Rescuing bileptons from Landau pole

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Based on arXiv:2505.15785
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The issue

A class of minimal 331 Extensions of the Standard Model predicts a new doubly-charged gauge boson, the Bilepton.

These extensions

- suffer from a Landau pole at the TeV scale,
- impose stringent bounds on the bilepton's mass

What's the plan?

- Exploring extensions of minimal 331 aimed at shifting the pole at higher energies
- Classifying potentially viable models based on the associated energy ranges of the bilepton mass.

Overview

$$SU_C(3) \times SU_L(2) \times U_Y(1) \to SU_C(3) \times SU_L(3) \times U_X(1)$$

$$\mathcal{L}^{331} = \mathcal{L}_{\text{strong}} + i \sum_j \bar{\psi}_{L,j}\gamma_\mu D^{331\,\mu}_{\text{ew}}\psi_{L,j} + \sum_j (D_\mu\phi_j)^\dagger D^\mu\phi_j - V(\phi) + \text{Yukawa}$$

$$D^{331\,\mu}_{\text{ew}} = \partial^\mu - i\,g_L\,W^{a,\mu}T^a \;- ig_X X B_\mu \mathbb{I}.$$

Quarks	Leptons
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$

Requires one spontaneous symmetry breaking

$$SU_C(3) \times SU_L(2) \times U_Y(1) \to SU_C(3) \times U_{\text{em}}(1) \Rightarrow \text{ one scalar doublet}$$

gauge bosons = 3 + 1	
massive	massless
$W^\pm_\mu, Z_\mu$	A_μ

Gell-Mann Nishijima relation: $Q = T_3 + \frac{Y}{2}$

Quarks	Leptons
$\begin{pmatrix} u_L \\ d_L \\ D_L \end{pmatrix}$	$\begin{pmatrix} \nu_L \\ \ell_L \\ E_L \end{pmatrix}$

Requires two spontaneous symmetry breakings

$$SU_C(3) \times SU_L(3) \times U_X(1) \to SU_C(3) \times SU_L(2) \times U_Y(1) \to SU_C(3) \times U_{\text{em}}(1)$$

$\Rightarrow$ at least two scalar triplets

gauge bosons = 8 + 1	
massive	massless
$W^\pm_\mu, Z_\mu, W'^\pm_\mu, Z'_\mu, Y^{\pm\pm}_\mu$	A_μ

Gell-Mann–Nishijima relation: $Q = T_3 + \beta T_8 + X$

$$\frac{Y}{2} = \beta T_8 + X$$

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Gell-Mann–Nishijima relation: $Q = T_3 + \beta T_8 + X$

$\beta = \pm\sqrt{3}, \pm\frac{1}{\sqrt{3}}$ $\qquad \frac{Y}{2} = \beta T_8 + X$

Minimal 331 Extension ($\beta = \sqrt{3}$)

- Gauge anomaly cancellation:

$$N_F \equiv N_q = N_\ell,$$

$$N_q^3 = \frac{2N_F - N_\ell^3}{3}.$$

$$N_q^3 = 2, \quad N_q^{\bar{3}} = 1, \quad N_\ell^3 = 0, \quad N_\ell^{\bar{3}} = 3.$$

- Asymptotic freedom:

$$n_{\mathrm{quark}} = 3N_F < \frac{33}{2} \qquad N_F = 3, 4, \not{5}$$

- SSB: $SU_L(3) \times U_X(1) \xrightarrow{\mu_{331}} SU_L(2) \times U_Y(1)$

- Bilepton:

arXiv:2312.02287

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ \frac{u}{\sqrt{2}} \end{pmatrix} \rightarrow m_Y \simeq u\,\frac{g_{2L}}{2} \gtrsim 1.3 \text{ TeV at } 90\% \text{ C.L}$$

$$\mu_{331} \sim u \gtrsim 3850\,\mathrm{GeV}$$

		Q	$SU_L(3)$	$U_X(1)$
Quarks L	u_L	2/3		
	d_L	-1/3	3	-1/3
	D_L^1	-4/3		
	c_L	2/3		
	s_L	-1/3	3	-1/3
	D_L^2	-4/3		
	b_L	-1/3		
	$-t_L$	2/3	$\bar{3}$	2/3
	T_L	5/3		
Leptons L	ℓ_L	-1		
	$-\nu_L^\ell$	0	$\bar{3}$	0
	ℓ_R^c	+1		

		Q	$SU_L(3)$	$U_X(1)$
Quarks R	u_R, c_R	2/3		2/3
	d_R, s_R	$-1/3$		$-1/3$
	D_R^1, D_R^2	$-4/3$	1	$-4/3$
	b_R	$-1/3$		$-1/3$
	t_R	2/3		2/3
	T_R	5/3		5/3

Scalar	$SU_L(3)$	$U_X(1)$
χ	3	+1
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Table 1: Matter content of minimal 331 model with $\beta = \sqrt{3}$

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Minimal 331 Extension $(\beta = \sqrt{3})$

- One-loop renormalization:

$$\alpha_i(\mu) = \frac{\alpha_i(\mu_0)}{1 - \frac{b_i}{2\pi}\alpha_i(\mu_0)\log\left(\frac{\mu}{\mu_0}\right)} + \mathcal{O}(\alpha_i^2)$$

$$b_i = \frac{2}{3}\sum_{fermions} \mathrm{Tr}(T_a T_a) + \frac{1}{3}\sum_{scalars} \mathrm{Tr}(T_a T_a) - \frac{11}{3}C_2$$

- Below μ_{331}: SM or 2HDM?

-SM $\mu_{331} \lesssim \mu_{331}^{\max} = 3620\ \mathrm{GeV}$

-2HDM $\mu_{331} \lesssim \mu_{331}^{\max} = 3908\ \mathrm{GeV}$ $\rightarrow 3850\ \mathrm{GeV} \lesssim \mu_{331} \lesssim 3908\ \mathrm{GeV}$

- Matching condition:

$$g_{2L}(\mu_{331}) = g_{3L}(\mu_{331}), \qquad \frac{1}{g_X^2(\mu_{331})} = \frac{1}{6}\left(\frac{1}{g_Y^2} - \beta^2\frac{1}{g_L^2}\right)\Bigg|_{\mu=\mu_{331}}$$

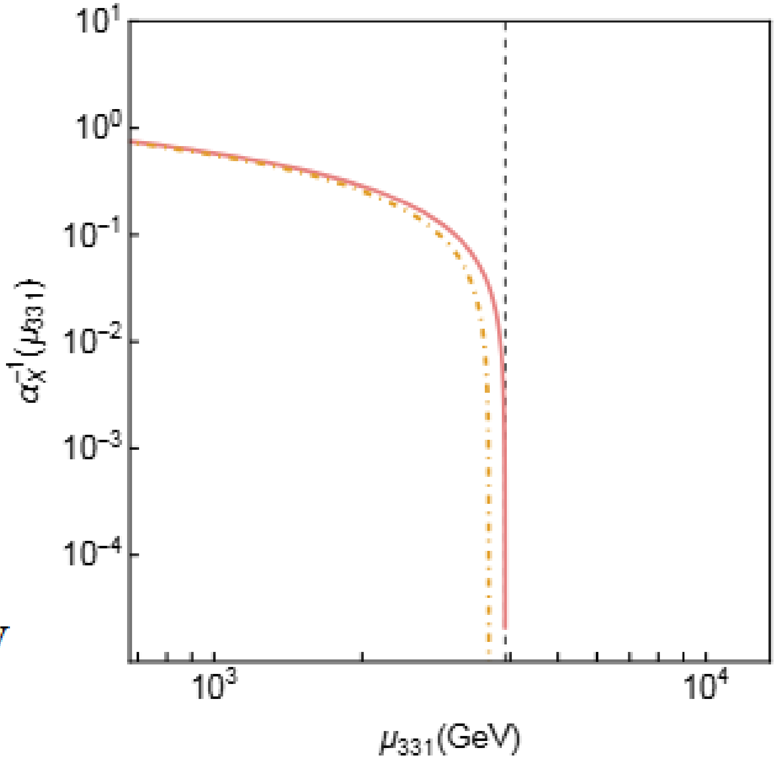

Fig.1: Running of α_X^{-1} on the energy scale μ331. The dot-dashed and continuous lines correspond respectively to the cases where one and two Higgs doublets. The vertical dashed line represents the Landau pole at 3908 GeV.

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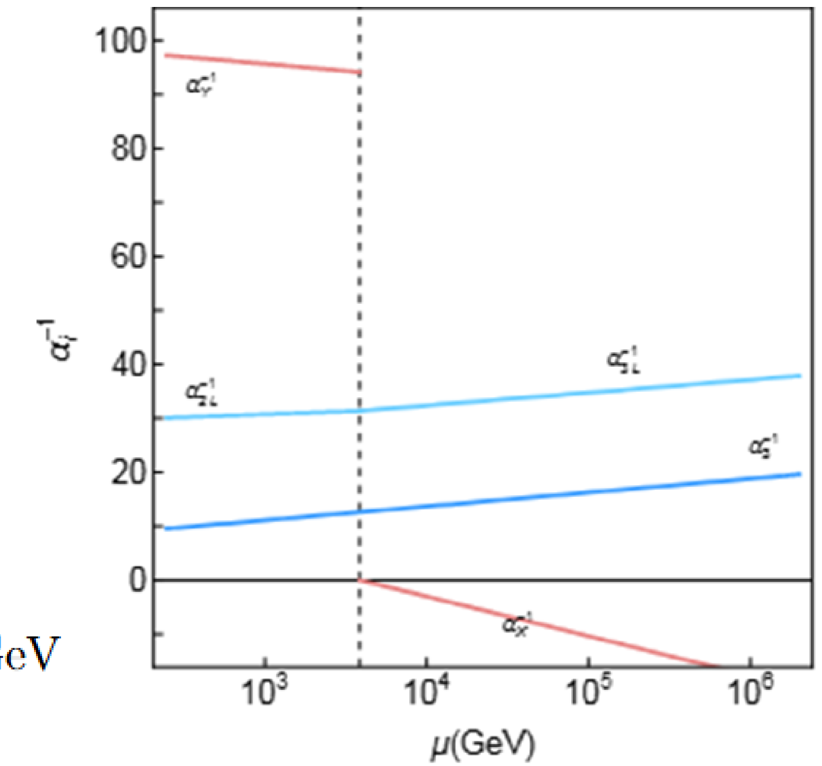

Fig.2: Running of α_i^{-1} with the energy scale μ. The vertical dashed line is the arbitrary 331 breaking scale μ_{331}.

Extension of scalar sector: Multi-Higgs

- Neutral X-charge bosons help!

$$b_Y = 7 + \frac{N_\rho}{6}\,, \qquad b_{2L} = -3 + \frac{N_\rho}{6}\,, \qquad b_X = 22\,, \qquad b_{3L} = -\frac{13}{2} + \frac{N_\rho}{6}$$

- Many Higgs picture:

  Critical value for $N_\rho < 18 \; \rightarrow N_H = 1 \; + N_\rho < 19$

N_H	3	6	9	12	15	18
$\mu_{331}^{\max}$(GeV)	$4.2 \cdot 10^3$	$5.5 \cdot 10^3$	$7.4 \cdot 10^3$	$1.1 \cdot 10^4$	$1.6 \cdot 10^4$	$2.6 \cdot 10^4$

- Case of interest: $N_H = 6$

$$E_6 \rightarrow SU(2) \times SU(6) \rightarrow SU_c(3) \times SU_L(3) \times U(1)$$

Colourless representations: $SU(6) \supset SU_C(3) \times SU(3) \times U(1)$

$$\mathbf{6}, \mathbf{84} \supset (\mathbf{1}, \mathbf{3})(\mathrm{Z})$$

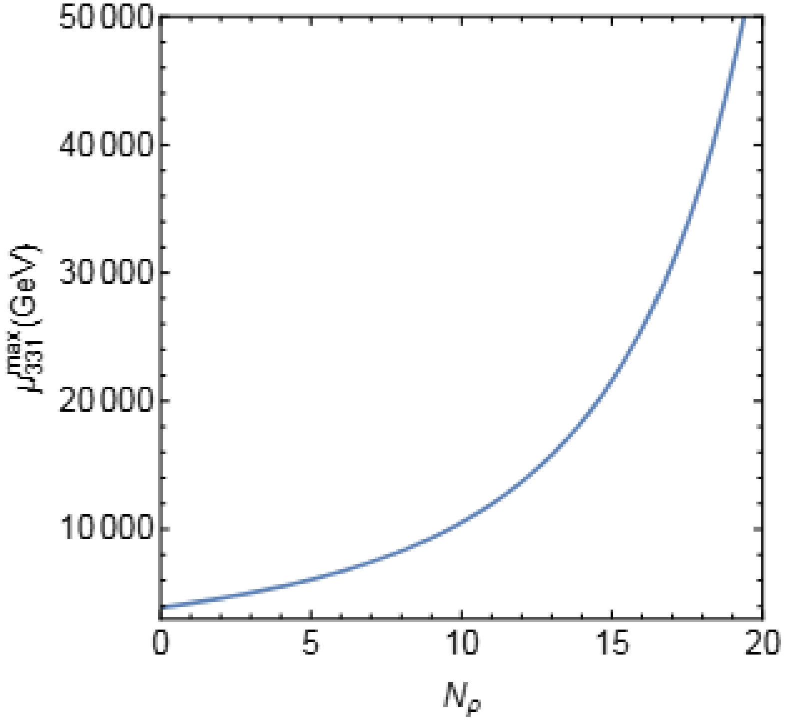

Fig.3: Maximal value of the 331 breaking scale as a function of the number of Higgs doublets.

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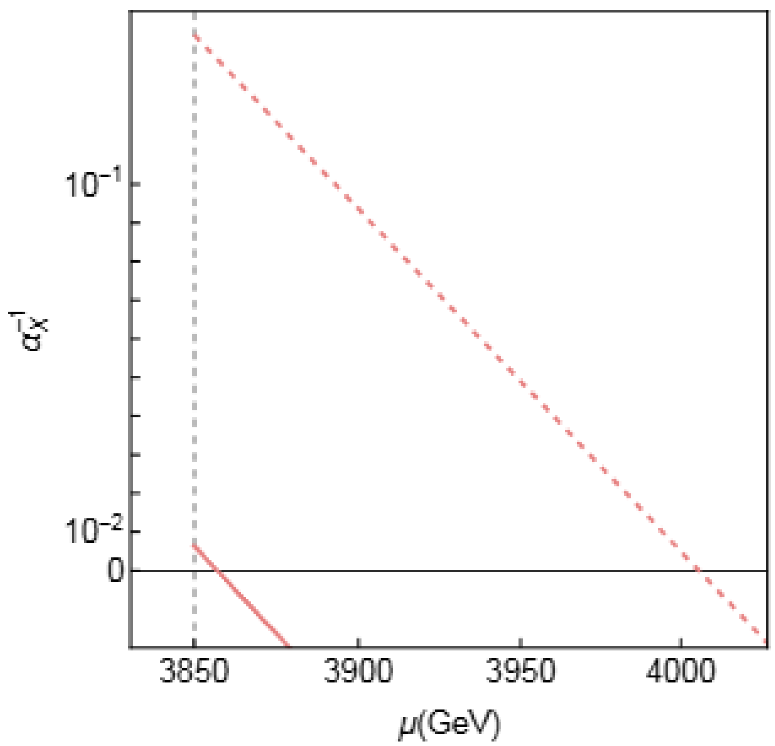

Fig.4: Comparison of minimal (continuous) with the multi-Higgs case with $N\rho = 4$ (dashed line).

Extension of scalar sector: Sextet

- Mass term for neutrinos through type II seesaw
  - -in SM we need $SU_L(2)$ triplet:

$$\mathcal{L}_Y = Y^{ij} \bar{L^c}_i S L_j$$

- **6** rep. of $SU_L(3)$ → $\mathbf{3} \oplus \mathbf{2} \oplus \mathbf{1}$

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\,\Delta^0 & \Delta^- & \Phi^0 \\ \Delta^- & \sqrt{2}\,\Delta^{--} & \Phi^- \\ \Phi^0 & \Phi^- & \sqrt{2}\,\sigma^0 \end{pmatrix},$$

- Assumptions on masses: heavy singlet

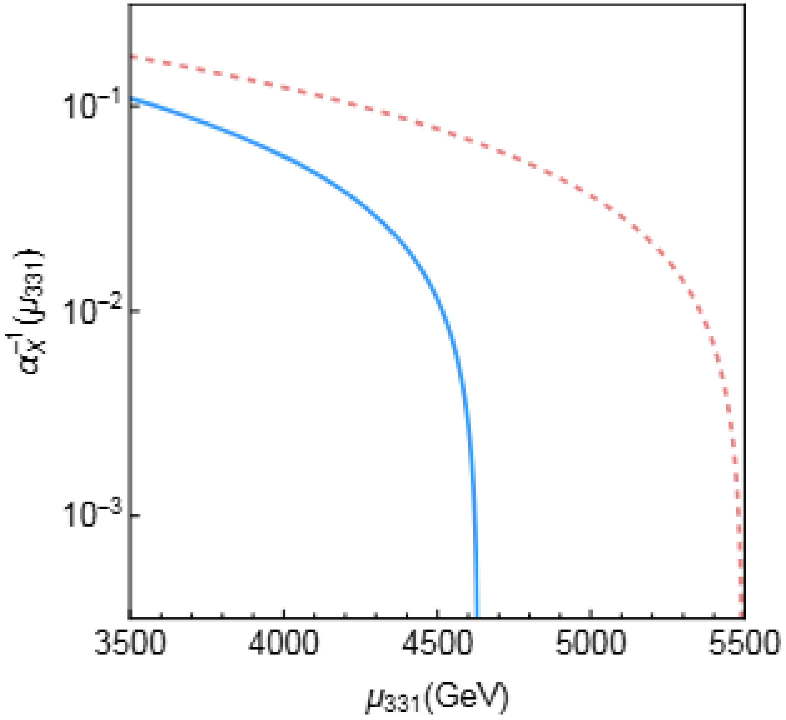

Fig.5: : Running of $\alpha_X^{-1}$with the energy scale μ_{331} in the sextet extension for different values of the mass of the doubletand triplet: electroweak scale (dashed line), 600 GeV (continuous line).

4th Family Extension

- Exclusion of SM4 : arXiv:1005.3505
- With Two Higgs Doublets things change!
  (e.g. arXiv:1205.5580, arXiv:1208.3195)
- Experimental bounds (VLQ)

-$m_{\tau'} > 700\ GeV$, $m_{b'} > 1390 \div 1570\ GeV$ (CMS)

- $m_{t'} > 715 \div 950\ GeV, m_{b'} > 1000 \div 2000\ GeV$ (ATLAS)

$$N_q^3 = 2, N_q^{\bar{3}} = 2, N_l^3 = 2, N_l^{\bar{3}} = 2$$

		Q	$SU_L(3)$	$U_X(1)$	$U_Y(1)$
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	d_L	-1/3	3	-1/3	1/6
	D_L^1	-4/3			-4/3
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	D_L^2	-4/3			-4/3
	b_L	-1/3			1/6
	$-t_L$	2/3	$\bar{3}$	2/3	1/6
	T_L^1	5/3			5/3
	b'_L	-1/3			1/6
	$-t'_L$	2/3	$\bar{3}$	2/3	1/6
	T_L^2	5/3			5/3
Leptons L	ν_{eL}	0			-1/2
	e_L	-1	3	-1	-1/2
	E_L^1	-2			-2
	$\nu_{\mu L}$	0			-1/2
	μ_L	-1	3	-1	-1/2
	E_L^2	-2			-2
	τ_L	-1			-1/2
	$-\nu_{\tau L}$	0	$\bar{3}$	0	-1/2
	E_L^3	+1			+1
	τ'_L	-1			-1/2
	$-\nu'_{\tau L}$	0	$\bar{3}$	0	-1/2
	E_L^4	+1			+1

		Q	$SU_L(3)$	$U_X(1)$
Quarks R	u_R, c_R	2/3		2/3
	d_R, s_R	$-1/3$		$-1/3$
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	b_R, b'_R	$-1/3$		$-1/3$
	t_R, t'_R	2/3		2/3
	T_R^1, T_R^2	5/3		5/3
Leptons R	$\nu_{eR}, \nu_{\mu R}$	0		0
	e_R, μ_R	-1		-1
	E_R^1, E_R^2	-2	1	-2
	$\nu_{\tau R}, \nu'_{\tau R}$	0		0
	τ_R, τ'_R	-1		-1
	E_R^3, E_R^4	1		1

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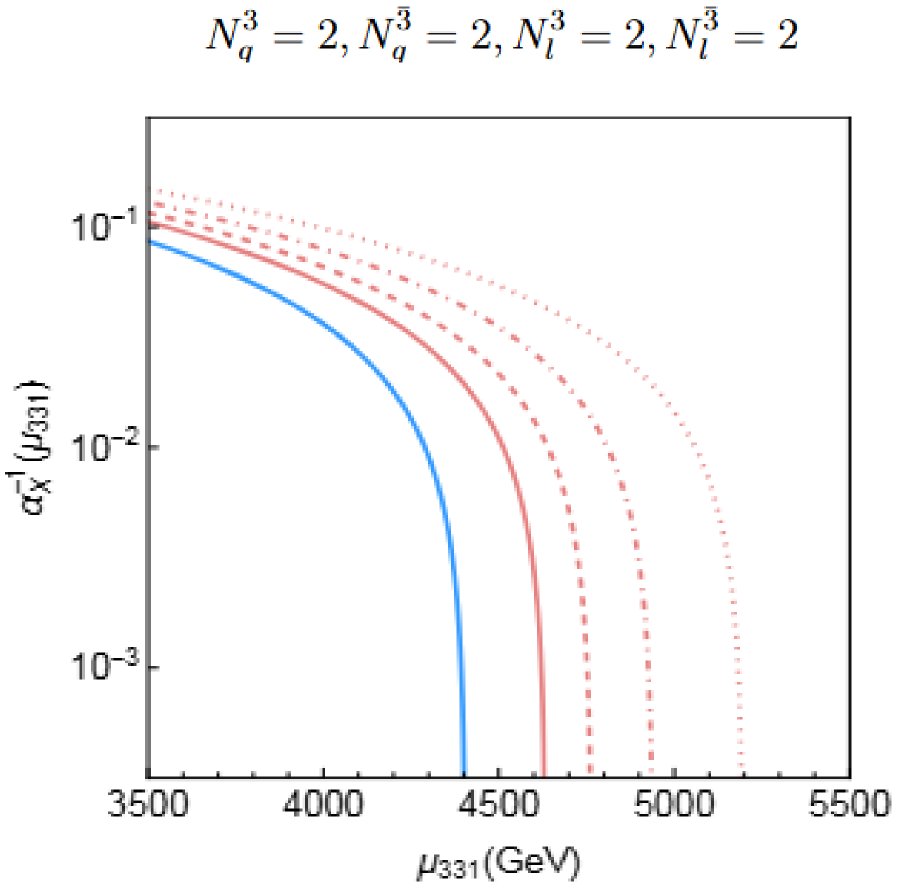

Fig.6: Running of $\alpha_X^{-1}$with the energy scale μ_{331}, 4th family model for different values of the mass scale m_{NP}: 1500GeV (blue continuous), 1000GeV (red continuous), 800GeV (dashed), 600GeV (dot-dashed) and 400GeV (dotted).

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- Addition of scalar sextet

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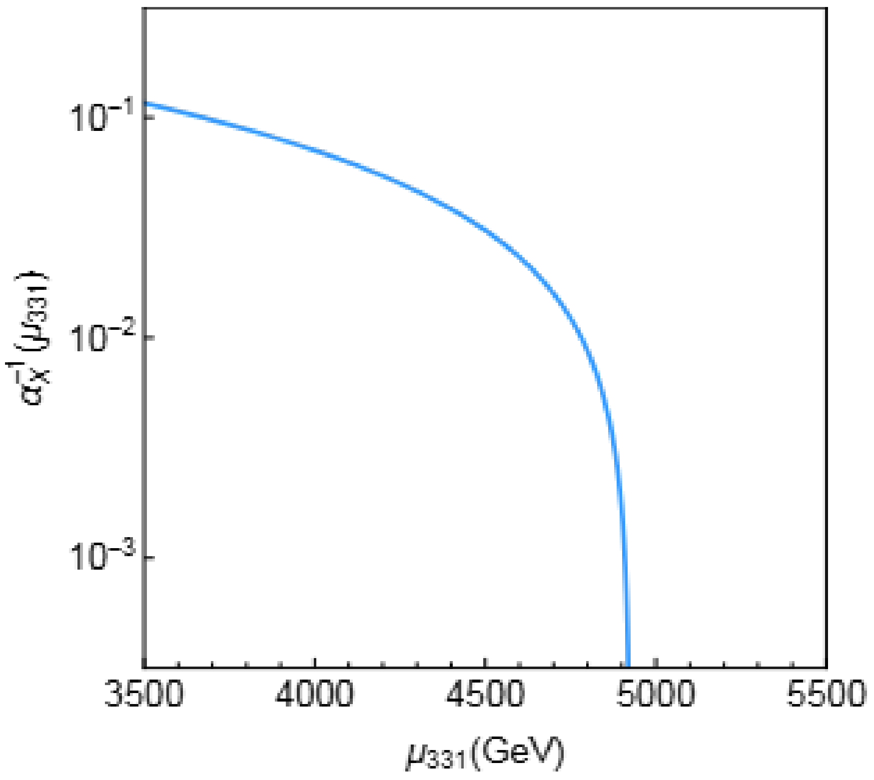

Fig.7: Running of $\alpha_X^{-1}$with the energy scale μ_{331}, 4th family model + sextet with $m_{NP} \sim 1500$ GeV.

Results and summary

- Minimal 331 extension predicts Bileptons
- Bilepton's mass has an experimental lower bound of $\sim 1.3\ TeV$
    - Below SSB scale: 2HDM → exihbits a Landau Pole at $\sim 4\ TeV$
    - Stringent bounds on bilepton's mass 3850 GeV $\lesssim \mu_{331} \lesssim$ 3908 GeV (through $m_Y \sim g_L \frac{u}{2}$)

- To overcome these limitations we consider extensions of the scalar and fermionic sectors of minimal 331.
    - By adding a scalar sextet of $SU_L(3)$ we obtain a Majorana mass term for neutrinos
    - In the Multi-Higgs case we study a case which could stem from a GUT (E_6)
    - Addition of a sequential fourth family at $\sim 1.5\ TeV$ is compatible with available experimental bounds

	b_Y	$-b_{2L}$	$\mu_{\max}$ (GeV)	m_Y (GeV)
331 minimal	7	3	3908	1270
Sextet	49/6	13/6	4531	1472
$N_H = 6$	47/6	13/6	5500	1788
4th fam. ($m_{NP} \sim 1500$ GeV)	83/9	5/3	4400	1430
4th fam. + sextet ($m_{NP} \sim 1500$ GeV)	187/18	5/6	4919	1599

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Thank you :)

Back Up

Scalar potential

$$V = m_1^2\rho^*\rho + m_2^2\eta^*\eta + m_3^2\chi^*\chi + \sqrt{2}f_{\rho\eta\chi}\rho\,\eta\,\chi$$
$$+\lambda_1(\rho^*\rho)^2 + \lambda_2(\eta^*\eta)^2 + \lambda_3(\chi^*\chi)^2$$
$$+\lambda_{12}\rho^*\rho\,\eta^*\eta + \lambda_{13}\rho^*\rho\,\chi^*\chi + \lambda_{23}\eta^*\eta\,\chi^*\chi$$
$$+\zeta_{12}\rho^*\eta\,\eta^*\rho + \zeta_{13}\rho^*\chi\,\chi^*\rho + \zeta_{23}\eta^*\chi\,\chi^*\eta,$$

Yukawa

$$L_{Yuk} = \lambda^d_{i,a}\bar{Q}_i\rho d_{a,R} + \lambda^d_{3,a}\bar{Q}_3\eta^\star d_{a,R}$$
$$+\ \lambda^u_{i,a}\bar{Q}_i\eta u_{a,R} + \lambda^u_{3,a}\bar{Q}_3\rho^\star u_{a,R}$$
$$+\ \lambda^J_{i,j}\bar{Q}_i\chi J_{j,R} + \lambda^J_{3,3}\bar{Q}_3\chi^\star T_R + h.c.$$

FCNC at tree level

$$\mathcal{L}^{Z'} = J_\mu Z'^\mu\,,$$

$$J_\mu = \bar{u}_L\gamma_\mu U_L^\dagger \begin{pmatrix} a & & \\ & a & \\ & & b \end{pmatrix} U_L u_L + \bar{d}_L\gamma_\mu V_L^\dagger \begin{pmatrix} a & & \\ & a & \\ & & b \end{pmatrix} V_L d_L\,,$$

Why $\beta = \sqrt{3}$?

$$Q_{\text{gauge bosons}} \in \mathbb{Z}$$

$$W_\mu = W_\mu^a T^a = \frac{1}{2}\begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}W_\mu^+ & \sqrt{2}Y_\mu^{Q_Y} \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}V_\mu^{Q_V} \\ \sqrt{2}Y_\mu^{-Q_Y} & \sqrt{2}V_\mu^{-Q_V} & -\frac{2}{\sqrt{3}}W_\mu^8 \end{pmatrix}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i\, W_\mu^2), \quad \text{(SM)}$$
$$Y_\mu^{\pm Q_Y} = \frac{1}{\sqrt{2}}(W_\mu^4 \mp i\, W_\mu^5),$$
$$V_\mu^{\pm Q_V} = \frac{1}{\sqrt{2}}(W_\mu^6 \mp i\, W_\mu^7)$$

$$\hat{Q}\, W_\mu = [Q, W_\mu] = [T^3 + \beta T^8 + X, W_\mu] = Q_W W_\mu .$$

$$Q_W = \begin{pmatrix} 0 & 1 & \frac{1}{2} + \beta\frac{\sqrt{3}}{2} \\ -1 & 0 & -\frac{1}{2} + \beta\frac{\sqrt{3}}{2} \\ -\frac{1}{2} - \beta\frac{\sqrt{3}}{2} & \frac{1}{2} - \beta\frac{\sqrt{3}}{2} & 0 \end{pmatrix}$$

$$Q_W \in \mathbb{Z} \Rightarrow \beta = (2n+1)/\sqrt{3} \quad n \in \mathbb{Z}.$$

$$g_i = g_i^*$$

From spontaneous symmetry breaking

$$SU_C(3) \times SU_L(3) \times U_X(1) \to SU_C(3) \times SU_L(2) \times U_Y(1)$$

$$\frac{g_X}{g_L} = \frac{6\sin^2\theta_W}{\sqrt{1-(1+\beta^2)\sin^2\theta_W}}, \quad \text{with } \sin^2\theta_W = \frac{g_Y}{\sqrt{g_Y^2 + g_L^2}}$$

$$\frac{g_X}{g_{3L}} \in \mathbb{R} \iff |\beta| < \frac{\cos\theta_W}{\sin\theta_W}$$

At tree level

$$|\beta| < \cos\theta_W = \frac{m_W}{m_Z} \simeq 1.83$$

$$\downarrow$$

$$\to \quad \beta = \pm\sqrt{3}, \pm\frac{1}{\sqrt{3}}$$