropic behaviours

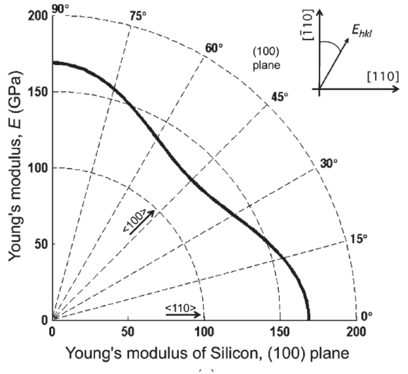
c-Si

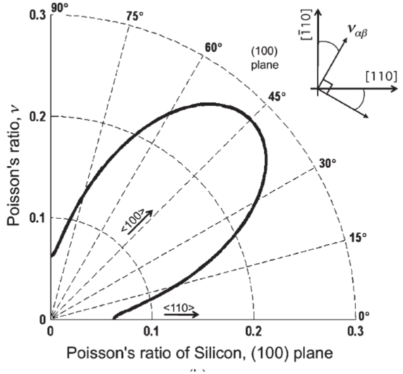


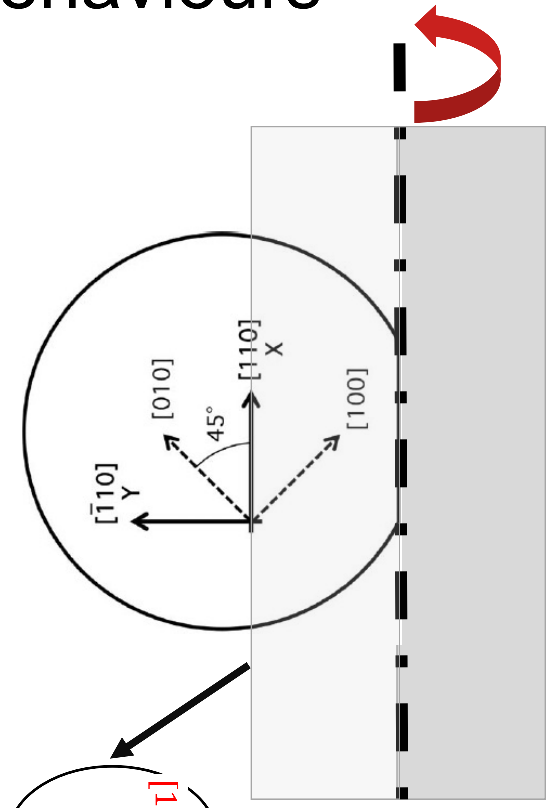
Typical c-Si <100>



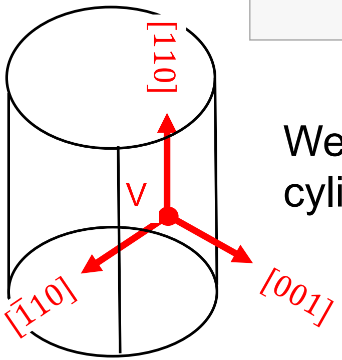








Metal cylinder for the wafer stitching



We must define a local cylindrical coordinate system.

A proper rotation of the elasticity matrix should be considered in the vibrational simulations.

Journal of Micromechanical Systems Hopcroft 2010

Setting the silicon anisotropic behaviours

E_{110} E_{100}

$E_x = E_y = 169$ GPa $E_z = 130$ GPa

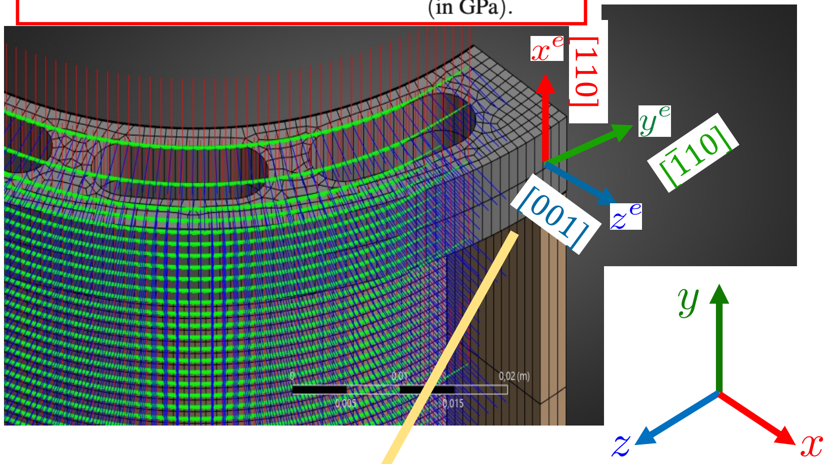
$\nu_{yz} = 0.36$ $\nu_{zx} = 0.28$ $\nu_{xy} = 0.064$

$G_{yz} = G_{zx} = 79.6$ GPa $G_{xy} = 50.9$ GPa

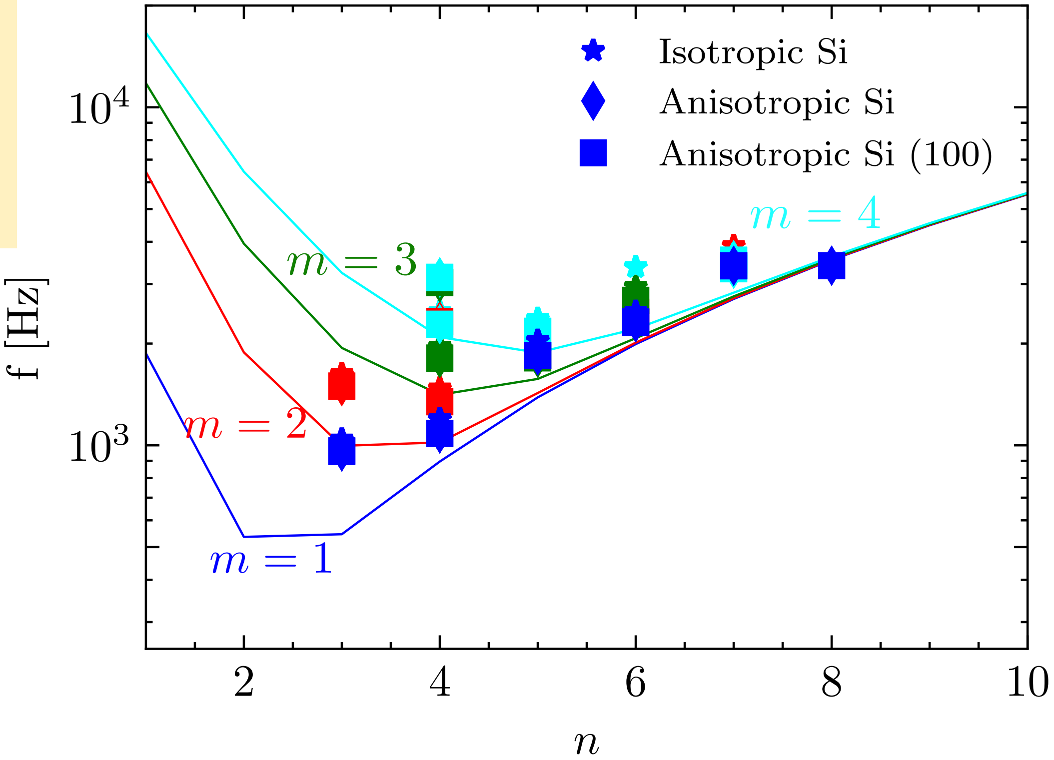
$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix} = \begin{bmatrix} 194.5 & 35.7 & 64.1 & 0 & 0 & 0\\ 35.7 & 194.5 & 64.1 & 0 & 0 & 0\\ 64.1 & 64.1 & 165.7 & 0 & 0 & 0\\ 0 & 0 & 0 & 79.6 & 0 & 0\\ 0 & 0 & 0 & 0 & 79.6 & 0\\ 0 & 0 & 0 & 0 & 0 & 50.9 \end{bmatrix} \begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{bmatrix}$$

(in GPa).

c-Si elasticity matrix in the frame of reference of a standard <100> silicon wafer



Element coordinate system for the correct representation of the orthotropic behaviours of the c-Si shell layer.



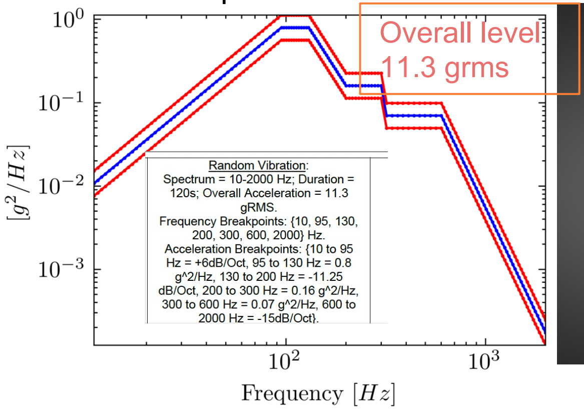
Frequencies are shifted by 10-100 Hz w.r.t. isotropic case

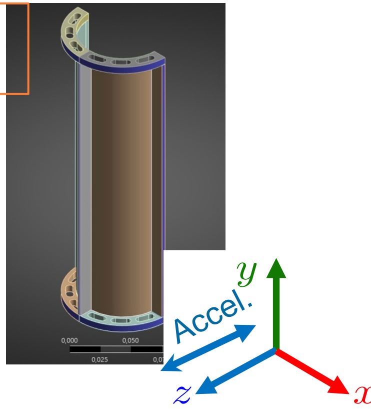
Random vibration test of the SVT-L0 model

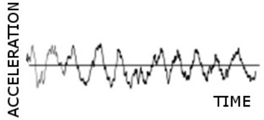
Ansys WB Workflow

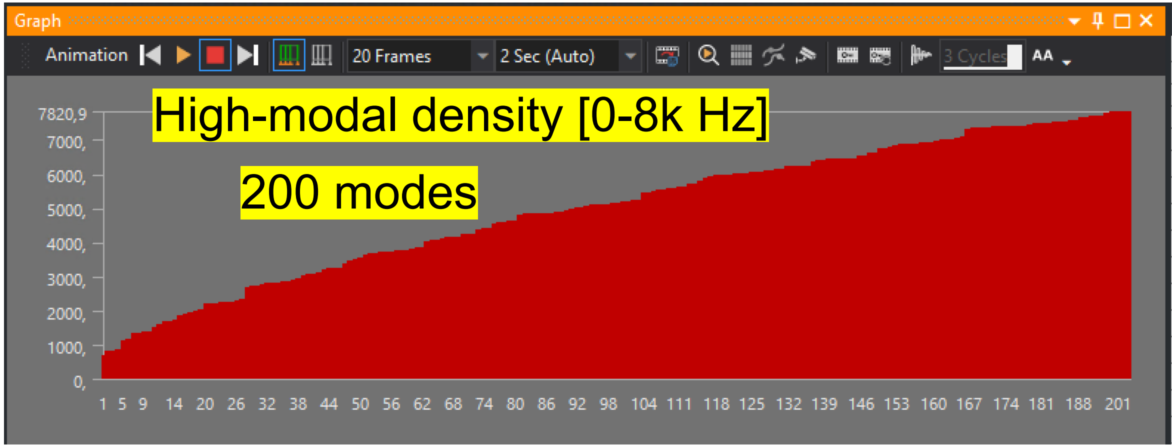
Partecipation factor in X,Y,Z 64% , 86%, 72% to have a good representation of the dynamics of the system.

Aerospace standard PSD







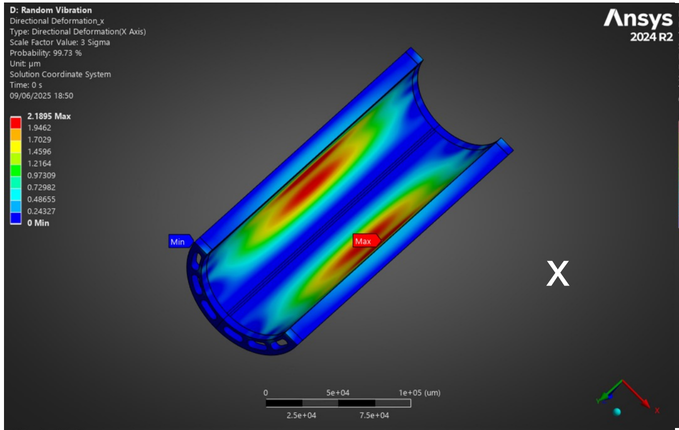


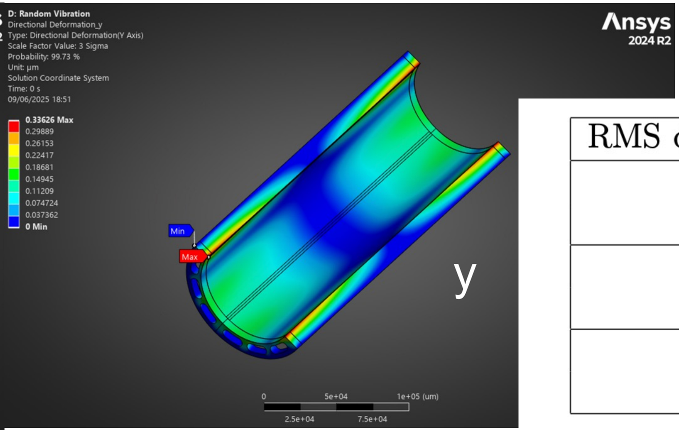
Tabular Data

	Mode	Frequency [Hz]
1	1,	686,9
2	2,	803,5
3	3,	817,17
4	4,	845,9
5	5,	1103,6
6	6,	1170,8
7	7,	1338,5
8	8,	1356,1
9	9,	1361,6
10	10,	1364,9
11	11,	1503,5
12	12,	1610,5
13	13,	1677,9
14	14,	1686,1
15	15,	1744,5

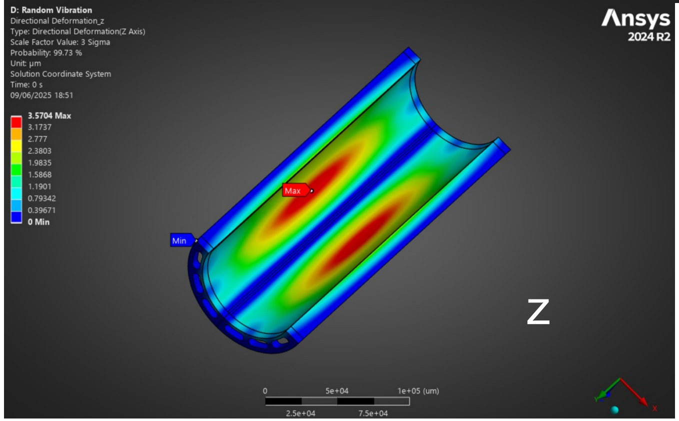
PSD spectrum used for vibrational tests with electronic equipment in the aerospace industry

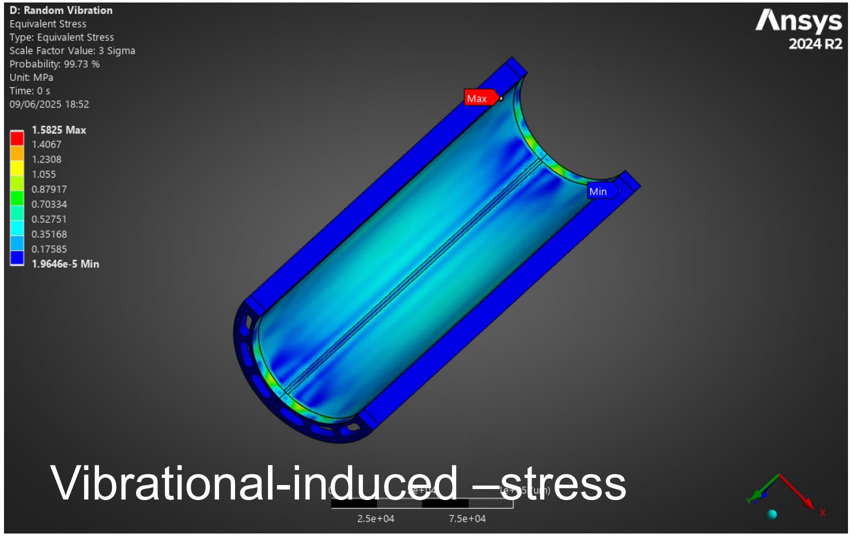
Directional deformation & stress



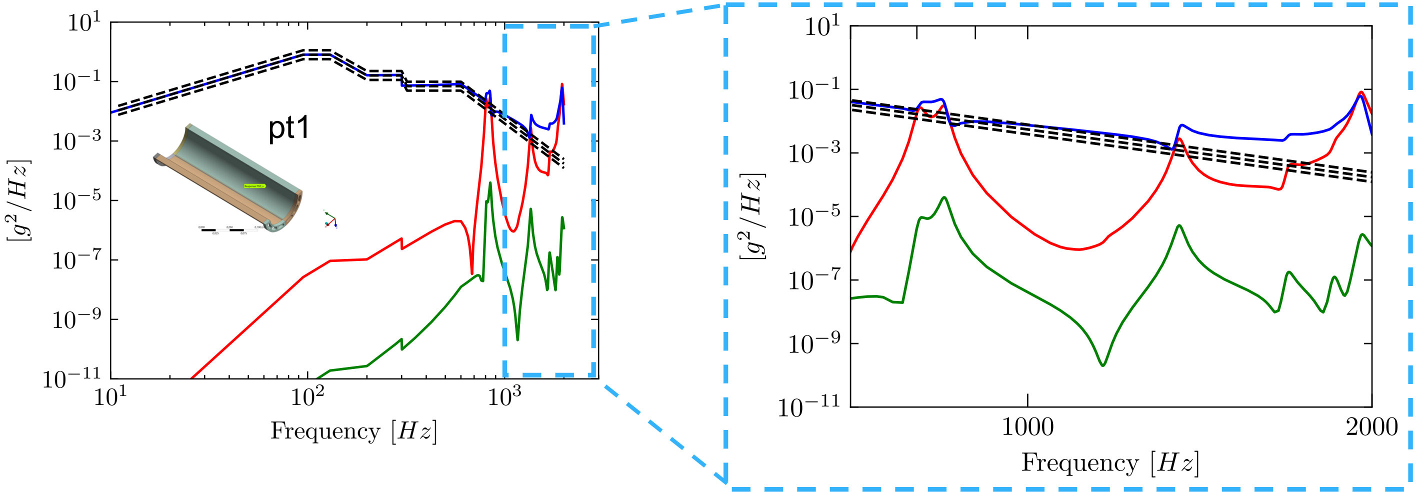


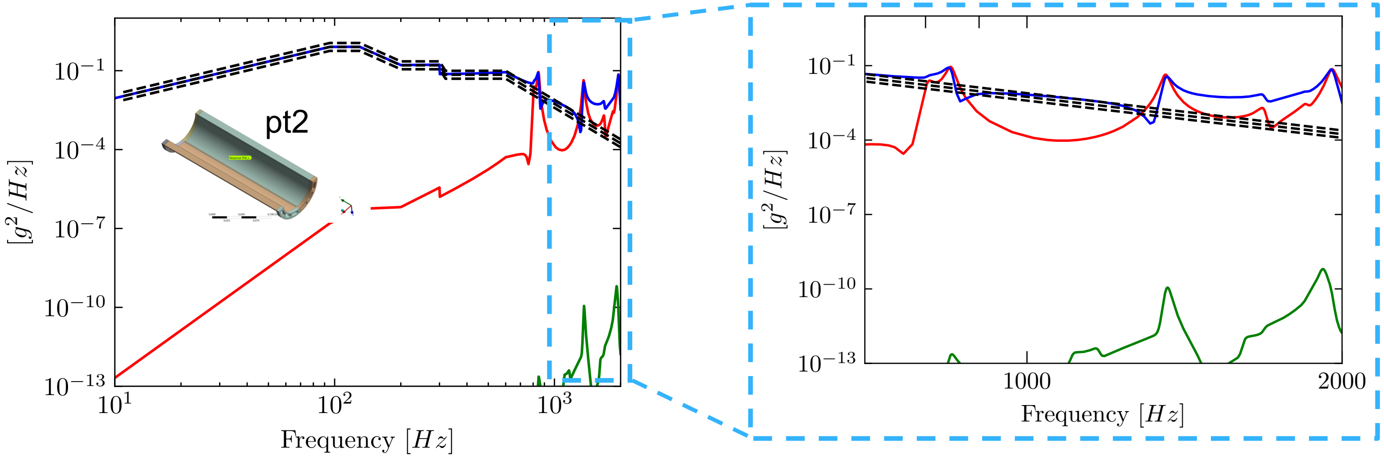
RMS displacement (maximum/averaged)	
u_x	$2.19\ \mu m$
$\bar{u}_x$	$0.69\ \mu m$
u_y	$330\ nm$
$\bar{u}_y$	$78\ nm$
u_z	$3.57\ \mu m$
$\bar{u}_z$	$1.51\ \mu m$





Acceleration in two control points

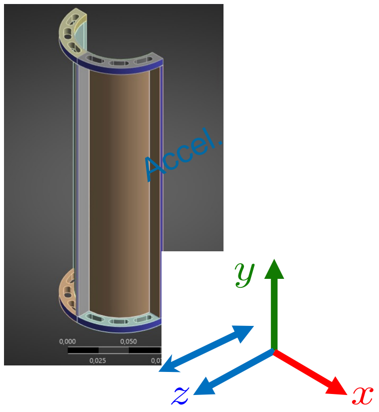




Critical damping

$$\zeta = 1\%$$

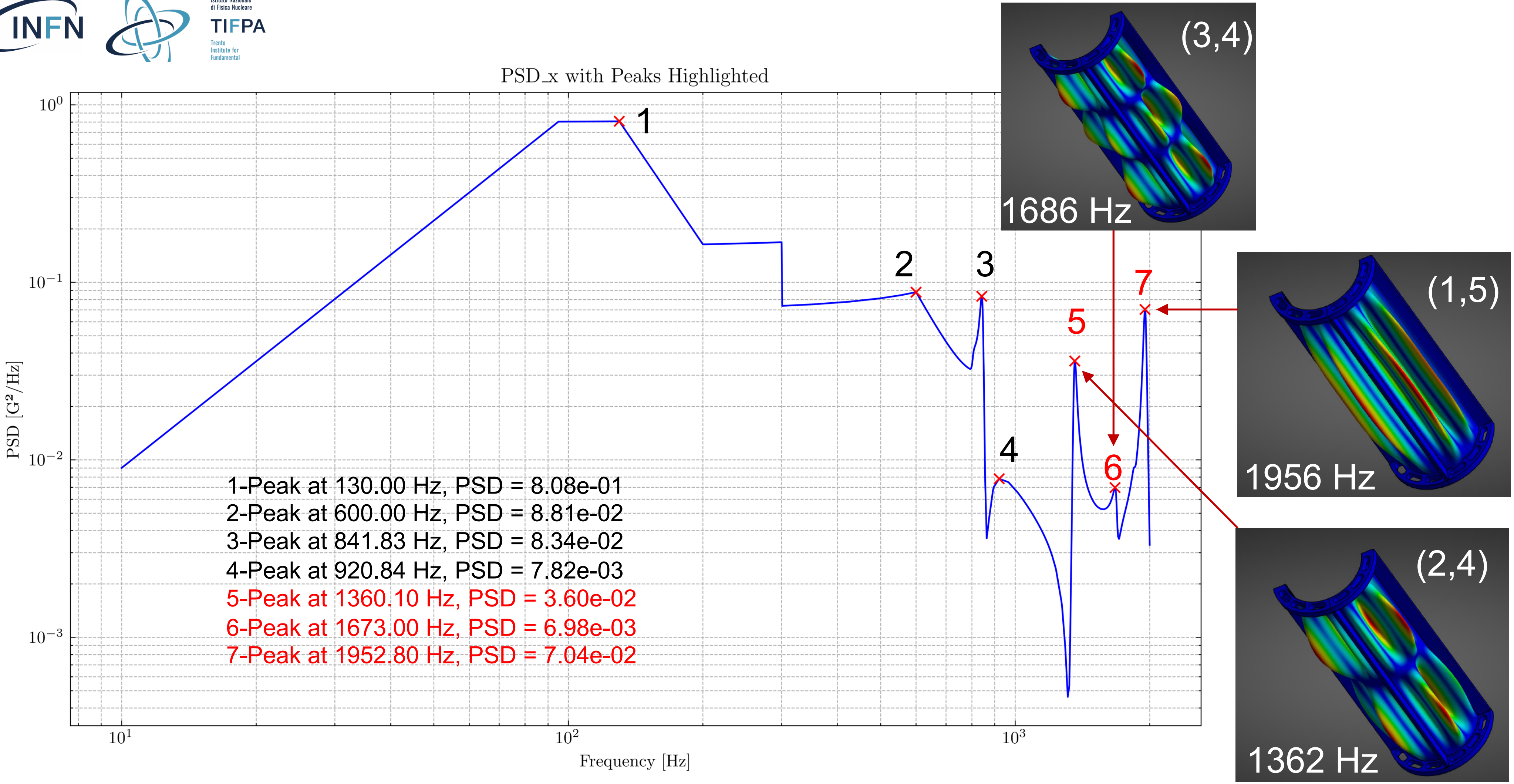
$$Q = \frac{1}{2\zeta} = 50$$



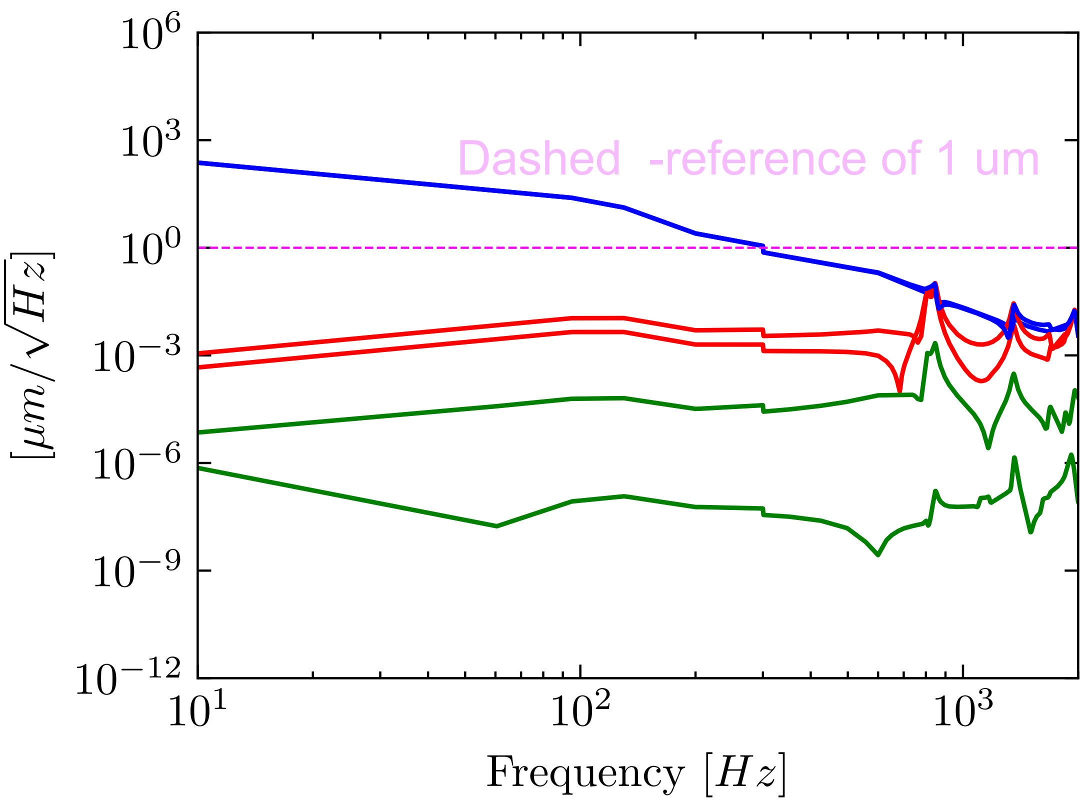
$$\mathrm{PSD}_{g^2/\mathrm{Hz}} = \frac{\mathrm{PSD}_{(m/s^2)^2/\mathrm{Hz}}}{9.81^2}$$

INFN

Istituto Nazionale di Fisica Nucleare
TIFPA
Trento Institute for Fundamental

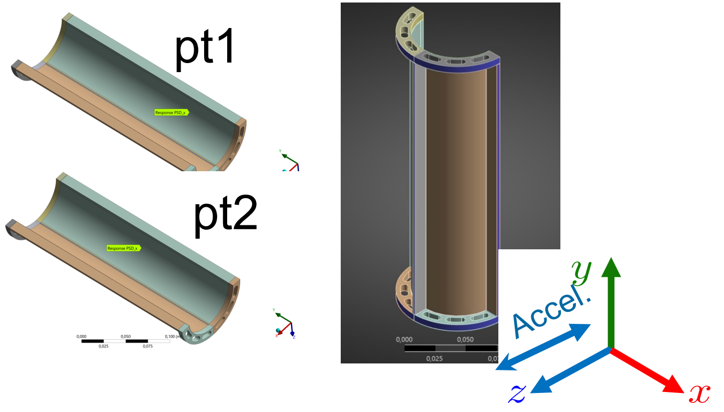


Displacement ASD



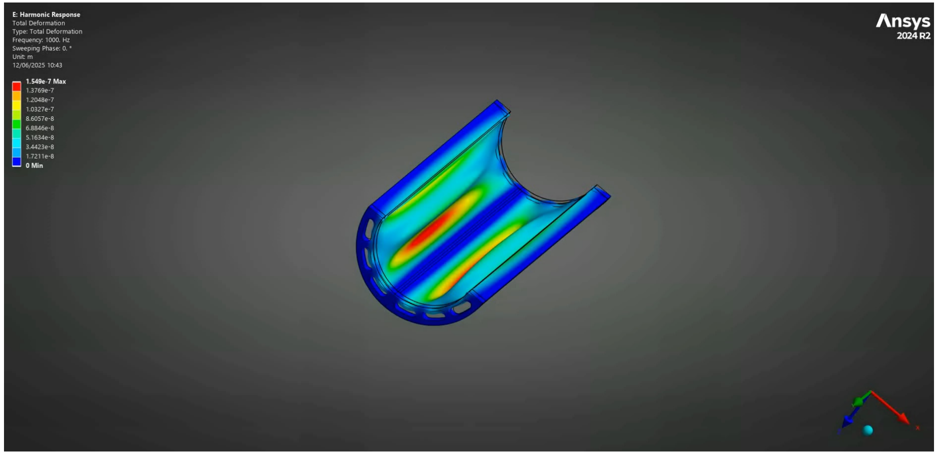
$$\zeta = 1\%$$

$$Q = \frac{1}{2\zeta} = 50$$



The system already developed appears to withstand severe random vibrational test.

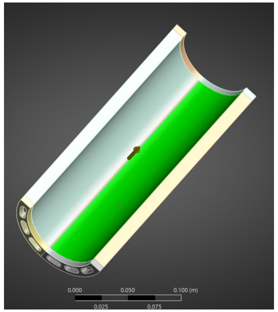
Harmonic Analysis at 1g **excitation**



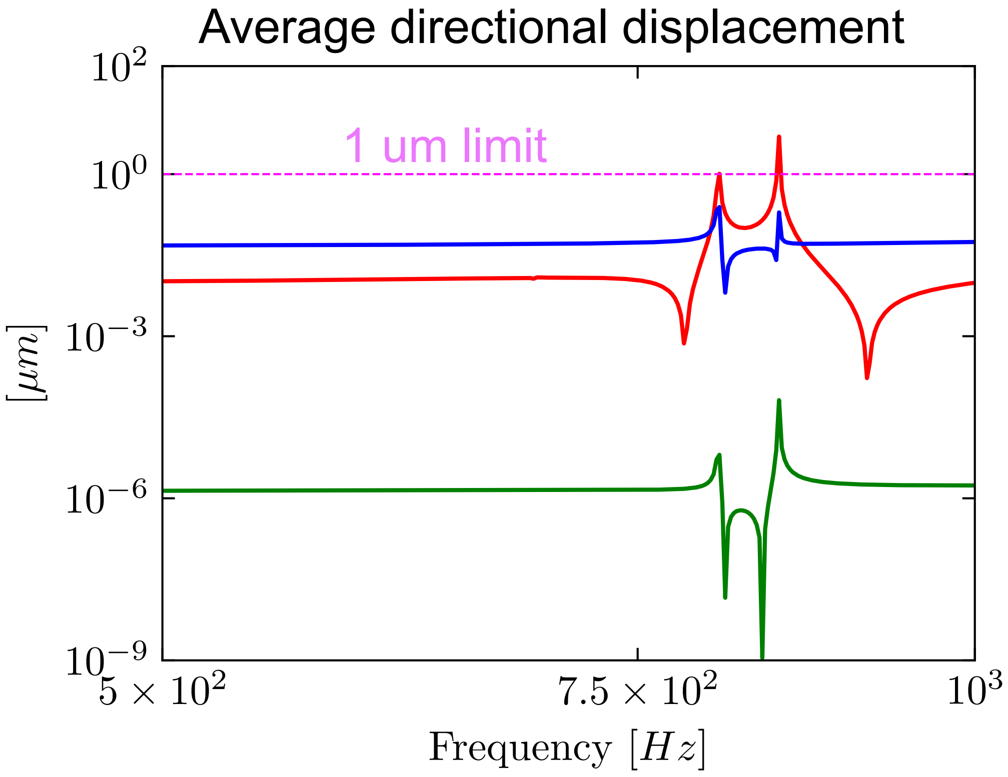
Acceleration 1g along Z axis

$$\zeta = 1\%$$

$$Q = \frac{1}{2\zeta} = 50$$



Reference surface for evaluating the average displacement



Conclusions

- **We have developed a modelling strategy** to analyse the vibrational behaviour of thin silicon shell structures for the Silicon Vertex Tracker (SVT).

- **We have validated the Finite Element Method (FEM) modal analysis** against analytical models to ensure high accuracy and reliability of the simulations.

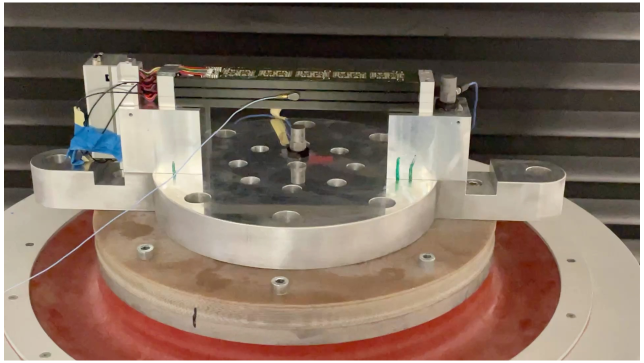
- **We have perform a first FEM random vibrational** test with PSD aerospace spectrum to assess the structural integrity and mechanical resilience of the silicon shells under severe transport conditions.

Prospectives

- **Developing a FEM-based model of the whole SVT apparatus** for estimating the displacement noise in the Si sensors due to multiple sources of vibrations (air-flow, seismic/cultural, thermal)

- **Configuring a dedicated experimental apparatus** for performing extensive vibrational tests at PROM facility in Trento

[Pro]M MECHATRONICS PROTOTYPING FACILITY https://promfacility.eu Trento

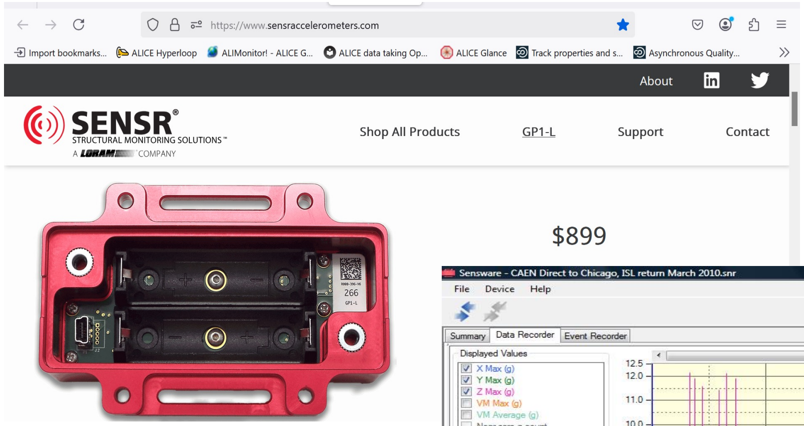


Vibrational test of ALPIDE sensors mounted on a CFRPs stave.

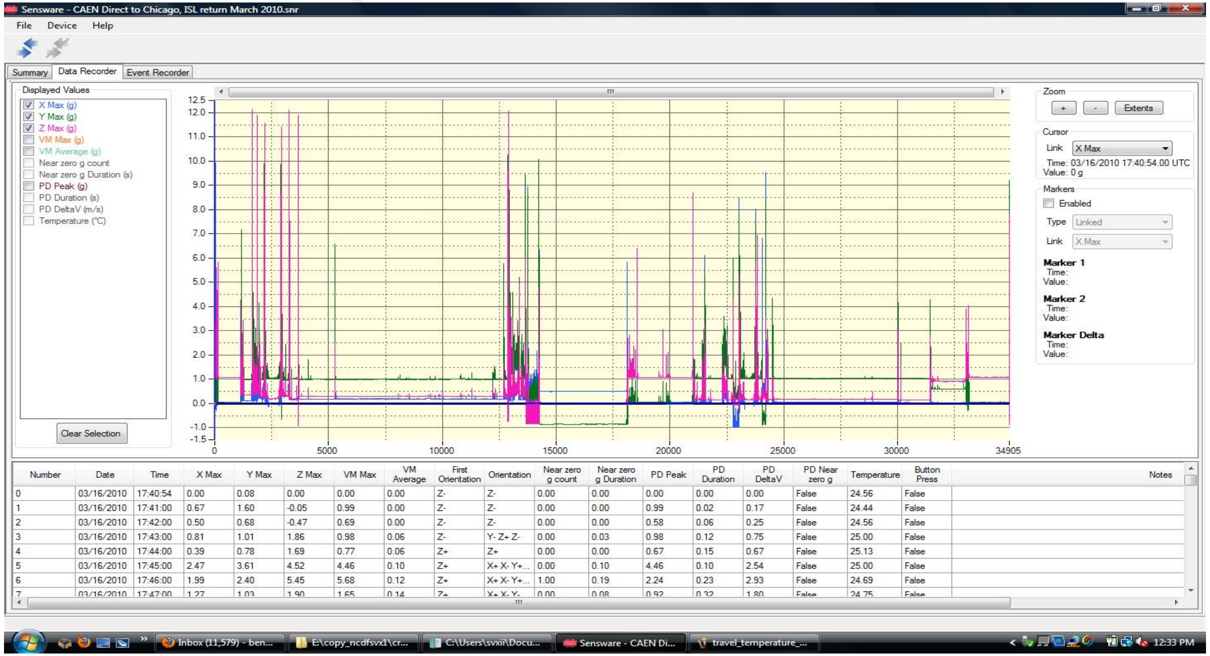
Thank you for your attention!

Back-up slides

Lessons learned from the CDF Silicon Detector experience



Transportation in the US



Acceleration >11g but commercial flight 2.5 g

Courtesily from dr. Benedetto di Ruzza

Seismic noise (Miles' formula)

The RMS acceleration response of an SDOF system to a white noise input is given by:

$$G_{RMS} = \sqrt{\frac{\pi}{2} f_n Q \cdot \mathrm{ASD_{input}}}$$

Where:

- G_{RMS}: Root Mean Square acceleration (e.g., in g_{RMS} or $\mathrm{m/s}^2_{RMS}$)
- f_n: Natural frequency of the SDOF system (in Hz)
- Q: Quality factor, representing the amplification at resonance. It is related to the damping ratio (ζ) by $Q = \frac{1}{2\zeta}$.
- $\mathrm{ASD_{input}}$: Input Acceleration Spectral Density (e.g., in g^2/Hz or $(\mathrm{m/s}^2)^2/\mathrm{Hz}$). For white noise between f_1 and f_2, this is the constant value of the PSD within that band, assuming f_n is within this range.
- Y_{RMS}: Root Mean Square displacement (e.g., in inches_{RMS} or meters_{RMS})
- All other terms are as defined for the acceleration formula.

Note on Units: If $\mathrm{ASD_{input}}$ is provided in g^2/Hz, the calculated G_{RMS} will be in g_{RMS}. To obtain Y_{RMS} in meters, G_{RMS} must first be converted to $\mathrm{m/s}^2$ using the standard acceleration due to gravity ($g_N \approx 9.806\,65\,\mathrm{m/s}^2$). If $\mathrm{ASD_{input}}$ is already in $(\mathrm{m/s}^2)^2/\mathrm{Hz}$, then Y_{RMS} will directly be in meters.

Applying this relationship to the RMS acceleration formula, the RMS displacement (Y_{RMS}) is:

$$Y_{RMS} = \frac{G_{RMS}}{(2\pi f_n)^2}$$

Substituting the expression for G_{RMS}:

$$Y_{RMS} = \frac{1}{(2\pi f_n)^2}\sqrt{\frac{\pi}{2} f_n Q \cdot \mathrm{ASD_{input}}}$$

This expression can be simplified to:

$$Y_{RMS} = \sqrt{\frac{Q \cdot \mathrm{ASD_{input}}}{32\pi^3 f_n^3}}$$

Two insulated modes

$$f_0 = 55.74Hz / f_0 = 686.9Hz$$

$$ASD = 1.61 \times 10^{-8} g_N^2/Hz \quad Q = 20$$

$$\delta_{RMS} = 424.52[nm] / \delta_{RMS} = 9.812[nm]$$

External (seismic) acceleration levels observed typically inside stationary particle physics experiments are low 10^{-14} - 10^{-16} $[m^4/s^2/Hz]$ in [0.1 -100] Hz.

GW detectors' seismic noise