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Bubble wall dynamics at the EW phase transition

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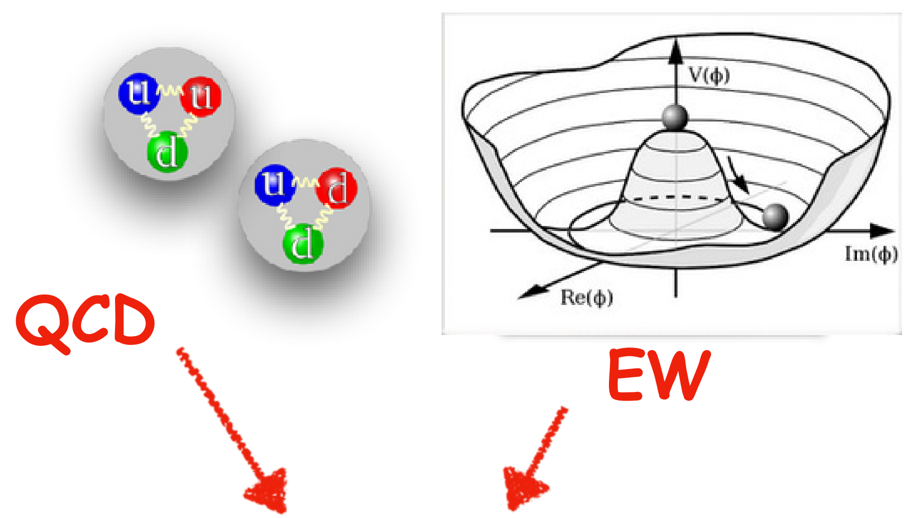
Progetto PRIN 2022 PNRR "SOPHYA"
Progetto PRIN 2022 "Bubble Dynamics in Cosmological Phase Transitions"

In collaboration with L. Delle Rose, C. Branchina, S. De Curtis arXiv:2504.21213

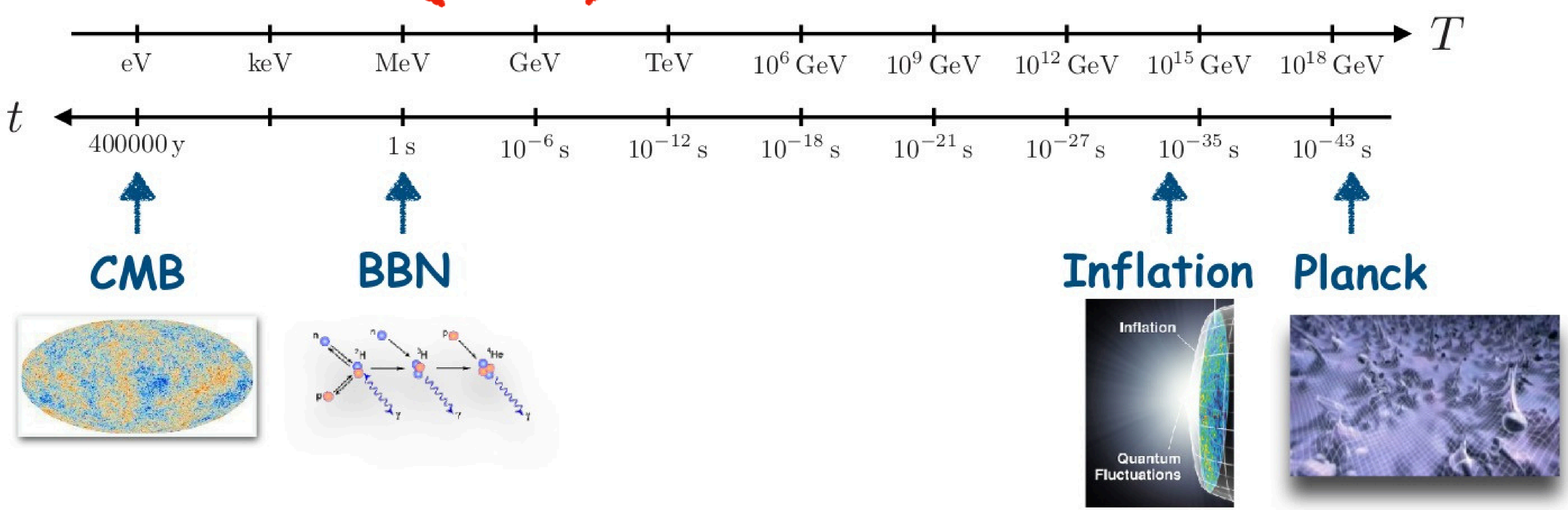
Thermal History of the Universe

Phase transitions are important events in the evolution of the Universe

- the SM predicts two of them (crossover)



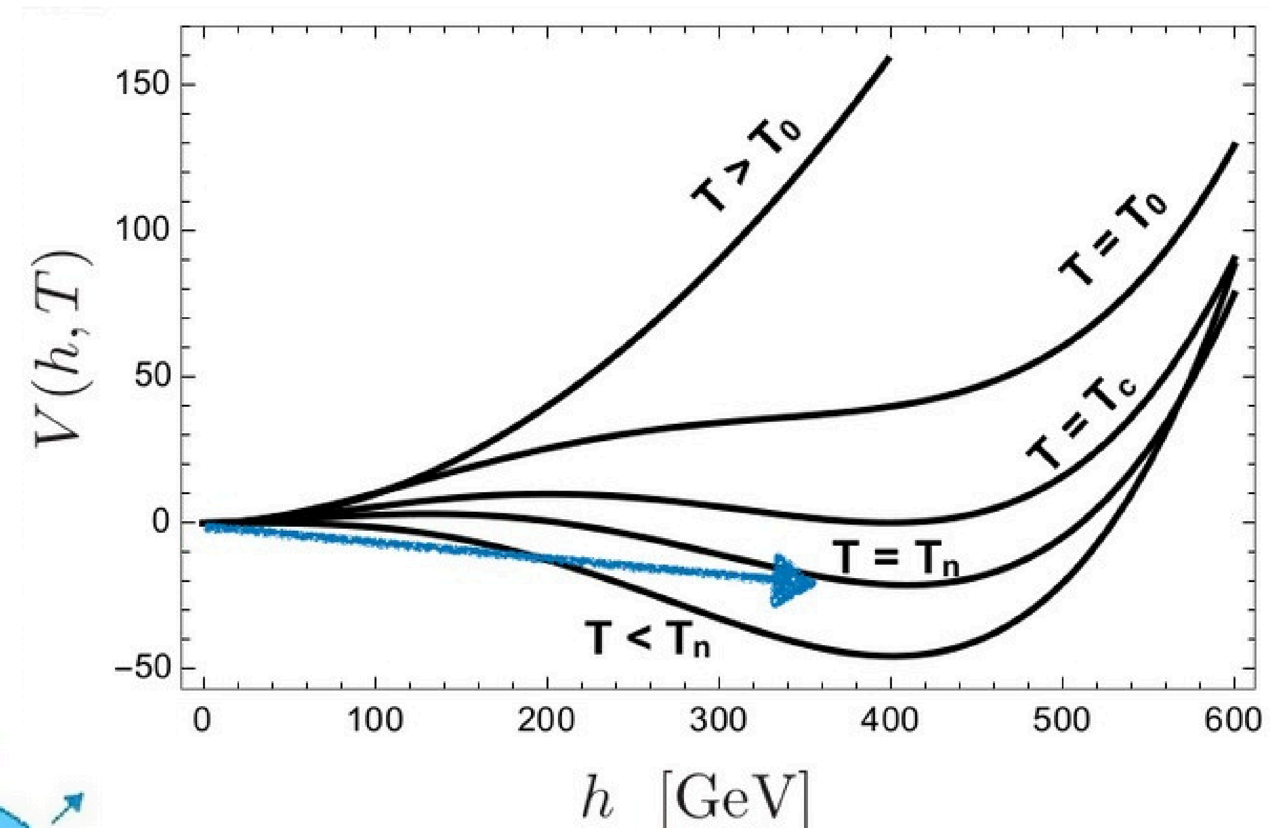
no strong breaking of thermal equilibrium
no distinctive experimental signatures

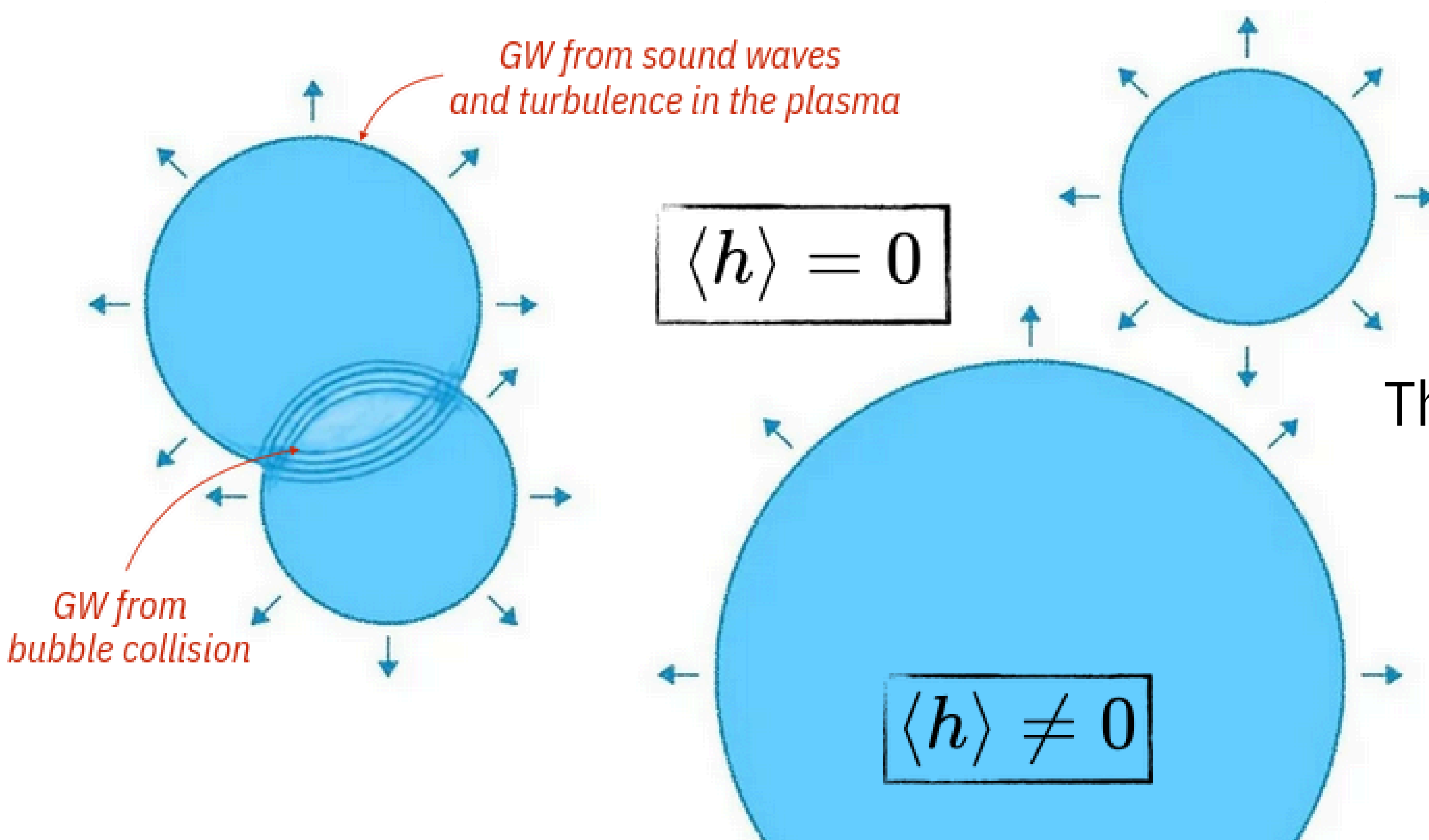


EWPhT and new physics

New physics may provide **first order** phase transitions

- h field **tunnels** from false to true minimum at $T_n < T_c$
- transition proceeds through **bubble nucleation**





The bubble wall in the plasma can produce

1. **breaking of thermal equilibrium** (BG)
2. **experimental signatures** (GW)

Toy example: SSM tree-level potential

Higgs + singlet scalar potential (Z2 symmetric)
in the high-temperature limit

$$V(h,s,T) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_s^2}{2}s^2 + \frac{\lambda_s}{4}s^4 + \frac{\lambda_{hs}}{4}h^2s^2 + \left(c_h\frac{h^2}{2} + c_s\frac{s^2}{2}\right)T^2$$

important to create a barrier in the potential

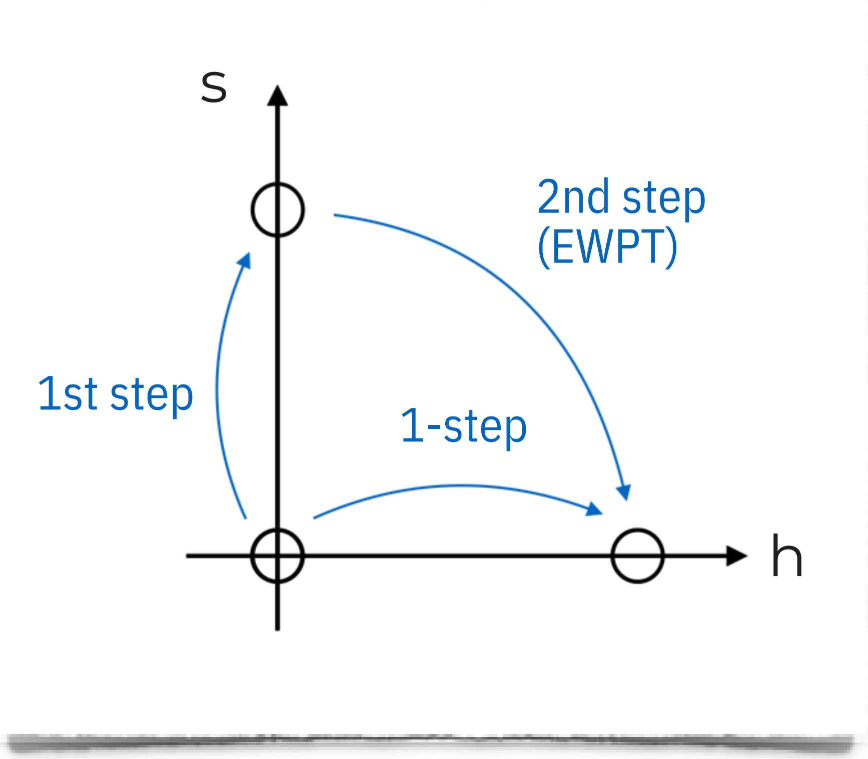
with thermal masses

$$c_h = \frac{1}{48}(9g^2 + 3g'^2 + 12y_t^2 + 24\lambda_h + \lambda_{hs}) \qquad c_s = \frac{1}{12}(2\lambda_{hs} + \lambda_s)$$

- Two interesting patterns of symmetry breaking (as the Universe cools down)

1-step PhT $(0,0) \to (v,0)$

2-step PhT $(0,0) \to (0,w) \to (v,0)$



- 2-step naturally realised since singlet is destabilised before the Higgs

Dynamics of the bubble wall

Plasma conveniently described as a mixture of **three components**

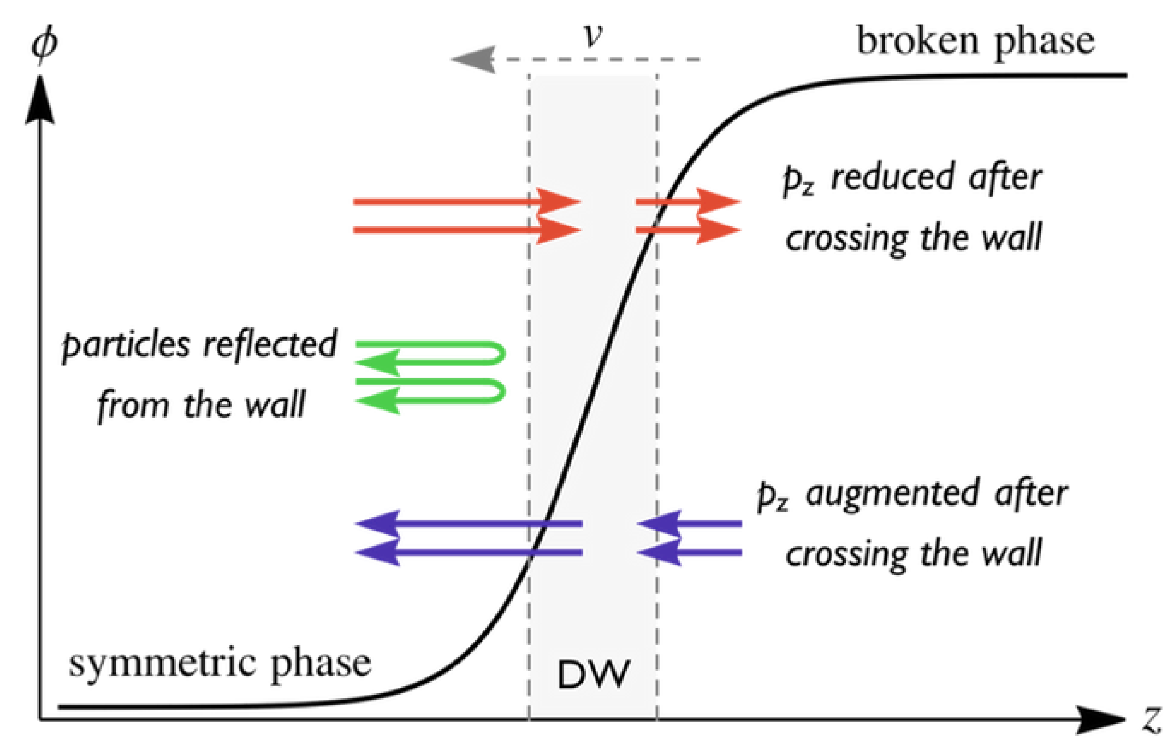
1. Scalar fields participating in the transition
2. Background species: ~ LTE
3. Species directly coupled to the scalars: OOE contributions relevant

- Scalar field EOMs

$$\phi' \Box \phi - V_T' = \sum N_i \frac{dm^2}{dz} \int \frac{d^3p}{(2\pi)^3 2E_p} \delta f(p)$$

- Boltzmann equation for OOE

$$\left(\frac{p_z}{E} \partial_z - \frac{(m^2)'}{2E} \partial_{p_z} \right) (f_v + \delta f) = -C[f_v + \delta f]$$



- We assume a planar wall and a steady state regime

Hydrodynamics of the plasma in LTE

Equations for the plasma are obtained from the **conservation of the stress-energy tensor**

$$T^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{eq} + \cancel{T^{\mu\nu}_{out}}$$

scalar field — plasma at equilibrium — plasma out of equilibrium

$$\partial_z T^{zz} = \partial_z T^{z0} = 0$$

Hydrodynamic equations provide solutions to the **temperature and velocity profiles**

$$T^{30} = w\gamma^2 v = c_1$$

$$T^{33} = \frac{(\partial_z \phi)^2}{2} - V(\phi, T) + w\gamma^2 v^2 = c_2$$

The constants are determined from the hydrodynamic regimes

Hydrodynamic regimes

The boundary conditions $T_\pm, v_{p\pm}$ depend on the speed of the wall

Detonation $(v_w > v_J)$

– the wall hits an unperturbed plasma in front of it

$(T_+ = T_n, \;\; v_{p+} = v_w)$

– rarefaction wave behind the wall

Deflagration $(v_w < c_s^-)$

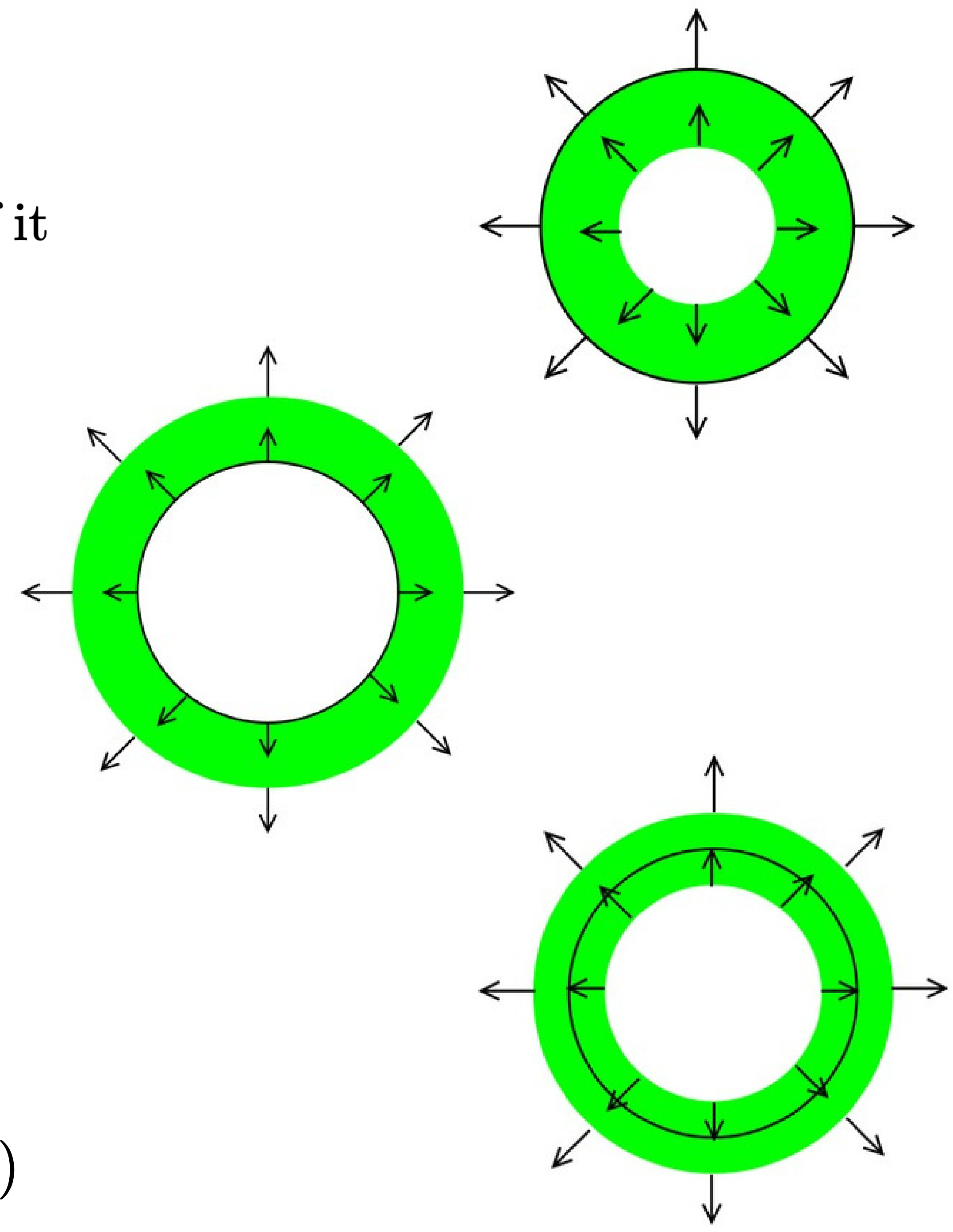
– plasma at rest behind the wall$(v_{p-} = v_w)$

– shock wave precedes the wall$(T_+^{SW} = T_n)$

Hybrid $(c_s^- < v_w < v_J)$

– shock wave precedes the wall $(T_+^{SW} = T_n)$

–rarefaction wave behind the wall $(v_{p-} = c_s^-)$



Espinosa, Konstandin, No, Servant, 2010

Determination of the wall speed

- we solve the EOM of the scalar fields

$$E_h = -\partial_z^2 h + \frac{\partial V}{\partial h} + \frac{F(z)}{h'} = 0$$
$$E_s = -\partial_z^2 s + \frac{\partial V}{\partial s} = 0$$

approx solutions $\longrightarrow$

$$h(z) = \frac{h_-}{2}\left(1 + \tanh\left(\frac{z}{L_h}\right)\right)$$
$$s(z) = \frac{s_+}{2}\left(1 + \tanh\left(\frac{z}{L_s} + \delta s\right)\right)$$

four parameters to be determined: v_w, L_h, L_s, δ_s

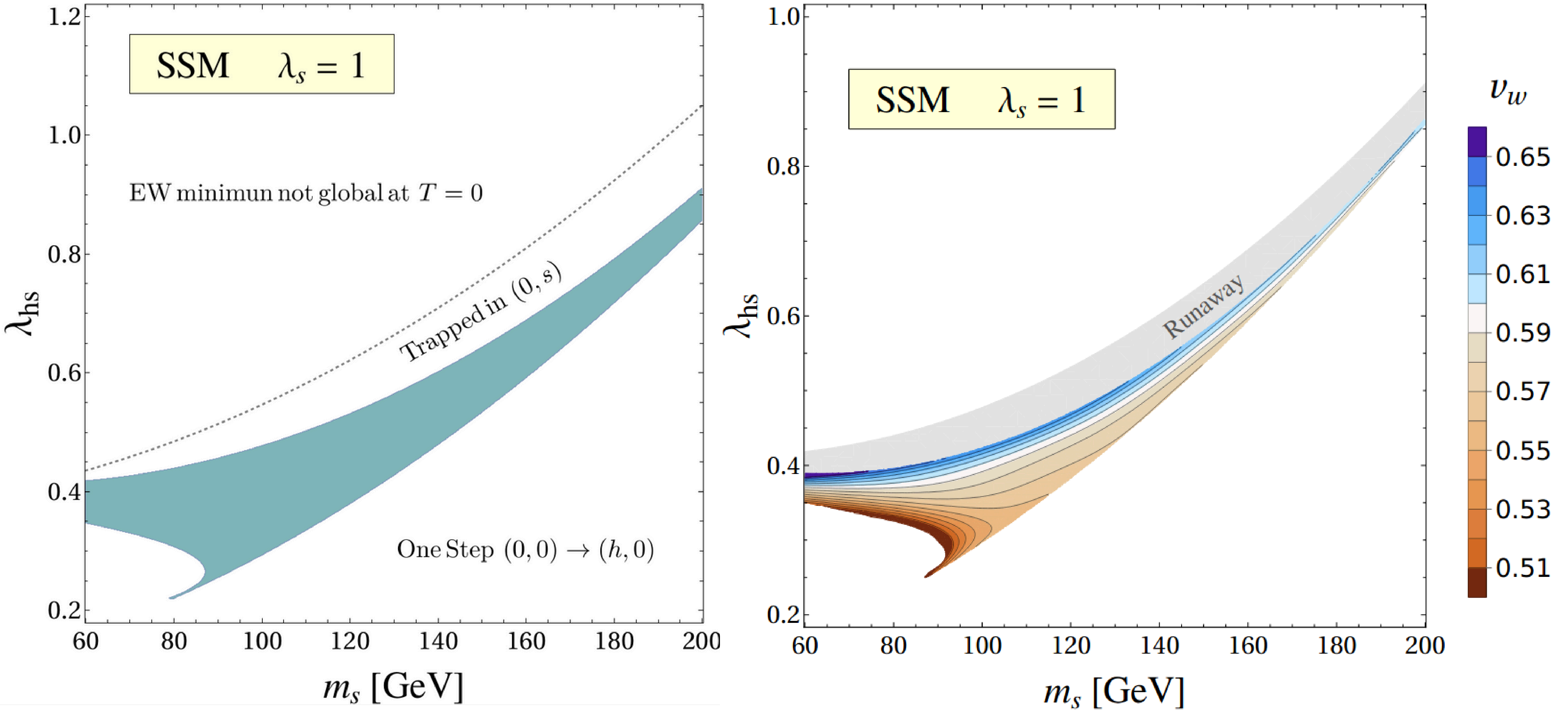
- the four parameters are determined by taking momenta of the EOMs

$$P_h = \int dz E_h h' = 0, \quad G_h = \int dz E_h \left(\frac{2h}{h_-}\right) h' = 0,$$
$$P_s = \int dz E_s s' = 0, \quad G_s = \int dz E_s \left(\frac{2s}{s_+}\right) s' = 0.$$

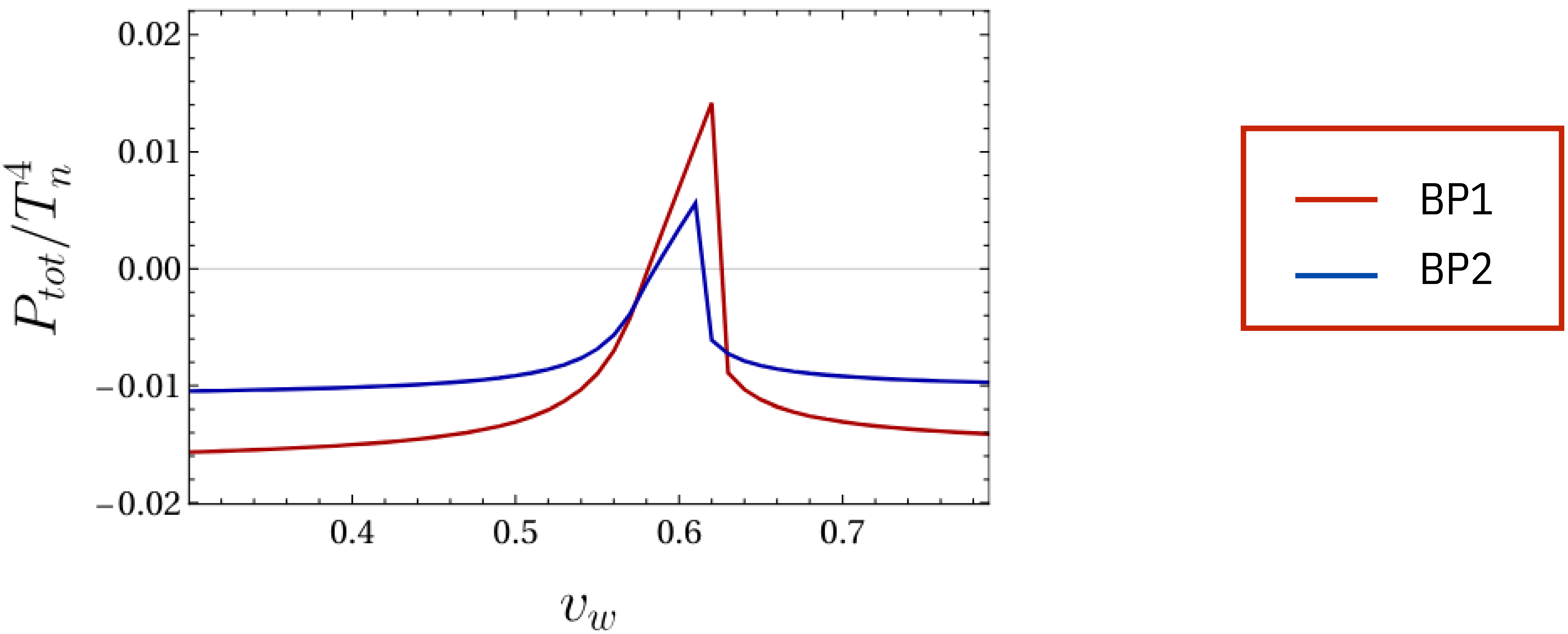
Results

We consider three models: singlet extension of the SM (SSM), triplet extension of the SM (RTSM), inert 2HDM (IDM).

SSM, $\lambda_s = 1$

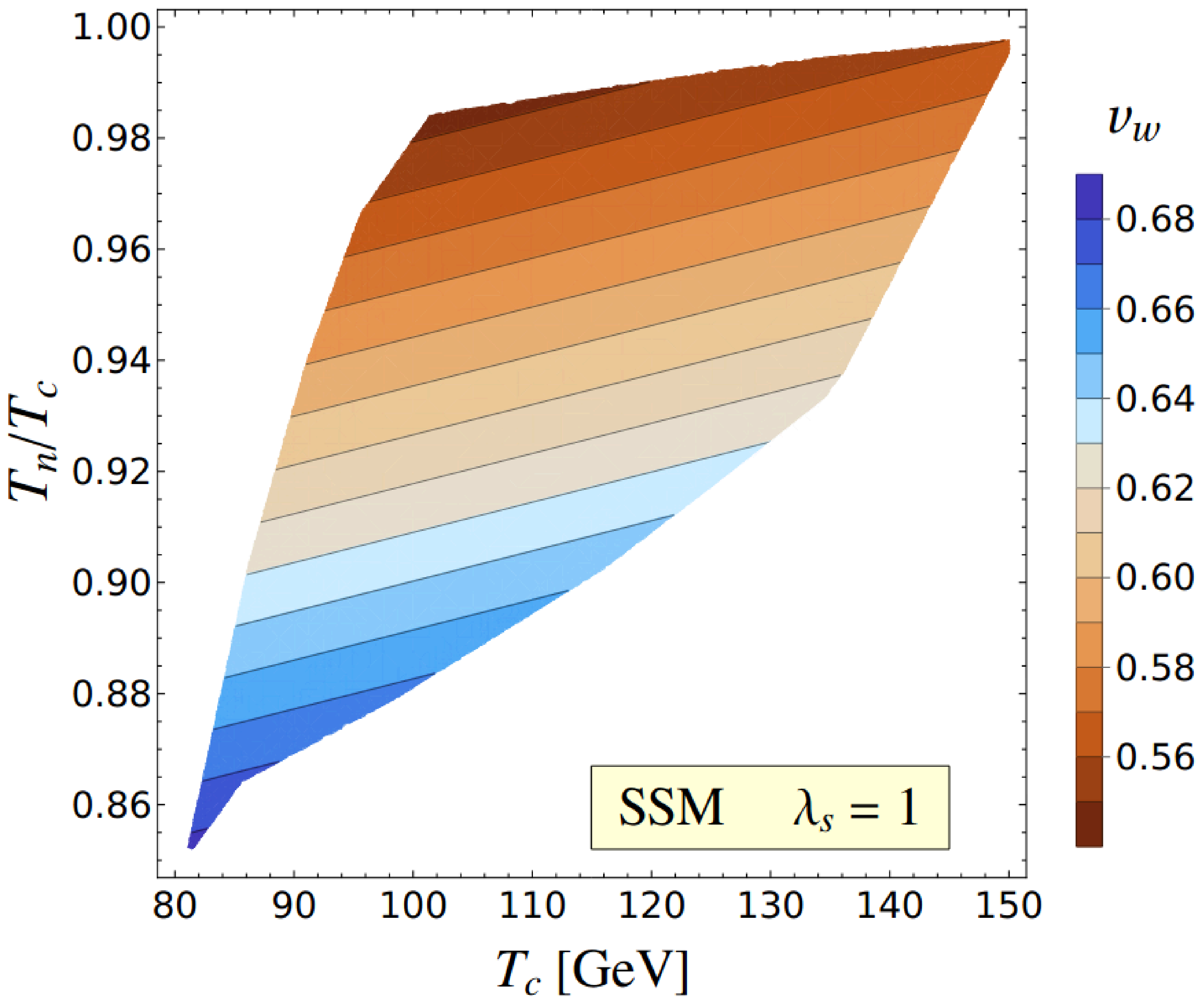


Constraints: Ptot= Ph + Ps



- The region on the peak, in the assumption used, is not accurately described
- Second zero of Ptot **NOT** a detonation

Results in terms of PhT parameters



v_w turns out to depend (~) **linearly** on $T_c, T_n/Tc$

- We extrapolated linear fits and verified that they are model-independent
- Particularly useful fits for calculating GW spectra

Conclusions and outlook

Conclusions:

- First order EWPT: theoretically and experimentally compelling
- Strategy put forward to provide full solution of the (steady-state) wall dynamics
- In LTE: complete solution in the parameter space of BSM models

- Only deflagration solutions

- Linear fit of $v_w(T_c, T_n/T_c)$

- Weak model dependence

Future perspectives:

- Inclusion of the out-of-equilibrium, already studied in *arXiv:2201.08220* *arXiv:2303.05846* *arXiv:2401.13522*

Back up slides

Key features of a first-order PhT

- the nucleation temperature T_n
- the strength α
- the (inverse) time duration of the transition β/H

} *equilibrium quantities*

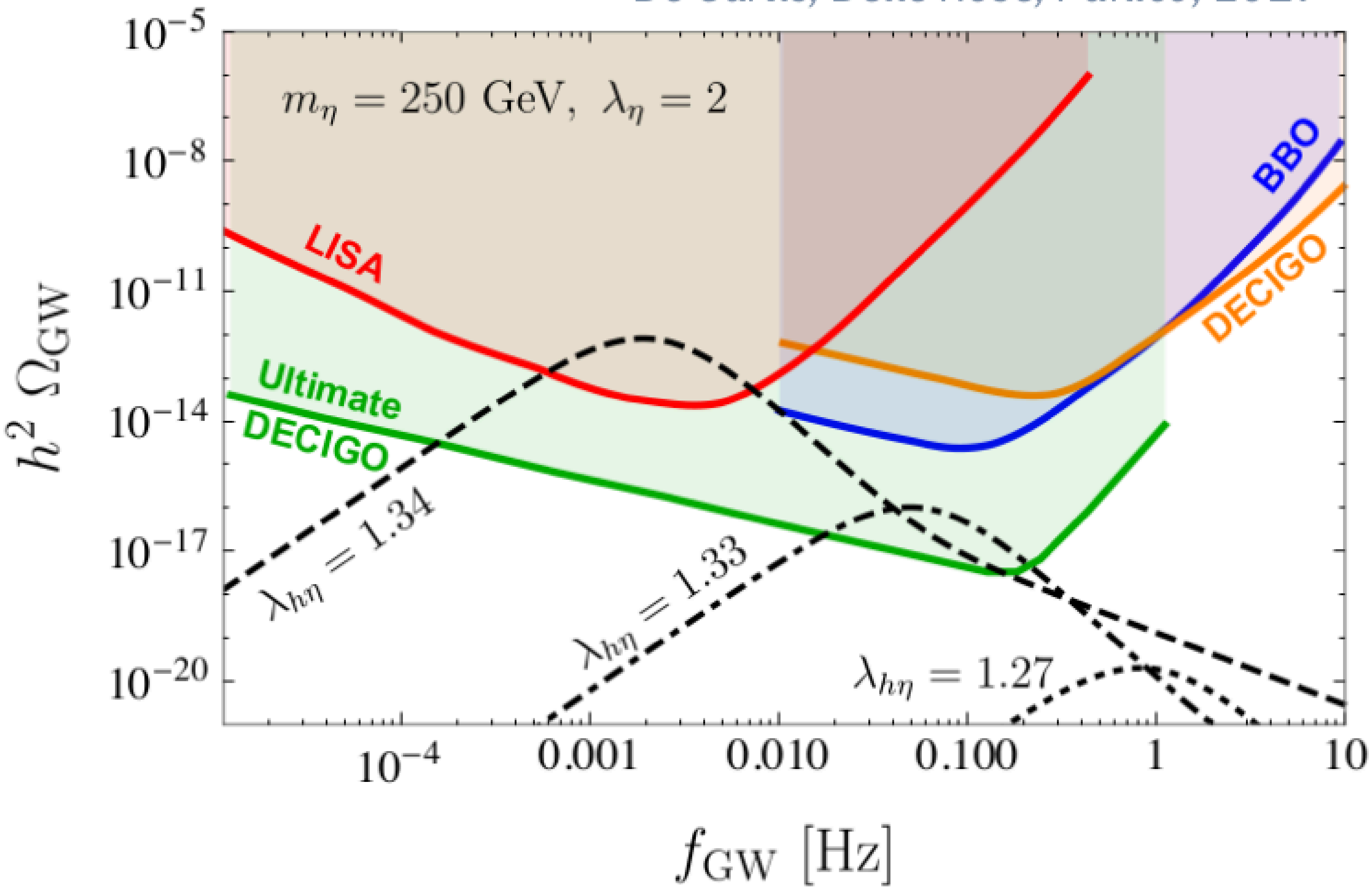
- the speed of the bubble wall v_w
- the thickness of the bubble wall L_w

} *non-equilibrium quantities*

GW from a first-order PhT

First-order PhTs produce stochastic background of gravitational waves

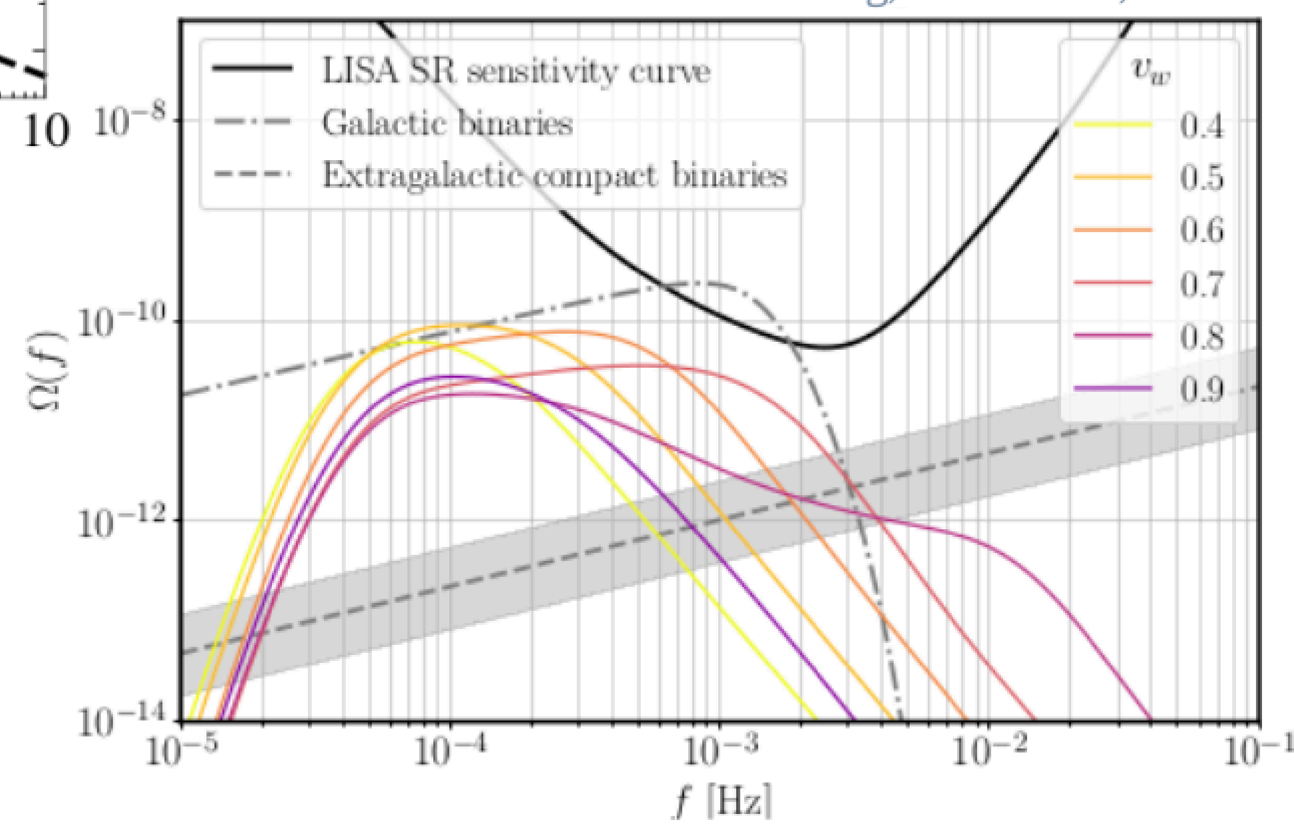
De Curtis, Delle Rose, Panico, 2019



for the EWPhT the peak frequency is within the range of future experiments

- wall speed has a strong effect on the shape of the power spectrum
- wall speed will be the best determined parameter

Gowling, Hindmarsh, 2019



The Boltzmann equation

$$\left(\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right)(f_v + \delta f) = -C[f_v + \delta f]$$

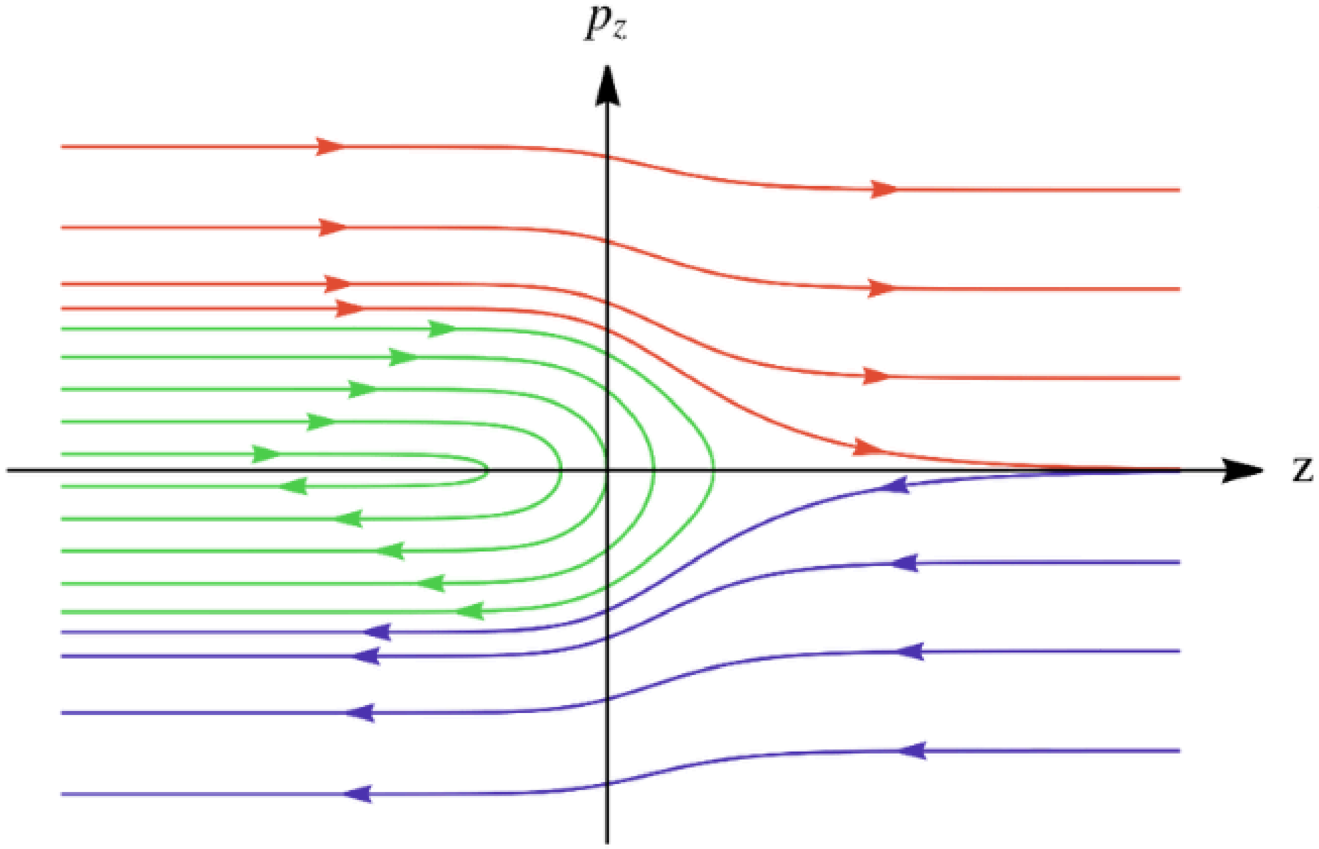
Assumptions on the plasma:

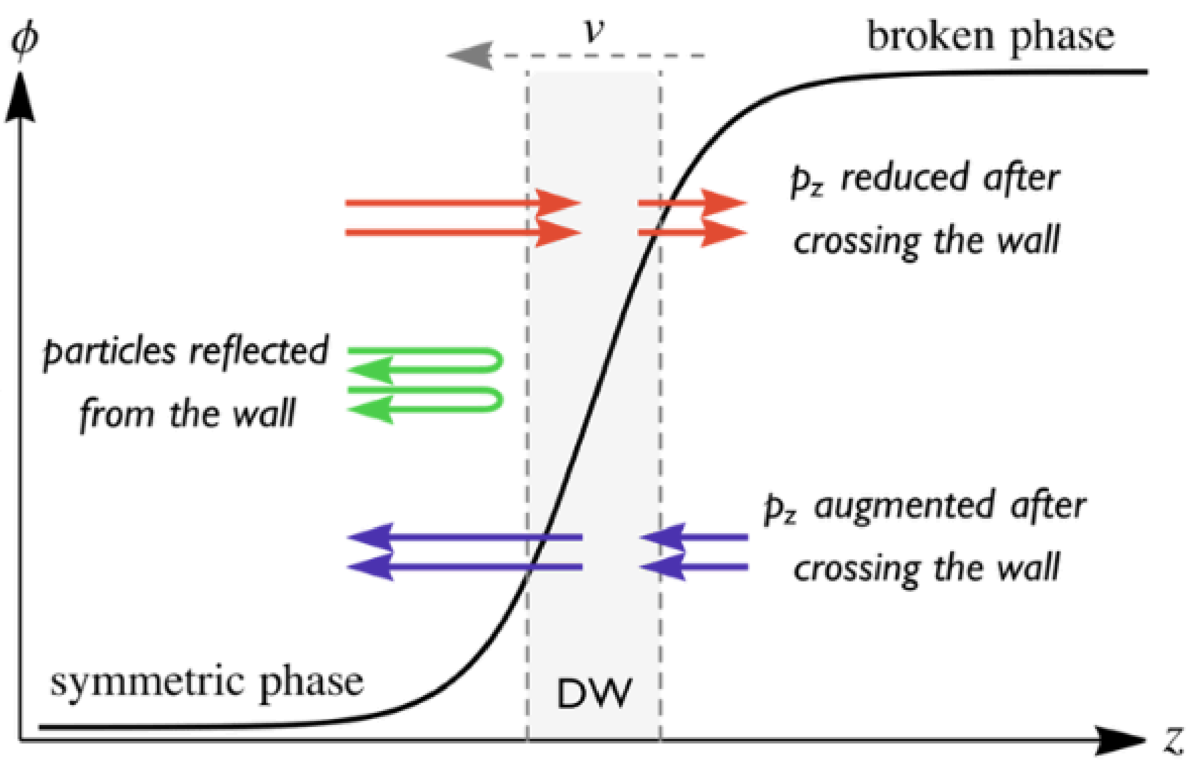
- High temperature, weakly coupled plasma
- Only 2→2 processes in the plasma are considered (*assumption valid for the computation of the collision integral*)
- Plasma made of two different kind of species
  - Top quark and W/Z bosons (main contributions)
  - All the other SM particles (background, assumed to be in local equilibrium)

The Liouville operator

Liouville operator is a derivative along flow paths

$$\mathcal{L}[f] = \left(\frac{p_z}{E}\partial_z - \frac{(m_z^2)'}{2E}\partial_{p_z}\right)f \longrightarrow \frac{p_z}{E}\frac{df}{dz}$$





$E,\ p_\perp$ and $c = \sqrt{p_z^2 + m_z^2}$ are conserved along the flow paths

The Collision term

The collision term is the challenging part of the Boltzmann equation

$$C[f_v + \delta f] = \frac{1}{4N_i E_i} \sum_j \int \frac{d^3k\, d^3p'\, d^3k'}{(2\pi)^5 2E_k 2E_{p'} E_{k'}} |\mathcal{M}_j|^2 \mathcal{P}[f_v + \delta f] \delta^4(p + k - p' - k')$$

for $2 \leftrightarrow 2$ *processes*

Boltzmann equation is an **integro-differential** equation

Typical setup:

- friction contributions from the top quark and W/Z bosons
- background is not perturbed
- infrared divergences regularised by thermal masses
- only leading-log terms are considered

Examples of processes:

process	$\lvert\mathcal{M}\rvert^2$
$t\bar{t} \to gg$	$\frac{128}{3} g_s^4 \left[\frac{ut}{(t - m_q^2)^2} + \frac{ut}{(u - m_q^2)^2}\right]$
$tg \to tg$	$-\frac{128}{3} g_s^4 \frac{su}{(u - m_q^2)^2} + 96 g_s^4 \frac{s^2 + u^2}{(t - m_g^2)^2}$
$tq \to tq$	$160 g_s^4 \frac{s^2 + u^2}{(t - m_g^2)^2}$

Structure of the collision integral

The linearised collision integral

$$\bar{\mathcal{C}}[\delta f_i] = \frac{1}{2N_i E_i}\sum_j \int \frac{d^3k\, d^3p'\, d^3k'}{(2\pi)^5\, 2E_k 2E_{p'} 2E_{k'}} |\mathcal{M}_j|^2 \bar{\mathcal{P}}[f]\delta^4(p+k-p'-k')$$

the population factor

$$\bar{\mathcal{P}}[f] = f_v(p) f_v(k)(1 \pm f_v(p'))(1 \pm f_v(k')) \sum \mp \frac{\delta f}{f_v'}$$

the collision integral yields two classes of terms:

$$\bar{\mathcal{C}}[\delta f] = \mathcal{Q}\frac{\delta f}{f_v'(p)} + (\langle \delta f(k)\rangle - \langle \delta f(p')\rangle - \langle \delta f(k')\rangle)$$

- the perturbation does not appear inside the integral: easy to handle

- perturbation is integrated (*bracket*): very challenging

Full solution to the Boltzmann equation

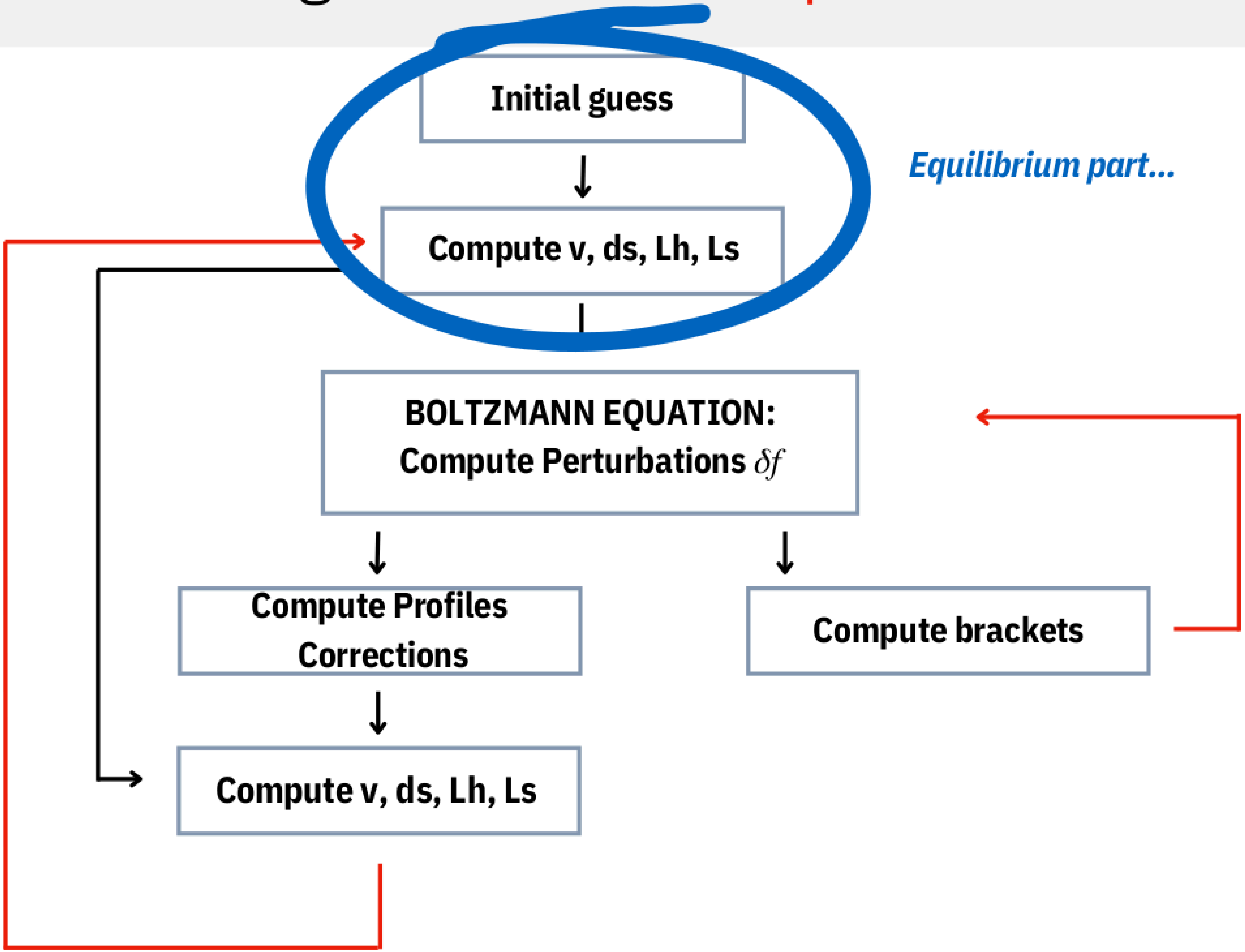
We propose a new method to solve the Boltzmann equation without imposing any ansatz for δf

De Curtis, Delle Rose, Guiggiani, Gil Muyor, Panico, 2022

Key features

- No term in the Boltzmann equation is neglected
- New approach to deal with collision integrals
- Iterative routine where convergence is achieved in few steps

Work flow algorithm: iterative procedure



Spectral decomposition of the collision integral

Structure of the collision integral: *the bracket*

$$\langle \delta f \rangle = -\frac{f_v(p/\beta(z))}{\beta(z)E_p} \int |\bar{\mathbf{k}}| d|\bar{\mathbf{k}}| d\cos\theta_{\bar{k}} f_0(|\bar{\mathbf{k}}|) \tilde{\mathcal{K}}(|\bar{\mathbf{p}}|, \cos\theta_{\bar{p}}, |\bar{\mathbf{k}}|, \cos\theta_{\bar{k}}) \frac{\delta f(k_\perp/\beta(z), k_z/\beta(z), z)}{f_0'(|\bar{\mathbf{k}}|)}$$

the bracket can be seen as the an application of a hermitian operator on the perturbations

$$\mathcal{O}[g] \equiv \int \mathcal{D}\bar{k} \tilde{\mathcal{K}}_{\bar{p},\bar{k}} g(|\bar{\mathbf{k}}|, \cos\theta_{\bar{k}})$$

main idea: decompose the bracket operator into its eigenfunctions ψ

$$\tilde{\mathcal{K}}_{\bar{p},\bar{k}} = \sum_l \lambda_l \psi_l(|\bar{\mathbf{p}}|, \cos\theta_{\bar{p}}) \psi_l(|\bar{\mathbf{k}}|, \cos\theta_{\bar{k}})$$

- kernels can be (numerically) evaluated only once
- huge improvement in time performance (~ 2 orders of magnitude)

De Curtis, Delle Rose, Guiggiani, Gil Muyor, Panico, 2023

Full solution to the Boltzmann equation

Structure of the Boltzmann equation

$$\frac{d}{dz}\delta f - \frac{\mathcal{Q}}{p_z}\frac{\delta f}{f_v'} = \frac{(m^2)'}{2p_z}\partial_{p_z} f_v + (\langle \delta f(k)\rangle - \langle \delta f(p')\rangle - \langle \delta f(k')\rangle)$$

Iterative procedure

- initial guess of the perturbation $\delta f0$
- next step of the iteration is found by solving

$$\frac{d}{dz}\delta f_n - \frac{\mathcal{Q}}{p_z}\frac{\delta f_n}{f_v'} = \frac{(m^2)'}{2p_z}\partial_{p_z} f_v + (\langle \delta f_{n-1}(k)\rangle - \langle \delta f_{n-1}(p')\rangle - \langle \delta f_{n-1}(k')\rangle)$$

De Curtis,Delle Rose , Guiggiani, Gil Muyor, Panico, 2022