Electroweak (composite) Dark Matter

Dario Buttazzo

Istituto Nazionale di Fisica Nucleare

2107.09688, 2205.04486 with Bottaro, Costa, Franceschini, Panci, Redigolo, Vittorio
work in progress with Di Lecce, Franceschini, Panci

XXXVIII Convegno Nazionale di Fisica Teorica — Cortona, 22.05.2025

Why physics at the TeV scale?

- What causes EWSB?

  i.e. *does the SM hold up to accessible energy scales?*

- What's the origin of Flavor and CP violation?

- What is Dark Matter?

- Unification
- Inflation
- Dark Energy
- Quantum gravity

  ...

Why physics at the TeV scale?

- What causes EWSB?

  i.e. *does the SM hold up to accessible energy scales?*

Clearly points to few TeV scale

- What's the origin of Flavor and CP violation?

- What is Dark Matter?

Might point to few TeV scale, esp. if related to previous point

WIMP

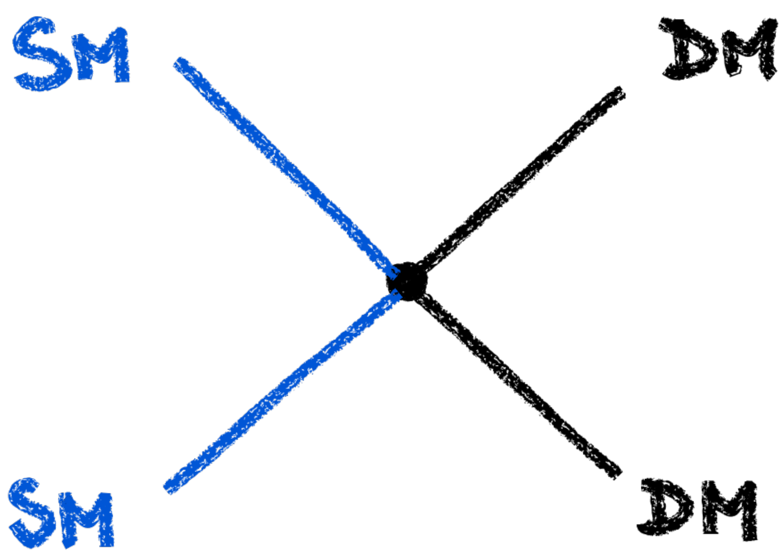
- Unification
- Inflation
- Dark Energy
- Quantum gravity

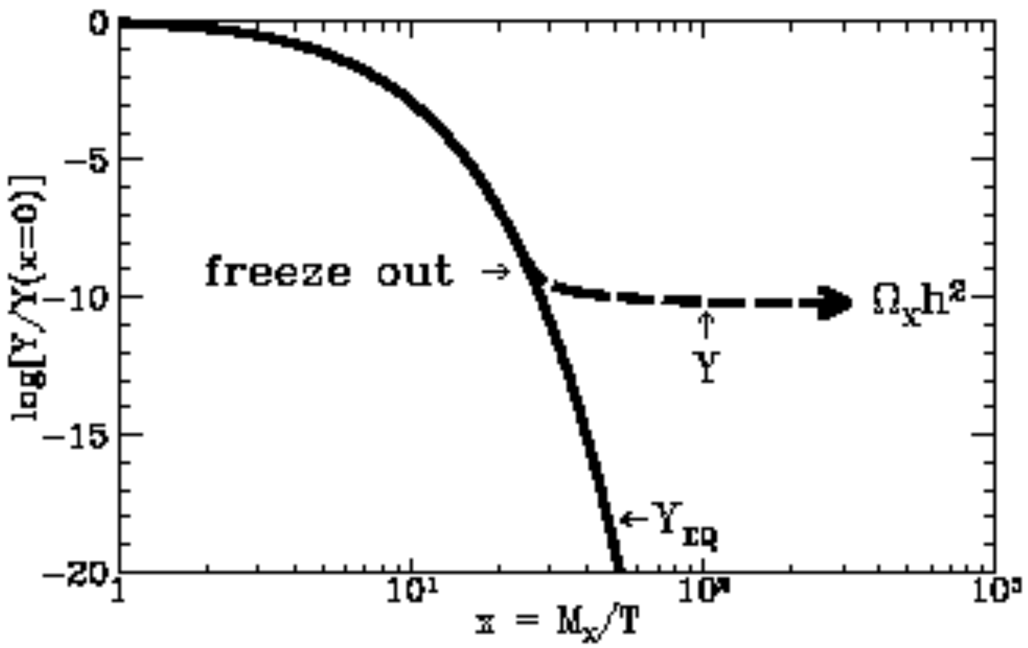
  …

Not clear if the solution lies at a scale accessible by experiments, and/or within particle physics…

The case for WIMPs

- Production in early Universe: thermal freeze-out of $2 \to 2$ scatterings





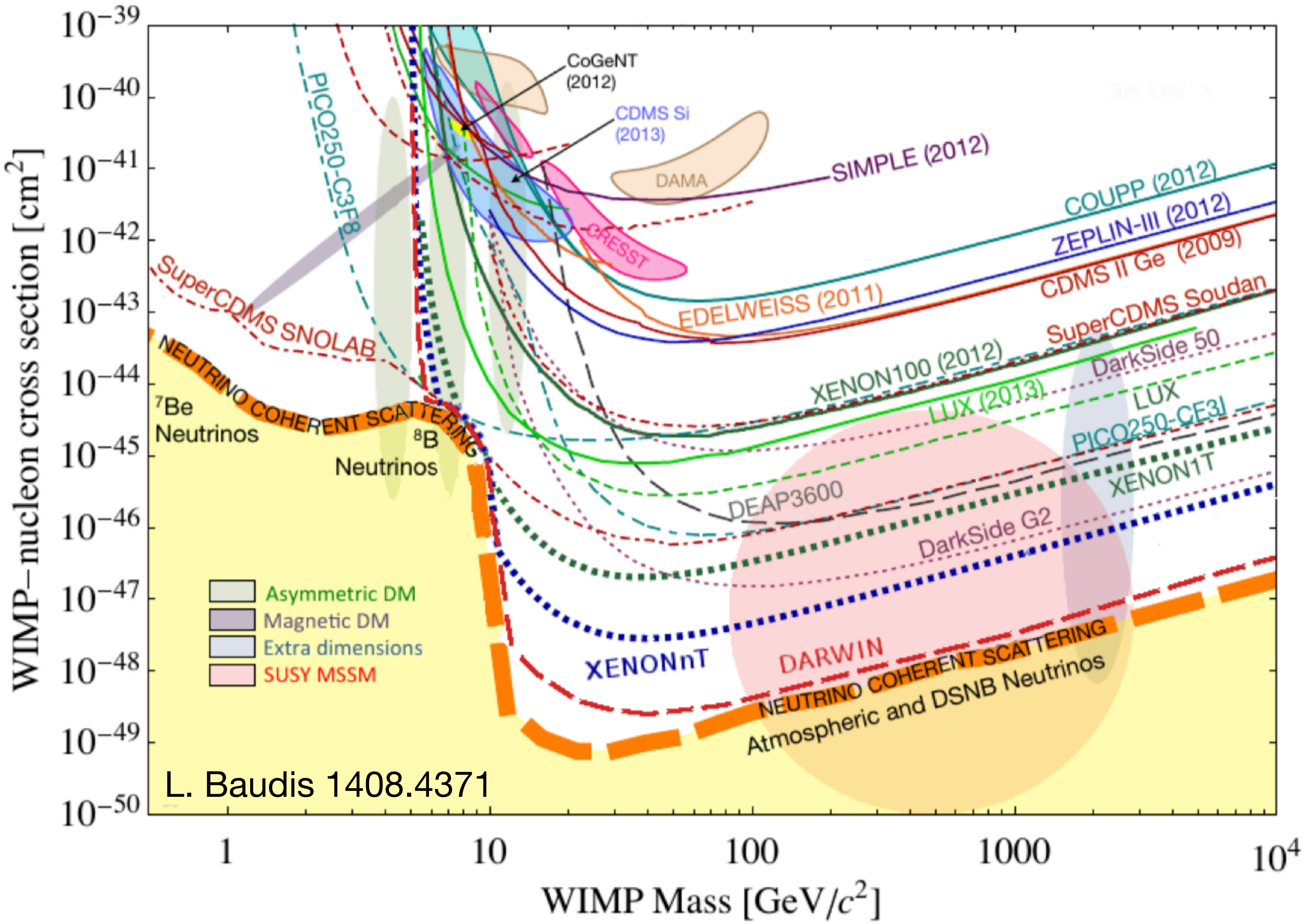
- For each value of the DM-SM coupling $g_\star$ the DM mass is predicted.

$$\boxed{g_\star \sim g_{EW} \;\Rightarrow\; M_{DM} \sim \text{TeV}}$$

- **WIMP miracle:** simple explanation for the observed Dark Matter abundance ($\Omega_{DM} \sim 0.26$) and a connection to naturalness of EW scale.

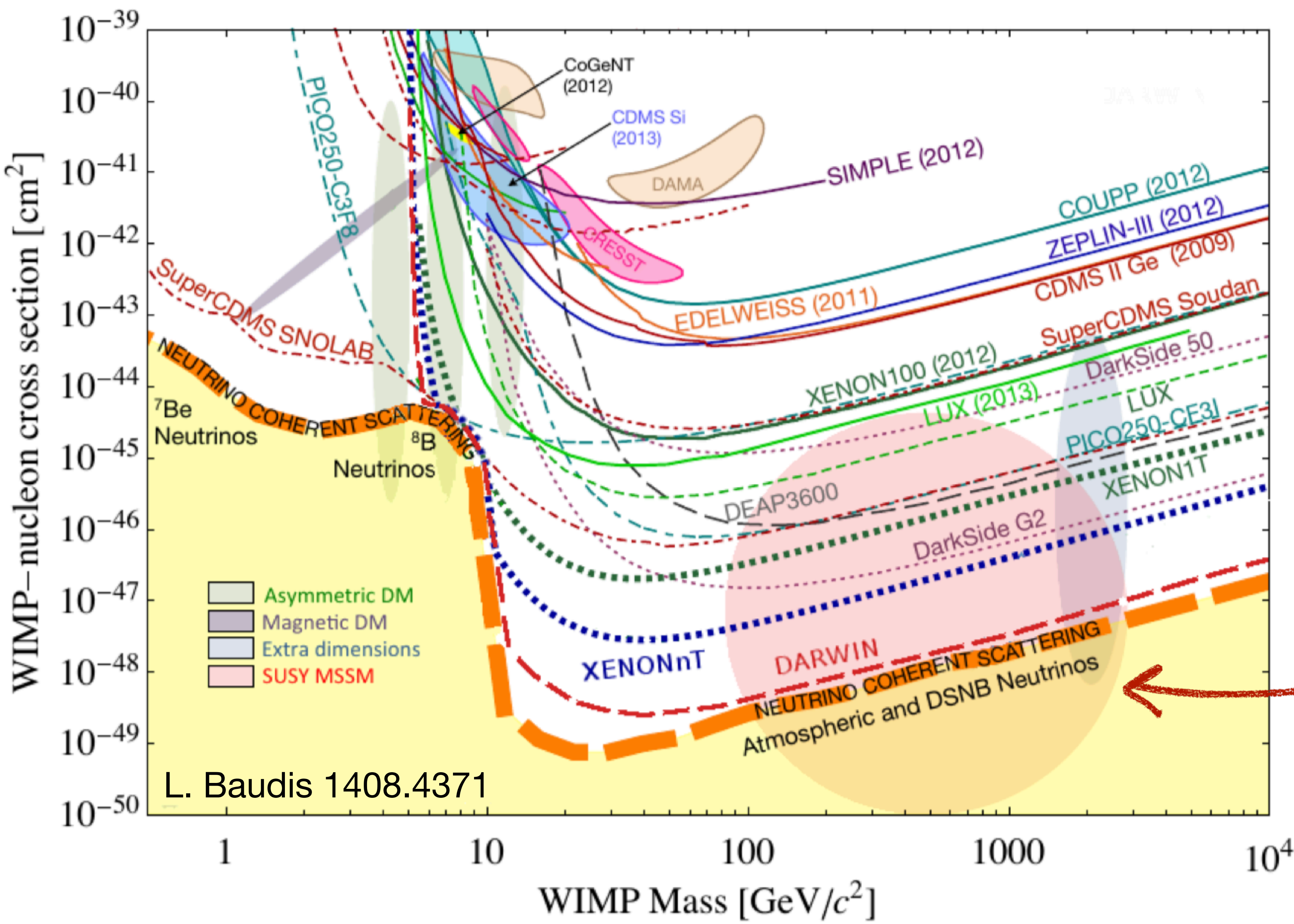
Ideal target for nuclear recoils & colliders!

Are WIMPs almost dead?



- Large fraction of the “standard” WIMP parameter space ruled out?

Are WIMPs almost dead?



WIMP param. space in specific models e.g. MSSM

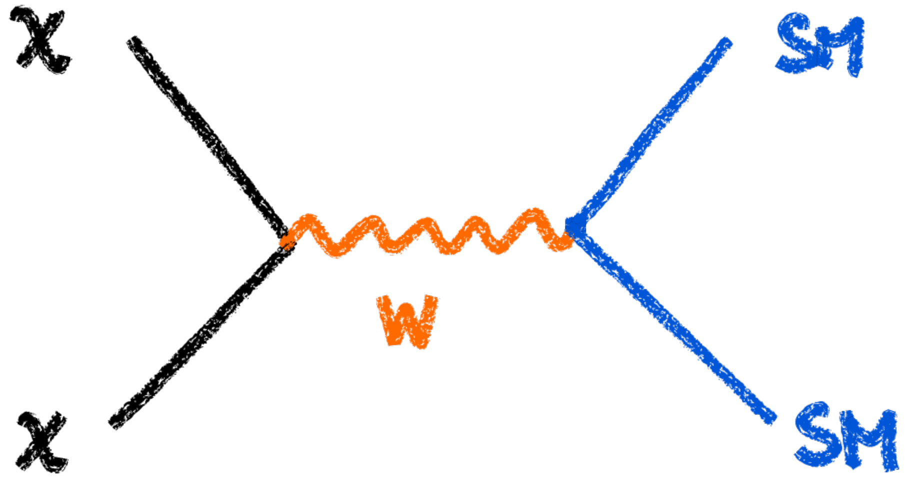
- Large fraction of the “standard” WIMP parameter space ruled out?

Not quite yet…

Which WIMP?

Consider generic EW multiplet: interacts w/ SM through W, Z

"Minimal Dark Matter": Cirelli, Fornengo, Strumia 2005



- No EM charge: DM is the neutral component

$$\chi_n = (\cdots, \chi^-, \chi^0, \chi^+, \cdots)$$

- DM needs to be stable: χ^0 lightest state in the multiplet

- Strong bounds from Direct Detection: no Z coupling @ tree-level

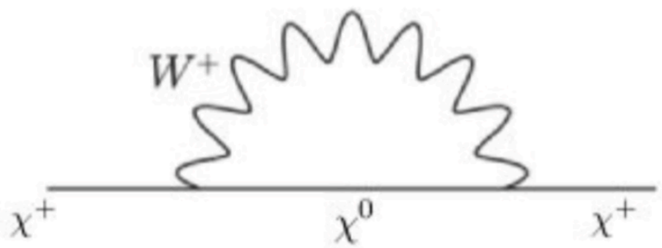
- Calculable: the WIMP EFT (without other states) must not break down when computing the relic abundance

➡ ∀ n-plet, single parameter sets the DM abundance: mass M_{DM}

Which WIMP?

- Minimal scenario: real multiplet (n odd), Y = 0 $\qquad \chi_n = (\cdots, \chi^-, \chi^0, \chi^+, \cdots)$
  - ✔ The neutral component χ^0 is automatically the lightest state: mass splitting induced at loop-level after EWSB

$$\Delta M_Q = \delta_{\rm EW} Q^2 \approx (167\,{\rm MeV})\, Q^2$$



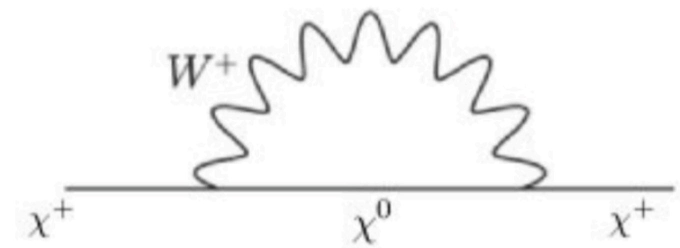
  - ✔ No coupling to Z boson: $(T^3_L)_{00} = 0$, Y = 0.

Which WIMP?

- Minimal scenario: real multiplet (n odd), Y = 0 $\chi_n = (\cdots, \chi^-, \chi^0, \chi^+, \cdots)$

  ✔ The neutral component χ^0 is automatically the lightest state: mass splitting induced at loop-level after EWSB

$$\Delta M_Q = \delta_{\rm EW} Q^2 \approx (167\,\text{MeV})\, Q^2$$



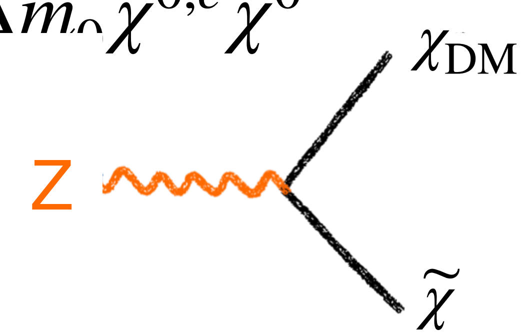
  ✔ No coupling to Z boson: $(T^3_L)_{00} = 0$, Y = 0.

- Complex WIMPs, any n and Y ≠ 0: non-minimal

  - Charged-neutral splitting $\mathcal{O}_+ = (\bar{\chi} T^a \chi)(H^\dagger \sigma^a H)$
    to make χ^0 the lightest state

  - Inelastic splitting $\mathcal{O}_0 = \left(\bar{\chi}(T^a)^{2Y}\chi^c\right)\left(H^{c,\dagger}\sigma^a H\right)^{2Y} \supset \Delta m_0 \chi^{0,c}\chi^0$
    needed to suppress Z-mediated direct detection

$$\left(\chi^0, \chi^{0,c}\right) \longrightarrow \left(\chi_{\rm DM}, \tilde{\chi}^0\right)$$ mass eigenstates



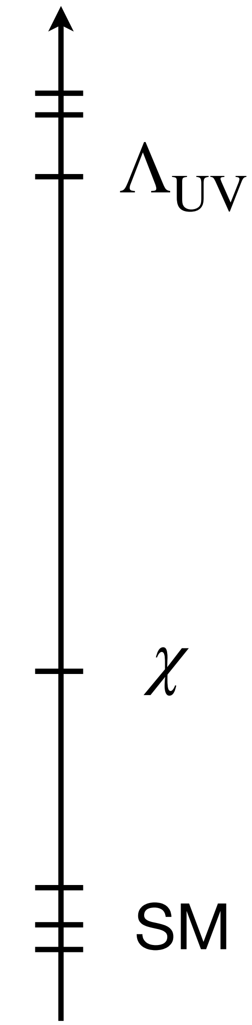
WIMP stability

- Plenty of other operators at the UV scale

$$\mathscr{L}_f \supset \frac{\chi H L}{\Lambda_{\mathrm{UV}}^{n-3}} (H^\dagger H)^{\#} W^{\#\#} + \frac{\chi^3}{\Lambda_{\mathrm{UV}}^3} H L$$

and similar for scalars

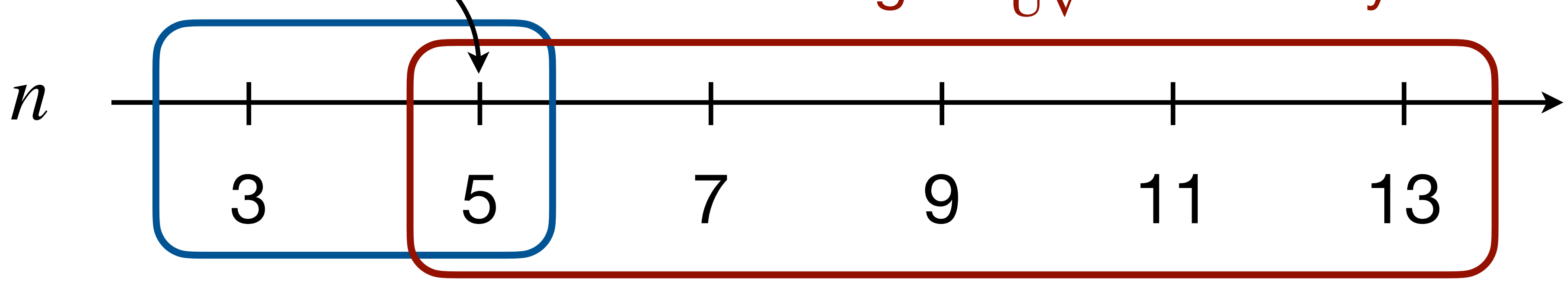
(for triplet: $\chi HL, \chi H^\dagger H$ are renormalizable)



… some can induce DM decay!

can be accidentally stable

need large Λ_{UV} for stability



Λ_{UV} can be ~ Planck

Landau pole gets closer $\longrightarrow$

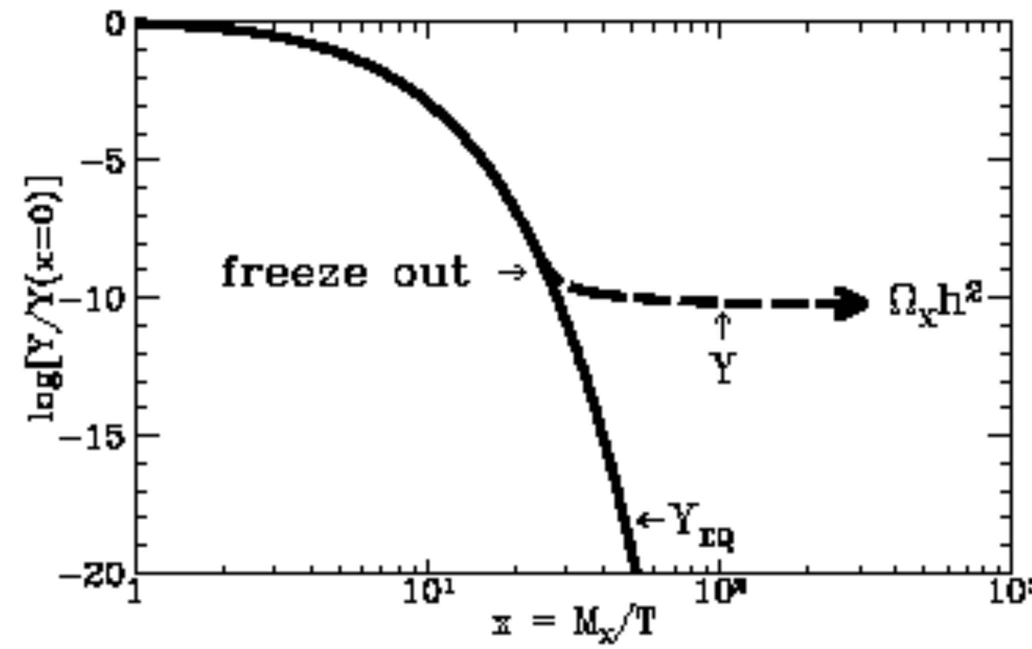
in all other cases need to impose stability in UV theory

Computing the relic abundance

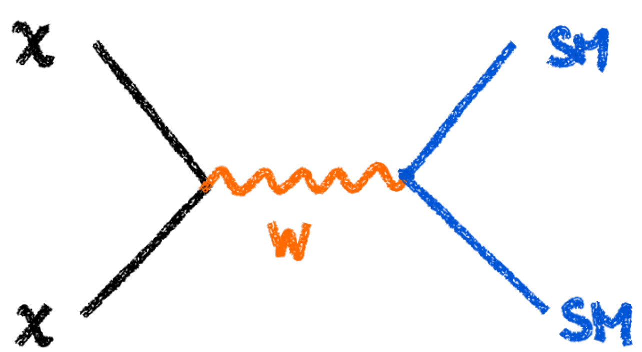
- Consider generic EW multiplet: interacts w/ SM through W, Z

$$\frac{dY}{dx} \propto \langle \sigma v \rangle (Y^2 - Y_{\rm eq}^2)$$

which cross-section?



- Tree-level EW cross-section…

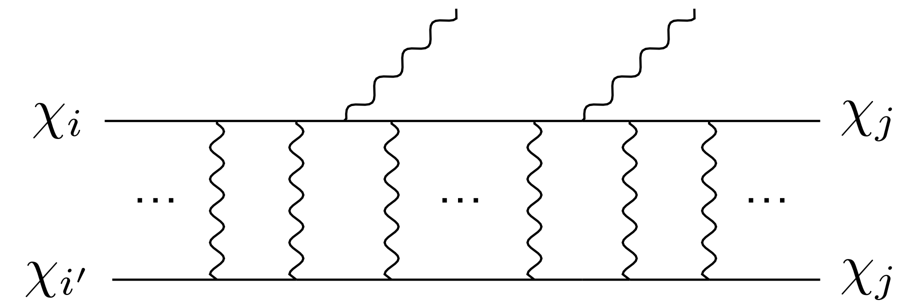


$$\langle \sigma v \rangle_0 = \frac{\pi \alpha_2^2 (2n^4 + 17n^2 - 19)}{16 g_\chi M_\chi^2}$$

… is inaccurate!

Large non-perturbative, non-relativistic effects

- Sommerfeld enhancement
- Bound state formation



Bound state formation

- Coupled Boltzmann eq. for DM and bound states:

$$z\frac{\mathrm{d}Y_{\mathrm{DM}}}{\mathrm{d}z} = -\frac{2s}{H}\langle\sigma_{\mathrm{ann}}v_{\mathrm{rel}}\rangle\left[Y_{\mathrm{DM}}^2-(Y_{\mathrm{DM}}^{\mathrm{eq}})^2\right] - \frac{2s}{Hz}\sum_{B_I}\langle\sigma_{B_I}v_{\mathrm{rel}}\rangle\left[Y_{\mathrm{DM}}^2-(Y_{\mathrm{DM}}^{\mathrm{eq}})^2\frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right],$$

$$z\frac{\mathrm{d}Y_{B_I}}{\mathrm{d}z} = Y_{B_I}^{\mathrm{eq}}\left\{\frac{\langle\Gamma_{B_I,\mathrm{break}}\rangle}{H}\left[\frac{Y_{\mathrm{DM}}^2}{(Y_{\mathrm{DM}}^{\mathrm{eq}})^2}-\frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right]+\frac{\langle\Gamma_{B_I,\mathrm{ann}}\rangle}{H}\left[1-\frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right]+\sum_{B_J}\frac{\langle\Gamma_{B_I\to B_J}\rangle}{H}\left[\frac{Y_{B_J}}{Y_{B_J}^{\mathrm{eq}}}-\frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right]\right\}$$

BS breakup in thermal plasma (negligible for tight bound states)

annihilation

decay into other BS

Bound state formation

- Coupled Boltzmann eq. for DM and bound states:

$$z\frac{\mathrm{d}Y_{\mathrm{DM}}}{\mathrm{d}z} = -\frac{2s}{H}\langle\sigma_{\mathrm{ann}}v_{\mathrm{rel}}\rangle\left[Y_{\mathrm{DM}}^2 - (Y_{\mathrm{DM}}^{\mathrm{eq}})^2\right] - \frac{2s}{Hz}\sum_{B_I}\langle\sigma_{B_I}v_{\mathrm{rel}}\rangle\left[Y_{\mathrm{DM}}^2 - (Y_{\mathrm{DM}}^{\mathrm{eq}})^2\frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right],$$

$$z\frac{\mathrm{d}Y_{B_I}}{\mathrm{d}z} = Y_{B_I}^{\mathrm{eq}}\left\{\frac{\langle\Gamma_{B_I,\mathrm{break}}\rangle}{H}\left[\frac{Y_{\mathrm{DM}}^2}{(Y_{\mathrm{DM}}^{\mathrm{eq}})^2} - \frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right] + \frac{\langle\Gamma_{B_I,\mathrm{ann}}\rangle}{H}\left[1 - \frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right] + \sum_{B_J}\frac{\langle\Gamma_{B_I\to B_J}\rangle}{H}\left[\frac{Y_{B_J}}{Y_{B_J}^{\mathrm{eq}}} - \frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right]\right\}$$

- if BS decay/annihilate quickly

$$\boxed{\frac{\mathrm{d}Y_{\mathrm{DM}}}{\mathrm{d}z} = -\frac{\langle\sigma_{\mathrm{eff}}v_{\mathrm{rel}}\rangle s}{Hz}(Y_{\mathrm{DM}}^2 - Y_{\mathrm{DM}}^{\mathrm{eq},2})}$$

$$\langle\sigma_{\mathrm{eff}}v_{\mathrm{rel}}\rangle \equiv S_{\mathrm{ann}}(z) + \sum_{B_J} S_{B_J}(z)$$

Bound state formation

- Coupled Boltzmann eq. for DM and bound states:

$$z\frac{\mathrm{d}Y_{\mathrm{DM}}}{\mathrm{d}z} = -\frac{2s}{H}\langle\sigma_{\mathrm{ann}}v_{\mathrm{rel}}\rangle\left[Y_{\mathrm{DM}}^2 - (Y_{\mathrm{DM}}^{\mathrm{eq}})^2\right] - \frac{2s}{Hz}\sum_{B_I}\langle\sigma_{B_I}v_{\mathrm{rel}}\rangle\left[Y_{\mathrm{DM}}^2 - (Y_{\mathrm{DM}}^{\mathrm{eq}})^2\frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right],$$

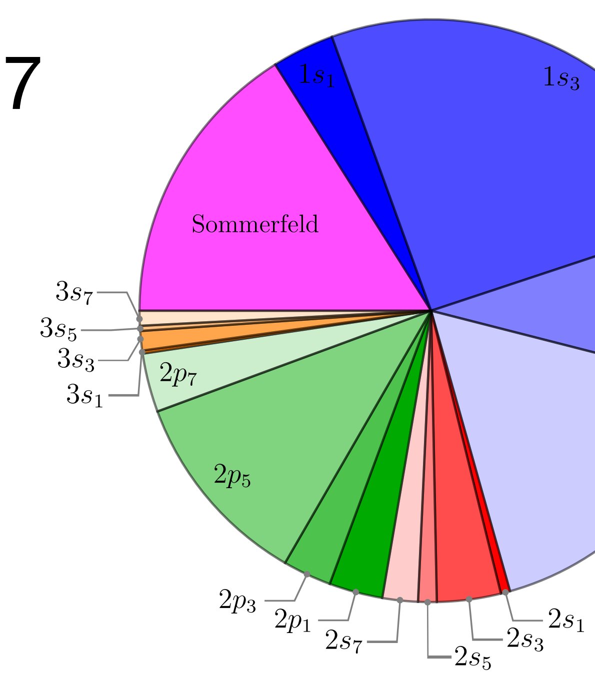
$$z\frac{\mathrm{d}Y_{B_I}}{\mathrm{d}z} = Y_{B_I}^{\mathrm{eq}}\left\{\frac{\langle\Gamma_{B_I,\mathrm{break}}\rangle}{H}\left[\frac{Y_{\mathrm{DM}}^2}{(Y_{\mathrm{DM}}^{\mathrm{eq}})^2} - \frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right] + \frac{\langle\Gamma_{B_I,\mathrm{ann}}\rangle}{H}\left[1 - \frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right] + \sum_{B_J}\frac{\langle\Gamma_{B_I\to B_J}\rangle}{H}\left[\frac{Y_{B_J}}{Y_{B_J}^{\mathrm{eq}}} - \frac{Y_{B_I}}{Y_{B_I}^{\mathrm{eq}}}\right]\right\}$$

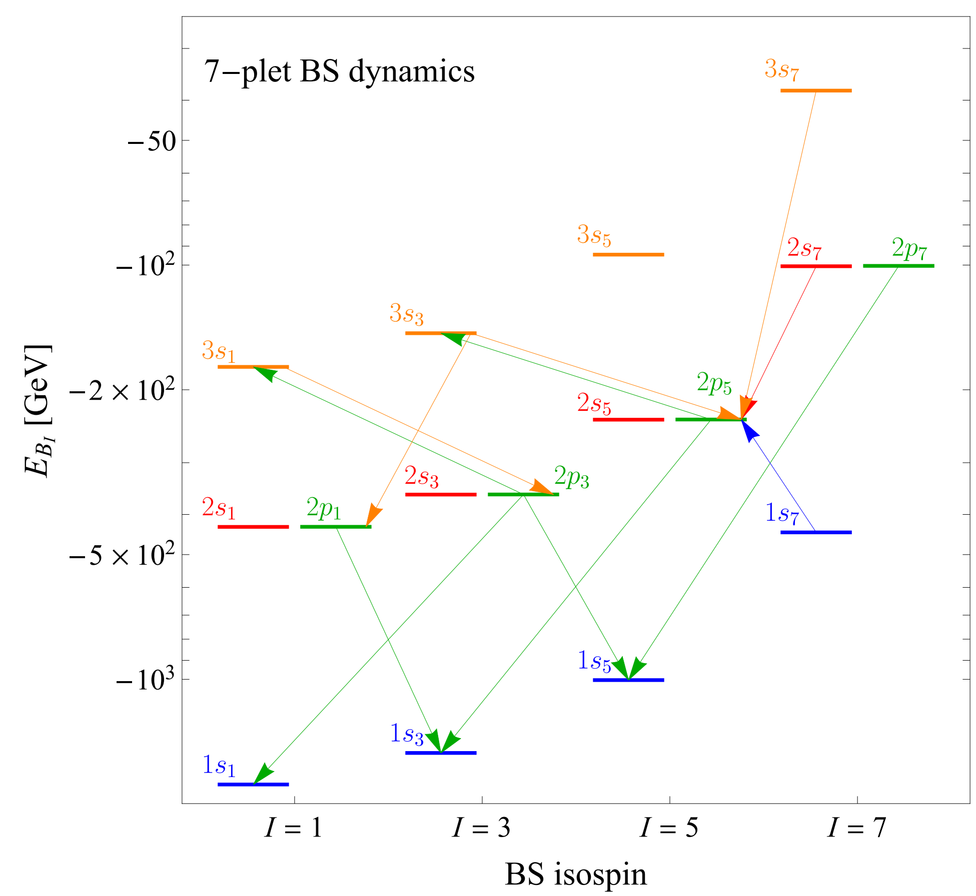
- if BS decay/annihilate quickly

$$\boxed{\frac{\mathrm{d}Y_{\mathrm{DM}}}{\mathrm{d}z} = -\frac{\langle\sigma_{\mathrm{eff}}v_{\mathrm{rel}}\rangle s}{Hz}\left(Y_{\mathrm{DM}}^2 - Y_{\mathrm{DM}}^{\mathrm{eq},2}\right)}$$

$$\langle\sigma_{\mathrm{eff}}v_{\mathrm{rel}}\rangle \equiv S_{\mathrm{ann}}(z) + \sum_{B_J}S_{B_J}(z)$$

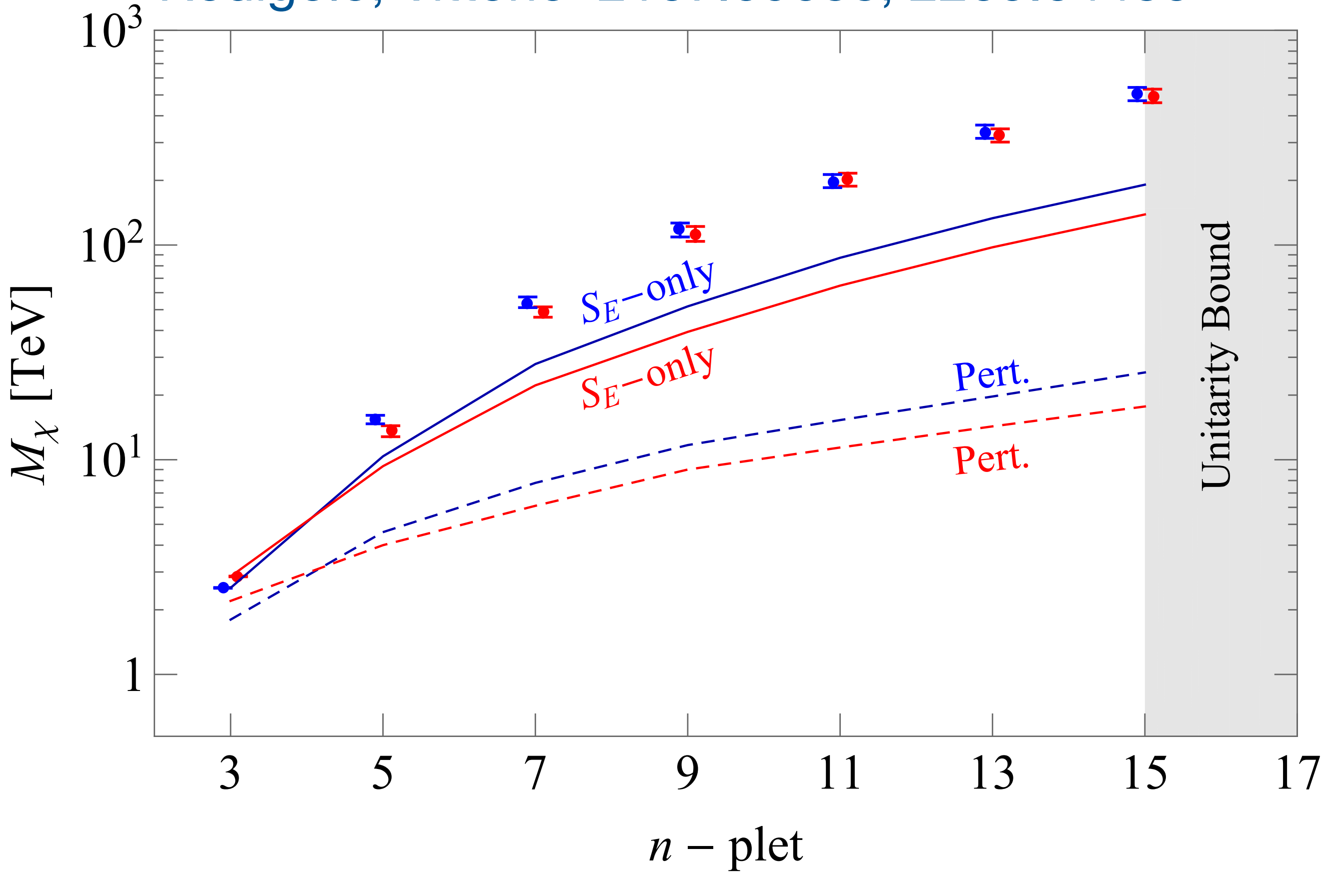
- Example: n = 7





Thermal freeze-out masses

Bottaro, DB, Costa, Franceschini, Panci, Redigolo, Vittorio 2107.09688, 2205.04486



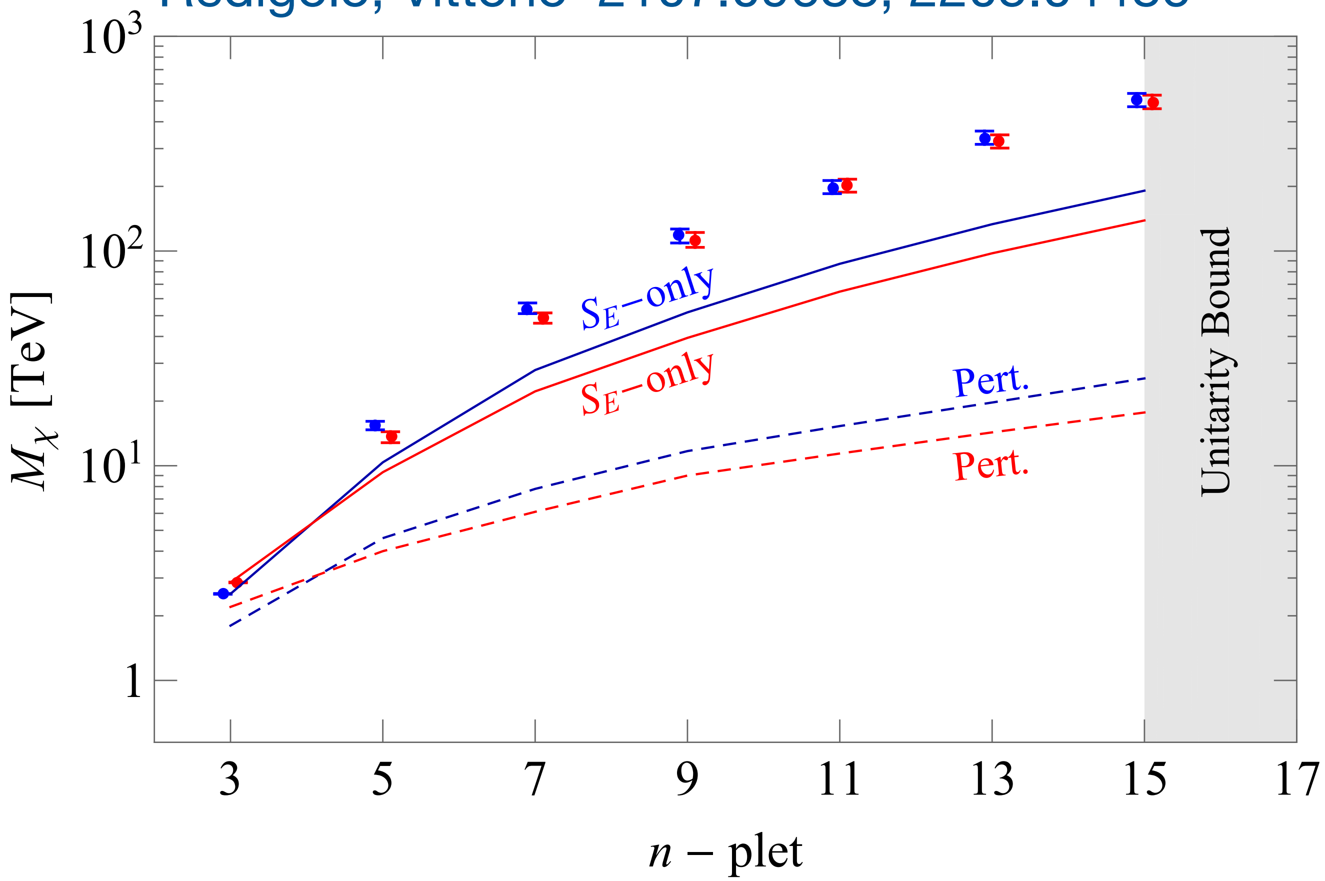
(and similar for scalars)

	EW n-plet	Mass [TeV]
Majorana fermion	3_0	2,86
	5_0	13,6
	7_0	48,8
	9_0	113
	11_0	202
	13_0	324,6
Dirac fermion	$2_{1/2}$	1,08
	3_1	2,85
	$4_{1/2}$	4,8
	5_1	9,9
	$6_{1/2}$	31,8
	$8_{1/2}$	82
	$10_{1/2}$	158
	$12_{1/2}$	253

How do we probe these states?

Thermal freeze-out masses

Bottaro, DB, Costa, Franceschini, Panci,
Redigolo, Vittorio 2107.09688, 2205.04486



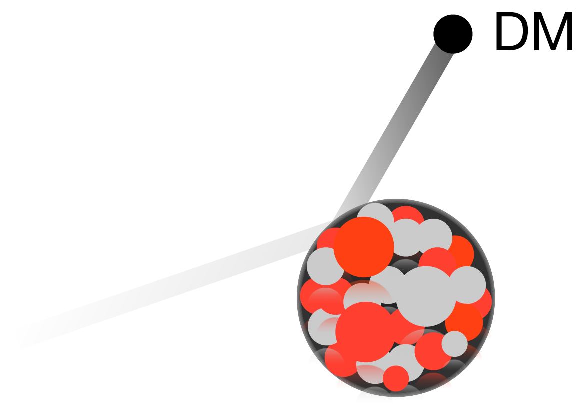
(and similar for scalars)

Potentially in reach of a high-energy collider!

How do we probe these states?

	EW n-plet	Mass [TeV]
Majorana fermion	3_0	2,86
	5_0	13,6
	7_0	48,8
	9_0	113
	11_0	202
	13_0	324,6
Dirac fermion	$2_{1/2}$	1,08
	3_1	2,85
	$4_{1/2}$	4,8
	5_1	9,9
	$6_{1/2}$	31,8
	$8_{1/2}$	82
	$10_{1/2}$	158
	$12_{1/2}$	253

Direct detection

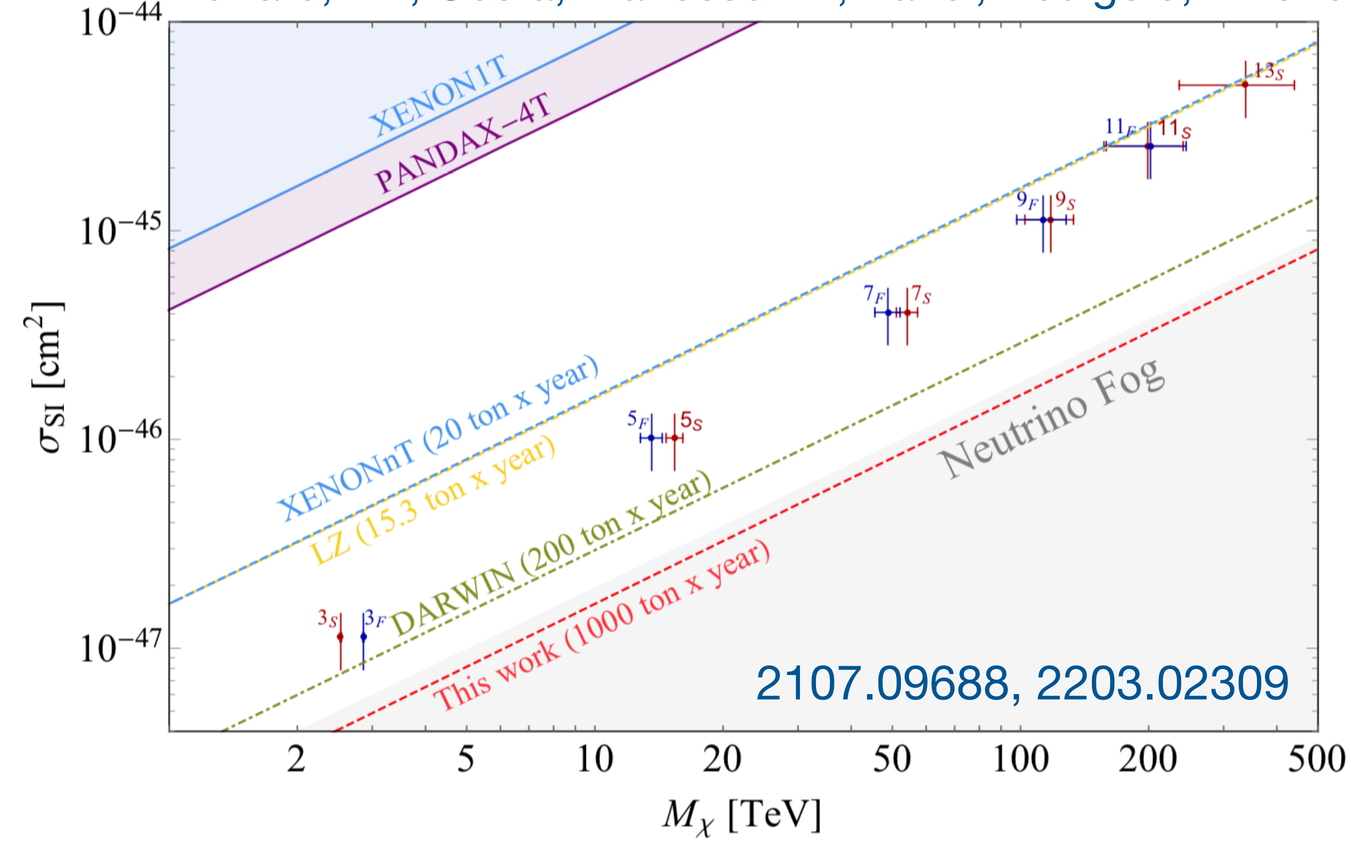


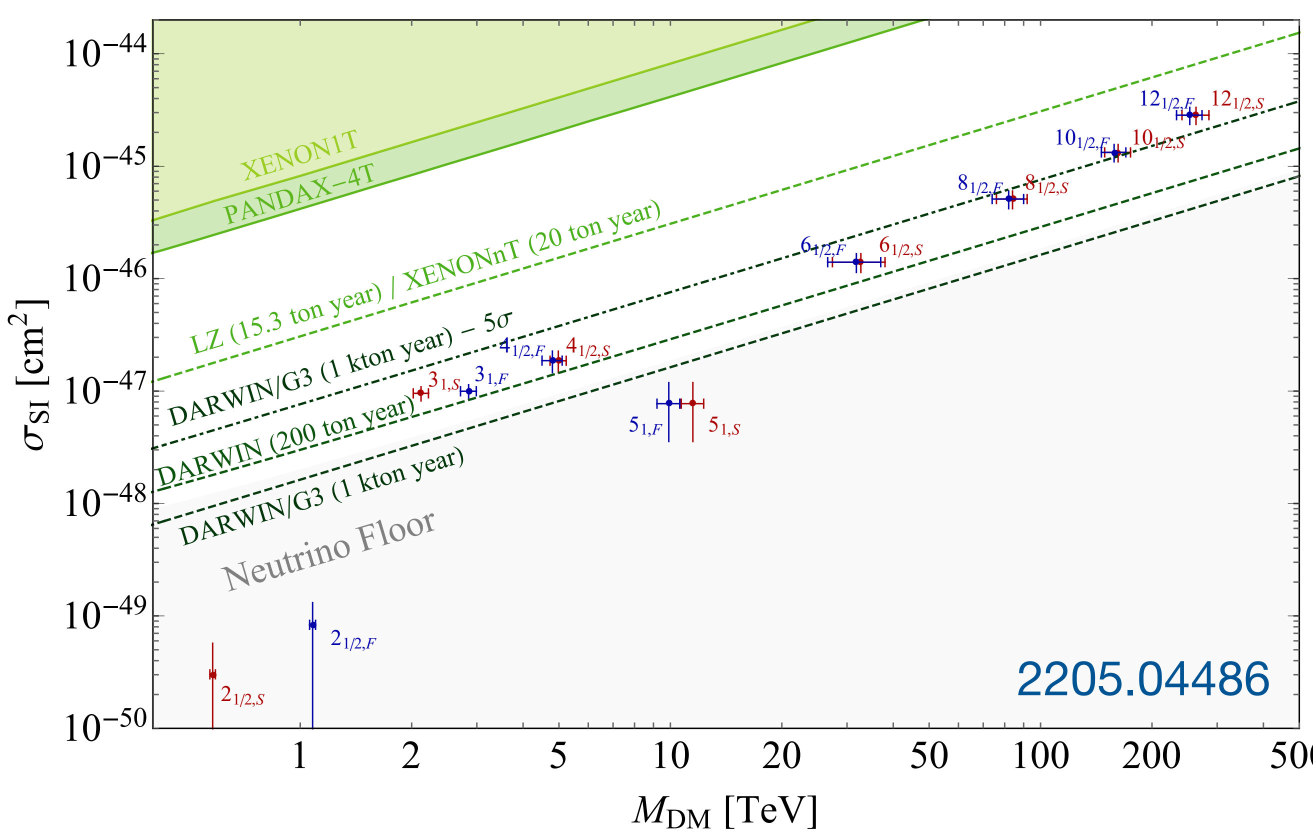
$$\mathscr{L}^{\mathrm{SI}}_{\mathrm{eff}} = \bar{\chi}\chi\left(f_q m_q \bar{q}q + f_G G^a_{\mu\nu}G^{\mu\nu,a}\right) + \frac{g_q}{M_\chi}\left(\bar{\chi}i\partial^\mu\gamma^\nu\chi\right)\mathcal{O}^q_{\mu\nu}$$

$$f_i \approx \frac{\alpha^2_{\mathrm{EW}}}{m^3_{\mathrm{EW}}}(n^2-1)$$

All WIMP candidates (except doublet!) above the neutrino floor, but need a very large exposure to be probed

Bottaro, DB, Costa, Franceschini, Panci, Redigolo, Vittorio

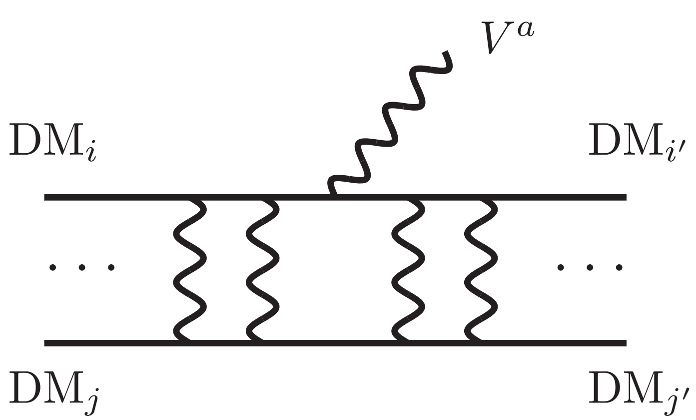




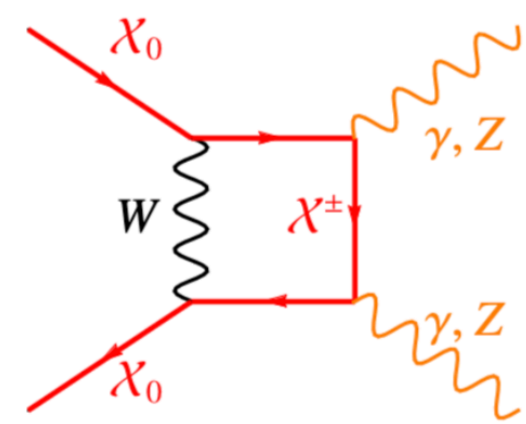
Indirect detection

- Searches for high-energy gamma-ray lines with Cherenkov telescopes are a powerful constraint for high-mass WIMP DM

(Large multiplets are more easily probed due to increased annihilation cross-section)



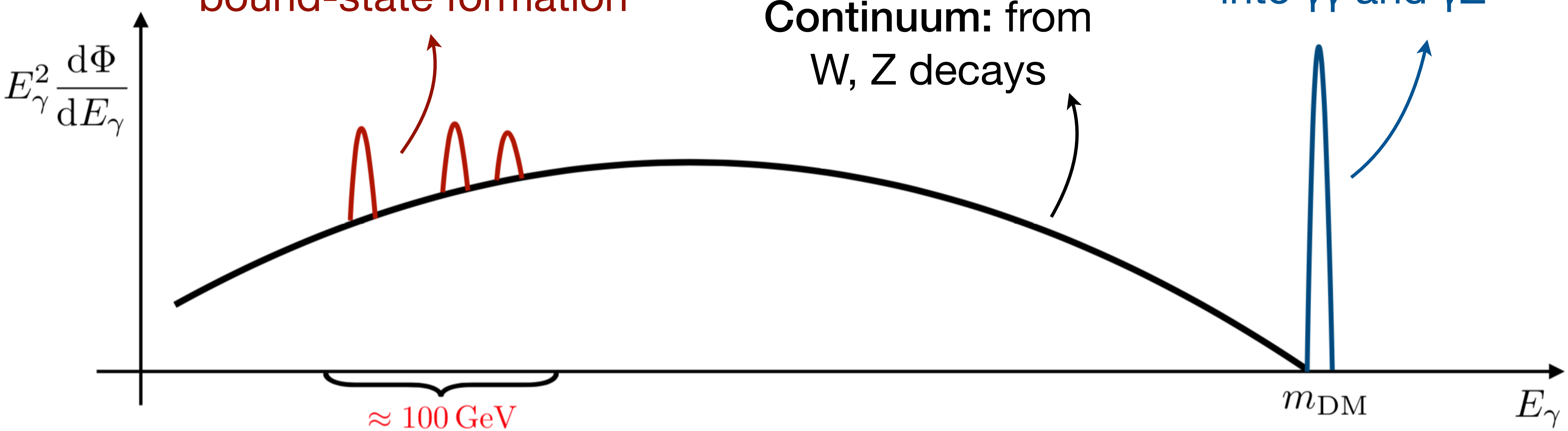
➡ talk by Giulio



Series of γ-lines: due to bound-state formation

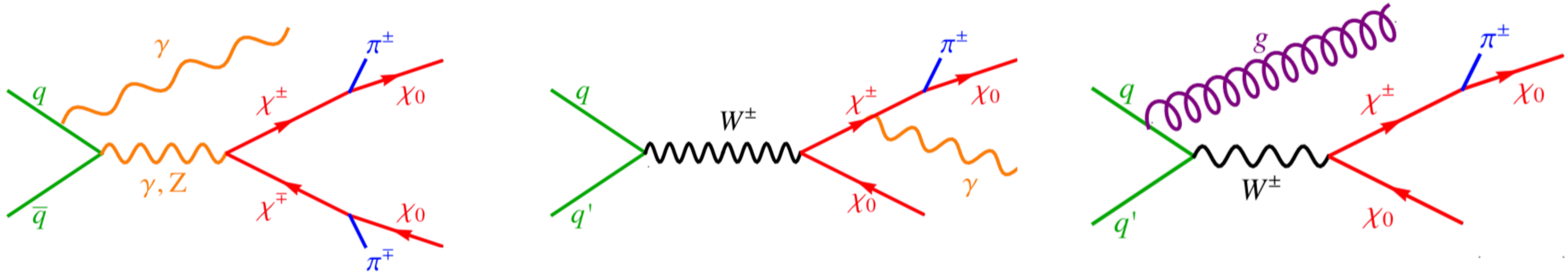
γ-ray line: annihilation into γγ and γZ

Continuum: from W, Z decays

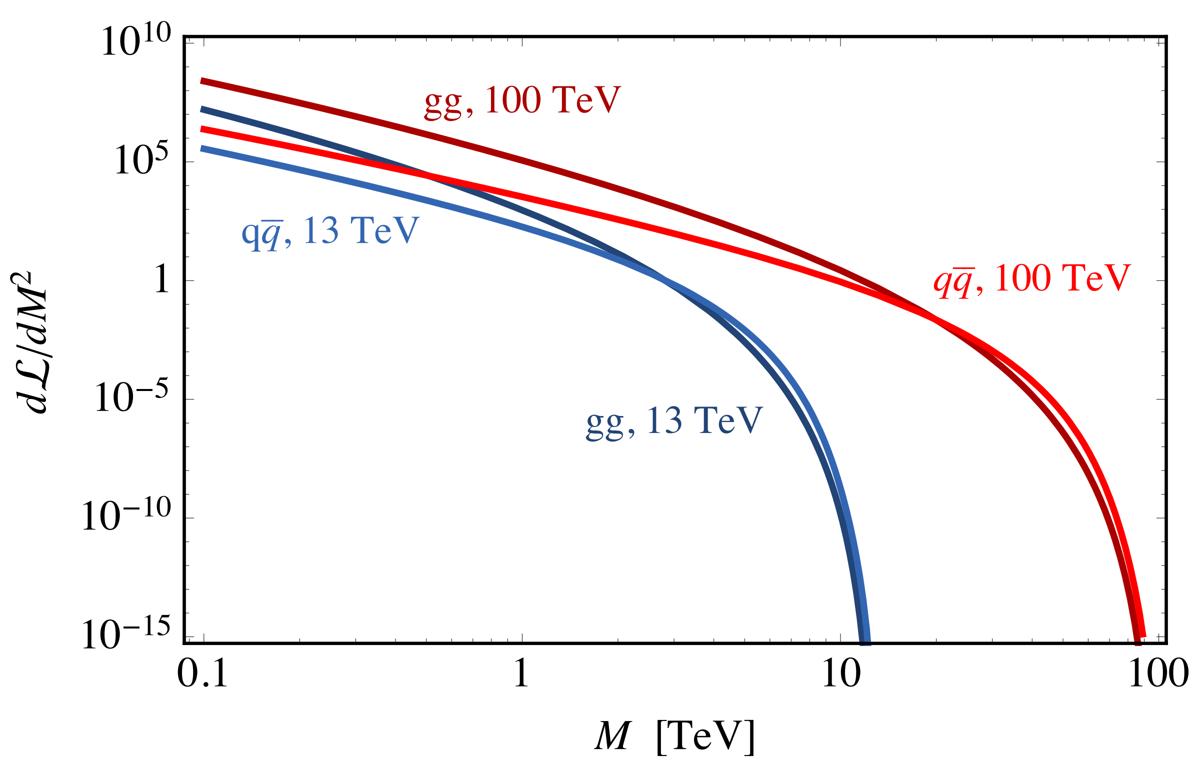


Colliders: missing energy searches

- $2 \to 2$ production of invisible χ pair + event tag, e.g. mono-γ

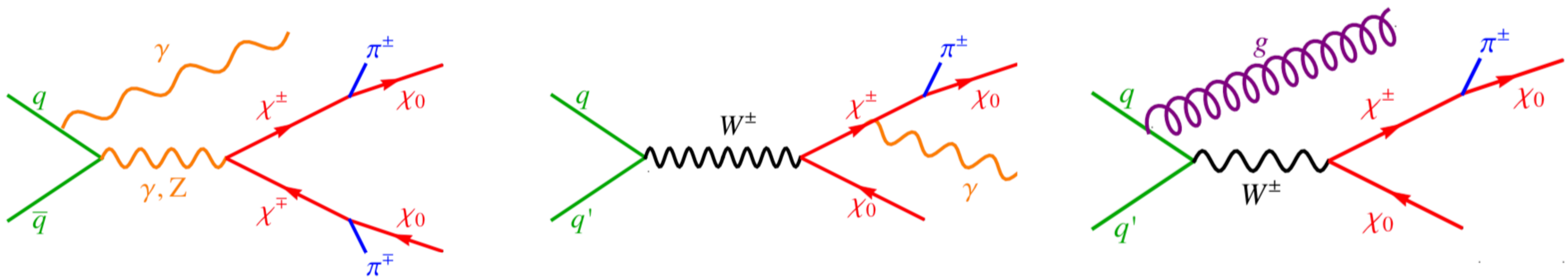


Very difficult at hadron colliders: large backgrounds, and strong PDF suppression at high partonic c.o.m. energies (large invariant masses)



Colliders: missing energy searches

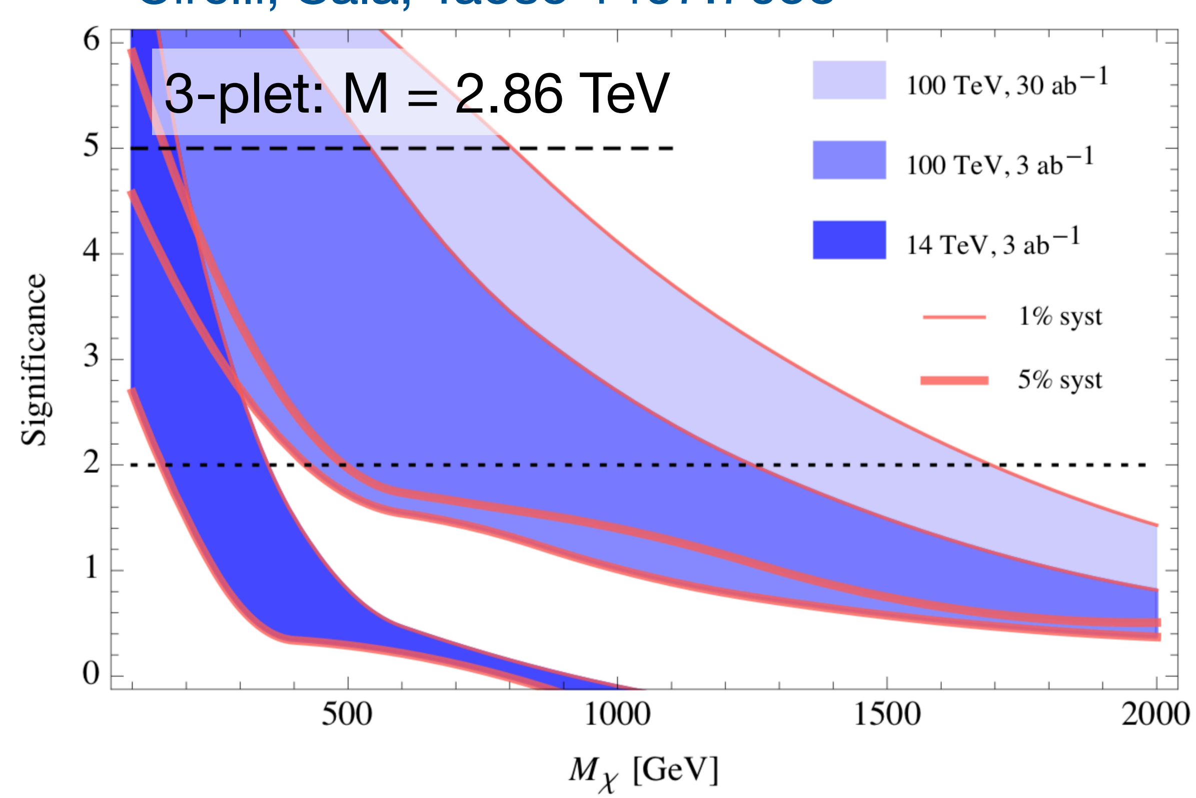
- 2 $\to$ 2 production of invisible χ pair + event tag, e.g. mono-γ



Very difficult at hadron colliders: large backgrounds, and strong PDF suppression at high partonic c.o.m. energies (large invariant masses)

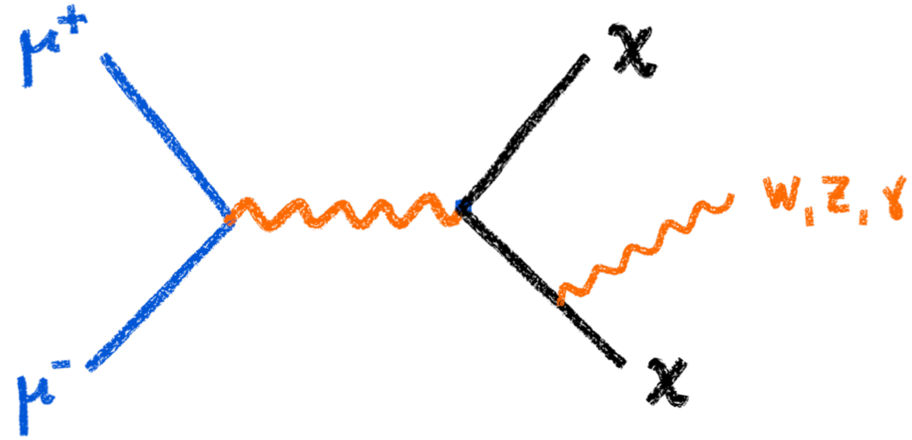
- LHC sensitive to DM masses ~ O(100 GeV)
- 100 TeV collider needed to reach thermal freeze-out targets for triplet & doublet

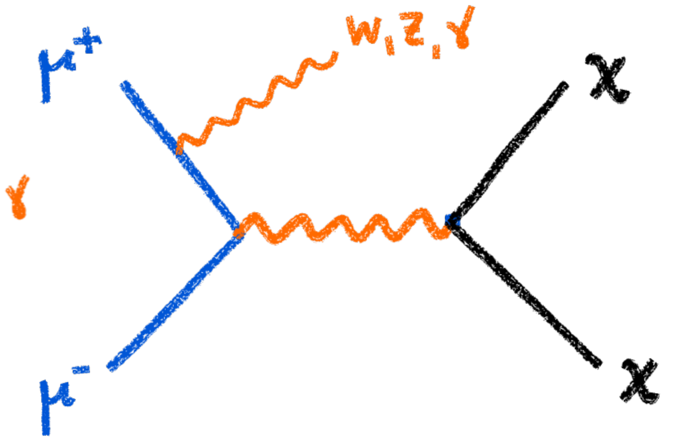
Cirelli, Sala, Taoso 1407.7058



Missing mass searches at μ collider

$2 \to 2$ production of χ pair





- Full energy available

  in the center of mass:

  ability to discover particles up to kinematical threshold $\sqrt{s}/2$

- Full event reconstruction: **missing invariant mass** (not just pT)

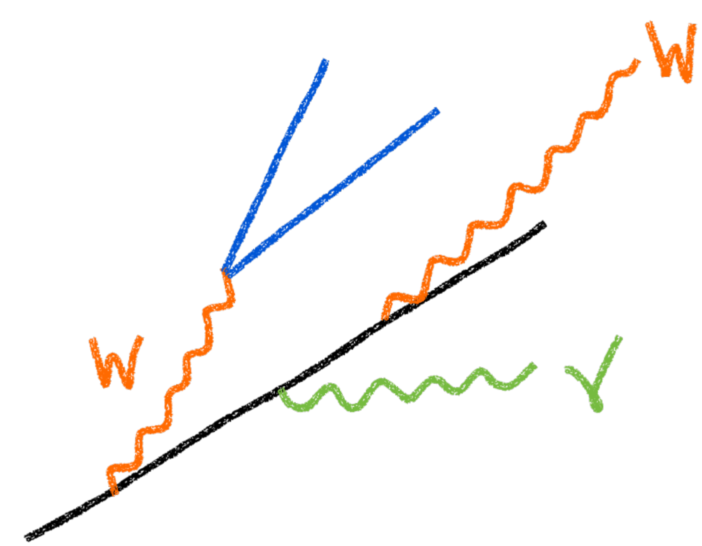
- No QCD backgrounds: ideal for EW physics

- EW radiation becomes important at multi-TeV energies!

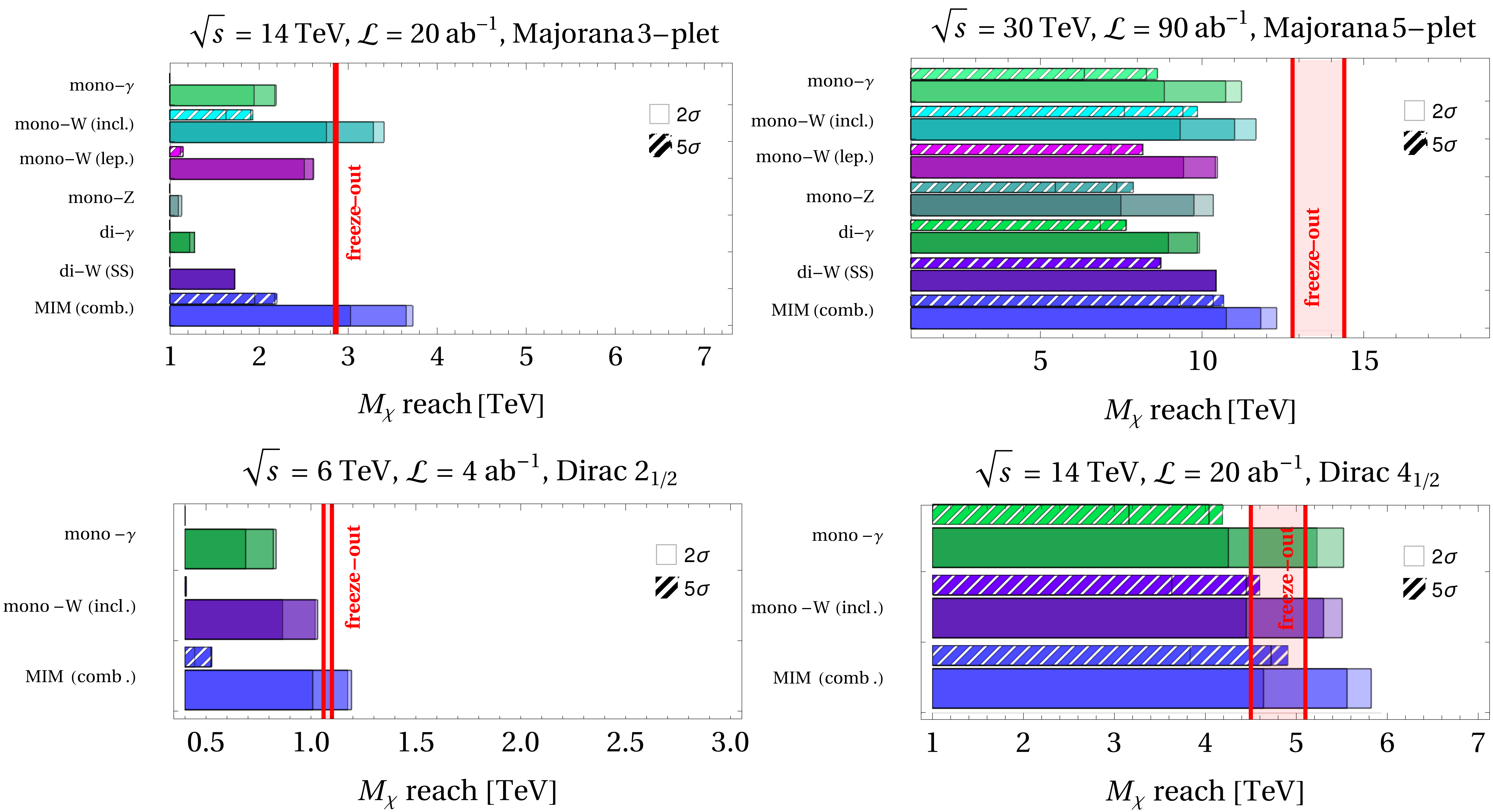
  Sudakov factor $\dfrac{\alpha}{4\pi}\log^2\left(E/m_W\right) \approx 1$ for E ~ 10 TeV

  - mono-γ, mono-W, mono-Z are all similar!
  - (Precise) resummation of double logs needed.

    Goal: % or ‰ precision



Missing mass searches at μ collider



Bottaro, DB, Costa, Franceschini, Panci, Redigolo, Vittorio 2203.02309, 2205.04486
Han, Liu, Wang, Wang 2009.11287

* shadings = different assumptions about systematic errors
typically low signal/background ➔ requires good control of systematics

Composite WIMPs

- Higgs hierarchy problem main motivation for TeV BSM physics!
  Embed WIMP candidates in a model that addresses EW naturalness

- Difficult to embed large SU(2) representations in a realistic model
  (even worse with grand unification)

- UV cut-off close-by for larger reps: strongly coupled UV completion

➡ Try to embed WIMPs in Composite Higgs models

Composite WIMPs

- Higgs hierarchy problem main motivation for TeV BSM physics!
  Embed WIMP candidates in a model that addresses EW naturalness

- Difficult to embed large SU(2) representations in a realistic model
  (even worse with grand unification)

- UV cut-off close-by for larger reps: strongly coupled UV completion

➡ Try to embed WIMPs in Composite Higgs models

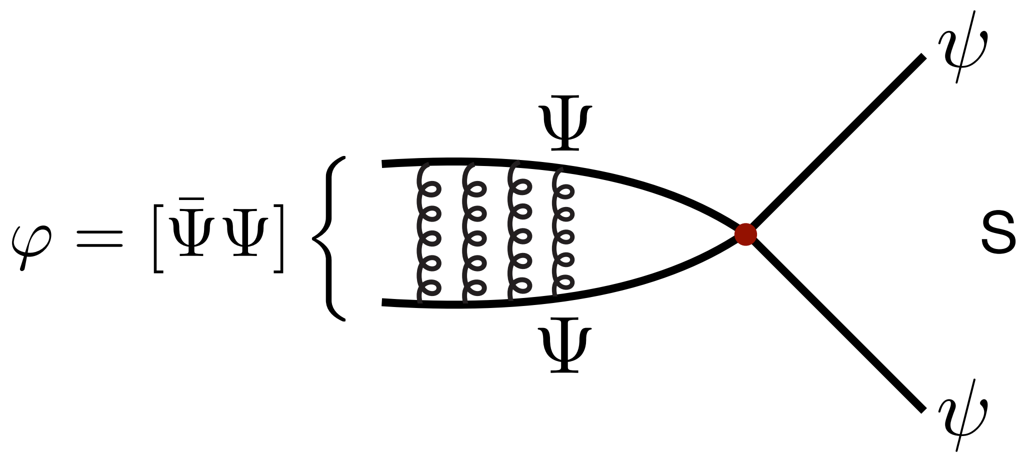
Several examples of composite Dark Matter in the literature:

- QCD-like fundamental models
  Antipitin et al. 1410.1817, 1503.08749
  Mitridate et al. 1702.01141, 1707.05380
  Contino et al. 1811.06975, 2008.10607
  DB, Di Luzio, et al. 1907.11228, 1911.04502
- Goldstone-boson Dark Matter in non-minimal Composite Higgs
  Cheng et al. 2110.10691
  Balkin et al. 1707.07685

➡ Consider just the low-energy chiral Lagrangian + partial compositeness

Composite Higgs

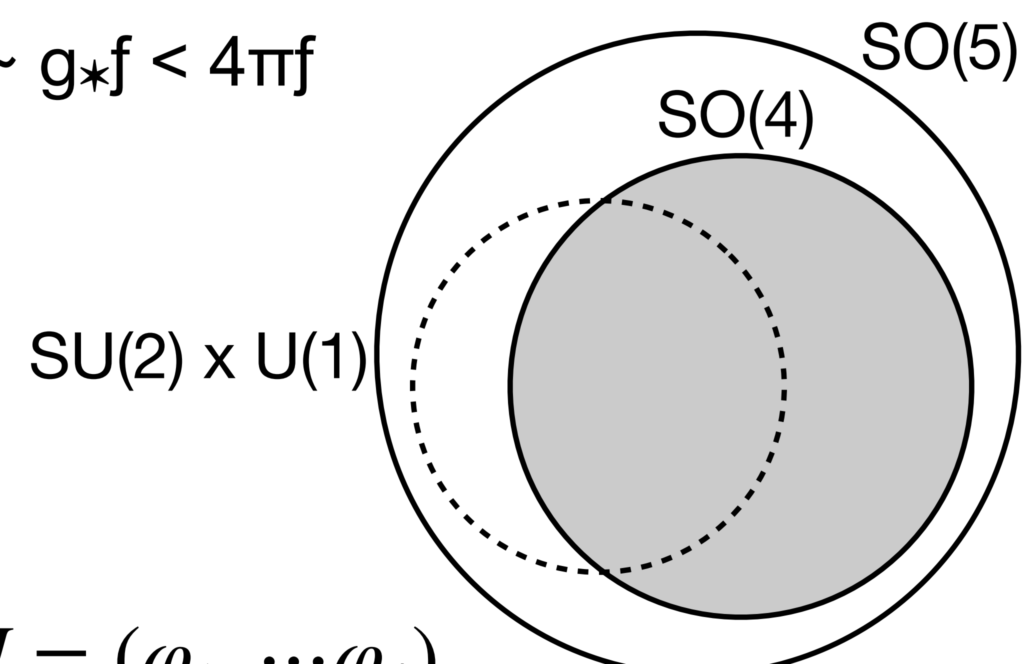
- Higgs is a composite pseudo-Goldstone boson, analogous to QCD pion



EW scale dynamically generated when $g_* \to 4\pi$

- Strong dynamics spontaneously breaks a global symmetry G → H
  - ➡ Natural separation between EW scale v and compositeness scale $\Lambda \approx 4\pi f$
  - ➡ Other composite resonances with mass $m_\rho \sim g_* f < 4\pi f$

$$\mathscr{L} = \frac{f^2}{4}\mathrm{Tr}[D_\mu \pi D^\mu \pi] + \cdots$$



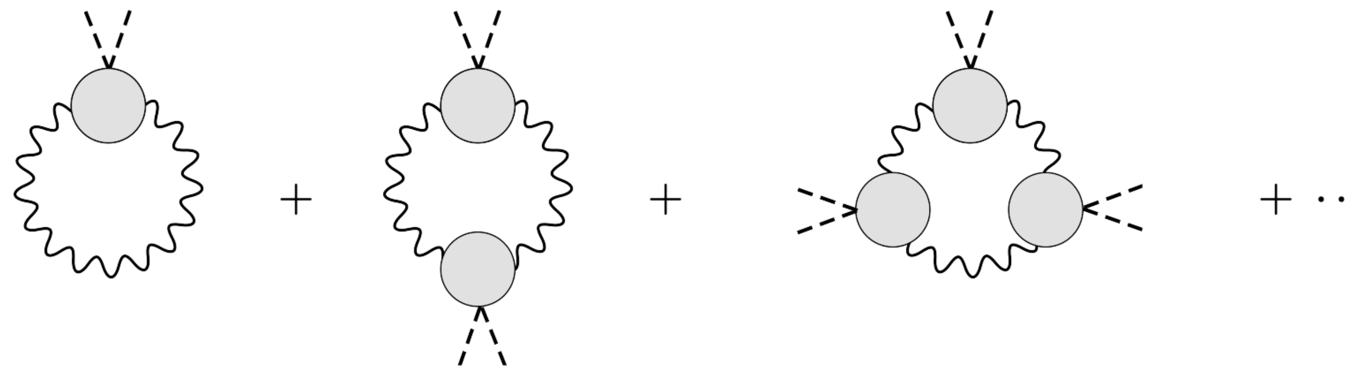
- Minimal realization (MCHM): SO(5) → SO(4),

  4 Goldstone bosons of the Higgs doublet $H = (\varphi_1, \cdots \varphi_4)$

Composite Higgs

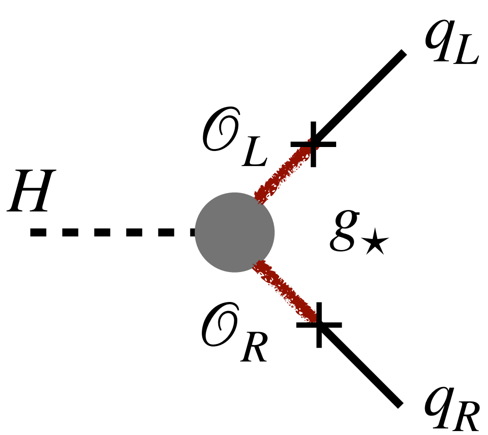
- Higgs has non-zero mass and potential: arises from explicit breaking of global symmetry G (like pion mass difference in QCD)

$$m_H^2 \approx \frac{g_{\rm SM}^2}{16\pi^2} m_\star^2$$



- Partial compositeness: Yukawa couplings arise through linear mixings between elementary fermions and composite states (they break the global symmetry)

$$\mathscr{L}_{\rm mix} = \mathscr{L}_{\rm strong} + g_\star \mathcal{O}_L H \mathcal{O}_R + \lambda_L \bar{q}_L \mathcal{O}_L + \lambda_R \bar{t}_R \mathcal{O}_R$$

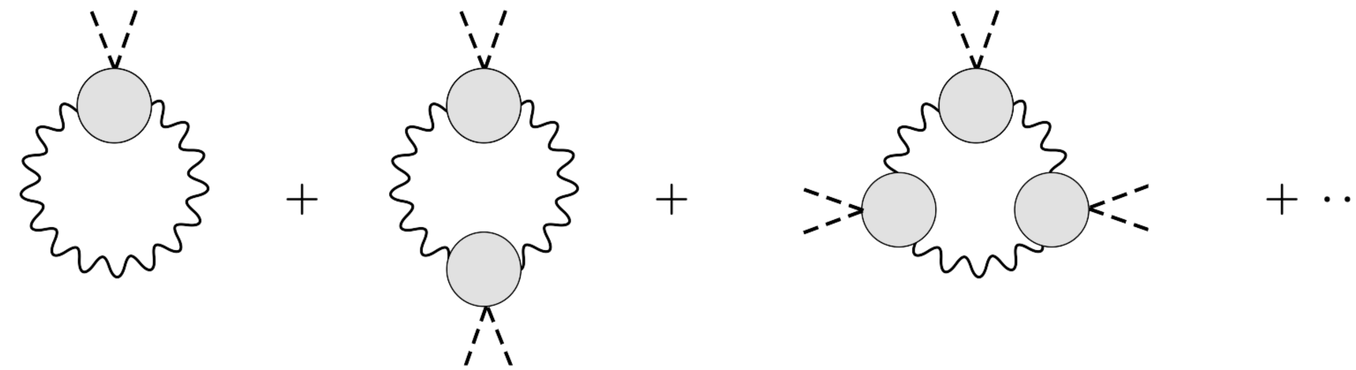


- Different choices of $\mathcal{O}_{L,R}$ rep.s under G define different models

Composite Higgs

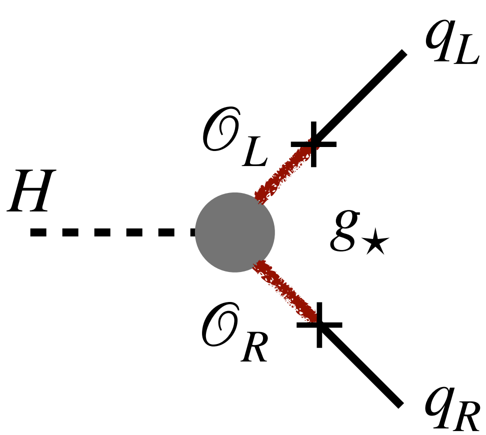
- Higgs has non-zero mass and potential: arises from explicit breaking of global symmetry G (like pion mass difference in QCD)

$$m_H^2 \approx \frac{g_{\rm SM}^2}{16\pi^2} m_\star^2$$



- <u>Partial compositeness:</u> Yukawa couplings arise through linear mixings between elementary fermions and composite states (they break the global symmetry)

$$\mathscr{L}_{\rm mix} = \mathscr{L}_{\rm strong} + g_\star \mathcal{O}_L H \mathcal{O}_R + \lambda_L \bar{q}_L \mathcal{O}_L + \lambda_R \bar{t}_R \mathcal{O}_R$$



  - Different choices of $\mathcal{O}_{L,R}$ rep.s under G define different models

- If DM is a composite resonance: include $\mathcal{O}_\chi$ that excites a neutral state

- Can also have partial compositeness: $\lambda_\chi \bar{\chi} \mathcal{O}_\chi$ with χ elementary

Make the DM stable again!

- How can we make the DM candidate stable in this model?

  In the UV theory, accidental symmetries like baryon number can give stable particles

- Global symmetry of the strong sector: $\mathrm{SO}(5) \times \mathrm{U}(1)_X \to \mathrm{SO}(4) \times \mathrm{U}(1)_X$

  $\mathrm{U}(1)_X$ unbroken, needed to reproduce hypercharges of SM fermions:

  $Y = T_R^3 + X$

- For suitable U(1) charges, can have accidental $\mathbb{Z}_n$ that stabilize DM

  <u>Example:</u> $e^{i\pi X}\chi = \pm\chi$ for X integer defines a $\mathbb{Z}_2$

Scalar	DM	SM
$\mathbb{Z}_2'$	+1	+1
	+1	-1
	-1	+1
	-1	-1

,

Fermion	DM	SM
$\mathbb{Z}_2'$	+1	+1
	+1	-1
	-1	+1
	-1	-1

see also Frigerio et al. 2212.11918

Make the DM stable again!

- An example that doesn't work: hypercharge

Field	$U(1)_Y$
L	$-1/2$
e_R	-1
q_l	$1/6$
u_R	$2/3$
d_R	$-1/3$
H	$1/2$
χ	$1/n$

$\xrightarrow{n\,=\,4}$

$12\,Y$	$\mathbb{Z}_2 \subset U(1)_Y$
-6	$+$
-12	$+$
2	$+$
8	$+$
-4	$+$
6	$+$
3	-

(similarly, have accidental $\mathbb{Z}_n$ when $Y_\chi = 1/6n$)

But $Q = Y + T_L^3$, i.e. χ can't be electrically neutral for $n \neq k/2$

- $U(1)_X$ charges can do the job in various models

Models

DOUBLET MODEL

Field	$SU(2)_L \times SU(2)_R$	X	Z_2
Q_L	(2,1)	1/6	-
Q_R	(1,2)	1/6	-
L_L	(2,1)	-1/2	-
L_R	(1,2)	-1/2	-
H	(2,2)	0	+
χ	(3,1)	0	+

- Resonances are 4-plets of SO(5) doublets of $SU(2)_{L(R)}$ are coupled to LH (RH) fermions
- X-charge corresponds to (B - L)/2: conserved by all SM interactions!
- RH mixings break custodial symmetry: strong bounds from Z properties

$$\Delta\rho \sim \lambda_R^2$$

$$\Gamma(Z \to \bar{b}b) \sim \lambda_L^2 \sim y_t^2/\lambda_R^2$$

- This model could still be viable for compositeness scale > few TeV, as predicted by thermal WIMP masses $\rightarrow$ fine-tuning

Models

BI-DOUBLET MODEL

Field	$SU(2)_L$ x $SU(2)_R$	X	Z_2
Q	(2,2)	2/3	+
U	(1,1)	2/3	+
D	(1,1)	-1/3	+
L	(2,2)	-1	+
E	(1,1)	-1	+
H	(2,2)	0	+
χ	(3,2)	1/2	-

- Resonances are 5-plets of SO(5)

$$\left\{ Q = \begin{pmatrix} T & X_{5/3} \\ B & X_{2/3} \end{pmatrix}, \quad U \right\}$$

- LH mixings break custodial symmetry:

$$\Gamma(Z \to \bar{b}b) \sim \Delta\rho \sim \lambda_L^2$$

ok if λ_L small

- t_R can be fully composite state of strong sector, since singlet of SO(5)

- $U(1)_X$ is explicitly broken by down-quark Yukawa couplings.

  $\mathbb{Z}_2$ is however accidentally conserved, DM is stable at all orders.

Models

TRIPLET MODEL

Field	$SU(2)_L \times SU(2)_R$	X	Z_2
Q	(2,2)	2/3	+
T	(1,3)	2/3	+
L	(2,2)	-1	+
E	(1,3)	-1	+
H	(2,2)	0	+
χ	(3,2)	1/2	-

- Resonances transform in **10** of SO(5)

$$\left\{ Q = \begin{pmatrix} T & X_{5/3} \\ B & X_{2/3} \end{pmatrix}, \quad T = (X_{5/3}, U, D) \right\}$$

- LH mixings break custodial symmetry:

$$\Gamma(Z \to \bar{b}b) \sim \Delta\rho \sim \lambda_L^2$$

ok if λ_L small

- $U(1)_X$ unbroken (~ combination of B, L)

Interactions with the composite Higgs

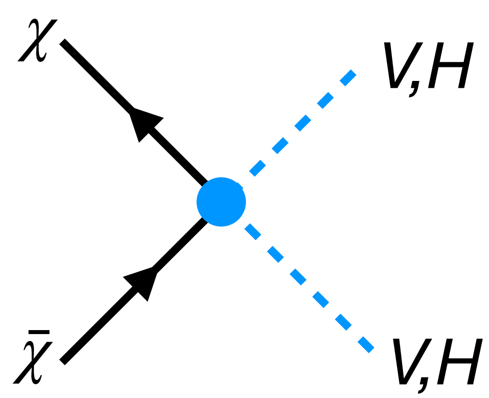
- χ is part of the strong sector, interacts with the composite Higgs

2 operators SO(4)-invariant:

$$\mathcal{O}_1 = \mathrm{Tr}[\bar{\chi}\chi]\mathrm{Tr}[H^\dagger H]$$

$$\mathcal{O}_2 = \mathrm{Tr}[\bar{\chi}t^a\chi t^b]\mathrm{Tr}[H^\dagger t^a H t^b]$$

- These interactions contribute to DM annihilation



$$\sigma_{\mathrm{ann}} \approx \left(\epsilon_\chi g_\star^2/m_\star\right)^2$$

$\epsilon_\chi \ll 1$ explicit breaking of SO(5)
(shift symmetry of the Goldstone)

Interactions with the composite Higgs

- χ is part of the strong sector, interacts with the composite Higgs

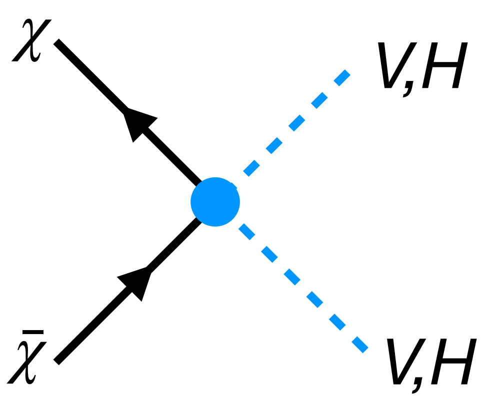
2 operators SO(4)-invariant:

$$\mathcal{O}_1 = \mathrm{Tr}[\bar{\chi}\chi]\mathrm{Tr}[H^\dagger H]$$

$$\mathcal{O}_2 = \mathrm{Tr}[\bar{\chi}t^a\chi t^b]\mathrm{Tr}[H^\dagger t^a H t^b]$$

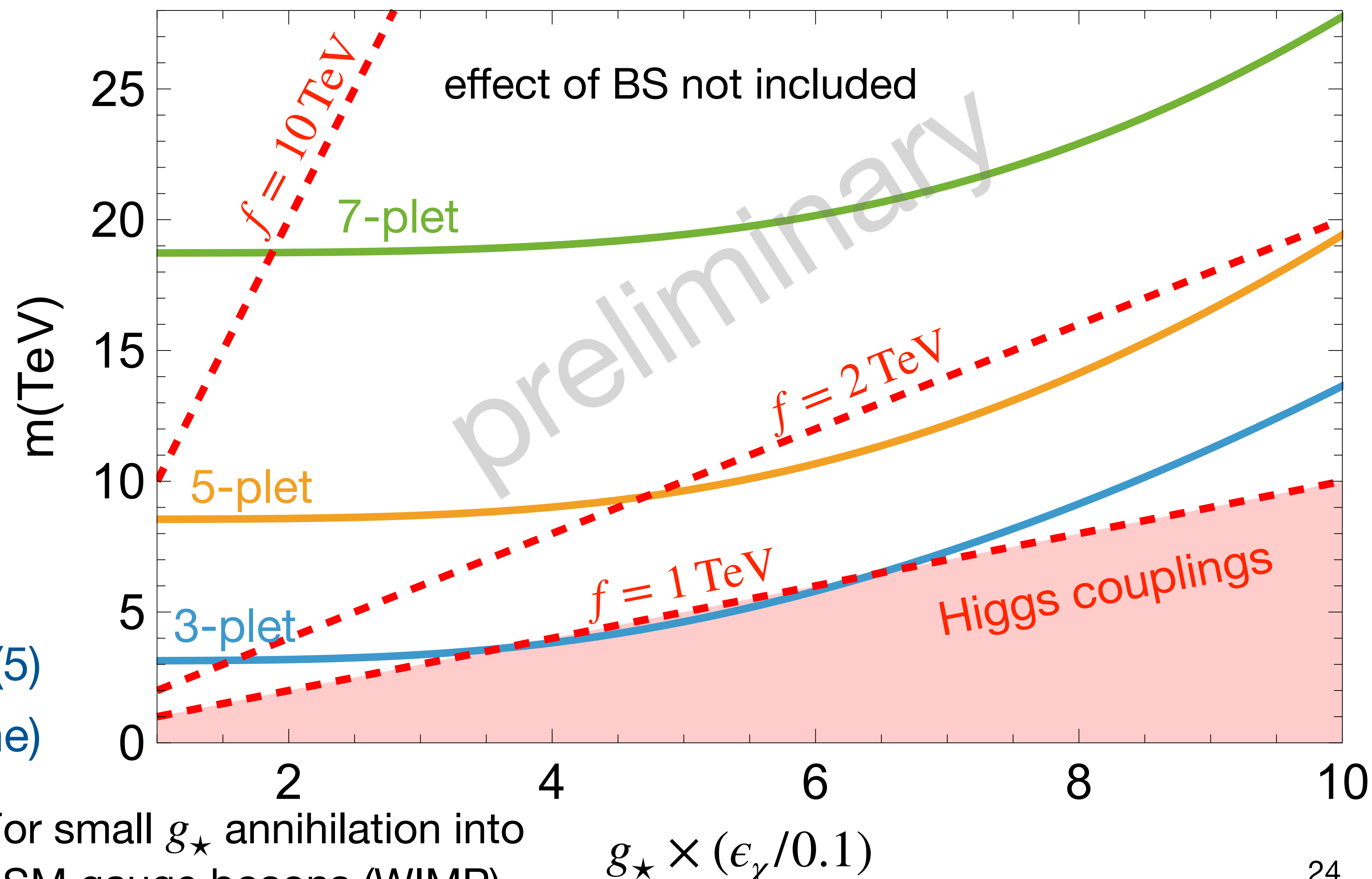
- These interactions contribute to DM annihilation

For large $g_\star$ strong interaction dominates



$$\sigma_{\mathrm{ann}} \approx \left(\epsilon_\chi g_\star^2/m_\star\right)^2$$

$\epsilon_\chi \ll 1$ explicit breaking of SO(5) (shift symmetry of the Goldstone)



For small $g_\star$ annihilation into SM gauge bosons (WIMP)

Interactions with the composite Higgs

- χ is part of the strong sector, interacts with the composite Higgs

2 operators SO(4)-invariant:

$$\mathcal{O}_1 = \mathrm{Tr}[\bar{\chi}\chi]\mathrm{Tr}[H^\dagger H]$$

$$\mathcal{O}_2 = \mathrm{Tr}[\bar{\chi} t_L^a \chi t_R^b]\mathrm{Tr}[H^\dagger t_L^a H t_R^b]$$

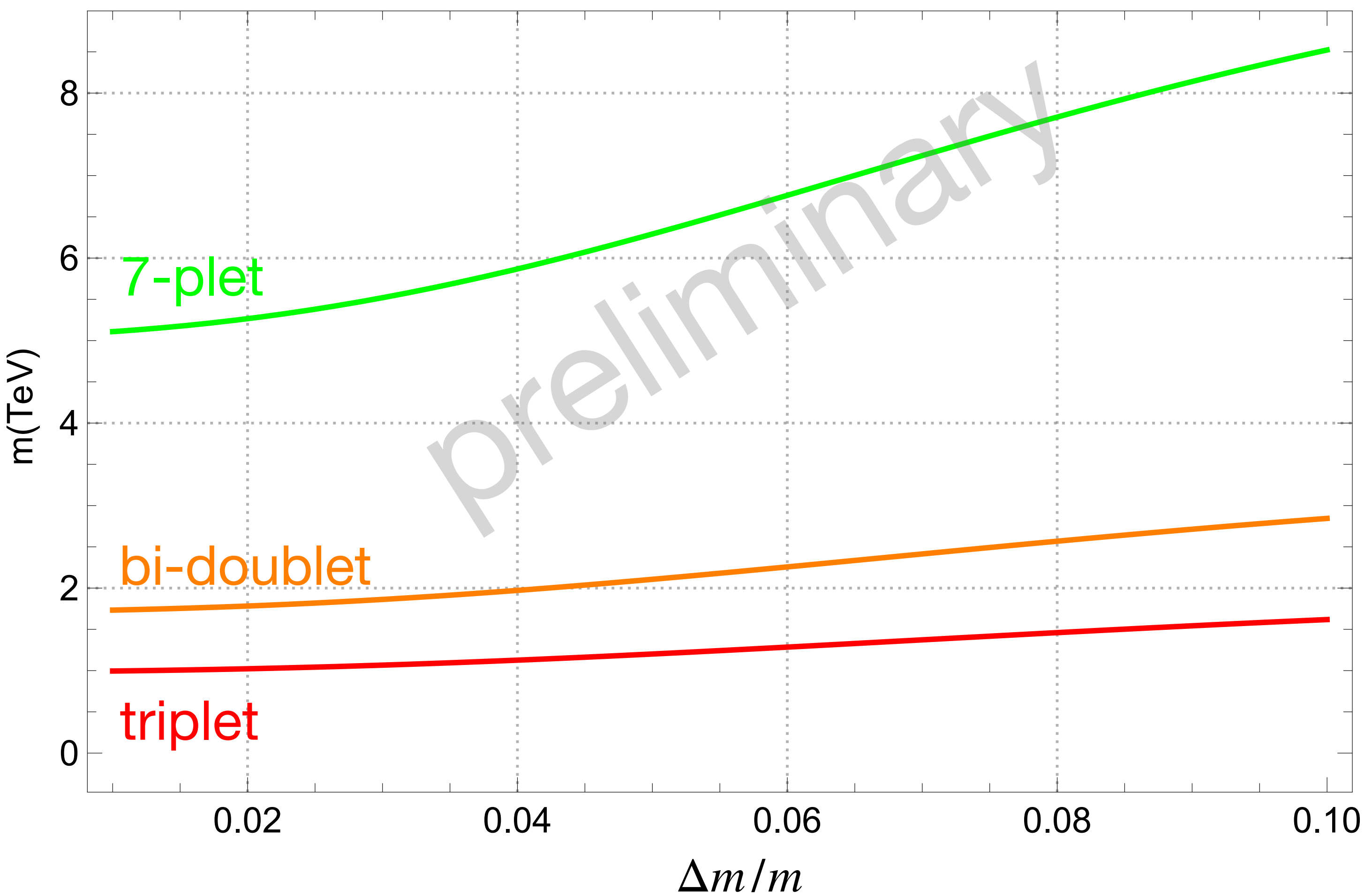
- After EWSB, $\mathcal{O}_2$ also contributes to splittings inside the multiplet: e.g. for a complex Higgsino ~ (2,2)

$$\mathcal{O}_2 \equiv \mathcal{O}_+ + \mathcal{O}_0$$

charged-neutral splitting

"inelastic" splitting

$$\frac{\Delta m}{m} \sim \epsilon_\chi \times \frac{v^2}{f^2}$$



effects of splittings on relic abundance are negligible, given suppression from v/f

Interactions with the composite Higgs

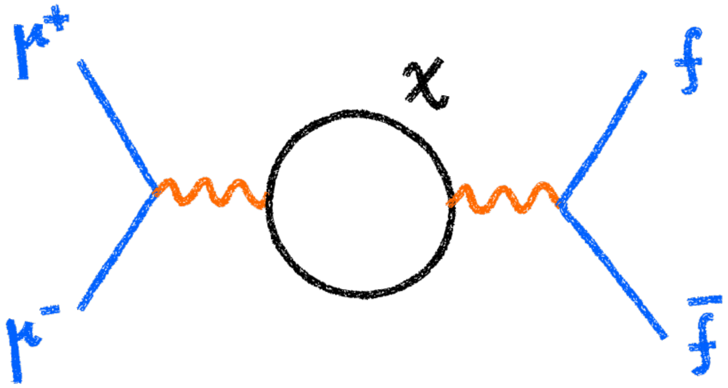
- χ is part of the strong sector, interacts with the composite Higgs

  2 operators SO(4)-invariant: $\mathcal{O}_1 = \mathrm{Tr}[\bar{\chi}\chi]\mathrm{Tr}[H^\dagger H]$

  $$\mathcal{O}_2 = \mathrm{Tr}[\bar{\chi}t_L^a \chi t_R^b]\mathrm{Tr}[H^\dagger t_L^a H t_R^b]$$

- After EWSB, $\mathcal{O}_2$ also contributes to splittings inside the multiplet: e.g. for a complex Higgsino ~ (2,2)

- SO(4)-breaking operators are also possible, give contributions to EWPT



- SO(5)-preserving (derivative) interactions of higher dimension

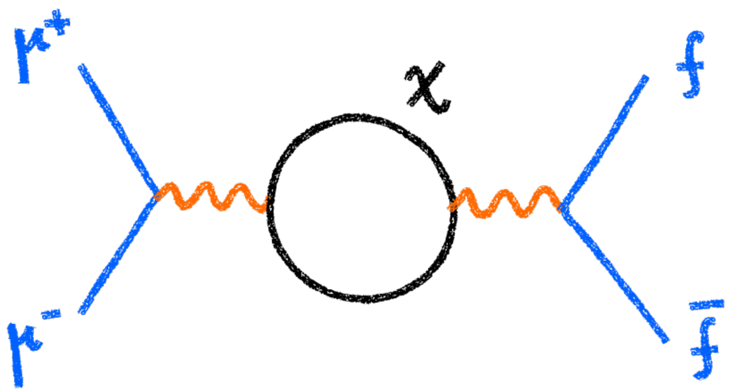
  $$\mathrm{Tr}[\bar{\chi}\gamma^\mu \chi]\mathrm{Tr}[H^\dagger D_\mu H] \qquad \mathrm{Tr}[\bar{\chi}\chi]\mathrm{Tr}[D_\mu H^\dagger D^\mu H]$$

  effects are suppressed by $T/m_\star$ in the non-rel. limit

A. Di Lecce

Indirect effects at colliders

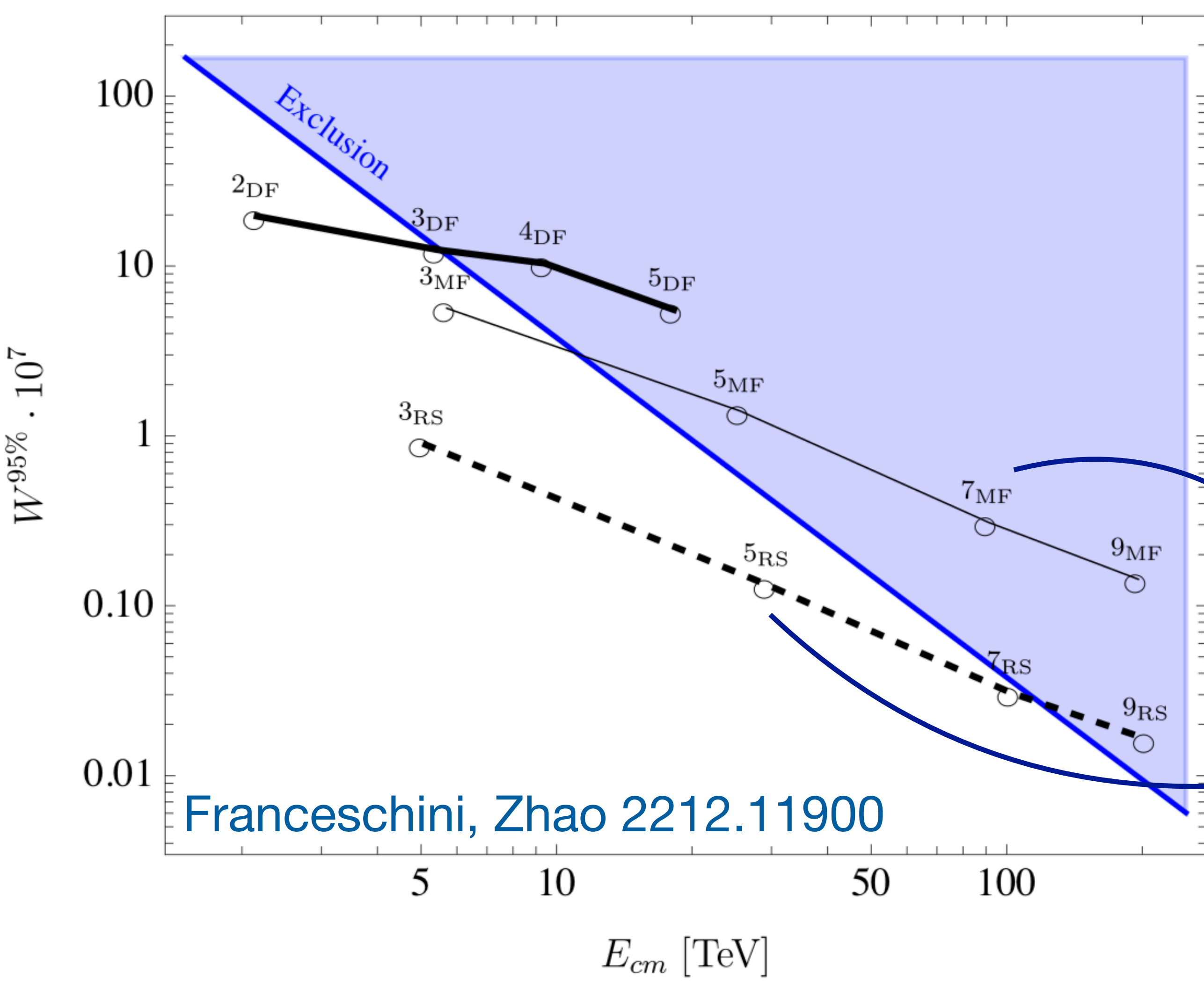
- All EW multiplets contribute to high-energy 2 → 2 fermion scattering: effects that grow with energy, can be tested at colliders



$$\hat{W} \approx 10^{-7} \times \left(\frac{1\,\text{TeV}}{M_{\text{DM}}}\right)^2 n^3 \propto 1/n^2$$

$$\hat{Y} \approx 10^{-7} \times \left(\frac{1\,\text{TeV}}{M_{\text{DM}}}\right)^2 Y^2 n \propto 1/n^4$$

Di Luzio, Gröber, Panico 1810.10993

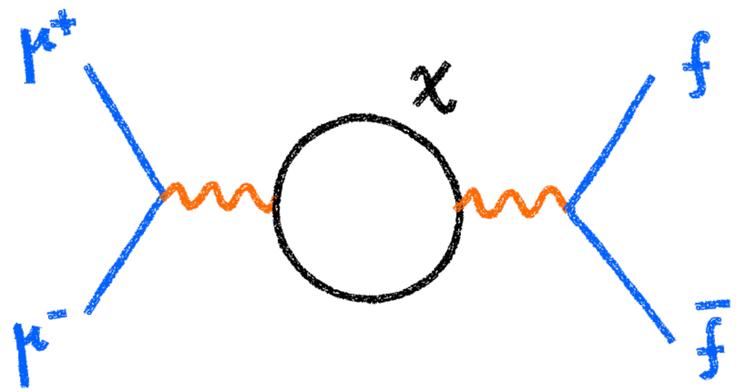


right of blue line: can be tested indirectly

left of blue line: can be tested directly

Indirect effects at colliders

- All EW multiplets contribute to high-energy $2 \to 2$ fermion scattering: effects that grow with energy, can be tested at colliders



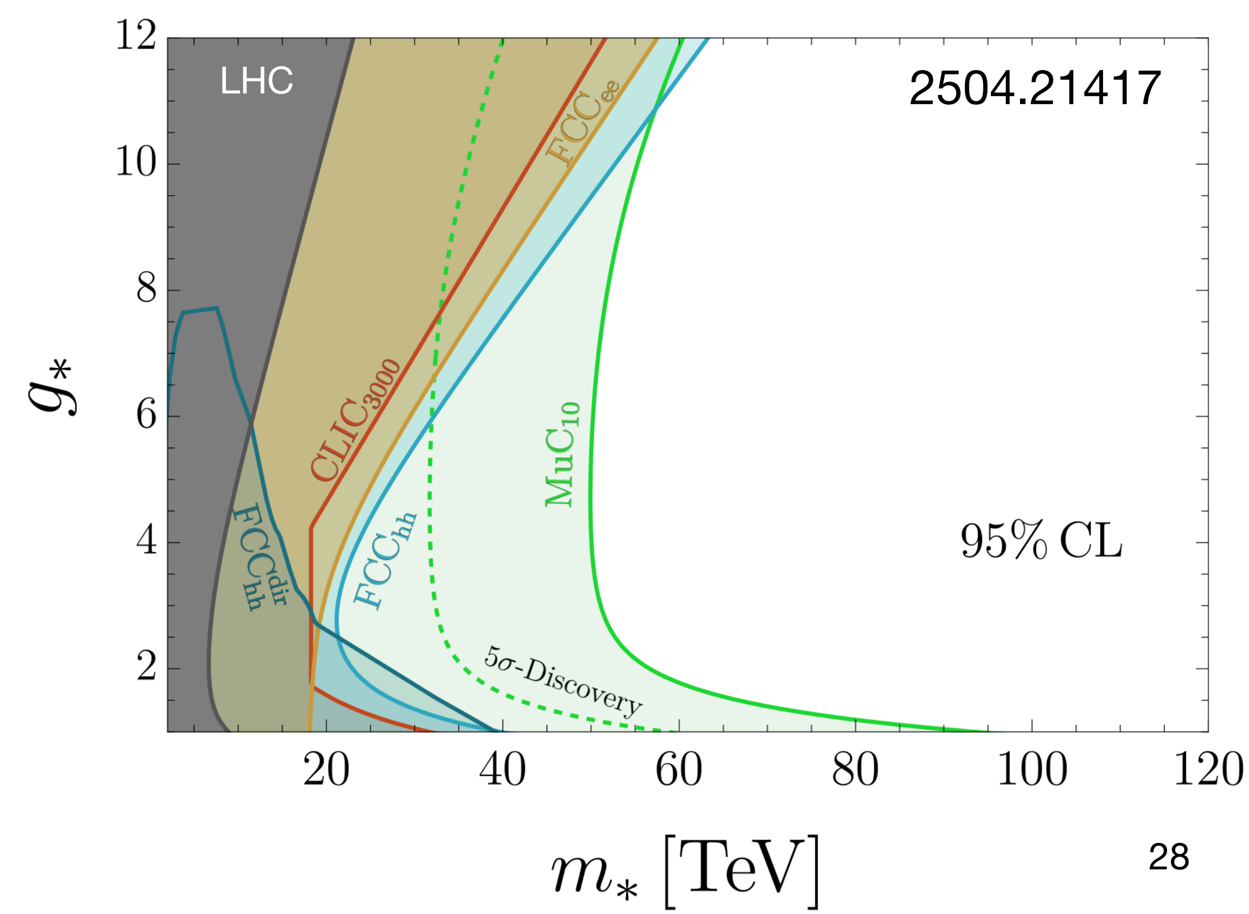
$$\hat{W} \approx 10^{-7} \times \left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{DM}}}\right)^2 n^3 \propto 1/n^2$$

$$\hat{Y} \approx 10^{-7} \times \left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{DM}}}\right)^2 Y^2 n \propto 1/n^4$$

Di Luzio, Gröber, Panico 1810.10993

- If WIMPs are composite, usual probes of compositeness in Higgs couplings, EWPT and high-energy scattering.

The scale $m_\star$ is related to the DM mass, set by thermal freeze-out.



Summary

- Thermal, weakly interacting Dark Matter points to multi-TeV scales

- Not probed yet, but in reach of future (futuristic) experiments

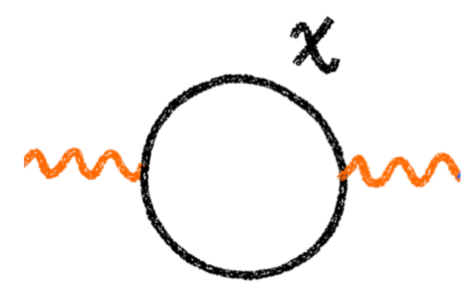
- Can be related to models of EWSB at TeV scale:
  not only SUSY, but also composite Higgs.

- There is a set of models where the symmetries of Composite Higgs can provide a stable Dark Matter candidate.
  Interesting interplay of Higgs and DM phenomenology.

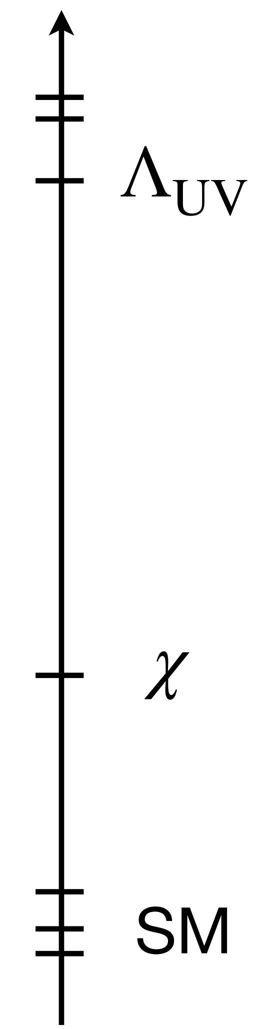
Calculability: the unitarity bound

- Cut-off scale Λ_{UV} must be high enough to not affect Ω_{DM} calculation

- Upper bound on cut-off scale from running of SU(2) gauge coupling



$$\Lambda_{\mathrm{UV}} \leq \Lambda_{\mathrm{Landau}}$$



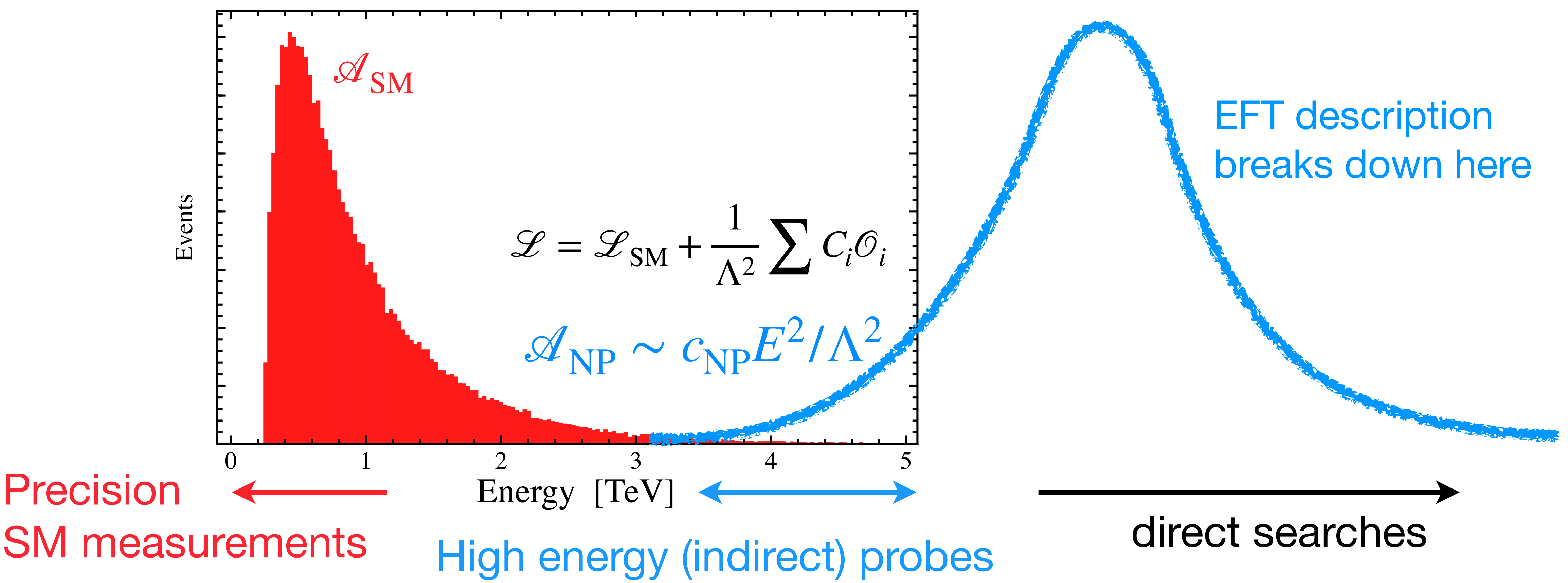
Actually, impose partial-wave unitarity of 2 → 2 scattering:

$$(\sigma_{\mathrm{eff}} v)^J \leq \frac{4\pi(2J+1)}{M_\chi^2 v}$$

- Real WIMPs: n ≤ 13
- Complex WIMPs: Y ≤ 1
  - n ≤ 12 for Y = 1/2
  - n ≤ 5 for Y = 1

High-energy probes

- NP effects are more important at high energies



- As simple as this: $$\frac{\Delta\sigma(E)}{\sigma_{SM}(E)} \propto \frac{E^2}{\Lambda^2_{BSM}} \approx \begin{cases} 10^{-6}, & E \sim 100\,\mathrm{GeV} \\ 10^{-2}, & E \sim 10\,\mathrm{TeV} \end{cases}$$

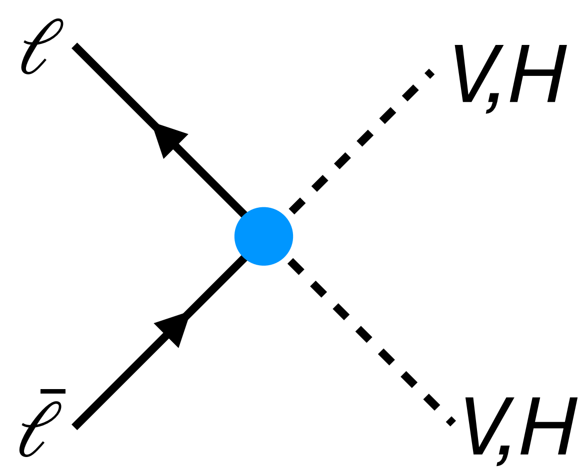
- Effective at LHC, FCC-hh, CLIC: “energy helps accuracy”…

Farina et al. 1609.08157, Franceschini et al. 1712.01310, …

… taken to the extreme at a μ-collider with 10’s of TeV!

Example: high-energy di-bosons

- Longitudinal $2 \to 2$ scattering amplitudes at high energy:



$$\ell^+\ell^- \to W_L^+ W_L^-$$
$$\ell^+\ell^- \to Z_L H$$

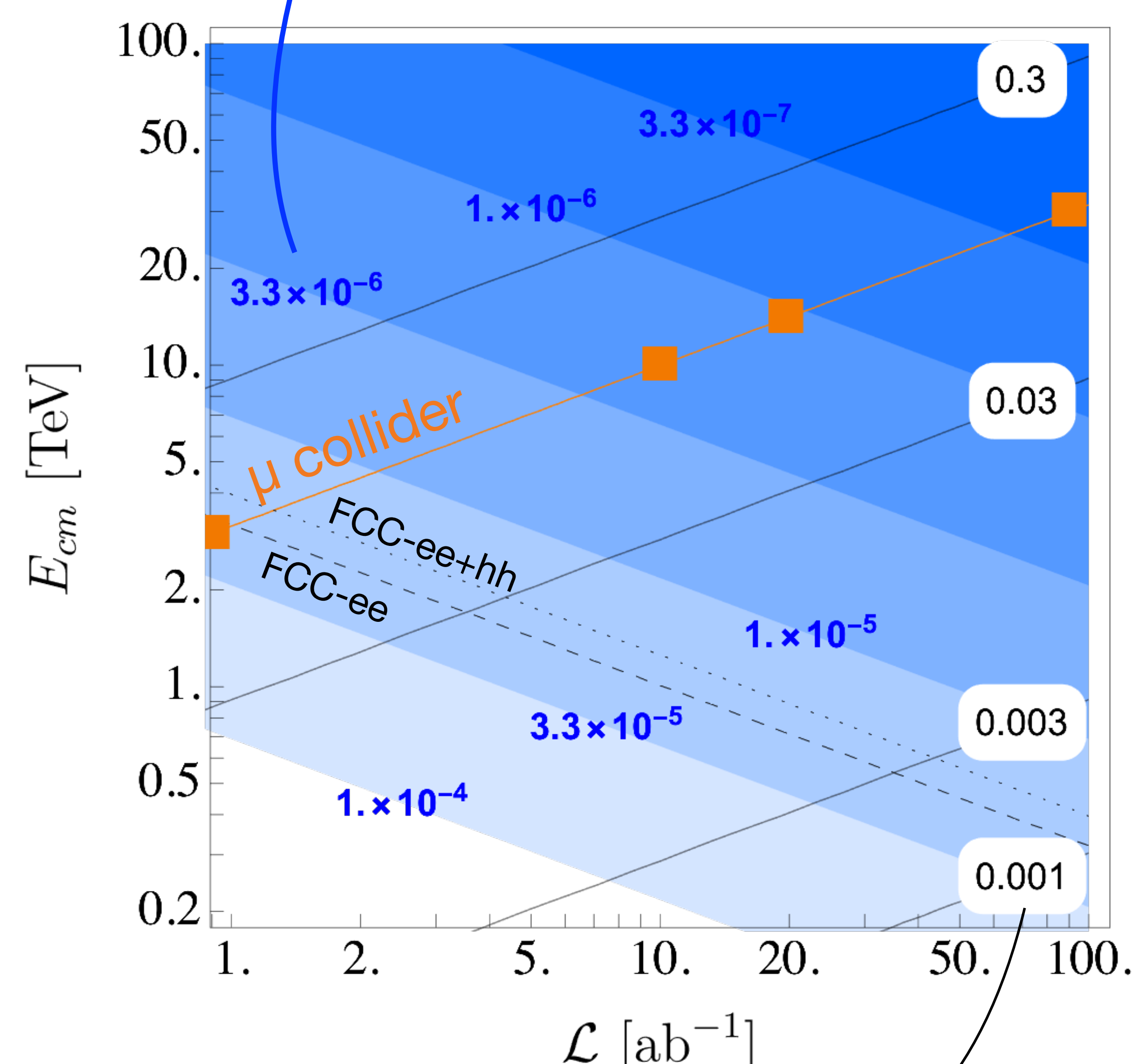
"high-energy primary effects"

related with Z-pole observables

$$\hat{S} = m_W^2 (C_W + C_B)$$

Determined by two dim. 6 operators (in flavor-universal theories):

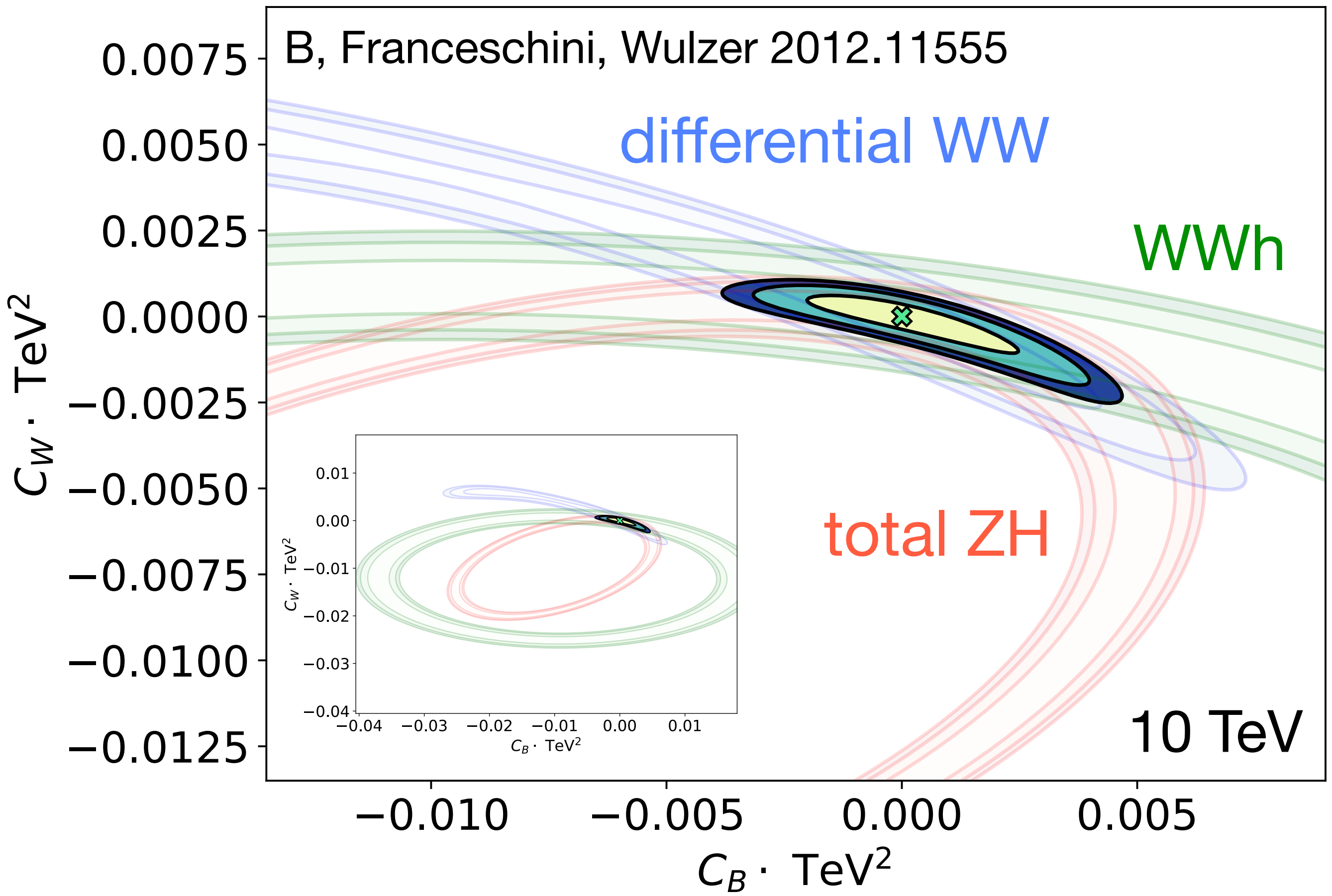
$$\boxed{\begin{aligned} \mathcal{O}_W &= \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu} \\ \mathcal{O}_B &= \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu} \end{aligned}}$$

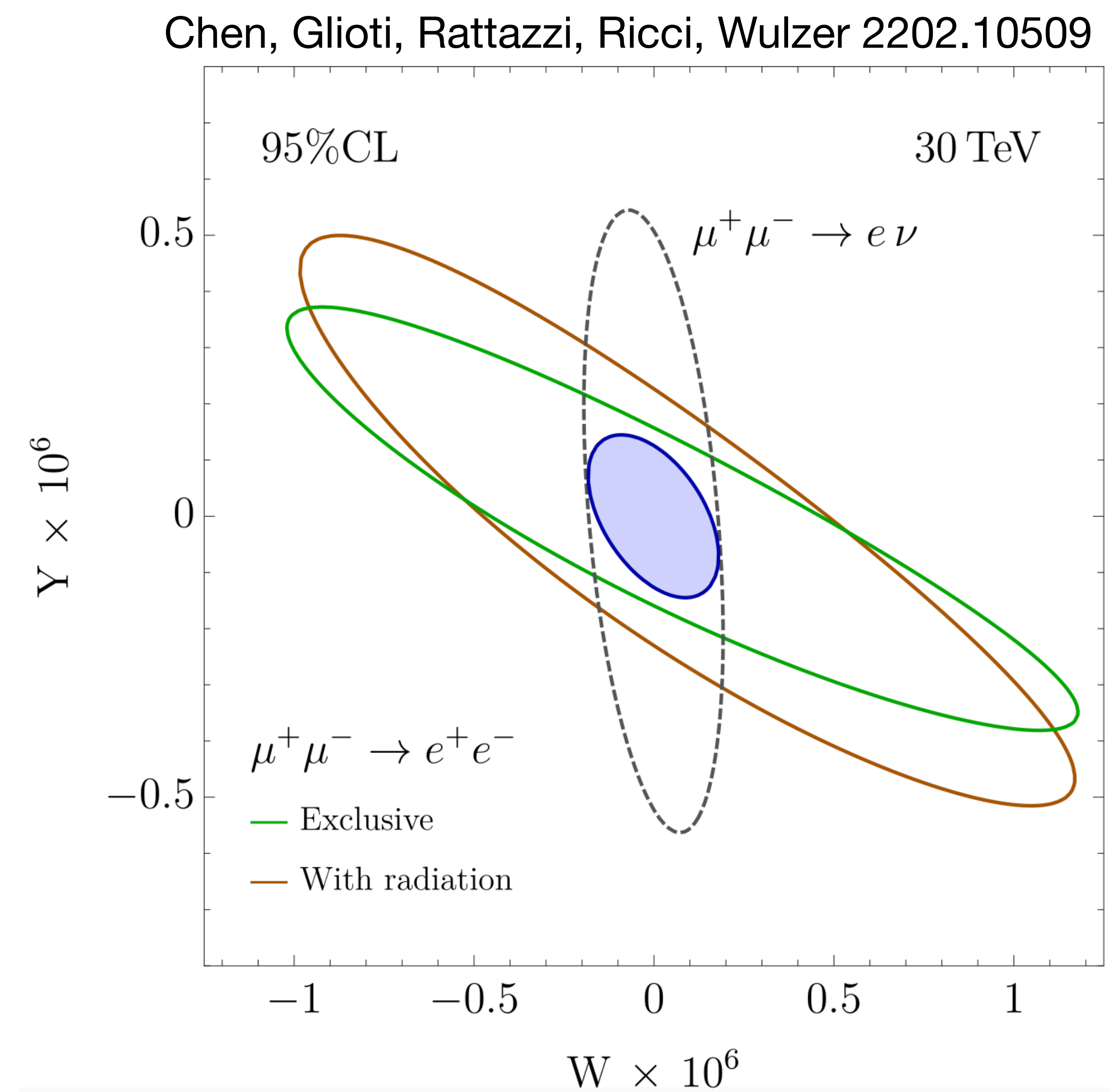


precision of measurement

LEP: 10^{-3}, FCC: few 10^{-5}, MuC: 10^{-6}

High-energy probes: radiation

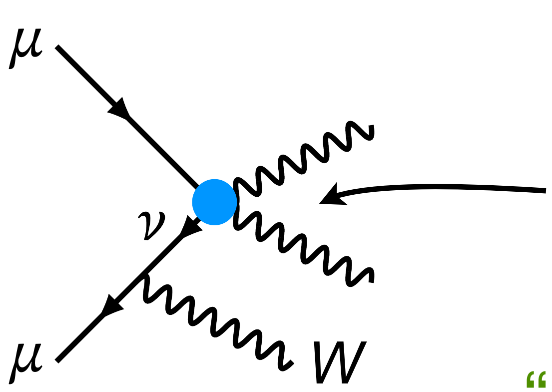




Gauge boson radiation important:

soft W emission allows to access

charged processes $\ell\nu \to W^\pm Z, W^\pm H$



"effective neutrino approximation"

- contains new physical information!
- need to properly define inclusive observables, resummation of logs, …

Results: real WIMPs

DM spin	EW n-plet	M_χ (TeV)	$(\sigma v)^{J=0}_{\rm tot}/(\sigma v)^{J=0}_{\rm max}$	$\Lambda_{\rm Landau}/M_{\rm DM}$	$\Lambda_{\rm UV}/M_{\rm DM}$
Real scalar	3	2.53 ± 0.01	–	2.4×10^{37}	$4 \times 10^{24}*$
	5	15.4 ± 0.7	0.002	7×10^{36}	3×10^{24}
	7	54.2 ± 3.1	0.022	7.8×10^{16}	2×10^{24}
	9	117.8 ± 8.8	0.088	3×10^{4}	2×10^{24}
	11	199 ± 14	0.25	62	1×10^{24}
	13	338 ± 24	0.6	7.2	2×10^{24}
Majorana fermion	3	2.86 ± 0.01	–	2.4×10^{37}	$2 \times 10^{12}*$
	5	13.6 ± 0.8	0.003	5.5×10^{17}	3×10^{12}
	7	48.8 ± 2.7	0.019	1.2×10^{4}	1×10^{8}
	9	113 ± 9	0.07	41	1×10^{8}
	11	202 ± 14	0.2	6	1×10^{8}
	13	324.6 ± 23	0.5	2.6	1×10^{8}

$$\mathscr{L}_{\rm s} \supset \frac{C_1^{(s)}}{\Lambda_{\rm UV}^{n-4}}\chi(H^\dagger H)^{\frac{n-1}{2}} + \frac{C_2^{(s)}}{\Lambda_{\rm UV}^{n-4}}\chi W_{\mu\nu}W^{\mu\nu}(H^\dagger H)^{\frac{n-5}{2}} + \cdots + \frac{C_w^{(s)}}{\Lambda_{\rm UV}^{n-4}}\chi(W_{\mu\nu}W^{\mu\nu})^{\frac{n-1}{4}} + \frac{C_{3\chi}^{(s)}}{\Lambda_{\rm UV}}\chi^3 H^\dagger H,$$

$$\mathscr{L}_{\rm f} \supset \frac{C_1^{(f)}}{\Lambda_{\rm UV}^{n-3}}(\chi HL)(H^\dagger H)^{\frac{n-3}{2}} + \frac{C_2^{(f)}}{\Lambda_{\rm UV}^{n-3}}(\chi\sigma^{\mu\nu}HL)W_{\mu\nu}(H^\dagger H)^{\frac{n-5}{2}} + \cdots + \frac{C_w^{(f)}}{\Lambda_{\rm UV}^{n-3}}(\chi HL)(W_{\mu\nu}W^{\mu\nu})^{\frac{n-3}{4}} + \frac{C_{3\chi}^{(f)}}{\Lambda_{\rm UV}^{3}}\chi^3 HL,$$

Results: complex WIMPs

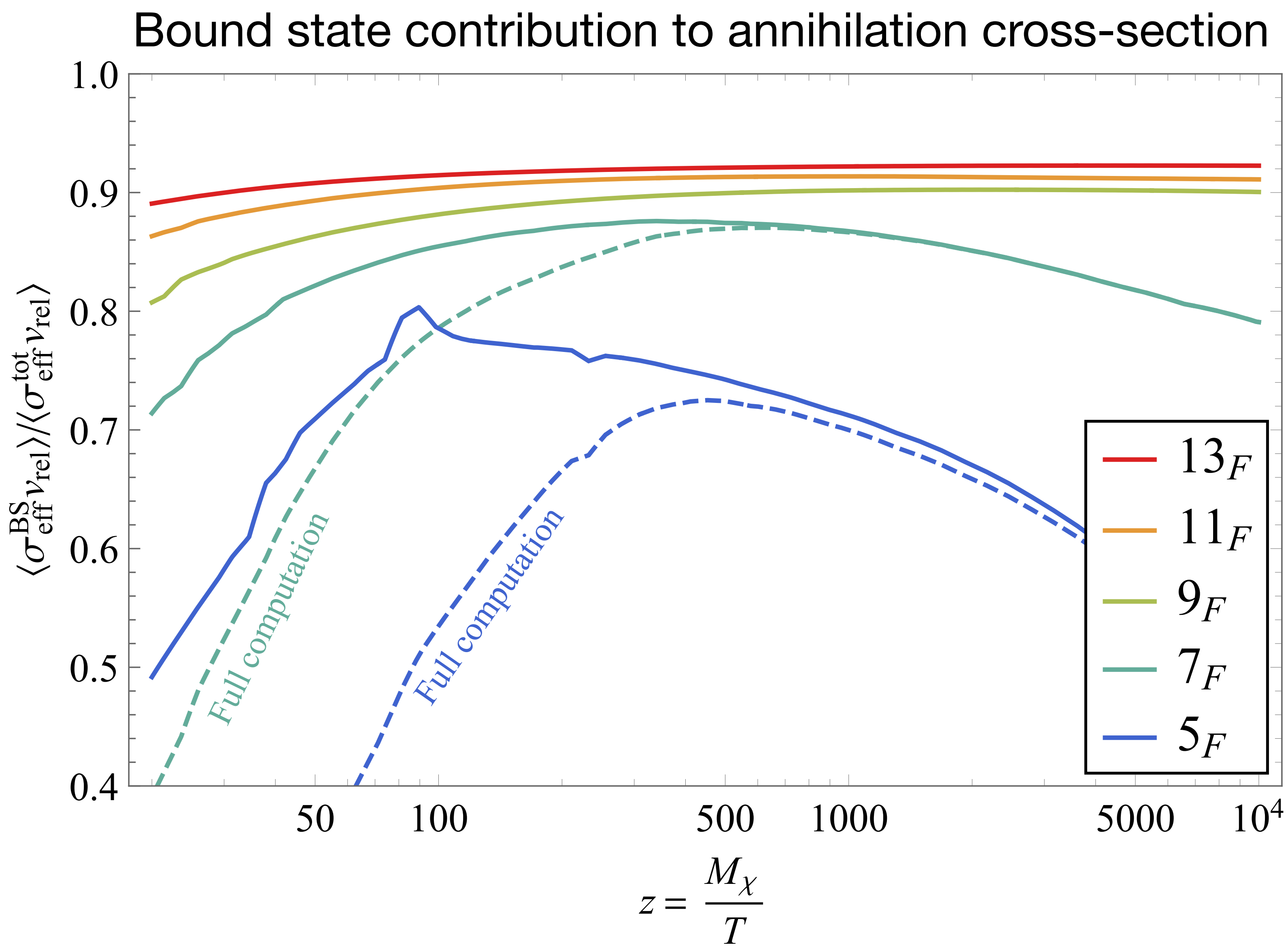
DM spin	n_Y	$M_{\rm DM}$ (TeV)	$\Lambda_{\rm Landau}/M_{\rm DM}$	$(\sigma v)^{J=0}_{\rm tot}/(\sigma v)^{J=0}_{\rm max}$	δm_0 [MeV]	$\Lambda^{\rm max}_{\rm UV}/M_{\rm DM}$	δm_{Q_M} [MeV]
Dirac fermion	$2_{1/2}$	1.08 ± 0.02	$> M_{\rm Pl}$	-	0.22 - 2×10^4	10^7	4.8 - 10^4
	3_1	2.85 ± 0.14	$> M_{\rm Pl}$	-	0.22 - 40	60	312 - 1.6×10^4
	$4_{1/2}$	4.8 ± 0.3	$\simeq M_{\rm Pl}$	0.001	0.21 - 3×10^4	5×10^6	20 - 1.9×10^4
	5_1	9.9 ± 0.7	3×10^6	0.003	0.21 - 3	25	$10^3 - 2\times10^3$
	$6_{1/2}$	31.8 ± 5.2	2×10^4	0.01	0.5 - 2×10^4	4×10^5	100 - 2×10^4
	$8_{1/2}$	82 ± 8	15	0.05	0.84 - 10^4	10^5	440 - 10^4
	$10_{1/2}$	158 ± 12	3	0.16	1.2 - 8×10^3	6×10^4	1.1×10^3 - 9×10^3
	$12_{1/2}$	253 ± 20	2	0.45	1.6 - 6×10^3	4×10^4	2.3×10^3 - 7×10^3
Complex scalar	$2_{1/2}$	0.58 ± 0.01	$> M_{\rm Pl}$	-	4.9 - 1.4×10^4	-	4.2 - 7×10^3
	3_1	2.1 ± 0.1	$> M_{\rm Pl}$	-	3.7 - 500	120	75 - 1.3×10^4
	$4_{1/2}$	4.98 ± 0.25	$> M_{\rm Pl}$	0.001	4.9 - 3×10^4	-	17 - 2×10^4
	5_1	11.5 ± 0.8	$> M_{\rm Pl}$	0.004	3.7 - 10	20	650 - 3×10^3
	$6_{1/2}$	32.7 ± 5.3	$\simeq 6\times10^{13}$	0.01	4.9 - 8×10^4	-	50 - 5×10^4
	$8_{1/2}$	84 ± 8	2×10^4	0.05	4.9 - 6×10^4	-	150 - 6×10^4
	$10_{1/2}$	162 ± 13	20	0.16	4.9 - 4×10^4	-	430 - 4×10^4
	$12_{1/2}$	263 ± 22	4	0.4	4.9 - 3×10^4	-	10^3 - 3×10^4

$$\mathscr{L}_{\rm D} = \overline{\chi}\left(i\not{D} - M_\chi\right)\chi + \frac{y_0}{\Lambda_{\rm UV}^{4Y-1}}\mathcal{O}_0 + \frac{y_+}{\Lambda_{\rm UV}}\mathcal{O}_+ + \text{h.c.}\ ,$$

$$\mathcal{O}_0 = \frac{1}{2(4Y)!}\left(\overline{\chi}(T^a)^{2Y}\chi^c\right)\left[(H^{c\dagger})\frac{\sigma^a}{2}H\right]^{2Y}\ ,$$

$$\mathcal{O}_+ = -\overline{\chi}T^a\chi H^\dagger\frac{\sigma^a}{2}H\ ,$$

Impact of bound state formation



Sommerfeld

$$S_{\rm ann} = \sum_I \left\langle \sigma_{\rm ann}^I v_{\rm rel} \frac{2\pi\alpha_{\rm eff}}{v_{\rm rel}} \right\rangle$$

$$\langle \sigma_{\rm eff} v_{\rm rel}\rangle \equiv S_{\rm ann}(z) + \sum_{B_J} S_{B_J}(z)$$

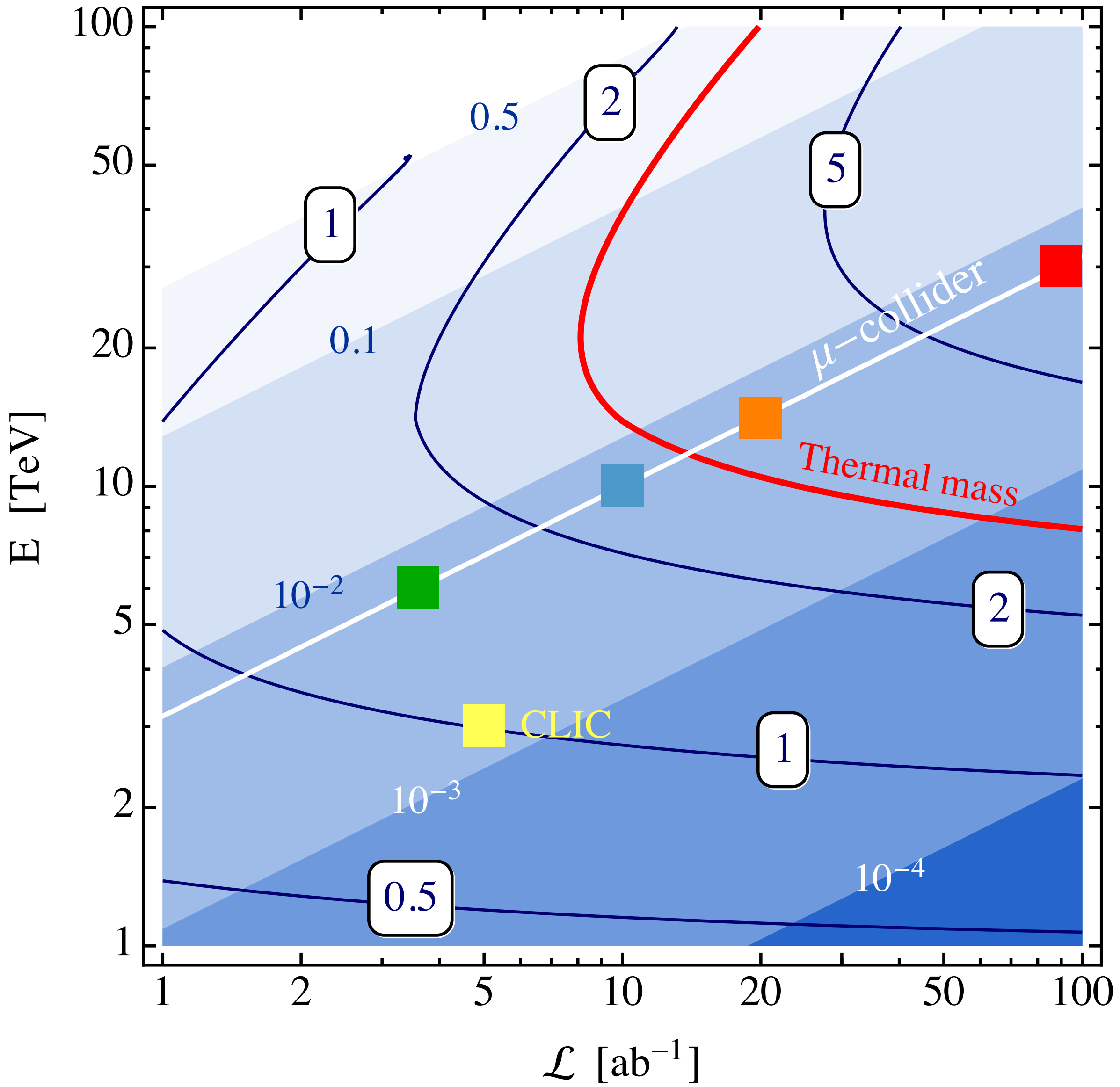
$$S_{B_J} = \sum_{I,l} \sigma_{B_J}^{I,l} \frac{2\pi\alpha_{\rm eff}}{v_{\rm rel}} R_J$$

branching ratio into SM

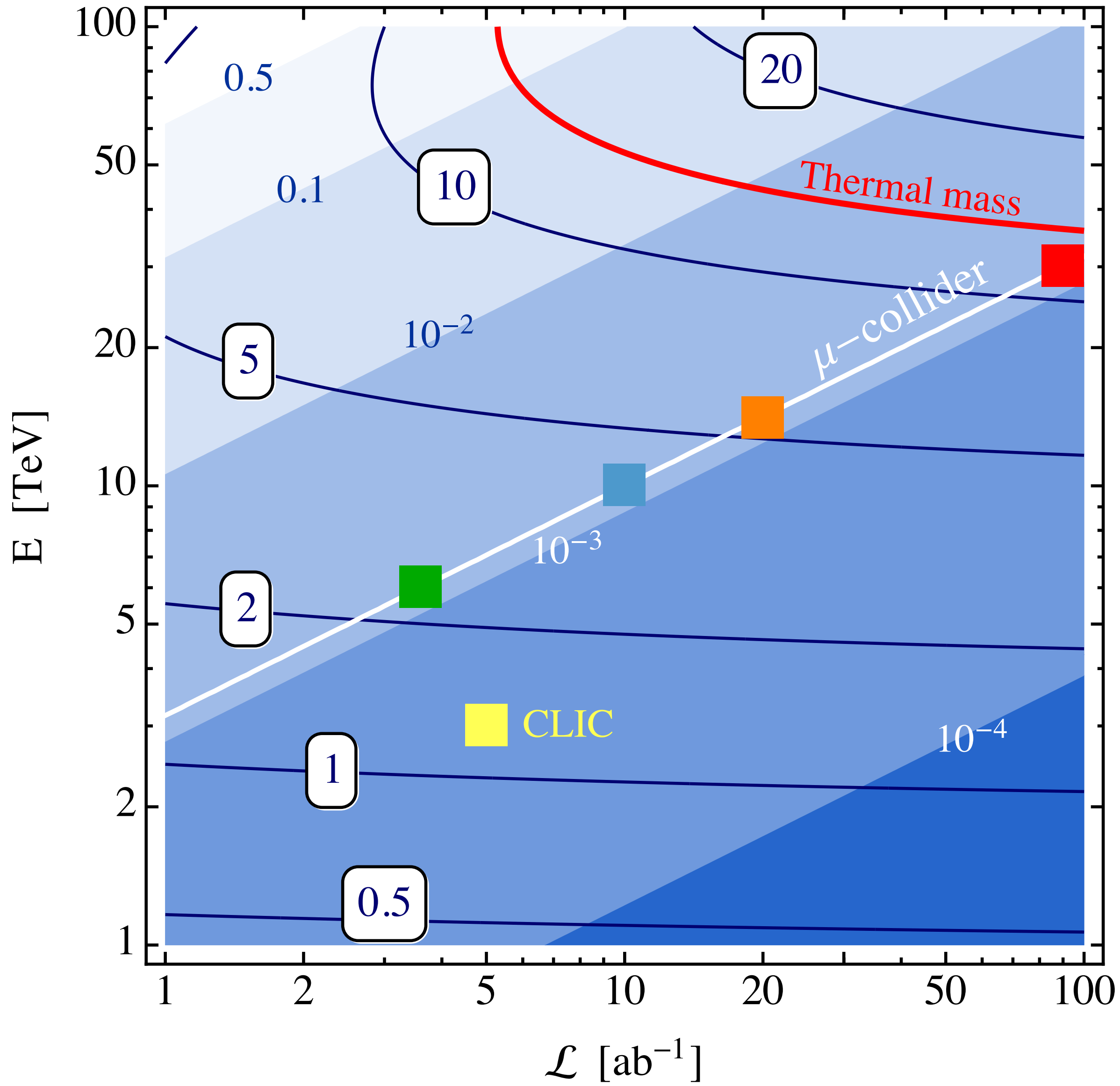
Reach at muon colliders

mono-W searches

Majorana 3-plet (Wino)

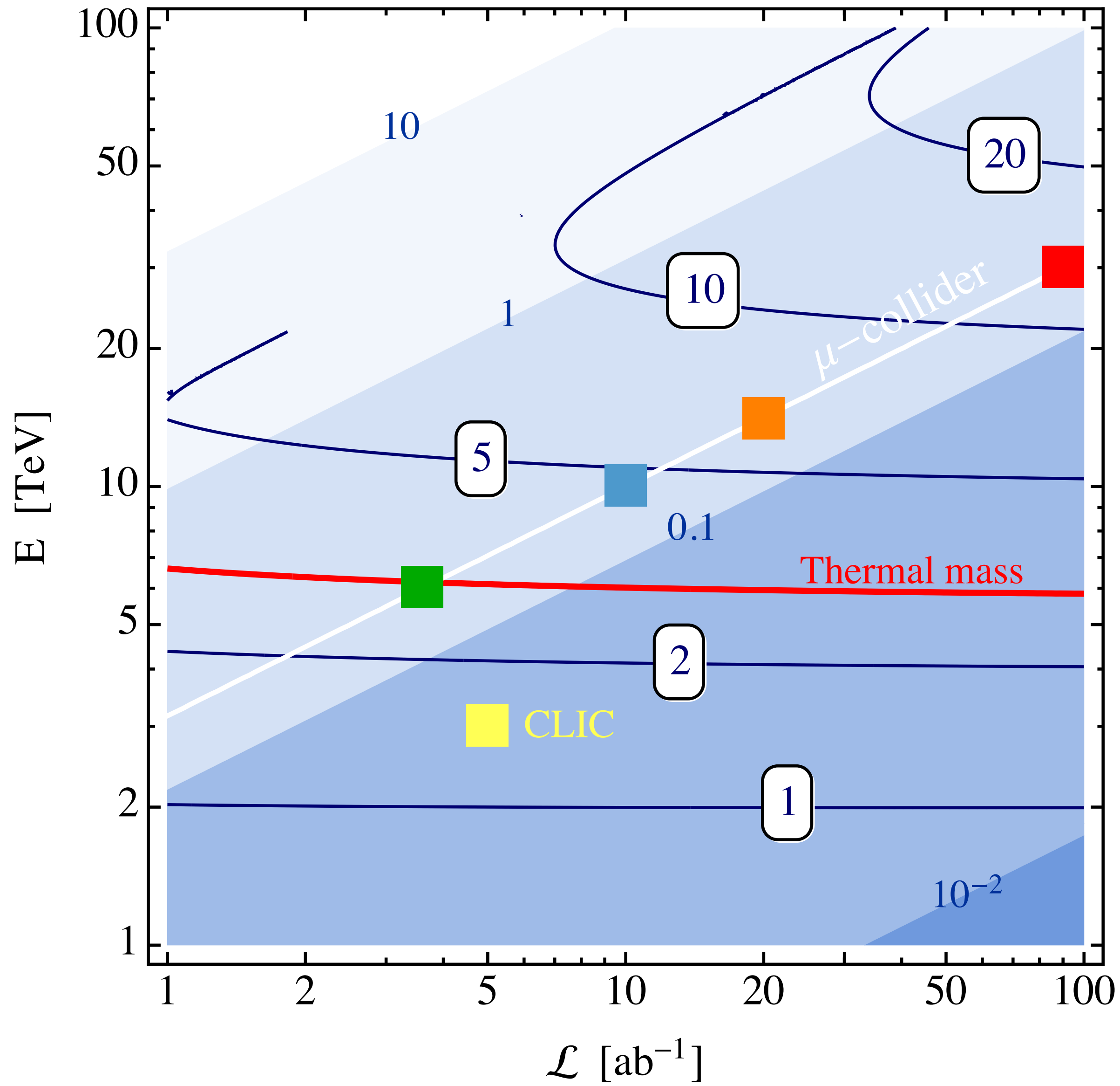


Majorana 5-plet (MDM)

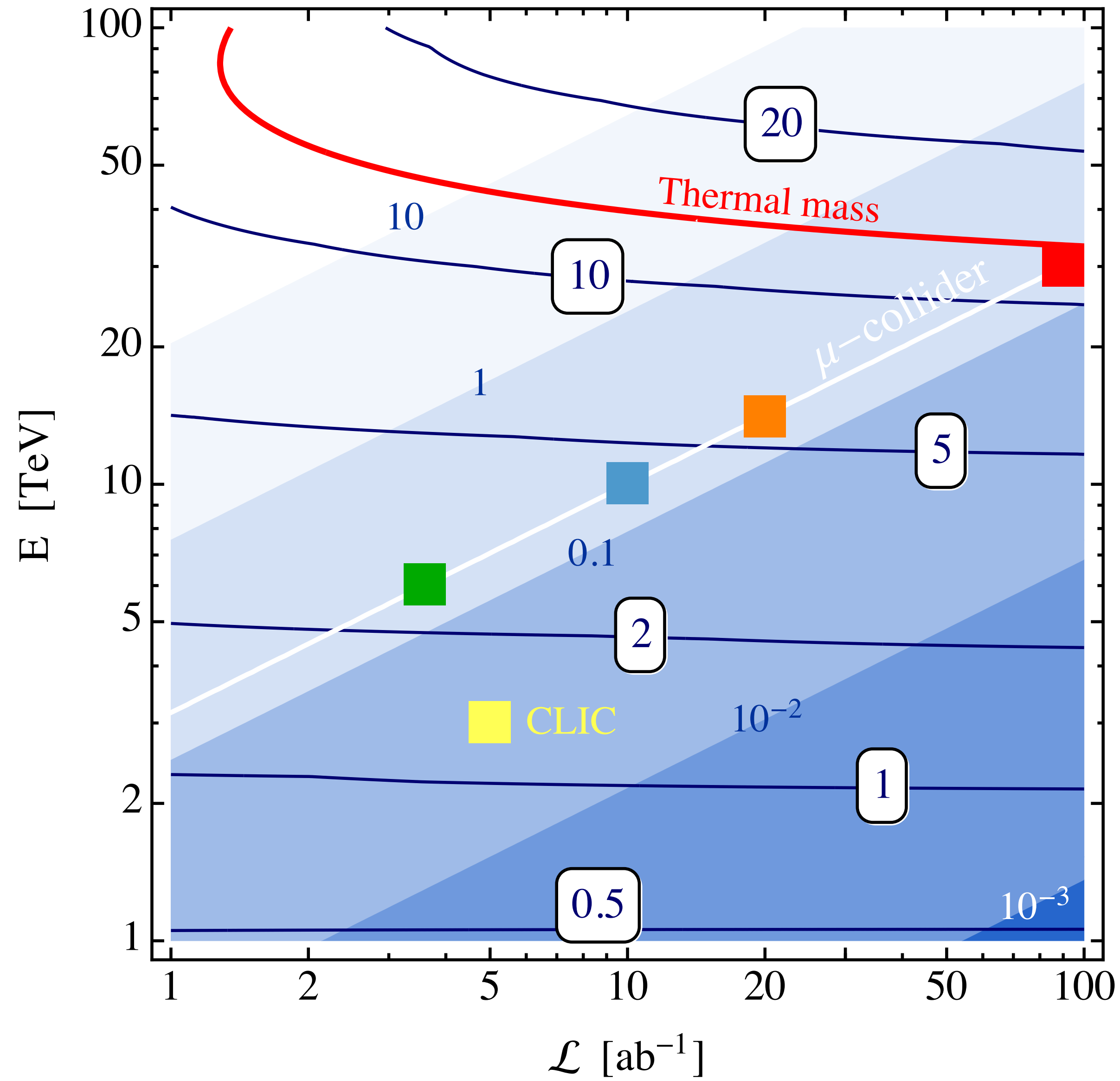


Disappearing track searches (mono-γ)

Majorana 3-plet (Wino)



Majorana 5-plet (MDM)



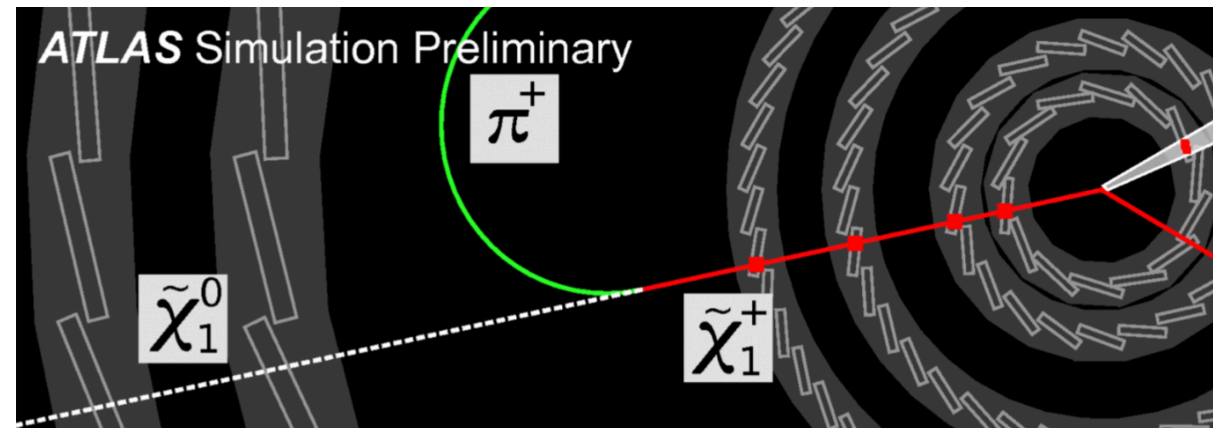
Mass splittings and disappearing tracks

- Dark Matter is part of a multiplet that includes also charged states

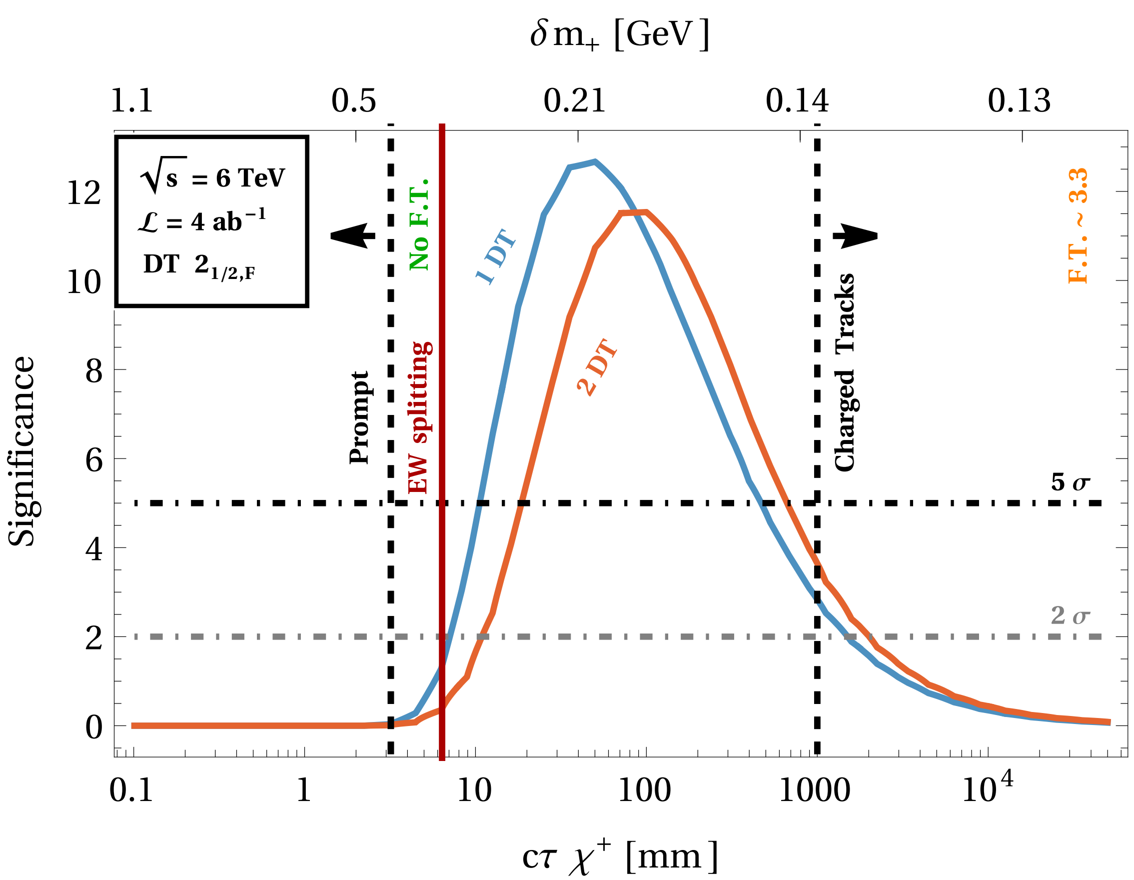
$$\chi_n = (\cdots, \chi^-, \chi^0, \chi^+, \cdots)$$

$\chi^\pm$ decays into DM inside the detector

- Look for the disappearing tracks of the charged particles to isolate the DM signal from the SM background (mainly neutrinos)



Capdevilla, Meloni, Simoniello, Zurita 2102.11292



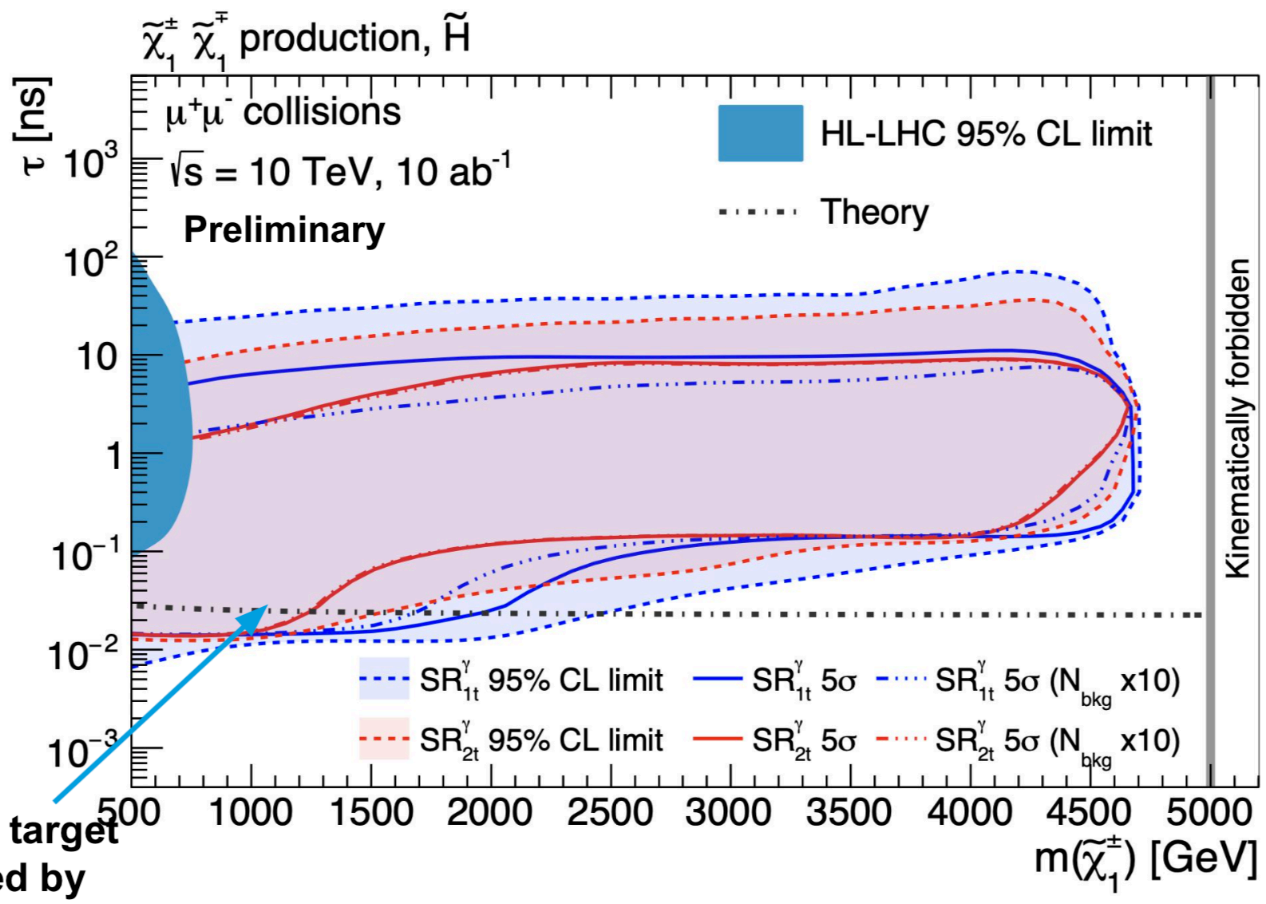
- Real WIMPs (Y = 0): mass splitting fixed by gauge interactions

$$M_Q - M_0 \approx Q^2 \alpha_{\rm em} m_W$$

$$c\tau_{\chi^\pm} \approx 50\,\mathrm{cm}/(n^2-1)$$

- Complex WIMPs: additional splitting needed to make DM stable

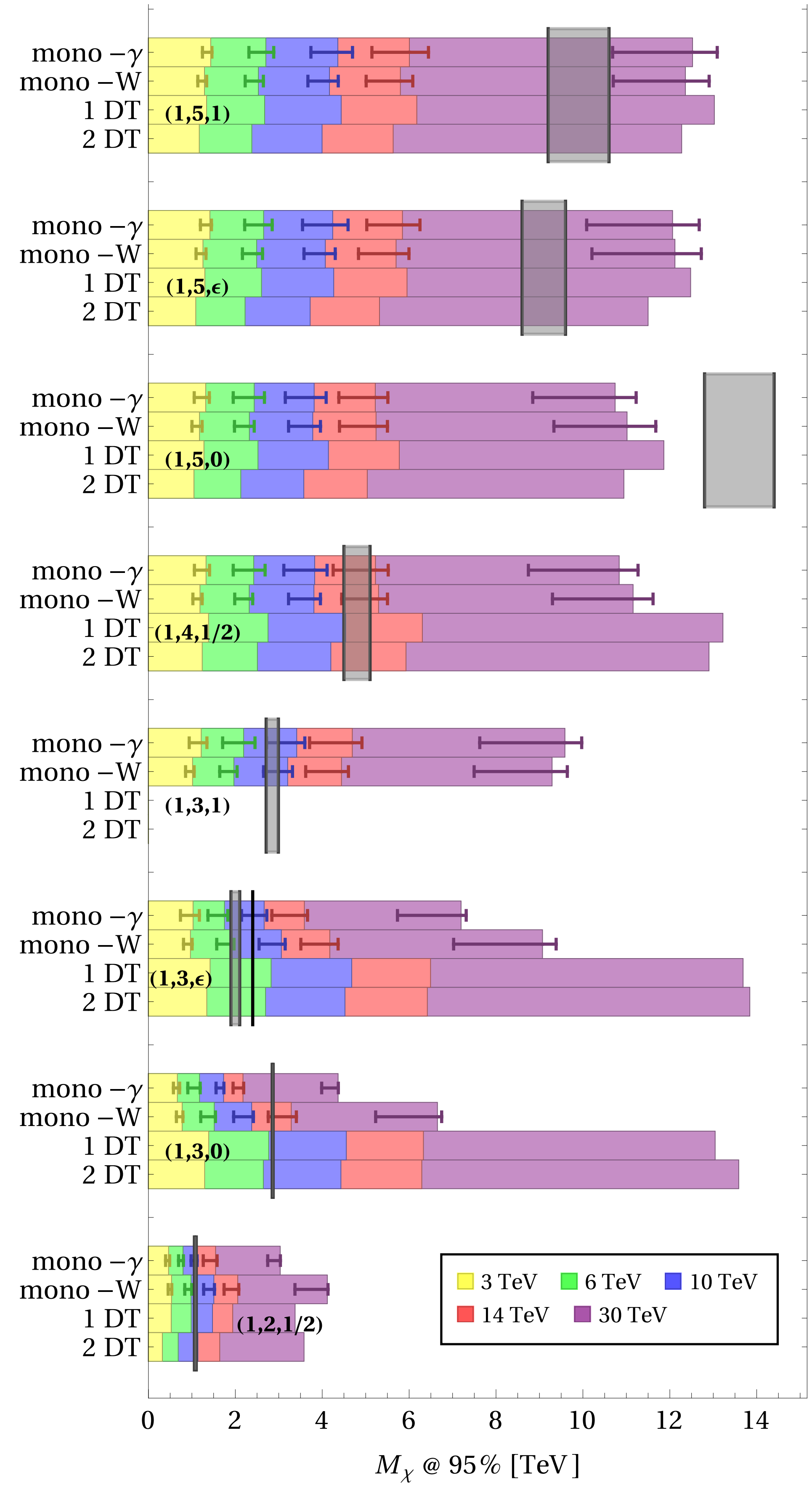
Disappearing tracks at μ collider



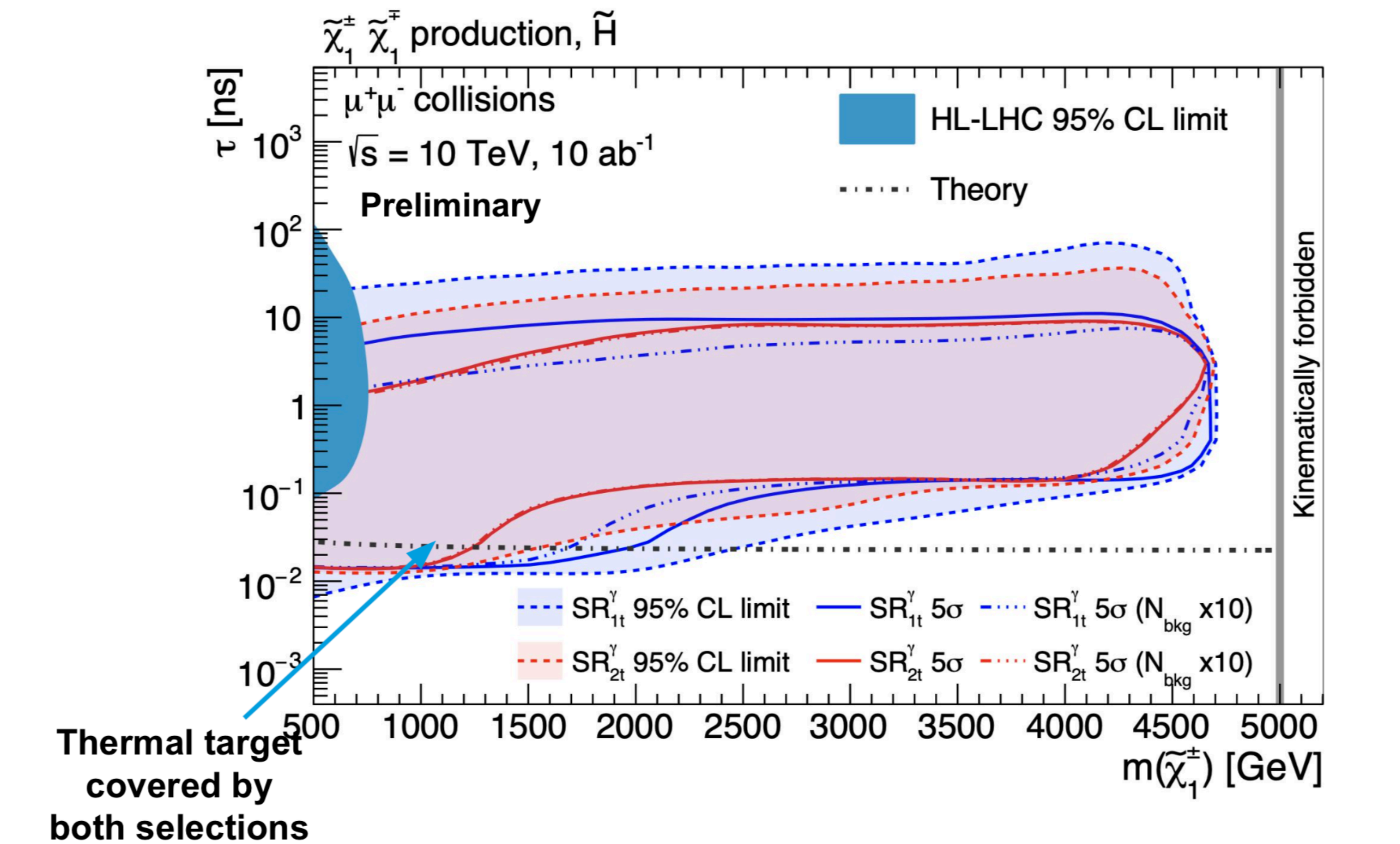
Thermal target covered by both selections

Capdevilla, Meloni, Simoniello, Zurita 2102.11292

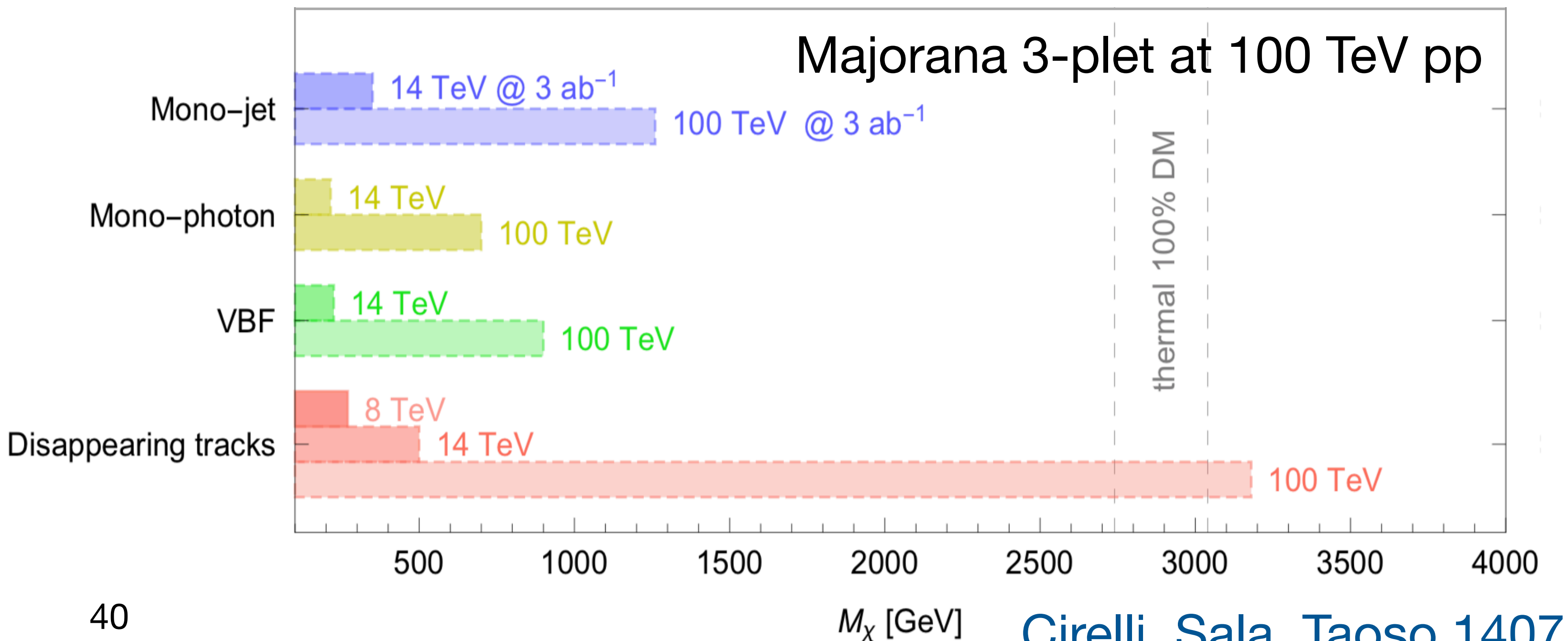
Bottaro et al. 2205.04486



Disappearing tracks at μ collider

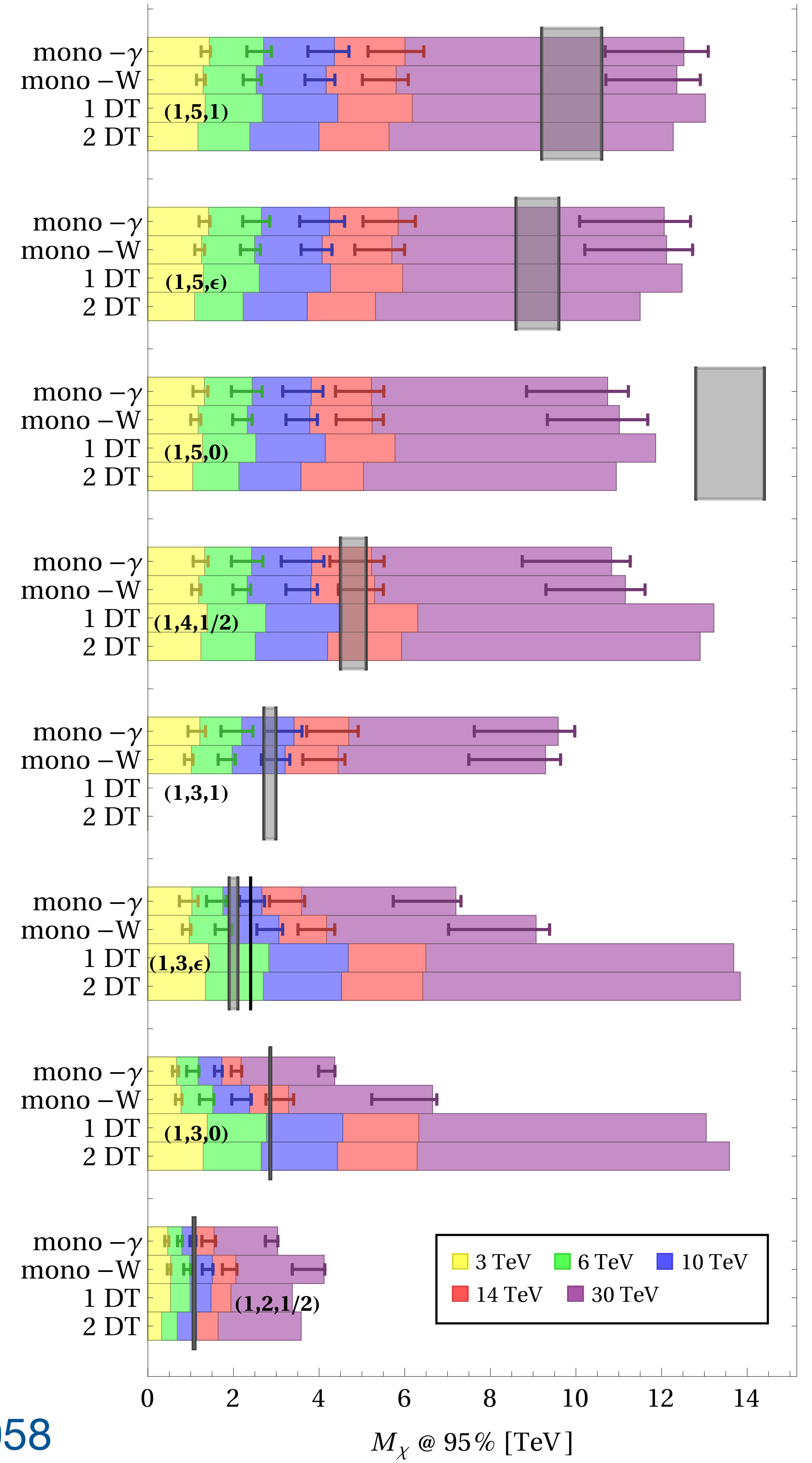


Capdevilla, Meloni, Simoniello, Zurita 2102.11292



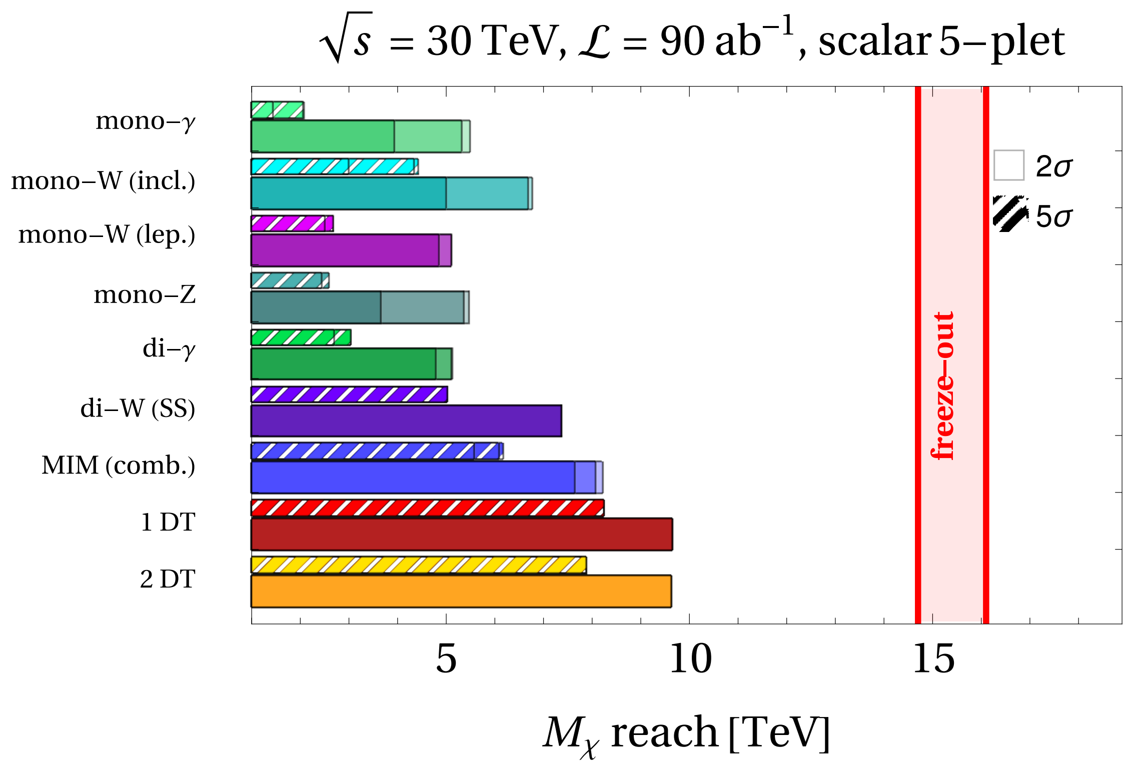
Cirelli, Sala, Taoso 1407.7058

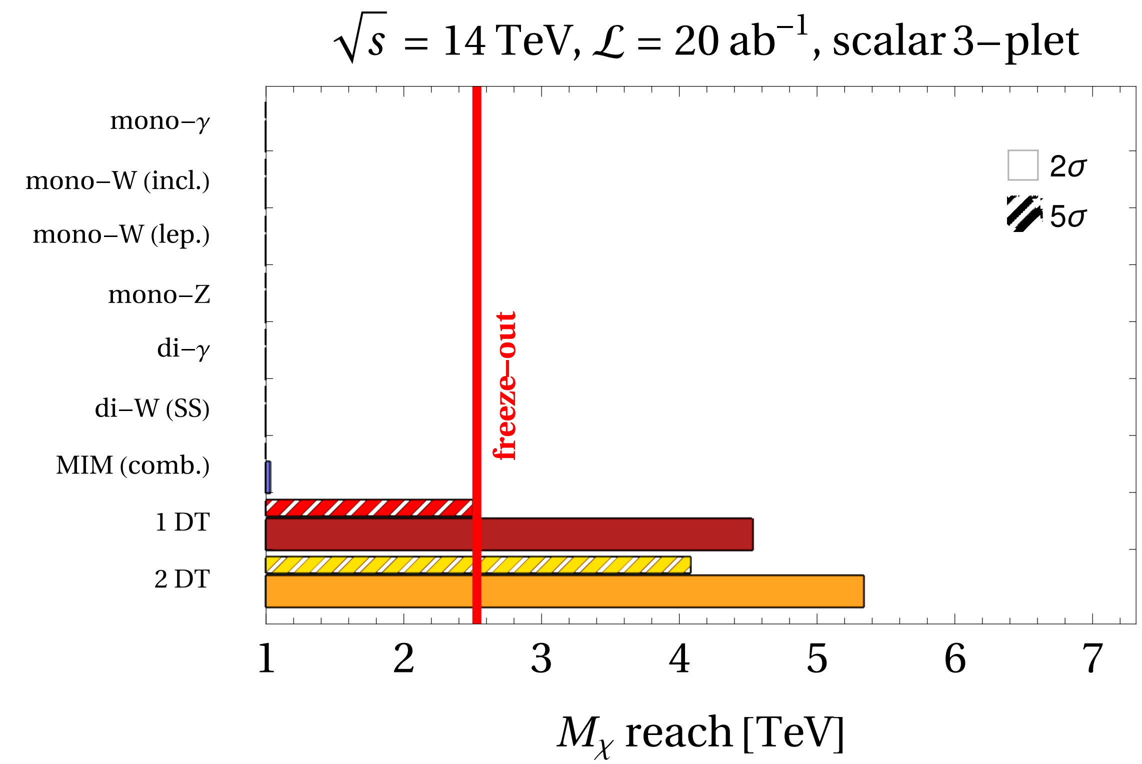
Bottaro et al. 2205.04486



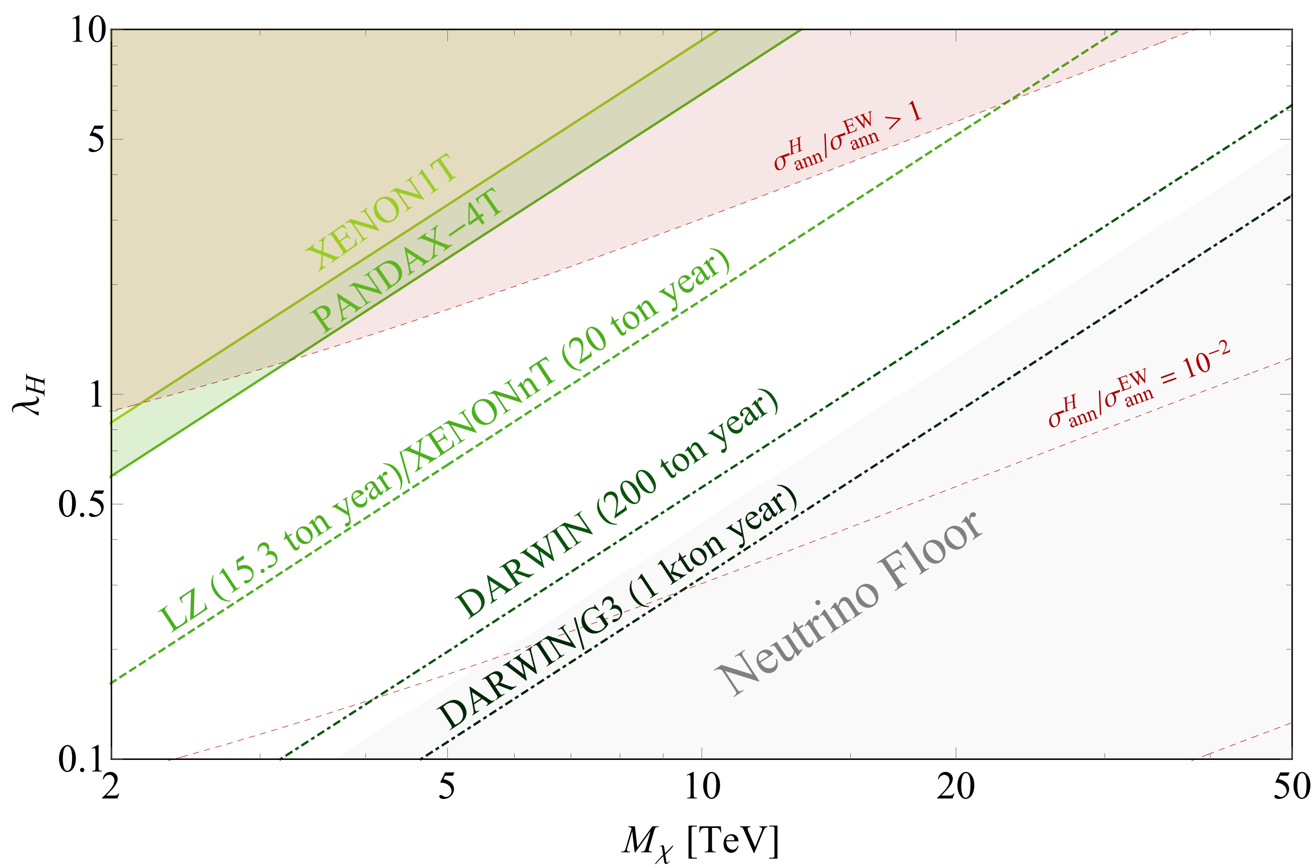
Scalar WIMPs

- Scalars have lower cross-sections





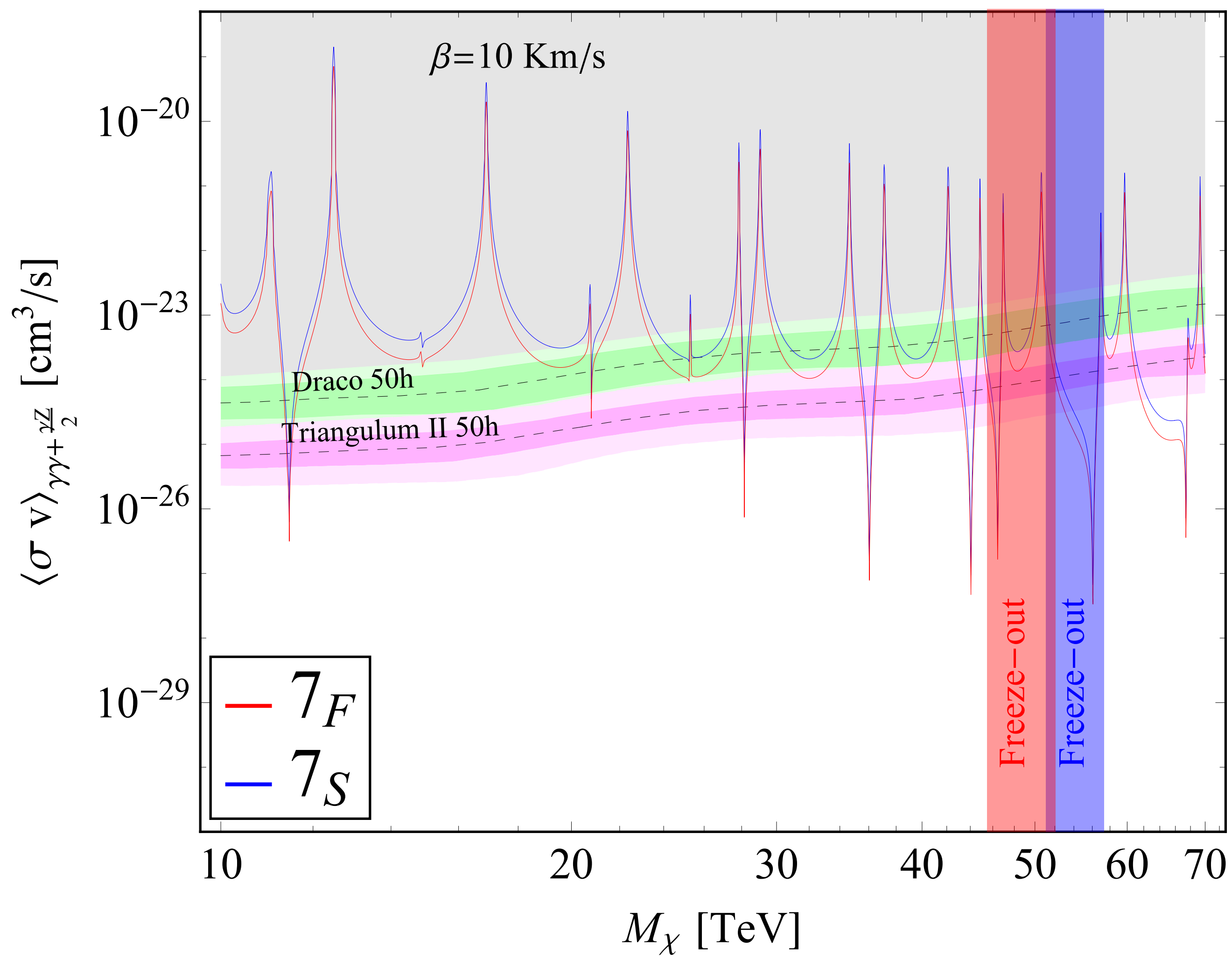
- Higgs portal coupling
  - ➔ direct detection

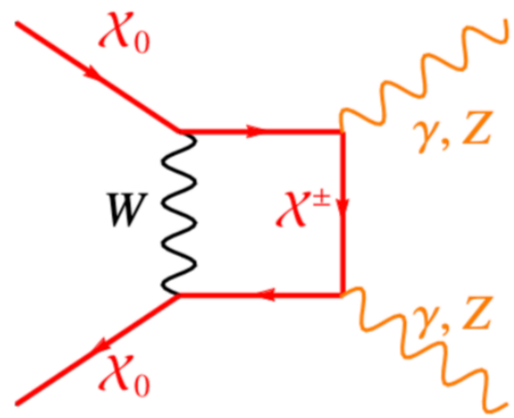


Indirect detection

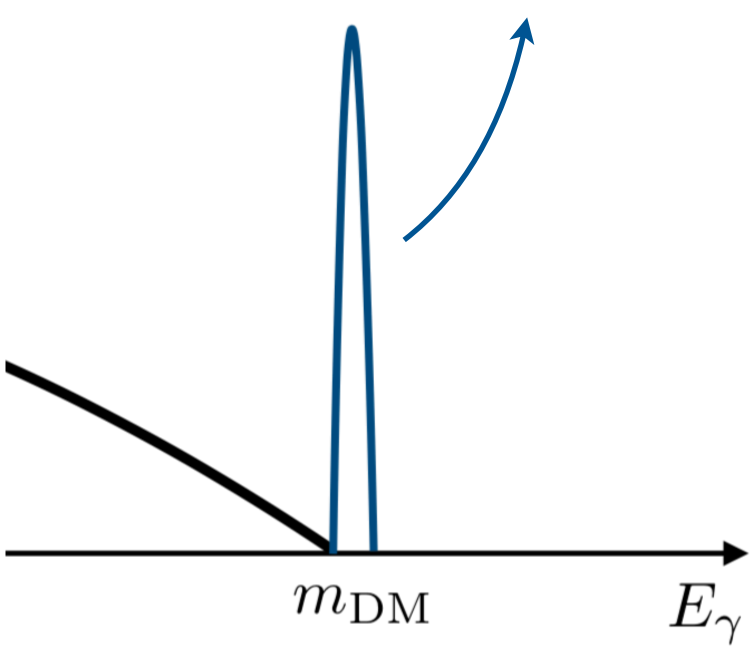
- Searches for high-energy gamma-ray lines with Cherenkov telescopes are a powerful constraint for high-mass WIMP DM

(Large multiplets are more easily probed due to increased annihilation cross-section)





γ-ray line: annihilation into γγ and γZ



➡ see Baumgart, Rodd, Slatyer, Vaidya 2309.11562