3D Parton Imaging with TMDs and Gluon Saturation at ePIC

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EIC at a Glance

Machine Snapshot

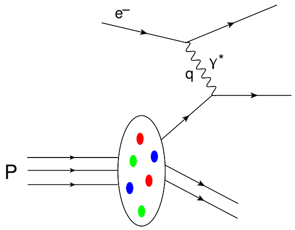
- **Polarised beams**
  e^- ($\sim$ 80%), $p^\uparrow$ ($\sim$ 70%)
- **Energy range**
  $\sqrt{s} = 20$–$140\,$GeV
- **Design luminosity**
  10^{33}–$10^{34}\,$cm^{-2}s^{-1}

Three Main Questions

- **Mass Emergence**
  How do gluon fields and confinement generate the proton's 1 GeV mass?
- **Spin Decomposition**
  Resolve $\Delta\Sigma$, ΔG, and orbital L_{q+g} with polarised DIS.
- **3-D Imaging**
  Map partons in momentum (TMD), position (GPD), and phase space; explore saturation at small x.

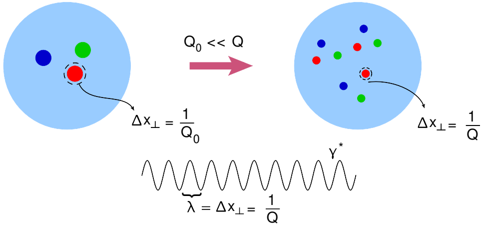
incoherent interaction in DIS: Bjorken limit

Bjorken limit: $-q^2 = Q^2 \to \infty,\ (P+q)^2 = s \to \infty \quad x_B = \frac{Q^2}{s+Q^2}$ fixed



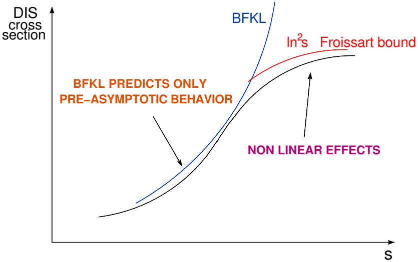
resum $\alpha_s \ln \frac{Q^2}{\Lambda^2_{QCD}} \simeq 1$

Scaling violation $\Rightarrow$ PDFs evolve with DGLAP



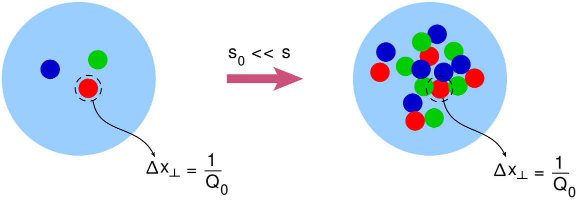
DIS cross section at Leading Log Approximation

BFKL: Leading Logarithmic Approximation $\alpha_s << 1 \qquad (\alpha_s \ln s)^n \sim 1$



- pQCD at LLA: $\sigma_{\rm tot} \propto s^{\Delta}$
- Parton density cannot rise forever
- Froissart bound: $\sigma_{\rm tot} \propto \ln^2 s$

Q_0 fixed $\quad (q+P)^2 = s \to \infty \quad \Rightarrow \quad x_B = \frac{Q^2}{s+Q^2} \to 0$ small-x_B

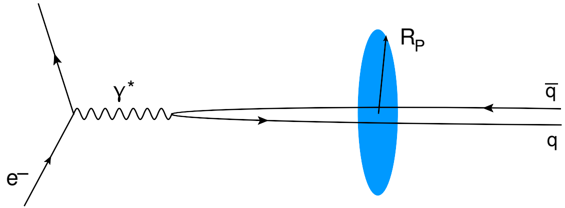


Dipole frame: coherent interactions

$$x_B = \frac{Q^2}{s} \to 0$$

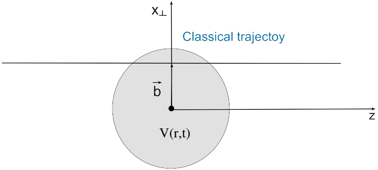
Formation time of the $q\bar{q}$ pair: $t_f \sim \frac{1}{\Delta E} \sim \frac{1}{M x_B}$

Compare with typical partons' interaction time: $t_{int} \sim R_P$



High-energy scattering in quantum mechanics and QCD

High-energy: $E >> V(x)$ WKB approximation.



$$\Psi_{free}(\vec{r},t) \to U(r_\perp)\Psi_{free}(\vec{r},t)$$

In QM $$U(r_\perp) = e^{\frac{-i}{v\hbar}\int_{-\infty}^{+\infty} dz' V(z'+r_\perp)}$$

In QCD $$U(x_\perp) = \mathrm{P}e^{\frac{-ig}{c\hbar}\int_{-\infty}^{+\infty} dt\, \dot{x}_\mu A^\mu(x(t))}$$

rotation in color space $U_{ij}(x_\perp)q_j^{free}(x)$ with $i,j = r,g,b$

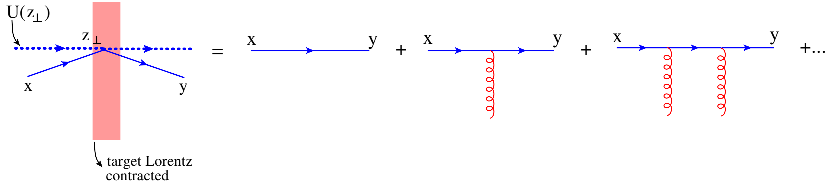
$$\mathrm{P}e^{\int_{-\infty}^{+\infty} dt A(t)} = 1 + \int_{-\infty}^{+\infty} dt\, A(t) + \int_{-\infty}^{+\infty} dt\, A(t)\int_{-\infty}^{t} dt'\, A(t') + \ldots$$

High-energy scattering in quantum mechanics and QCD

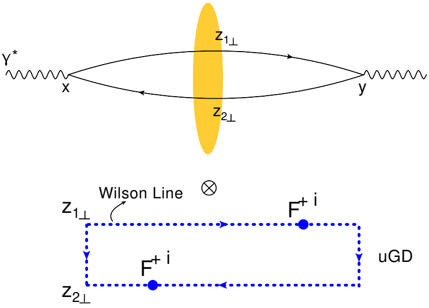
In QCD $\qquad U(x_\perp) = \mathrm{P}e^{\frac{-ig}{c\hbar}\int_{-\infty}^{+\infty} dt\ \dot{x}_\mu A^\mu(x(t))}$

rotation in color space $\qquad U_{ij}(x_\perp)q_j^{free}(x) \qquad$ with $i,j = r,g,b$

$$\mathrm{P}e^{\int_{-\infty}^{+\infty} dt A(t)} = 1 + \int_{-\infty}^{+\infty} dt\, A(t) + \int_{-\infty}^{+\infty} dt\, A(t) \int_{-\infty}^{t} dt'\, A(t') + \dots$$



DIS in the Shock-wave/Wilson lines formalism

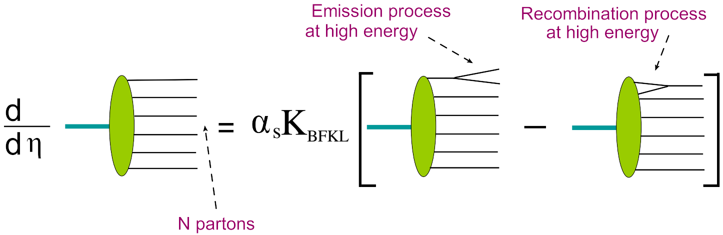


$$\frac{d\sigma^{DIS}}{d\eta} = I(k_\perp) \otimes G(k_\perp, \eta)$$

η rapidity

Factorization in terms of Unintegrated Gluon Distribution: $G(k_\perp, \eta)$

BK non-linear evolution equation



- At high-energy recombination process tames the emission process and unitarity is restored.

$$\frac{dN(x_B, k_\perp)}{\ln x_B} = \alpha_s K_{BFKL}\left[N(x_B, k_\perp) - [N(x_B, k_\perp)]^2\right]$$

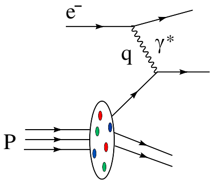
NLO BK   Balitsky and G.A.C. (2007)

NNLO BK exploiting conformal invariance   Balitsky and G.A.C. work in progress

Incoherent-vs-Coherent

- Do DGLAP equations describe high parton-density dynamics?
- DGLAP is evolution equation towards dilute regime.

Incoherent Interactions



Bjorken Limit

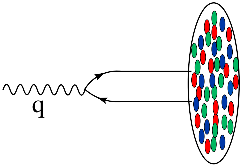
$$-q^2 = Q^2 \to \infty, \ (P+q)^2 = s \to \infty$$

$$x_B = \frac{Q^2}{s+Q^2} \ \text{fixed}$$

$$\text{resum} \ \ \alpha_s \ln \frac{Q^2}{\Lambda^2_{QCD}}$$

vs.

Coherent Interactions



Regge Limit

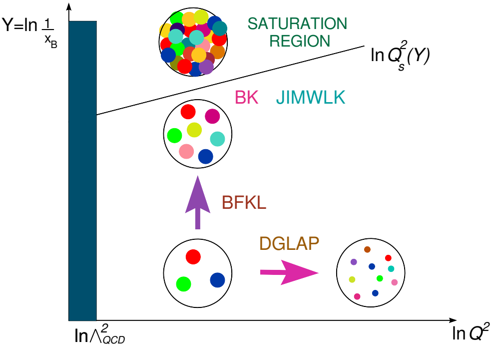
$$Q^2 \ \text{fixed}, \ s \to \infty$$

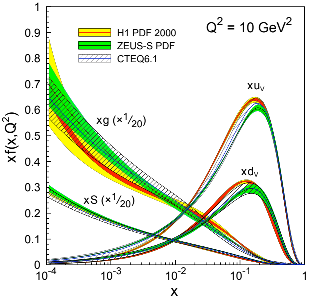
$$x_B = \frac{Q^2}{s} \to 0$$

$$\text{resum} \ \ \alpha_s \ln \frac{1}{x_B}$$

DGLAP vs. BFKL

$x_B \sim \frac{Q^2}{s}$, $\Delta x_\perp \sim \frac{1}{Q}$





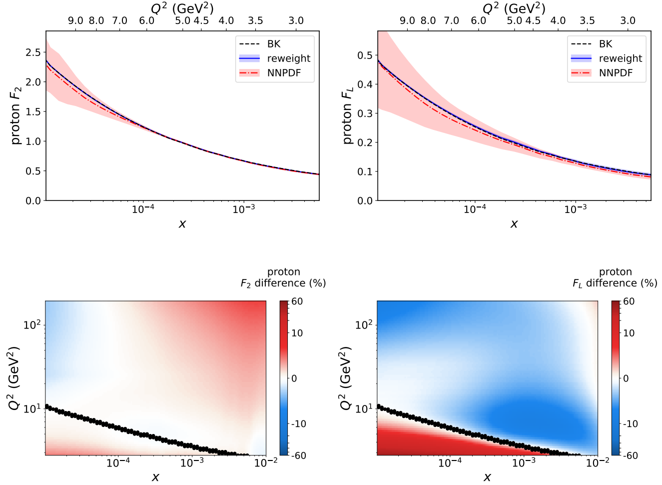
$Q_s^2 \sim \left(\frac{A}{x_B}\right)^{1/3}$

uGD: $F(x, k_\perp)$ depends independently on x and $k_\perp$.

Geometric scaling: $F(x, k_\perp) \sim \frac{1}{Q_s^2} f\left(\frac{k_\perp}{Q_s(x)}\right)$

proton's F_2 and F_L as a function of x at $Q^2 = 10Q_s^2(x)$

percent-level tension already visible; ePIC data $\times$ 20 stats

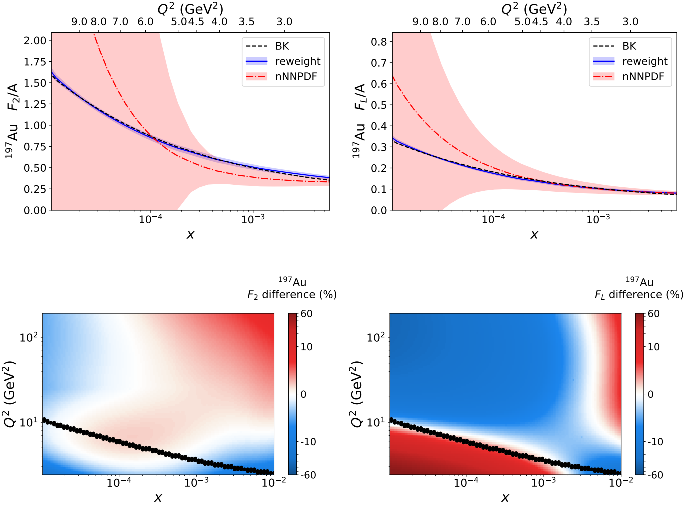


Relative difference $(F_{2,L}^{\mathrm{BK}} - F_{2,L}^{\mathrm{Rew}})/F_{2,L}^{\mathrm{BK}}$

Armesto et al. (2022)

^{197}Au F_2 and F_L as a function of x at $Q^2 = 10Q_s^2(x)$

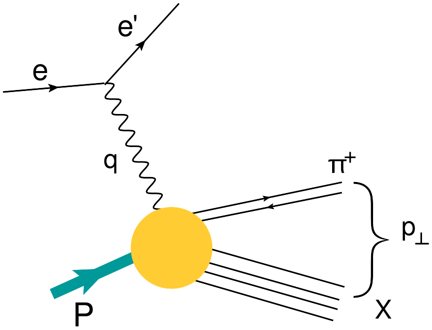
re-weighting is simply a way to make the initial conditions "fair"



Relative difference $(F_{2,L}^{\mathrm{BK}} - F_{2,L}^{\mathrm{Rew}})/F_{2,L}^{\mathrm{BK}}$

Armesto et al. (2022)

Semi-inclusive DIS (SIDIS)

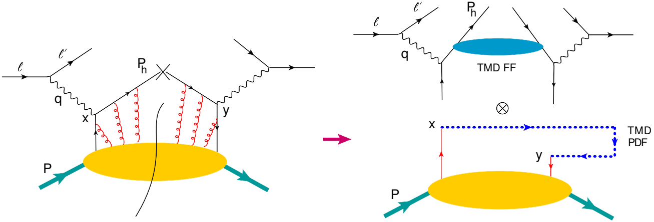


$-q^2 = Q^2 \gg p_\perp^2 \sim \Lambda_{\rm QCD}^2 \quad \Rightarrow$ TMD factorization

$Q^2 \gg p_\perp^2 \gg \Lambda_{\rm QCD}^2 \quad \Rightarrow$ Sudakov resummation

$Q^2 \sim p_\perp^2 \gg \Lambda_{\rm QCD}^2 \quad \Rightarrow$ Collinear factorization

Semi-inclusive DIS (SIDIS): incoherent interactions



- Cross-section is factorized in terms of Transverse Momentum Distributions

$$E' E_h \frac{d\sigma_{ep\to e'hX}}{d^3P_h d^3\ell'} = \hat{\sigma} \otimes f(x_B, k_\perp) \otimes D(z, q_\perp)$$

3D imaging of partons in momentum space

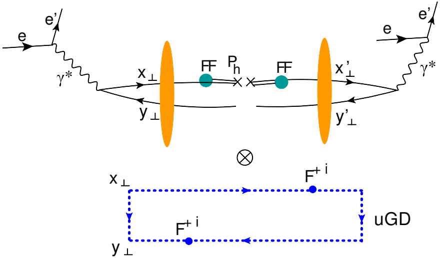
Evolution equations of TMDs

Different formalisms are available

- Collin-Soper-Sterman formalism
  - 2 couple evolution equations in rapidity and UV regulator
  - difficult to extend at small-x
- Soft-Collinear-Effective Theory
  - difficult to extend at small-x
- Rapidity-only evolution equation (à la small-x)
  - suitable to bridge small and moderate x_B regimes
  - Not a renrm. group equation $\Rightarrow$ needs running coup. corrections
  - running coup. corrections in Sudakov approximation I. Balitsky and G. A. C. (2022)

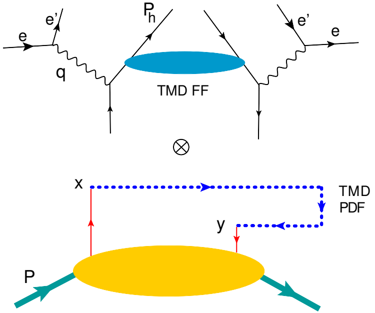
SIDIS coherent vs incoherent interactions

coherent interactions



vs

incoherent interactions



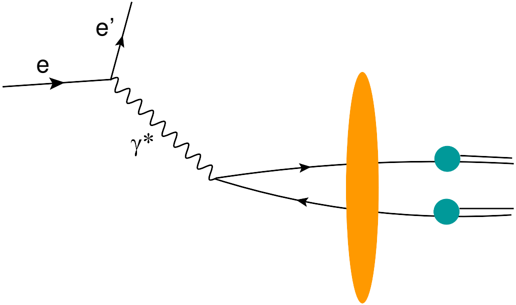
how to bridge the TMD and the uGD formalism?

Would sub-eikonal corrections help?

Need dictionary between higher-twist and sub-eikonal corrections

Back-to-back di-hadron suppression in SIDIS

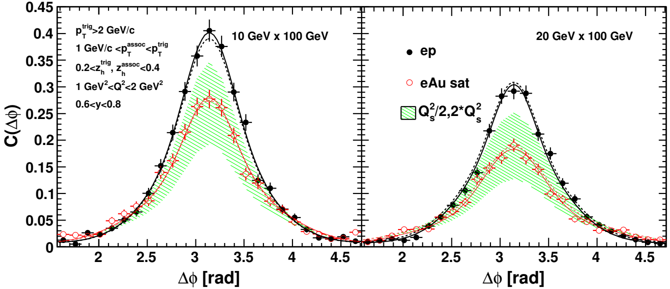
Di-hadron production: Weiszäcker Williams distribution



$$C(\Delta\phi) = \frac{1}{N^{\rm trig}}\frac{N^{\rm pair}}{d\Delta\phi} = \frac{1}{\frac{d\sigma_{\rm SIDIS}^{\gamma^*+A\to h_1+X}}{dz_{h1}}}\frac{d\sigma_{\rm tot}^{\gamma^*+A\to h_1+h_2+X}}{dz_{h1}dz_{h2}d\Delta\phi}$$

Back-to-back di-hadron suppression in SIDIS

Di-hadron production: Weiszäcker Williams distribution



Lower peak
⇒ $k_\perp$ broadening
⇒ Saturation

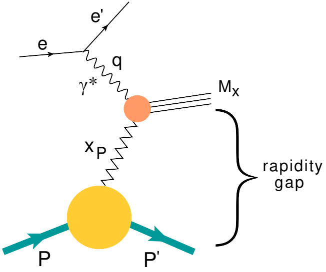
$$C(\Delta\phi) = \frac{1}{N^{\rm trig}}\frac{N^{\rm pair}}{d\Delta\phi} = \frac{1}{\frac{d\sigma_{\rm SIDIS}^{\gamma^*+A\to h_1+X}}{dz_{h1}}}\frac{d\sigma_{\rm tot}^{\gamma^*+A\to h_1+h_2+X}}{dz_{h1}dz_{h2}d\Delta\phi}$$

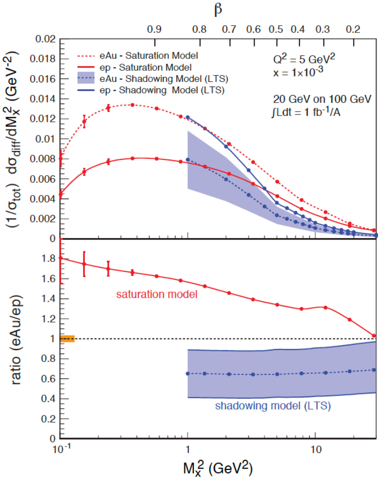
$\frac{dN^{\rm pair}}{d\Delta\phi}$: distribution of particle pairs
$N^{\rm trig}$: the number of trigger particles

Zheng, Aschenauer, Lee, Xiao (2014)

10% of HERA ($0.5\,\mathrm{fb}^{-1}$) events
ePIC ($10\,\mathrm{fb}^{-1}$): $\times 20$

$$\beta = Q^2/(Q^2 + M_X^2)$$





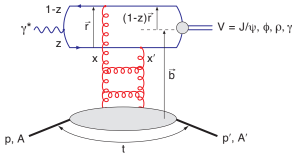
Exchange vacuum quantum number
Sensitive to saturation phenomena

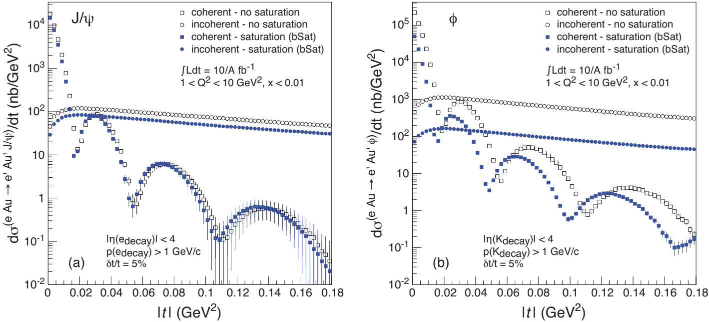
Discriminate between saturation models and Leading-twist shadowing

Coherent j/ψ production in the dipole model

Probing the spatial distribution:

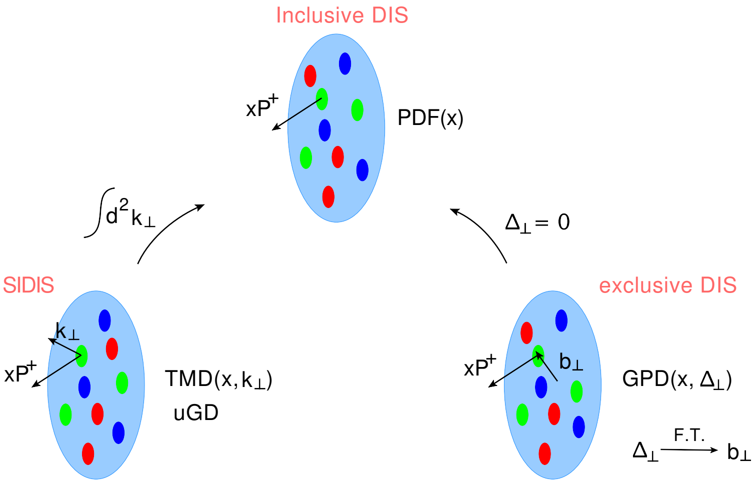
$|t| = \Delta_\perp^2 \qquad \Delta_\perp \overset{F.T.}{\leftrightarrow} b_\perp$





from Toll, Ullrich arXiv-1211-3048

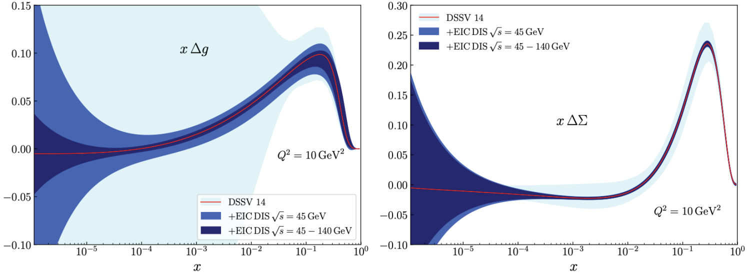
3D parton imaging



pseudo data on the gluon helicity and quark singlet helicity

Yellow Report 2103.05419 [hep-ph]

gluon helicity (left panel) and quark singlet helicity (right panel) distributions as a function of x for $Q^2 = 10\ GeV^2$. Large uncertainty at small-x_B



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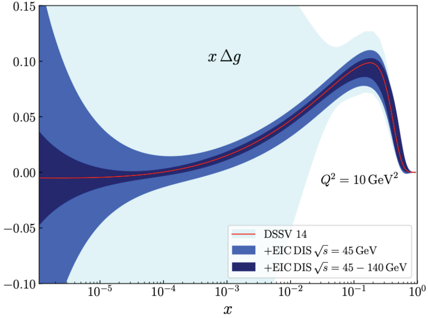
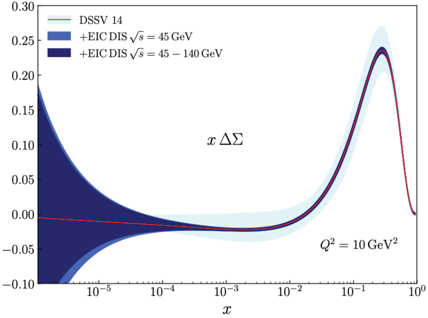
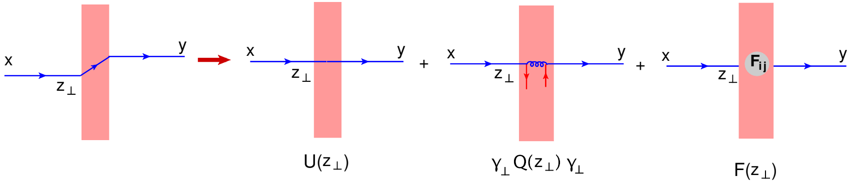
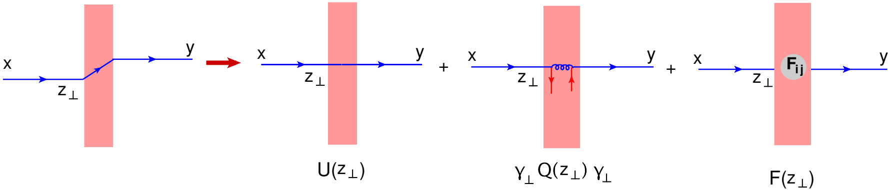




small-x_B: Eikonal + sub-eikonal interactions



Quark (and gluon) propagator with sub-eikonal corrections



Quark propagator for g_1 structure function

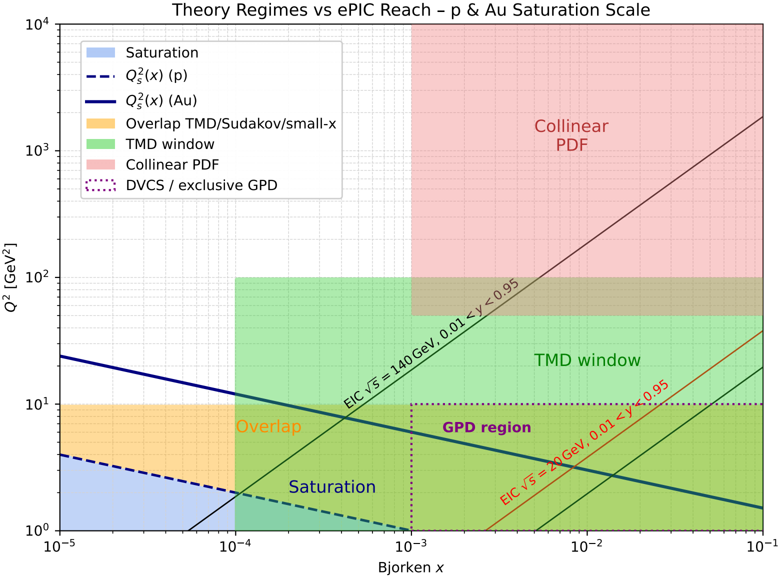
$$\langle \mathrm{T}\{\psi(x)\bar{\psi}(y)\}\rangle_{A,\psi,\bar{\psi}} = -\frac{2}{\pi^3 s^2 x^{+2} y^{2+}} \int \frac{d^2z}{(\mathcal{Z}+i\epsilon)^3}\left(\sqrt{\frac{2}{s}}x^+ \not{p}_1 + \not{X}_\perp\right)$$

$$\times \left\{ i\not{p}_2 U(z_\perp) + \frac{\mathcal{Z}}{8}\left(\gamma^\mu_\perp Q(z_\perp)\gamma^\perp_\mu + \frac{1}{s}\not{p}_2\gamma^5\mathcal{F}(z_\perp)\right)\right\}\left(\sqrt{\frac{2}{s}}y^+\not{p}_1 + \not{Y}_\perp\right)$$

- Eikonal interaction
- Sub-eikonal interaction G.A.C (2019-2021)
- work done in this direction: Kovchegov et al (2017-2025); Altinoluk, Beuf; Armesto et al (2014-2024)

- **Gluon saturation** – forward dijets in $ep(eA)$; measure $C(\Delta\phi)$ ($\sim 10^7$ events)
- **Small-x BK/JIMWLK** – coherent J/ψ in e+Au; t–slope $\Rightarrow$ ($\sim 10^6$)
- **TMD Sivers/Collins** – polarised SIDIS $ep^{\uparrow} \to e\,h\,X$; asymmetry $A_{UT}^{\sin(\phi_h-\phi_S)}$ ($\sim 10^8$)
- **3-D imaging (GPD)**– DVCS $ep \to ep\gamma$; $d\sigma/dt \Rightarrow$ transverse density ($\sim 10^6$)
- **Proton spin at small x** – inclusive $g_1(x, Q^2)$ in polarised ep; extract $\Delta g + \Delta\Sigma$ ($\sim 10^7$)

3D parton imaging



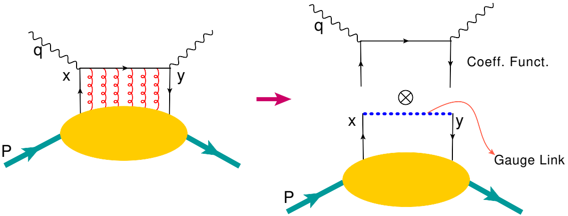
Conclusions

- EIC represents an opportunity to understand the origin of the main properties of the proton: its mass and its spin;
- Evolution equations at higher perturbative order are important to make reliable prediction for the experiment;
  - BK/JIMWLK non-linear evolution equation are relevant at small-x_B/high-parton density.
- At EIC we have a chance to observe parton saturation;
- The challenge is to develop a bridging formalism which can smoothly interpolate the Bjorken region and the Regge region;
  - To this end the sub-eikonal corrections represent a good starting point;
- Spin of the proton at small-x_B can be explored for the first time at the EIC;
  - Sub-eikonal corrections play a central role.

Back-up Slides

non-local Operator Product Expansion

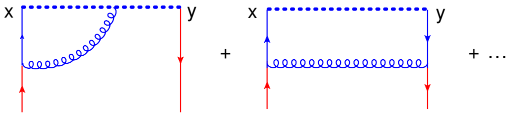
Balitsky-Braun (1988)



$$T\{j_\mu(x)j_\nu(y)\} = C_\xi(x,y)\ \bar{\psi}(x)\gamma_\mu\gamma^\xi\gamma_\nu[x,y]\psi(y) + O((x-y)^2)$$

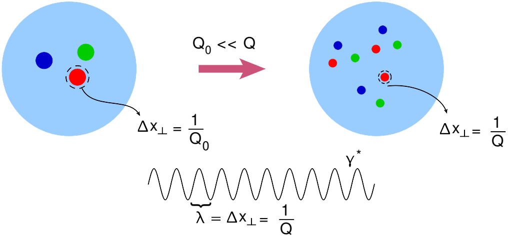
- $C_\xi(x,y)$ is the coefficient function calculable in pQCD
- $\bar{\psi}(x)\gamma_\mu\gamma^\xi\gamma_\nu[x,y]\psi(y)$ is the non local operator

DGLAP evolution equation



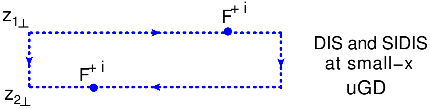
Renorm-group equation for light-ray operators: DGLAP evolution equation

$$\mu^2 \frac{d}{d\mu^2} \bar{\psi}(x)[x,y]\psi(y) \;=\; K_{\rm LO} \bar{\psi}(x)[x,y]\psi(y) + \dots$$

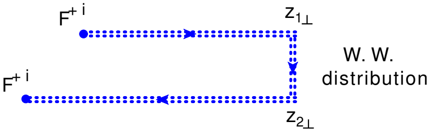


Gluon distributions at small-x

Dipole unintegrated gluon distribution: inclusive and single-inclusive DIS

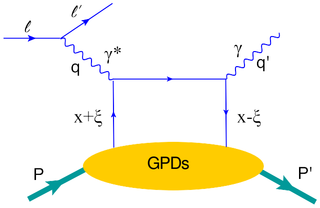


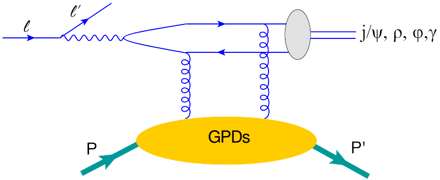
Weizsäcker Williams gluon distribution: di-hadron production in SIDIS



WW-distribution has a partonic interpretation in the light-cone gauge

Information on the spacial distribution of partons inside hadrons





$(p-p')^2 = t = -\Delta_\perp^2$ q-GPD defined through off-forward matrix elements of non-local operators

$$F^q = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p'|\bar{q}(-z/2)\gamma^+ q(z/2)|P\rangle\Big|_{z^+,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

$P = \frac{1}{2}(p+p')$, $\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$

D. Müller (1994), A. Radyushkin (1996), X. Ji (1997)