The Search for the String Theory Vacuum

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Introduction

String theory: all matter and forces are tiny strings

- no anomalies; well defined at high energies
- at low energies, GR + lots of interesting matter
- houses naturally several ideas: extra dimensions, susy, axions, inflation...

- too much ‘stuff’! extra dimensions, susy...
- too many models! give me a prediction

Today: focus on whether we can get a vacuum with positive, small cosmological constant [c.c.]

Broadly speaking:

EFT: 4d GR + matter

⇝

'optimists'

10d/11d GR + matter

⇝

'pessimists'

string theory

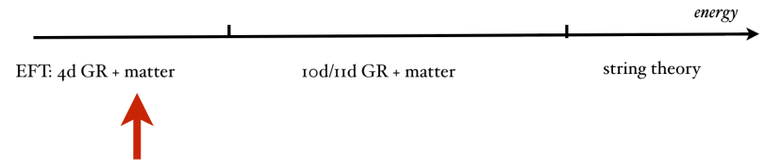
⇝

still working on it...

Plan

- Effective field theory approach
  - Higher-dimensional techniques
    - Towards a synthesis
      - [String field theory]

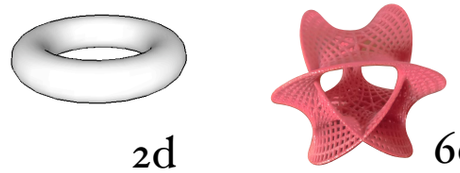
Effective field theory



- Let us first look at Minkowski (zero c.c.)

simplest possibility: matter fields= 0 ⇨ Ricci = 0 also in internal space

many examples: *Calabi–Yau* manifolds



2d 6d [a slice thereof]

- They typically can be deformed in many ways (*moduli*), at zero energy cost.

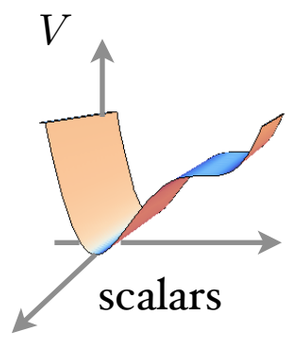
min.=2, max=502 [?]

⇩

'valleys' in the 4d potential.

⇩

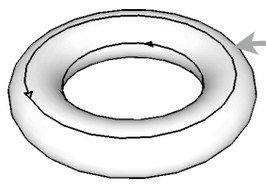
massless scalars



points along the valley= different CY metrics

- Hope: matter fields $\neq 0$ ⇨ fix moduli, introduce c.c.$\neq 0$?

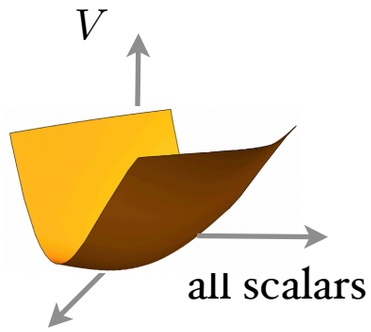
- Most matter fields are analogues of EM with many indices: $F_{\underbrace{m_1 \ldots m_k}_{\text{antisymmetric}}}$



they make changing shape cost energy

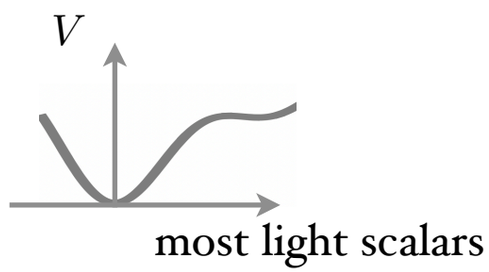
⇨ effective potential generated along former 'valley'

- a random choice of flux leads to a runaway,
  but there is a general way to avoid this.



EFT: ignore steep directions

(better: *integrate out*)



but some scalars remain massless at this stage.

- Fixing all scalars and changing the c.c. requires new ingredients.

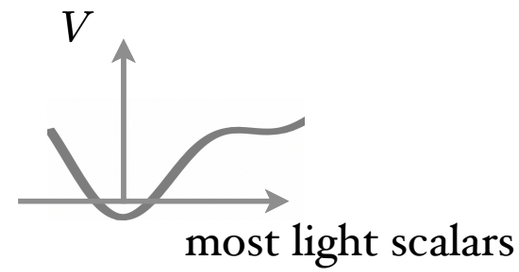
  - Quantum corrections (guaranteed to arise!)

    [Kachru, Kallosh, Linde, Trivedi '03]

  - (More) sources for the antisymmetric fields.
    Extended objects: *D-branes*, *O-planes*

    [DeWolfe, Giryavets, Kachru, Taylor '05]

    can easily make the c.c. small and negative.



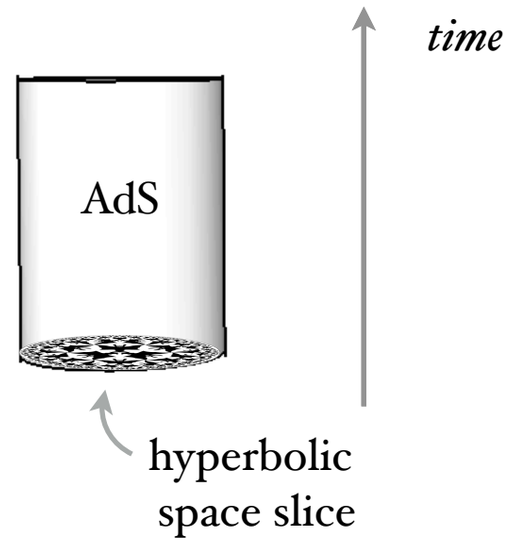
- A possible source of skepticism: $\mathrm{AdS}_4 \times M_6$ spacetime

  - small negative c.c.

    ⇨ large # fields

a CFT_3 should live on its boundary

[Maldacena '97]



Models with O-planes: $\#(\text{fields}) \sim N^{9/2}$

Models with instantons: $\#(\text{fields}) \sim e^N$

known CFT_3: N^2 or slower

- Positive c.c. also possible, but much harder. Classically:

$$V = ag^2 + bg^3 + cg^4$$

internal curvature ↑ | sources ↑ | flux ↑

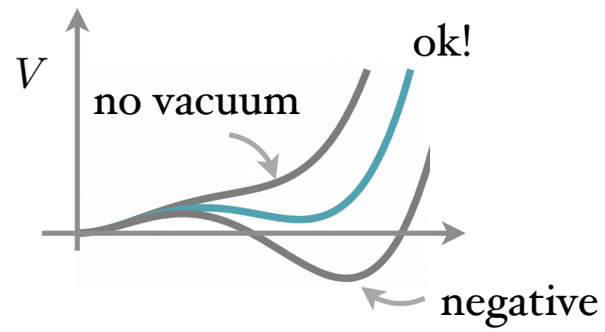
g = related to internal volume (and *dilaton* scalar)

a, b, c = functions of remaining scalars

[Silverstein '07]

for positive vacuum:

- $b < 0$
- $1 < \frac{4ac}{b^2} < \frac{9}{8}$



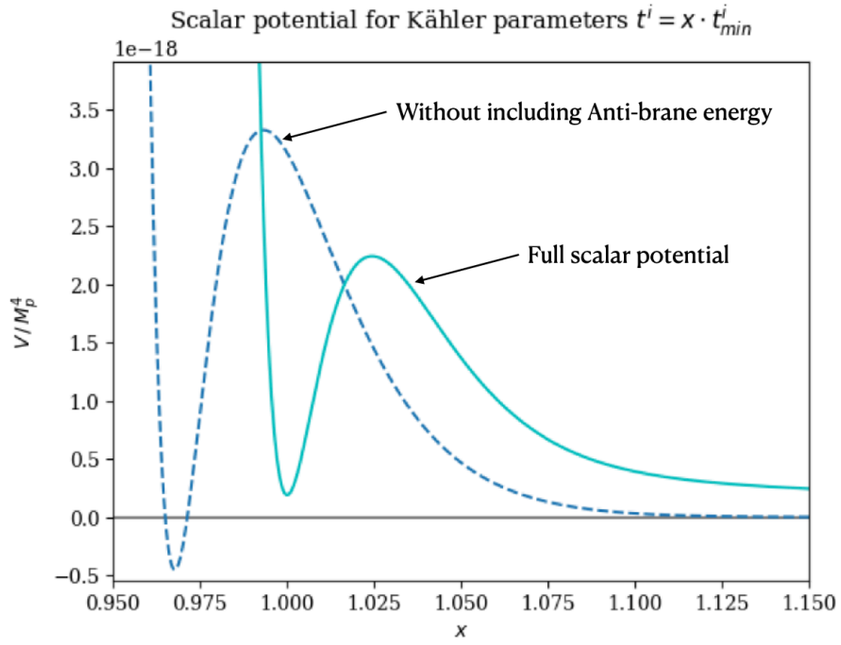
- KKLT model (and its variants): combine quantum effects with susy-breaking effect
  - original version: explicit anti-brane term

[Kachru, Kallosh, Linde, Trivedi '03
Balasubramanian, Berglund, Conlon, Quevedo '05;
Saltman, Silverstein '04; Burgess, Kallosh, Quevedo '03...]

  - recent search among 'computable' CYs:

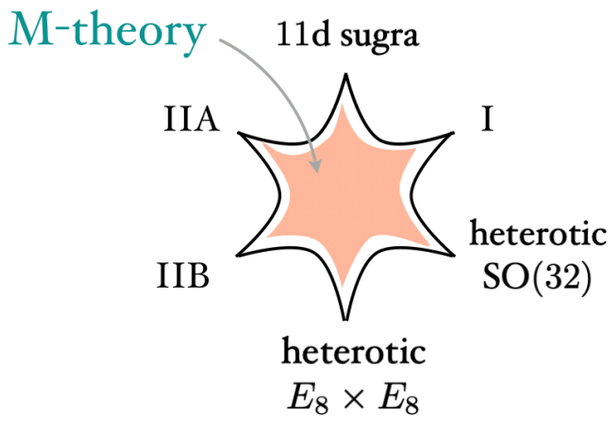
202073 CY EFTs ⇝ 30 vacua

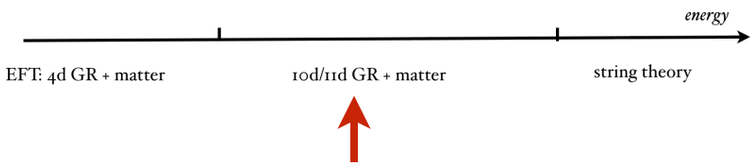
[McAllister, Moritz, Nally, Schachner '24]



10d methods

Various versions of string theory are perturbatively accessible





- Type II bosons:

$g_{\mu\nu}$ metric

ϕ dilaton

analogues of EM

$$\begin{cases} H_3 = \mathrm{d}B_2 & (H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}) \\ F_k = \mathrm{d}C_{k-1} - H \wedge C_{k-3} \end{cases}$$

antisymmetrized derivative

antisymmetrized product

RR fields: all k even (IIA) or odd (IIB)

- Bosonic action: $S = \frac{1}{2\kappa^2}\int_{M_{10}} \mathrm{d}^{10}x\sqrt{-g}\left(\mathrm{e}^{-2\phi}\left(R + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2\right) - \frac{1}{4}\sum_k |F_k|^2\right)$

- dS is hard because of extra dimensions:

vacuum metric: $\mathrm{d}s_{10}^2 = \underbrace{\mathrm{e}^{2A}}_{\text{warping}}\mathrm{d}s_4^2 + \mathrm{d}s_6^2$

[Gibbons '84]

[de Wit, Smit, Hari Dass '87]

$$R_{MN} = 8\pi G(T_{MN} - \tfrac{1}{2+d} g_{MN} T^P{}_P)$$

in particular $\mathrm{e}^{-4A} R_{00} = 8\pi G \mathrm{e}^{-4A}(T_{00} - \tfrac{1}{2+d} g_{00} T^P{}_P)$

$\mathrm{e}^{-4A} R_{00} = \mathrm{e}^{-4A} R^4_{00} - \nabla^2 \mathrm{e}^{-4A} g_{00}$

$R^4_{00} = \Lambda g^4_{00}$

total derivative

$\int_{\text{internal}} \searrow$

$\Lambda \leqslant 0$

$T_{00} - \tfrac{1}{2+d} g_{00} T^P{}_P \geqslant 0$

'strong energy condition' (SEC)

- used in GR singularity theorems

[Hawking, Penrose '70...]

- most matter fields satisfy it
- F_0 doesn't; another argument for this case

[Maldacena, Nuñez '00]

- In string theory:
  - O-planes: negative tension, violate SEC
  - Instantons: beyond classical EoM

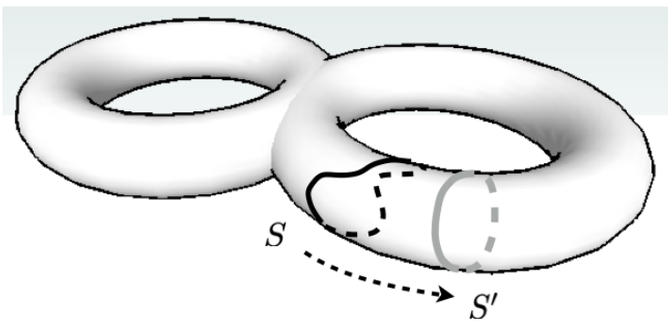
Several techniques to look for solutions: G-structures, generalized geometry...

[Hull '86; Strominger '86; Gauntlett, Pakis '02; Gauntlett, Martelli, Pakis, Waldram '02; Kaste, Minasian, Petrini, AT '02; Hitchin '02...]

- A common thread: describe metric in terms of antisymmetric 'fields' called *calibrations*

their flux gives the smallest attainable volume of a subspace.

$$\int_S \Phi = \mathrm{Vol}(S')$$



These objects transform well under string theory's additional symmetries [T-duality...]

- For supersymmetric solutions, schematically

[Graña, Minasian, Petrini, AT '05]

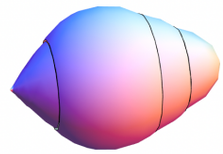
±: even/odd

$$(\mathrm{d} - H\wedge)\mathrm{e}^{-\phi}\Phi_{\mp} = *F_{\mathrm{int}}$$
$$(\mathrm{d} - H\wedge)\mathrm{e}^{-\phi}\Phi_{\pm} = 0$$

- Recently, more complicated versions for susy-breaking as well

[Lüst, Marchesano, Martucci, Tsimpis '08; Legramandi, AT '19; Menet '23]

- Very successful for AdS...
  - Lots of explicit analytic solutions
  - Often with several sources together
  - Full classifications for AdS7, AdS6...



[Apruzzi, Fazzi, Rosa, AT '13,
D'Hoker, Gutperle, Karch, Uhlemann '16...]

    ⇨ progress in CFT6, lower-dim. daughter models

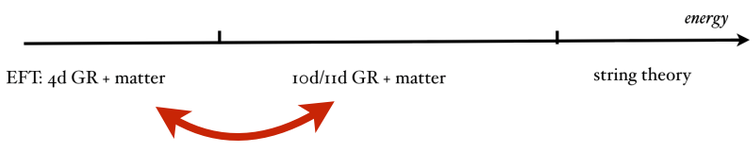
  - Progress towards existence theorems [i.e. no need to solve all PDEs]

[Ashmore, Strickland-Constable,
Tennyson, Waldram '19]

- ... but most explicit solutions so far are unrealistic:
  - Unbroken supersymmetry
  - c.c. negative and large! $(-\Lambda)^{-1/2} \sim \text{diam}(M_6)$

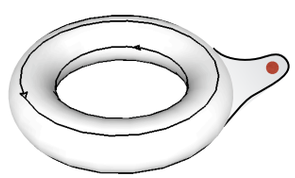
- Can we do better?

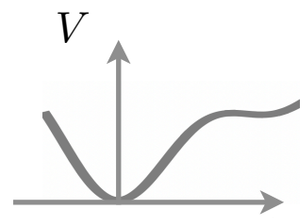
10d vs. 4d



- special case: point internal sources
  (*O3*-planes, *D3*-branes)

⇨ internal space is *conformal* to Calabi–Yau





most light scalars

$\exists$ complex coordinates: $\Phi_+ = \mathrm{e}^{\mathrm{i}J}$, $\Phi_- = \Omega$

locally $J_{i\bar{\jmath}} = \mathrm{i}g_{i\bar{\jmath}}$, $\Omega_{ijk} = \epsilon_{ijk}$ ← holomorphic indices

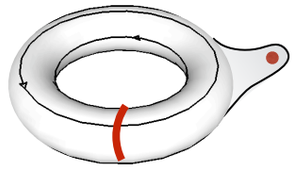
[Becker, Becker '96, Dasgupta, Rajesh, Sethi '99,
Graña, Polchinski '00, Giddings, Kachru, Polchinski '01]

$$*F_3 = \mathrm{e}^{-\phi} H, *F_5 = \mathrm{e}^{-\phi} \mathrm{d}(4A - \phi)$$

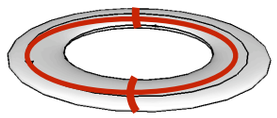
linear equations, easy to solve

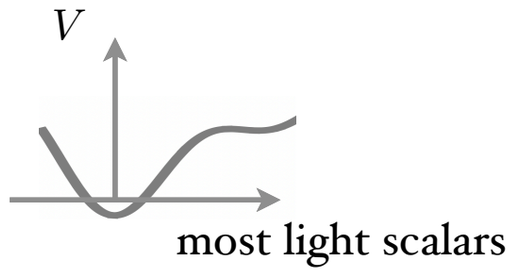
- also possible to include *D7*-branes:

  now conformal *Kähler*



- 3d internal sources (*O6-planes*)





⇨ nonlinear equations,
Calabi–Yau is more distorted

Approximate solutions: expansion in small c.c.

[Saracco, AT '12; Junghans '20;
Marchesano, Palti, Quirant, AT '20]

schematically $\Phi_+ \sim \mathrm{e}^{-\mathrm{i}J}, \Phi_- \sim \Omega + \sqrt{-\Lambda}(\mathrm{d}f + K_{2,1})$

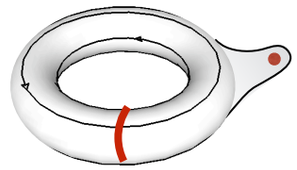
$$\Delta K = \mathrm{Re}\Omega + \delta_{\mathrm{O6}}, \Delta f = \mathrm{Re}\Omega \cdot K$$

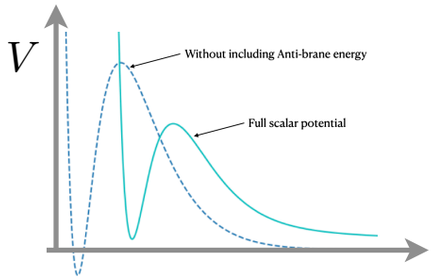
- Higher orders seem to still work [Emelin '24]

- Several similar solutions now exist; with fewer fluxes, susy breaking

[Cribiori, Junghans, Van Hemelryck,
Van Riet, Wrase '21;
Marchesano, Quirant, Zatti '22]

- How about positive c.c.?





one version of KKLT model:

non-perturbative effect induced by D7 gaugino condensation

$$(\mathrm{d} - H\wedge)\mathrm{e}^{-\phi}\Phi_{\mp} = *F_{\text{int}} + \delta$$

δ ← localized term in the susy equations

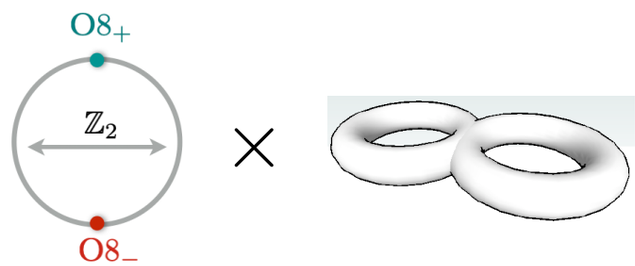
c.c. of solution obtained this way agrees with 4d EFT

[Koerber, Martucci '07, Dymarsky, Martucci '10; Bena, Graña, Kovensky, Retolaza '19; Kachru, Kim, McAllister, Zimet '19]

- but controversy remains: instabilities, 'throat fitting', 'tadpole problem'...

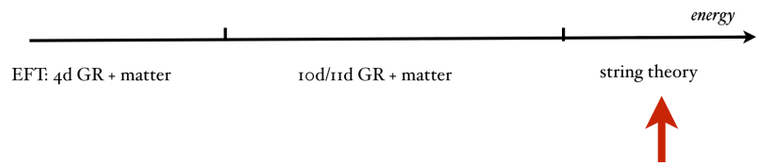
[Carta, Moritz, Westphal '19; Bena, Dudas, Graña, Lüst '18; Bena, Blåbäck, Graña, Lüst '21, Dall'Agata, Emelin, Farakos, Morittu '22...]

- many other proposals! 11d sugra, O-planes only, T-folds, Casimir energy...



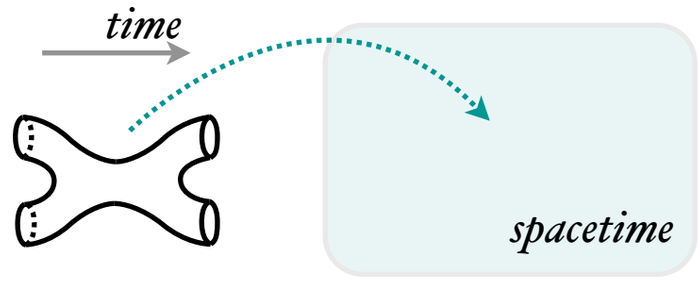
[Acharya, Bobkov, Kane, Kumar, Shao '07; Danielsson, Haque, Shiu, Van Riet '09; Cordova, De Luca, AT '18; de Carlos, Guarino, Moreno '09; De Luca, Silverstein, Torroba '21]

The string frontier



- Textbook version of string theory:

Field theory on the string
whose fields describe
its embedding in spacetime



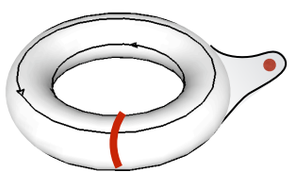
First-quantized; struggles to accommodate the RR fields although [Berkovits '99...]
[which are necessary for dS] [Kutasov, Maxfield, Melnikov,, Sethi '15]

- String Field Theory: second quantized. Its fields directly create strings

This program is finally coming to fruition [Pius, Rudra, Sen '14; Sen '14; de Lacroix, Erbin, Kashyap, Sen, Verma '17...]

- Recently, tentative description of the Conformal CY backgrounds in this language.

[Cho, Kim '24]



Conclusions

- A lot of progress on some fronts:
  - many classifications, some very explicit
  - solutions with localised sources are now quite common
  - some promising models
- Confusion on many others!
  - Quantum corrections are difficult to control
  - agreement on de Sitter remains elusive