# NLO corrections and Decoherence effects

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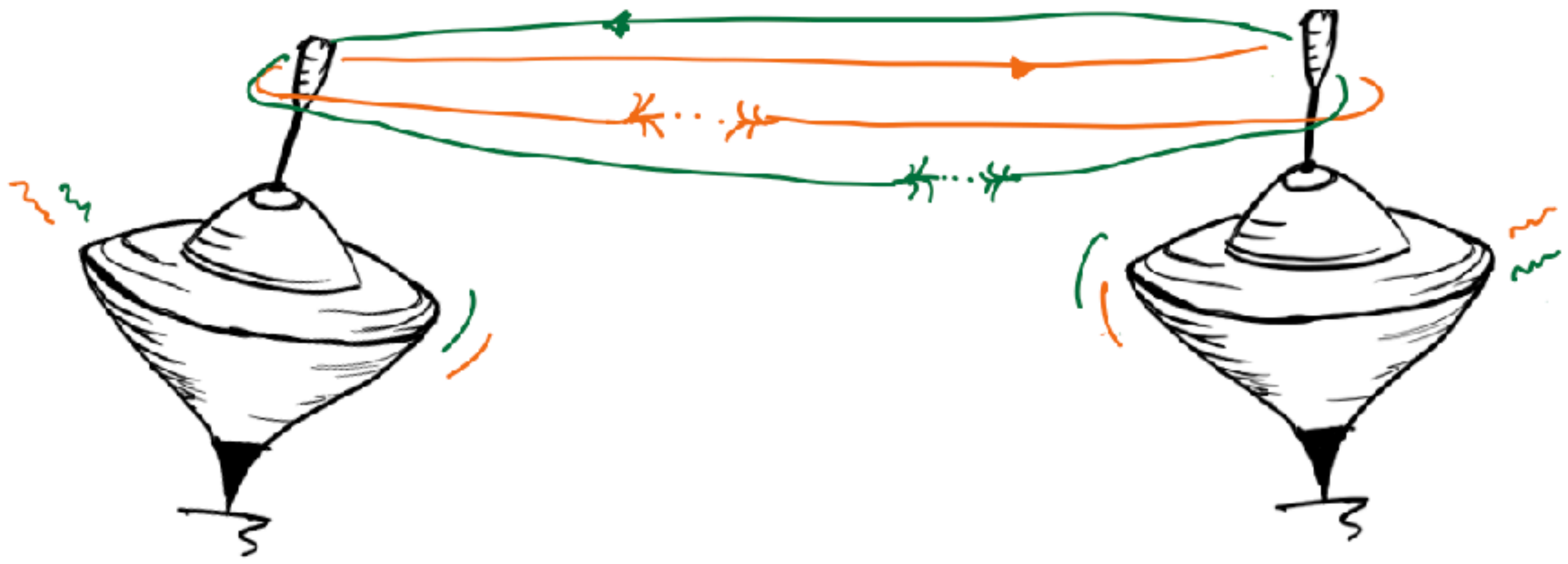


with Fabio Maltoni, Alan Barr and Leonardo Satrioni

THE UNIVERSITY of EDINBURGH

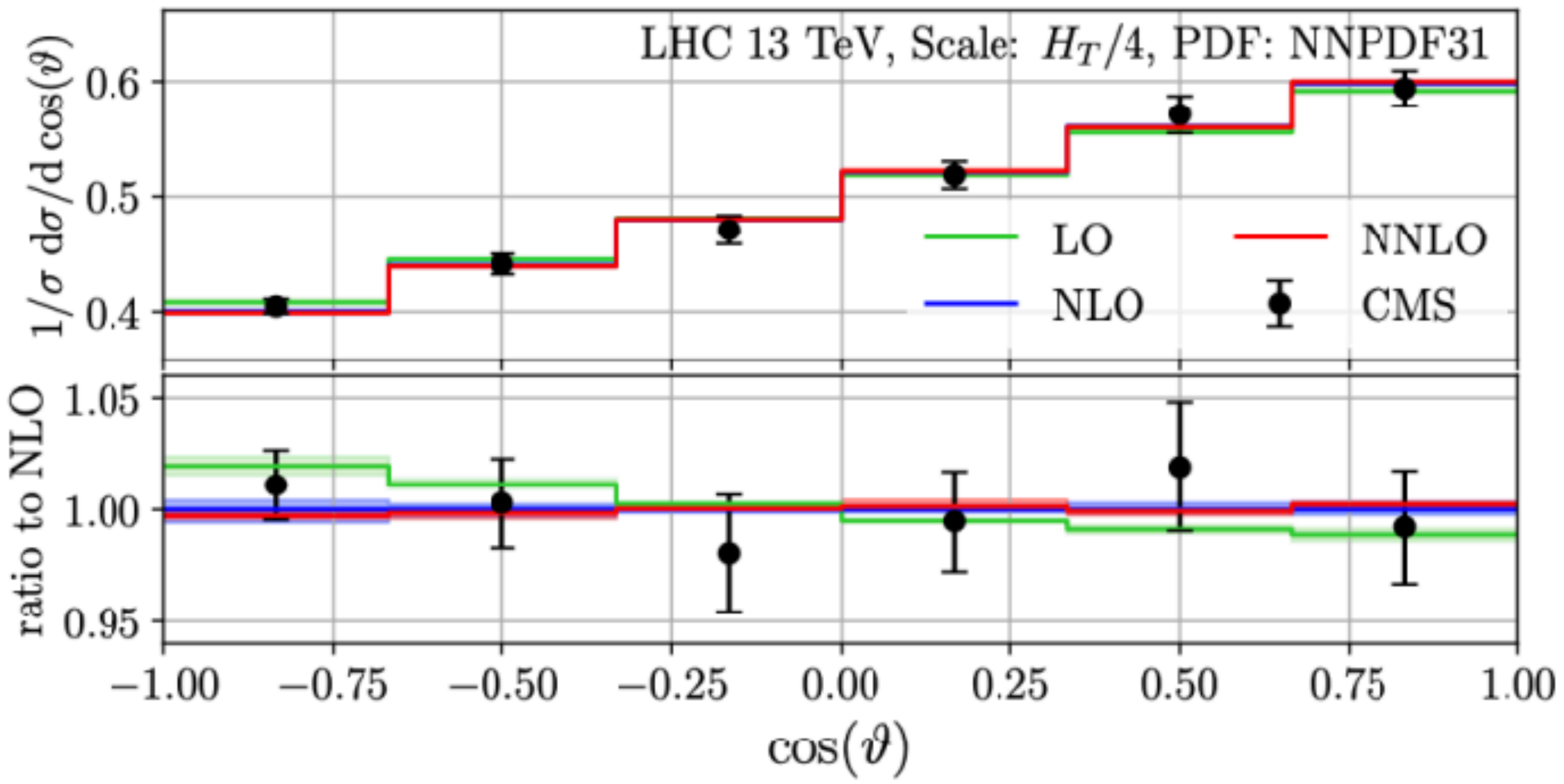
**GGI - Florence - 2025**

# Motivation

We assume that NLO corrections in the ttbar entanglement are small

Knowledge from spin-correlations studies

[Czakon, Mitov, Poncelet '08]




[Eleni's Talk at GGI '23]

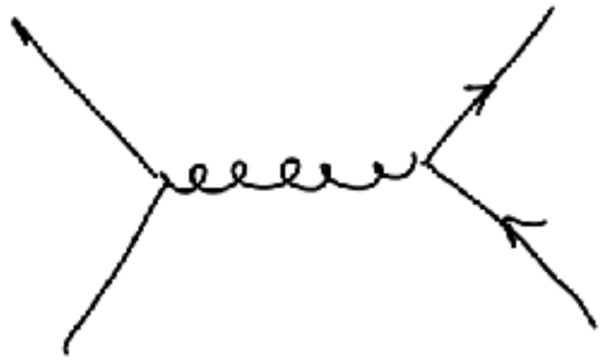
If we plan to measure spin entanglement at LHC with more and more precision [Talks by Giovanni and Priyanka]

Revisit this assumption with a more quantum info perspective

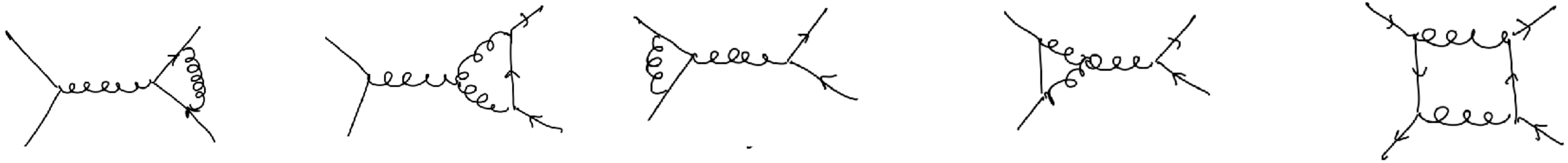
# NLO corrections

Formally, for ttbar production at NLO (in QCD) we need…

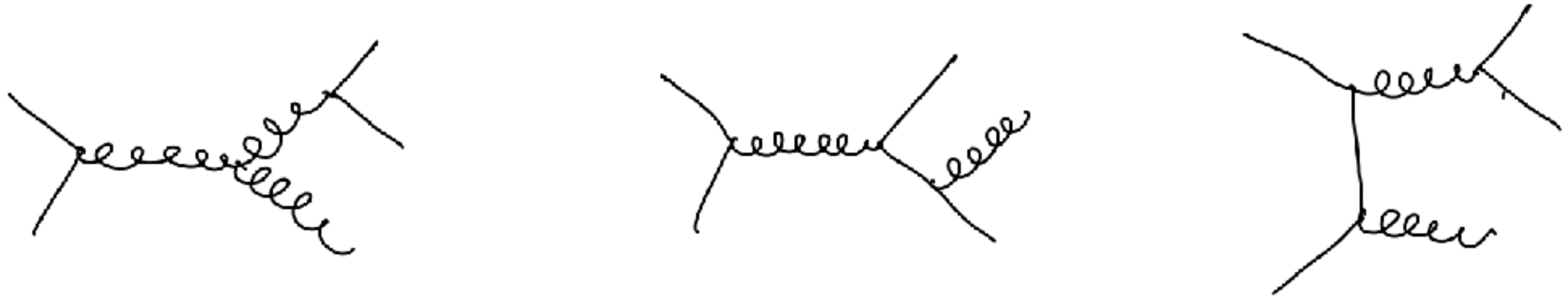
tree



virtual



real



Extra radiation?
Larger Hilbert space.

If the radiation is unresolvable, can we see this as a quantum map?

Decoherence!

# Some nice decoherence effects

Soft radiation decrease *momentum* entanglement

[Carney, Chaurette, Neuenfeld, Semenoff, ’17]
[Neuenfeld ’18]

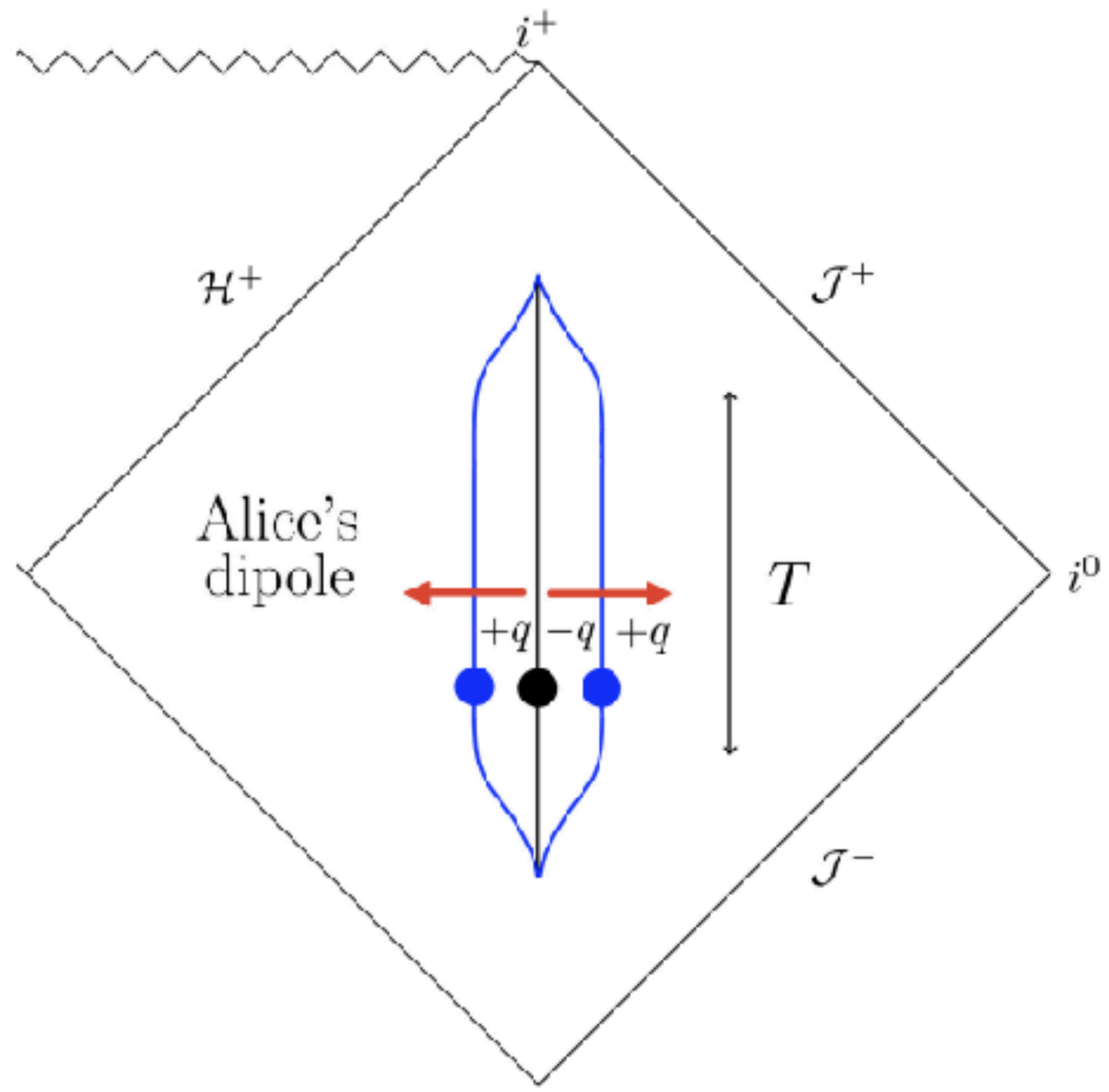
In K0K0 systems [Berltmann ’05]

and B mesons [Talks by Sven Vahsen in Pittsburg And Hans-Guenther Moser at GGI 23]

[See also poster by Mahood and Stoetzer]

Inflation [Burgess, Colas, Holman, Kaplanek and Rennin ’24]

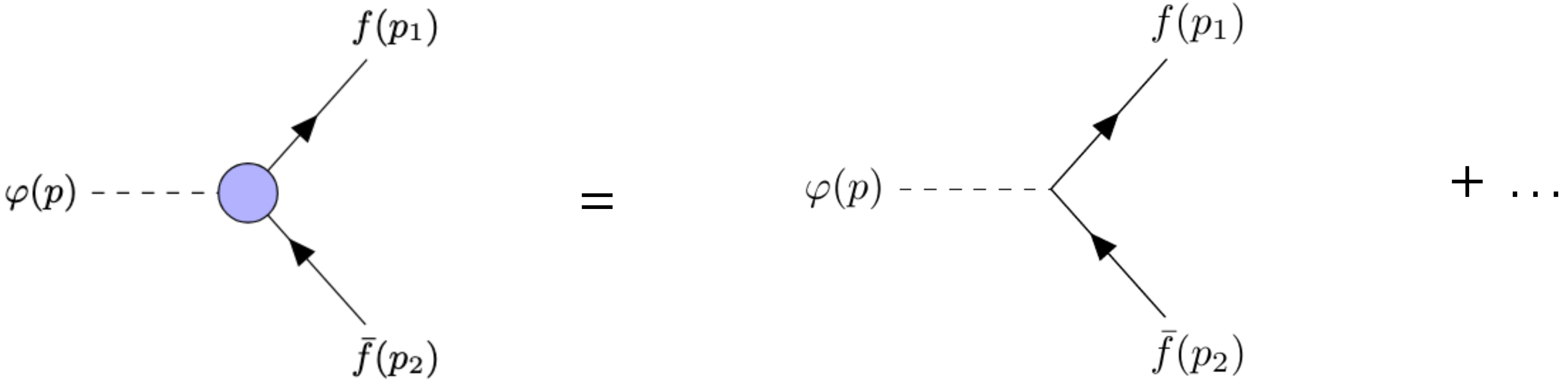
BH horizons decohere superpositions




[Danielson, Satischandran, Wald ’23]
[Biggs, Maldacena ’24]

# Let’s do an easier example…

Fermion pair from a scalar decay



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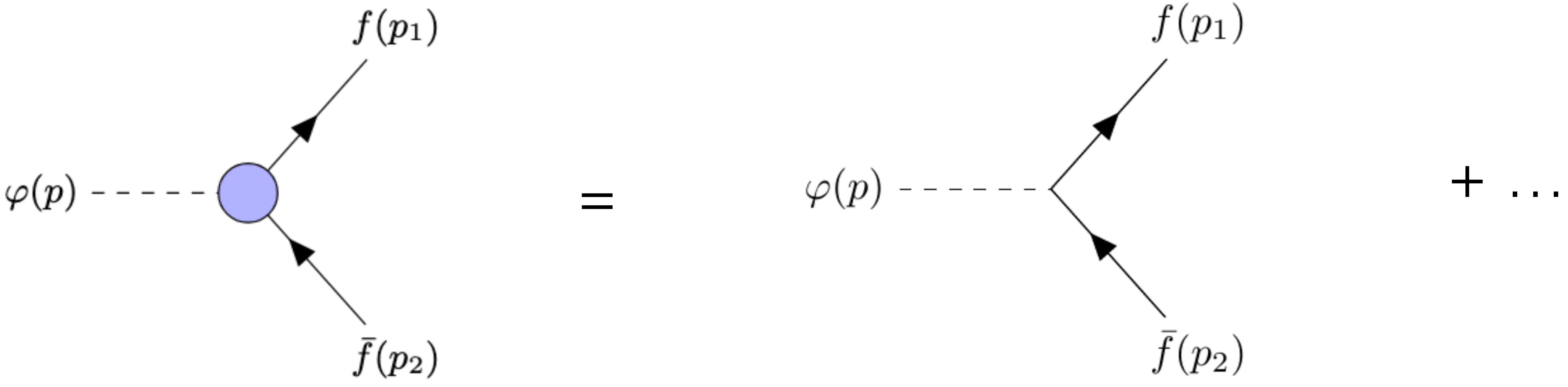



At tree-level: $R_{\mathrm{LO}} = \frac{4N_C y_f^2 m_f^2 \beta^2}{1-\beta^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\rho_{\mathrm{LO}} = \frac{1}{\mathrm{tr}[R_{\mathrm{LO}}]} R_{\mathrm{LO}} = |\Psi^+\rangle\langle\Psi^+|$

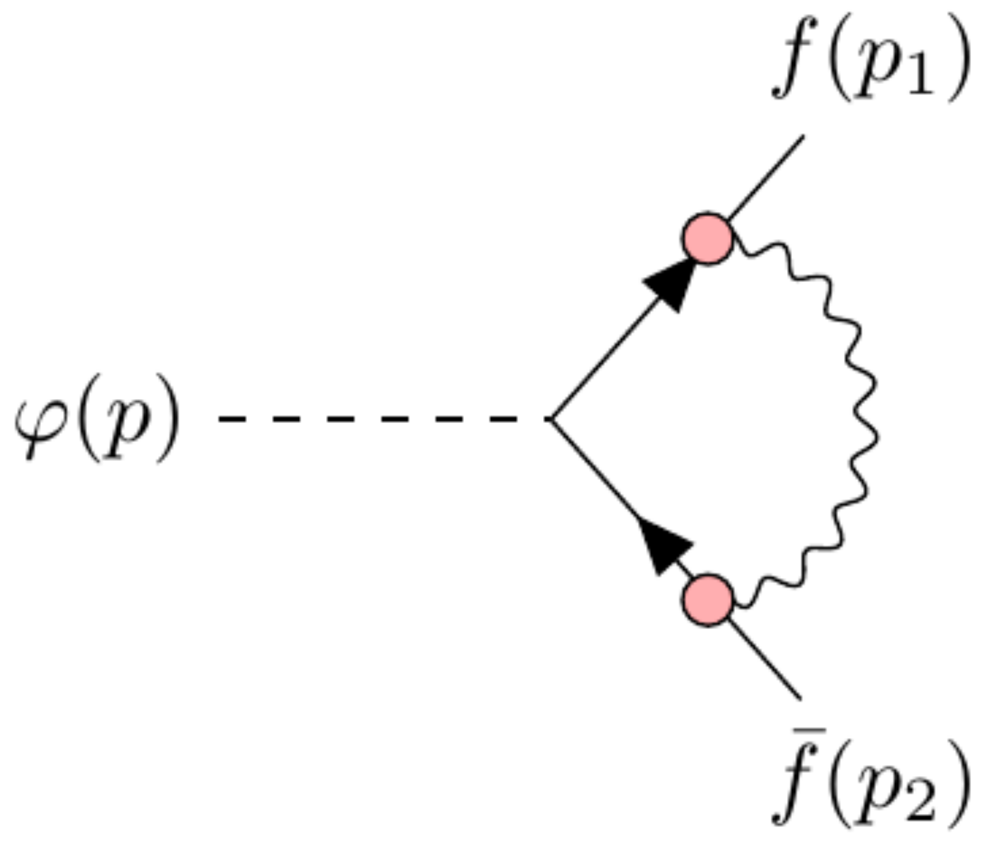
Maximally entangled: controlled place to study entanglement decrease

# NLO corrections

General interaction: Scalar, pseudo scalar, vector and axial

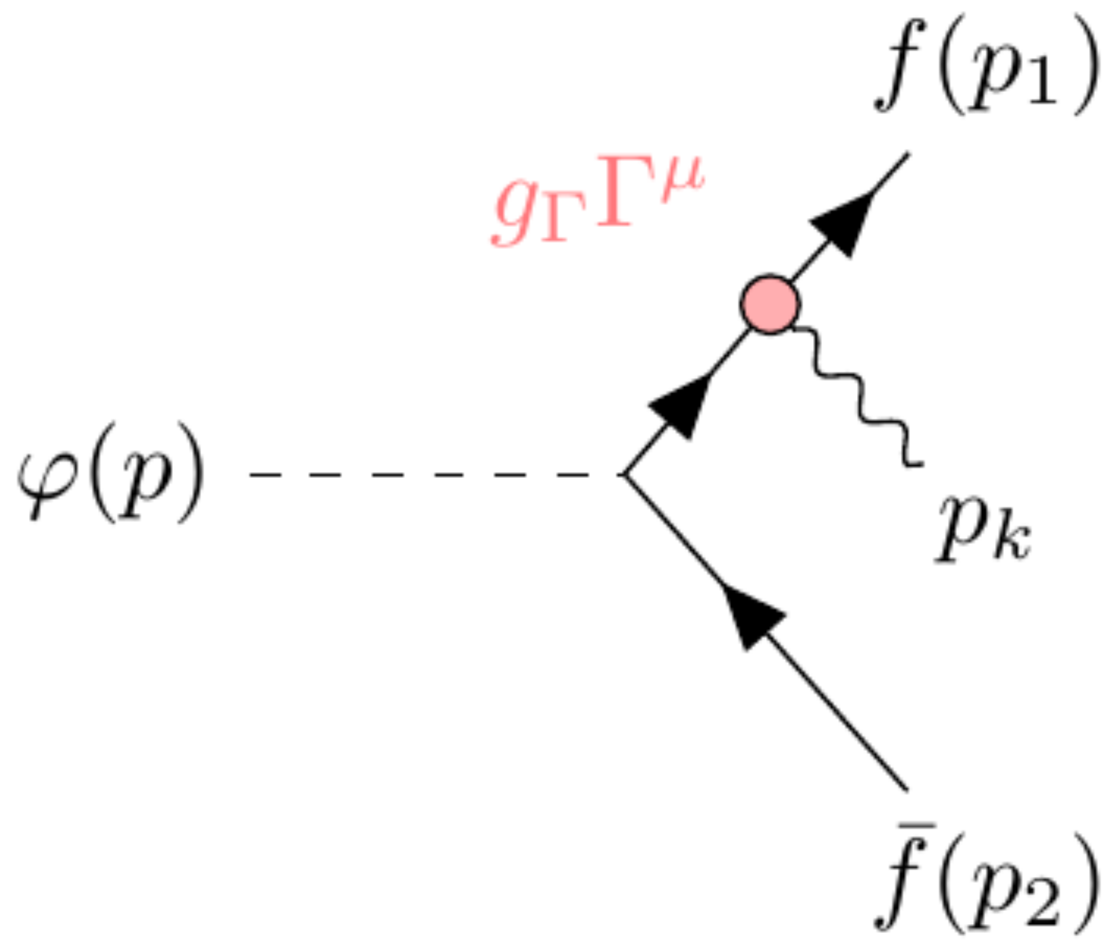
$$g_\Gamma \Gamma^\mu = \{g_S 1, g_P \gamma^5, g_V \gamma^\mu, g_A \gamma^\mu \gamma^5\}$$

Virtual correction: one-loop




+

Real emission



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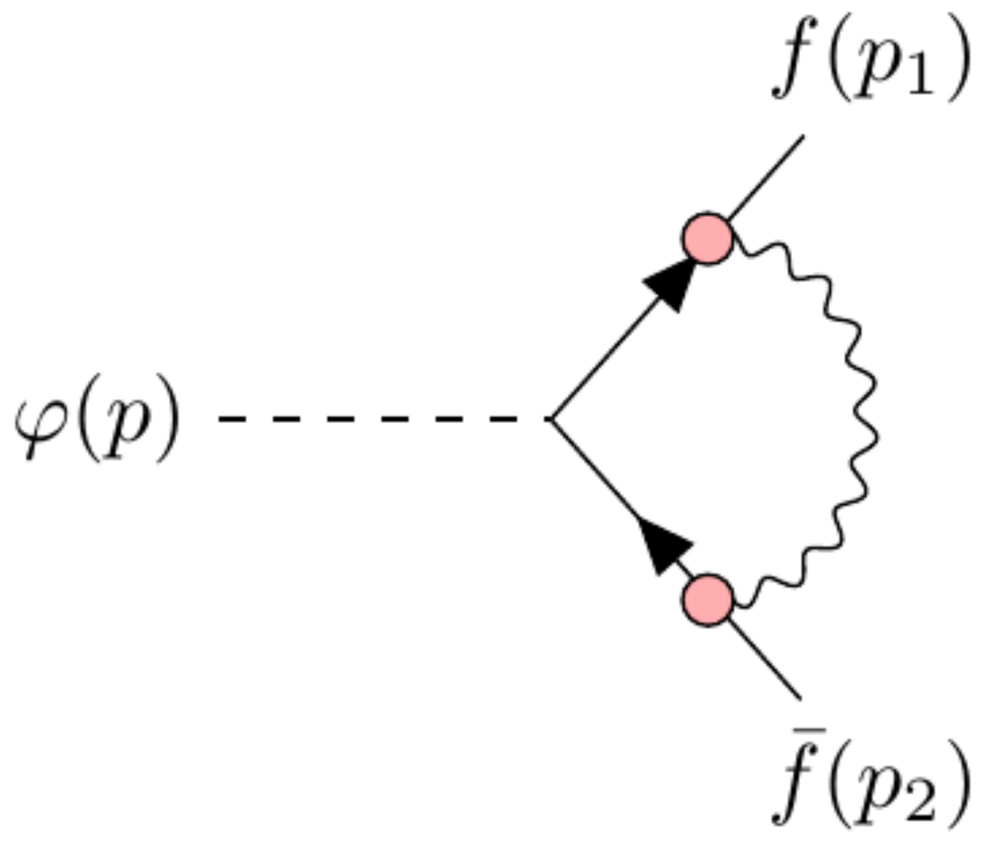
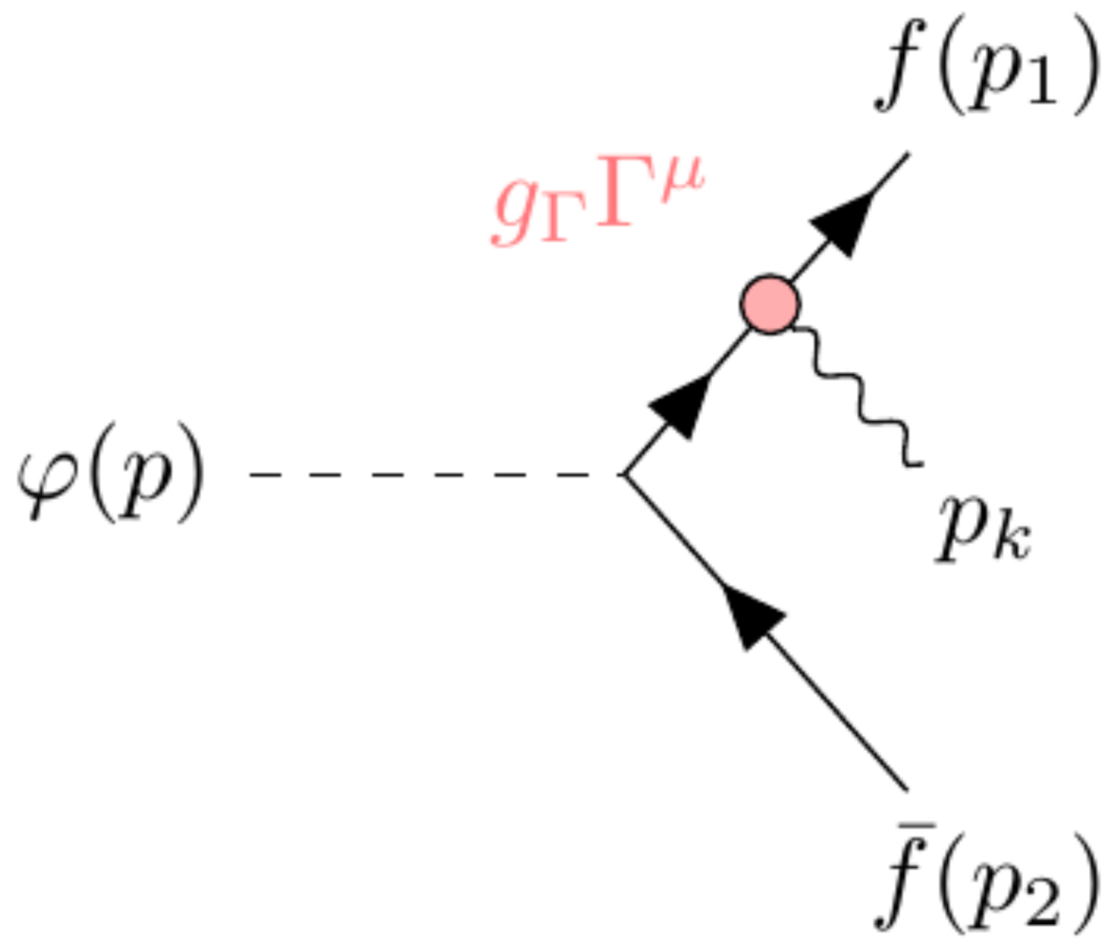



+

Real emission




Same Hilbert space

Trace over the extra d.o.f (environment)

↓

Quantum Map.
Open Quantum system

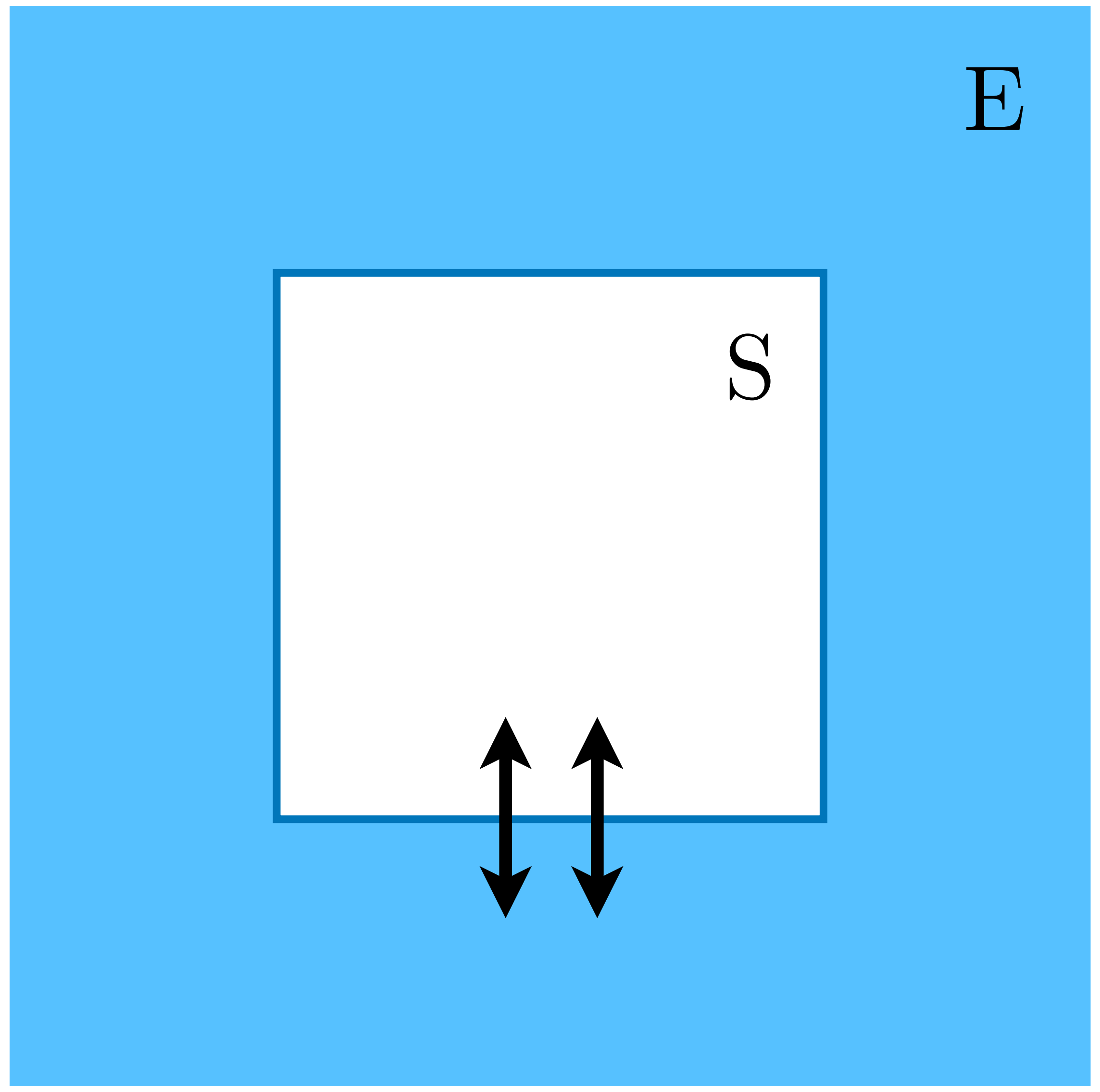
# Quantum Maps

The evolution of a system+environment is unitary

$$\rho'(t) = U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)$$

Tracing over the environment subsystem

$$\rho_{\mathrm{S}}(t) = \mathrm{tr}_E\left[U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)\right]$$



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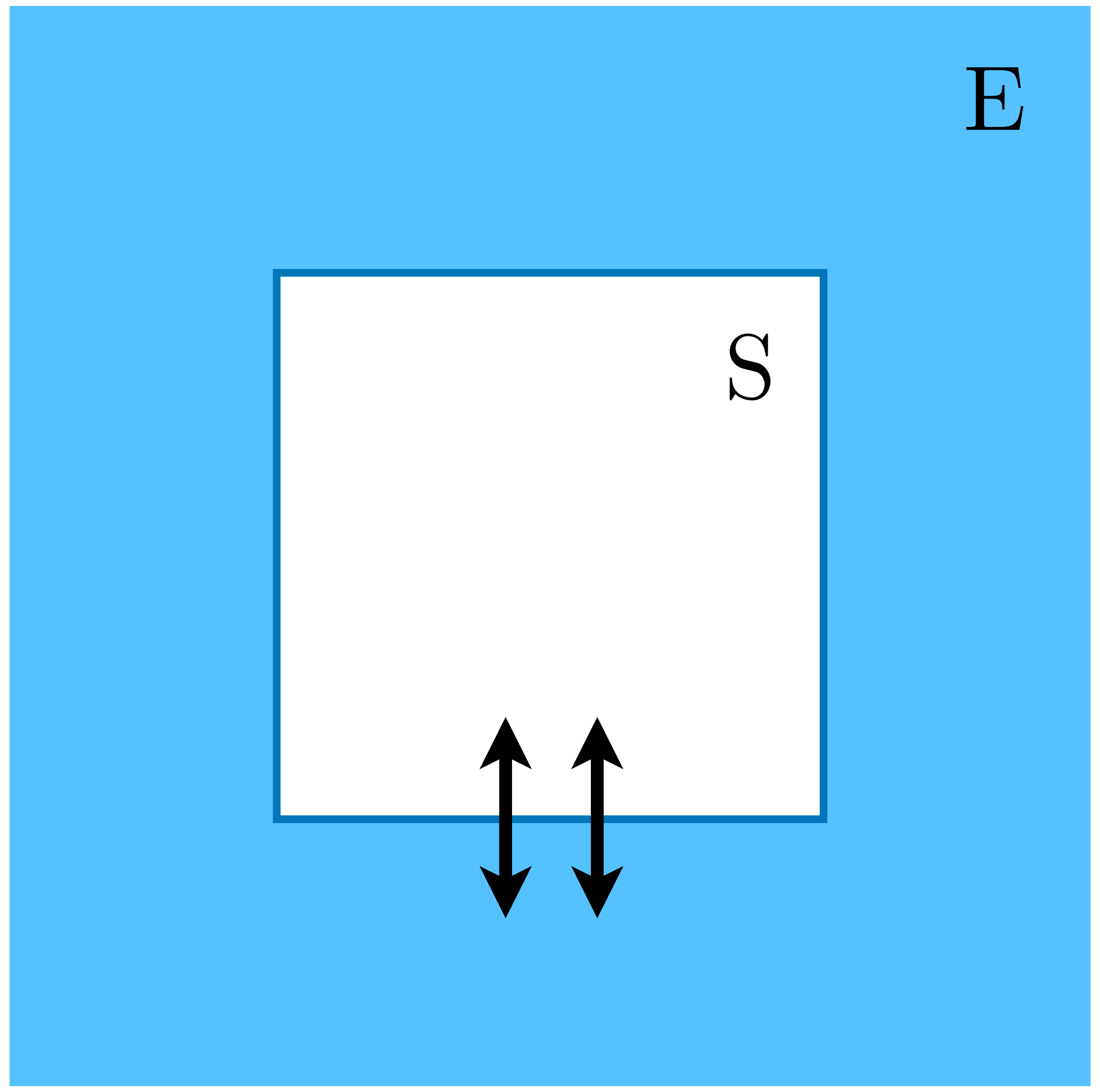


which we can write as a operator-sum representation (Kraus operators)

$$\rho_{\mathrm{S}}(t) = \sum_j K_j \rho_{\mathrm{S}}(0) K_j^\dagger \qquad \text{s.t} \qquad \sum_j K_j K_j^\dagger = 1$$

[See also poster by Grzelka and Altomonte]

For bipartite qubits: $K_j$ Tensor product of Pauli

# Environment as unresolved radiation

Tree-level $\mathcal{A}_{\alpha\beta}(\varphi \to t\bar{t})[\mathcal{A}_{\alpha'\beta'}(\varphi \to t\bar{t})]^\dagger \sim \mathcal{O}(y_t^2)$

One-loop $\mathcal{A}^{(1)}_{\alpha\beta}(\varphi \to t\bar{t}) \left[\mathcal{A}^{(0)}_{\alpha'\beta'}(\varphi \to t\bar{t})\right]^\dagger + \text{h.c} \quad \sim \mathcal{O}(y_t^2\, g^2)$

Real emission contribution to the R-matrix

$$\mathcal{A}^h_{\alpha\beta}(\varphi \to t\bar{t} + k)[\mathcal{A}^h_{\alpha'\beta'}(\varphi \to t\bar{t} + k)]^\dagger \sim \mathcal{O}(y_t^2\, g^2)$$

We trace out the **unresolved** interaction: soft or collinear

$$\text{tr}_{\mathcal{H}_k}[\cdot] = \int \text{d}\Phi(k) \sum_{\sigma=\pm} \langle k,\sigma| \cdot |k,\sigma\rangle$$

If it's resolved: three-body decay [See Morales and Horodecki Talks]

# NLO reduced density matrix

$$\rho^{\text{red}}_{\text{LO+NLO}} = \sum_j K_j \rho_{\text{LO}} K_j^\dagger$$

$$= \mathsf{p}_{\text{LO}}\, \mathbb{1} \rho_{\text{LO}} \mathbb{1} + \bar{\mathcal{E}}_{\text{V}}[\rho_{\text{LO}}] + \bar{\mathcal{E}}_{\text{R}}[\rho_{\text{LO}}]$$

LO contribution

"Map" of virtual emission

UV and IR divergent

"Map" of real emission

IR divergent

# Virtual radiation map

Scalar current at tree-level $\bar{u}(p_1, h_1)v(p_2, h_2)$

Virtual $\bar{u}(p_1, h_1)\mathbb{T}_{\text{virt.}}v(p_2, h_2) = \tilde{T}_{\text{virt.}}\bar{u}(p_1, h_1)v(p_2, h_2)$

One-loop
"tensor" integral (w/.gamma's)

Scalar integral (w/o gamma 's)
(Passarino Veltman B's and C's)

# Virtual radiation map

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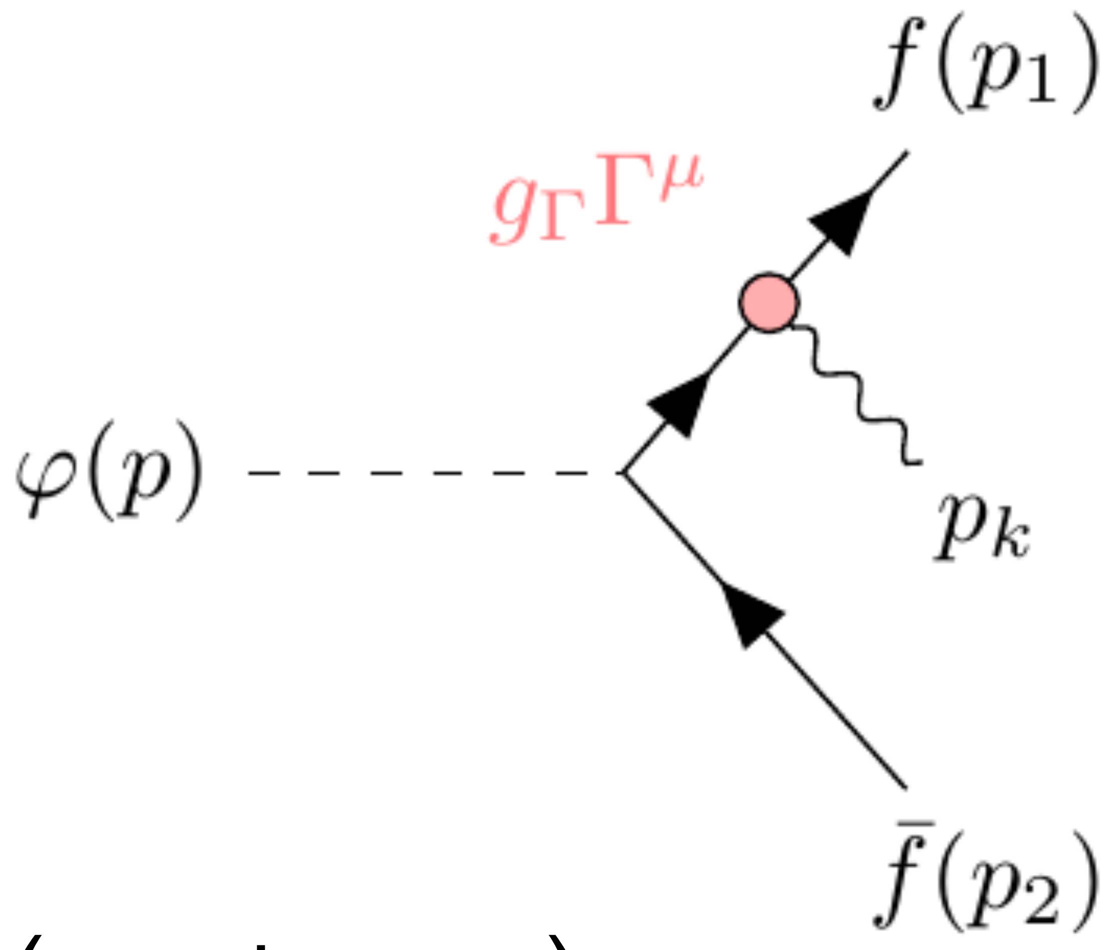
“tensor” integral (w/.gamma’s)

Scalar integral (w/o.gamma ’s)
(Passarino Veltman B’s and C’s)

The map is just an identity $\bar{\mathcal{E}}_{\text{V}}[\rho_{\text{LO}}] = \mathsf{p}_{\text{V}}\mathbb{1}\rho_{\text{LO}}\mathbb{1}$

This is special for scalar decay.
It would generalise for a vector current

# Real radiation map




Real emissions are different

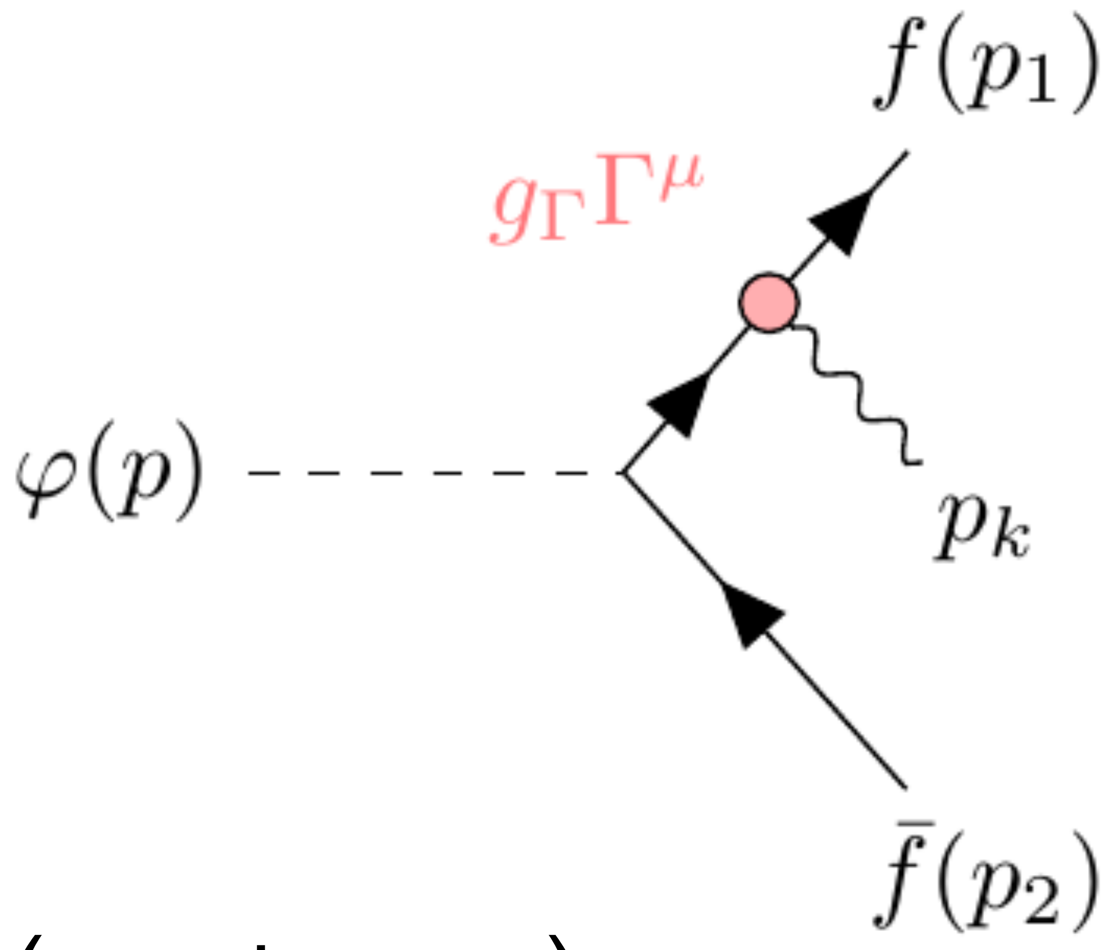
→ change the LO spin-structure

Let's split into a soft and hard emission (w.r.t. $\omega_0$ )

$$\bar{\mathcal{E}}_{\rm R}[\rho_{\rm LO}] = \bar{\mathcal{E}}_{\rm R}^{\rm soft}[\rho_{\rm LO}] + \bar{\mathcal{E}}_{\rm R}^{\rm hard}[\rho_{\rm LO}]$$

Unresolved radiation

# Real radiation map

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Unresolved radiation

In principle, both are written as operator-sum representation (Kraus)

$$\bar{\mathcal{E}}_{\rm R}[\rho_{\rm LO}] = \sum_j K_j \rho_{\rm LO} K_j^\dagger$$

↓ Built of Pauli matrices

# Soft part

We can use the soft theorem
$$\mathcal{M}_{n+1} = \sum_{i=1}^{n} \left[ \frac{p_i \cdot \varepsilon_h(k)}{p_i \cdot k} + \cdots \right] \mathcal{M}_n$$

Scalar function

Next-to-leading soft (change structures)

Leading-soft map
$$\bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{soft}}[\rho_{\mathrm{LO}}] = \underbrace{\mathsf{p}_{\mathrm{R}}^{\mathrm{soft}} \mathbb{1} \rho_{\mathrm{LO}} \mathbb{1}}_{\mathrm{scalar,vector}} + \overbrace{\mathsf{q}_{5}^{\mathrm{soft}} \sum_{j \neq \mathrm{id}} K_j \rho_{\mathrm{LO}} K_j^{\dagger}}^{\mathrm{pseudoscalar,axial}}$$

p’s cancel the IR divergence of virtual: KLN theorem

# Hard (collinear) emission part

- Now, this has a non-trivial Kraus operator part

$$\bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{hard}}[\rho_{\mathrm{LO}}] = \mathsf{p}_{\mathrm{R}}^{\mathrm{hard}} \mathbb{1} \rho_{\mathrm{LO}} \mathbb{1} + \mathsf{q}^{\mathrm{hard}} \sum_{j \neq \mathrm{id}} K_j \rho_{\mathrm{LO}} K_j^{\dagger}$$

without the identity

non-zero q = decoherence

Change in spin-structure: dipole-like interaction (IR finite)

Taking the collinear limit for the emission …

# Full NLO map

$$\mathcal{E}_{\text{full}}[\rho_{\text{LO}}] = \mathsf{p}_{\text{id}}\, \mathbb{1}\rho_{\text{LO}}\mathbb{1} + \mathsf{q} \sum_{j\neq\text{id}} K_j \rho_{\text{LO}} K_j^\dagger$$

$$\mathsf{p}_{\text{id}} = \left(\mathsf{p}_{\text{LO}} + \mathsf{p}_{\text{V}} + \mathsf{p}_{\text{R}}^{\text{soft}} + \mathsf{p}_{\text{R}}^{\text{hard}}\right)$$

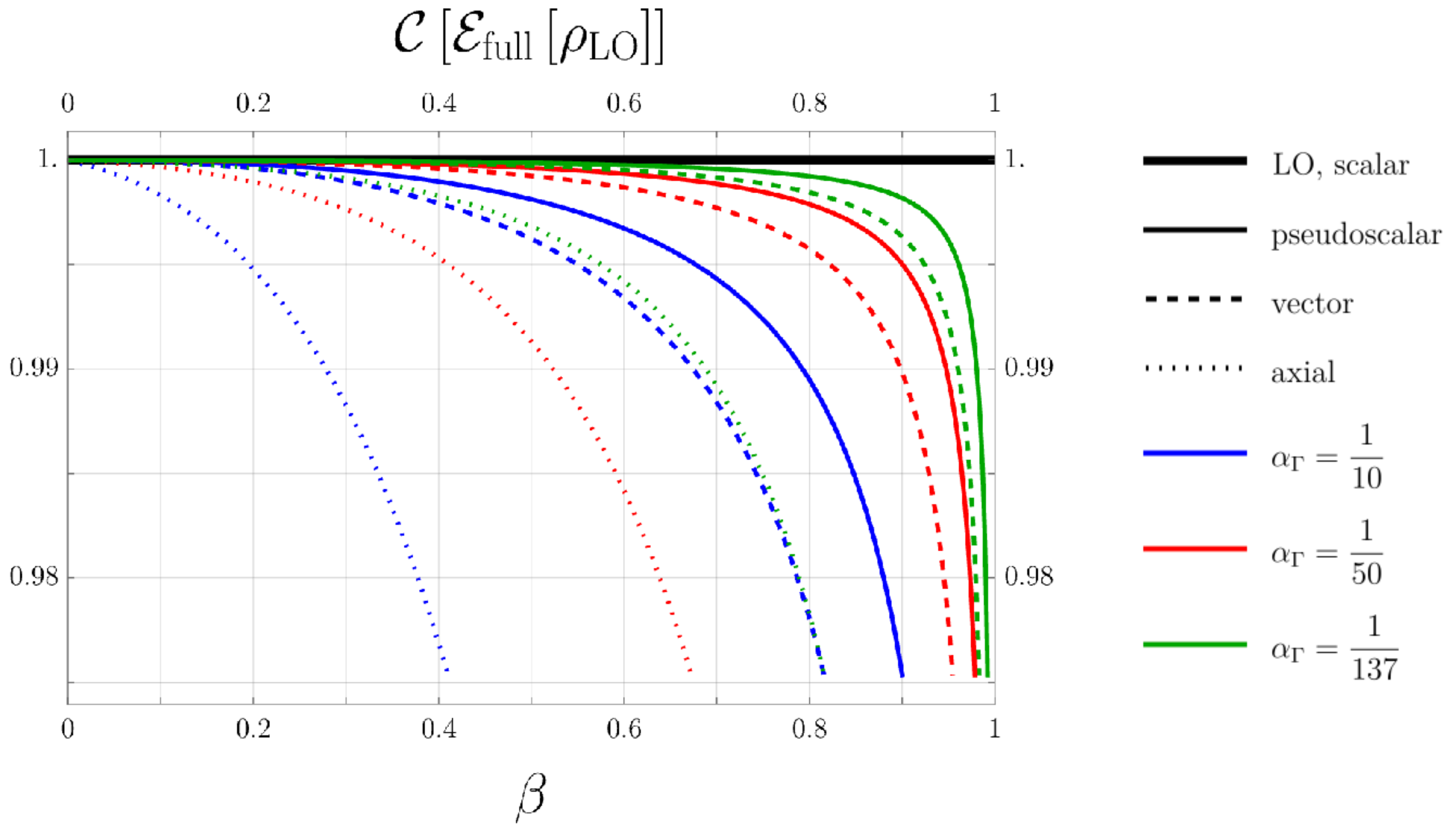
Identity part:
does not change entanglement

$$\mathsf{q} = \mathsf{q}^{\text{hard}} + \mathsf{q}_5^{\text{soft}}$$

Non-trivial Kraus part:
Decoherence!

*in the leading soft/collinear limit

# Decoherence

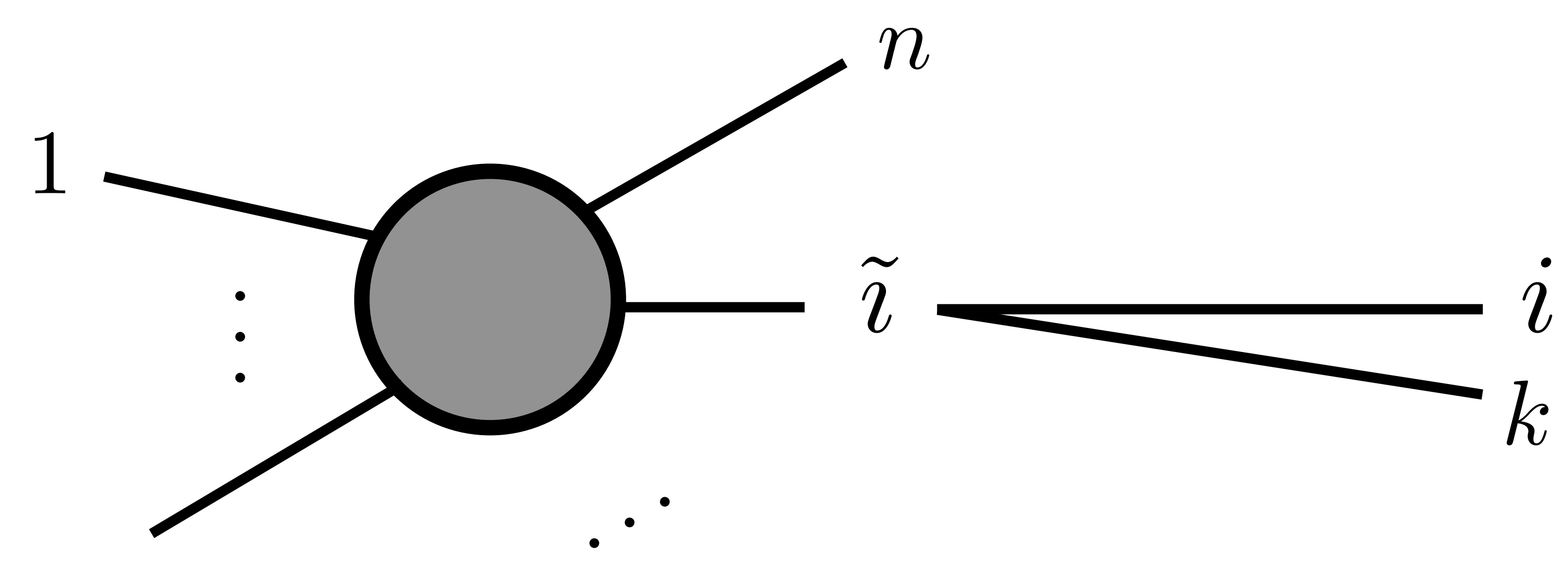



$\beta = \sqrt{1 - 4m_f^2/m_\varphi^2}$

Entanglement is lost mainly due to collinear emission ⟶ Small effects ~ 1%

# Collinear again…

In the collinear limit, when a n-parton system undergoes a splitting $\tilde{\imath} \to ik$




The amplitude factorize

$$\mathcal{M}_{n+1}^{\lambda_i \lambda_k}(\cdots, p_i, p_k, \cdots) = \mathcal{S}_{\tilde{\imath} \to ik}^{\lambda_{\tilde{\imath}} \lambda_i \lambda_k} \; \mathcal{M}_n^{\lambda_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)$$

→ Helicity dependent AP splitting function

used for spin correlations in Parton showers [Richardson, Webster ’18] [Hamilton, Karlberg, Salam, Scyboz, Verheyen ’21]

# Splitting functions as Kraus operators

Density matrix before the splitting $$\rho^{\lambda_{\tilde{\imath}}\lambda'_{\tilde{\imath}}} = \frac{1}{\mathcal{N}_i}\, \mathcal{M}^{\lambda_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)\, \overline{\mathcal{M}}^{\lambda'_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)\,.$$

After the splitting in the col. limit $$\rho^{(\lambda_i\lambda_k)(\lambda'_i\lambda'_k)} = \left[\mathcal{S}^{\lambda_{\tilde{\imath}}\lambda_i\lambda_k}_{\tilde{\imath}\to ik}\right]\, \rho^{\lambda_{\tilde{\imath}}\lambda'_{\tilde{\imath}}}\, \left[\mathcal{S}^{\lambda'_{\tilde{\imath}}\lambda'_i\lambda'_k}_{\tilde{\imath}\to ik}\right]^\dagger \left(\frac{\mathcal{N}_i}{\mathcal{N}_{\tilde{\imath}k}}\right)$$

Tracing over the unresolved d.o.f

$$\bar{\mathcal{E}}_{\text{col}}[\rho] = \rho^{\lambda_i\lambda'_i}_{\text{red}} = \sum_{\sigma=\pm}\int_{p_k} \mathcal{S}^{\lambda_{\tilde{\imath}}\lambda_i\sigma}_{\tilde{\imath}\to ik}\cdot \rho^{\lambda_{\tilde{\imath}}\lambda'_{\tilde{\imath}}}\cdot \mathcal{S}^{\lambda'_{\tilde{\imath}}\lambda'_i\sigma}_{\tilde{\imath}\to ik} = \mathsf{q}^{\text{hard}} \sum_{j\neq\text{id}} K_j \rho_{\text{LO}} K_j^\dagger$$

Splitting functions as Kraus (here: one emission)

# Conclusions

- NLO corrections are still expected to be small
- Full pheno study required to know the exact impact
- Three main effects drive the smallness of decoherence in ttbar

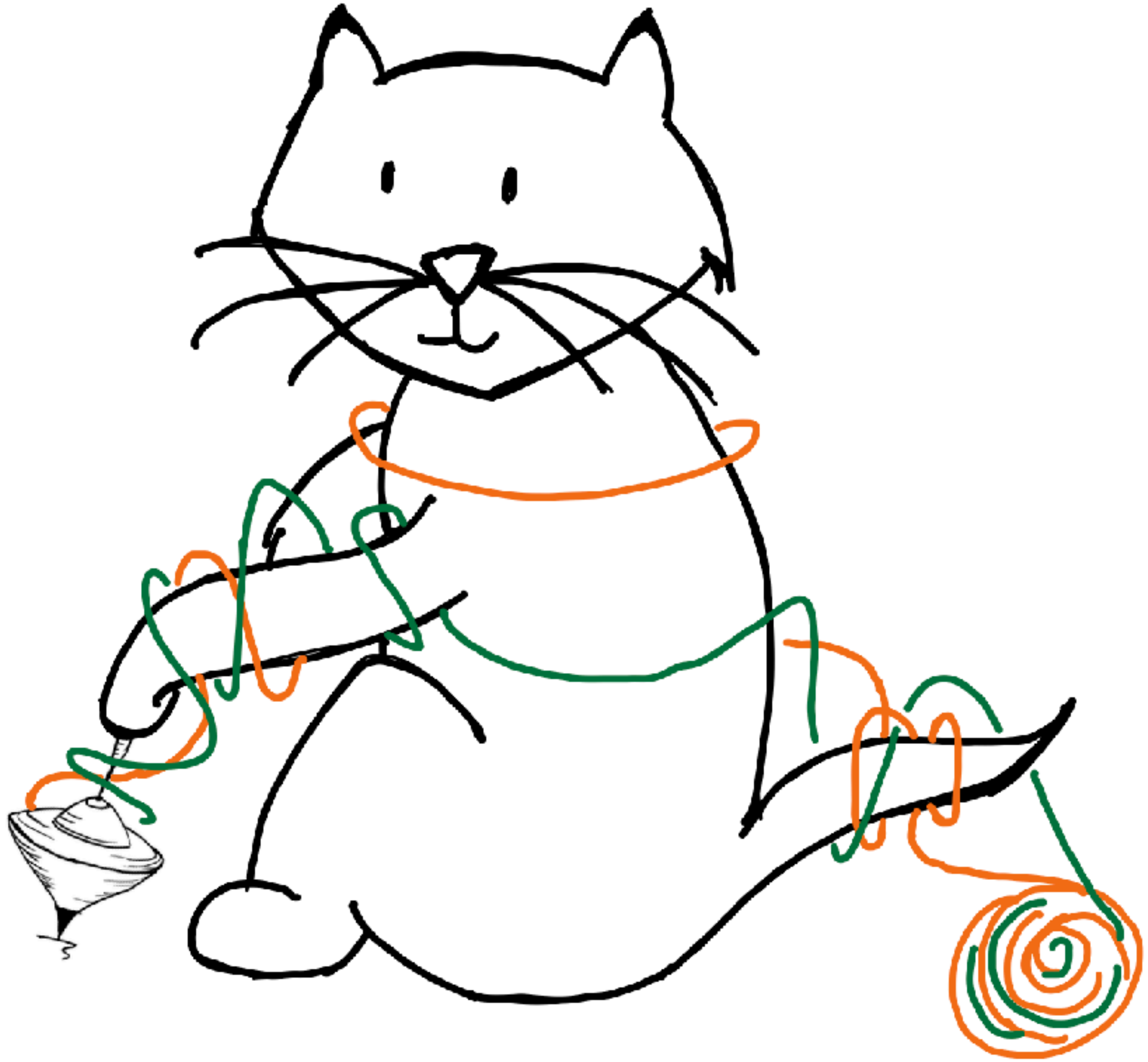
NLO: $\alpha$ is small

Leading soft radiation is a scalar function

Collinear emission: $1/m_t$

- Many new directions
    - Next-to-leading soft/collinear
    - Resummation
    - Gravity?
    - Pheno study
    - Qutrits
    - …

# Thank you!



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# IR Safe?

- IR cancellations are in the “identity part” of the map

Cancel as in the cross-section/decay: KLN theorem

- By power counting, one can see that the identity part contains integrals of

$$\int \mathrm{d}\Phi(k) \frac{1}{(p_i \cdot k)(p_j \cdot k)} \qquad i, j = 1, 2 \longrightarrow \text{IR divergent}$$

- While the non-trivial Kraus has $\int \mathrm{d}\Phi(k) \frac{k^{\mu_1} \dots k^{\mu_n}}{(p_i \cdot k)(p_j \cdot k)}$ IR finite for n > 0
(rank-n tensor integral)