Cortona, 22 maggio 2025

Stefano Scacco[a,b]

On the Atomki nuclear anomaly after the MEG-II result

Based on: *Barducci, Germani, Nardecchia, Scacco, Toni*, ***J. High Energ. Phys. 2025, 35 (2025)***

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Summary

- **Context**
  - Atomki experiment ⟵
  - Motivation
  - Phenomenology spin 0 and spin 1
- **Spin 2 model**
- **MEG-II experiment**
  - Spin parity 0^+
- **Latest developments**

Discovery of X17 anomaly

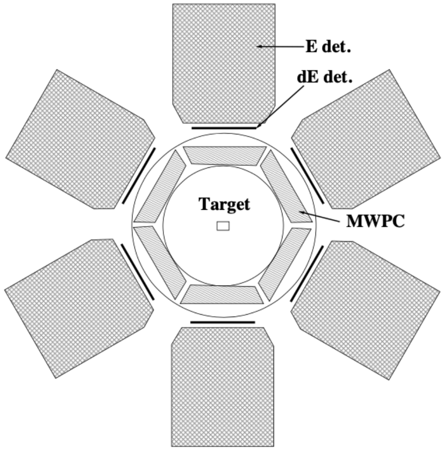
Atomki experiment (Hungary)

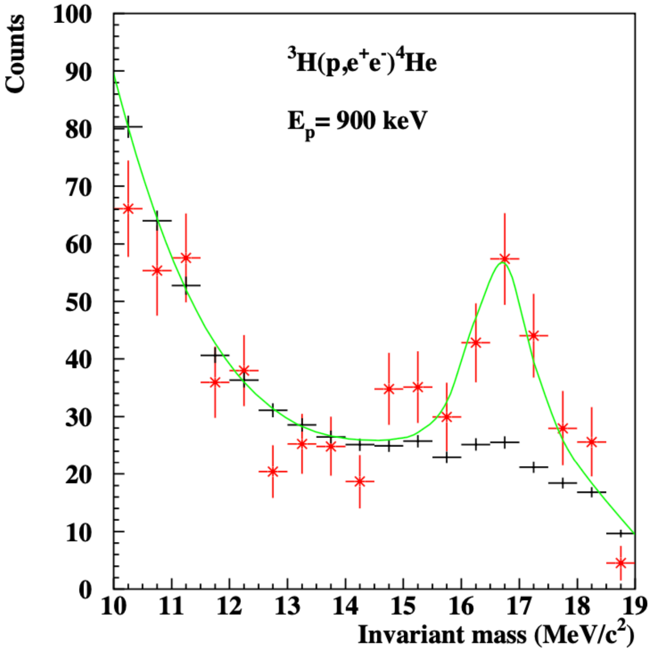
[1] A.J. Krasznahorkay et al. ***Phys. Rev. Lett. 116, 042501***
[2] A.J. Krasznahorkay et al. **arXiv:1910.10459**
[3] A.J. Krasznahorkay et al. **arXiv:2209.10795**
[4] J. Gulyś et al. **arXiv:1504.00489**

Observed process: $p + A \to N^* \to N + e^+e^-$.

Nuclear transition

Apparatus [4]:





Different nuclei tested ^{8}Be [1], ^{4}He [2], ^{12}C [3] → resonant peak at 17 MeV

Decay: $X \to e^+e^-$

Atomki signal observables

[1] Denton, Gehrlein, **Phys. Rev. D 108 (2023)**
[2] Zhang, Miller, **Phys. Lett. B 773 (2017) 159–165**

Mass [1]

$$m_X = (16.85 \pm 0.04)\,\mathrm{MeV}$$

Beryllium (R_{Be})

$$\frac{\Gamma(^8\mathrm{Be}(18.15) \to {}^8\mathrm{Be} + X)}{\Gamma(^8\mathrm{Be}(18.15) \to {}^8\mathrm{Be} + \gamma)}\,\mathrm{BR}(X \to e^+e^-) = (6 \pm 1) \times 10^{-6}.$$

Helium (R_{He})

$$\frac{\Gamma(^4\mathrm{He}(20.21) \to {}^4\mathrm{He} + X)}{\Gamma(^4\mathrm{He}(20.21) \to {}^4\mathrm{He} + e^+e^-)}\,\mathrm{BR}(X \to e^+e^-) = 0.20 \pm 0.03 \qquad S^\pi = 0^+, 1^-, 2^+, \ldots$$

$$\frac{\Gamma(^4\mathrm{He}(21.01) \to {}^4\mathrm{He} + X)}{\Gamma(^4\mathrm{He}(20.21) \to {}^4\mathrm{He} + e^+e^-)}\,\mathrm{BR}(X \to e^+e^-) = 0.87 \pm 0.14 \qquad S^\pi = 0^-, 1^+, 2^-, \ldots$$

Carbon (R_{C})

$$\frac{\Gamma(^{12}\mathrm{C}(17.23) \to {}^{12}\mathrm{C} + X)}{\Gamma(^{12}\mathrm{C}(17.23) \to {}^{12}\mathrm{C} + \gamma)}\,\mathrm{BR}(X \to e^+e^-) = 3.6(3) \times 10^{-6}$$

Physics Beyond Standard Model is required [2]

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  - Spin parity 0^+
- **Latest developments**

Philosophy

[1] Feng et al, **Phys. Rev. Lett. 117 (2016)**
[2] Feng et al, **Phys. Rev. Lett. D 95 (2017)**
[3] Alves, **Phys. Rev. D 103 (2021)**
[4] Wong, **arXiv:2201.09764**
[5] Barducci, Toni, **JHEP 02 (2023) 154**
[6] Mommers, Vanderhaeghen, **Phys. Lett. B 858 (2024)**

If X17 is a new particle

What is its nature? → **UV Models [1 - 4]**

Why haven't we seen it before? → **Phenomenology [5, 6]**

Philosophy

[1] Feng et al, **Phys. Rev. Lett. 117 (2016)**
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[3] Alves, **Phys. Rev. D 103 (2021)**
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Methodology

1) Choose X17 spin and parity.

2) Verify symmetries and conservations.

3) Write down most general Lagrangian.

4) Identify relevant couplings

5) Calculate Atomki observables.

6) Identify other relevant bounds.

7) Produce exclusion plots.

Spin 0

[1] Barducci, Toni, **JHEP 02 (2023) 154**

Spin 0 excluded by angular momentum conservation [1]

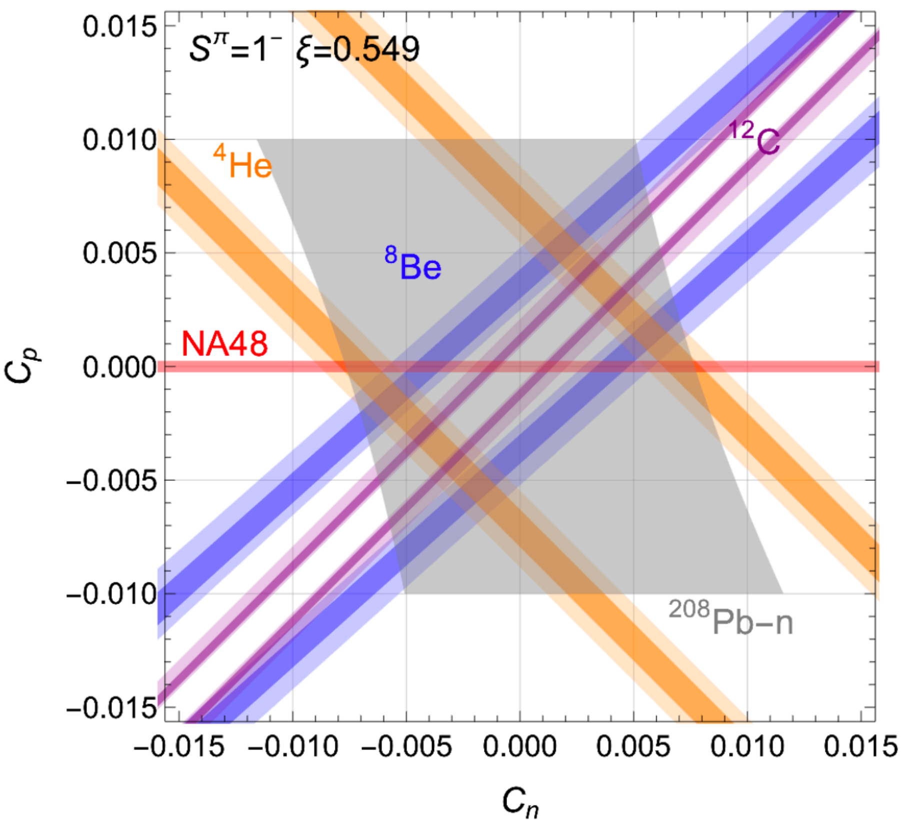
Process $N^* \to N$	X boson spin parity $S^\pi = 1^-$	$S^\pi = 1^+$	$S^\pi = 0^-$	$S^\pi = 0^+$
$^8\mathrm{Be}(18.15) \to {}^8\mathrm{Be}$	1	0, 2	1	/ ←
$^8\mathrm{Be}(17.64) \to {}^8\mathrm{Be}$	1	0, 2	1	/ ←
$^4\mathrm{He}(21.01) \to {}^4\mathrm{He}$	/	1	0	/
$^4\mathrm{He}(20.21) \to {}^4\mathrm{He}$	1	/	/	0
$^{12}\mathrm{C}(17.23) \to {}^{12}\mathrm{C}$	0, 2	1	/ ←	1

Angular momentum of X in every nuclear transition, in different spin parity models.

Spin 1

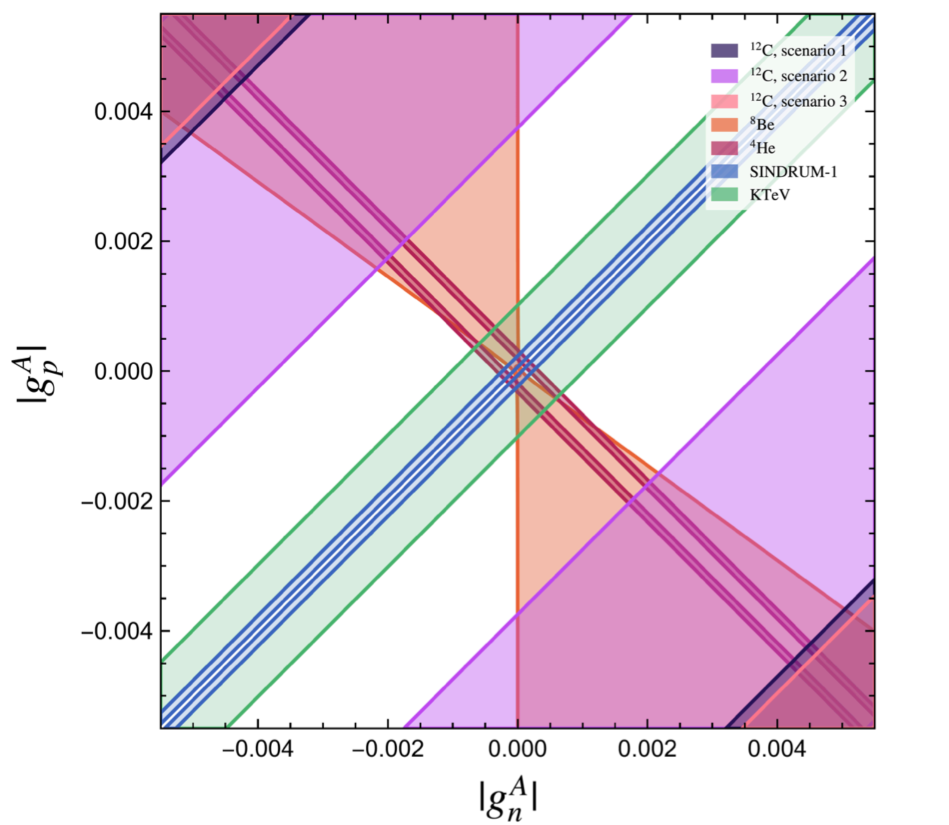
[1] Barducci, Toni, **JHEP 02 (2023) 154**
[2] Mommers, Vanderhaeghen, **Phys. Lett. B 858 (2024)**

$S^\pi = 1^-$



C_p, C_n nucleon couplings [1]

$S^\pi = 1^+$



g_p^A, g_n^A nucleon couplings [2]

At least 2σ tension

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Spin 2: amplitudes

[1] Panico, Vecchi, Wulzer, **JHEP 06 (2016) 184**

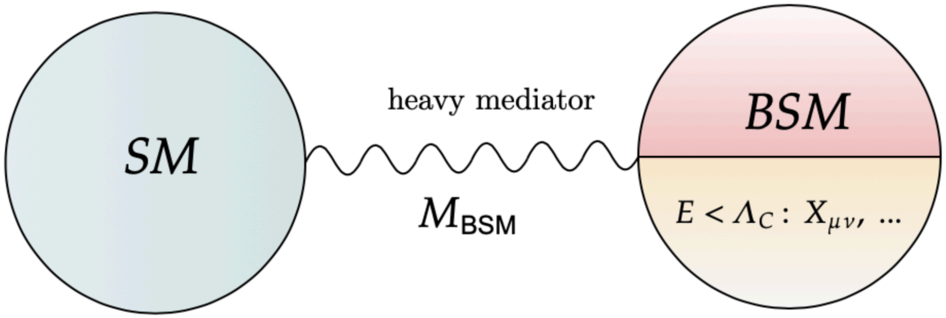
Contact off shell terms complicate Lagrangian

Following [1], most general real production amplitudes

$$
\begin{aligned}
\mathcal{A}(f \to f'X) = \overline{u}(p',\sigma')\Bigg\{ & C_f \left[\gamma_\mu \left(\frac{p'+p}{4}\right)_\nu + \gamma_\nu \left(\frac{p'+p}{4}\right)_\mu\right] \\
& + \tilde{C}_f \left[\gamma_\mu\gamma_5 \left(\frac{p'+p}{4}\right)_\nu + \gamma_\nu\gamma_5 \left(\frac{p'+p}{4}\right)_\mu\right] \\
& + D_f \left(p'+p\right)_\mu \left(p'+p\right)_\nu \\
& + \tilde{D}_f \left(p'+p\right)_\mu \left(p'+p\right)_\nu i\gamma_5 \Bigg\} u(p,\sigma)\ \left[\epsilon_a^{\mu\nu}(p-p')\right]^*
\end{aligned}
$$

Spin 2: free parameters

Spin 2 theories are EFTs



$$C_f \sim \tilde{C}_f \sim \mathcal{O}(M_{\text{BSM}}^{-1}) \quad \text{and} \quad D_f \sim \tilde{D}_f \sim \mathcal{O}(M_{\text{BSM}}^{-2}) \ .$$

At dimension 5, couplings are C_p, C_n, C_e for 2^+, e $\tilde{C}_p, \tilde{C}_n, \tilde{C}_e$ for 2^-.
No neutrino couplings.

Conservatively, $\Lambda_c = 4\pi m_X \approx 200$ MeV

Spin 2: Atomki signal observables

Final results

$$R_{\text{Be}} = \frac{k m_X^2}{18\pi} \left| \sqrt{\frac{4\pi}{3}} [(-\alpha_1 + \beta_1 \xi) M1^{\gamma}_{I=1} (C_p - C_n) + \beta_1 M1^{\gamma}_{I=0} (C_p + C_n)] - \frac{1}{2}(5C_p - 4C_n) \langle {}^8\text{Be} || \hat{\sigma}^{(p)} || {}^8\text{Be}(18.15) \rangle + \frac{1}{2}(4C_p - 5C_n) \langle {}^8\text{Be} || \hat{\sigma}^{(n)} || {}^8\text{Be}(18.15) \rangle \right|^2 \frac{\text{BR}(X \to e^+e^-)}{\Gamma({}^8\text{Be}(18.15) \to {}^8\text{Be} + \gamma)} ,$$

$$R_{\text{He}} = \frac{m_N^2}{\alpha^2} \frac{5}{4} \frac{k m_X^4}{\omega^5} (C_p + C_n)^2 \left| 1 - \left(3 - 2\frac{k^2}{m_X^2}\right) r_{\text{He}} \right|^2 \text{BR}(X \to e^+e^-) ,$$

$$R_{\text{C}} = \frac{m_N^2}{12\pi\alpha} \frac{m_X^4}{k\omega^3} [1 + 6 r_{\text{C}}^2] (C_p - C_n)^2 \, \text{BR}(X \to e^+e^-) ,$$

$S^\pi = 2^+$

$$R_{\text{Be}} = \frac{m_N^2 k^3}{18\pi m_X^2} \left| \tilde{C}_p \langle {}^8\text{Be} || \hat{\sigma}^{(p)} || {}^8\text{Be}(18.15) \rangle + \tilde{C}_n \langle {}^8\text{Be} || \hat{\sigma}^{(n)} || {}^8\text{Be}(18.15) \rangle \right|^2 \times \left(1 + \frac{2}{3} \frac{\omega^2}{m_X^2}\right) \frac{\text{BR}(X \to e^+e^-)}{\Gamma({}^8\text{Be}(18.15) \to {}^8\text{Be} + \gamma)} ,$$

$$R_{\text{C}} = \frac{m_N^2}{32\pi\alpha} \frac{k^5}{m_X^2 \omega^3} \left(\tilde{C}_p - \tilde{C}_n\right)^2 |\tilde{r}_{\text{C}}|^2 ,$$

$$R_{\text{He}} = \frac{80 m_N^2}{\alpha^2} \frac{\sigma_- \Gamma_+}{\sigma_+ \Gamma_-} \left(\frac{k}{\omega}\right)^5 \left(\tilde{C}_p + \tilde{C}_n\right)^2 |\tilde{r}_{\text{He}}|^2 \text{BR}(X \to e^+e^-) ,$$

$S^\pi = 2^-$

Estimated parameters that were absent in literature

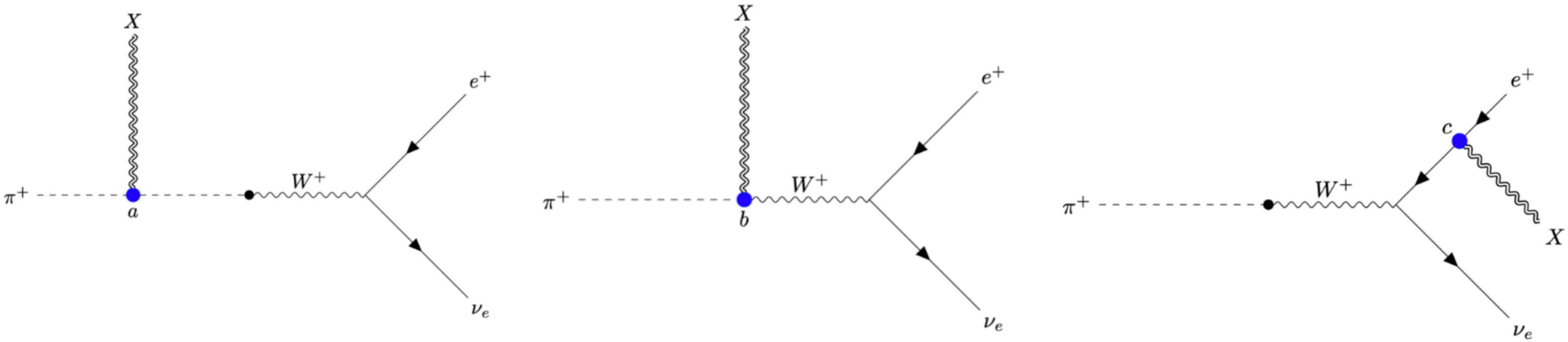
r_{He}	$\tilde{r}_{\text{He}}$	r_{C}	$\tilde{r}_{\text{C}}$
~ 4.6	~ 7.7	~ 5.5	~ 1

Spin 2: other bounds

SINDRUM Experiment [1, 2]: $\pi^+ \to e^+\nu_e(X \to e^+e^-)$

[1] **SINDRUM** Collab., **Phys. Lett. B 175 (1986) 101**
[2] Hostert, Pospelov, **Phys. Rev. D 108 (2023)**

$$\mathrm{BR}(\pi^+ \to e^+\nu_e X) \times \mathrm{BR}(X \to e^+e^-) < 6.0 \times 10^{-10}$$

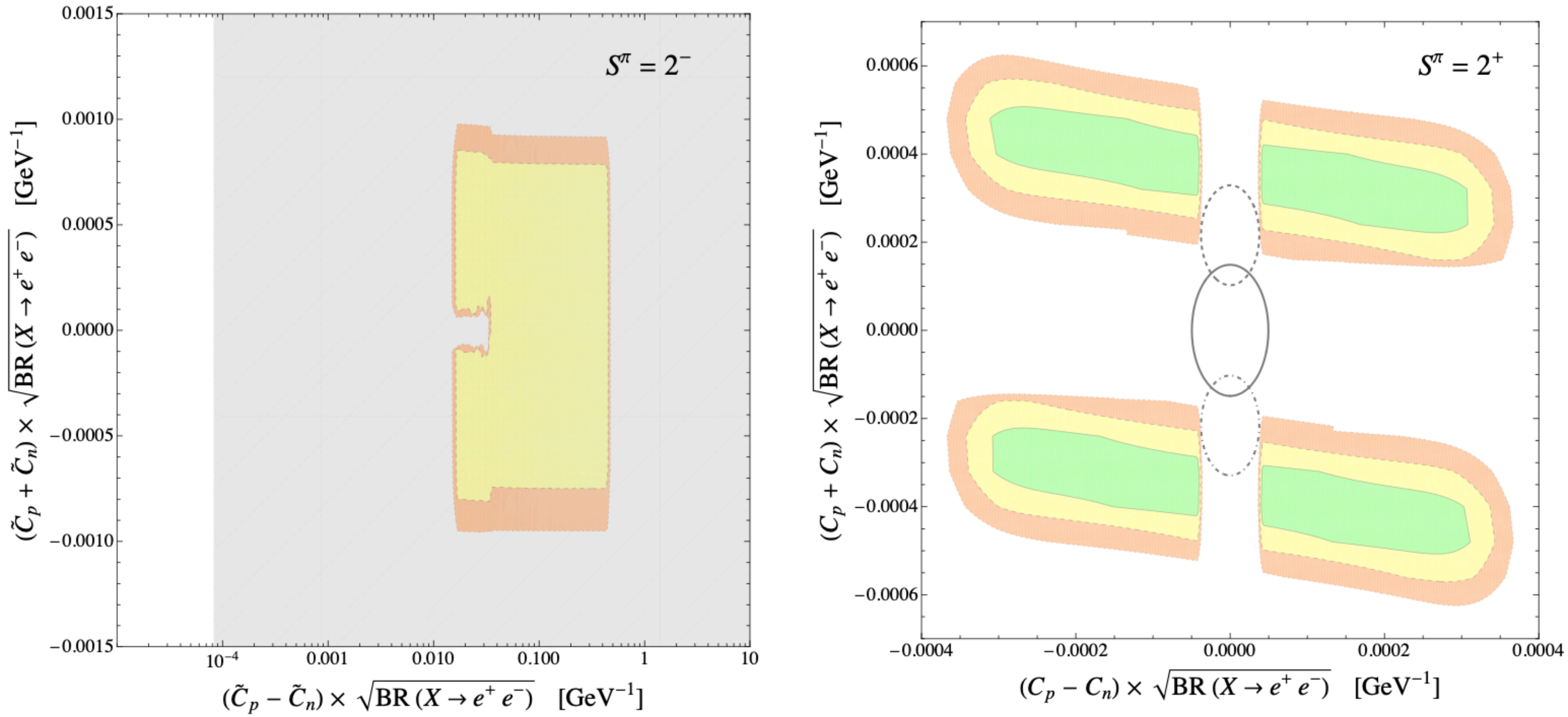


Charged pion bremsstrahlung X17 production diagrams.

$$\mathrm{BR}(\pi^+ \to e^+\nu_e X) = \frac{m_\pi^{12}\left(90\left(\eta_2^2(C_p - C_n)^2 + \eta_3^2(C_p + C_n)^2\right) + 3C_e^2 + 10C_e\eta_3(C_p + C_n)\right)}{2^8\, 3^3\, 5\pi^2 m_\mu^2 m_X^4 \left(m_\pi^2 - m_\mu^2\right)^2} \qquad S^\pi = 2^+$$

$$\mathrm{BR}(\pi^+ \to e^+\nu_e X) = \frac{m_\pi^{12}\left(54\eta_2^2(\tilde{C}_p - \tilde{C}_n)^2 + 5\tilde{C}_e^2\right)}{2^8\, 3^2\, 5^2\pi^2 m_\mu^2 m_X^4 \left(m_\pi^2 - m_\mu^2\right)^2} \qquad S^\pi = 2^-$$

Spin 2: exclusion plots



Exclusion plots. Legend: Atomki observables compatible at 1σ (Green), 2σ (Yellow), 3σ (Orange).
SINDRUM exclusion region: grey region to the left, outside of ellipse on the right (continuous line for $C_e = 0$, Dashed lines for $C_e = \pm 10^{-4}$).

Spin 2 is excluded

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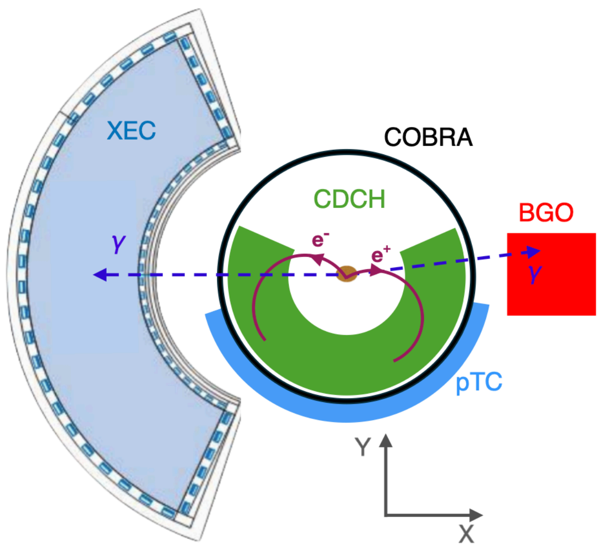
MEG-II null result

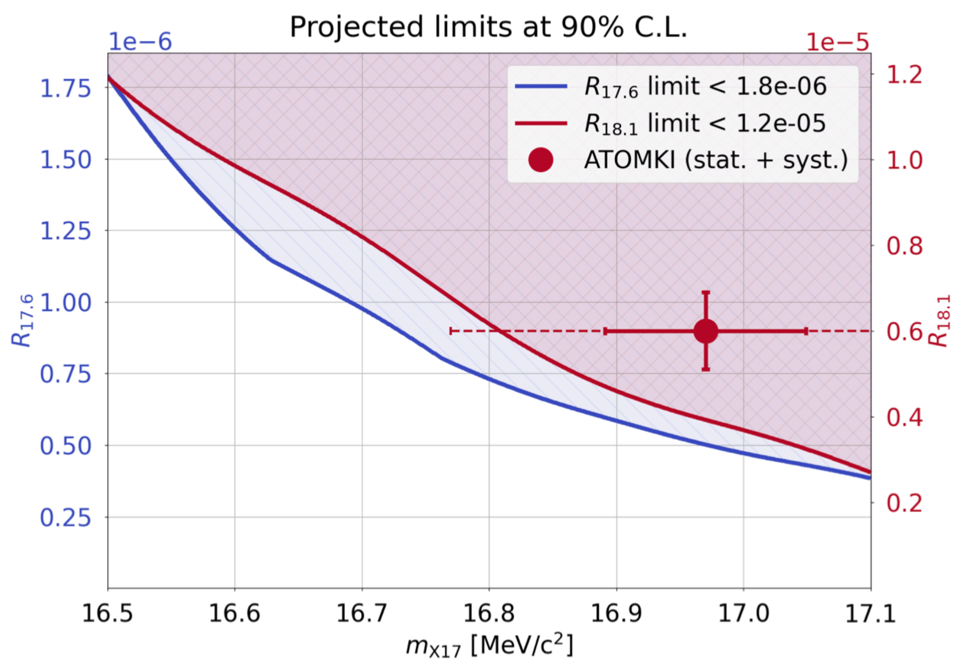
[1] **MEG-II** Collab., **arXiv:2411.07994**

MEG-II Experiment, PSI (Svizzera) [1]

X17 with Beryllium



Null result for Be → **1.5σ exclusion**



How to treat an inconclusive result?

Atomki signal is almost compatible with MEG-II

Either combine them…

	$R_{\mathrm{Be}}\ [10^{-6}]$
Atomki	6 ± 1 [1, 2]
MEG-II	< 5.3 at 90% CL [38]
Combined	5.5 ± 1.0

… or exclude Beryllium

↓

0^+ becomes possible

↓

Phenomenological analysis

Both scenarios explored

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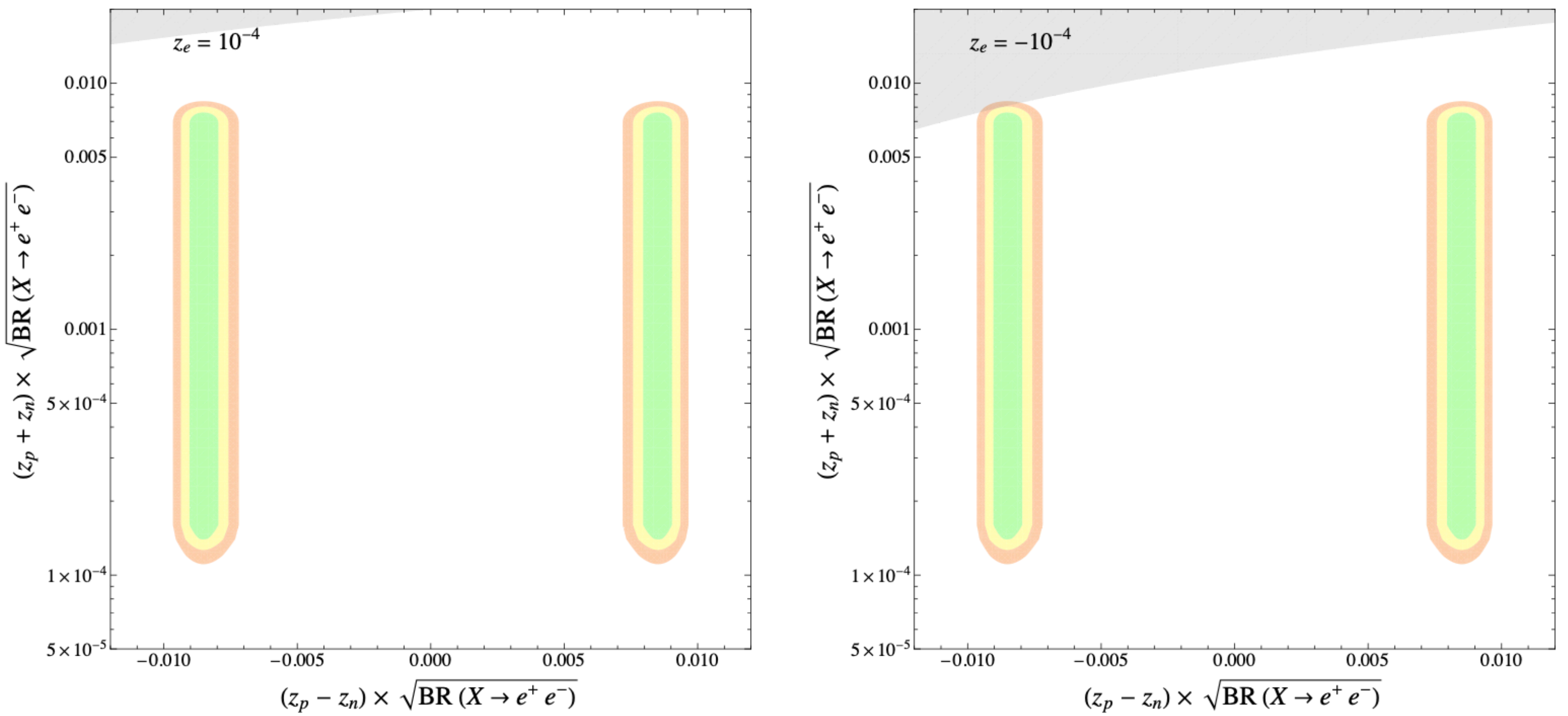
0^+ exclusion plot

Directly at renormalizable level $$\mathcal{L}_{\text{int}}^{d\leq 4} = z_p \bar{p} p X + z_n \bar{n} n X + z_e \bar{e} e X$$



Exclusion plots. Legend: Atomki observables compatible at 1σ (Green), 2σ (Yellow), 3σ (Orange). SINDRUM exclusion region in grey.

It works…

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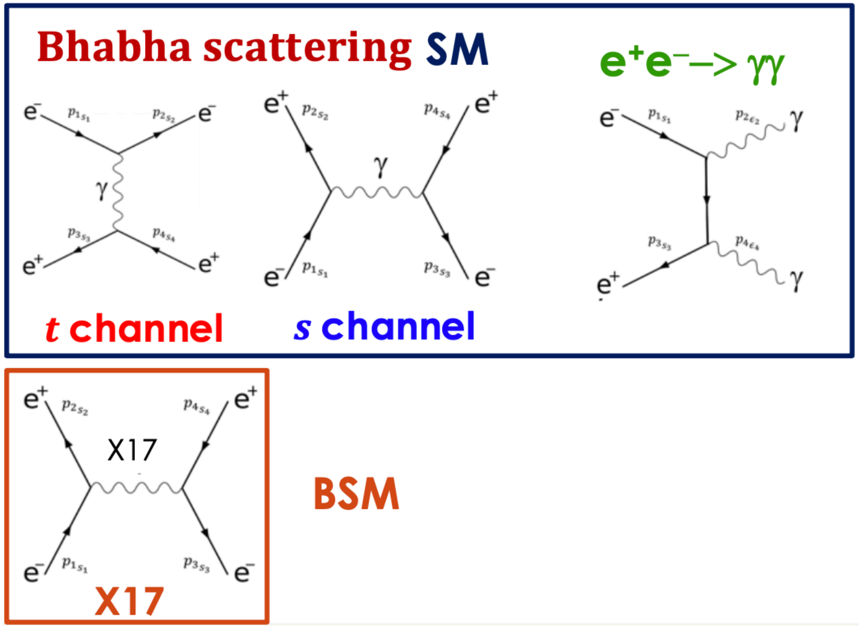
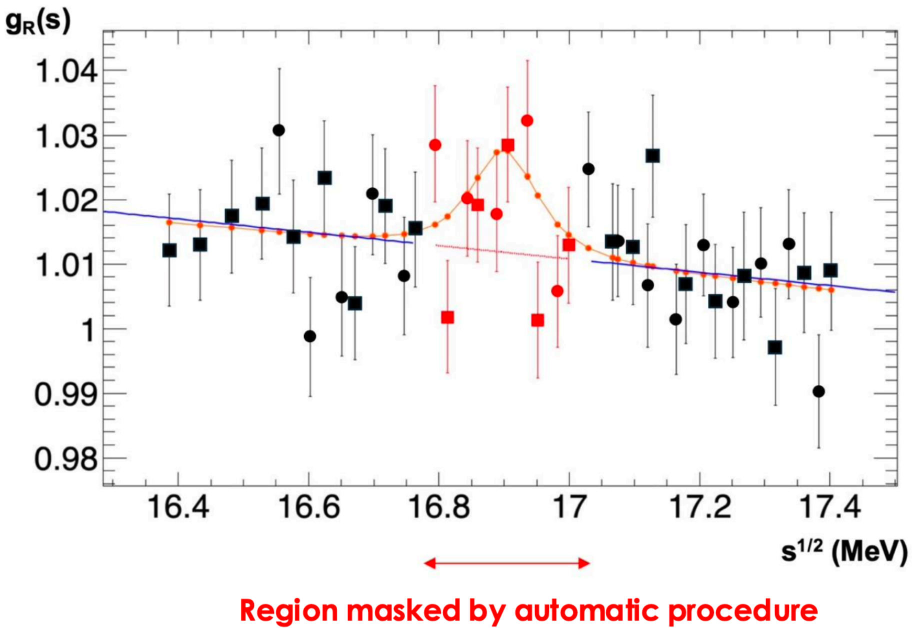
PADME's (unexpected) result

[1] **PADME** Collab., ***JHEP* 08 (2024) 121**

Positron beam against diamond target apparatus [1]:





PADME announced a local 2.5σ excess at 17 MeV at LDMA (April 2025)

Courtesy of Mauro Raggi

Conclusion

"X17 is dead! Long live X17!"

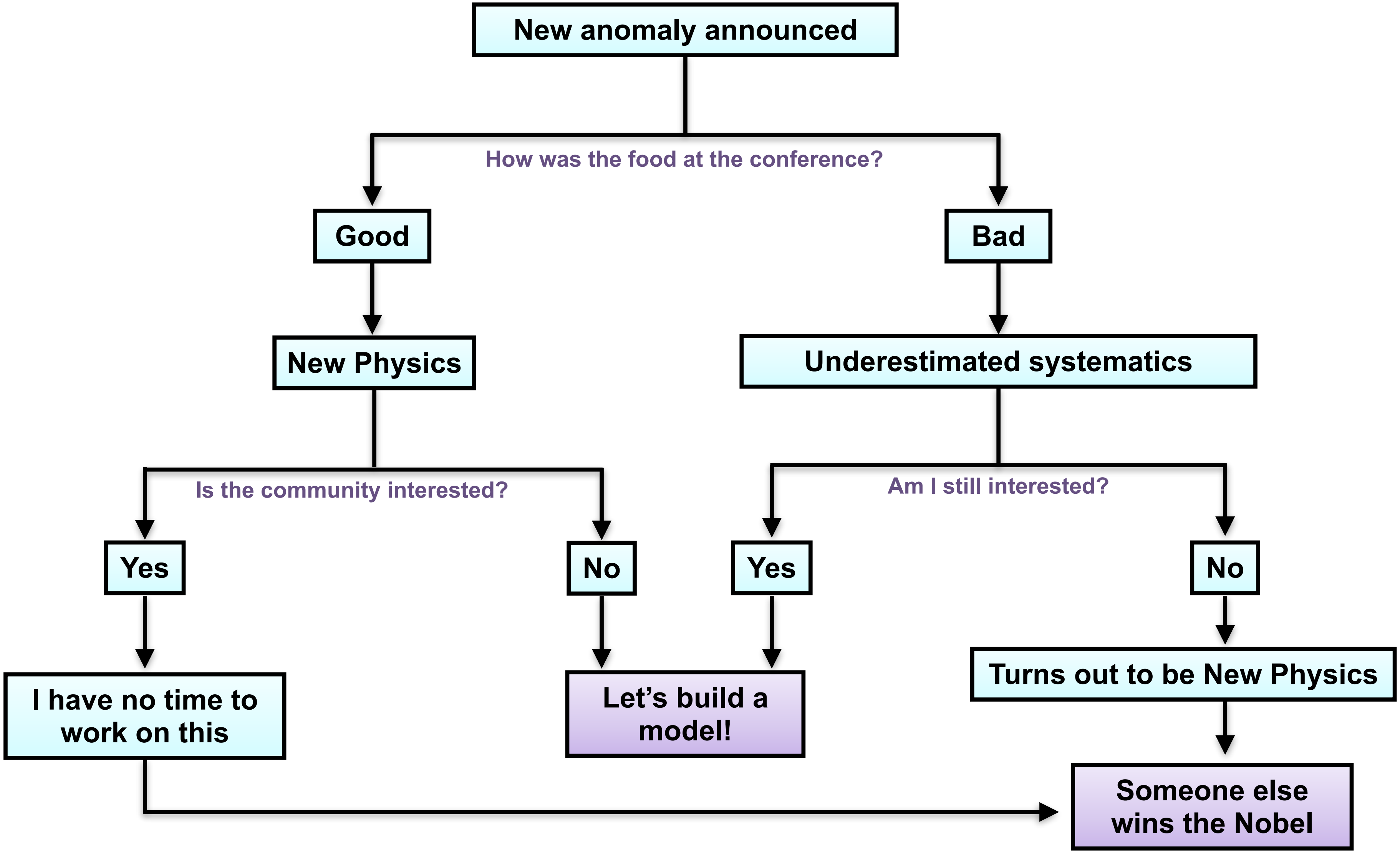
(Claudio Toni)

Excesses compatible X17 are observed left and right

BUT

No phenomenological model works so far

Procedural diagram of a theoretical physicist



Thanks for your attention

Backup slides

The independent X17 confirmation

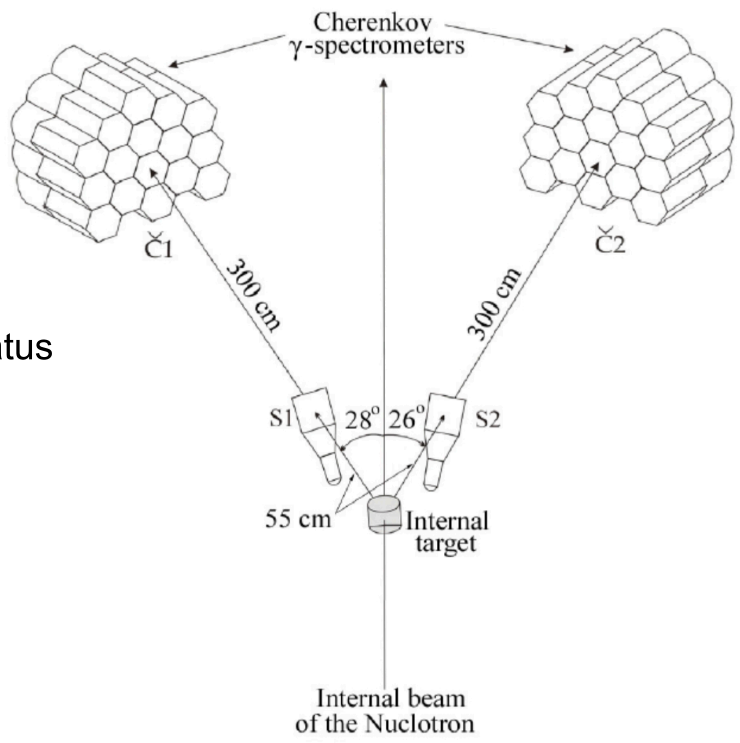
[1] Abraamyan et al, **Phys. Part. Nucl. 55(4):868-873**

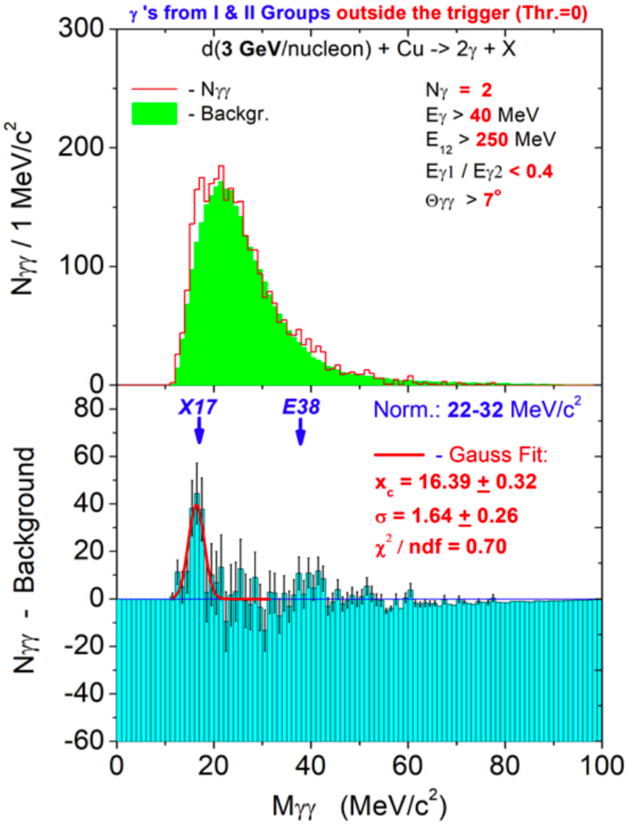
JINR Experiment [1] (Russia)

Observed process: $p + N \to \gamma\gamma + \dots$

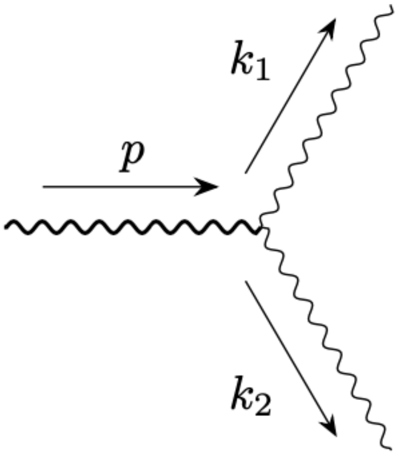
Decay: $X \to \gamma\gamma$

Apparatus





Landau-Yang theorem



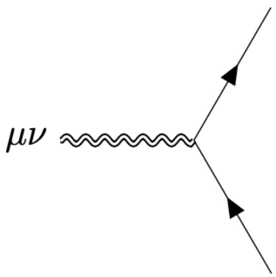
Theorem:

"A massive odd spin boson cannot decay into two photons."

It excludes spin 1

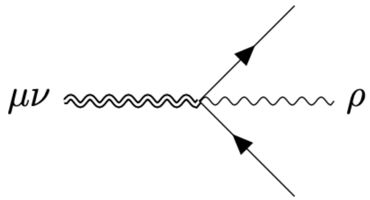
Spin 2?

Feynman rules spin 2



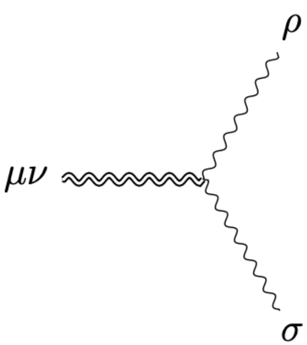
$$= \frac{i}{4\Lambda_e}\left[\gamma_\mu\left(k_\nu^- - k_\nu^+\right) + \gamma_\nu\left(k_\mu^- - k_\mu^+\right)\right]$$

$k_\mu^\pm$ outward fermion momentum



$$= \frac{-ie}{2\Lambda_e}\left[\gamma_\mu\,\delta_{\nu\rho} + \gamma_\nu\,\delta_{\mu\rho}\right]$$

From covariant derivative



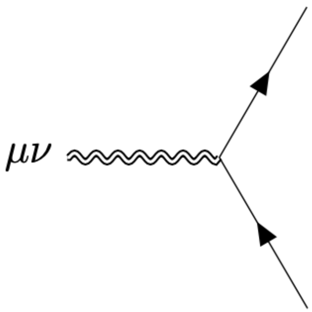
$$= \frac{1}{\Lambda_\gamma}\,\Pi^\xi_{\mu\nu\rho\sigma}(k_1, k_2)$$

$$\begin{aligned}\Pi^\xi_{\mu\nu\rho\sigma}(k_1,k_2) &= \delta_{\rho\sigma}\,\left(k_{1,\mu}\,k_{2,\nu} + k_{1,\nu}\,k_{2,\mu}\right) - \delta_{\mu\rho}\,k_{1,\sigma}\,k_{2,\nu} - \delta_{\mu\sigma}\,k_{1,\nu}\,k_{2,\rho}\\ &- \delta_{\nu\sigma}\,k_{1,\mu}\,k_{2,\rho} - \delta_{\nu\rho}\,k_{1,\sigma}\,k_{2,\mu} + k_1\cdot k_2\,\left(\delta_{\mu\sigma}\,\delta_{\nu\rho} + \delta_{\nu\sigma}\,\delta_{\mu\rho}\right)\\ &+\frac{1}{\xi}\left[-\delta_{\mu\rho}\,k_{2,\sigma}\,k_{2,\nu} - \delta_{\mu\sigma}\,k_{1,\nu}\,k_{1,\rho} - \delta_{\nu\sigma}\,k_{1,\mu}\,k_{1,\rho} - \delta_{\nu\rho}\,k_{2,\sigma}\,k_{2,\mu}\right]\end{aligned}$$

ξ gauge parameter

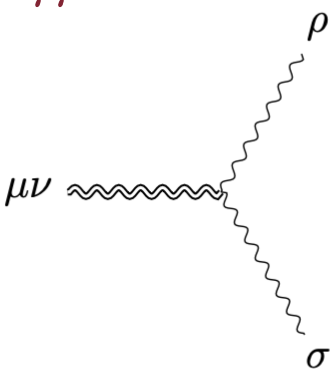
Spin 2 decays

$X \to e^+e^-$



$$\Gamma(X \to e^+e^-) = \frac{1}{\Lambda_e^2}\frac{m_X^3}{160\pi}\left[1 - \frac{4m_e^2}{m_X^2}\right]^{3/2}\left[1 + \frac{8}{3}\frac{m_e^2}{m_X^2}\right]$$

$X \to \gamma\gamma$



$$\Gamma(X \to \gamma\gamma) = \frac{1}{\Lambda_\gamma^2}\frac{m_X^3}{80\pi}$$

Atomki signal calculation (1)

Nuclear interaction from

$$H_{\rm int}^{s} = \int d^3\vec{r}\, \mathcal{H}_{\mu\nu}(\vec{r}) X^{\mu\nu}(\vec{r})$$

Interaction picture

$$\mathcal{T}_{fi}^{s} = \langle N, X | H_{\rm int}^{s} | N^\star \rangle = \langle N | \int d^3\vec{r}\, [\epsilon_a^{\mu\nu}(\vec{k})]^* \mathcal{H}_{\mu\nu}(\vec{r}) e^{-i\vec{k}\cdot\vec{r}} | N^\star \rangle$$

$\mathscr{H}_{\mu\nu}$ expanded in non-relativistic limit, in powers of $\langle p_N \rangle^2 / m_N^2 \approx 0.06$

$$\mathcal{H}^{\mu\nu} = \begin{pmatrix} \mathcal{E} & \vec{\mathcal{P}}^T \\ \vec{\mathcal{P}} & \hat{\mathcal{W}} \end{pmatrix}$$

$$\begin{aligned}
\mathcal{E}(\vec{r}) =& \sum_{s=1}^{A} m_s C_s \delta_{\vec{r},\vec{r}_s} \\
&+ \sum_{s=1}^{A} \sum_{i} \frac{C_s}{8m_s} \left[p_s^i p_s^i \delta_{\vec{r},\vec{r}_s} + 2 p_s^i \delta_{\vec{r},\vec{r}_s} p_s^i + \delta_{\vec{r},\vec{r}_s} p_s^i p_s^i \right] , \\
\vec{\mathcal{P}}(\vec{r}) =& \frac{1}{2} \sum_{s=1}^{A} C_s \left[\vec{p}_s \delta_{\vec{r},\vec{r}_s} + \delta_{\vec{r},\vec{r}_s} \vec{p}_s \right] + \vec{\nabla} \times \left(\frac{1}{4} \sum_{s=1}^{A} C_s \vec{\sigma}_s \delta_{\vec{r},\vec{r}_s} \right) , \\
\hat{\mathcal{W}}^{ij}(\vec{r}) =& \sum_{s=1}^{A} \frac{C_s}{4m_s} \left[p_s^i p_s^j \delta_{\vec{r},\vec{r}_s} + p_s^i \delta_{\vec{r},\vec{r}_s} p_s^j + p_s^j \delta_{\vec{r},\vec{r}_s} p_s^i + \delta_{\vec{r},\vec{r}_s} p_s^i p_s^j \right] \\
&- \sum_{s=1}^{A} \frac{C_s}{4m_s} (\vec{\sigma}_s \times \vec{\nabla})^j \left[p_s^i \delta_{\vec{r},\vec{r}_s} + \delta_{\vec{r},\vec{r}_s} p_s^i \right] \\
&- \sum_{s=1}^{A} \frac{C_s}{4m_s} (\vec{\sigma}_s \times \vec{\nabla})^i \left[p_s^j \delta_{\vec{r},\vec{r}_s} + \delta_{\vec{r},\vec{r}_s} p_s^j \right] ,
\end{aligned}$$

Tensor boson 2^+

$$\begin{aligned}
\mathcal{E}(\vec{r}) =& \frac{1}{2} \sum_{s=1}^{A} \tilde{C}_s \left[\vec{p}_s \delta_{\vec{r},\vec{r}_s} + \delta_{\vec{r},\vec{r}_s} \vec{p}_s \right] \cdot \vec{\sigma}_s , \\
\vec{\mathcal{P}}(\vec{r}) =& \frac{1}{2} \sum_{s=1}^{A} m_s \tilde{C}_s \vec{\sigma}_s \delta_{\vec{r},\vec{r}_s} , \\
\hat{\mathcal{W}}^{ij}(\vec{r}) =& \frac{1}{4} \sum_{s=1}^{A} \tilde{C}_s \left[p_s^i \delta_{\vec{r},\vec{r}_s} + \delta_{\vec{r},\vec{r}_s} p_s^i \right] \sigma_s^j \\
&+ \frac{1}{4} \sum_{s=1}^{A} \tilde{C}_s \left[p_s^j \delta_{\vec{r},\vec{r}_s} + \delta_{\vec{r},\vec{r}_s} p_s^j \right] \sigma_s^i .
\end{aligned}$$

Axial tensor boson 2^-

Atomki signal calculation (2)

Given $kr \approx 0.1$ → Long wavelength approximation

$$e^{-i\vec{k}\cdot\vec{r}} \approx 1 - i\vec{k}\cdot\vec{r} - \frac{1}{2}(\vec{k}\cdot\vec{r})^2 + \ldots$$

- Selection rules parity and angular momentum.
- Lowest order expansion.

Transition amplitude given as

$$\mathcal{T}^s_{fi} = \langle N| \sum_{\mathcal{O}} \sum_{JM} \mathcal{O}_{JM} |N^\star\rangle$$

Use Wigner-Eckart theorem

$$\langle J_f M_f | \mathcal{O}_{J,-M} | J_i M_i \rangle = \frac{(-1)^{J_i - M_i}}{\sqrt{2J+1}} C^{J,-M}_{J_f,M_f;J_i,-M_i} \langle J_f || \mathcal{O}_J || J_i \rangle$$

Use isospin conservation for Helium and Carbon

$$\sum_N m_N C_N \approx \frac{m_N}{2}\left(C_p + C_n\right)\mathbf{I} + \frac{m_N}{2}\left(C_p - C_n\right)\tau_z$$

$$\sum_N m_N \tilde{C}_N \approx \frac{m_N}{2}\left(\tilde{C}_p + \tilde{C}_n\right)\mathbf{I} + \frac{m_N}{2}\left(\tilde{C}_p - \tilde{C}_n\right)\tau_z$$

SINDRUM constraint (1)

Calculation method

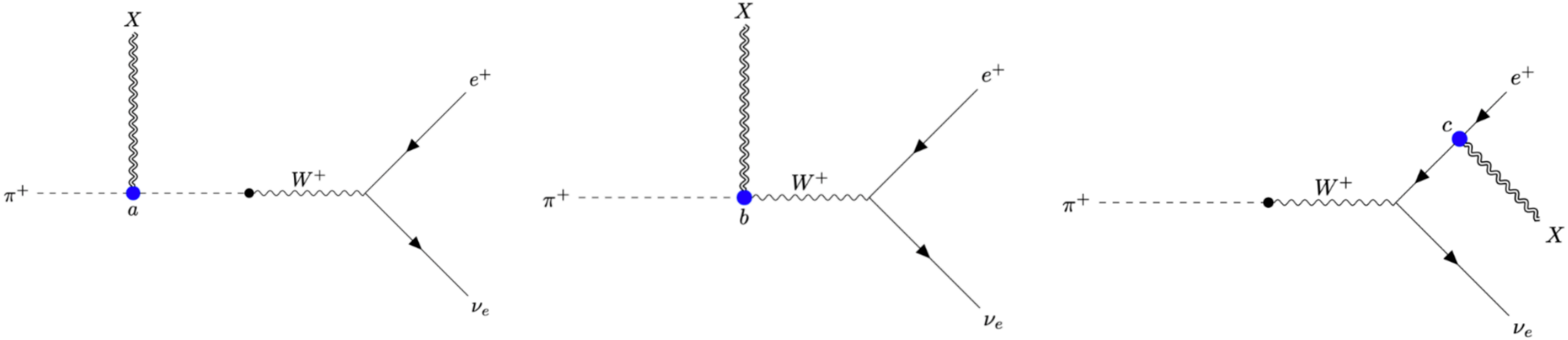
χPT used. At lowest order:

$$U = \exp\left\{\frac{i}{f_\pi}\begin{pmatrix}\pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0\end{pmatrix}\right\} \longrightarrow \mathcal{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[(D^\mu U)^\dagger D_\mu U\right] + \frac{f_\pi^2}{4}\mathrm{Tr}\left[U^\dagger\chi + \chi^\dagger U\right]$$

Spin 2 as external current $\chi_L^{\mu\nu} \to g_L \chi_L^{\mu\nu} g_L^\dagger$ e $\chi_R^{\mu\nu} \to g_R \chi_R^{\mu\nu} g_R^\dagger$

$$\begin{aligned}
\mathcal{L}_{\chi\mathrm{PT}} + \Delta\mathcal{L}_{\chi\mathrm{PT}}^{\text{spin-2}} \supset\ & (\partial_\mu\pi^+)(\partial^\mu\pi^-) - m_\pi^2\pi^+\pi^- \\
& + \eta_3(C_u + C_d)X^{\mu\nu}(\partial_\mu\pi^+)(\partial_\nu\pi^-) \\
& + \frac{gf_\pi}{2}\eta_3(C_u + C_d)X^{\mu\nu}(V_{ud}W_\mu^+\partial_\nu\pi^- + V_{ud}^*W_\mu^-\partial_\nu\pi^+) \\
& - i\frac{gf_\pi}{2}\eta_2(C_u - C_d)X^{\mu\nu}(V_{ud}W_\mu^+\partial_\nu\pi^- - V_{ud}^*W_\mu^-\partial_\nu\pi^+) \\
& + i\frac{gf_\pi}{2}\eta_2(\tilde{C}_u - \tilde{C}_d)X^{\mu\nu}(V_{ud}W_\mu^+\partial_\nu\pi^- - V_{ud}^*W_\mu^-\partial_\nu\pi^+) \\
& + \frac{gf_\pi}{2}(V_{ud}W_\mu^+\partial_\nu\pi^- + V_{ud}^*W_\mu^-\partial_\nu\pi^+) + \ldots\ ,
\end{aligned}$$

SINDRUM constraint (2)



$$\mathrm{BR}(\pi^+ \to e^+\nu_e X) = \frac{m_\pi^{12}\left(10\left(\eta_2^2(C_d - C_u)^2 + \eta_3^2(C_u + C_d)^2\right) + 3C_e^2 - 10C_e\eta_3(C_u + C_d)\right)}{2^8\ 3^3\ 5\pi^2 m_\mu^2 m_X^4\left(m_\pi^2 - m_\mu^2\right)^2}$$

Tensor boson 2^+

$$\mathrm{BR}(\pi^+ \to e^+\nu_e X) = \frac{m_\pi^{12}\left(10\eta_2^2(\tilde{C}_d - \tilde{C}_u)^2 + 3\tilde{C}_e^2\right)}{2^8\ 3^3\ 5\pi^2 m_\mu^2 m_X^4\left(m_\pi^2 - m_\mu^2\right)^2}$$

Axial tensor boson 2^-

SINDRUM constraint (3)

From quark to nucleon coupling

Static quark model [13].

Nucleons are 3 states quark $|q_1\uparrow, q_2\uparrow, q_3\downarrow\rangle$. Identical quarks in $J=1$.

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|1,1;\ \frac{1}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|1,0;\ \frac{1}{2},\frac{1}{2}\right\rangle$$

Tensor boson 2^+

$\mathcal{E} \approx m_s C_s \mathbf{1}$ Non-relativistically

$$C_p = \frac{1}{m_N}\langle p|\,\mathcal{E}\,|p\rangle = \frac{2}{3}\frac{2m_u^{\text{eff}}C_u + m_d^{\text{eff}}C_d}{m_N} + \frac{1}{3}\frac{2m_u^{\text{eff}}C_u + m_d^{\text{eff}}C_d}{m_N} = \frac{2}{3}C_u + \frac{1}{3}C_d$$

$$C_n = \frac{1}{m_N}\langle n|\,\mathcal{E}\,|n\rangle = \frac{2}{3}\frac{m_u^{\text{eff}}C_u + 2m_d^{\text{eff}}C_d}{m_N} + \frac{1}{3}\frac{m_u^{\text{eff}}C_u + 2m_d^{\text{eff}}C_d}{m_N} = \frac{1}{3}C_u + \frac{2}{3}C_d$$

Axial tensor boson 2^-

$\vec{\mathcal{P}} \approx m_s \tilde{C}_s \vec{\sigma}_s$ Non-relativistically

$$\tilde{C}_p = \frac{1}{m_N}\langle p|\,\mathcal{P}\,|p\rangle = \frac{2}{3}\frac{2m_u^{\text{eff}}\tilde{C}_u - m_d^{\text{eff}}\tilde{C}_d}{m_N} + \frac{1}{3}\frac{m_d^{\text{eff}}\tilde{C}_d}{m_N} = \frac{4}{9}\tilde{C}_u - \frac{1}{9}\tilde{C}_d$$

$$\tilde{C}_n = \frac{1}{m_N}\langle n|\,\mathcal{P}\,|n\rangle = \frac{2}{3}\frac{-m_u^{\text{eff}}\tilde{C}_u + 2m_d^{\text{eff}}\tilde{C}_d}{m_N} + \frac{1}{3}\frac{m_u^{\text{eff}}\tilde{C}_u}{m_N} = -\frac{1}{9}\tilde{C}_u + \frac{4}{9}\tilde{C}_d$$

$$\text{BR}(\pi^+ \to e^+\nu_e X) = \frac{m_\pi^{12}\left(90\left(\eta_2^2(C_p - C_n)^2 + \eta_3^2(C_p + C_n)^2\right) + 3C_e^2 + 10C_e\eta_3(C_p + C_n)\right)}{2^8\ 3^3\ 5\pi^2 m_\mu^2 m_X^4\left(m_\pi^2 - m_\mu^2\right)^2}$$

Tensor boson 2^+

$$\text{BR}(\pi^+ \to e^+\nu_e X) = \frac{m_\pi^{12}\left(54\eta_2^2(\tilde{C}_p - \tilde{C}_n)^2 + 5\tilde{C}_e^2\right)}{2^8\ 3^2\ 5^2\pi^2 m_\mu^2 m_X^4\left(m_\pi^2 - m_\mu^2\right)^2}$$

Axial tensor boson 2^-

Lagrangian for 0^+

Renormalizable level

$$\mathcal{L}_{\rm int}^{d\leq 4} = z_p \bar{p} p X + z_n \bar{n} n X + z_e \bar{e} e X$$

With effective photon coupling

$$\mathcal{L}_{\rm int}^{d=5} = \frac{\alpha}{8\pi} \frac{X}{f_\gamma} F_{\mu\nu} F^{\mu\nu}$$

Relevant couplings z_e, z_p, z_n.
No neutrino couplings.

Decay rate

$$\Gamma = \Gamma(X \to e^+ e^-) = \frac{z_e^2 m_X}{8\pi} \left(1 - \frac{4m_e^2}{m_X^2}\right)^{3/2}$$

Atomki signal and other constraints

Prompt decay in Atomki

Decay in apparatus (geometrically)

$$\Gamma \geq 1.3 \times 10^{-4}\ \text{eV} \longrightarrow |z_e| \geq 1.4 \times 10^{-5}$$

Electron $g-2$

$$\delta a_e^{\text{BSM}} \approx \frac{z_e^2}{4\pi^2}\frac{m_e^2}{m_X^2}\left[\ln\frac{m_X}{m_e} - \frac{7}{12}\right] \longrightarrow |z_e| \leq 10^{-4}$$

Atomki observables

Same method as before

$$R_{\text{He}} = \frac{1}{\alpha^2}\frac{15}{8}\left(\frac{k}{\omega}\right)^5 (z_p + z_n)^2 \left|1 + 3r_{\text{He}}\right|^2 \text{BR}(X \to e^+e^-) \,,$$

$$R_{\text{C}} = \left(\frac{k}{\omega}\right)^3 \frac{(z_p - z_n)^2}{8\pi\alpha_e}\text{BR}(X \to e^+e^-) \,,$$

Spin 0 SINDRUM bound (1)

Scalar boson included in $\chi = 2B_0(s + ip)$

$$s + ip = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} - X \begin{pmatrix} z_u & 0 \\ 0 & z_d \end{pmatrix}$$

Need $\mathcal{O}(p^4)$ Lagrangian

$$\begin{aligned}
\mathcal{L}_{\chi\mathrm{PT}}^{\mathrm{NLO}} =& L_1 \mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right]^2 + L_2 \mathrm{Tr}\left[D_\mu U^\dagger D_\nu U\right] \mathrm{Tr}\left[D^\mu U^\dagger D^\nu U\right] \\
&+ L_3 \mathrm{Tr}\left[D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\right] + L_4 \mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] \mathrm{Tr}\left[U^\dagger \chi + \chi^\dagger U\right] \\
&+ L_5 \mathrm{Tr}\left[D_\mu U^\dagger D^\mu U (U^\dagger \chi + \chi^\dagger U)\right] + L_6 \mathrm{Tr}\left[U^\dagger \chi + \chi^\dagger U\right]^2 \\
&+ L_7 \mathrm{Tr}\left[U^\dagger \chi - \chi^\dagger U\right]^2 + L_8 \mathrm{Tr}\left[U^\dagger \chi U^\dagger \chi + \chi^\dagger U \chi^\dagger U\right] + \dots\ ,
\end{aligned}$$

After NLO correction to kinetic and m_π

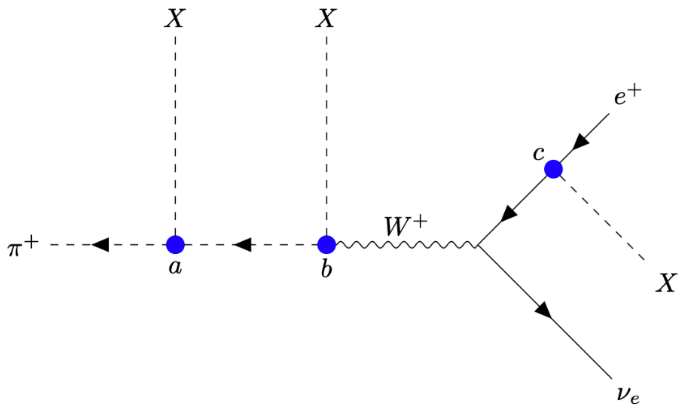
$$\begin{aligned}
\mathcal{L}_{\chi\mathrm{PT}} + \mathcal{L}_{\chi\mathrm{PT}}^{\mathrm{NLO}}\,|_{\mathrm{rescaled}} \supset\ & (\partial_\mu \pi^+)(\partial^\mu \pi^-) - m_\pi^2 \pi^+ \pi^- \\
& + (1 + \delta_1) m_\pi^2 \frac{z_u + z_d}{m_u + m_d} X \pi^+ \pi^- \\
& - 2\delta_2 \frac{z_u + z_d}{m_u + m_d} X (\partial_\mu \pi^+)(\partial^\mu \pi^-) \\
& - \delta_2 g f_\pi \frac{z_u + z_d}{m_u + m_d} X (V_{ud} W_\mu^+ \partial^\mu \pi^- + V_{ud}^* W_\mu^- \partial^\mu \pi^+) \\
& + \frac{g f_\pi}{2} (1 + \delta_2)(V_{ud} W_\mu^+ \partial_\nu \pi^- + V_{ud}^* W_\mu^- \partial_\nu \pi^+) + \dots\ ,
\end{aligned}$$

$$\delta_1 = 16 \frac{m_\pi^2}{f_\pi^2} \left[2L_6(m_\pi) + L_8(m_\pi)\right]$$

$$\delta_2 = 4 \frac{m_\pi^2}{f_\pi^2} \left[2L_4(m_\pi) + L_5(m_\pi)\right]$$

Spin 0 SINDRUM bound (2)

Same bremsstrahlung diagrams



Final result

$$\Gamma(\pi^+ \to e^+ \nu_e X) = \frac{G_F^2 f^2 |V_{ud}|^2 m_\pi^3}{32(2\pi)^3} \left[(z_u + z_d)^2 F_1 + z_e (z_u + z_d) F_2 + z_e^2 F_3 \right]$$

$F_1 \cong 0.024 \qquad F_2 \cong -0.143 \qquad F_3 \cong 0.676$