Toponium physics at the LHC

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- **PRD 104 (2021) 034023**
- **EPJC 85 (2025) 157**

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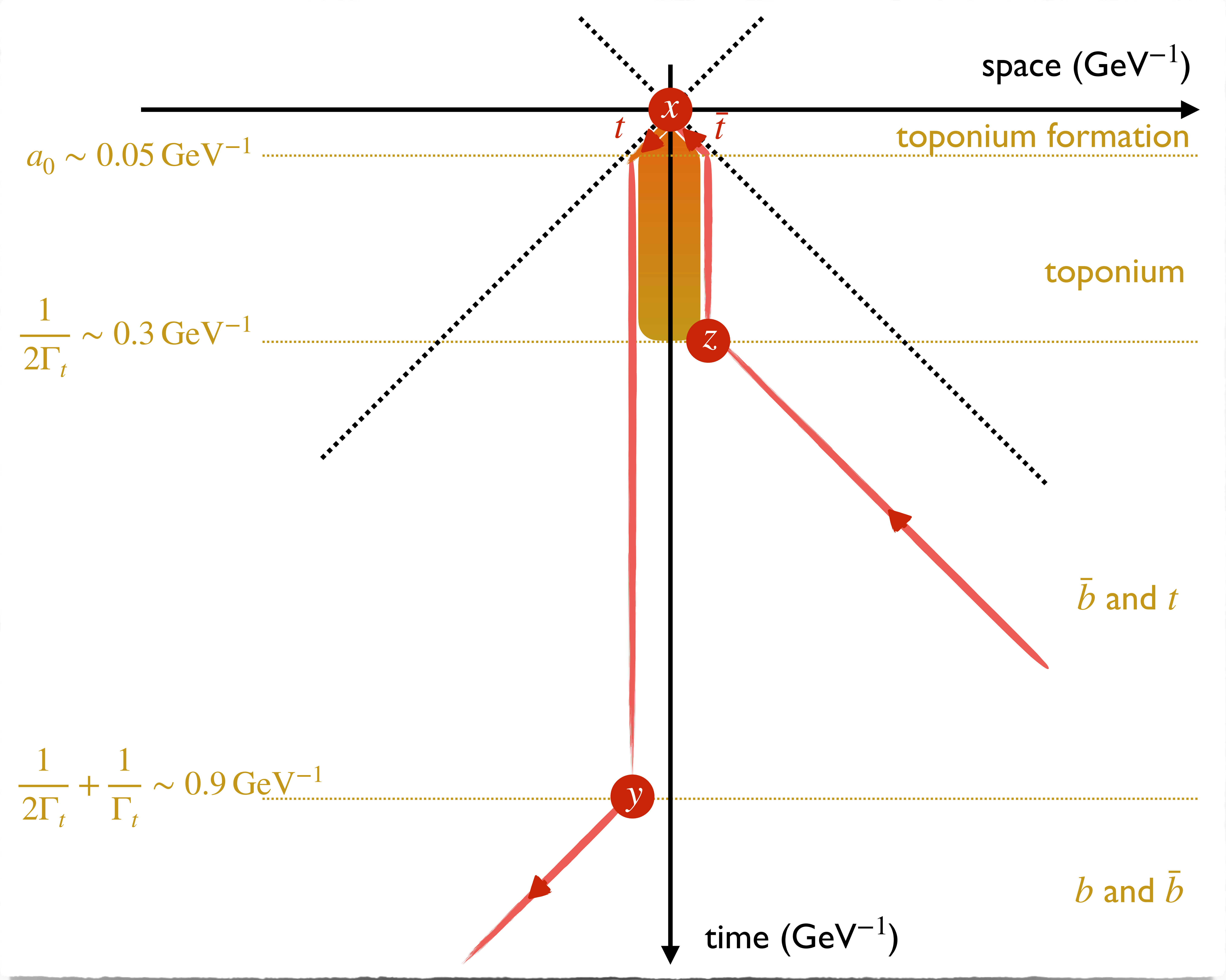
Top-antitop production near threshold



[Fadin & Khoze (JETP Lett`87)]
[Fadin, Khoze & Sjöstrand (Z.Phys.C`90)]
[Sumino, Fujii, Hagiwara, Murayama & Ng (PRD`93)]

A position-space picture

- Two-particle state created at $x = (t_0, \vec{x})$

  ≃ wave packet propagation to the $V_{\rm QCD}$ barrier

  ➙ Typical scale: the Bohr radius

  $$a_0 = \frac{1}{C_F\,\alpha_s\,m_t/2} \sim (20\ \mathrm{GeV})^{-1}$$

- Oscillations within the barrier until system decay

  ➙ Top $[y = (t_1, \vec{y})]$ or antitop $[z = (t_2, \vec{z})]$ decay

  ➙ Typical scale ≃ $(2\Gamma_t)^{-1} \sim (3\ \mathrm{GeV})^{-1}$

- Probe of the QCD potential

  ➙ Toponium effects

  ➙ Currently not included in MC simulations

The toponium Green's function

Three-point correlation function in the non-relativistic limit

$$K_{abcd}(x,y,z) = \Big\langle 0 \Big| T\{t_c(y)\bar{t}_d(z) : \bar{t}_a(x) t_b(x) :\} \Big| 0 \Big\rangle$$

Antitop-decay first

Top-decay first

$$= \frac{(1+\gamma^0)_{ca}}{2}\,\frac{(1-\gamma^0)_{bd}}{2} \int \mathrm{d}^3 r \Big[K_1\big(y;(z^0,\vec{r})\big)\, K_2(z^0,\vec{r},\vec{z};x^0,\vec{x},\vec{x}) + K_1\big(z;(y^0,\vec{r})\big)\, K_2(y^0,\vec{y},\vec{r};x^0,\vec{x},\vec{x}) \Big]$$

Non-relativistic spin projection operators

1-particle-state and 2-particle-state propagators

Green's function ratio as a seed for toponium modelling

- Valid near threshold ➙ non-relativistic matrix elements
- Conventional MC generators: standard matrix elements
  - ➙ Assumption 1: relativistic effects negligible
  - ➙ Assumption 2: higher-spin contributions negligible
- Matrix-element re-weighting
  - ➙ Colour singlet projection
  - ➙ Validity: $E < 4$ GeV and $p^* < 50$ GeV

$$i\mathcal{M} \;\to\; i\mathcal{M} \times \frac{\widetilde{G}(E;p^*)}{\widetilde{G}_0(E;p^*)}$$

The toponium Green's function (from K_2)

- Solution to the Lippmann-Schwinger equation
  - ➙ Fourier transform of the QCD potential
  - ➙ S-wave contributions
- To be solved numerically

$$\widetilde{G}(E;p) = \widetilde{G}_0(E;p) + \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, \widetilde{V}_{\mathrm{QCD}}(\vec{p}-\vec{q})\, \widetilde{G}(E;q)$$

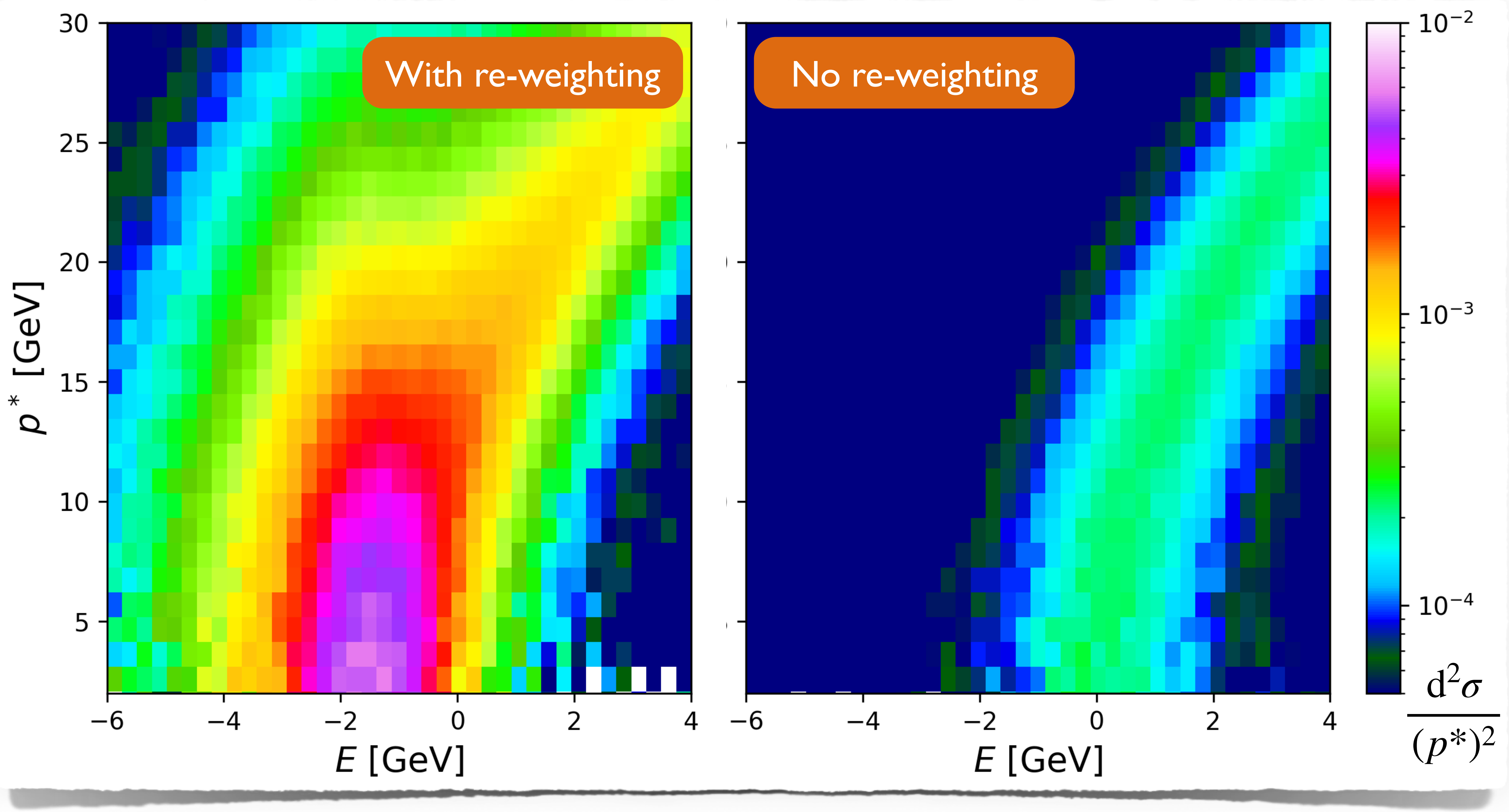
Free Green's function

[Jezabek, Kuhn & Teubner (Z.Phys.C`92)]

$t\bar{t}$ production near threshold with MG5aMC

Process: $gg \to t\bar{t} \to b\ell^+\nu_\ell \bar{b}\ell'^- \bar{\nu}'_\ell$

- Six-body final state: spin correlations included
- Projection on the colour singlet
- Matrix-elements with and without re-weighting
- No matching with parton showers



Without re-weighting

- Rates ≡ phase space x Breit-Wigner
- Similar heat map as for $\widetilde{G}_0$

With re-weighting

- Similar heat map as for $\widetilde{G}$
- Normalisation and shape affected
- Access to the QCD Green's function
  → ratio of the re-weighted/non-re-weighted predictions

Typical top momentum in the toponium rest frame

$$\langle p(E) \rangle = \frac{\int p\, d^3p\, \frac{d\sigma}{p^2 dp dE}}{\int d^3p\, \frac{d\sigma}{p^2 dp dE}}$$ ↔ for $E \simeq -2$ GeV: 20 GeV (the Bohr radius!)

[Hagiwara, BF, Ma & Zheng (EPJC'25)]

Invariant masses: tops & toponium

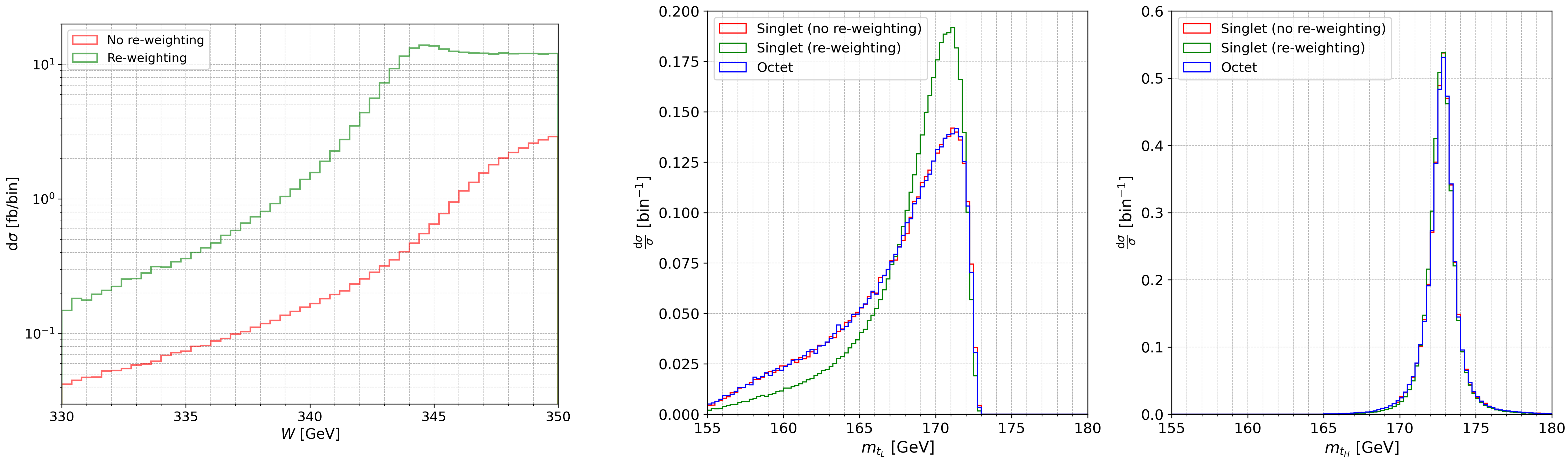
$m_{t\bar{t}}$ invariant mass distribution

- Peak at $E \simeq -2$ GeV ⊕ extended bound state effects
- Shape in agreement with pNRQCD
  ⚠️ Comparison (relativistic pieces, octet, α_s, PDFs, *etc.*)

[Sumino & Yokoya (JHEP`10)]

Top mass distributions

- Light top ⬌ invariant mass shifted to lower values
  ➞ Governed by the QCD Green's function
  ➞ Distortion of the Breit-Wigner
  ➞ Bound state effects within the Coulomb potential generated by t_H
- Heavy top ⬌ Breit-Wigner
  ➞ Effectively stable until t_L decays
  ➞ No QCD potential from the decayed top (free propagation)



[Hagiwara, BF, Ma & Zheng (EPJC'25)]

Comparison with pseudo-scalar toy models

A toponium toy Lagrangian with a pseudo-scalar state

$$\mathcal{L}_{\eta_t} = \frac{1}{2}\partial_\mu \eta_t \, \partial^\mu \eta_t - \frac{1}{2} m_{\eta_t}^2 \eta_t^2 - \frac{1}{4} g_{gg}\, \eta_t \, G_a^{\mu\nu}\, \tilde{G}^a_{\mu\nu} - i g_{tt}\, \eta_t \, \bar{t}\gamma_5 t$$

- No free parameters

$$\sigma(\eta_t) \equiv \int_{-8\text{ GeV}}^{4\text{ GeV}} dE \left(\frac{d\sigma_\text{full}}{dE} - \frac{d\sigma_\text{NLO}}{dE} \right) \quad \rightarrow \sigma_\eta(13\text{ TeV}) \approx 6.5\text{ pb}$$

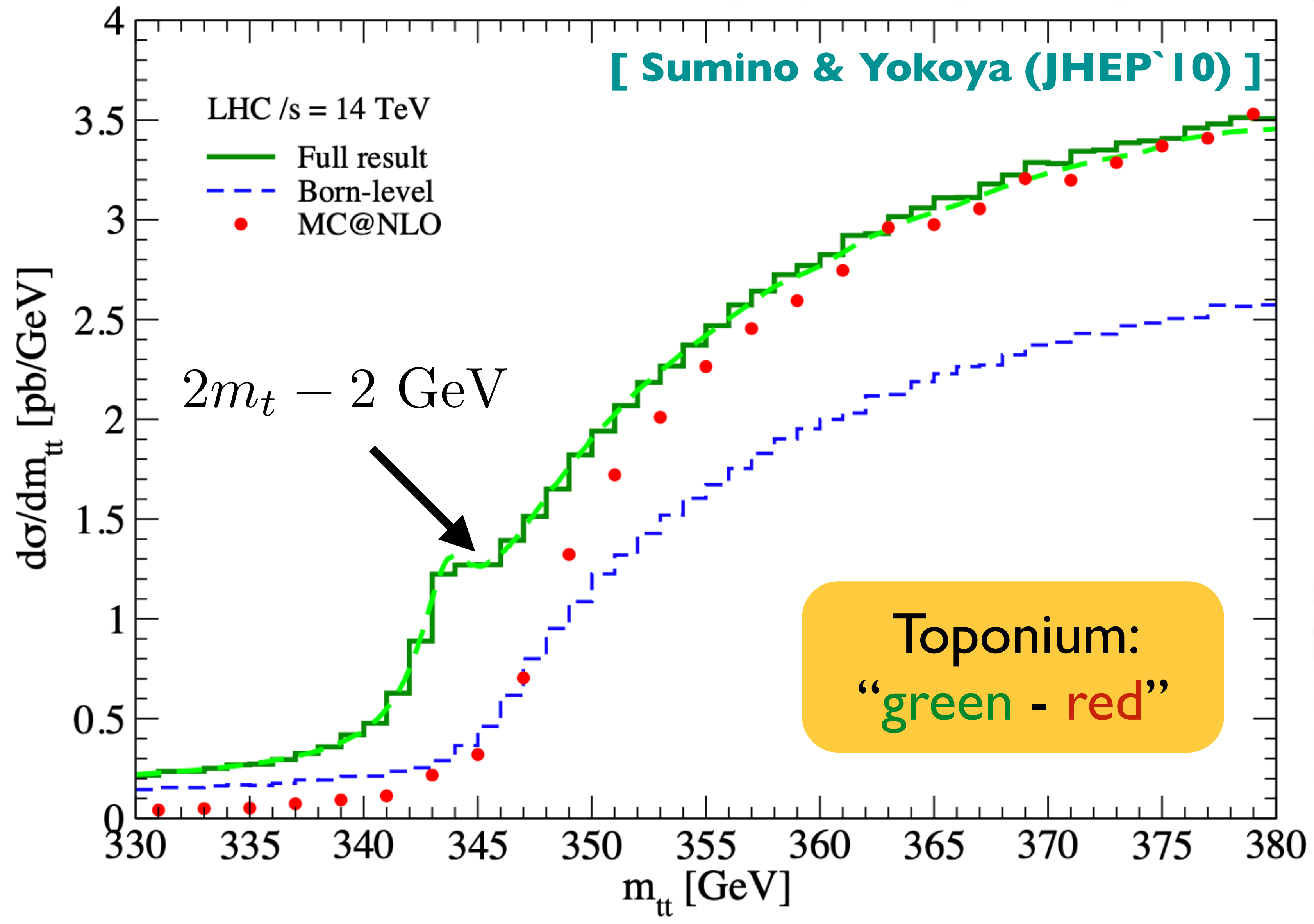
→ To be compared with $\sigma_{t\bar{t}} \approx 810\,\text{pb}$

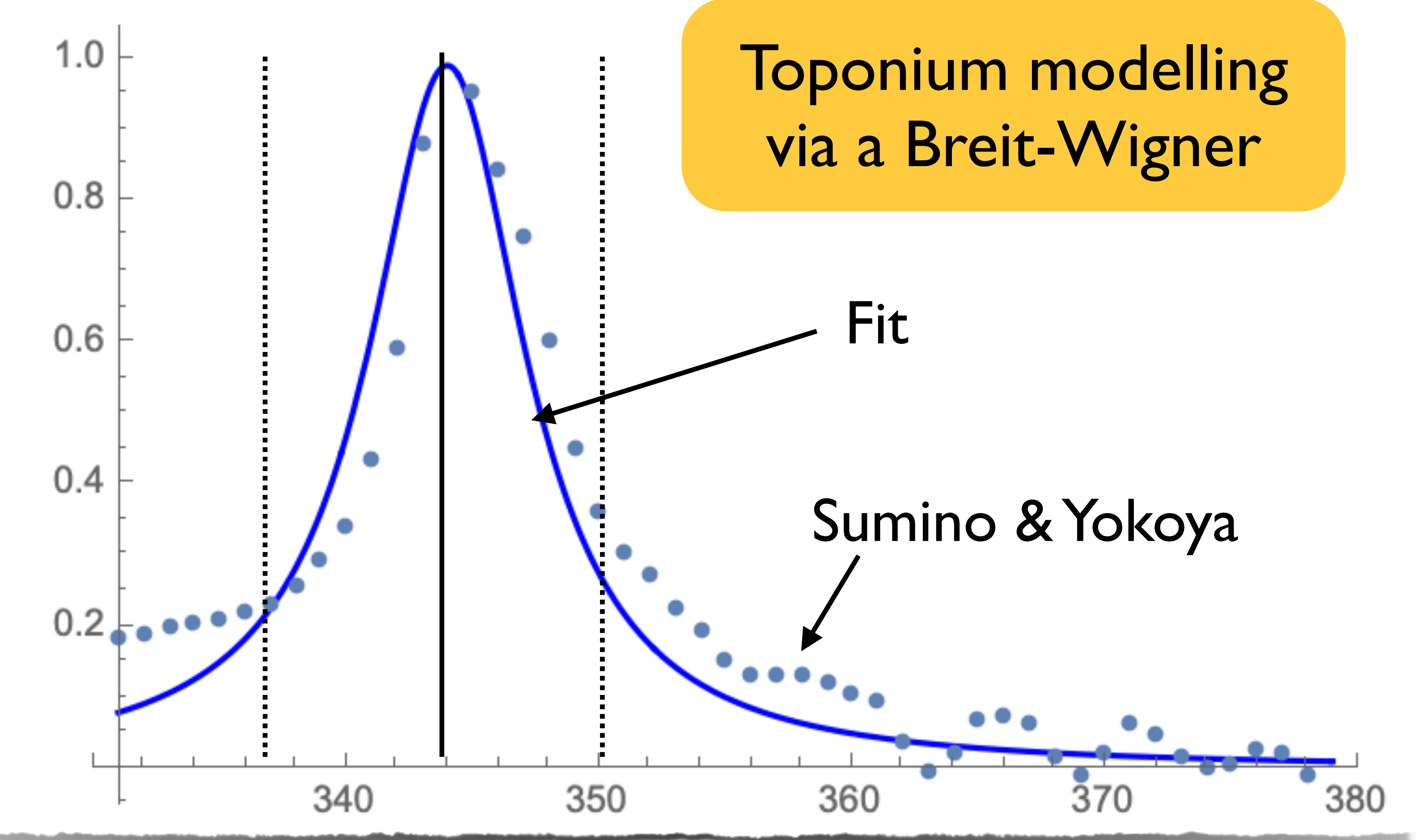
(Non-perfect) Breit-Wigner fit → $m_{\eta_t} \simeq 344\,\text{GeV}; \quad \Gamma_{\eta_t} \simeq 7\,\text{GeV}$

⊕ Green's functions re-weighting

⊕ cut on the $t\bar{t}$ invariant mass

[Hagiwara, BF, Ma & Zheng (PRD`21)]

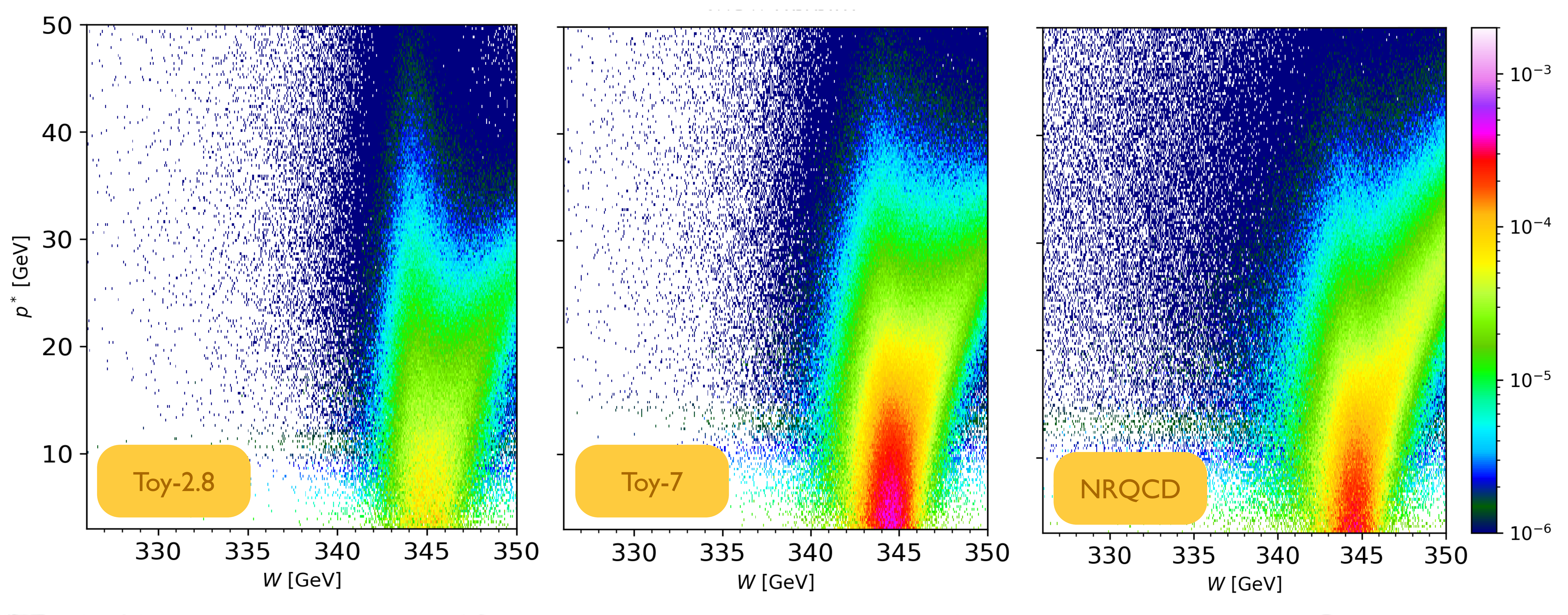




A ground-state toponium toy Lagrangian with a pseudo-scalar state

- Only capturing the vicinity of the ground state
  → OK for $E \lesssim -0.5\,\text{GeV}$
- Radial excitations and P-wave states in $-0.5\,\text{GeV} \leq E \leq 0\,\text{GeV}$
  → Not in Green's function (yet)
  → Not captured in the ground-state approximation
- Mixed contributions around threshold ($\Gamma_{\eta_t} \simeq 3\,\text{GeV}$)
- Fit for E < 0 only: $\sigma_\eta(13\,\text{TeV}) \approx 4.4\,\text{pb}$

Toy models *versus* NRQCD: $\mathrm{d}^2\sigma/(p^*)^2$



Imperfections of the toy model visible in the key distribution

- The W spectrum seems OK in all cases
  → only the peak is well reproduced…
- Strong distortion with the p^* distribution

Toy models should not be used anymore!

Towards discovery: spin density matrices

Di-leptonic signal properties from $t\bar{t}$ production and $t/\bar{t}$ decay density matrices

- Distribution of the angles of the leptons in their parent top rest frames

$$\sum_{\sigma,\bar\sigma,\sigma',\bar\sigma'} \rho^{\sigma\bar\sigma,\sigma'\bar\sigma'}_{gg\to(t\bar t)_1}\, \mathrm{d}\rho^{t\to b\ell^+\nu_\ell}_{\sigma,\sigma'}\, \mathrm{d}\rho^{\bar t\to\bar b\ell'^-\bar\nu'_\ell}_{\bar\sigma',\bar\sigma'} \quad\Rightarrow\quad \frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta\,\mathrm{d}\varphi\,\mathrm{d}\cos\bar\theta\,\mathrm{d}\bar\varphi} \propto \frac{1+\sin\bar\theta\sin\theta\cos(\bar\varphi-\varphi)+\cos\bar\theta\cos\theta}{2}$$

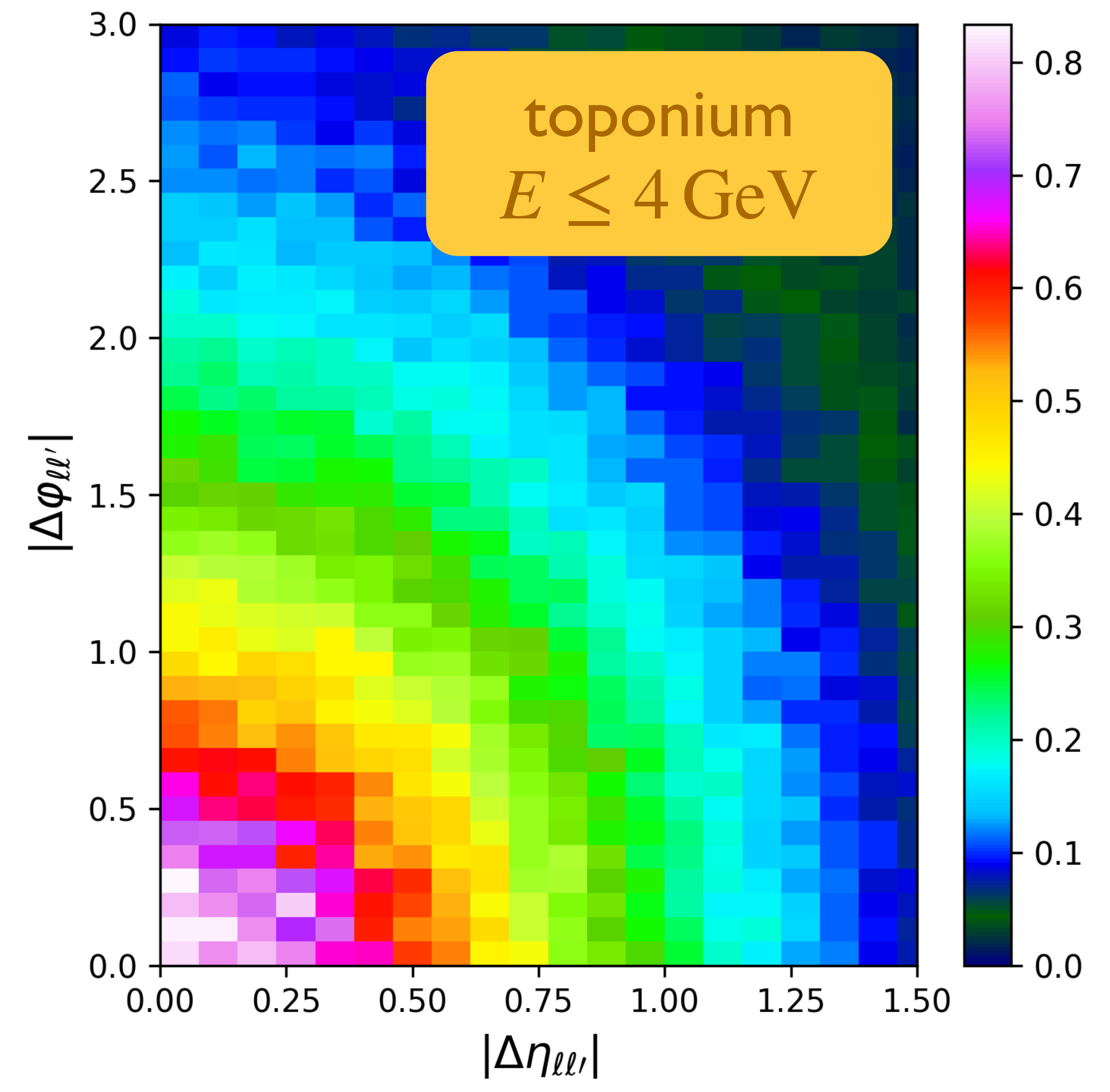
[**Hagiwara, Yokoya & Zheng (JHEP'18)**]

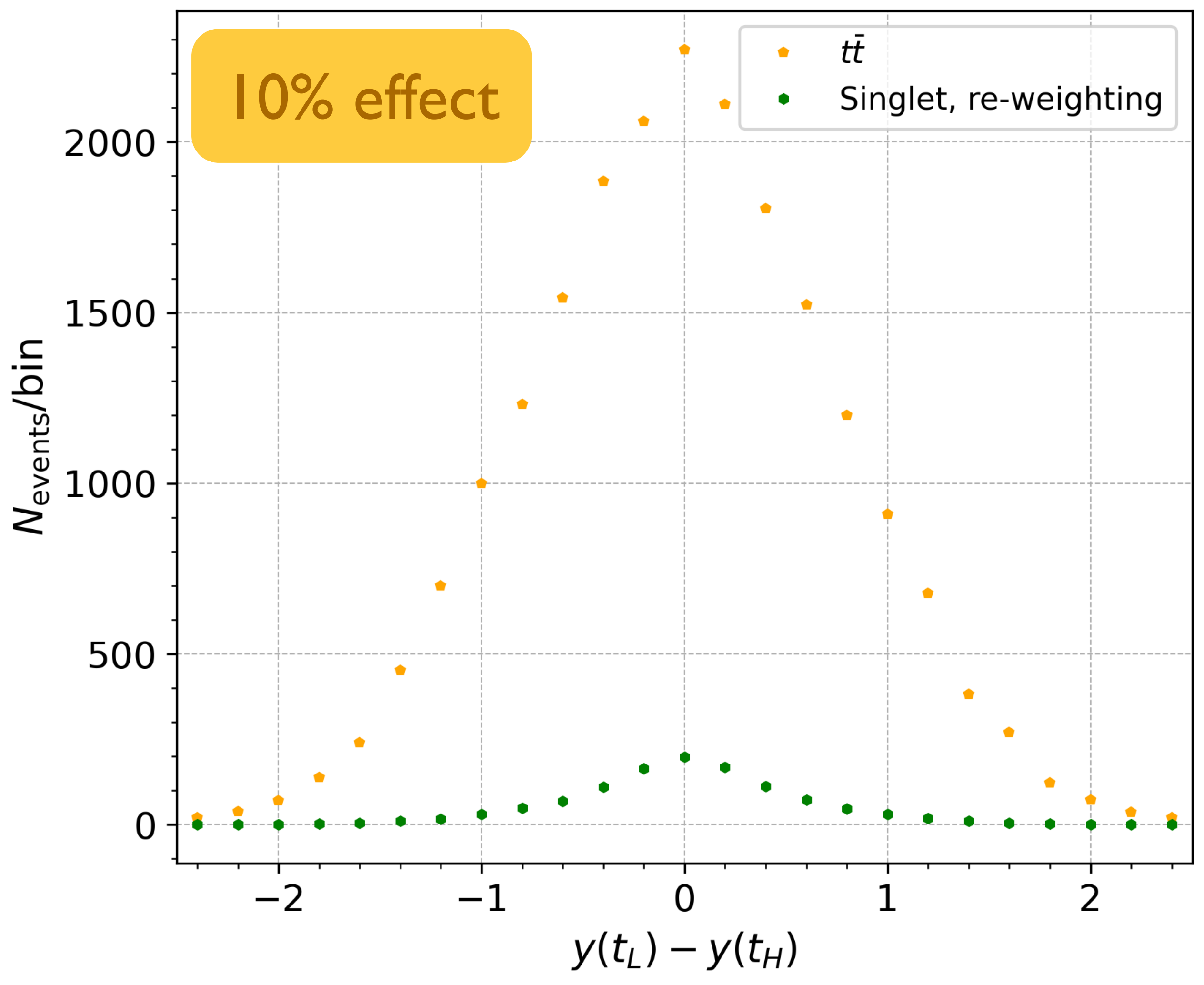
- Toponium characteristics: small $\Delta\varphi_{\ell\ell'}$, small $|\Delta\eta_{\ell\ell'}|$ and small $m_{\ell\ell'}$
  - ➙ Small azimuthal angle separation (survives the lab frame boost)
  - ➙ Small di-lepton invariant mass *(cf. tension with data?)*

$$m^2_{\bar\ell\ell'} = 2E_{\bar\ell}E_{\ell'}\Big(1-\sin\bar\theta\sin\theta\cos(\bar\varphi-\varphi)-\cos\bar\theta\cos\theta\Big)$$

Bulk of toponium events

- *Option A:* $\Delta\varphi_{\ell\ell'} < 0.9;\ \Delta\eta_{\ell\ell'} < 0.9$
  - ➙ $N_{t\bar t} = 127{,}000;\ N_{\text{toponium}} = 3{,}520$ [sensitivity of 9.7σ (S/B ~ 2.8%)]
- *Option B:* $\Delta\varphi_{\ell\ell'} < \pi/5;\ m_{\ell\ell'} < 40$ GeV
  - ➙ $N_{t\bar t} = 77{,}100;\ N_{\text{toponium}} = 2{,}200$ [sensitivity of 7.8σ (S/B ~ 2.9%)]
- *Option C:* $\Delta R^2_{\ell\ell'} < 0.8$
  - ➙ $N_{t\bar t} = 99{,}400;\ N_{\text{toponium}} = 2{,}980$ [sensitivity of 9.3σ (S/B ~ 3%)]





A few last words…

Top-antitop production near the threshold

- Emergence of a toponium system at a scale of 0.05 GeV^{-1}
- Decay at a time scale of ~ 0.3 GeV^{-1}
- Occurs well before hadronisation at 5 GeV^{-1}

Possibility to probe the toponium wave function

- 'The smallest hadron in the SM'
- The story just begins…

To-do list

- Higher spins
- Colour octet (no resonant enhancement)
- Matching NRQCD to NLO-QCD + parton showers
- Impact on top mass measurements