No-go theorems for higher-spin charges in AdS_2

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based on 2502.06937 w/ A. Antunes & N. Levine

21/05/2025 Cortona

One sentence summary:

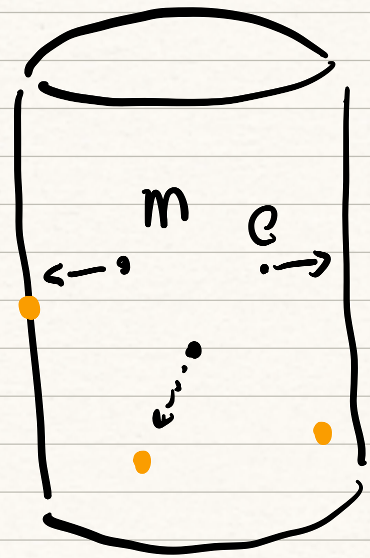
It is impossible to non-trivially* deform
a) A free field in AdS
b) CFTs in AdS*
while preserving higher-spin charges

* trivial deformation = change the mass

Outline:

1. Motivation and basics
2. Higher-spin currents and charges in AdS
3. Example: free massive boson
4. No-go theorems
5. Outlook

1. Motivation and basics



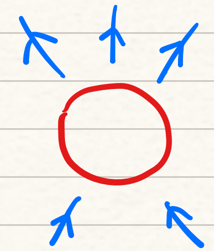
- Integrability is a powerful set of ideas:
  it allows to solve interacting QFTs

- Anti de-Sitter space is a great playground:
  - massive QFT in $AdS_d \rightarrow$ conformal theory in $\mathbb{R}^{d-1}$
    (critical points of long range models)
  - most symmetric box:
    solve QFTs via the conformal bootstrap ?

[Paulos, Penedones, Toledo, van Rees, Vieira, 2016]

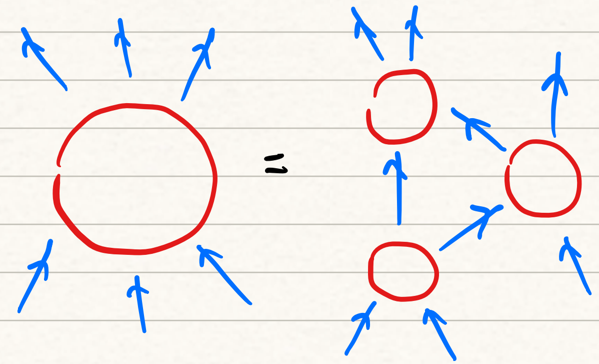
- Integrability in 2d flat space

  - No particle production



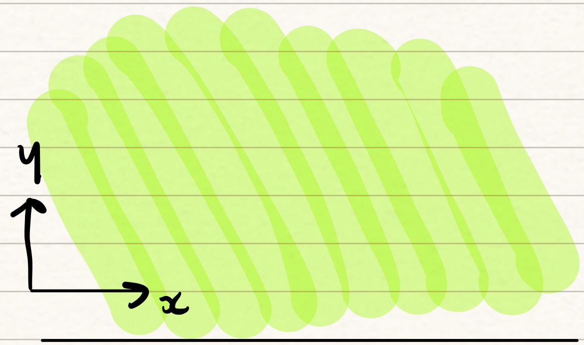
$= 0$

  - Factorized scattering (Yang-Baxter)

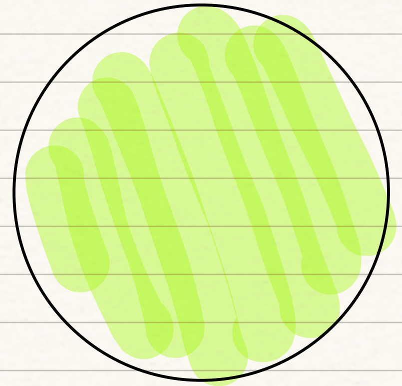


- Wealth of exact results: S-matrix, (finite size) spectra, form factors
- Examples:
  - Sine-Gordon: $$S = \int (\partial\phi)^2 + \cos(\beta\phi)$$
  - Deformed Minimal Models $$S = S_{CFT} + \int \phi_{(1,3)}$$

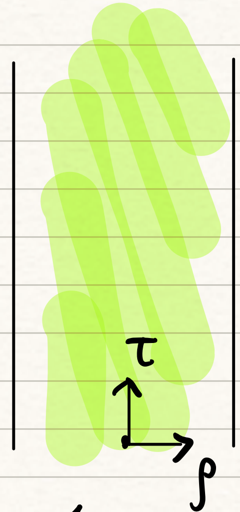
• (Euclidean) Anti de-Sitter space



$$ds^2 = \frac{1}{y^2}\left(dx^2 + dy^2\right)$$



$$ds^2 = \frac{4}{(1-r^2)^2}\left(dr^2 + r^2 d\theta^2\right)$$



$$ds^2 = \frac{1}{(\cos\rho)^2}\left(d\tau^2 + d\rho^2\right)$$

- Maximally symmetric space : can rotate around any point
  can translate in any direction

- Killing vectors (ξ^A $A = 0, 1, 2$) form $SL(2,\mathbb{R}) \simeq SO(2,1)$

translations: $\left[p = \partial_x\right]$ dilatations: $\left[d = x\partial_x + y\partial_y\right]$ sp. conformal: $\left[K = (x^2-y^2)\partial_x + 2xy\partial_y\right]$

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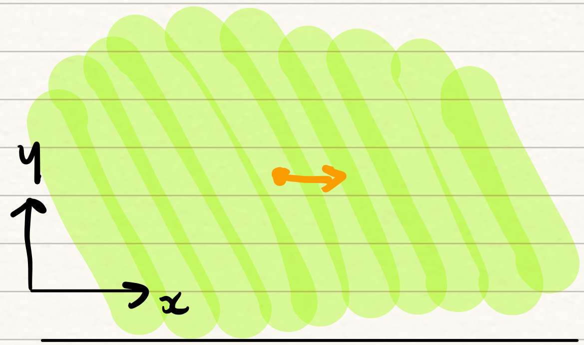

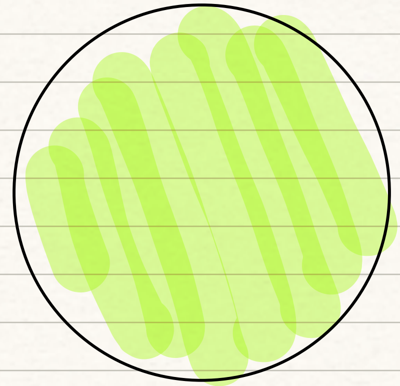

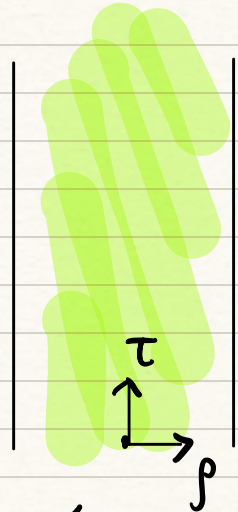

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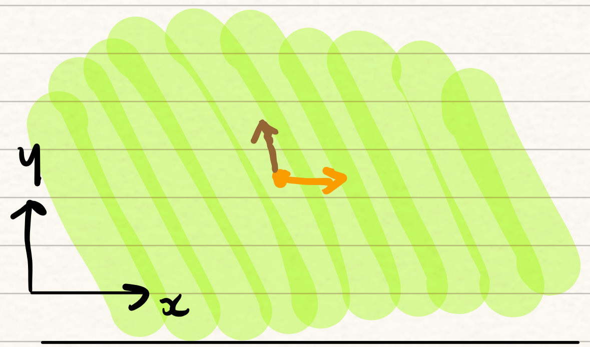

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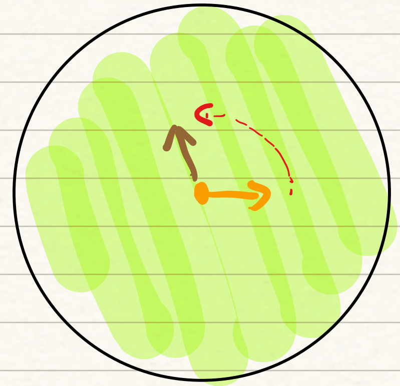

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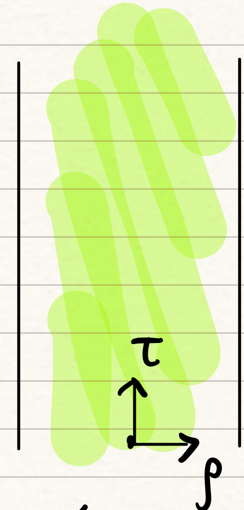

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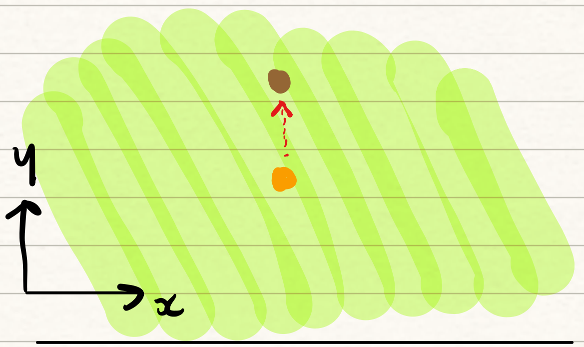

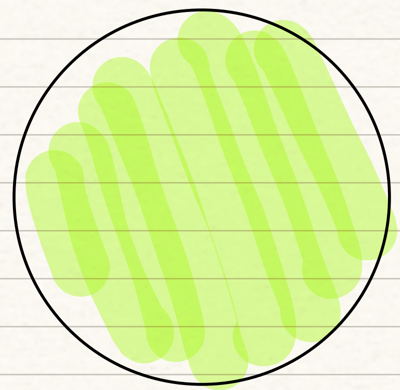

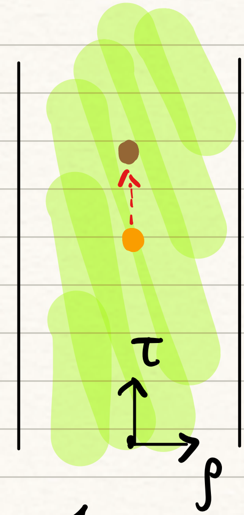

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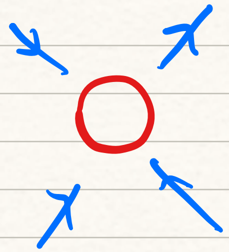
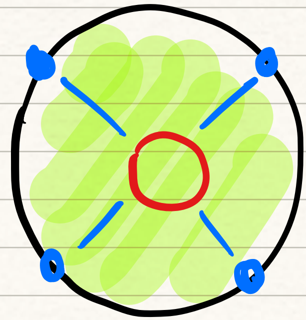
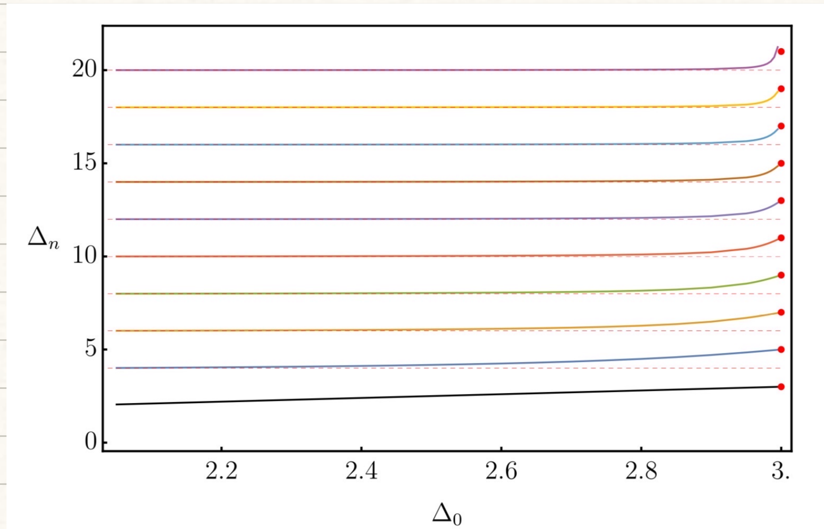
Integrability in AdS_2 ?

- S-matrix elements $\rightsquigarrow$ correlators of boundary ops.





" $\langle \phi \phi \phi^n \rangle = 0 \quad n > 2$ " ?



[Paulos, Zan, 2020]

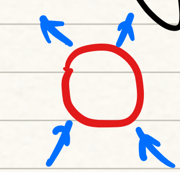
Integrability from higher-spin symmetry

- Need ∞ many conserved charges

$\boxed{\text{higher-rank currents}}$ Non-local

Flat space: $\boxed{\partial_\mu T^{\mu}{}_{\underbrace{z\cdots z}_{s}} = 0} \Rightarrow Q_s$

$Q_s|p\rangle = p^s|p\rangle \Rightarrow \sum_i p_i^s = 0 \Rightarrow$ factorized scattering

but non trivial 

What are the consequences of HS charges in AdS?

2. Higher-spin currents and charges

$$Q_\zeta^{(j)}(\Sigma) = \int_\Sigma d\Sigma^\mu \; \zeta^{\mu_2 \cdots \mu_j} \; T_{\mu \mu_2 \cdots \mu_j}$$

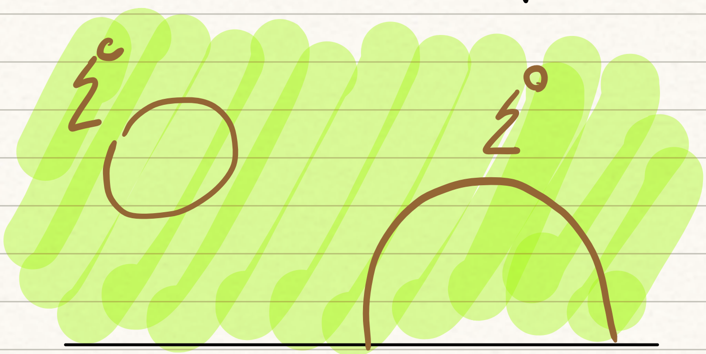
charge

rank-j operator tensor

Killing tensor

$$\nabla_{(\mu} \zeta_{\nu_1 \cdots \nu_{j-1})} = 0$$

codimension-1 surface:

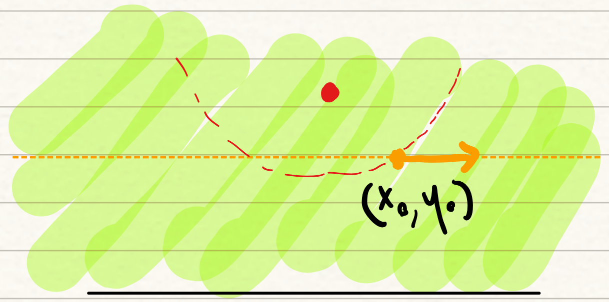


Minimal requirement:

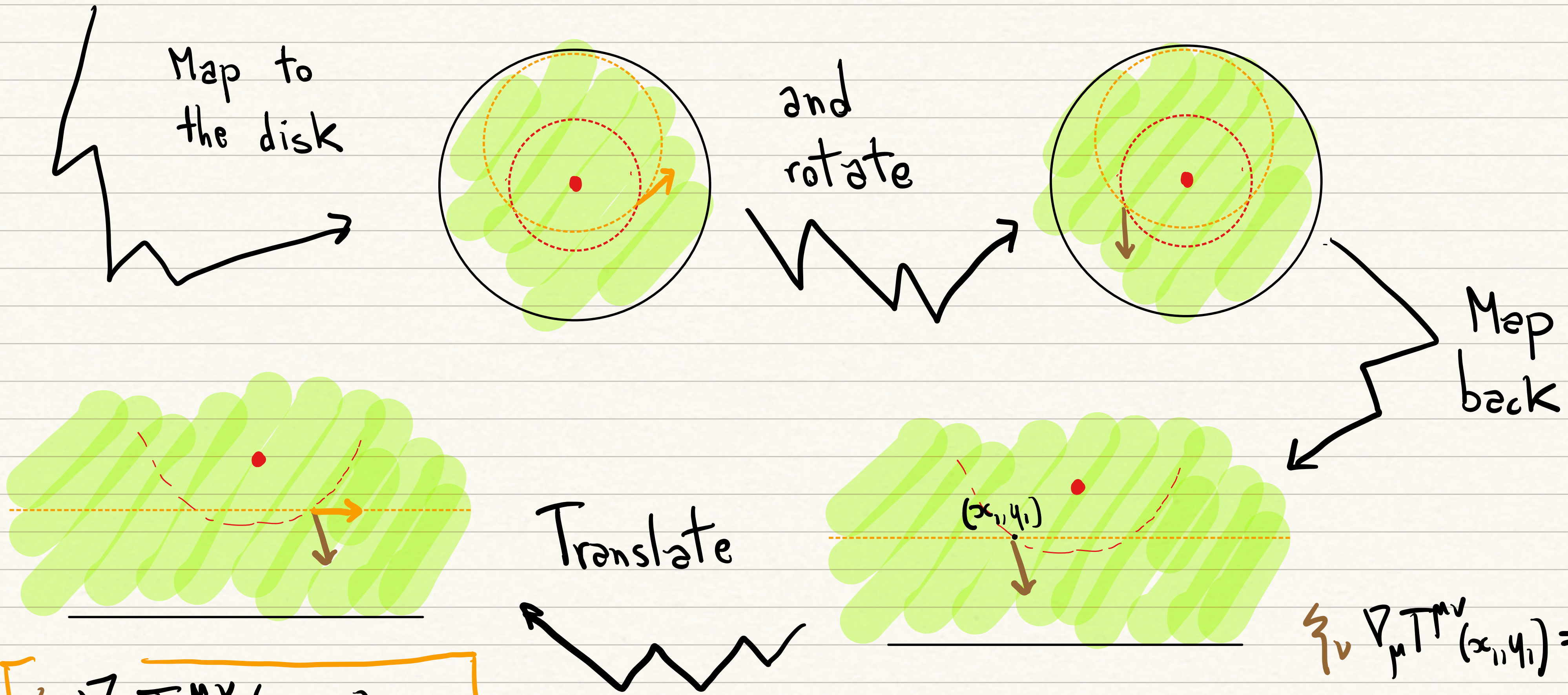
* $\nabla^\mu \left(\zeta^{\mu_2 \cdots \mu_j} T_{\mu \mu_2 \cdots \mu_j} \right) = 0$

* Extra "Cardy" conditions on boundary OPE for $Q_\zeta^{(j)}(\Sigma^0)$

How many components must we conserve?



$$\boxed{\nabla_\mu T^{\mu x}(x_0, y_0) = 0}$$



$$\xi_\nu \nabla_\mu T^{\mu\nu}(x_1, y_1) = 0$$

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How many components must we conserve?

In general: $\left[\nabla^{\mu_1}\left(\zeta^{\mu_2\cdots\mu_J} T_{\mu_1\cdots\mu_J}\right) = 0 \implies \nabla^{\mu} T_{\mu\mu_2\cdots\mu_J} = 0\right]$

for 1 fixed ζ

Mechanism: $\zeta^{\mu_2\cdots\mu_J}\nabla^{\mu_1} T_{\mu_1\cdots\mu_J} = 0 \xRightarrow[\text{under isometries}]{\text{invariance}} (\mathcal{L}_\xi \zeta)^{\mu_2\cdots\mu_J}\nabla^{\mu_1} T_{\mu_1\cdots\mu_J} = 0$

Lie derivative
ξ = Killing vector

Space of AdS_2 Killing tensors:

- $\zeta_{\mu_1\cdots\mu_J} = \alpha_{A_1\cdots A_J}\,\xi^{A_1}\cdots\xi^{A_J}$ homogeneous poly. of ξ^A [Thompson 1986]
- Form spin J IRREPS of $SL(2,\mathbb{R})$ under isometries $\mathcal{L}_\xi \zeta$ (+ traces)

$$p^J \underset{K}{\overset{P}{\rightleftarrows}} p^{J-1}d \underset{K}{\overset{P}{\rightleftarrows}} \cdots \underset{K}{\overset{P}{\rightleftarrows}} K^J$$

How many components must we conserve?

$$(\mathcal{L}_\xi \zeta)^{\mu_2\cdots\mu_j} \nabla^{\mu_1} T_{\mu_1\cdots\mu_j} = 0 \quad \rightsquigarrow \quad \zeta_w^{\mu_2\cdots\mu_j} \nabla^{\mu_1} T_{\mu_1\cdots\mu_j} = 0$$

$w = -(j-1) \dots j-1$ full $sl(2,\mathbb{R})$ irrep

- Rank $j-1$ Killing tensors form (overcomplete) basis of rank-$(j-1)$ symmetric tensors

$$\left[\nabla^{\mu_1}\left(\zeta^{\mu_2\cdots\mu_j} T_{\mu_1\cdots\mu_j}\right) = 0 \quad \Longrightarrow \quad \nabla^{\mu} T_{\mu\mu_2\cdots\mu_j} = 0 \right]$$

for 1 fixed ζ

Consequences:

- If Cardy conditions are satisfied, we get full multiplet of charges
- If Traces $(T^{\mu_1\cdots\mu_j}) \neq 0$ we also get all lower spin charges

3. Example: free massive boson

$$S = \int_{AdS_2} \frac{1}{2}(\partial\Phi)^2 + m^2\Phi^2$$

dual to GFF for φ:

$$m^2 = \Delta_\varphi(\Delta_\varphi - 1)$$

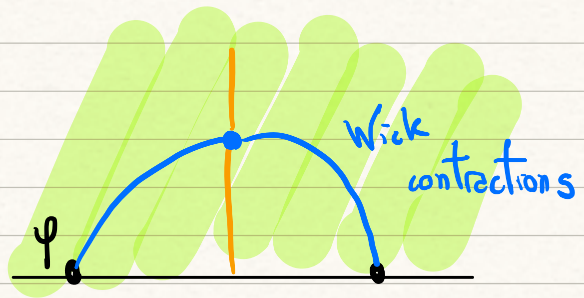
- Higher-spin currents for all even J, e.g.

$$T_{\mu\nu\rho\sigma} = \Phi \overleftrightarrow{\nabla}_{(\mu} \overleftrightarrow{\nabla}_{\nu} \overleftrightarrow{\nabla}_{\rho} \overleftrightarrow{\nabla}_{\sigma)} \Phi + 14\, g_{(\mu\nu} \Phi \overleftrightarrow{\nabla}_{\rho} \overleftrightarrow{\nabla}_{\sigma)} \Phi + 9\, g_{(\mu\nu} g_{\rho\sigma)} \Phi^2$$

[Bekaert, Meunier, 2010]

- We can build spin 3 charges $\underbrace{Q^{(3)}_{-3}}$, $Q^{(3)}_{-2}$, ... $Q^{(3)}_{3}$

$$Q^{(3)}_{-3} = \int_0^\infty dy\, T_{xxxx}$$



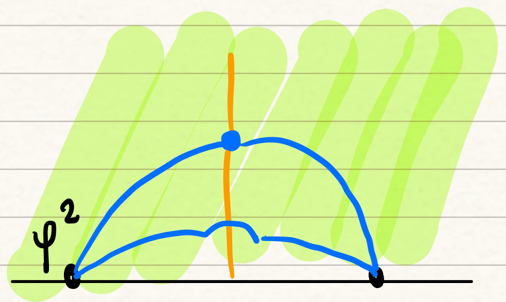
$$\boxed{[Q^{(3)}_{-3}, \varphi] = 16\, \partial_x^3 \varphi}$$

- Does $Q^{(3)}_{-3}$ just produce conformal descendants? No!

$$[Q^{(3)}_{-3}, \varphi^2] = \# \, \varphi \, \partial_x^3 \varphi = \alpha \, \partial_x^3 \varphi^2 + \beta \, \partial_x (\varphi \partial_x^2 \varphi + \ldots)$$

acts constituent-wise

New primary!



- All states @ fixed particle number live in a single higher spin multiplet
- Higher-spin symmetry implies integer spaced primary spectrum:

$$\Delta = n \Delta_\varphi + m$$

[Alkalaev, Bekaert, 2020]

easy to prove using constituent-wise action

- Constituent-wise action reminiscent of action on multi-particle states in flat space:

$$Q_J |p_1, \ldots p_n\rangle = \sum_{i=1}^{n} (p_i)^{J-1} |p_1, \ldots p_n\rangle$$

used to prove factorization

[Zamolodchikov^2, 1978]
[Dorey, 1996]

4. No-go theorems

Theorem: [Continuous interacting deformations of a free field cannot preserve any higher-spin charge]

- We focus on a free scaler, but free fermion works as well
- Similar Result for CFTs
- Theorem constrains both bulk and boundary interactions
  $\rightsquigarrow$ relevant for long-range models

[Benedetti, Lauria, Mazac, Van Vliet, 2024]

Proof.

- Step 1 : One h.s. charge $\xrightarrow{\text{isometries}}$ one full $SL(2,\mathbb{R})$ multiplet

- Step 2 : One multiplet $\xrightarrow{\text{commute}}$ higher-spin multiplets

  One multiplet $\xrightarrow{\text{commute or trace}}$ lower-spin multiplets

  This step uses the continuity assumption: $[Q, Q'] \neq 0$

- Step 3 : constraints of h.s. charges on correlators.

  Let's delve into this

• Step 3.1 : action of charges on boundary operator.

perturbing parameter (dimensionless)

$\lambda = \lambda_0 R$

$$\Theta_0(\lambda) = \varphi + \Theta(\lambda)$$

$$\left[Q^{(3)}_{-3}, \Theta_0\right] = \partial_x^3 \Theta_0 + \omega(\lambda)\, \partial_x^2 \Theta_1 + \alpha(\lambda)\, \partial_x \Theta_2 + \gamma(\lambda)\, \Theta_3$$

but $\Delta_n = \Delta_0 + n$ primaries do not exist as $\lambda \to 0$, @ generic Δ_φ

By continuity:

$$\boxed{\left[Q^{(J)}_{-J}, \Theta_0\right] = \partial_x^J \Theta_0} \qquad \lambda \in \text{h.s. symmetric range}$$

$$\Rightarrow \quad \left(\partial^J_{x_1} + \dots + \partial^J_{x_n}\right) \langle \Theta_0(x_1) \dots \Theta_0(x_n) \rangle = 0$$

- Step 3.2 : solving the constraint

[Maldacena, Zhiboedov, 2010]
[Zamolodchikov^2, 1978]

× $$\left(\sum_i \partial^J_{x_i}\right)\langle \mathcal{O}_0(x_1)\dots\mathcal{O}_0(x_n)\rangle = 0 \qquad J \text{ odd}$$

momentum space + famous argument for S-matrix factorization

$$\Rightarrow \quad \langle \mathcal{O}_0(p_1)\dots\mathcal{O}_0(p_{2n})\rangle = \delta(p_1+p_2)\,\delta(p_3+p_4)\dots\delta(p_{2n-1}+p_{2n})\,f(p_1,\dots p_{2n}) + \underset{\text{permutations}}{}$$

(2n+1 point-function $=0$)

- conformal invariance : $$\langle \mathcal{O}_0(p_1)\dots\mathcal{O}_0(p_{2n})\rangle = \sum_{\text{perm}} \frac{C_{(12)(34)\dots(2n-1\,2n)}}{x_{12}^{2\Delta_0}\,x_{34}^{2\Delta_0}\dots x_{2n-1,2n}^{2\Delta_0}}$$
- Permutation symmetry

$$\boxed{\langle \mathcal{O}_0(p_1)\dots\mathcal{O}_0(p_{2n})\rangle = \text{Generalized free boson}}$$

∎

$$\boxed{\langle \mathcal{O}_0(p_1) \cdots \mathcal{O}_0(p_{2n}) \rangle = \text{Generalized free boson}}$$

Comments:

- Generalized free fermion analogous up to statistics
- @ "special values" of Δ_φ result unchanged @ least at leading order in perturbation theory.
- Interesting leeway for a tower of fields w/ fine tuned masses: "$\Delta_{\varphi_i} = \Delta_{\varphi_0} + i$"

5. Summary

- Free fields and CFTs have higher spin charges which cannot be preserved under deformations

- AdS killing tensors are demanding: require full conservation

- Charge conservation can be broken at the boundary
  $\Rightarrow$ protected boundary operators (see paper)

- Higher spin charges imply sum rules for BOE data
  (see paper)

& Outlook

- Weakly broken charges & flat space limit
- Integrability in AdS via different mechanisms (e.g. non-local charges)
- Explicit examples : $\frac{1}{2}$ BPS Wilson loop in $\mathcal{N}=4$ SYM.

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[Thank you]