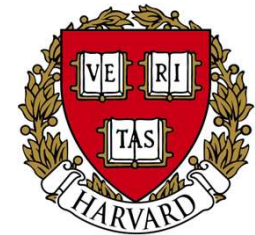


Dynamics across quantum phase transitions in Rydberg atom arrays

Simone Notarnicola, University of Padova



Dipartimento
di Fisica
e Astronomia
Galileo Galilei



New Frontiers in Theoretical Physics - XXXVIII Convegno Nazionale di Fisica Teorica

Palazzone della Scuola Normale Superiore di Pisa, Cortona

May, 21st 2025

<https://quantum.dfa.unipd.it/>
<https://qcsc.dfa.unipd.it/>

Introduction

**Main goal of
quantum
processors:**

- Assemble quantum many-body states
- Preserve and manipulate their information

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Applications:

- Preparation of phases of matter
- Quantum dynamics
- Solution of combinatorial problems

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Noisy Intermediate Scale Quantum era

Analog platforms

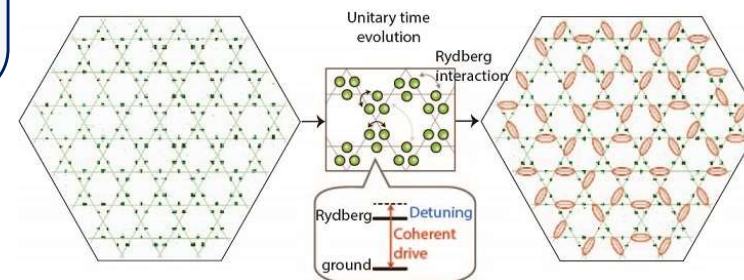
Dynamics under effective Hamiltonians

Digital platforms

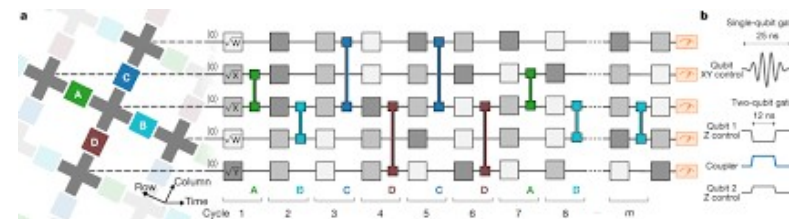
Gate-based state preparation

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Semeghini et al,
Science (2021)
Harvard



Omran et al,
Nature (2019)
Google

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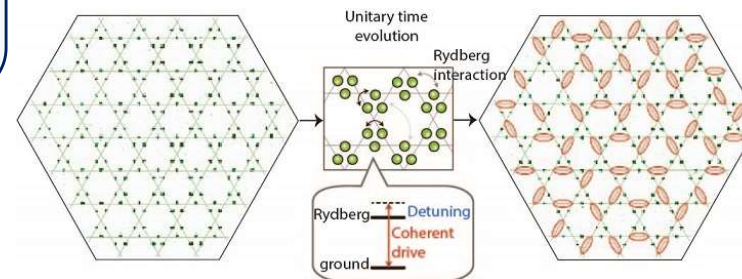
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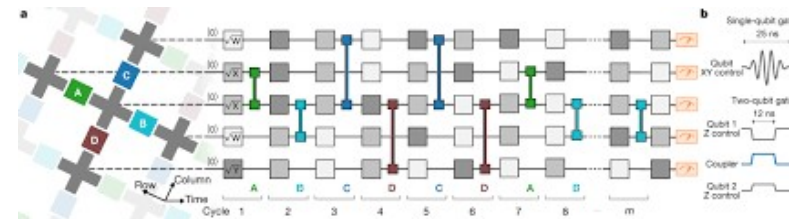
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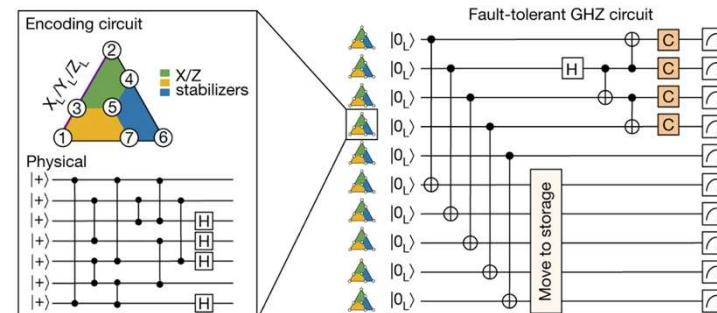


Omran et al,
Nature (2019)
Google

Early Fault Tolerant Quantum Computing era

Logical protocols

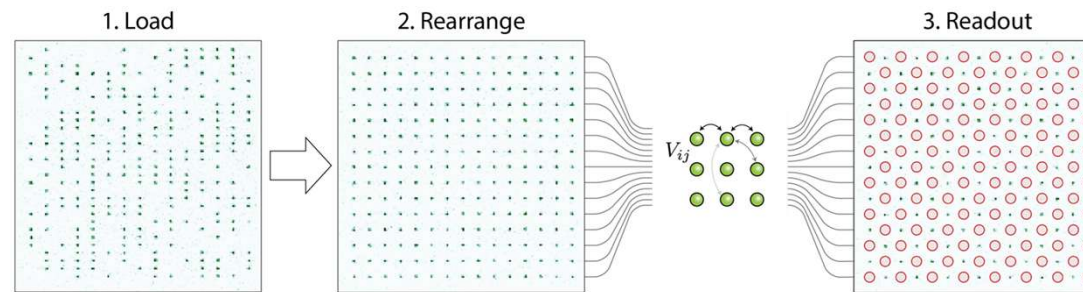
Operations on logical qubits



Bluvstein et al,
Nature (2024)
Harvard

Outline

✓ Rydberg atom arrays



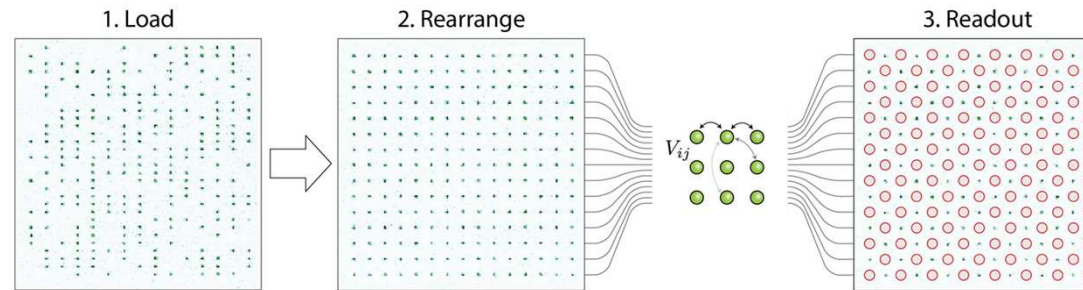
| QuEra >



quera.com

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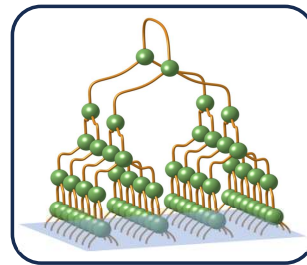


|QuEra>



quera.com

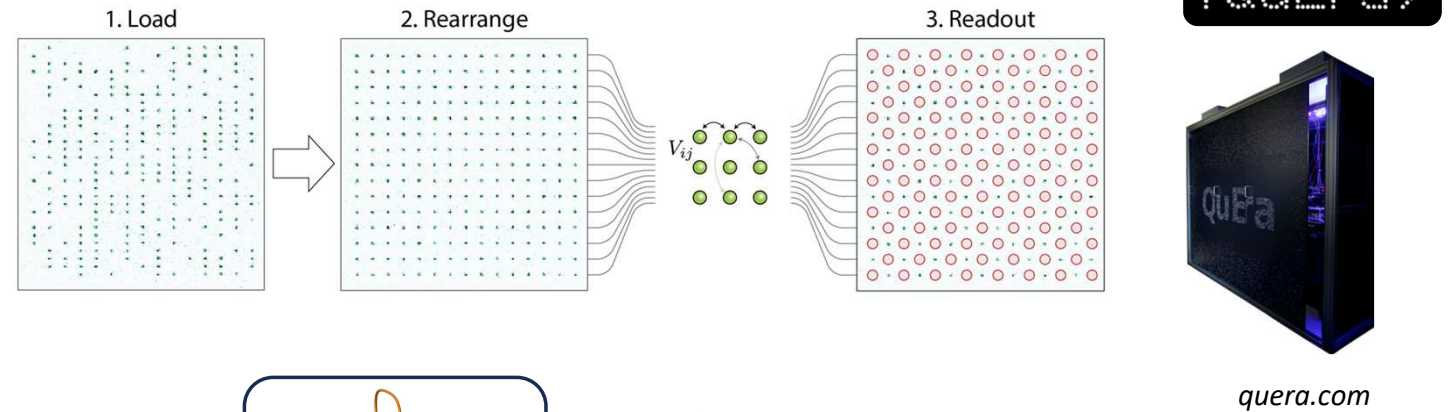
✓ Tree Tensor Networks



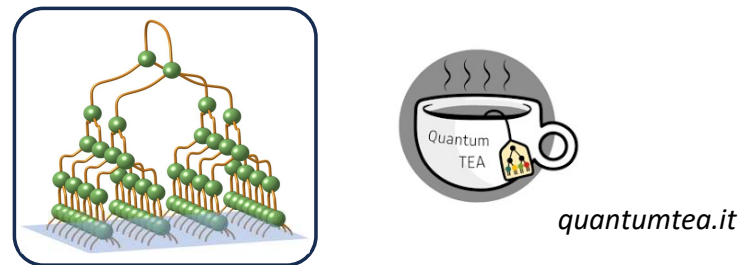
quantumtea.it

Outline

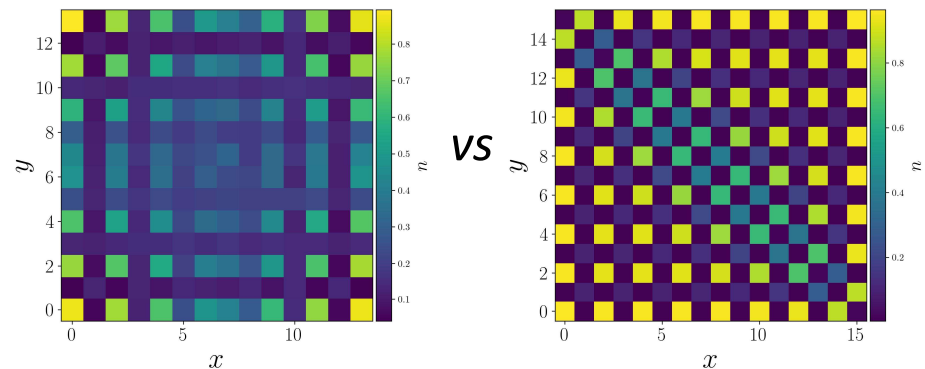
- ✓ Rydberg atom arrays



- ✓ Tree Tensor Networks



- ✓ Boundary frustration in the striated phase of the Rydberg atom square lattice



- ✓ Conclusions

Rydberg atom arrays

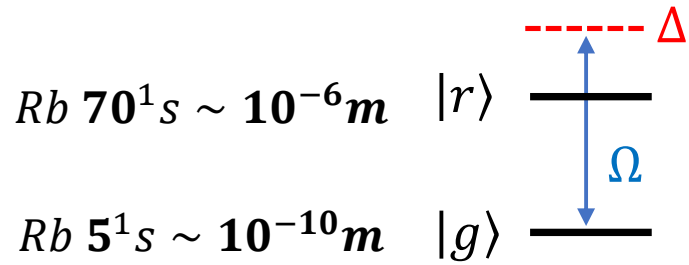
Qubit $\equiv$ atom

Rb $70^1s \sim 10^{-6}m$ $|r\rangle$ —

Rb $5^1s \sim 10^{-10}m$ $|g\rangle$ —

Rydberg atom arrays

Qubit $\equiv$ atom

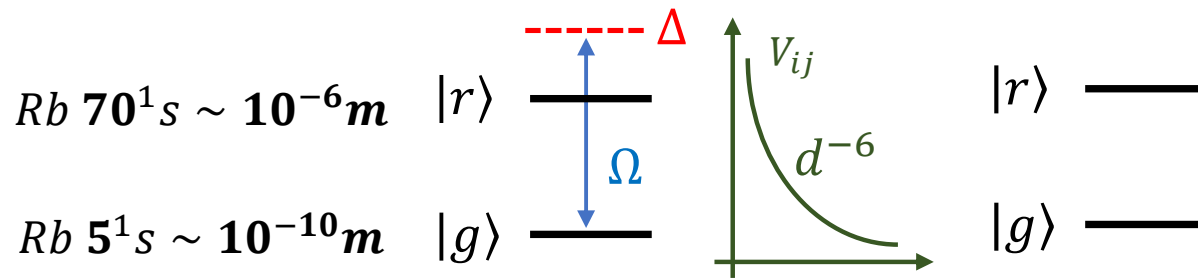


$$H_{Ryd} = \underbrace{\frac{\Omega}{2} \sum_i (|g_i\rangle\langle r_i| + h.c.)}_{\text{Quantum driver}} - \underbrace{\Delta \sum_i n_i}_{\text{Chemical potential}}$$

Rescaled Hamiltonian: $\frac{\Delta}{\Omega} \equiv \Delta$

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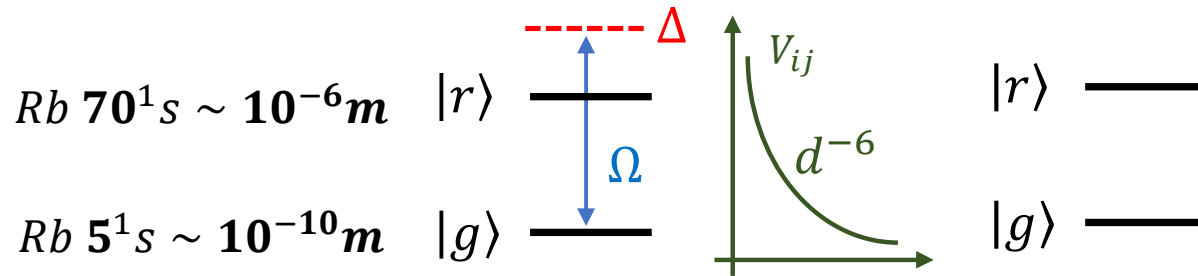


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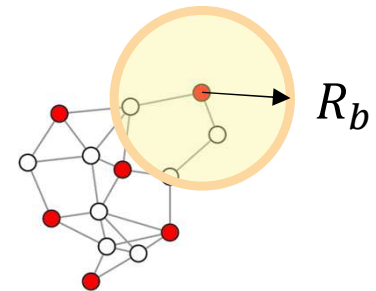
Rydberg atom arrays

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- Isotropic interactions
- $d \sim 4 \mu\text{m}$
- No simultaneous excitations if $d < R_b$ (blockade radius)

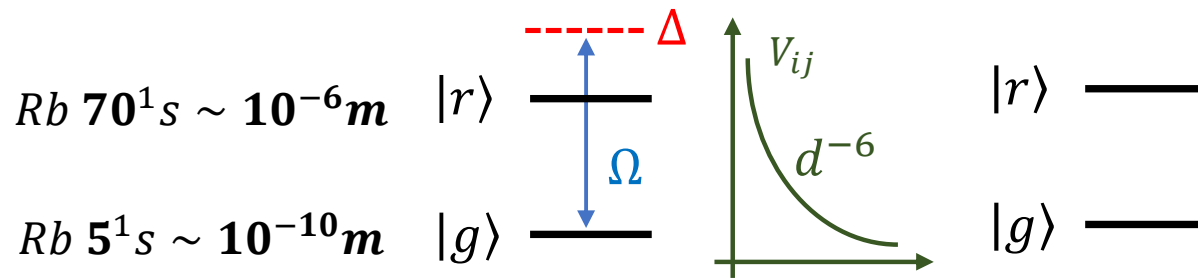
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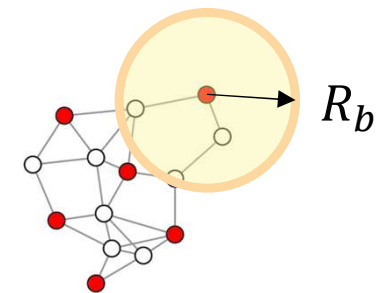
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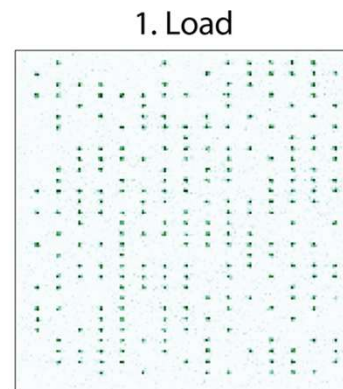
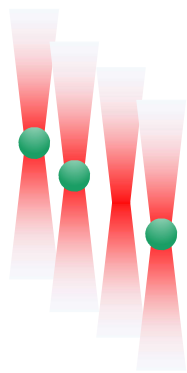
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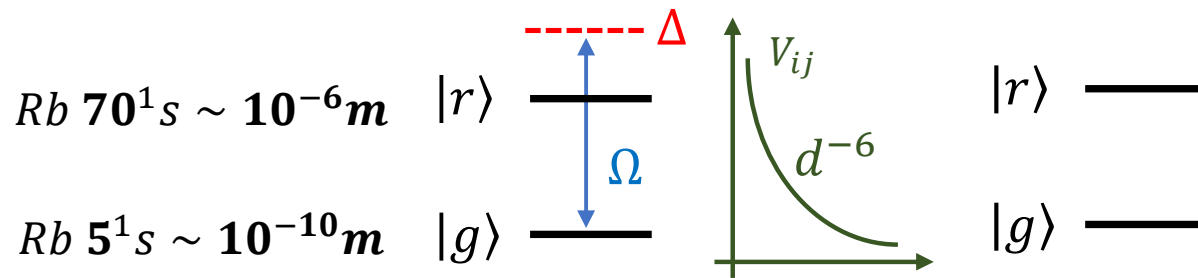
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Lattice loading
with one atom
per tweezer



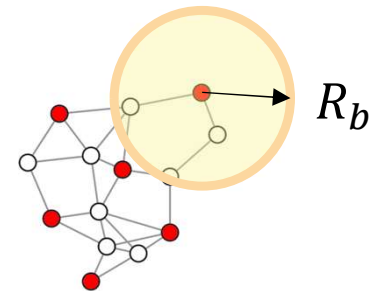
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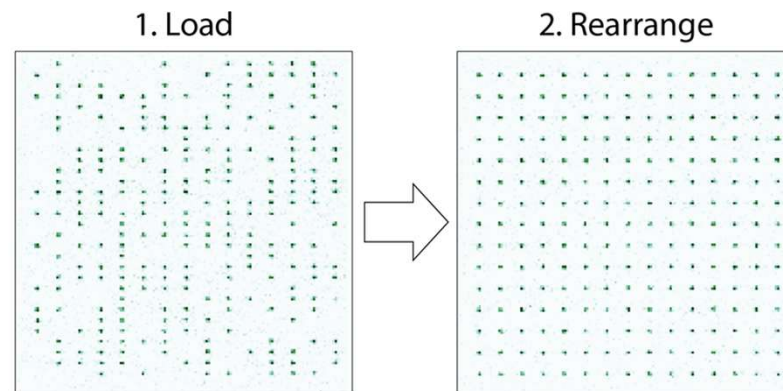
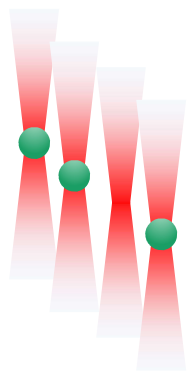
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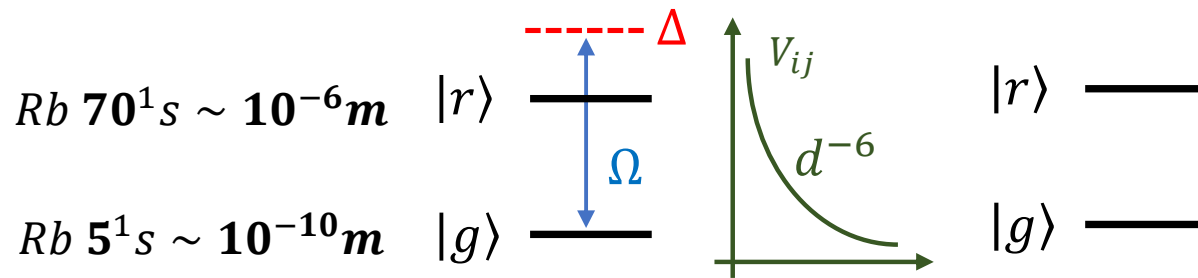
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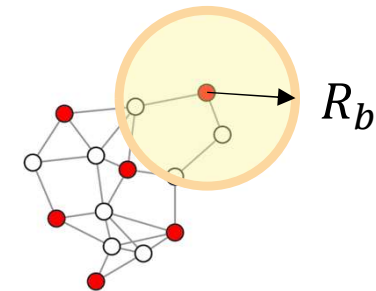
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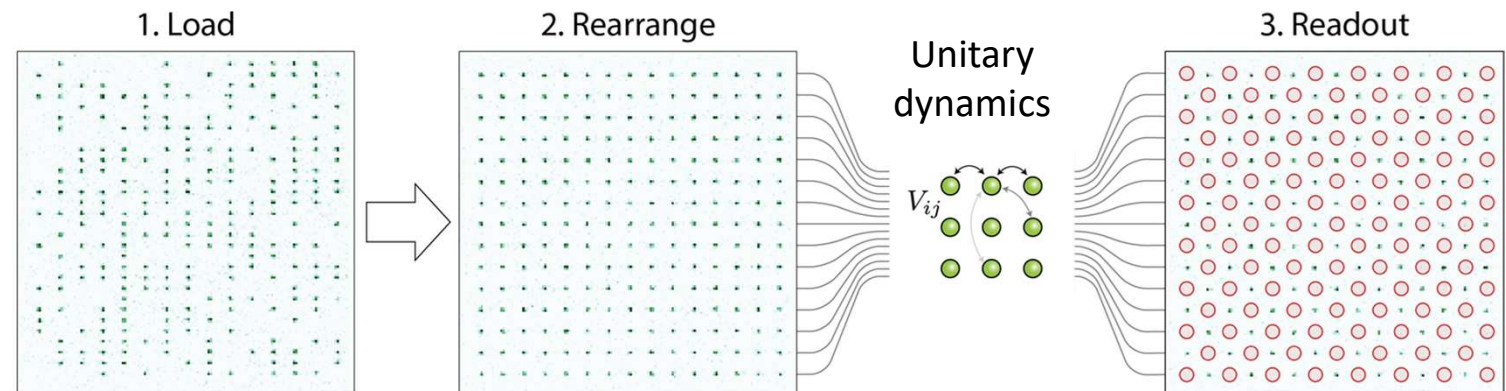
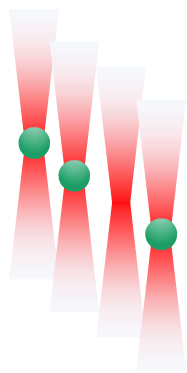
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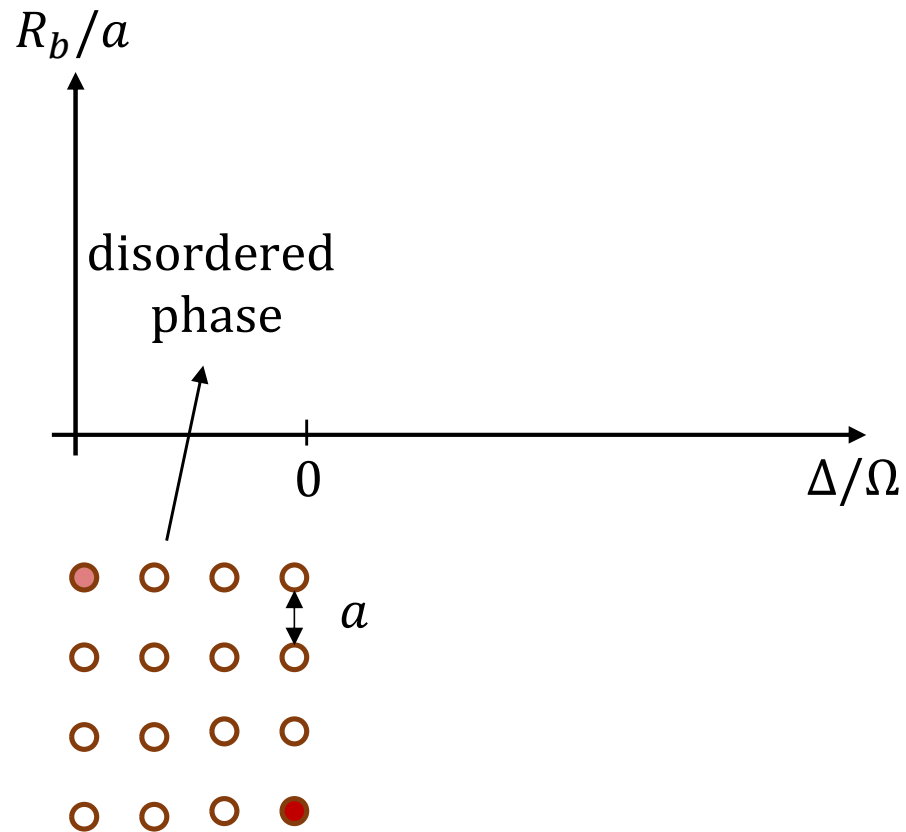
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Rydberg atom arrays

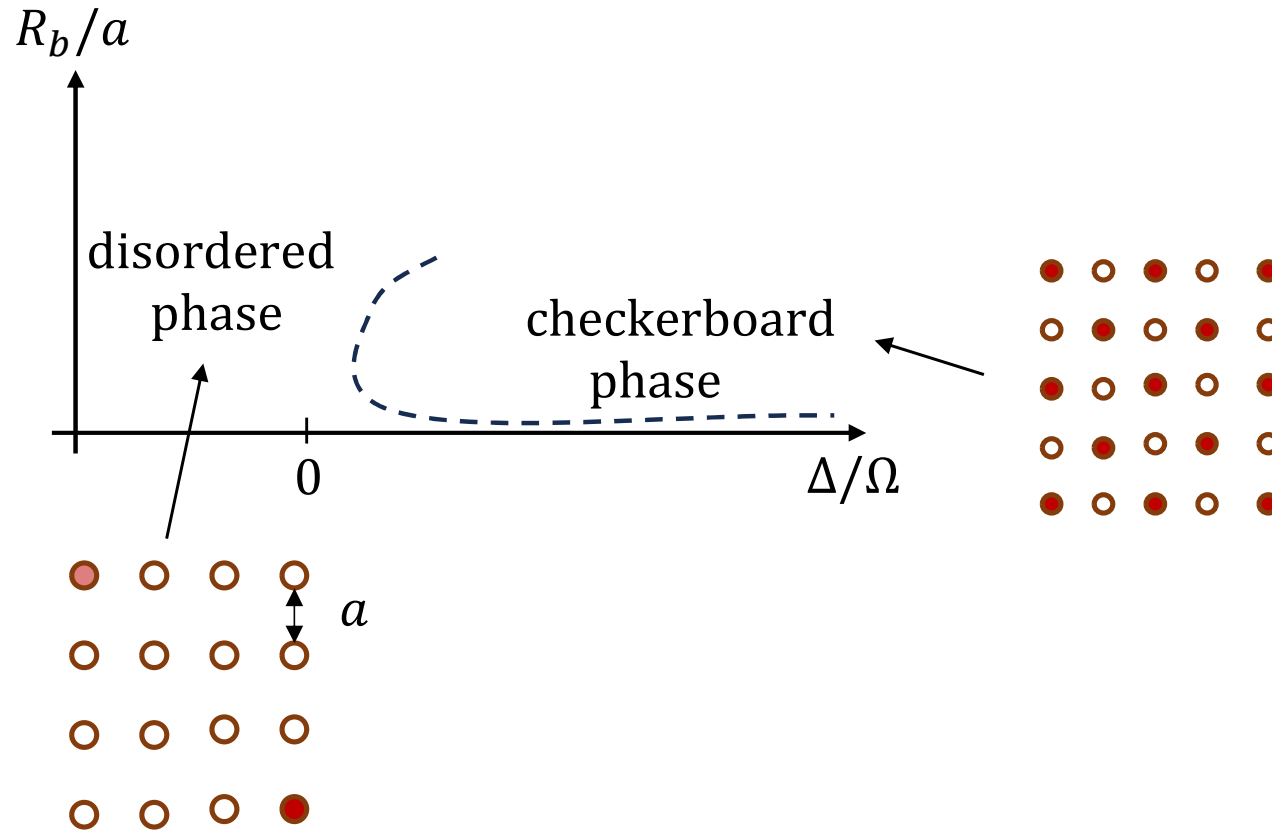
Ground state phase diagram



Negative detuning →
Disordered phase
With no localized excitations

Rydberg atom arrays

Ground state phase diagram

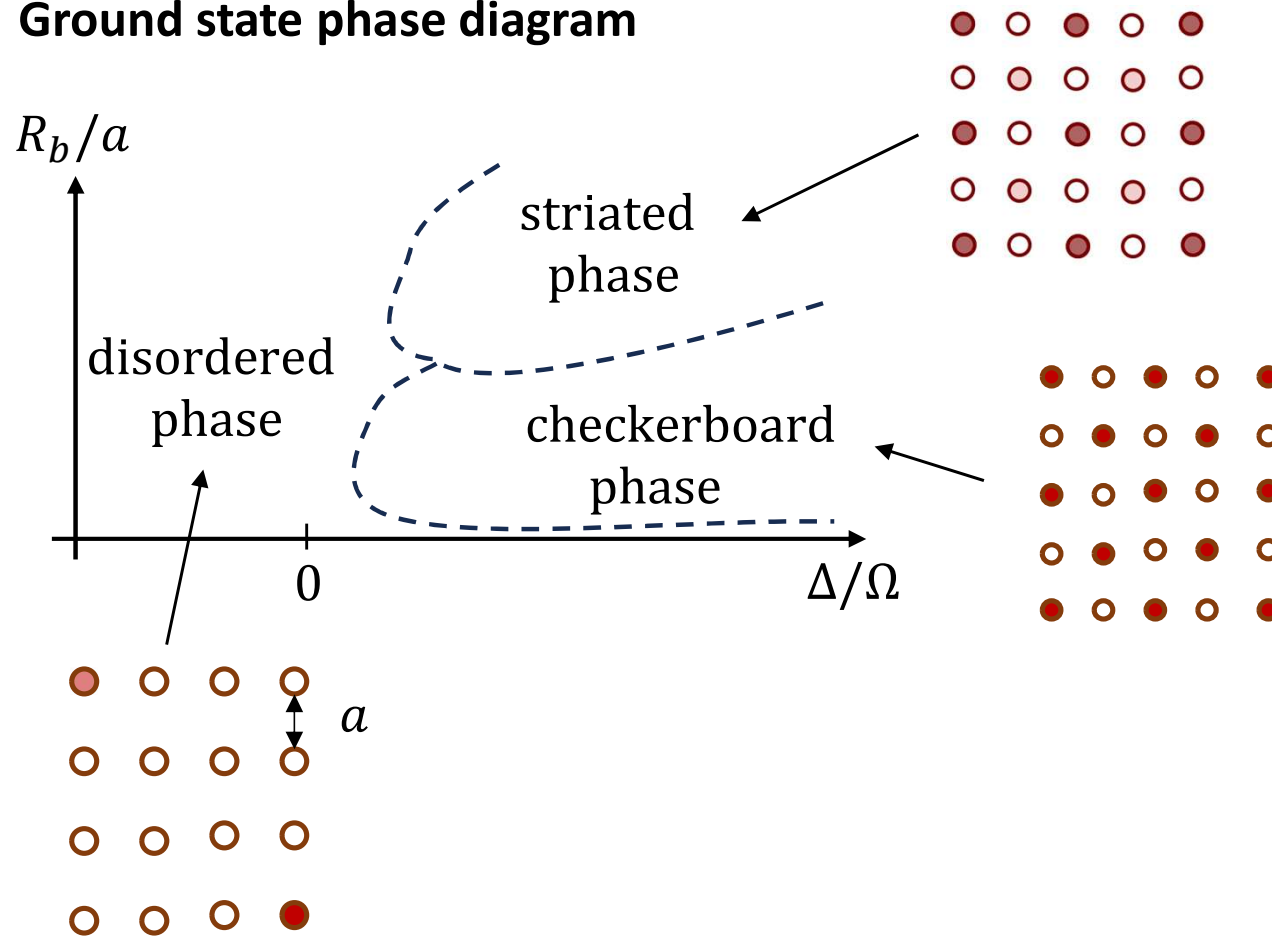


Negative detuning →
Disordered phase
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Positive detuning →
Charge density waves
With localized excitations

Rydberg atom arrays

Ground state phase diagram



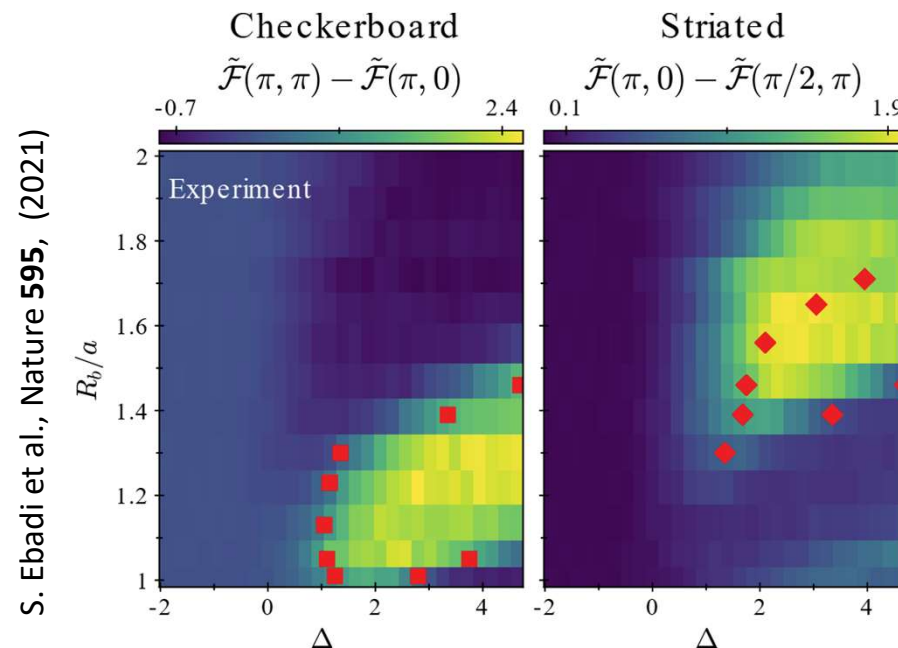
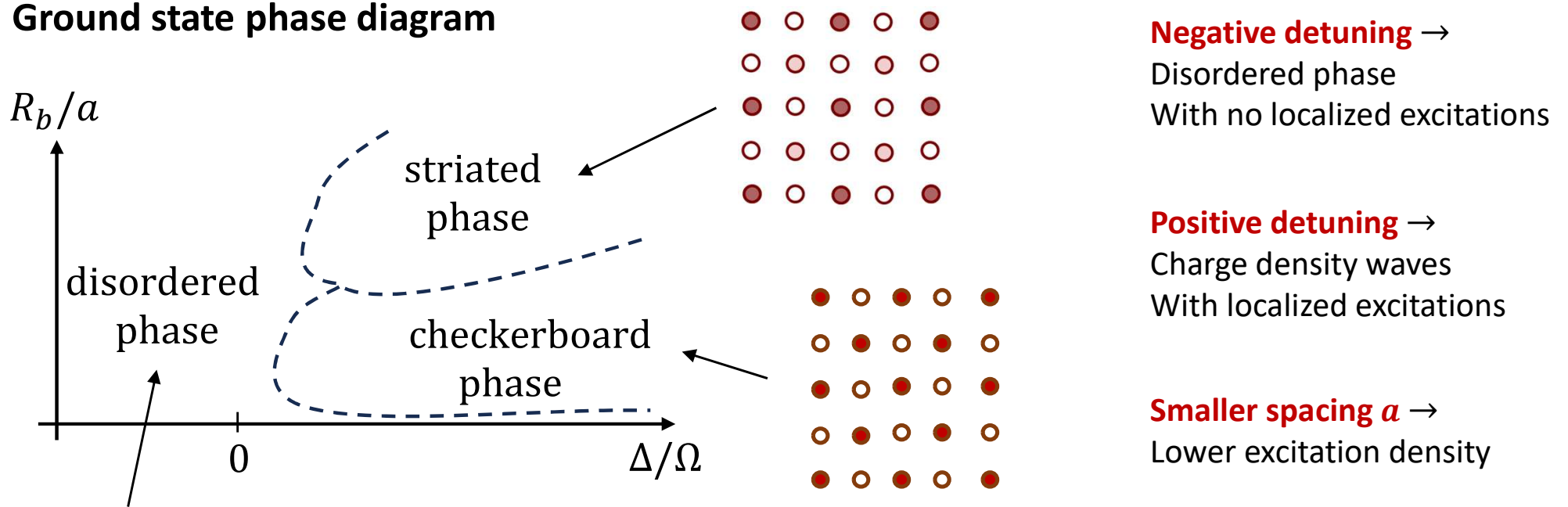
Negative detuning →
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Positive detuning →
Charge density waves
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Smaller spacing a →
Lower excitation density

Rydberg atom arrays

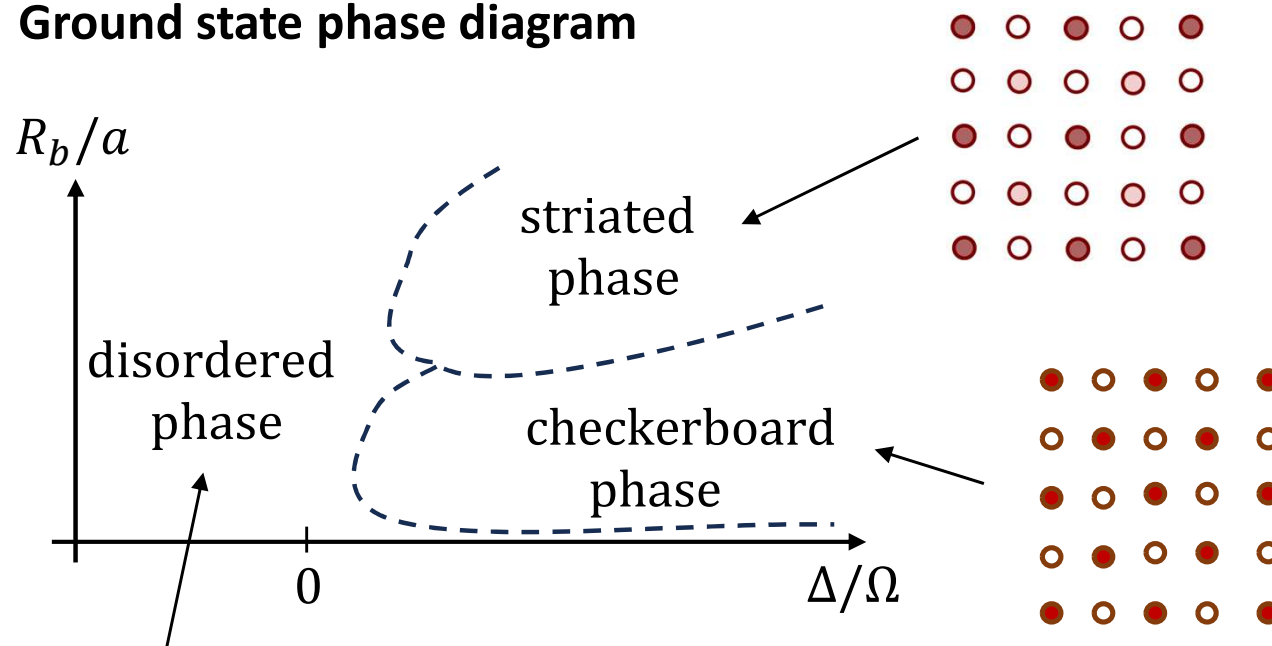
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S. Ebadi et al., Nature **595**, (2021)

Rydberg atom arrays

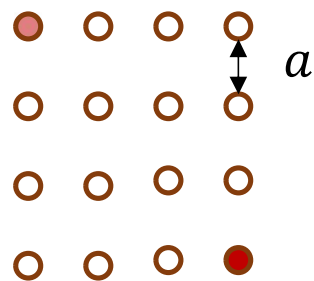
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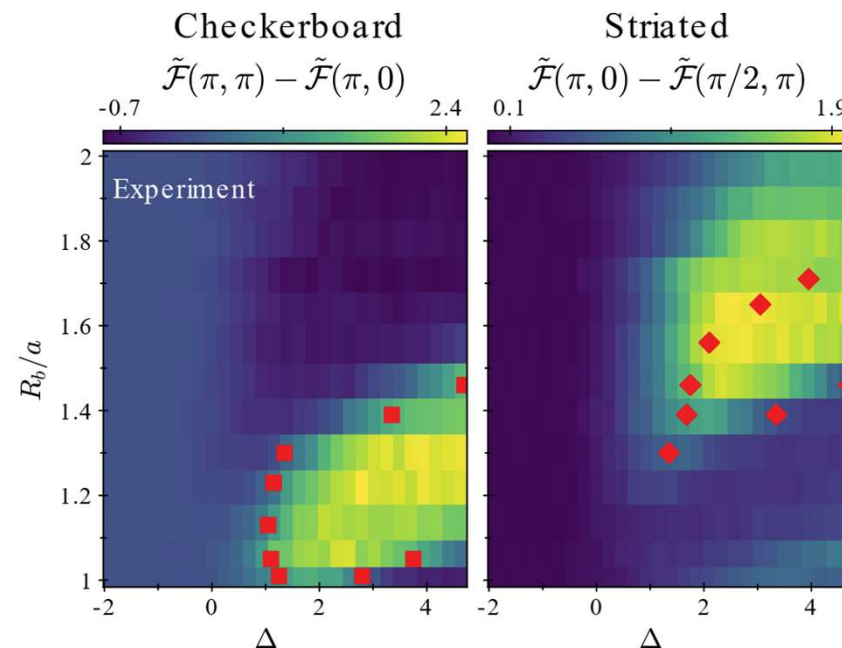
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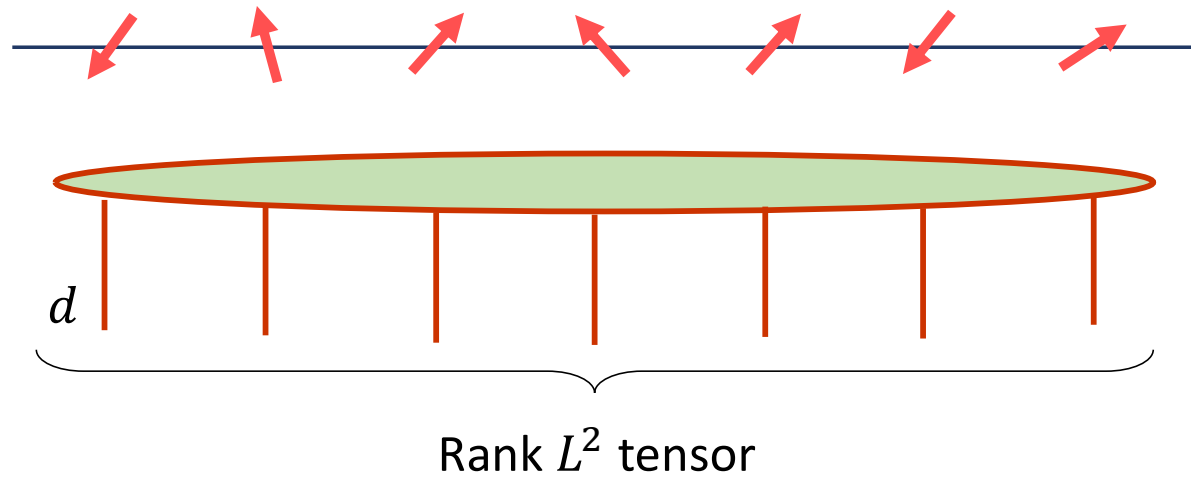
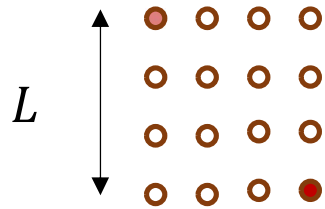
S. Ebadi et al., Nature 595, (2021)



Striated phase:
Frustration induced by
the size of the system

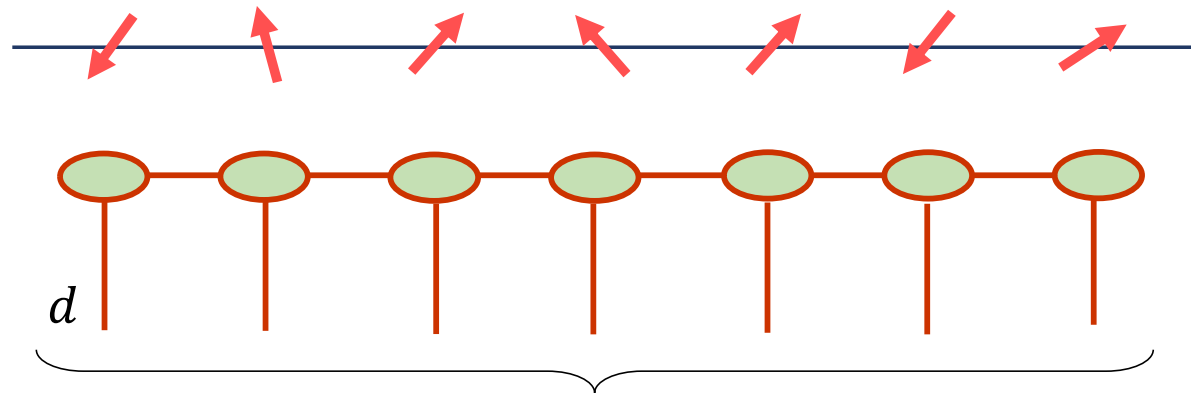
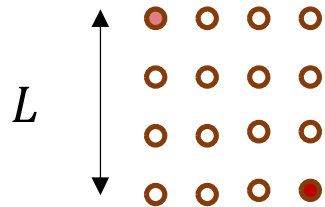
Tree Tensor Networks

$$|\Psi\rangle = \sum_{\alpha_1, \dots, \alpha_{L^2}} c_{\alpha_1, \dots, \alpha_{L^2}} |\alpha_1, \dots, \alpha_L\rangle$$



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L^2 Rank $d \cdot m^2$ tensors

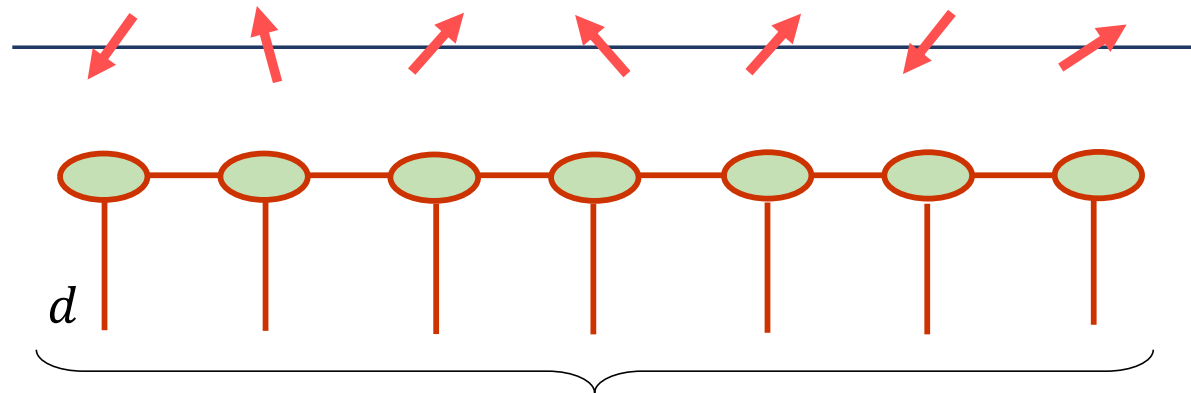
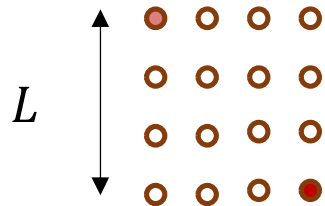
Efficient method to compress information when correlations fastly decay in space

Bond dimension m :

maximum amount of mutual information between
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Tree Tensor Networks

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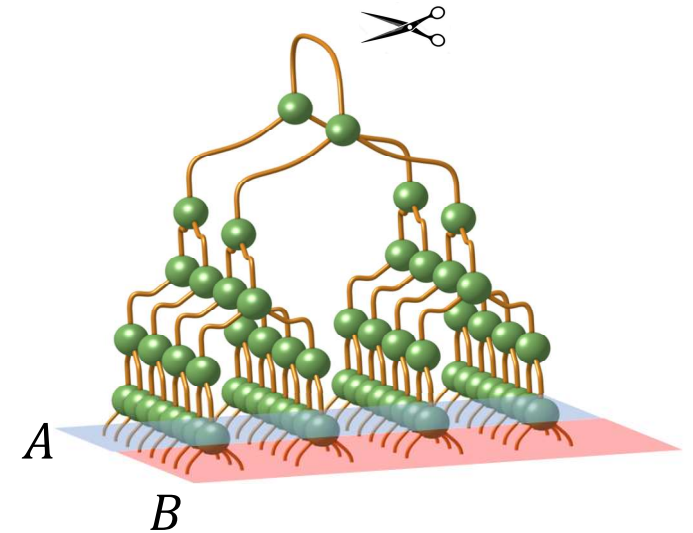
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Tree Tensor Networks: suited for high-dimensional systems

- Ground state, dynamics
- Closed/open systems
- Condensed matter, high energy physics, optimization problems



Experimental realization

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Cloud available analog quantum processor

- **256 entangled qubits**
Coherence throughout the full computation
- **Field-programmable qubit arrays (FPQA™)**
Programmable connectivity of near-arbitrary qubit layout configurations

Experimental realization

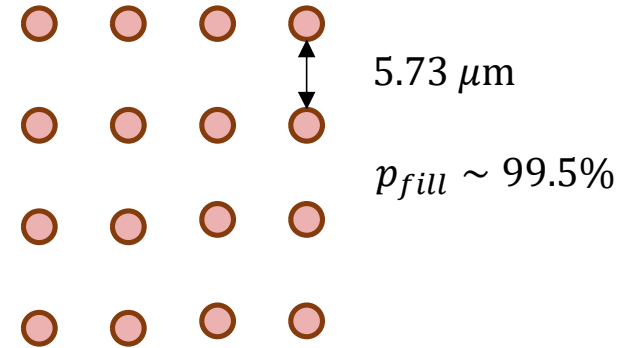
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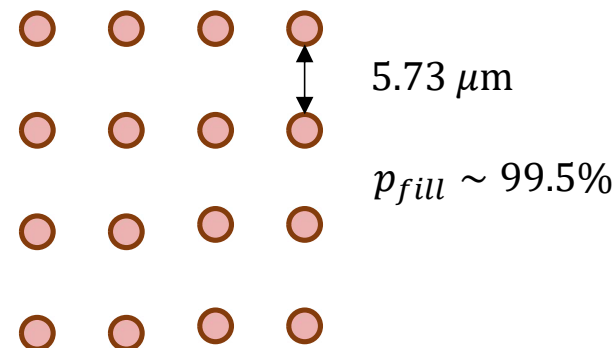
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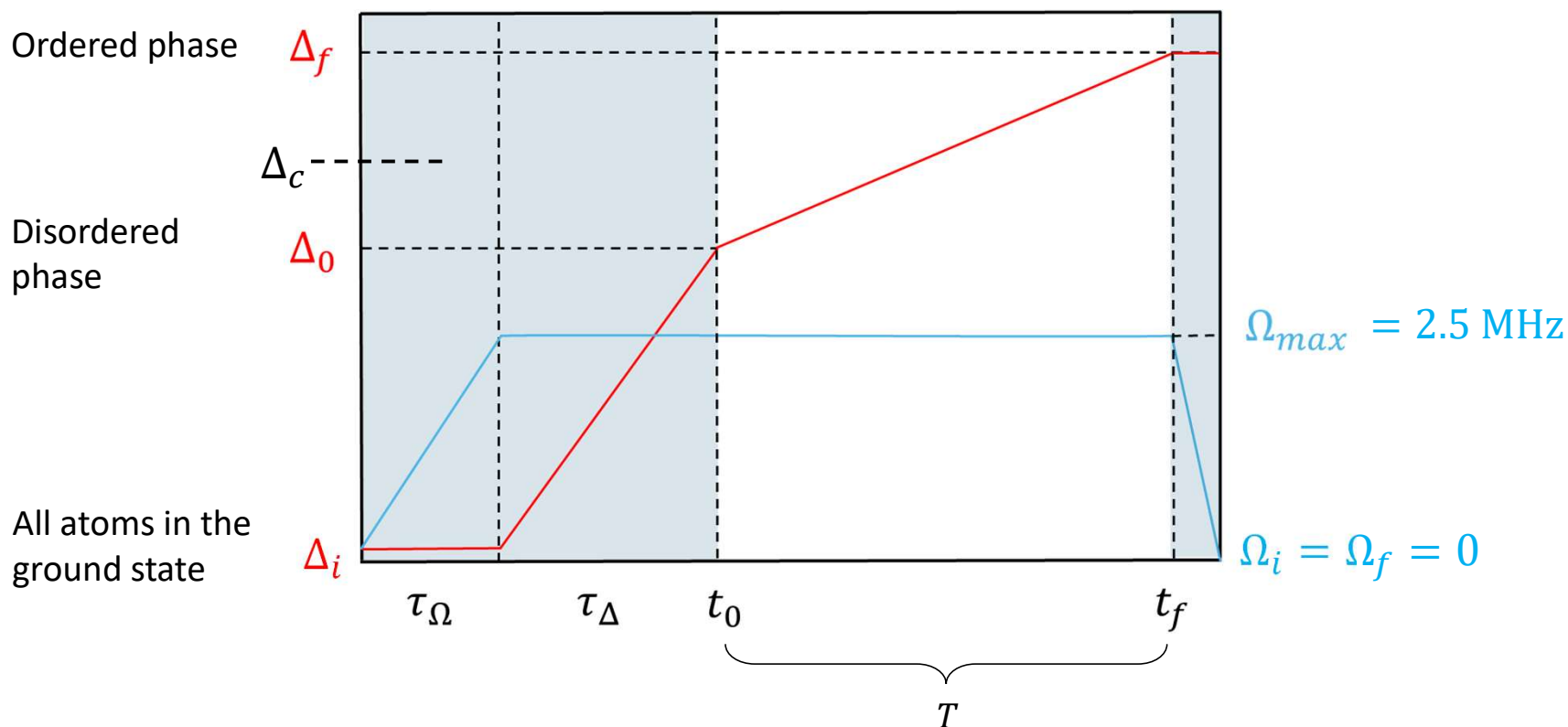
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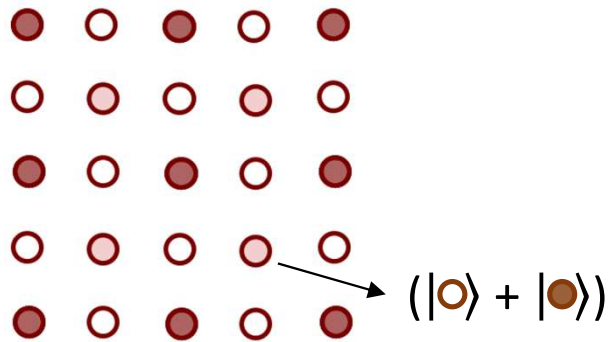


Fixed spacing a



Boundary frustration in the striated phase

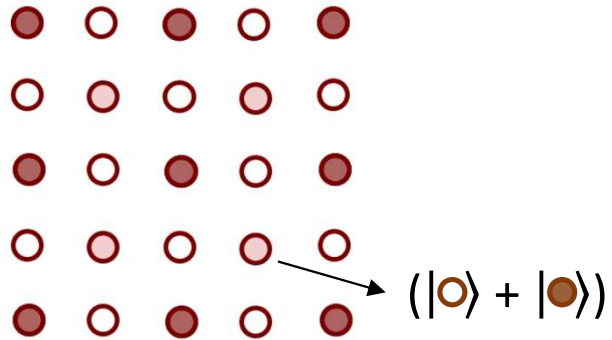
Striated phase Driven by long-range interactions



Boundary frustration in the striated phase

Striated phase

Driven by long-range
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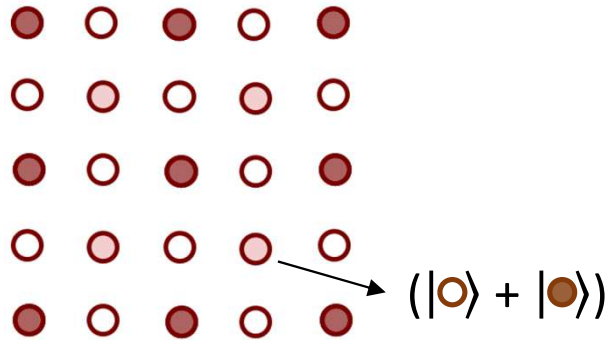


- Compute the ground state of H_{Ryd}
- Measure n local Rydberg excitation density

Boundary frustration in the striated phase

Striated phase

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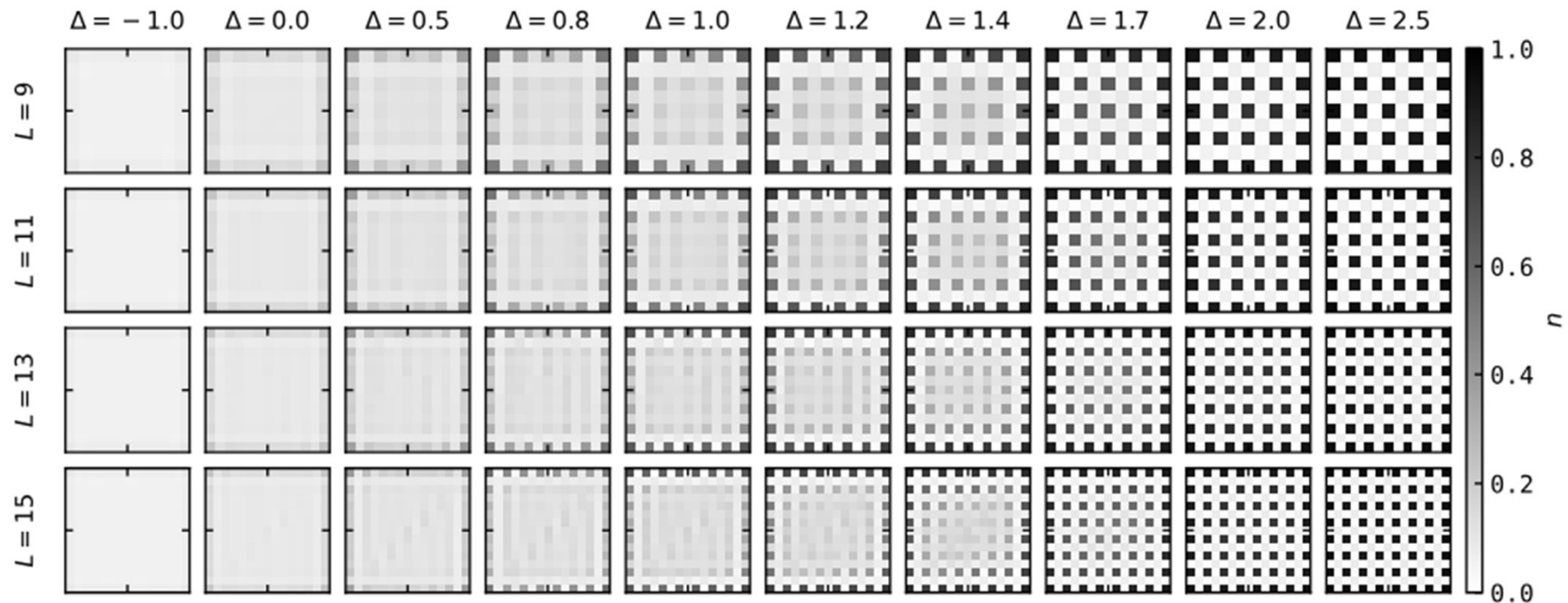
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Disordered phase

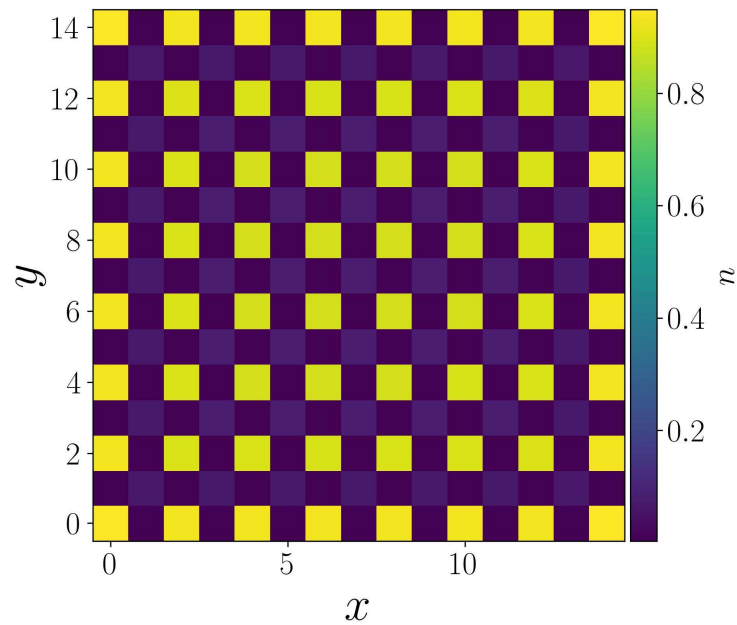
Low density of excitations

Striated phase

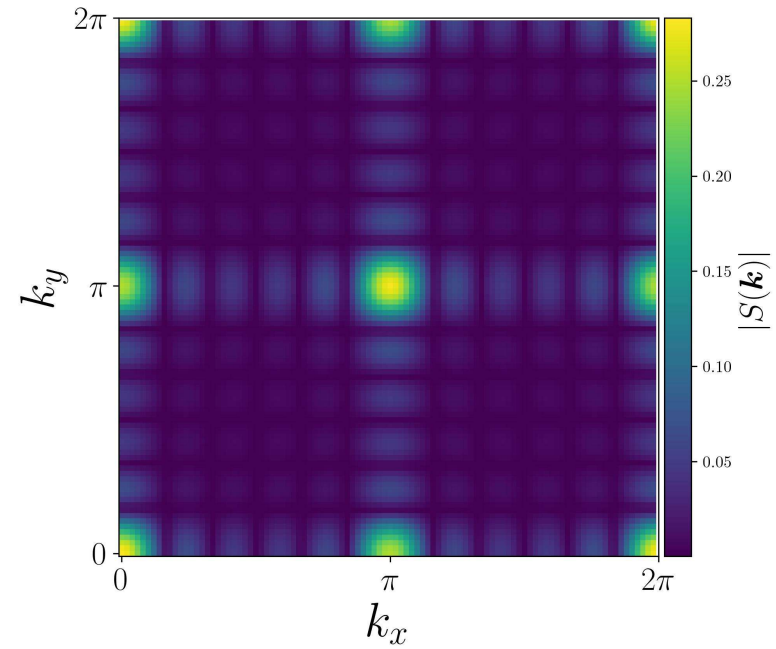
Density-wave phase



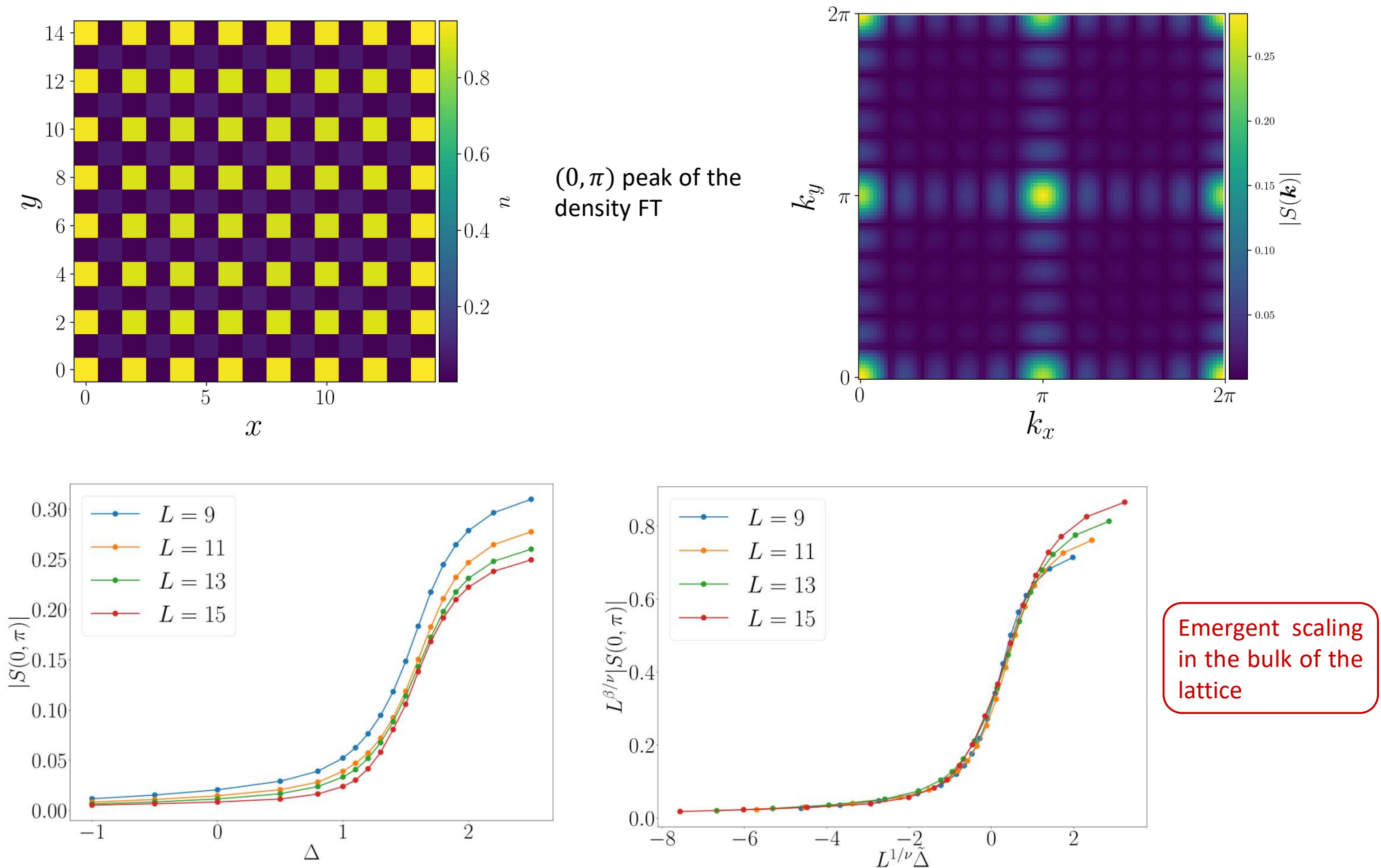
Boundary frustration in the striated phase



$(0, \pi)$ peak of the
density FT

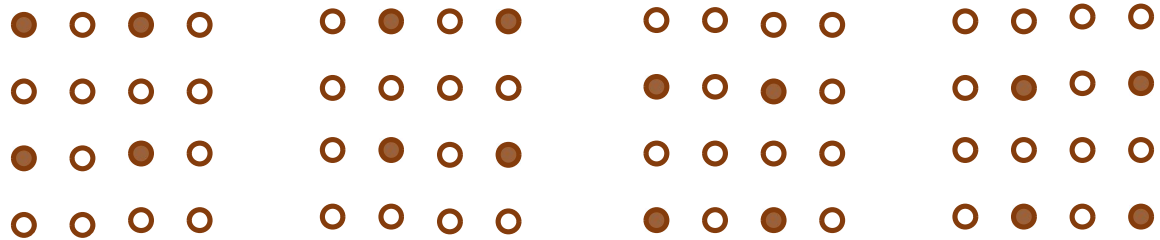


Boundary frustration in the striated phase



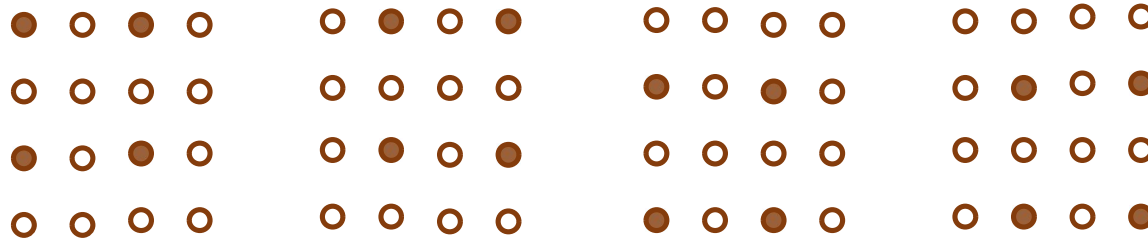
Boundary frustration in the striated phase

Degeneracy with
even x even
lattice sizes



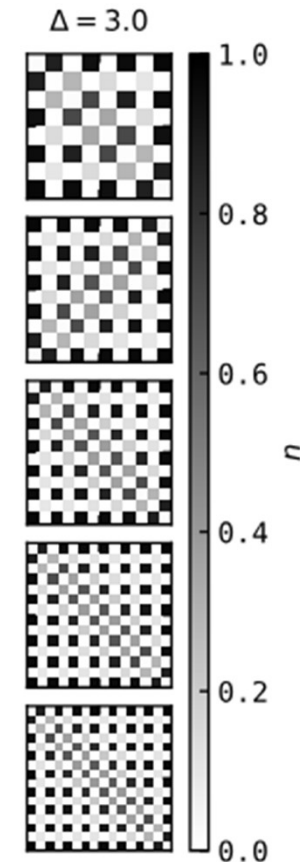
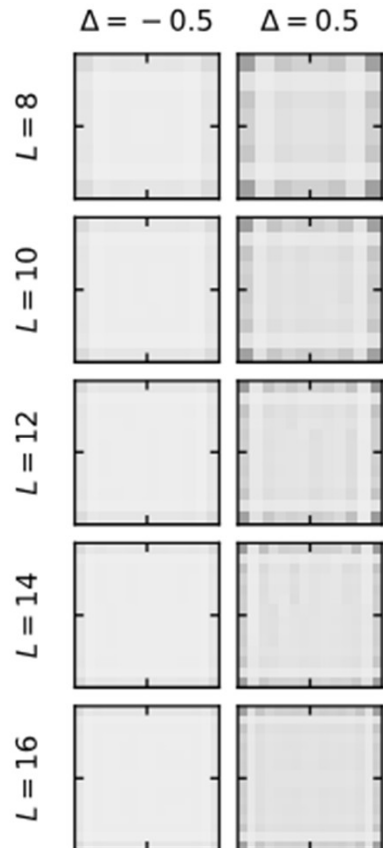
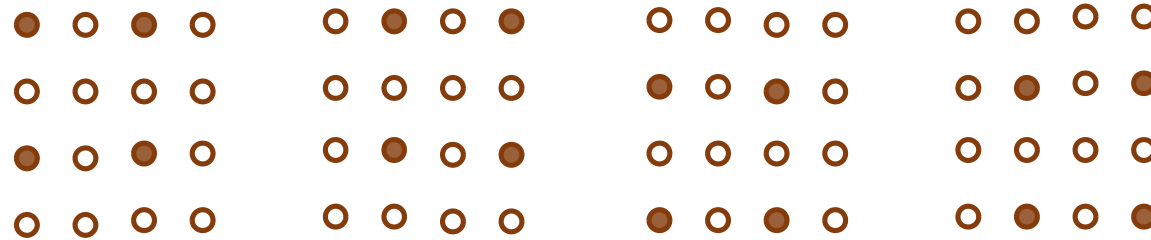
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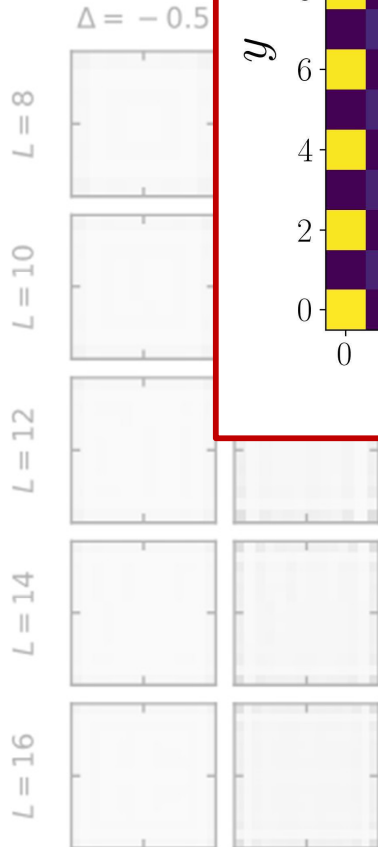
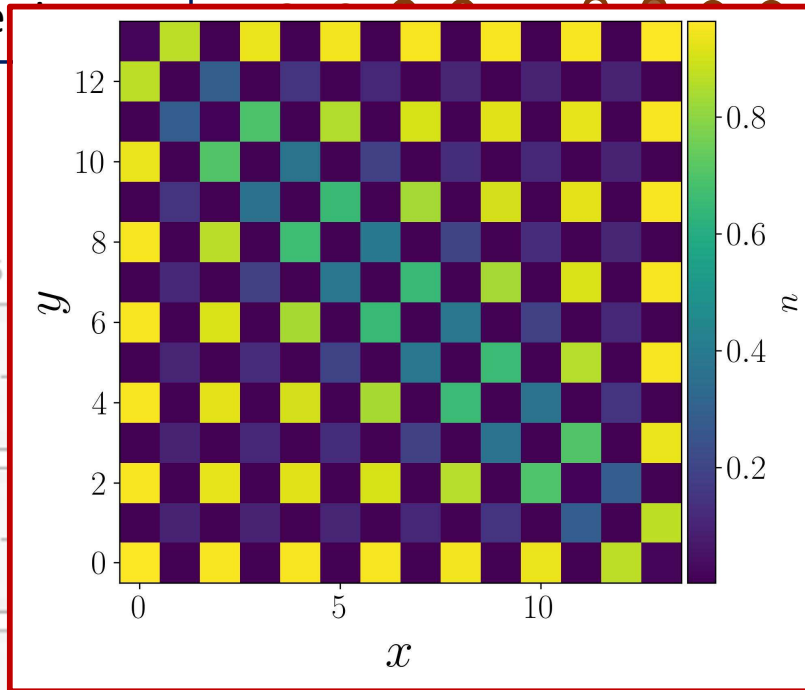
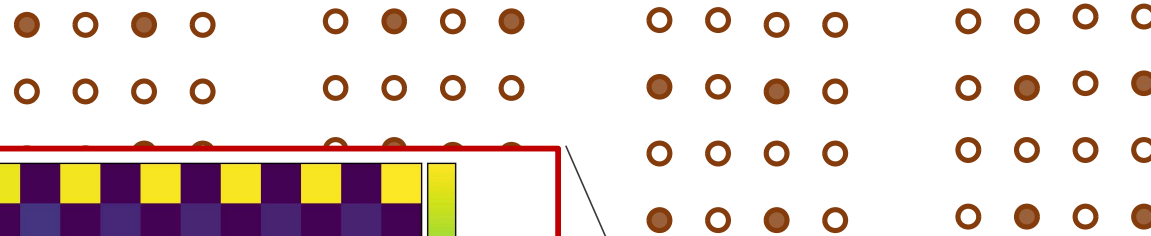
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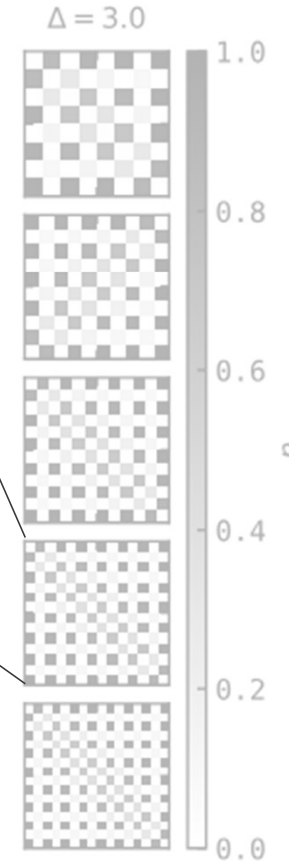
Ordered phase
Domains with
diagonal interface

Boundary frustration in the striated phase

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even x even
lattice

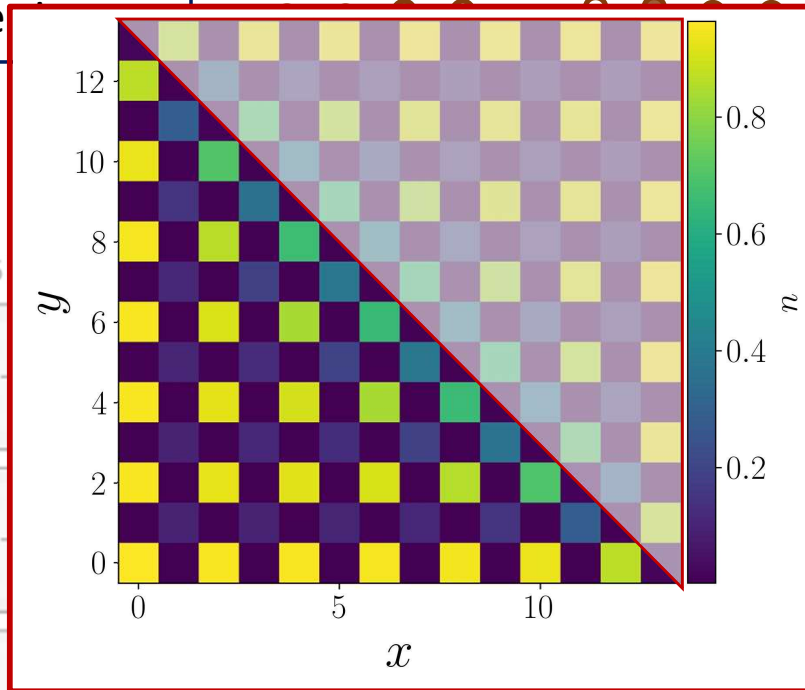
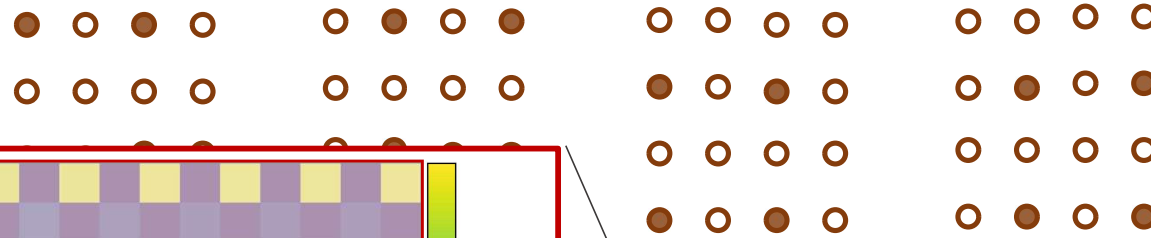


Ordered phase
Domains with
diagonal interface

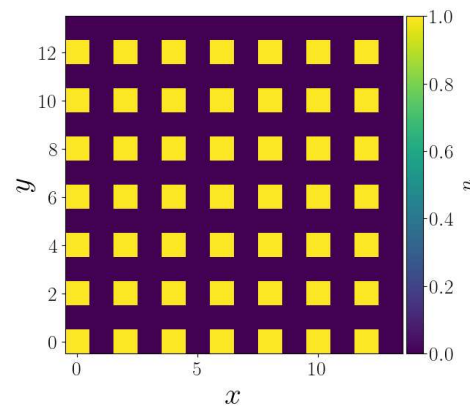
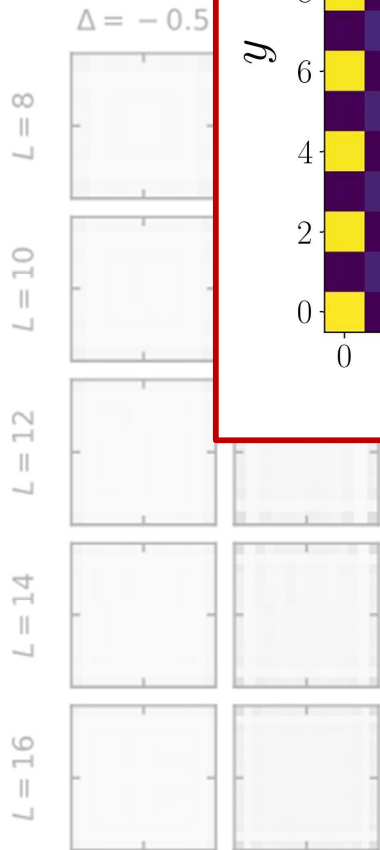
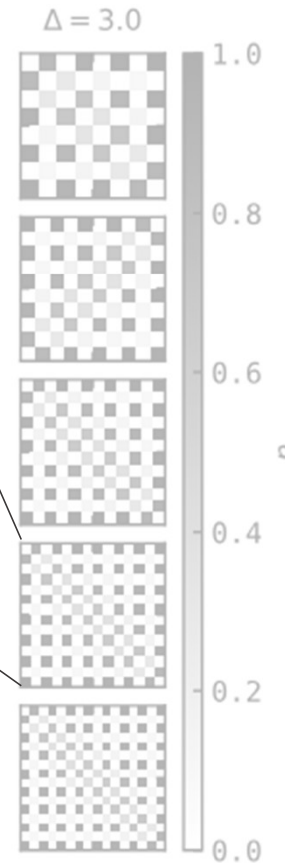


Boundary frustration in the striated phase

Degeneracy with
even x even
lattice

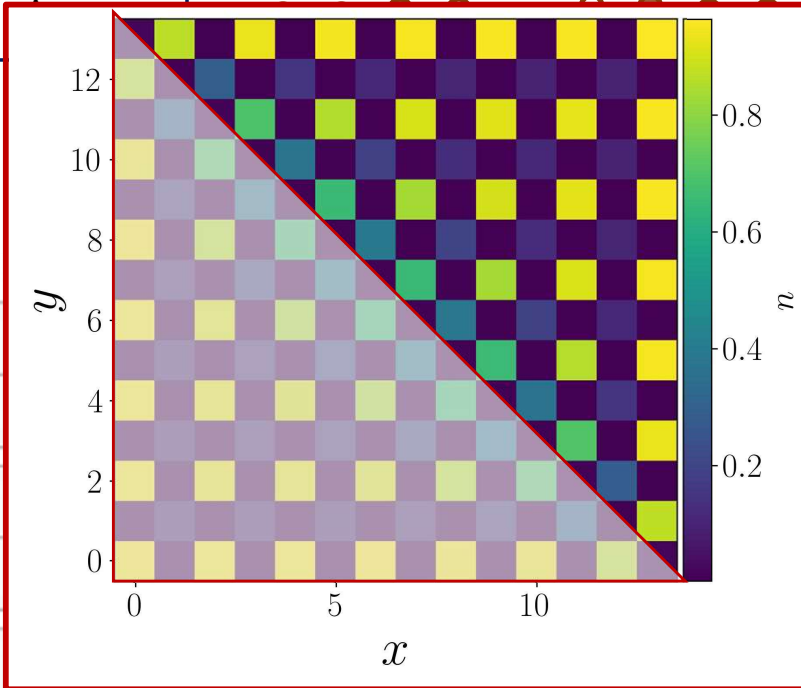
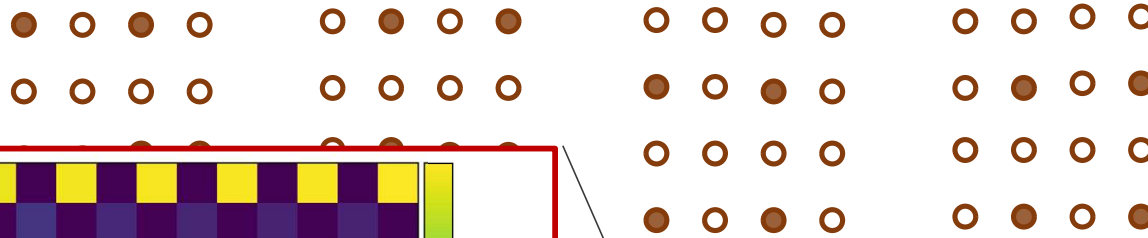


Ordered phase
Domains with
diagonal interface



Boundary frustration in the striated phase

Degeneracy with
even x even
lattice



Ordered phase
Domains with
diagonal interface

Competition between
- Boundary
- Interface
↓
Stable at large L

$\Delta = -0.5$

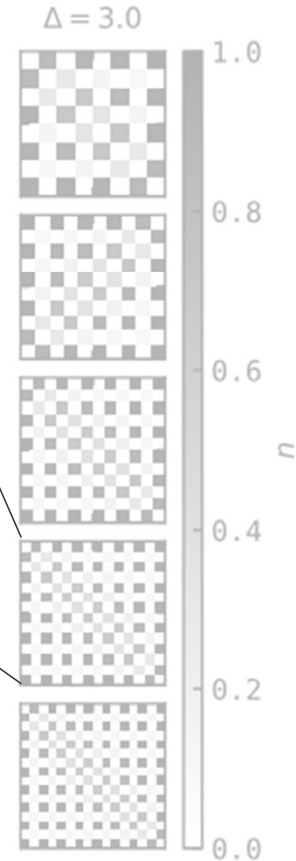
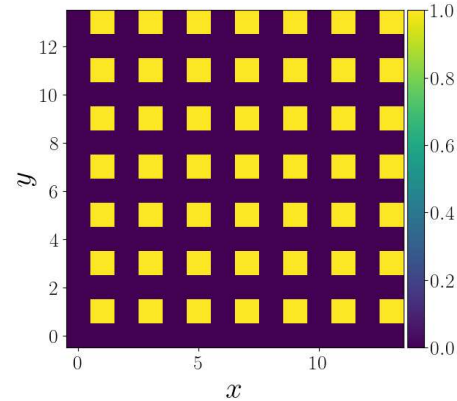
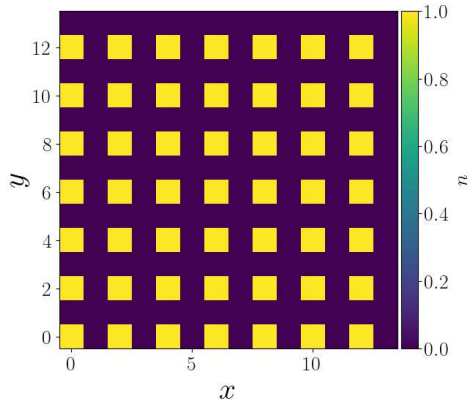
$L = 8$

$L = 10$

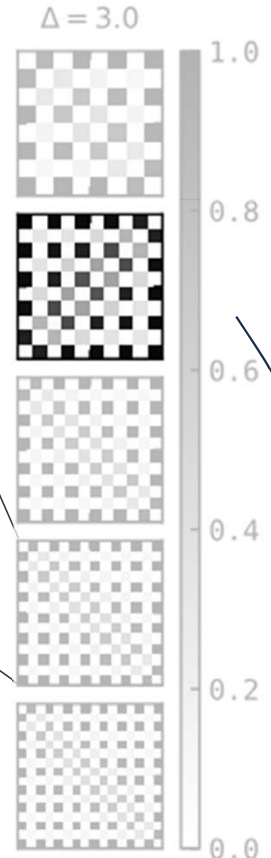
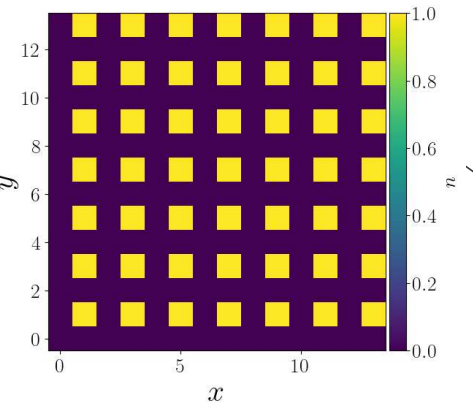
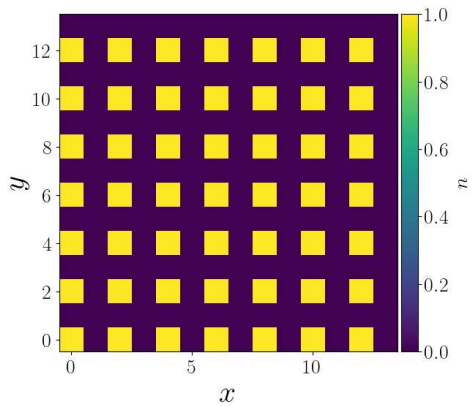
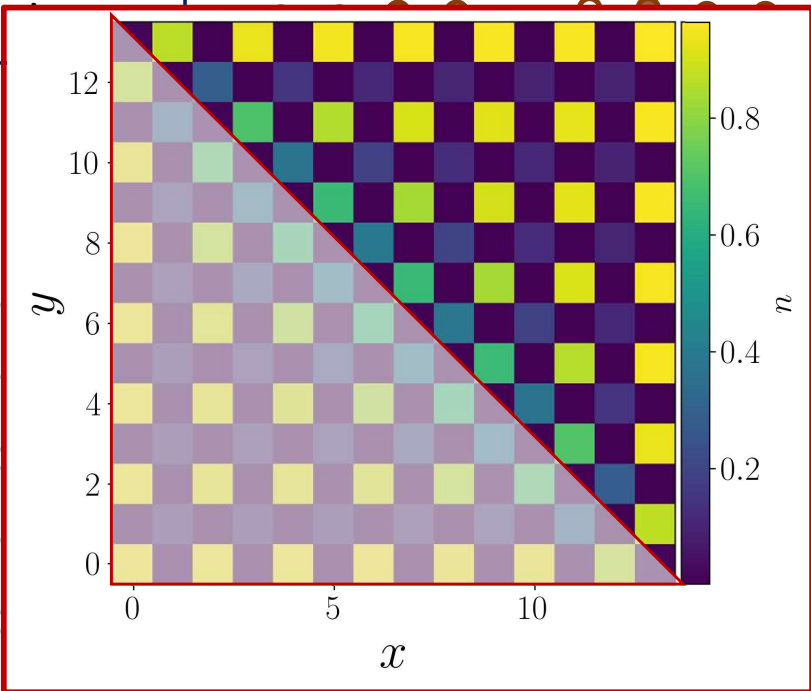
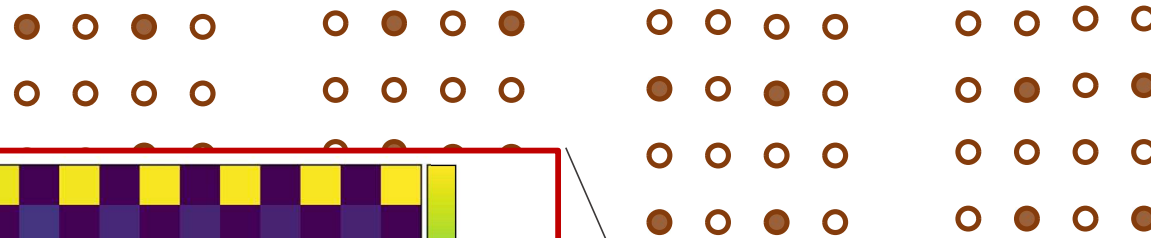
$L = 12$

$L = 14$

$L = 16$



Degeneracy with even x even lattice



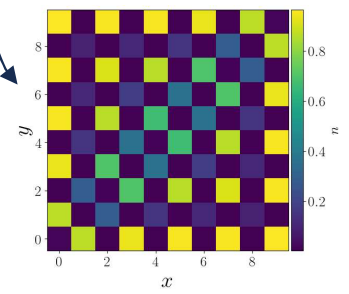
Ordered phase
Domains with
diagonal interface

Competition between

- Boundary
- Interface

↓

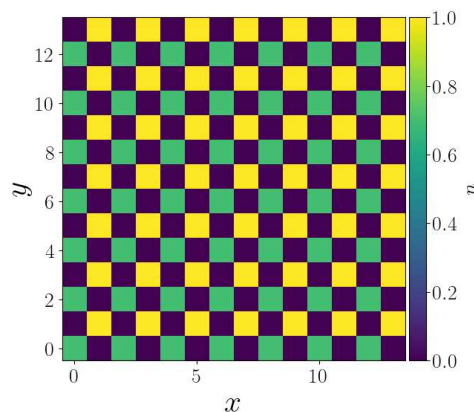
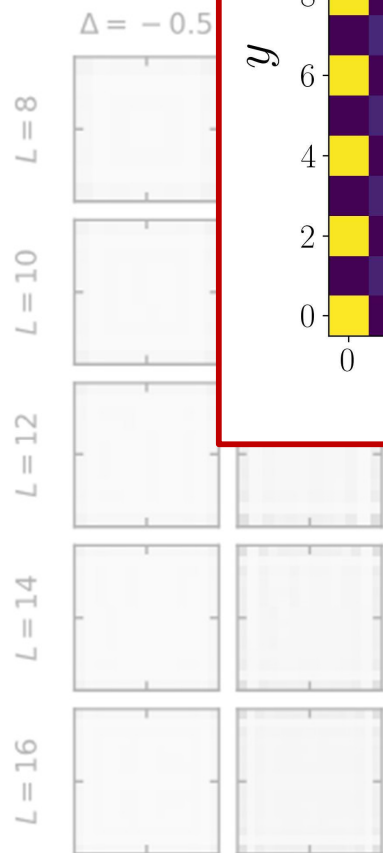
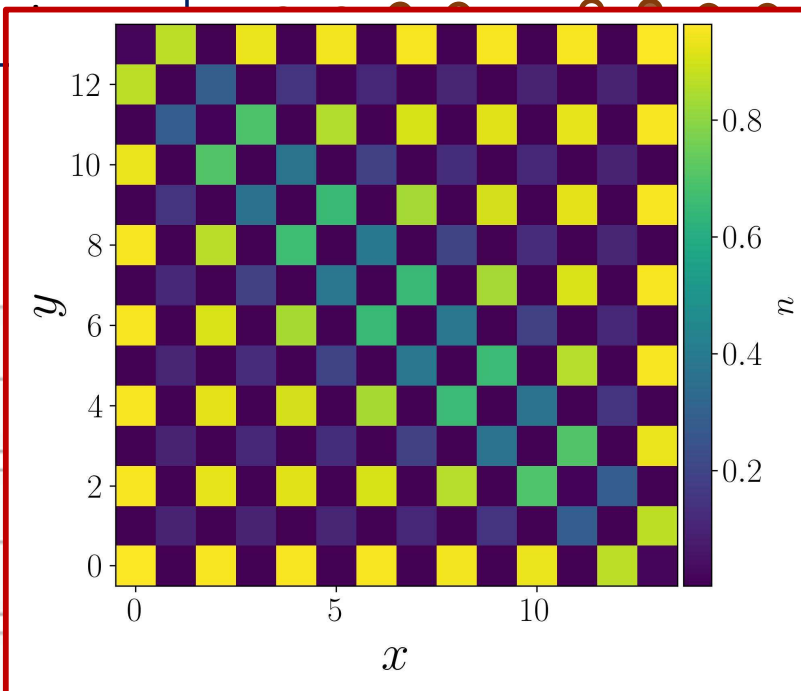
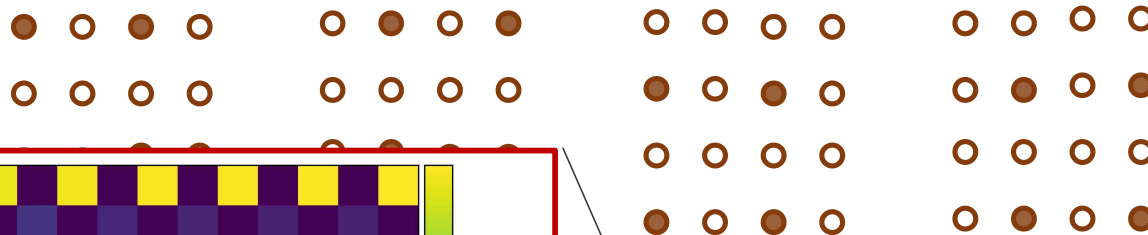
Stable at large L



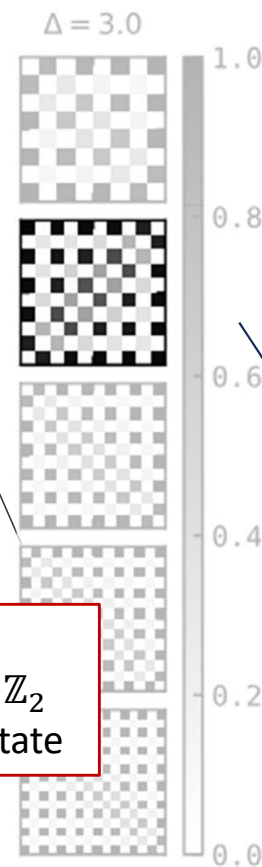
Double degenerate

Boundary frustration in the striated phase

Degeneracy with
even x even
lattice



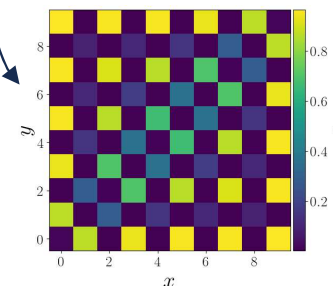
The two domains
belong to the same $\mathbb{Z}_2$
antiferromagnetic state



Ordered phase
Domains with
diagonal interface

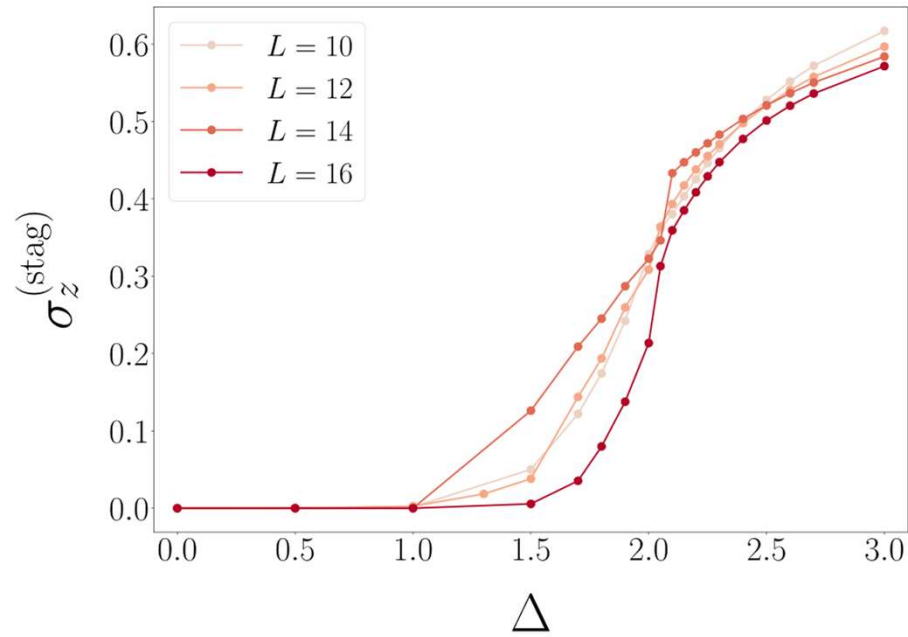
Competition between
- Boundary
- Interface

↓
Stable at large L

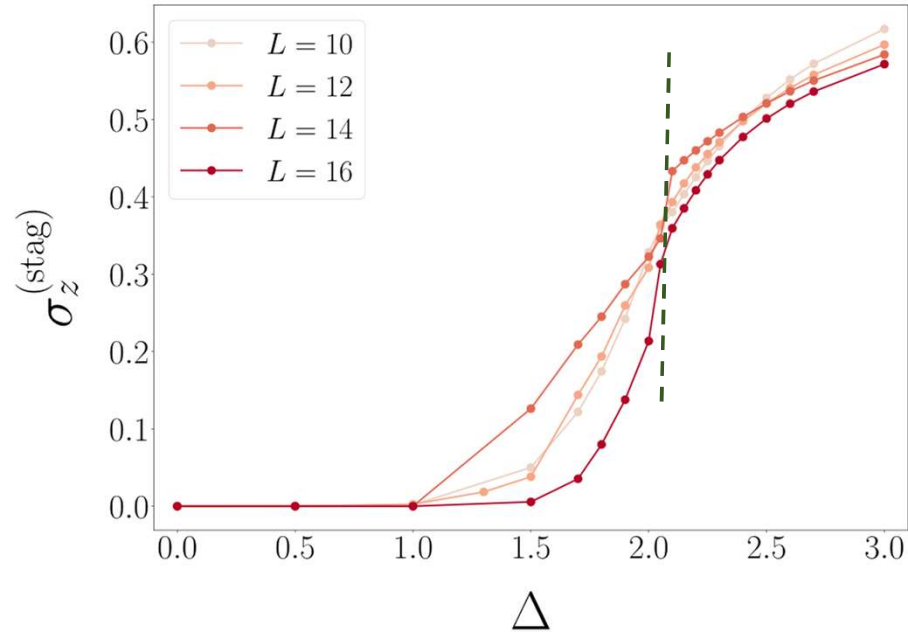


Double degenerate

Boundary frustration in the striated phase



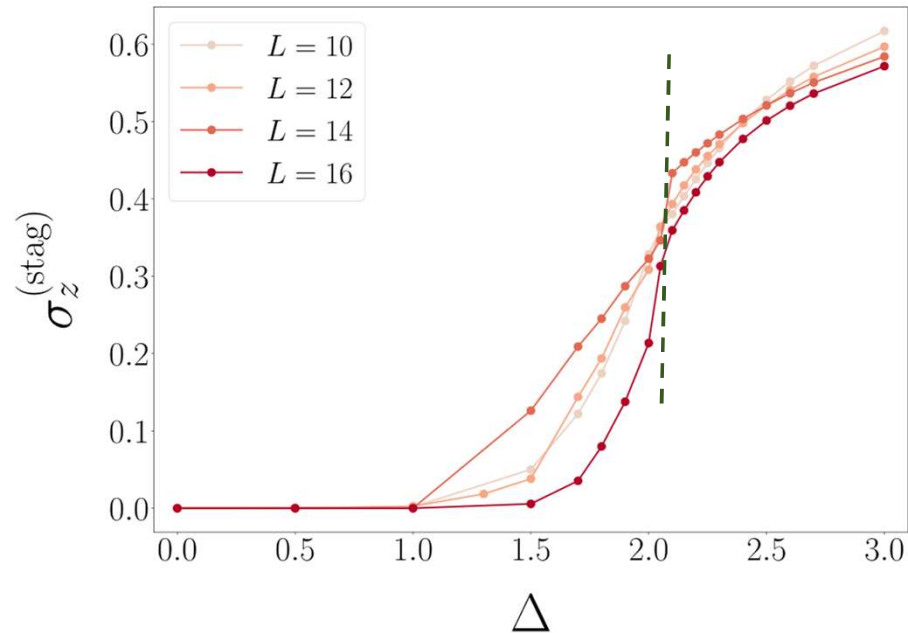
Boundary frustration in the striated phase



Even size:

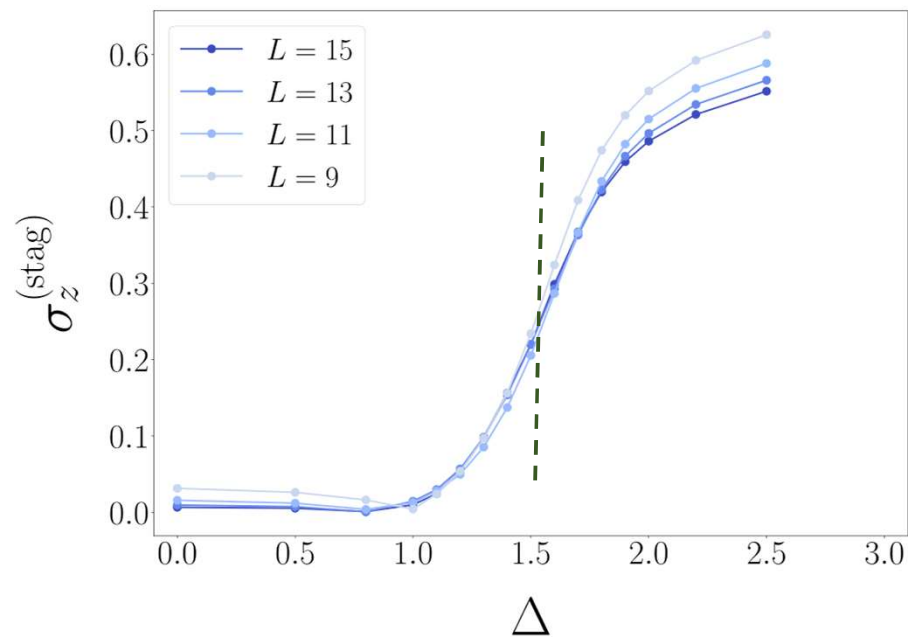
- Discontinuous staggered magnetization
- Singlet formed by the two degenerate ground states

Boundary frustration in the striated phase



Even size:

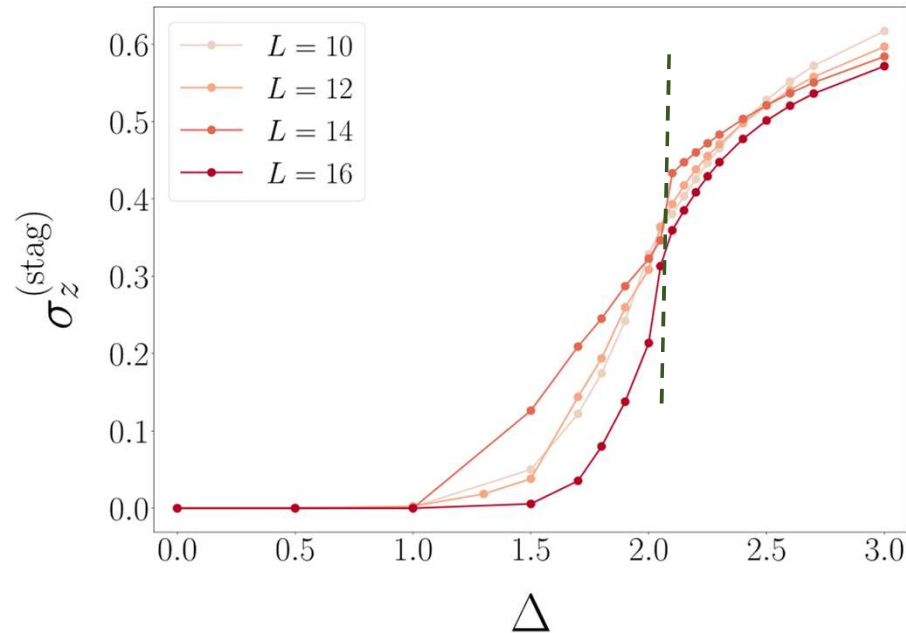
- Discontinuous staggered magnetization
- Singlet formed by the two degenerate ground states



Odd size:

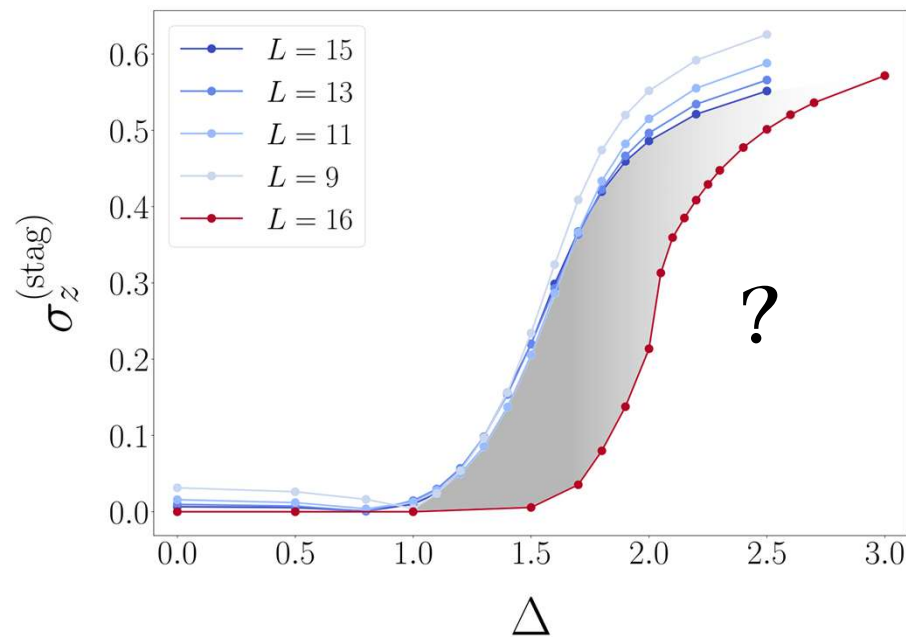
- Unique, ordered ground state

Boundary frustration in the striated phase



Even size:

- Discontinuous staggered magnetization
- Singlet formed by the two degenerate ground states



Odd size:

- Unique, ordered ground state

Boundary frustration in the striated phase

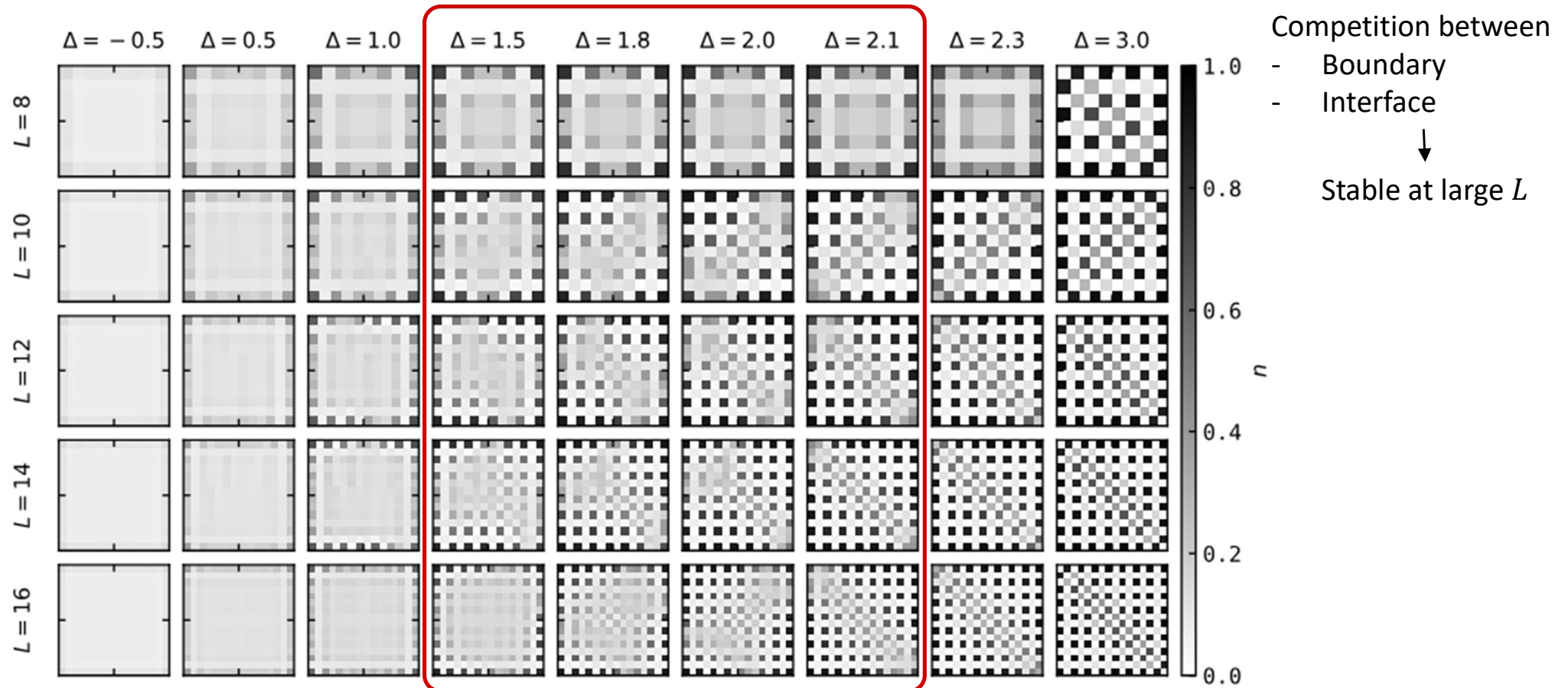
Degeneracy is not broken
anymore by the boundary

Disordered phase

Low density of excitations
No density-wave order

Ordered phase

Twofold degenerate state
Domains with diagonal interface



Boundary frustration in the striated phase

Degeneracy is not broken
anymore by the boundary

Disordered phase

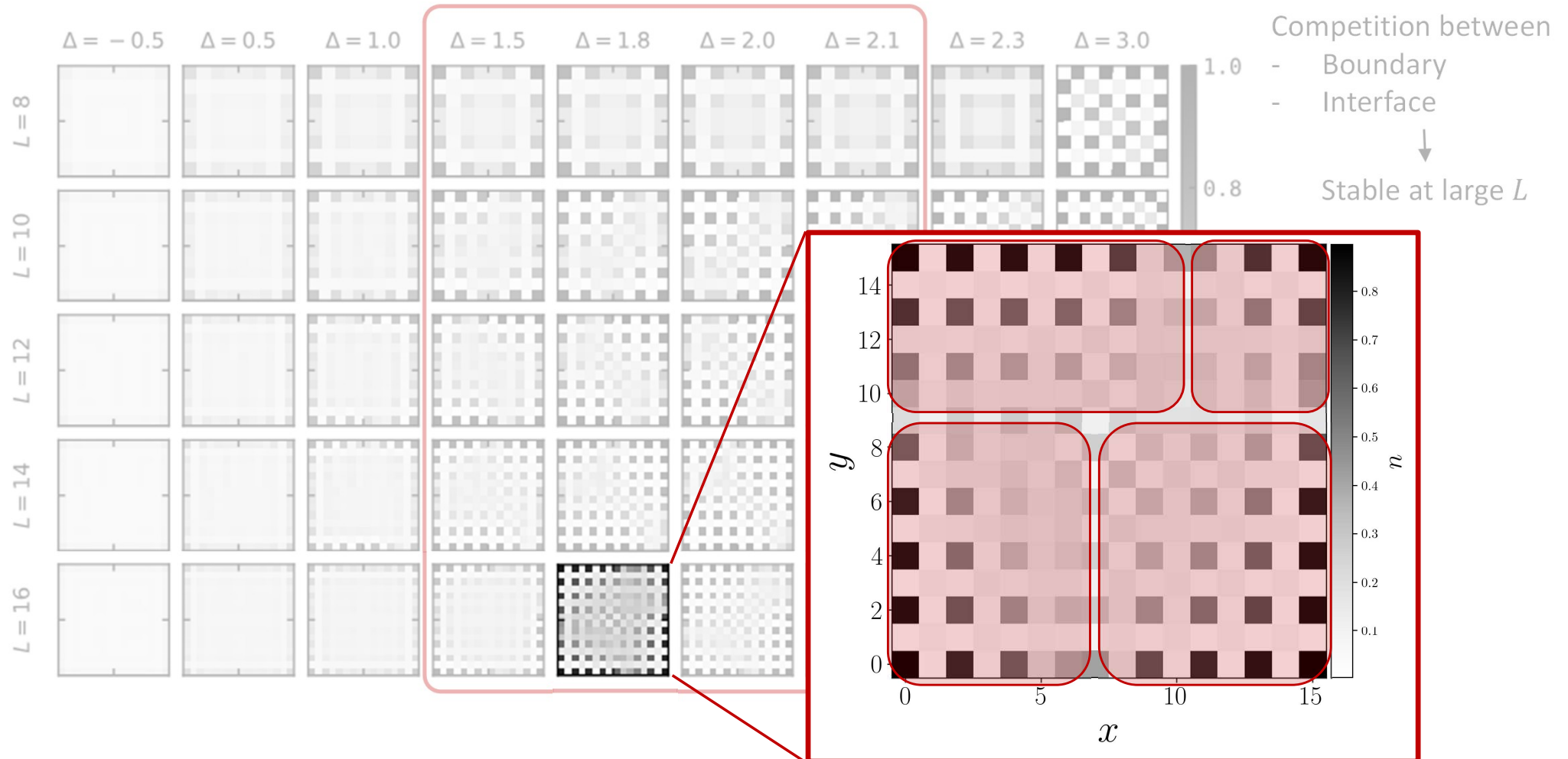
Low density of excitations
No density-wave order

Intermediate regime

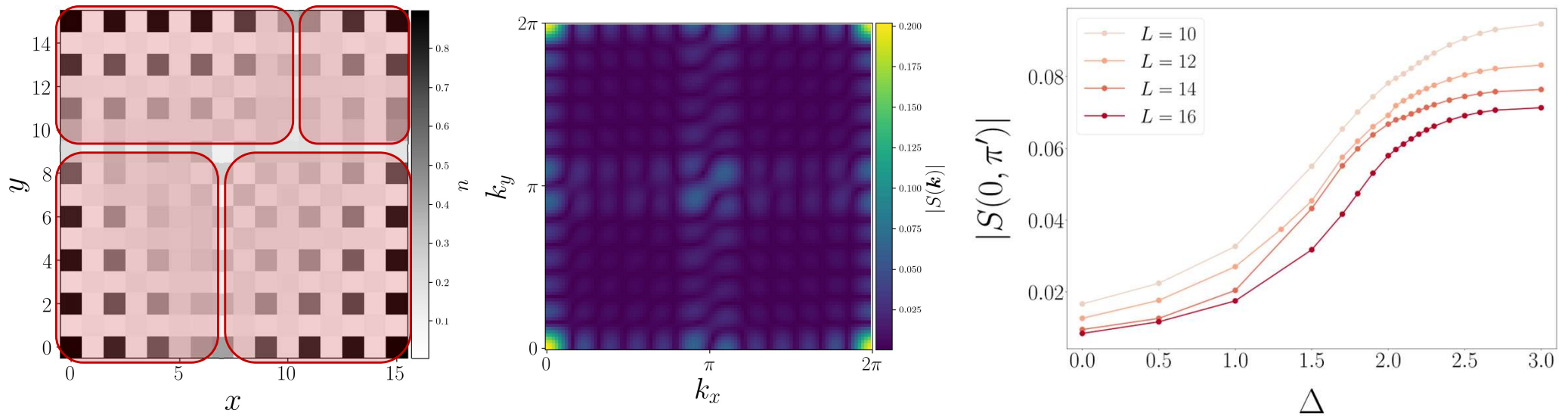
Competing domains
enucleated by the corners

Ordered phase

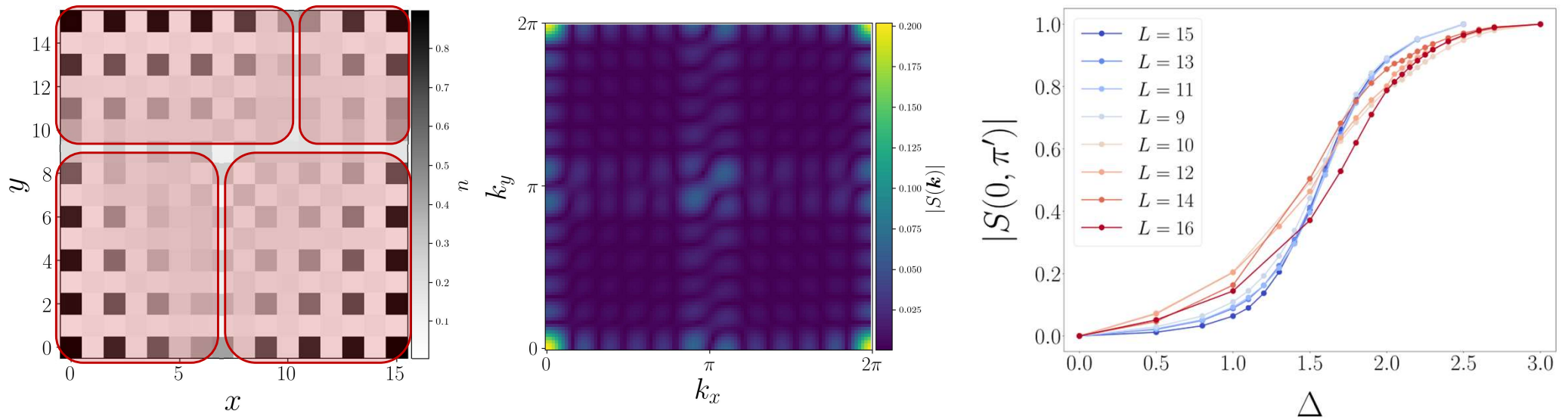
Twofold degenerate state
Domains with diagonal interface



Boundary frustration in the striated phase

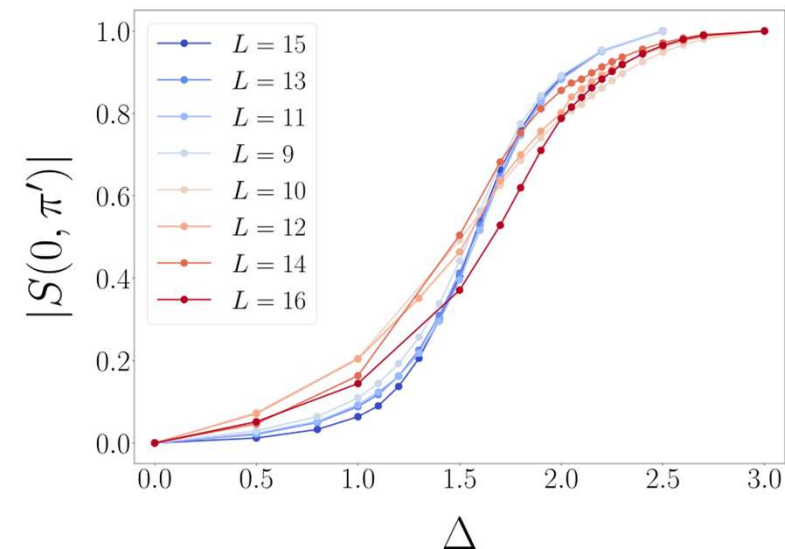
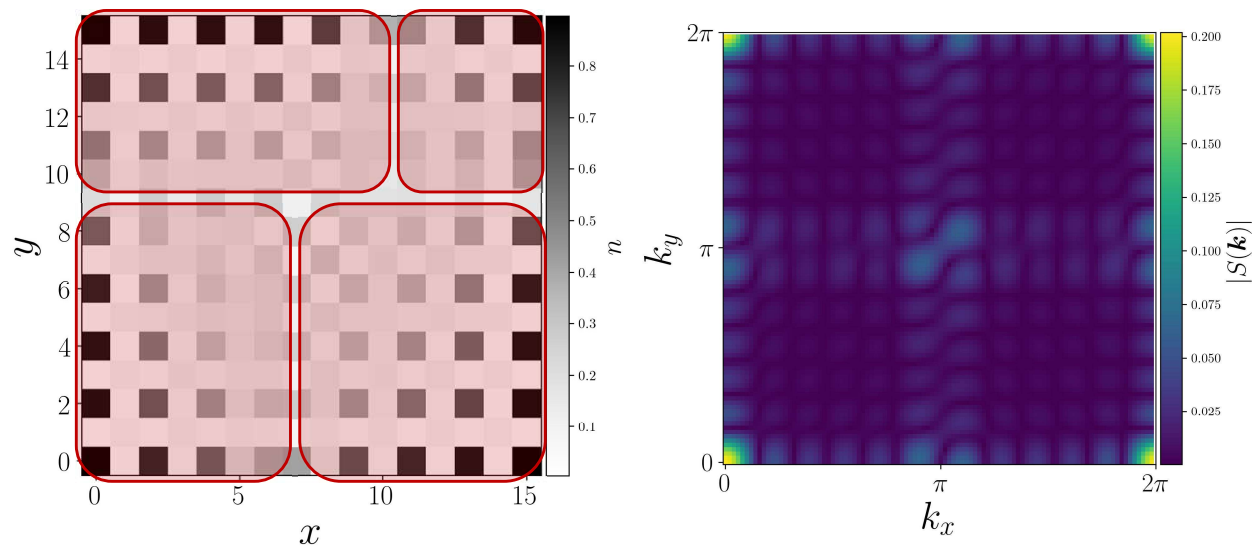


Boundary frustration in the striated phase

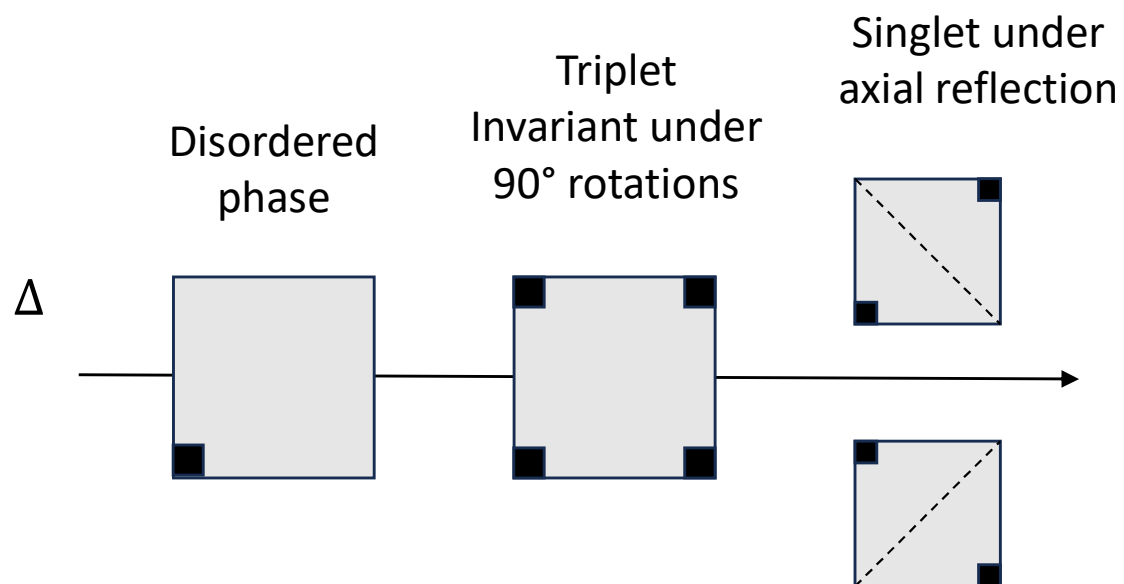


The striated order emerges around the same Δ in the even and odd case

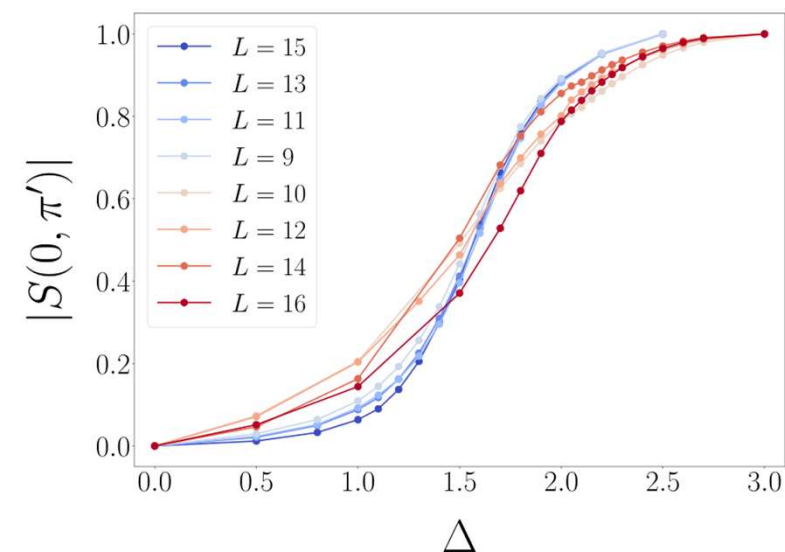
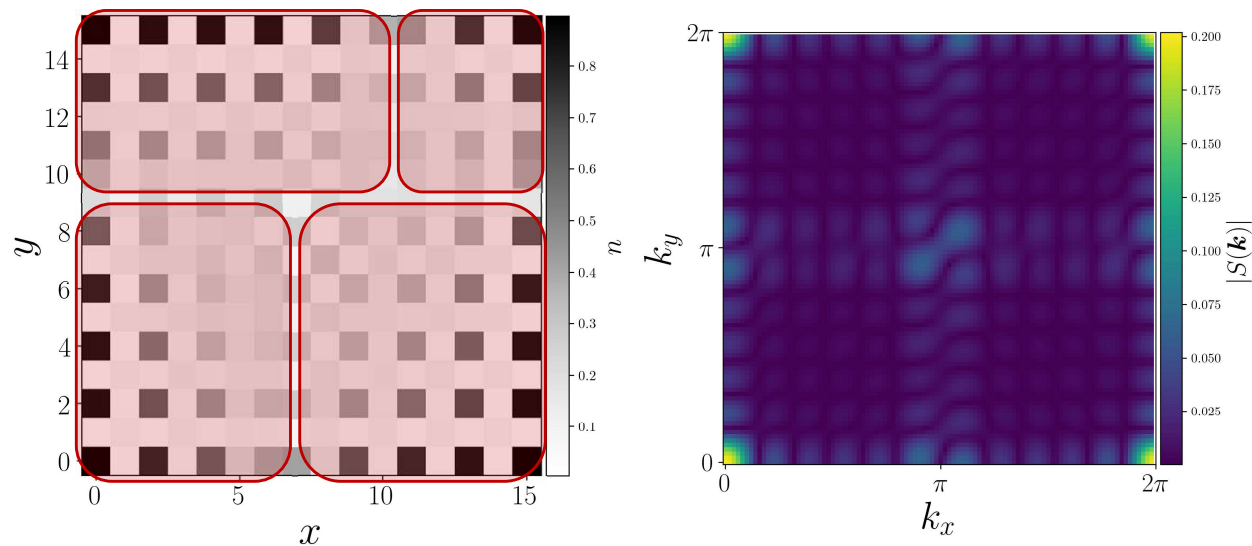
Boundary frustration in the striated phase



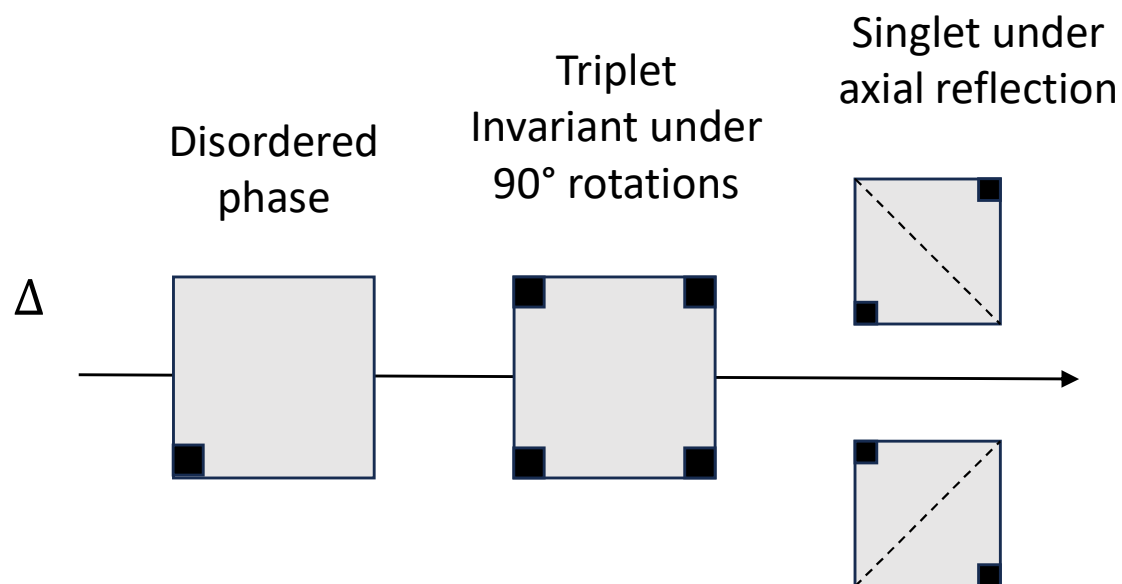
The striated order emerges around the same Δ in the even and odd case



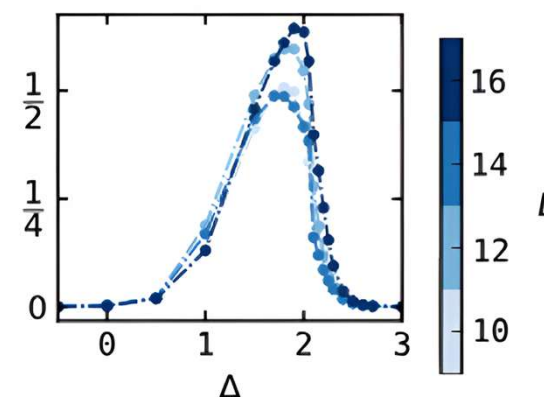
Boundary frustration in the striated phase



The striated order emerges around the same Δ in the even and odd case



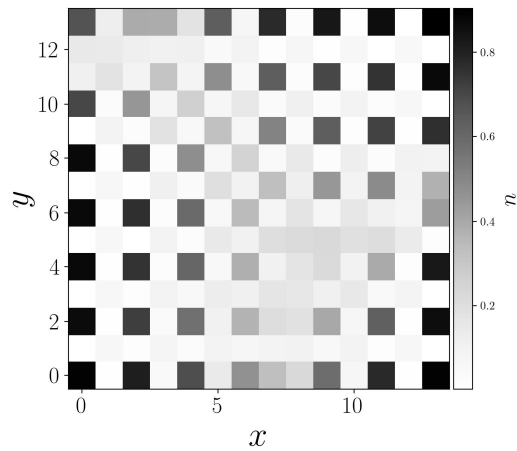
4-corners correlator



Boundary frustration in the striated phase

Ground state

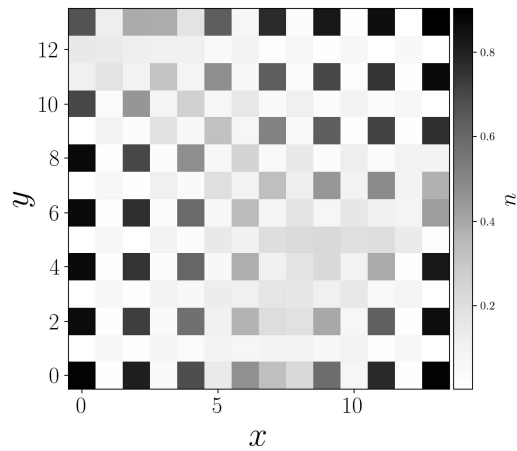
$$\Delta = 1.8$$



Boundary frustration in the striated phase

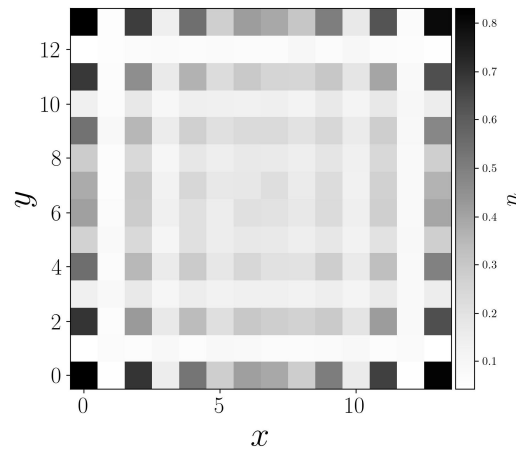
Ground state

$$\Delta = 1.8$$



Experiment

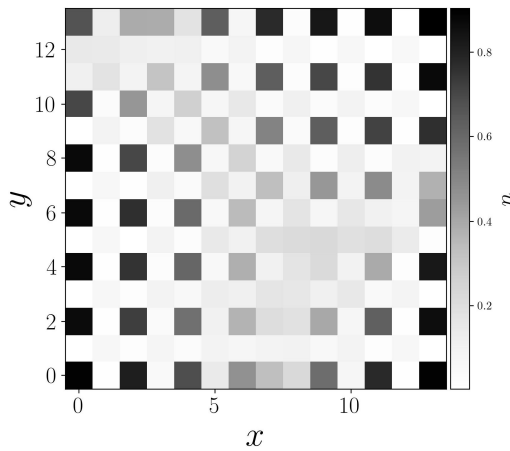
$$\Delta_f = 1.8$$



Boundary frustration in the striated phase

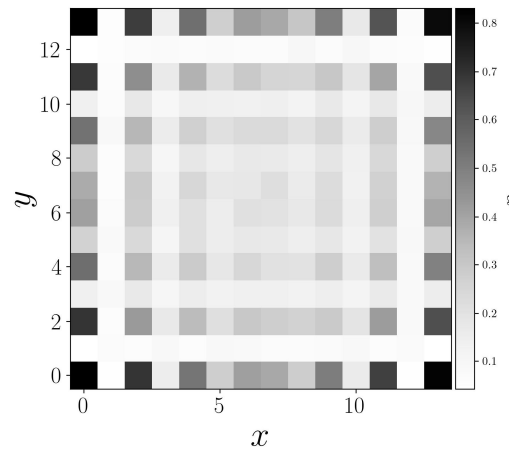
Ground state

$$\Delta = 1.8$$



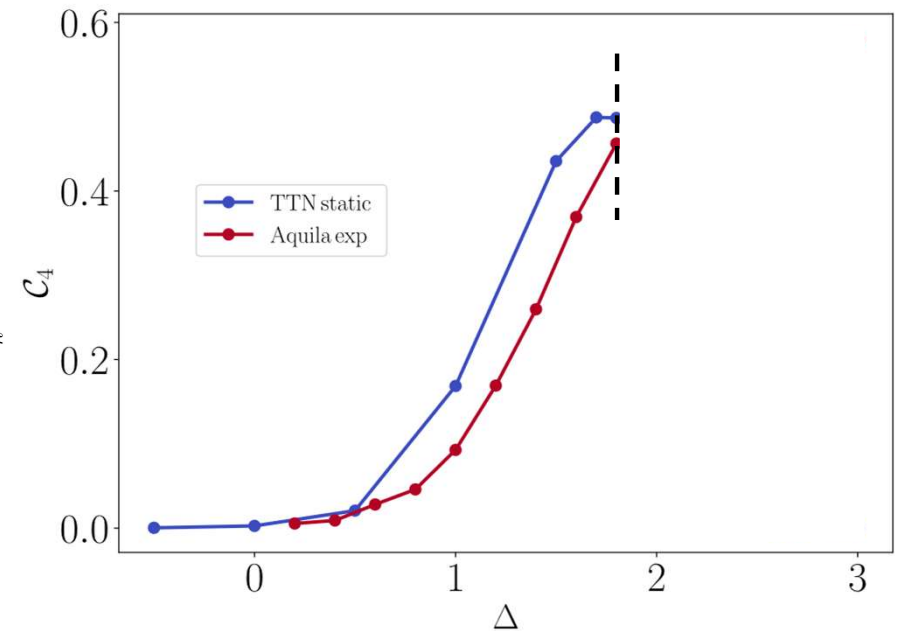
Experiment

$$\Delta_f = 1.8$$



Four corners correlator:

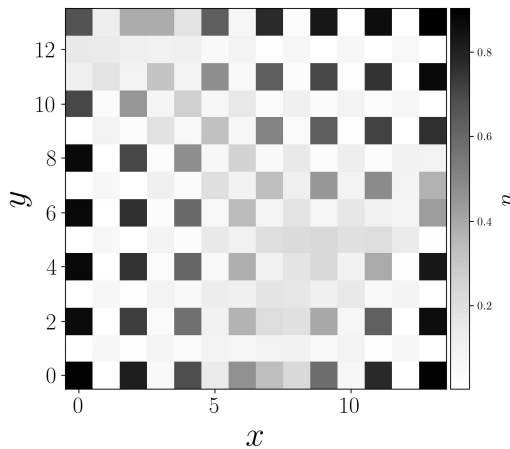
Qualitative agreement in the intermediate region



Boundary frustration in the striated phase

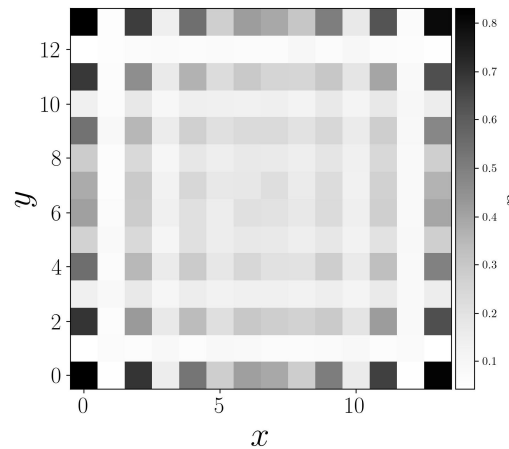
Ground state

$\Delta = 1.8$



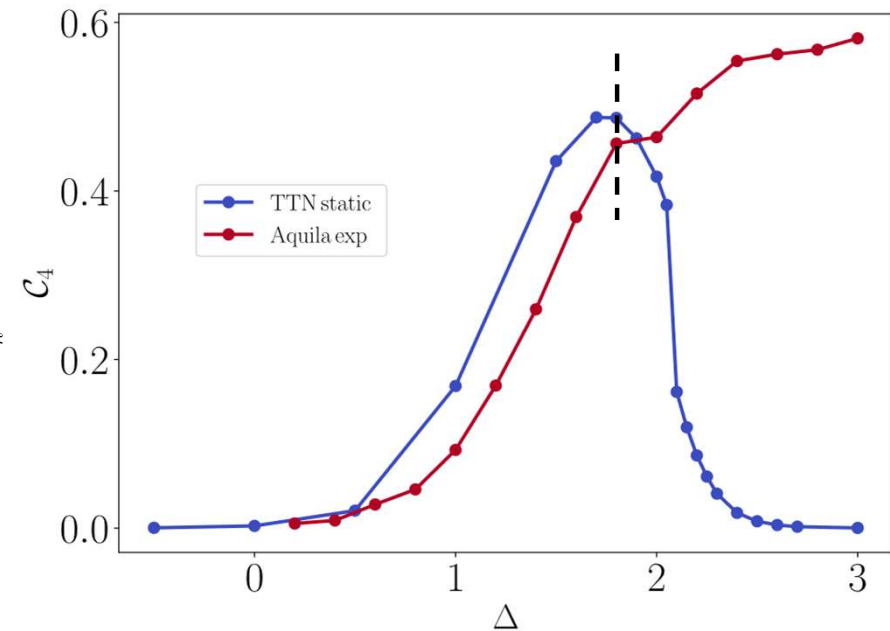
Experiment

$\Delta_f = 1.8$

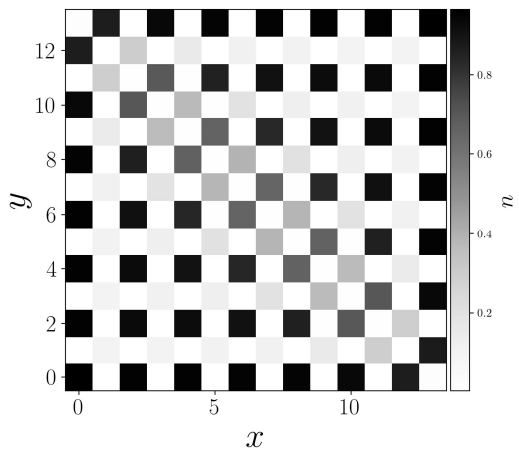


Four corners correlator:

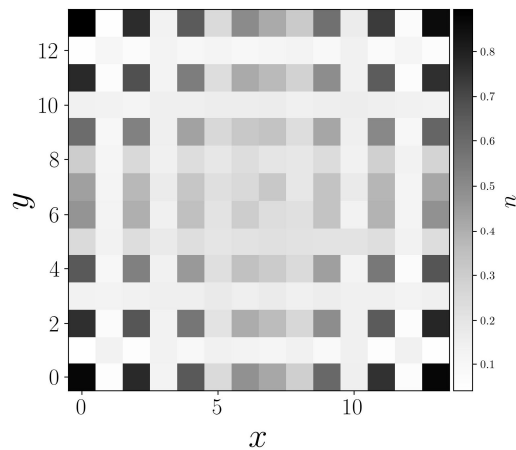
Qualitative agreement in the intermediate region



$\Delta = 3.0$



$\Delta_f = 3.0$

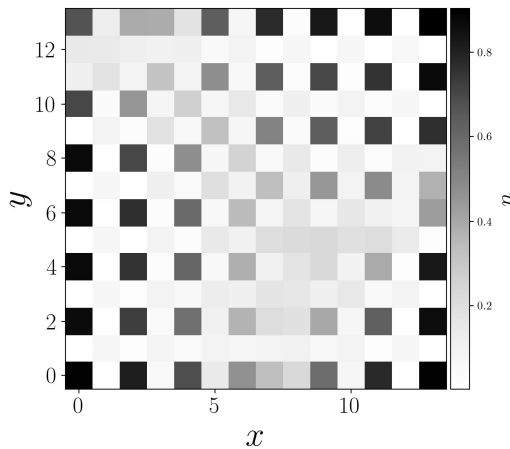


Disagreement in the singlet phase between numerical ground state and experiment!

Boundary frustration in the striated phase

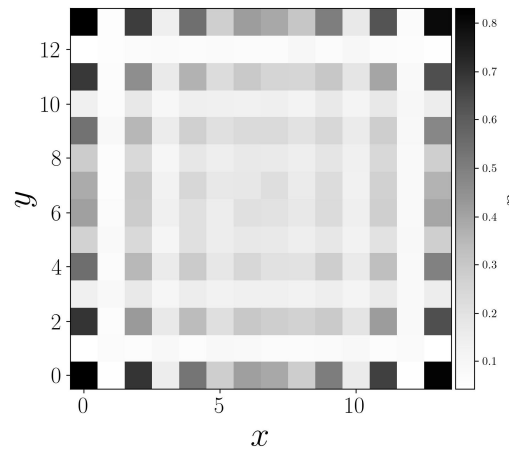
Ground state

$\Delta = 1.8$



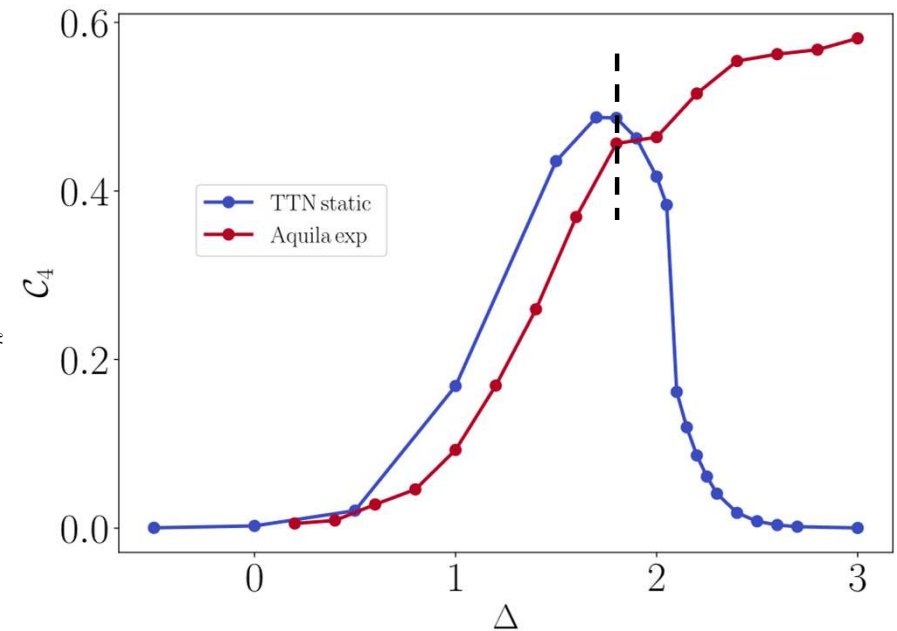
Experiment

$\Delta_f = 1.8$

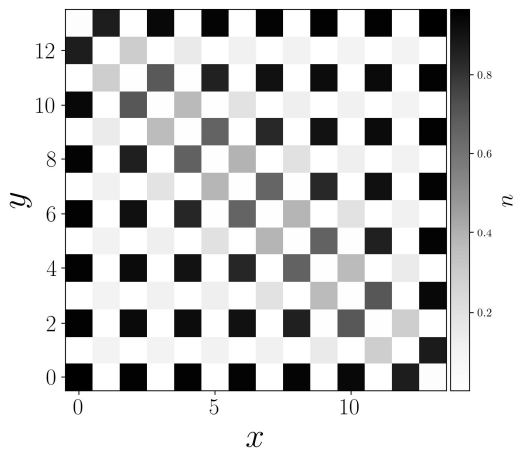


Four corners correlator:

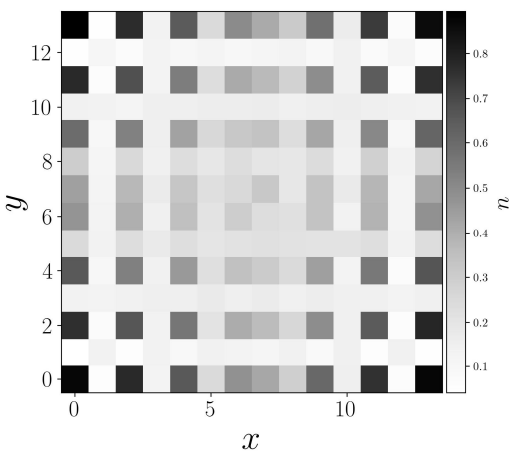
Qualitative agreement in the intermediate region



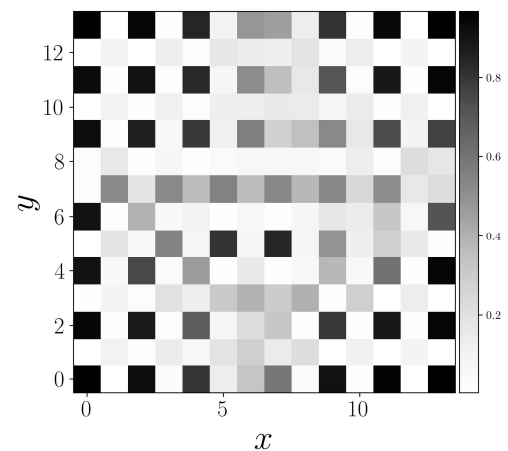
$\Delta = 3.0$



$\Delta_f = 3.0$



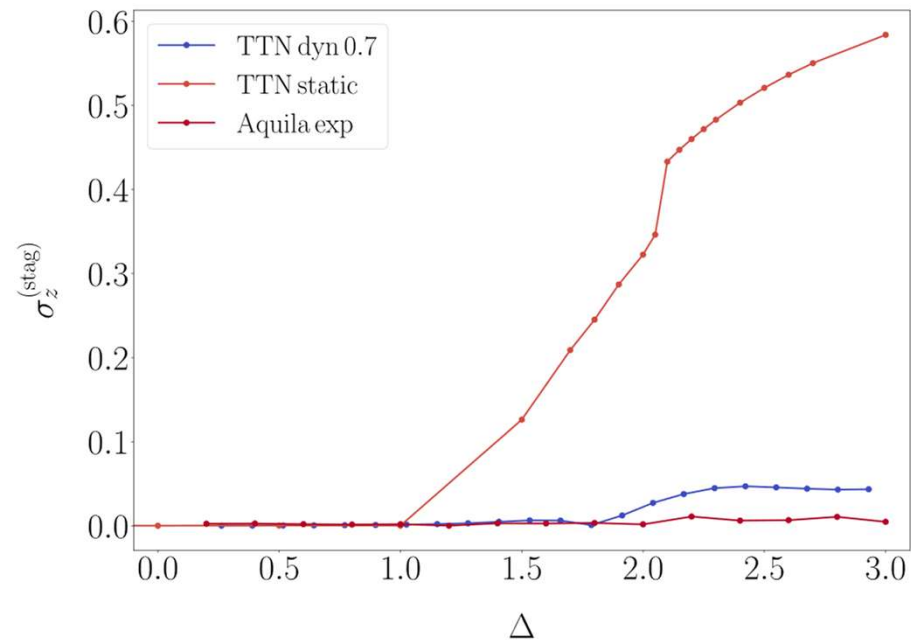
$\Delta_f = 3.0$



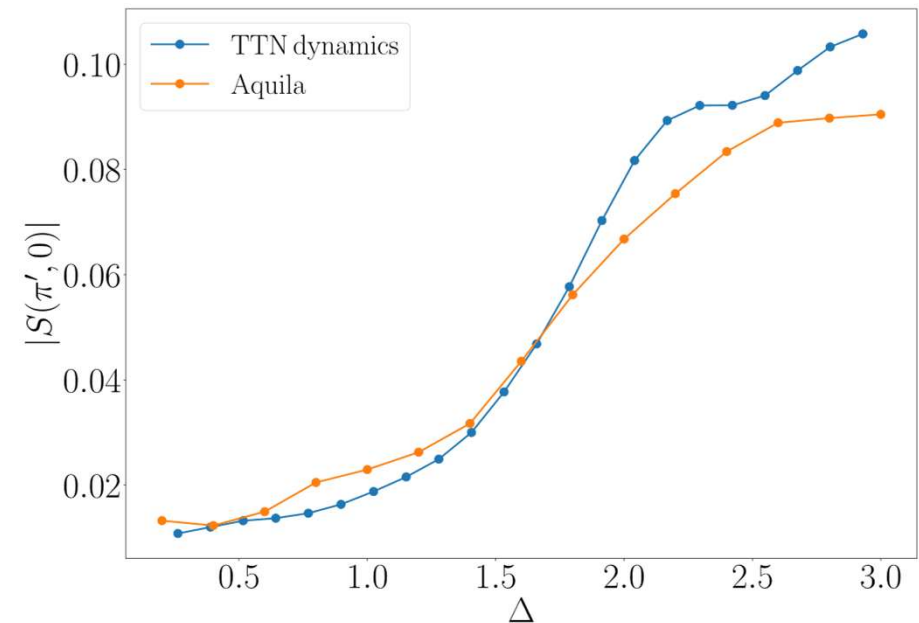
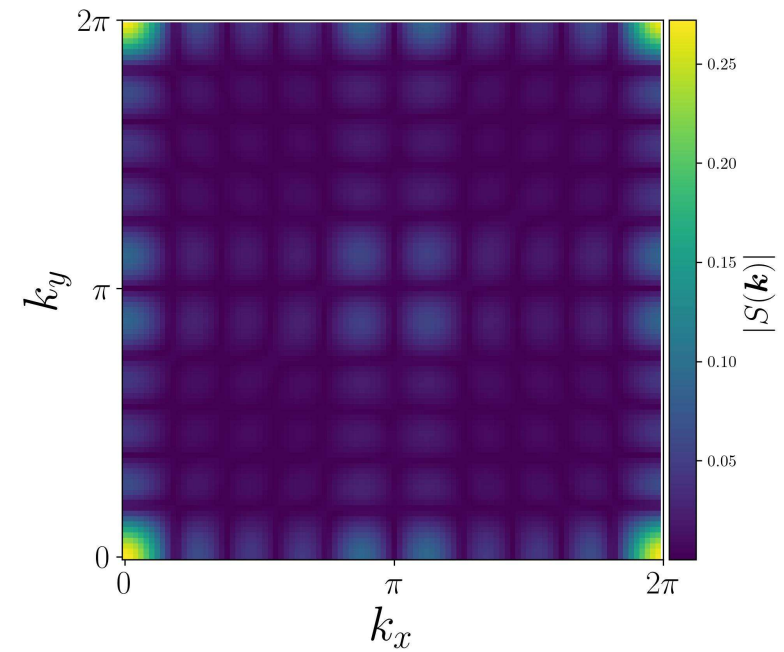
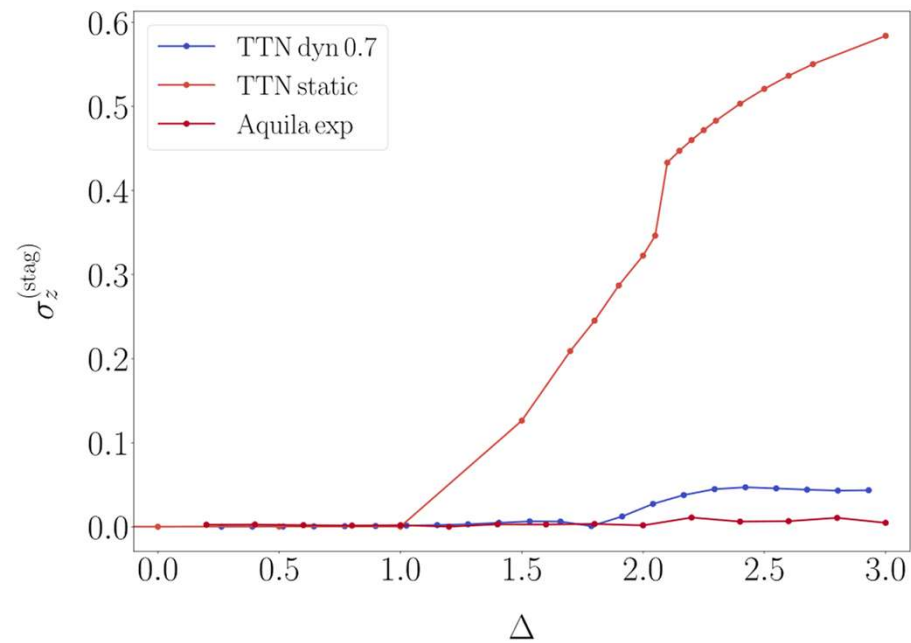
TTN dynamics

Agreement
Numerical dynamics -
Experiment

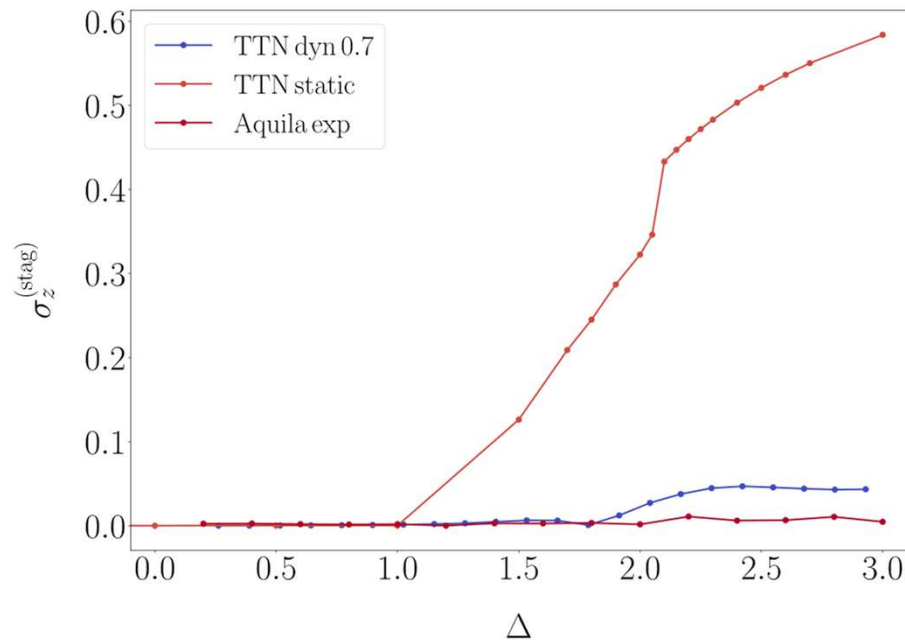
Boundary frustration in the striated phase



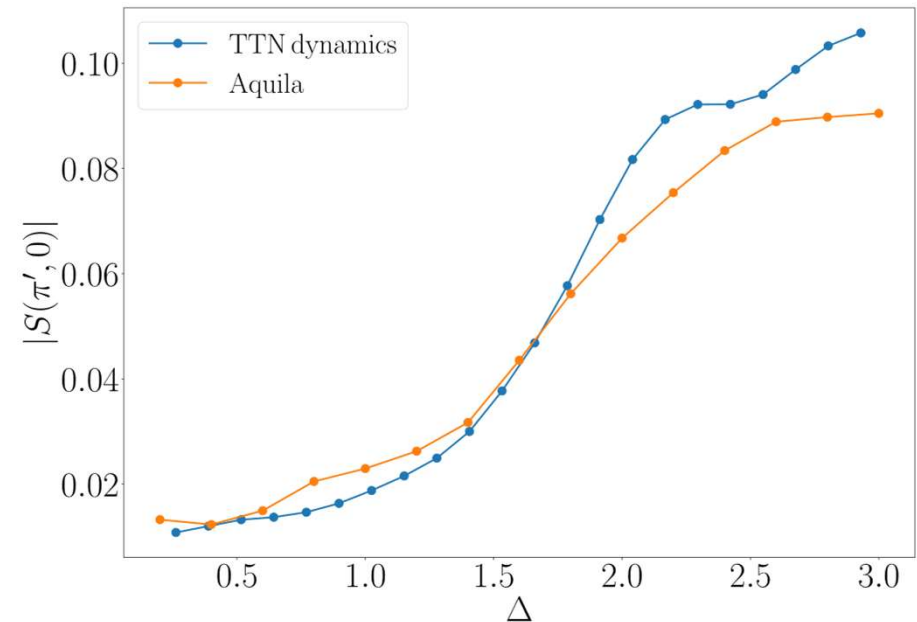
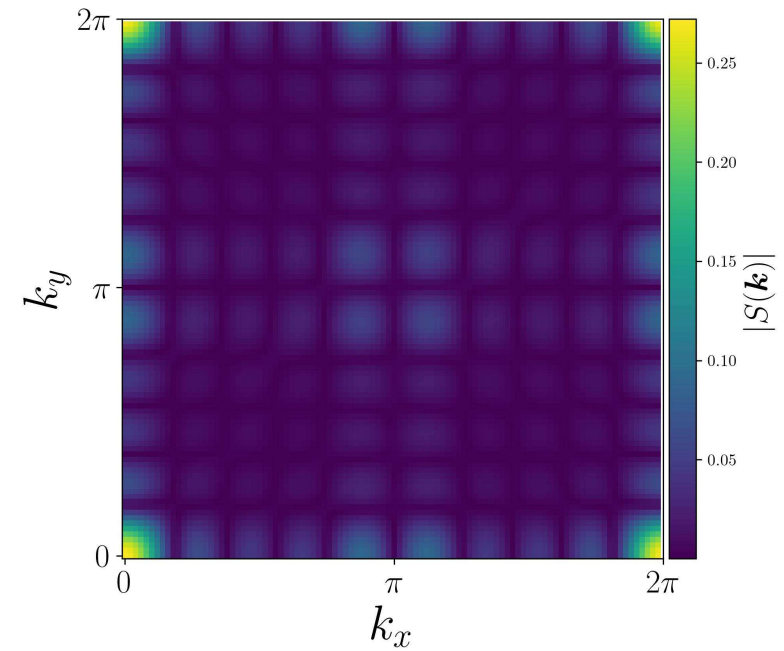
Boundary frustration in the striated phase



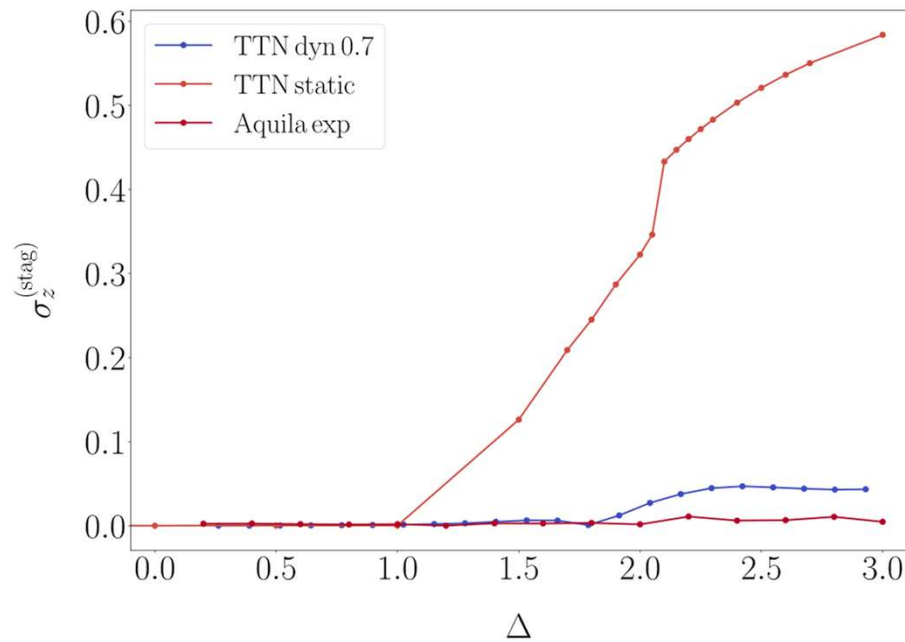
Boundary frustration in the striated phase



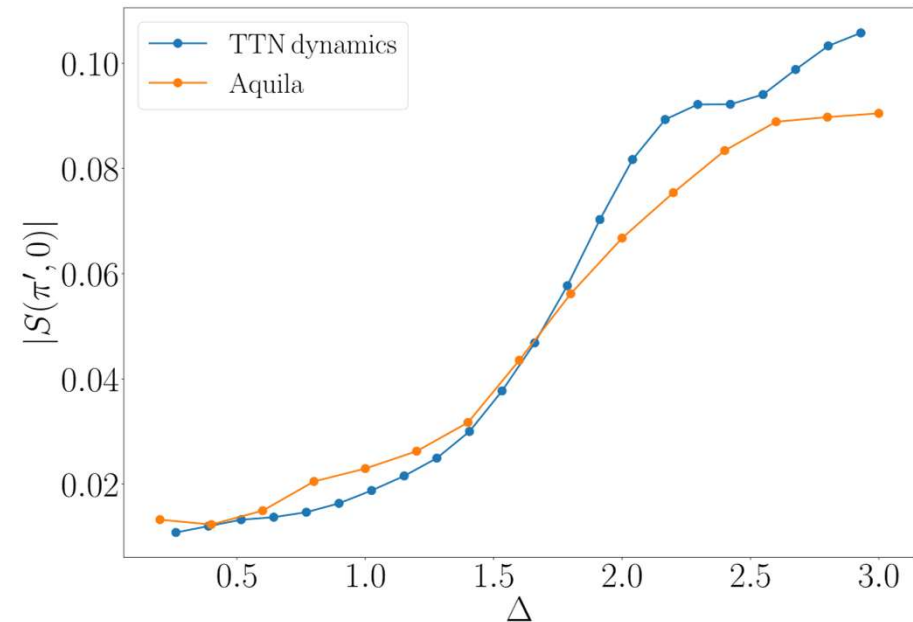
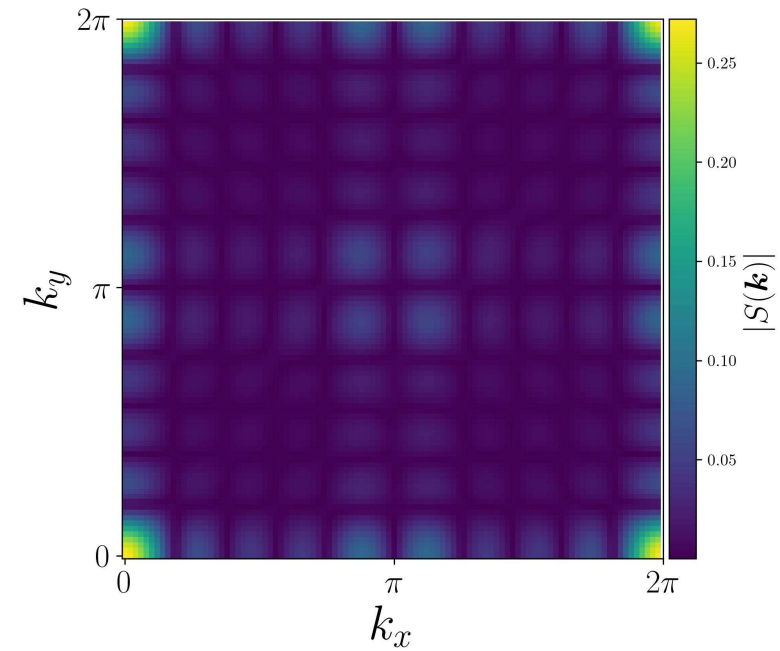
- Discontinuity in the staggered magnetization



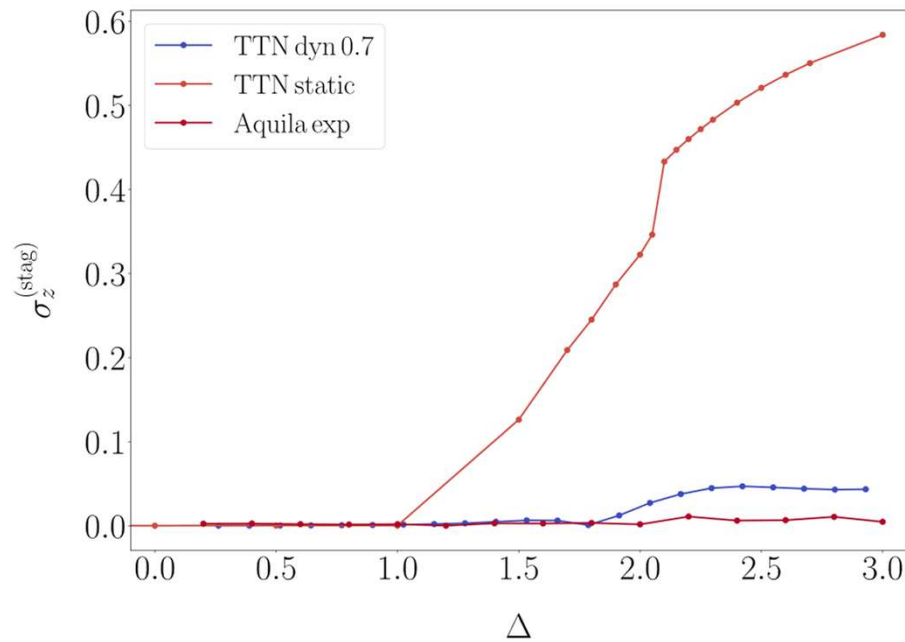
Boundary frustration in the striated phase



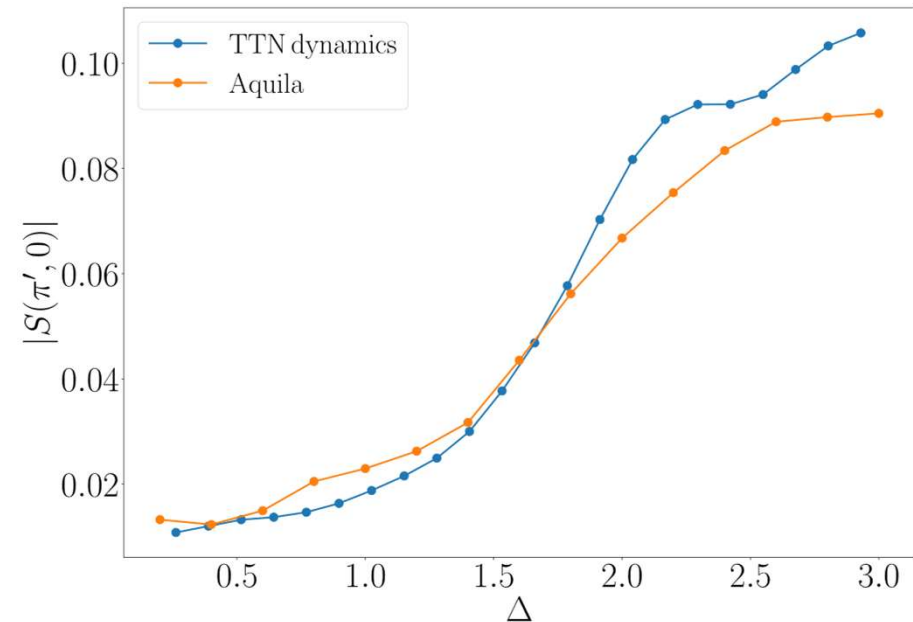
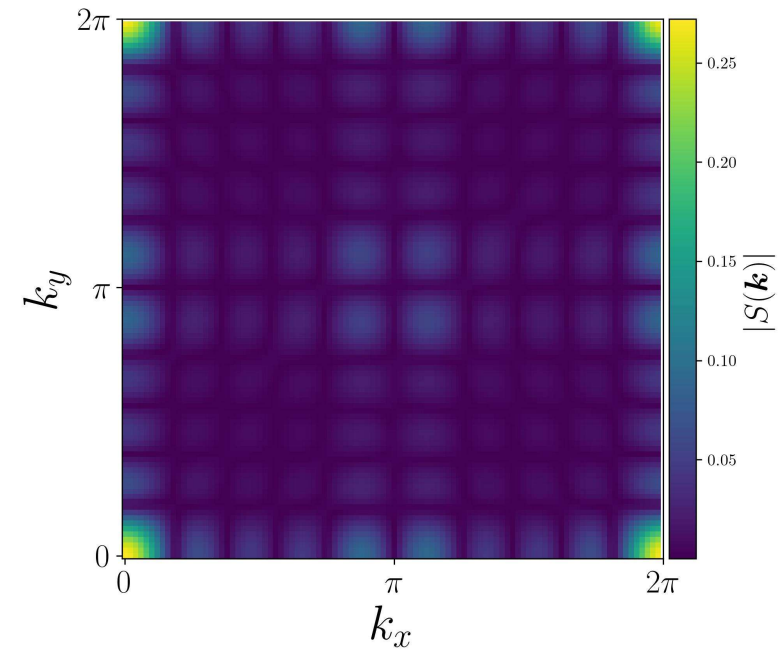
- Discontinuity in the staggered magnetization
- Transition between the singlet and the triplet striated ground state



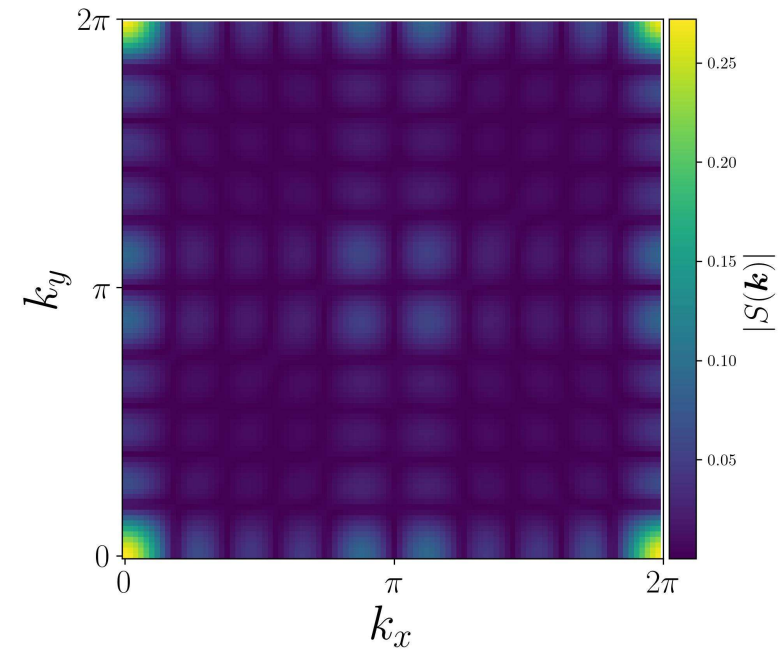
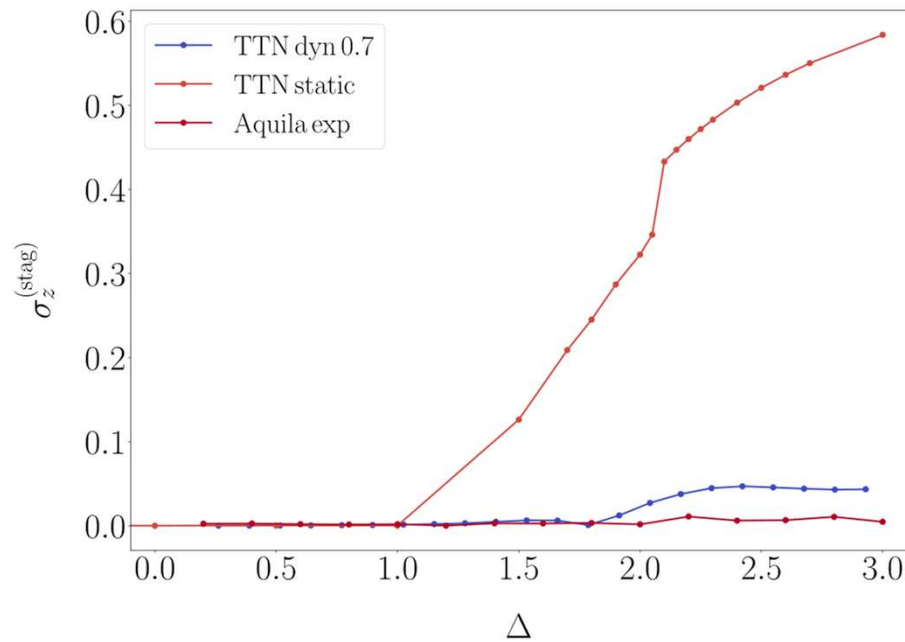
Boundary frustration in the striated phase



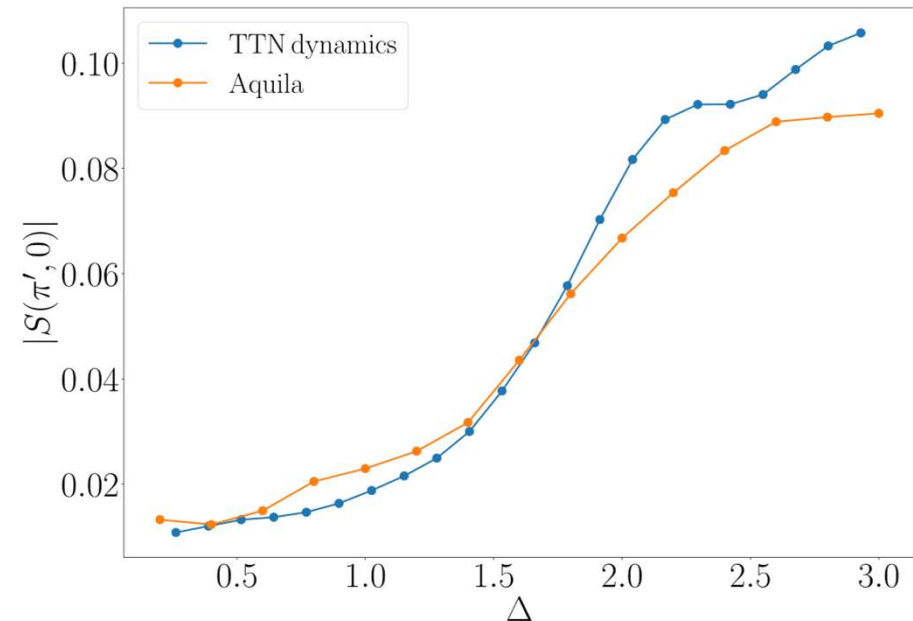
- Discontinuity in the staggered magnetization
- Transition between the singlet and the triplet striated ground state
- Avoided level crossing, exponentially small gap



Boundary frustration in the striated phase



- Discontinuity in the staggered magnetization
- Transition between the singlet and the triplet striated ground state
- Avoided level crossing, exponentially small gap
- The time scale to prepare the ordered ground state is not experimentally accessible



Conclusions

- Relevance of quantum simulators to observe quantum many body physics not accessible numerically

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- Two-dimensional square Rydberg lattice as a rich quantum simulation platform

Conclusions

preliminary

- Relevance of quantum simulators to observe quantum many body physics not accessible numerically
- Two-dimensional square Rydberg lattice as a rich quantum simulation platform
- Boundary frustration changes the nature of the transitions

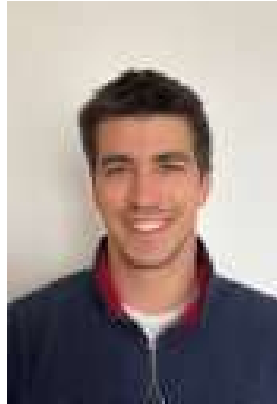
Conclusions

preliminary

- Relevance of quantum simulators to observe quantum many body physics not accessible numerically
- Two-dimensional square Rydberg lattice as a rich quantum simulation platform
- Boundary frustration changes the nature of the transitions
- Interplay between experiment and numerical simulations



Marco Rigobello



Luka Pavesic



Pietro Silvi



Simone
Montangero



Daniel
Jaschke



Marco Di Liberto

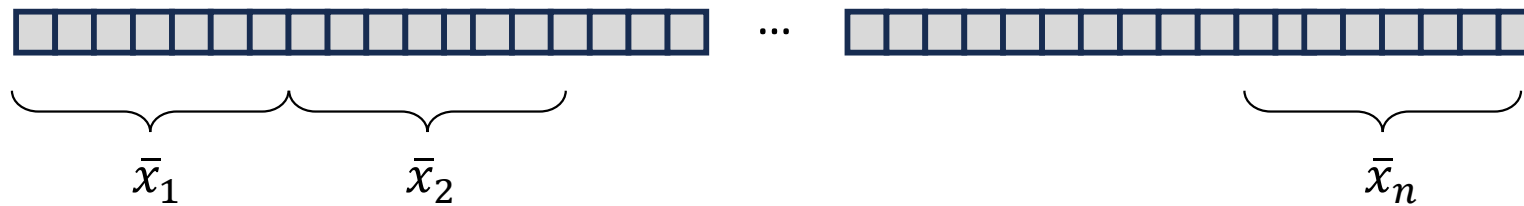
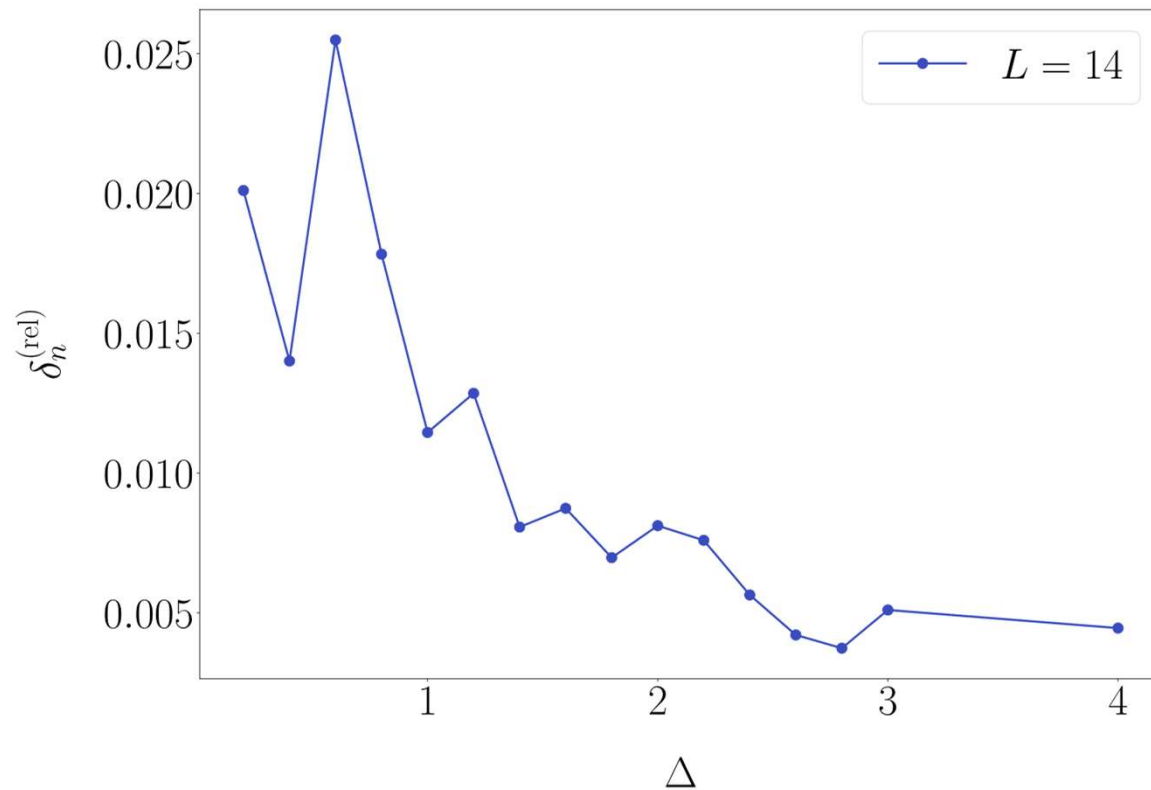


M. Lukin

Thank
you
for
your
attention



Striated phase – experimental results



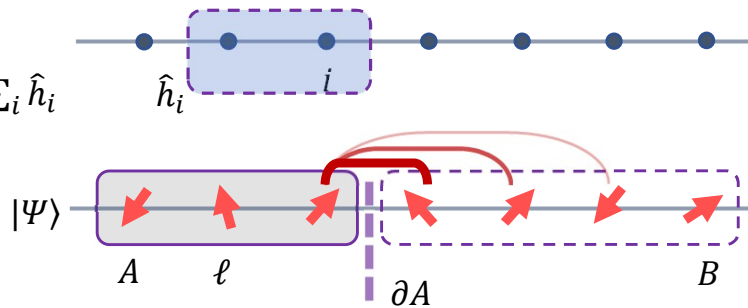
$$\delta_n^{(rel)} = \frac{\sqrt{\text{var}[\{\bar{x}_i\}]}}{\bar{n}}$$

Tensor Network methods

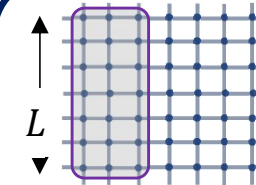
Area laws of entanglement

1-D local Hamiltonian $\hat{H} = \sum_i \hat{h}_i$

Local correlations
(exponential decaying)



TN in 2-D

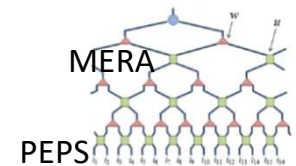
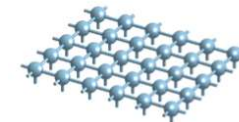


For a 2-D system
 $\partial A \sim L \rightarrow \mathcal{S} \sim \exp(L)$

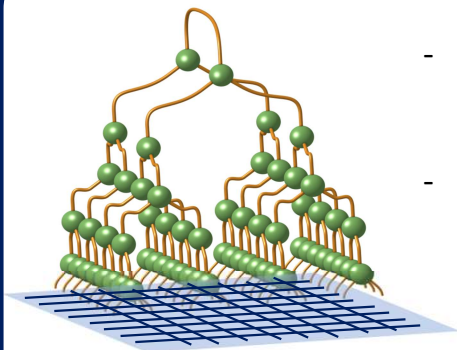
Needed compromise between:

- Computational complexity
- Accuracy

Several geometries:

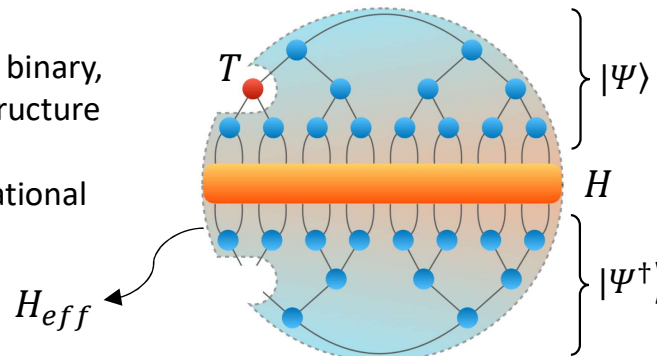


TTN algorithm



physical
lattice

- Tree-tensor network: binary, hierarchical tensor structure
- Competitive computational complexity



H_{eff}

PhD Thesis, T.
Felser

Sequential tensor manipulation

- Ground state:

$$\min\{T^\dagger H_{eff} T\}$$

- Dynamics:

$$T_{t+dt} \simeq \left[e^{-idt H_{eff} \left(t + \frac{dt}{2} \right)} \right]_{Kryl} T_t$$

Rydberg atom arrays – phase diagram

Physical Review B
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