

Measuring Quantum Discord at the LHC (for top quarks)

Navin McGinnis

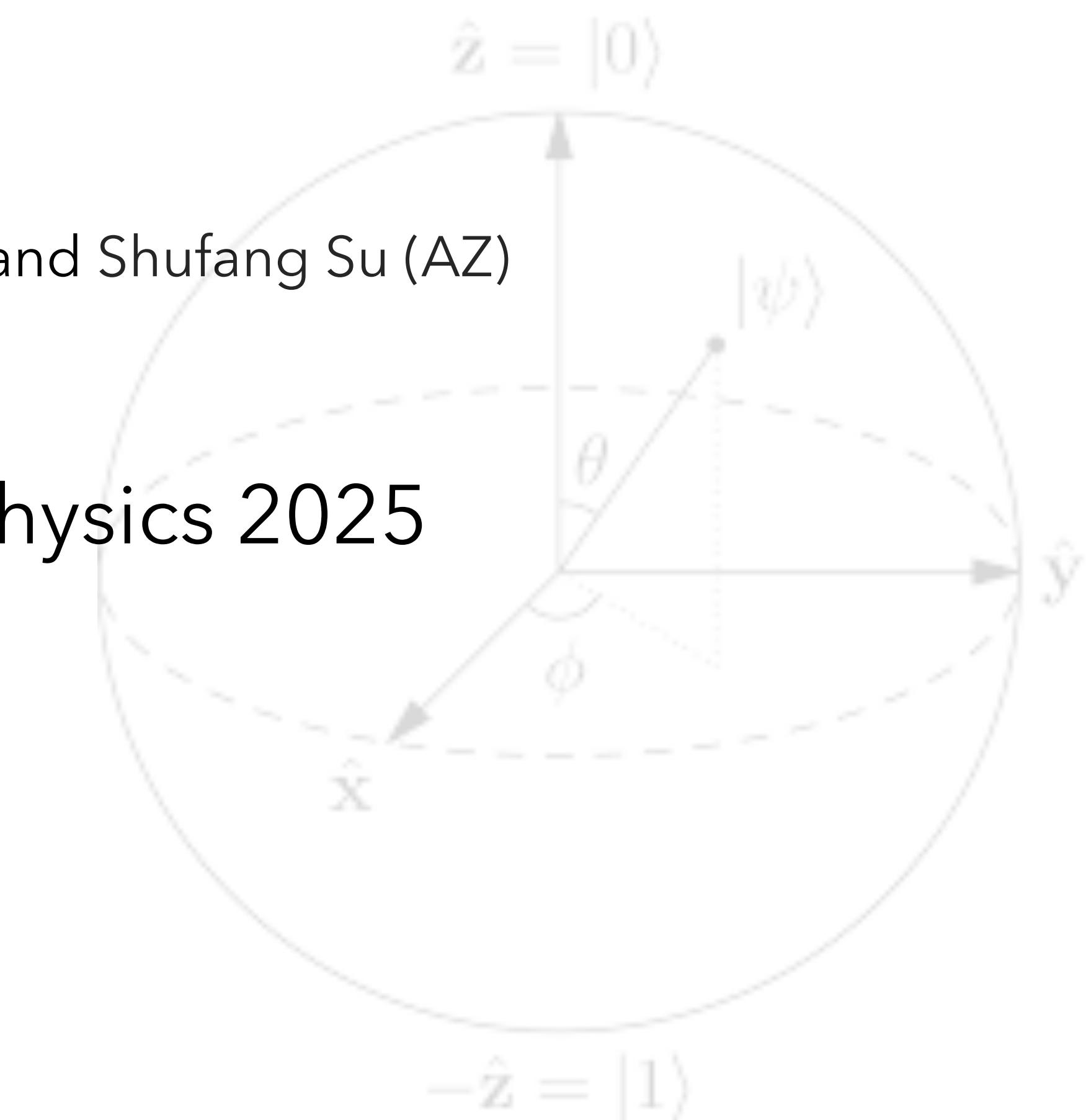
Based on collaboration w/ Tao Han (Pitt), Matt Low (Pitt), and Shufang Su (AZ)

[2412.21158](https://arxiv.org/abs/2412.21158) [hep-ph]

Quantum Observables for Collider Physics 2025

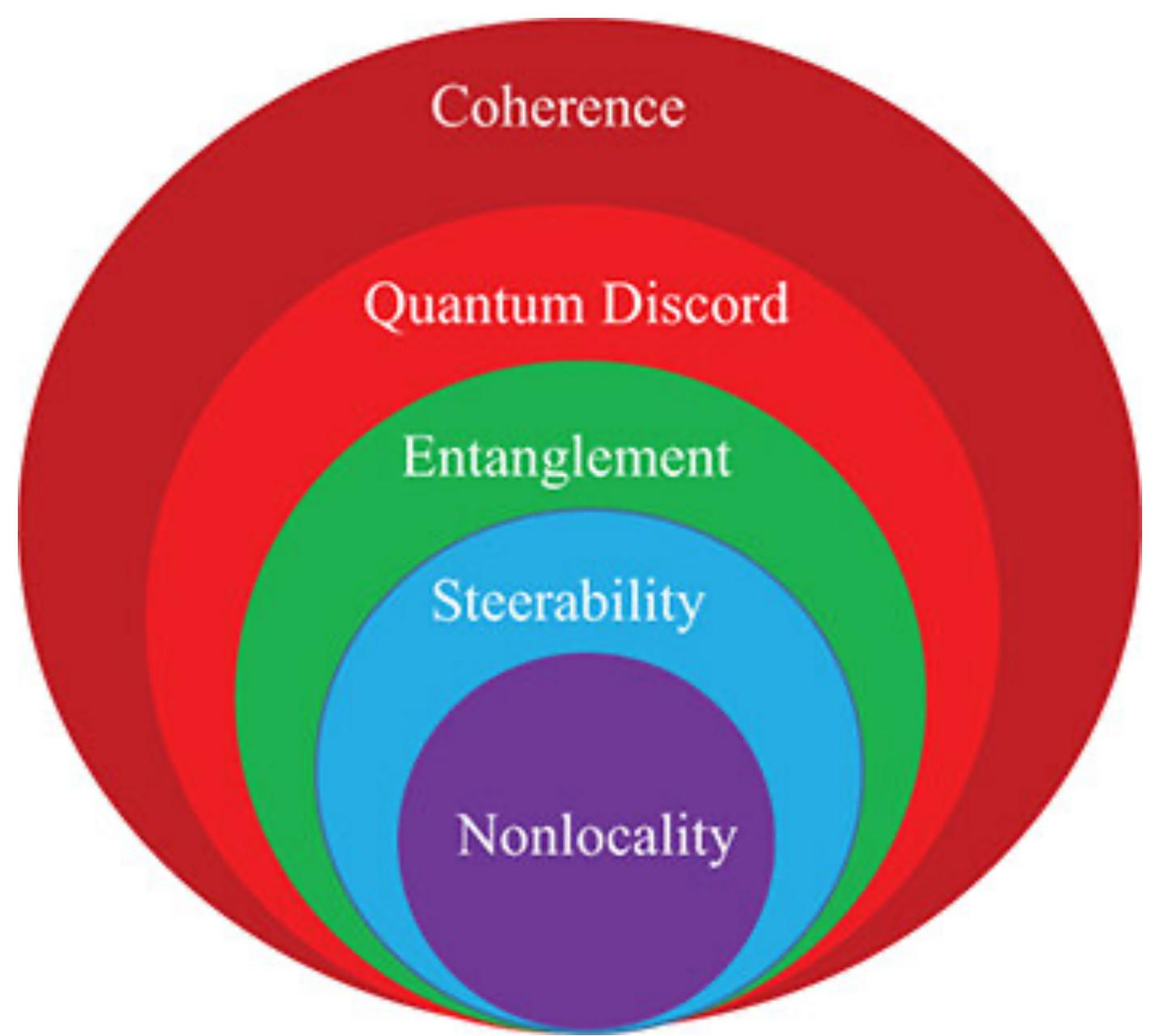
GGI

April 10, 2025



Exploring Quantum Mechanics in High Energy Physics @ PITT PACC - March 2024





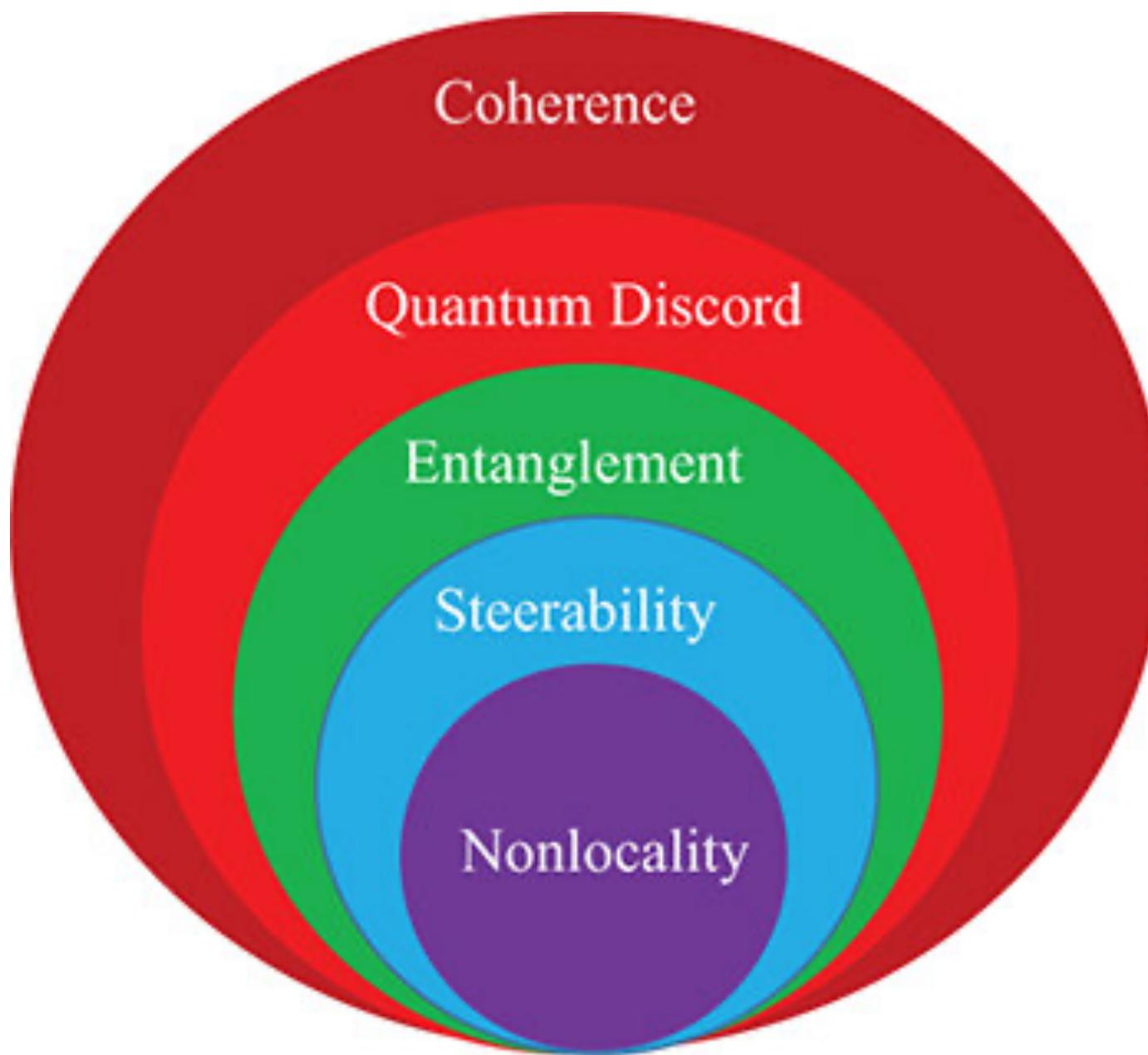
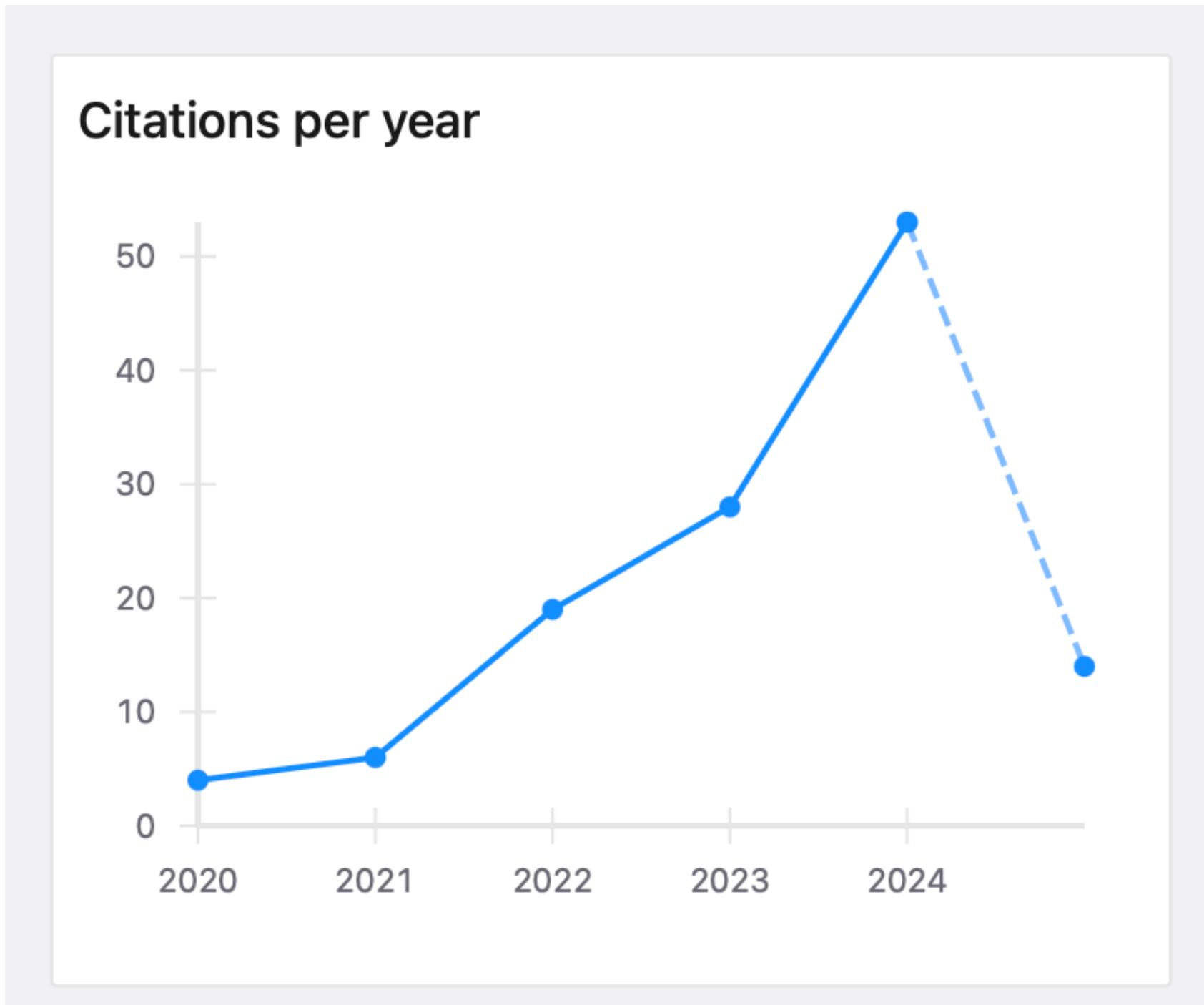
Entanglement and quantum tomography with top quarks at the LHC

Yoav Afik^{1,*} and Juan Ramón Muñoz de Nova^{2,†}

¹*Experimental Physics Department, CERN, 1211 Geneva, Switzerland*

²*Departamento de Física de Materiales, Universidad Complutense de Madrid, E-28040 Madrid, Spain*

(Dated: September 8, 2021)



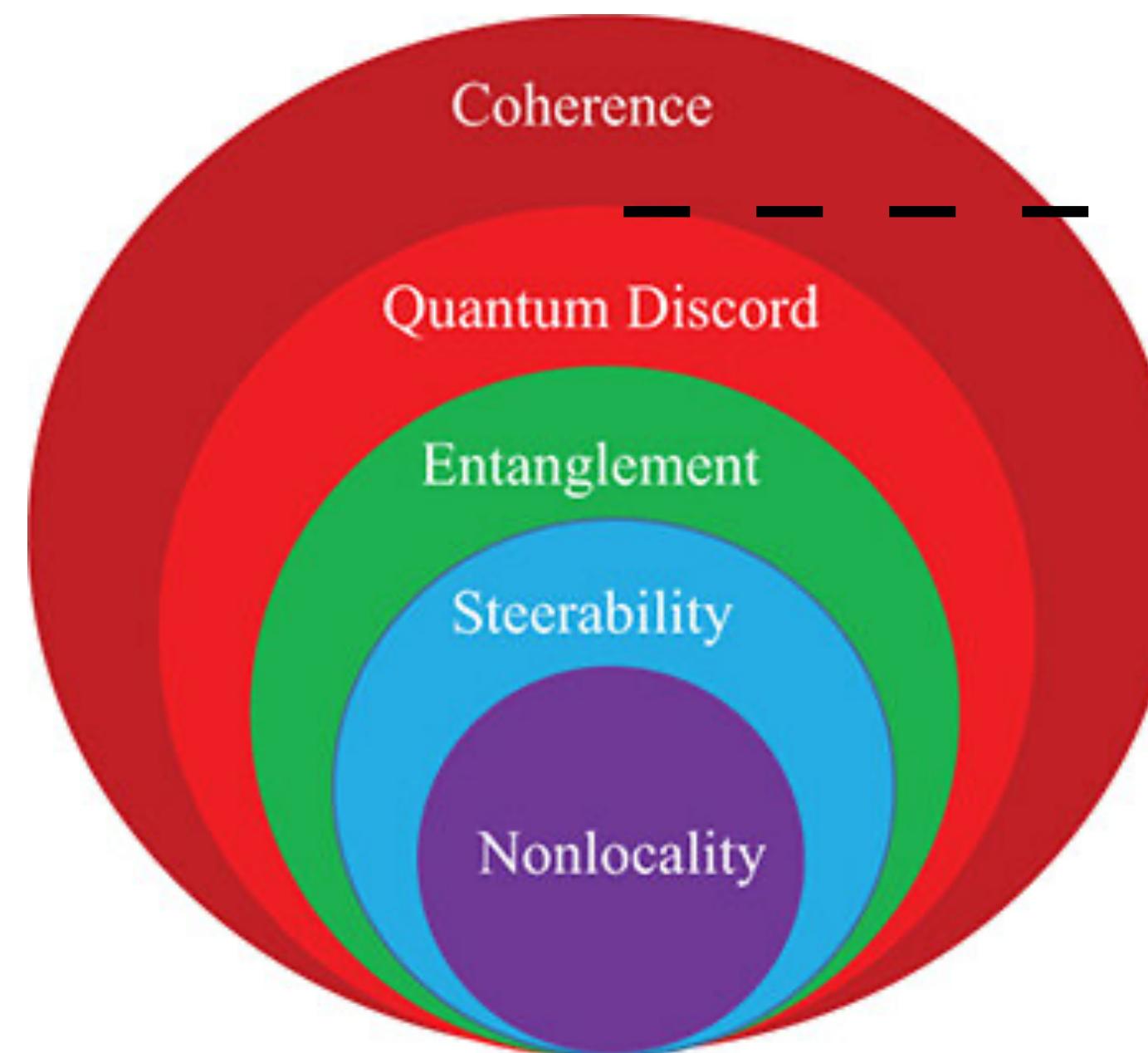
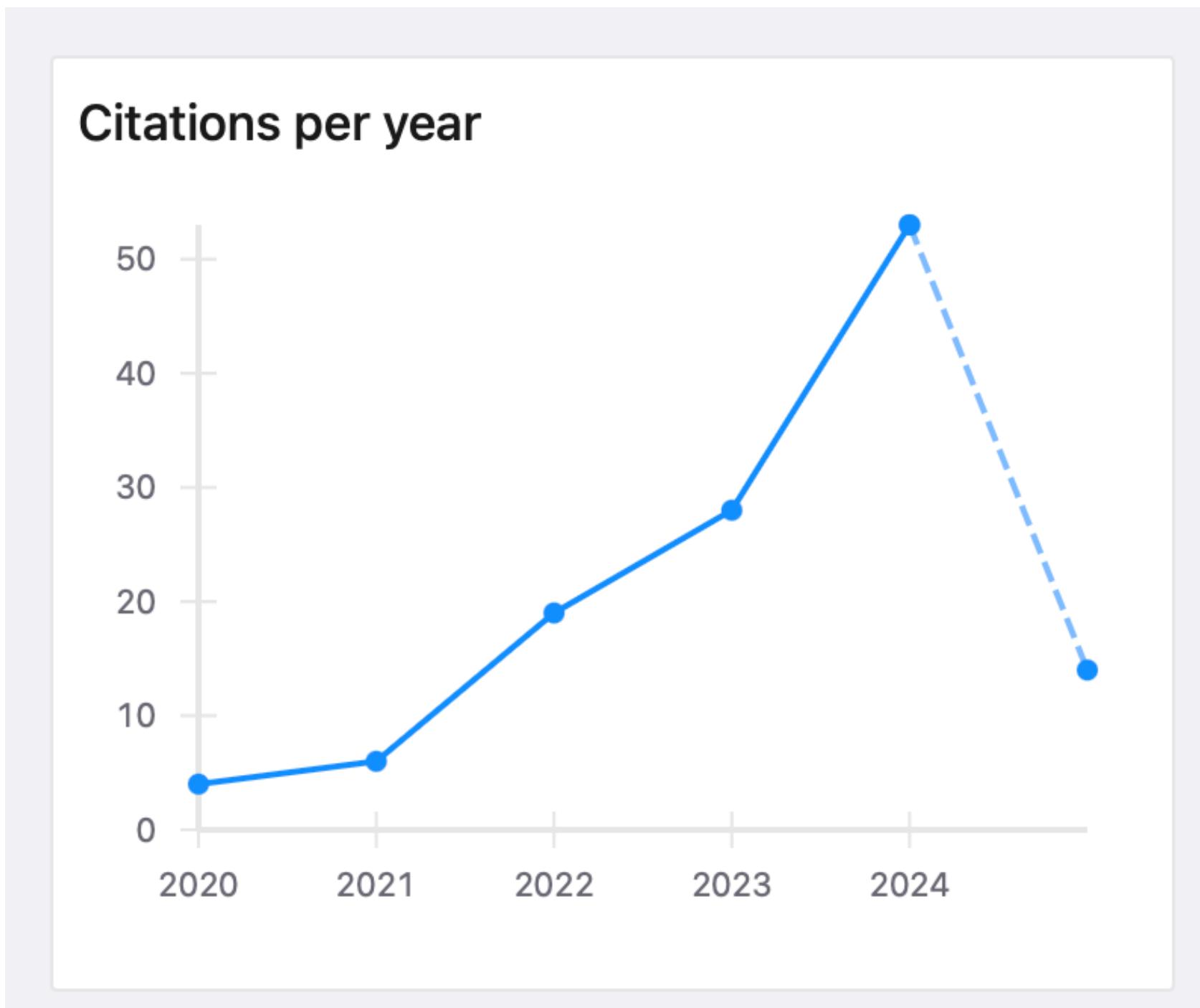
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Boundary of quantum
Information

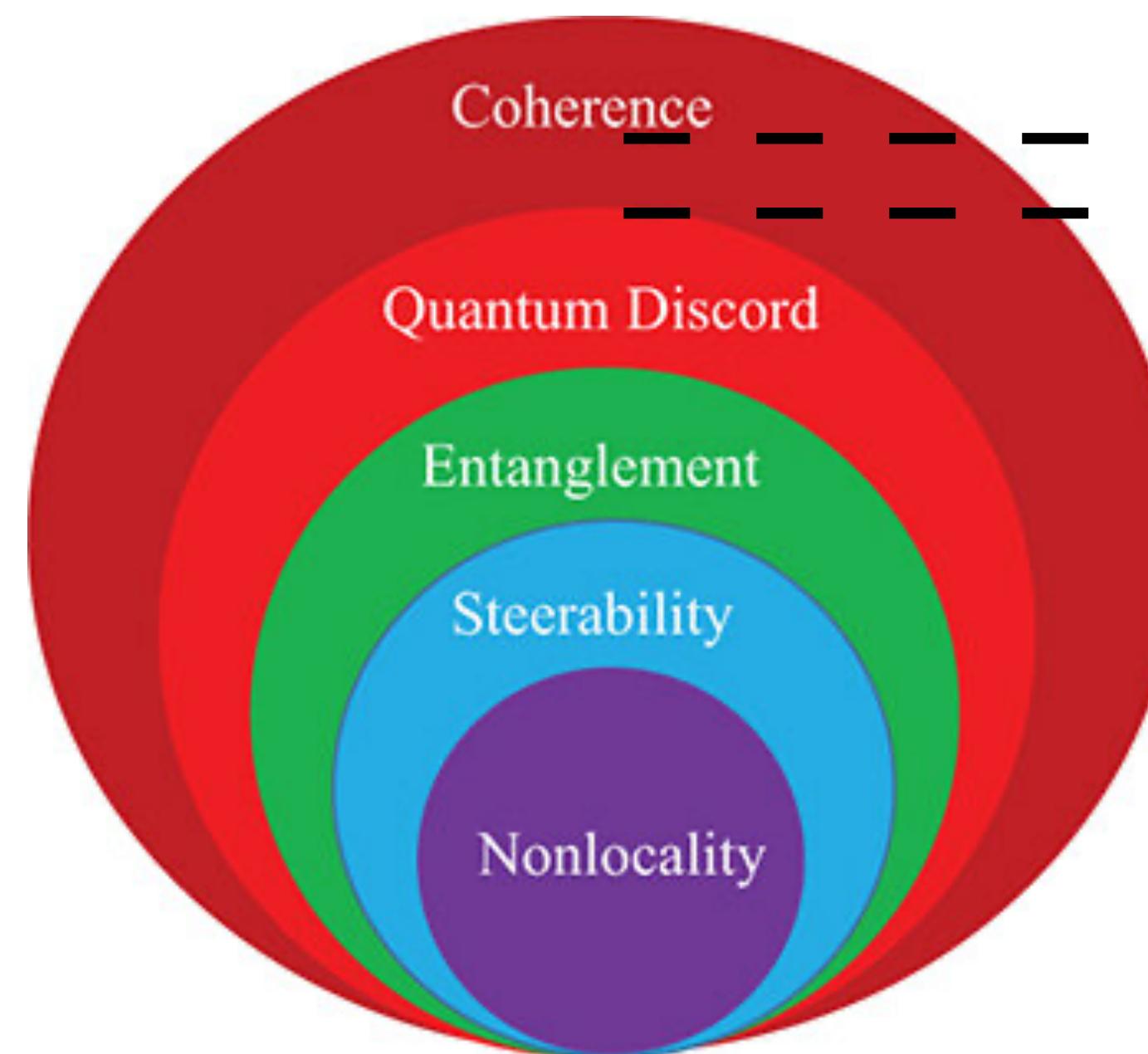
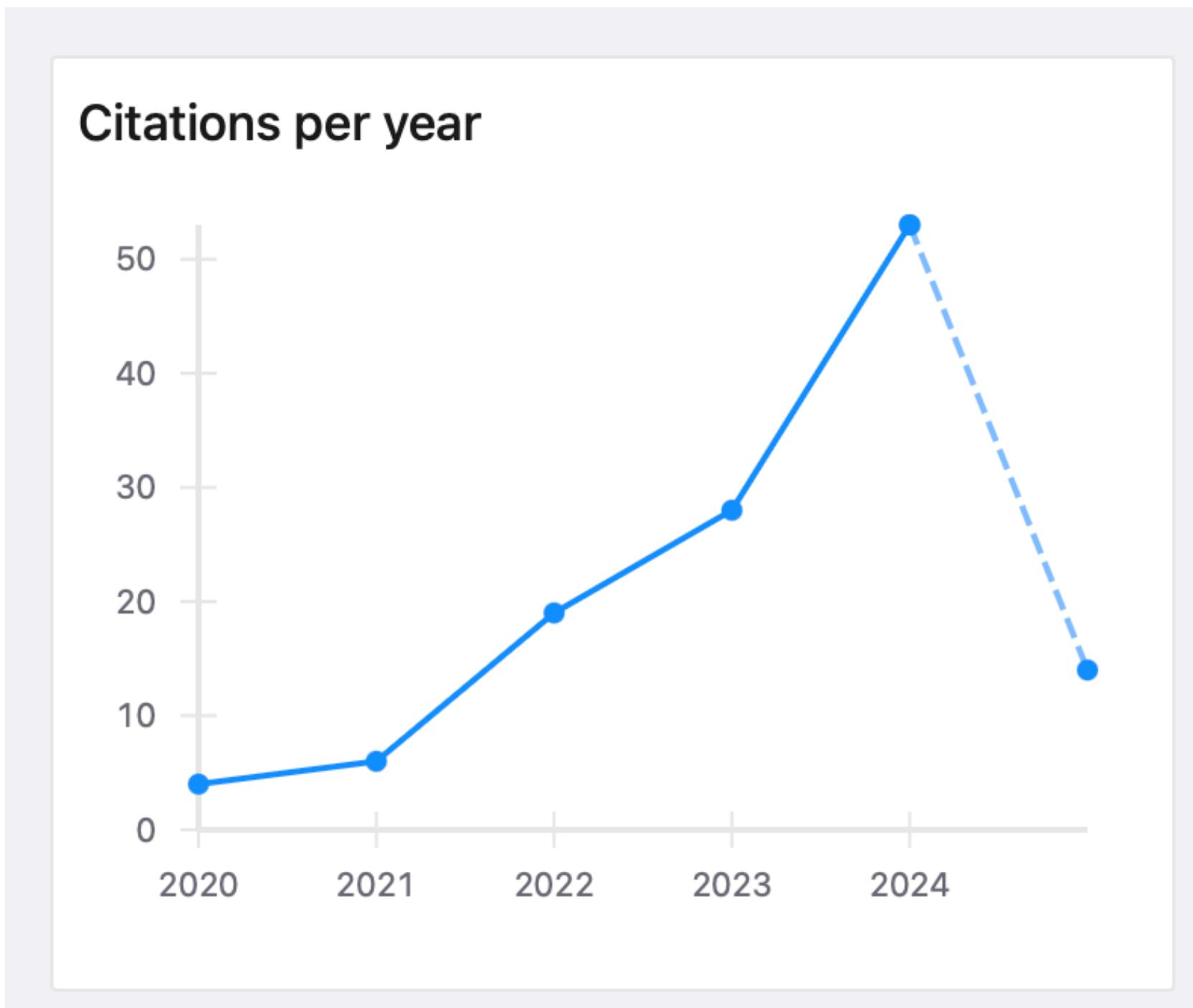
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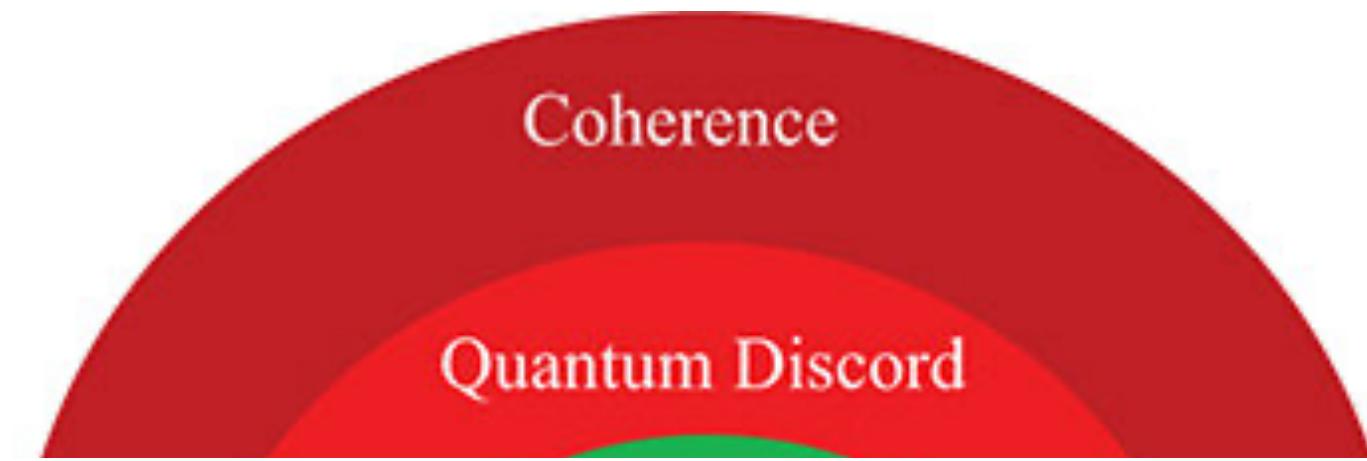
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Local quantum uncertainty
Boundary of quantum
Information



Quantum discord and steering in top quarks at the LHC

Yoav Afik^{1,*} and Juan Ramón Muñoz de Nova^{2,†}

¹*Experimental Physics Department, CERN, 1211 Geneva, Switzerland*

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(Dated: June 7, 2023)



Discord for top quarks

Quantum Discord

$$\rho = |A, B\rangle \langle A, B|$$

$$\mathcal{D}_A(\rho) = I(\rho) - J_A(\rho)$$

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}).$$

$$J_A(\rho_{AB}) = \max_{\hat{n}} J_A(\rho_{AB}; \hat{n}).$$

Discord for top quarks

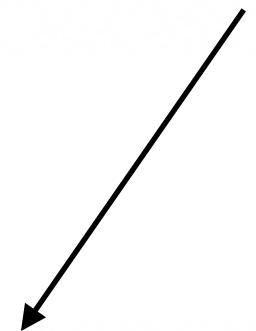
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$$J_A(\rho_{AB}; \hat{n}) = S(\rho_A) - S(\rho_A | \rho_B; \hat{n}).$$

Discord for top quarks

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Mutual Information
(all correlations)

$$J_A(\rho_{AB}) = \max_{\hat{n}} J_A(\rho_{AB}; \hat{n}).$$

Conditional information
(only classical correlations)

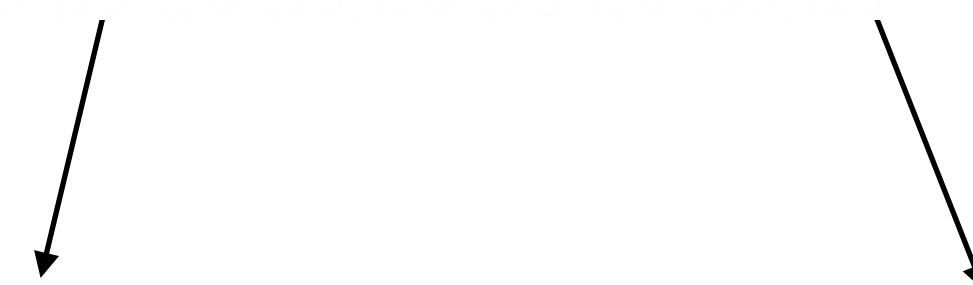
Discord for top quarks

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Mutual Information
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Conditional information
(only classical correlations)

$$\mathcal{D}_A(\rho) \xrightarrow{\quad} = 0 \quad \text{Classical}$$

Discord for top quarks

Quantum Discord

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$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}).$$



Mutual Information
(all correlations)

Conditional information
(only classical correlations)

$\mathcal{D}_A(\rho)$	$= 0$	Classical
	$\neq 0$	Quantum (Can have separable states w $D_A \neq 0$)

Discord for top quarks

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Discord for top quarks

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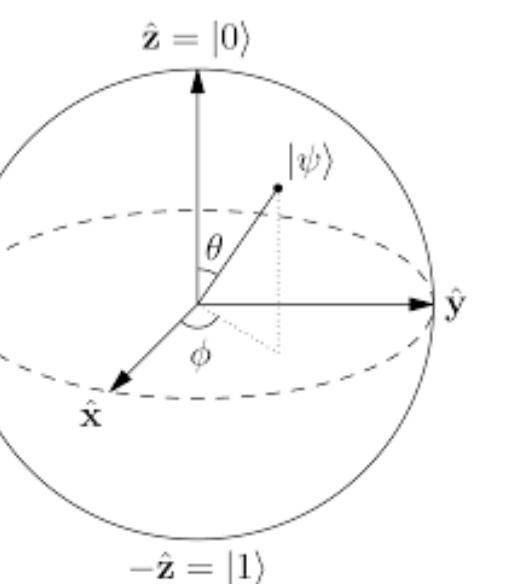
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$$\Pi_{\pm \hat{n}} = \mathbb{I}_2 \otimes |\pm n\rangle \langle \pm n|$$

$$(\vec{\sigma} \cdot \hat{n} |\pm n\rangle = \pm |\pm n\rangle)$$



Discord for top quarks

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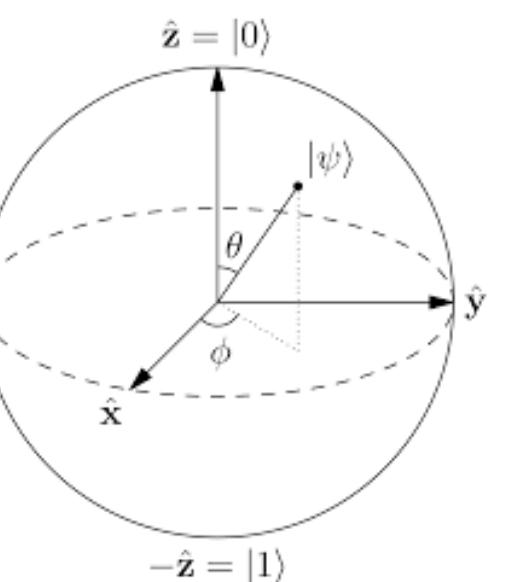
$$(\vec{\sigma} \cdot \hat{n} |\pm n\rangle = \pm |\pm n\rangle)$$

$$\xrightarrow{\hspace{1cm}}$$

$$S(\rho_A | \rho_B; \hat{n}) = p_{+\hat{n}} S(\rho_{+\hat{n}}) + p_{-\hat{n}} S(\rho_{-\hat{n}})$$

$$\rho_{\pm \hat{n}} = \frac{1}{p_{\pm \hat{n}}} \text{tr}_B (\Pi_{\pm \hat{n}} \rho_{AB} \Pi_{\pm \hat{n}})$$

$$p_{\pm \hat{n}} = \text{tr} (\Pi_{\pm \hat{n}} \rho_{AB} \Pi_{\pm \hat{n}})$$



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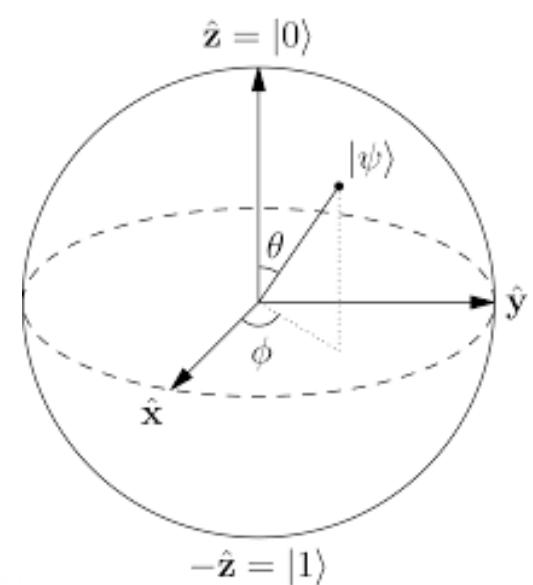
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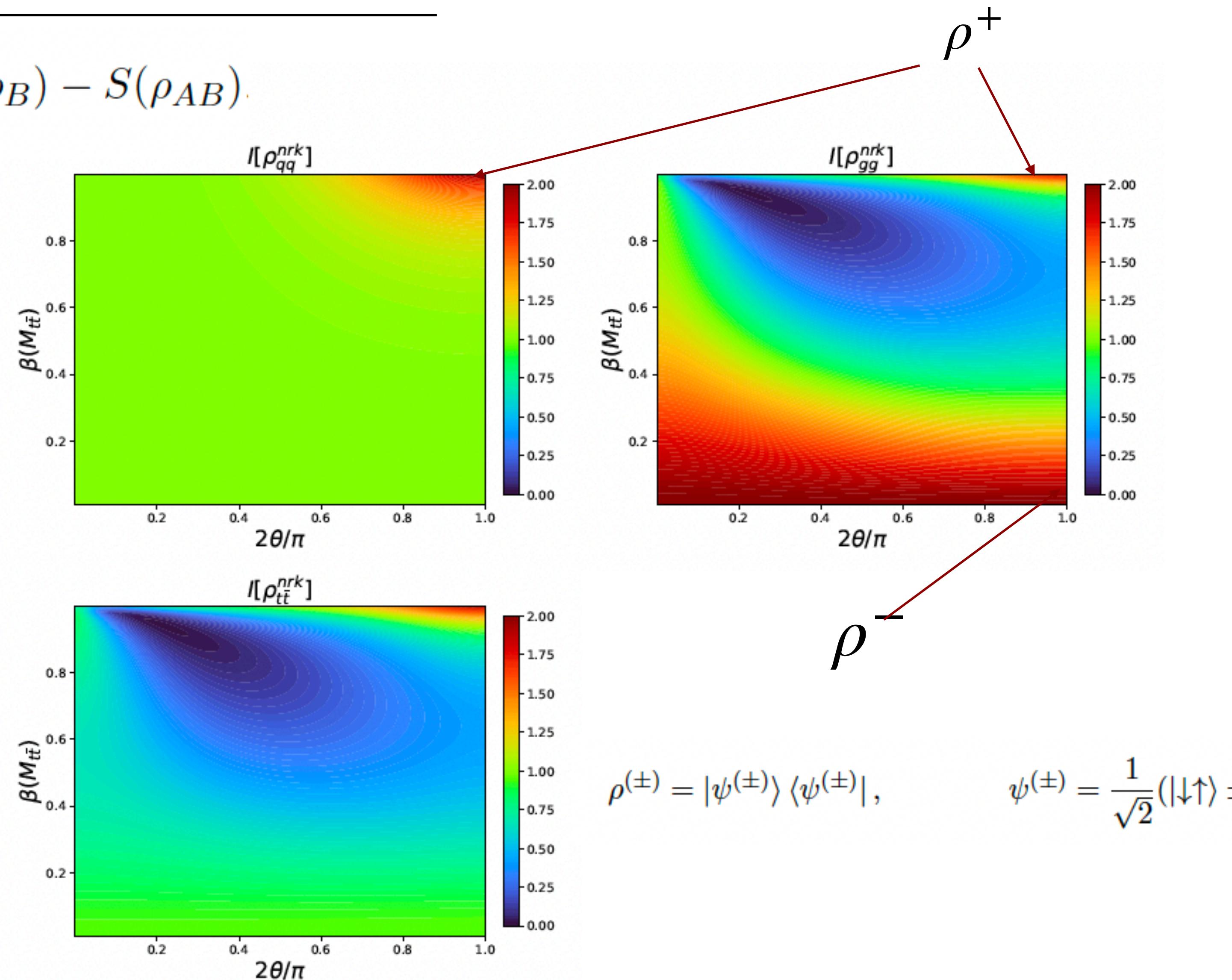
$$p_{\pm \hat{n}} = \text{tr} (\Pi_{\pm \hat{n}} \rho_{AB} \Pi_{\pm \hat{n}})$$



$$J_A(\rho_{AB}) = S(\rho_A) - \min_{\hat{n}} (p_{+\hat{n}} S(\rho_{+\hat{n}}) + p_{-\hat{n}} S(\rho_{-\hat{n}}))$$

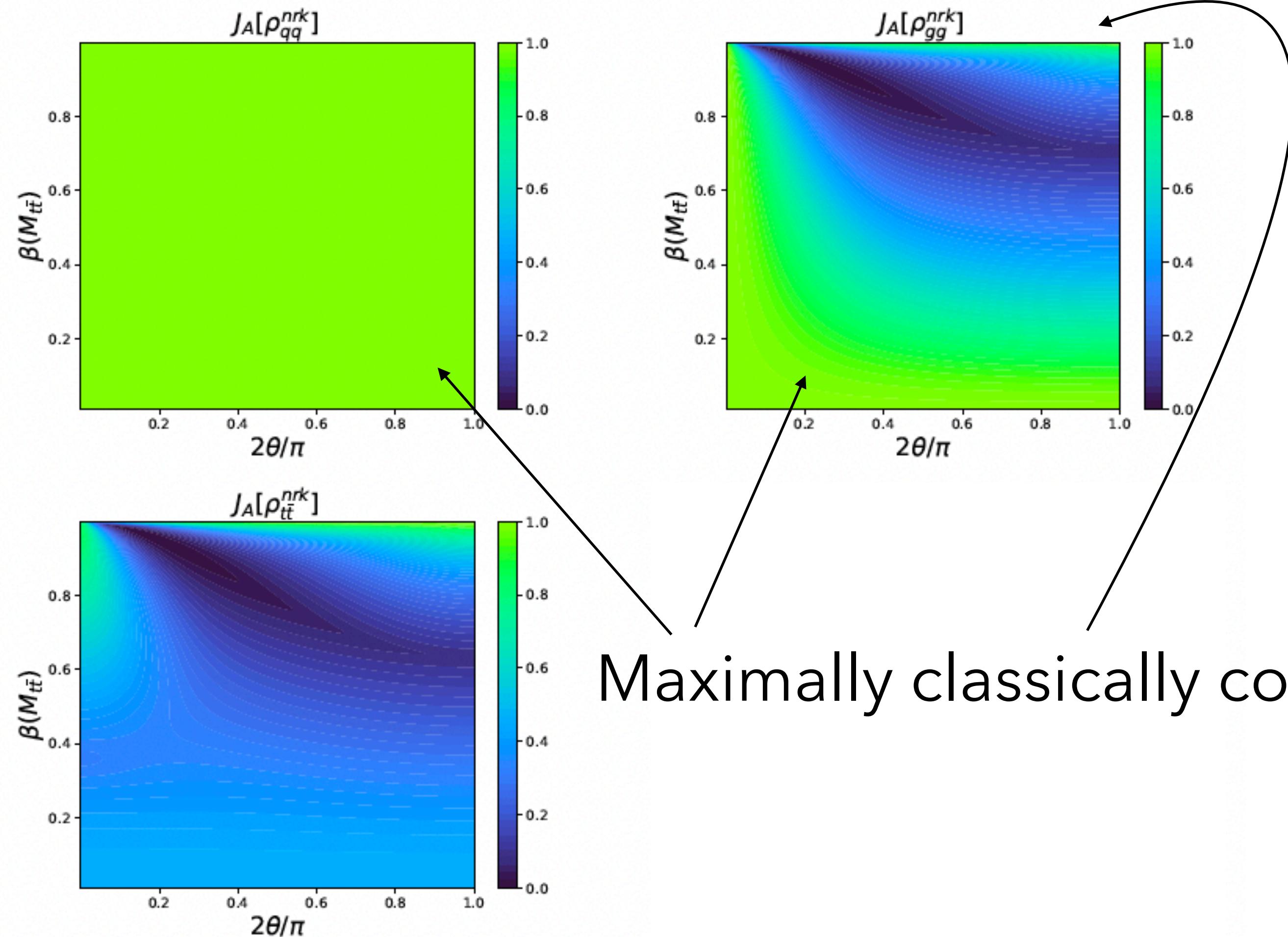
Discord for top quarks

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}).$$



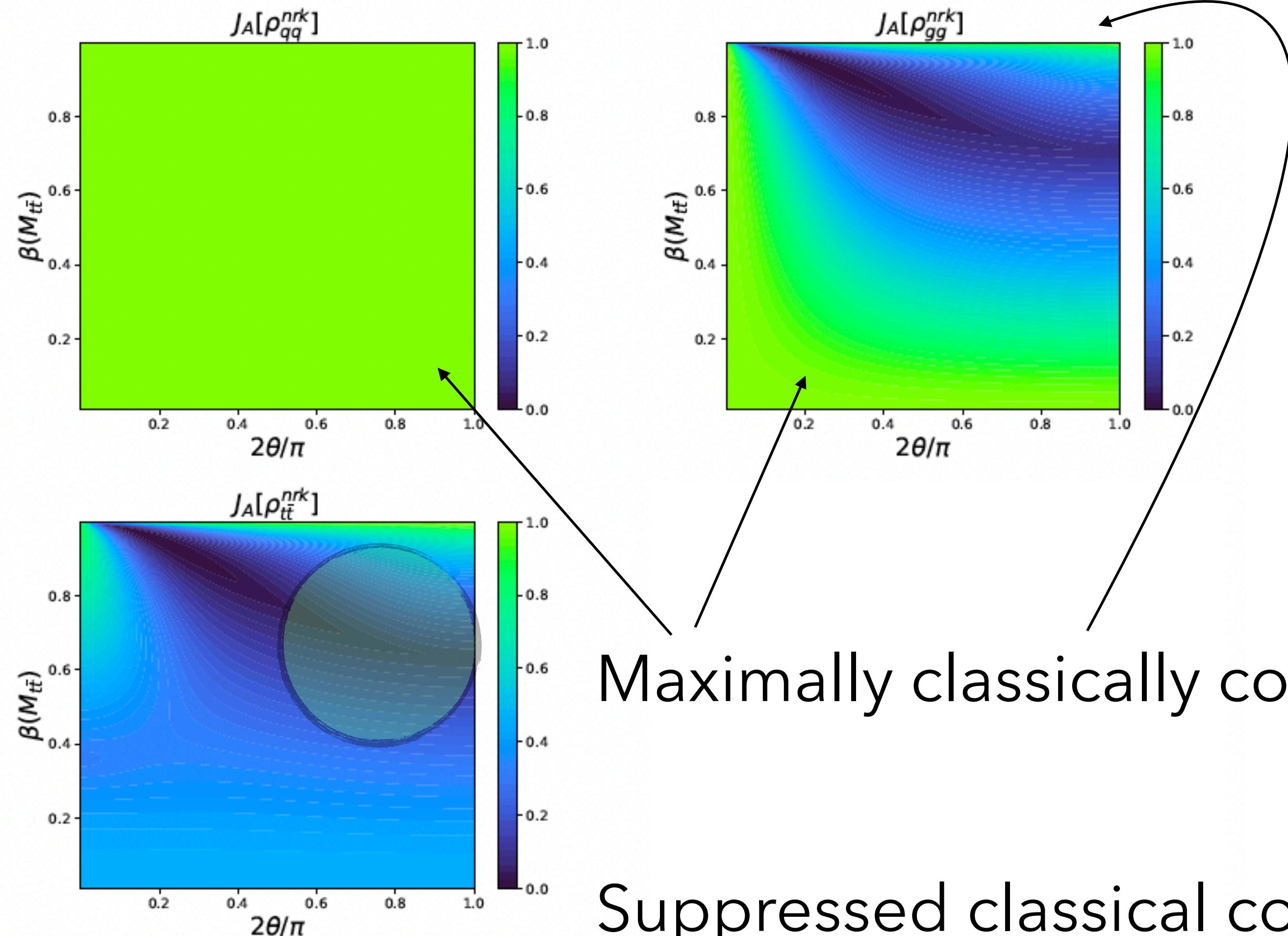
Discord for top quarks

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Discord for top quarks

$$J_A(\rho_{AB}) = S(\rho_A) - \min_{\hat{n}} (p_{+\hat{n}} S(\rho_{+\hat{n}}) + p_{-\hat{n}} S(\rho_{-\hat{n}}))$$



Maximally classically correlated

Suppressed classical correlations

Discord for top quarks

$$\mathcal{D}_A(\rho) = I(\rho) - J_A(\rho)$$

$$= S(\rho_B) - S(\rho_{AB}) + \min_{\hat{n}} \left(p_{+\hat{n}} S(\rho_{+\hat{n}}) + p_{-\hat{n}} S(\rho_{-\hat{n}}) \right)$$

$$\rho = \frac{1}{4} \left[\mathbf{1}_2 \otimes \mathbf{1}_2 + \sum_{i=1}^3 B_i^+ (\sigma_i \otimes \mathbf{1}_2) + \sum_{i=1}^3 B_j^- (\mathbf{1}_2 \otimes \sigma_j) + \sum_{i,j=1}^3 C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

$$p_{\pm\hat{n}} = \frac{1 \pm \hat{\mathbf{n}} \cdot \mathbf{B}^-}{2}, \quad \rho_{\pm\hat{n}} = \frac{\mathbb{I}_2 + \mathbf{B}_{\pm\hat{n}}^+ \cdot \boldsymbol{\sigma}}{2}, \quad \mathbf{B}_{\pm\hat{n}}^+ = \frac{\mathbf{B}^+ \pm \mathbf{C} \cdot \hat{\mathbf{n}}}{1 \pm \hat{\mathbf{n}} \cdot \mathbf{B}^-}.$$

Discord for top quarks

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$$\begin{array}{ccc} U_A \otimes V_B & \longrightarrow & U_A^\dagger \otimes V_B^\dagger \\ & \searrow & \swarrow \\ & \rho'_{AB} = \frac{1}{4} \left(\mathbb{I}_4 + \sum_i \Lambda_i \sigma_i \otimes \sigma_i \right) & \end{array}$$

Λ_i = Singular values of C_{ij} in any basis of choice

Discord for top quarks

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$$= S(\rho_B) - S(\rho_{AB}) + \min_{\hat{n}} (p_{+\hat{n}} S(\rho_{+\hat{n}}) + p_{-\hat{n}} S(\rho_{-\hat{n}}))$$

$$D_A(\rho'_{AB}) = 1 + \sum_i e_i \log_2 (e_i) - \frac{1}{2}(1 + \lambda) \log_2 \left(\frac{1 + \lambda}{2} \right) - \frac{1}{2}(1 - \lambda) \log_2 \left(\frac{1 - \lambda}{2} \right)$$

e_i = Eigenvalues of ρ_{AB}

λ = Largest singular value of C_{ij}

Discord for top quarks

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Calculation appeared before:

S. Luo, Quantum discord for two-qubit systems, Phys. Rev. A 77 (2008) 042303.

Discord for top quarks

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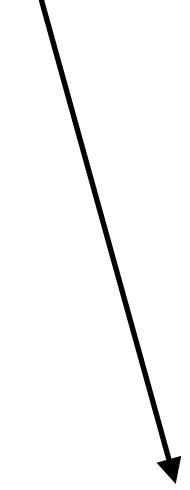
$$\begin{aligned} D_A(\rho_{AB}) &= 1 + \frac{1}{4}(1 - C_k - C_n - C_r) \log_2 \left(\frac{(1 - C_k - C_n - C_r)}{4} \right) \\ &\quad + \frac{1}{4}(1 + C_k - C_n + C_r) \log_2 \left(\frac{(1 + C_k - C_n + C_r)}{4} \right) \\ &\quad + \frac{1}{4}(1 + C_n - \Delta) \log_2 \left(\frac{(1 + C_n - \Delta)}{4} \right) + \frac{1}{4}(1 + C_n + \Delta) \log_2 \left(\frac{(1 + C_n + \Delta)}{4} \right) \\ &\quad - \frac{1}{2}(1 + \lambda) \log_2 \left(\frac{1 + \lambda}{2} \right) - \frac{1}{2}(1 - \lambda) \log_2 \left(\frac{1 - \lambda}{2} \right) \end{aligned}$$

where $\Delta = \sqrt{C_k^2 + 4C_{kr}^2 + C_r^2 - 2C_k C_r}$, and $\lambda = \max\{|C_n|, \frac{1}{2}|C_k + C_r - \Delta|, \frac{1}{2}|C_k + C_r + \Delta|\}$.



Discord at colliders

Experiments


$$\overline{C}_{ij} = \frac{1}{\sigma_{\Pi}} \int_{\Pi} d\Omega \frac{d\sigma}{d\Omega} C_{ij}(\Omega)$$

Fictitious states

Discord at colliders

Experiments

$$\bar{C}_{ij} = \frac{1}{\sigma_{\Pi}} \int_{\Pi} d\Omega \frac{d\sigma}{d\Omega} C_{ij}(\Omega)$$

Fictitious states

For entanglement, not a
big deal:

$$C[\bar{\rho}] \neq 0 \implies C[\rho] \neq 0$$

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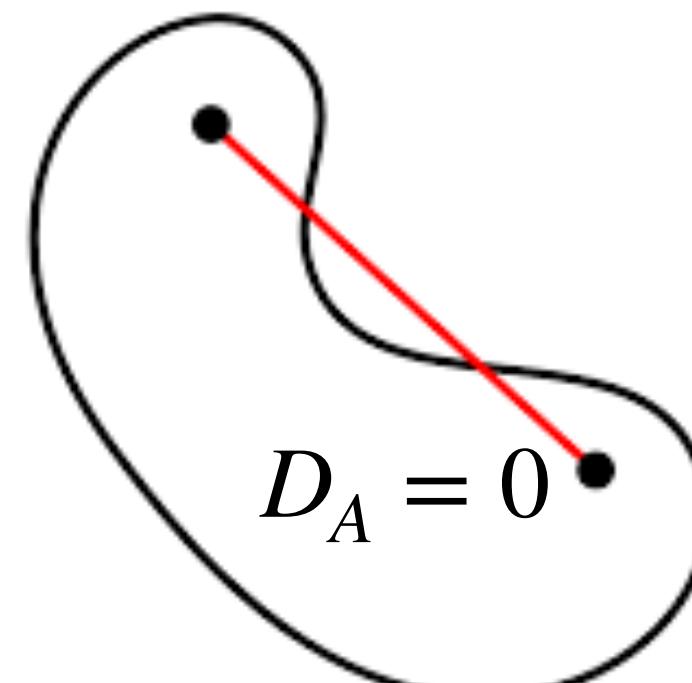
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- Set of zero-discord states not convex: mixtures of zero-discord states can lead to a fictitious states with $D_A[\bar{\rho}] \neq 0$.

$$\bar{\rho} = \lambda \rho_0 + (1 - \lambda) \rho'_0$$



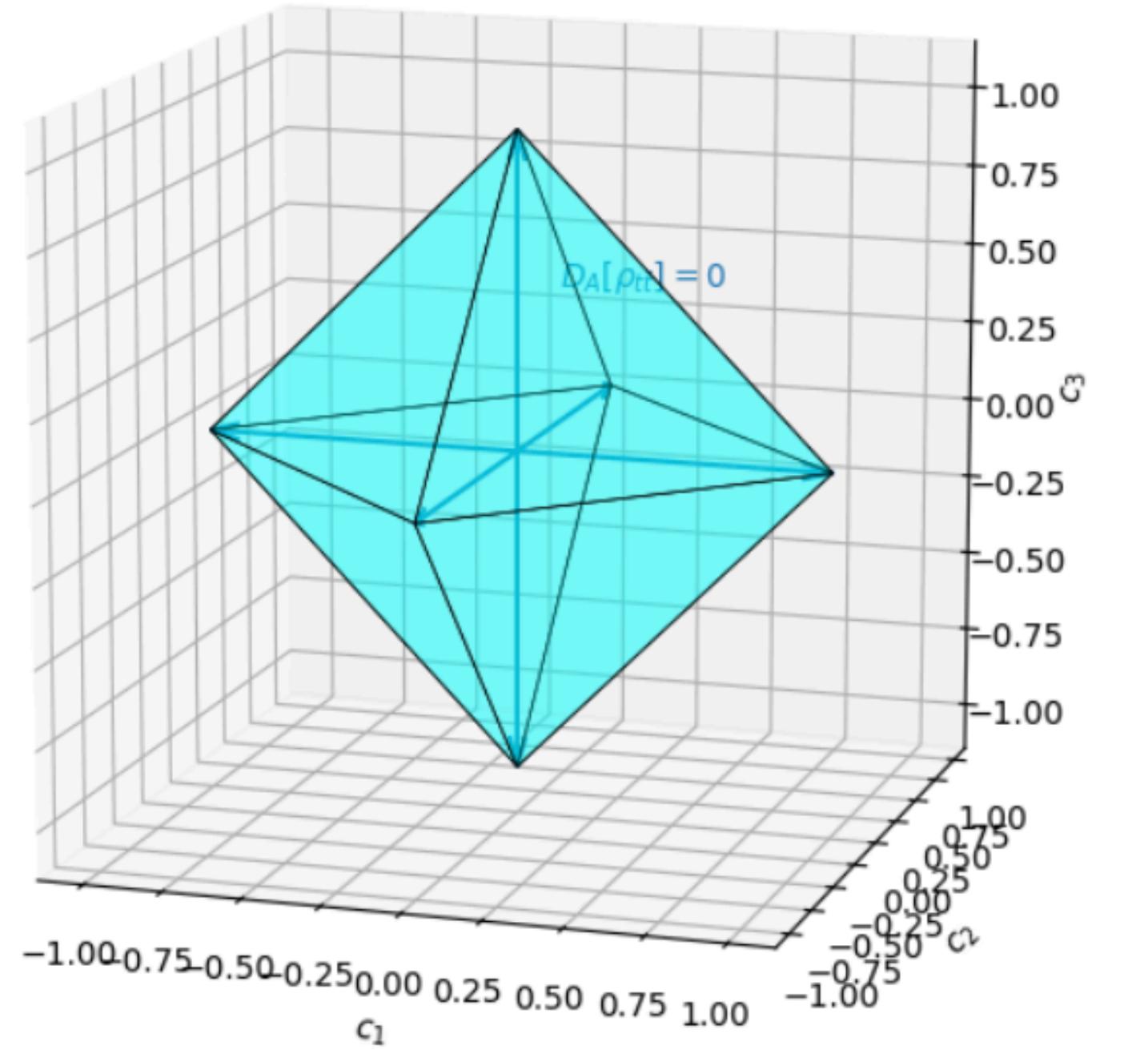
Discord at colliders

Quantum Discord and the Geometry of Bell-Diagonal States

Matthias D. Lang* and Carlton M. Caves

*Center for Quantum Information and Control, University of New Mexico,
MSC07-4220, Albuquerque, New Mexico 87131-0001, USA*

(Dated: July 25, 2018)



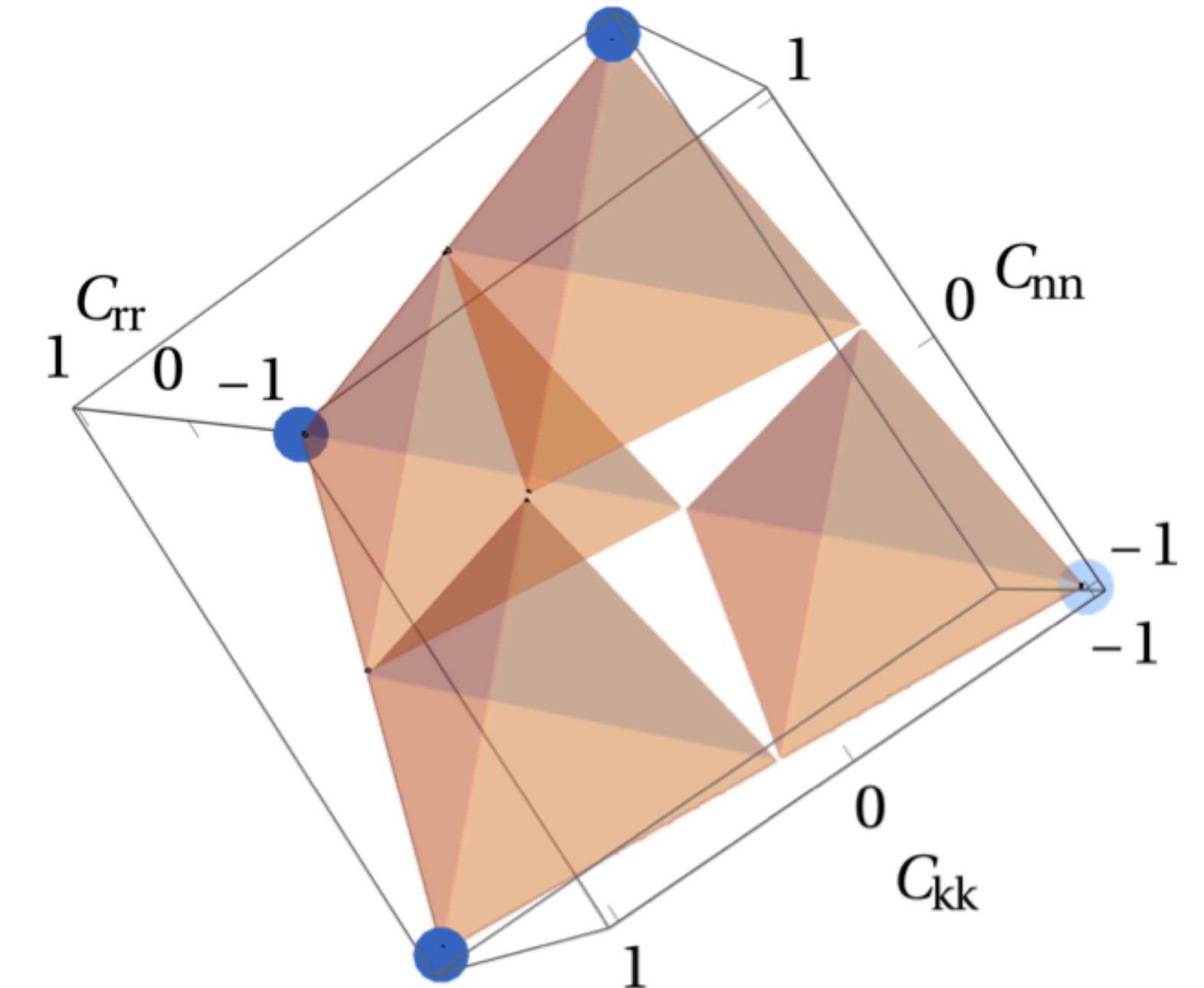
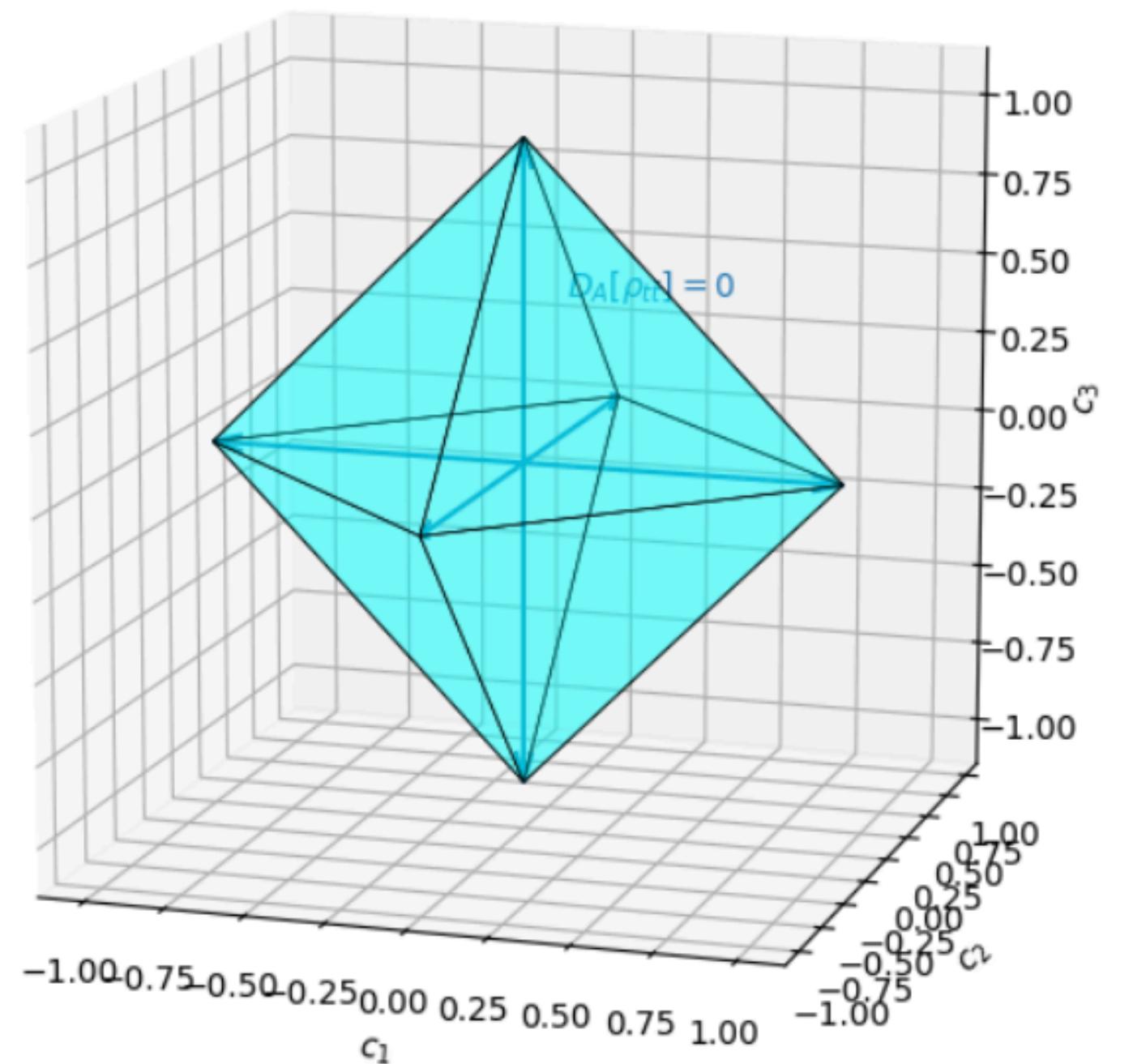
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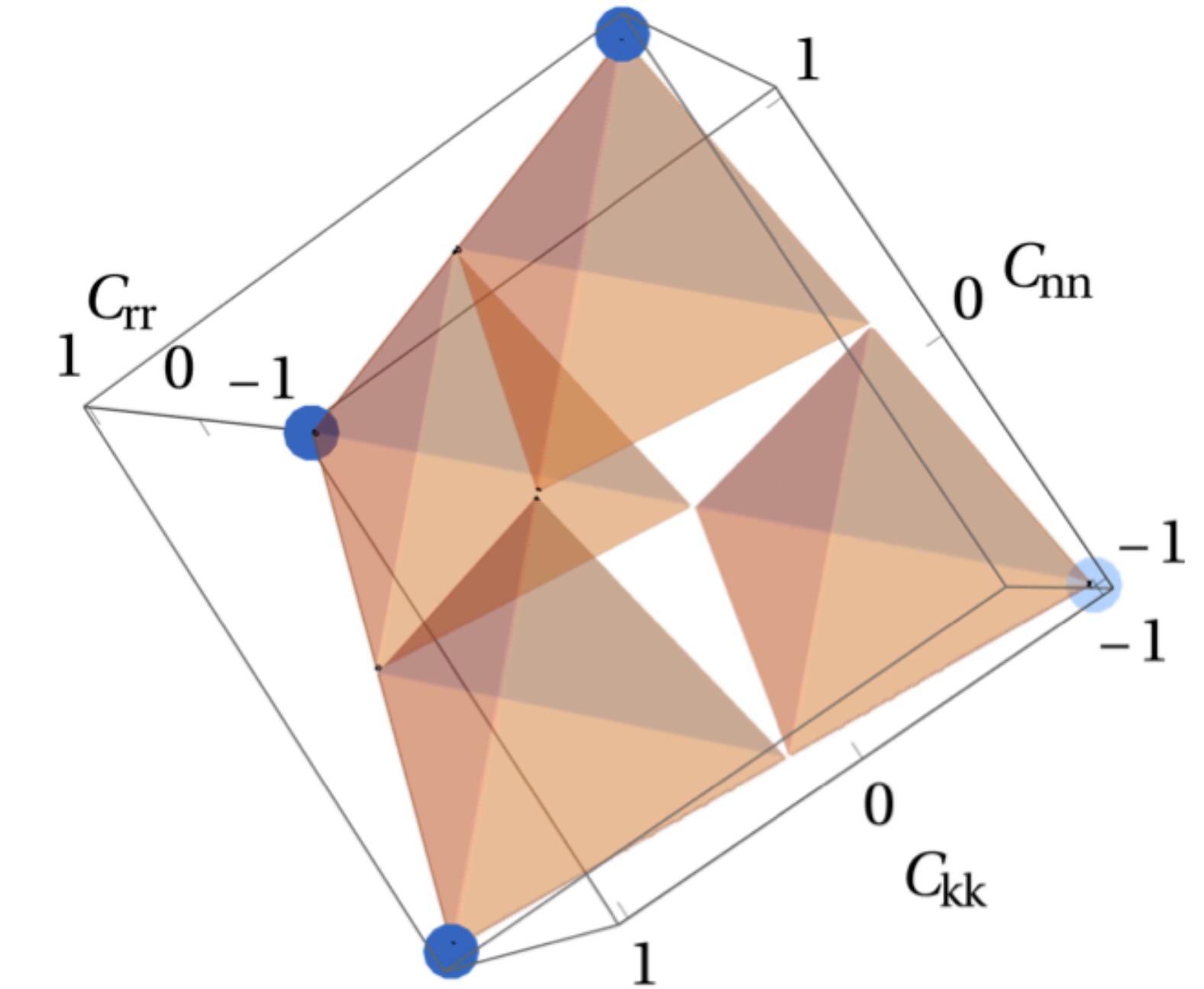
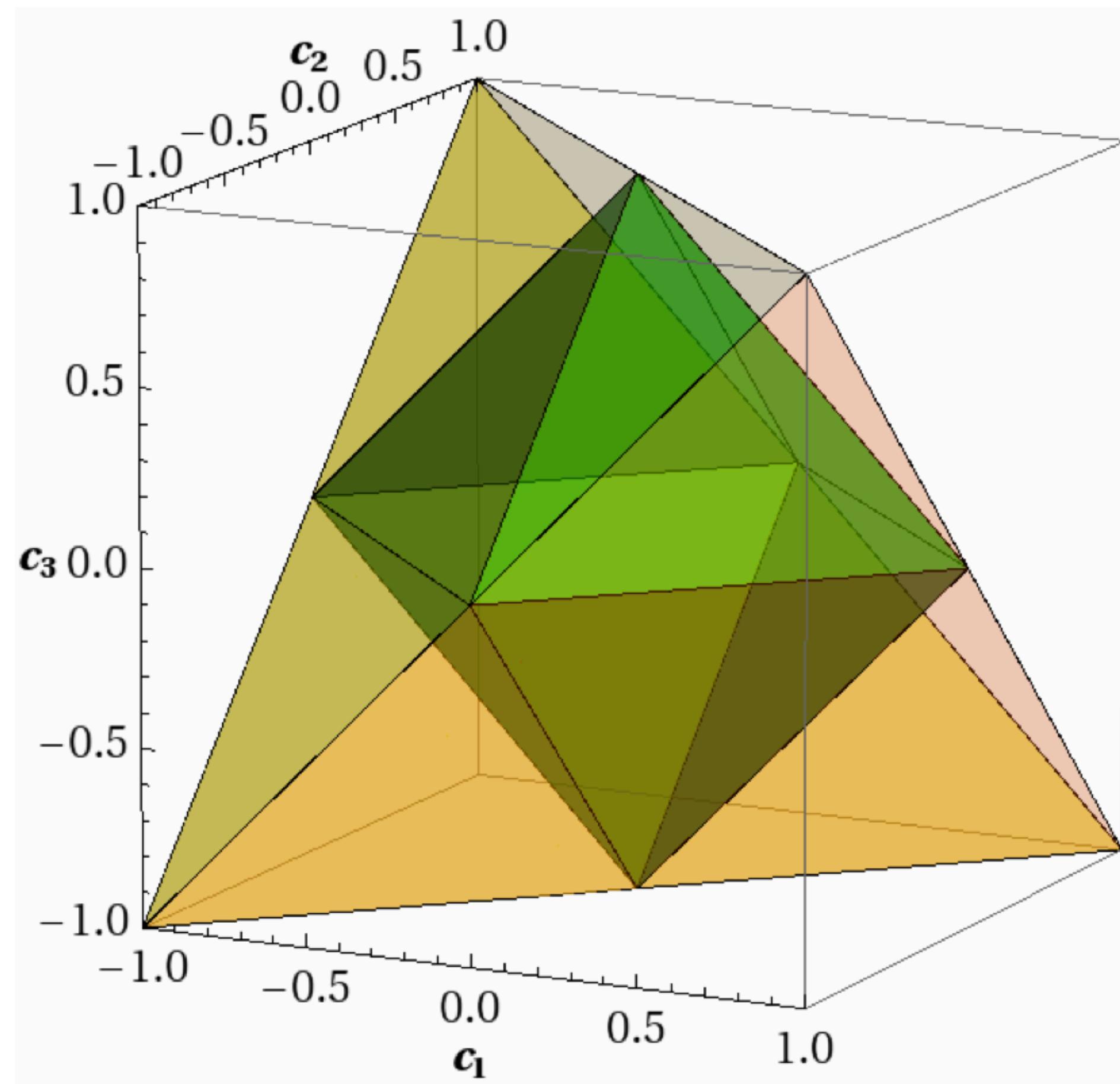
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Fictitious states

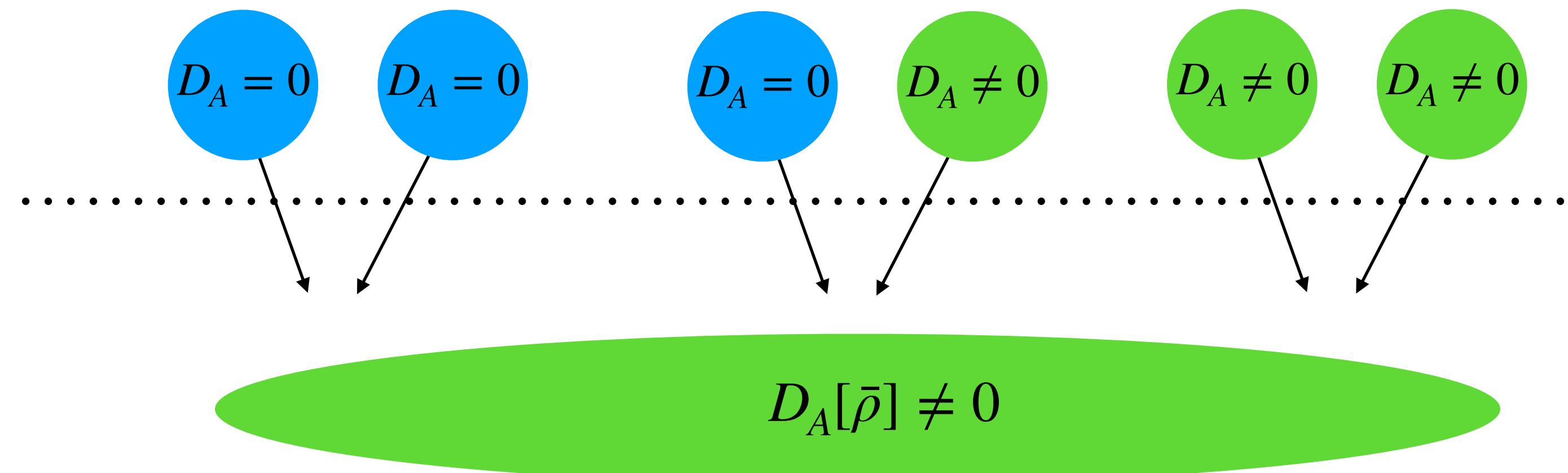
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Two solutions:

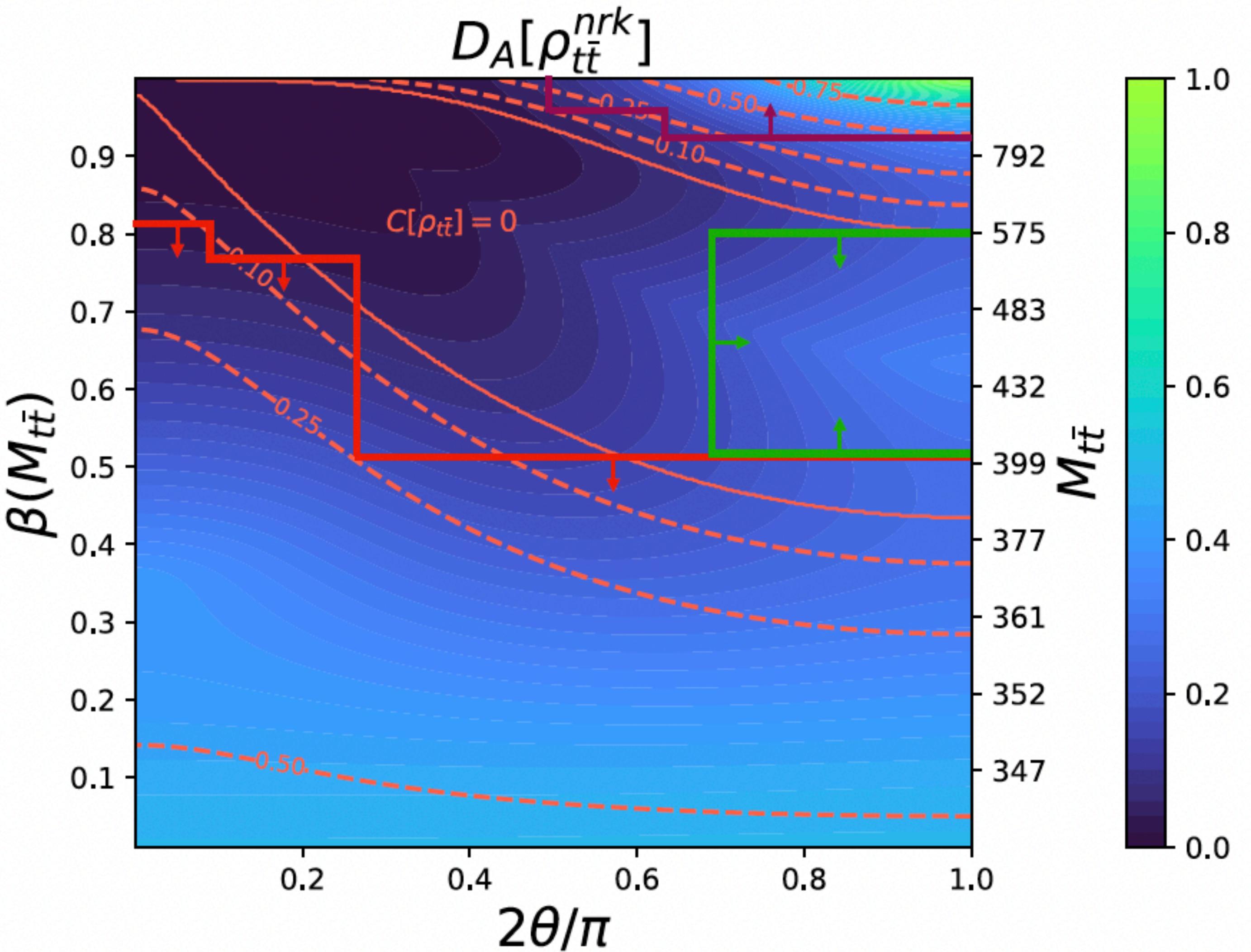
- Measure discord in regions of phase space which give an entangled fictitious state
- Restrict phase space to regions where all sub-states have $D_A[\rho] \neq 0$

$$D_A[\rho] \neq 0$$

Discord at colliders

Three signal regions:

- Threshold
- Separable
- Boosted

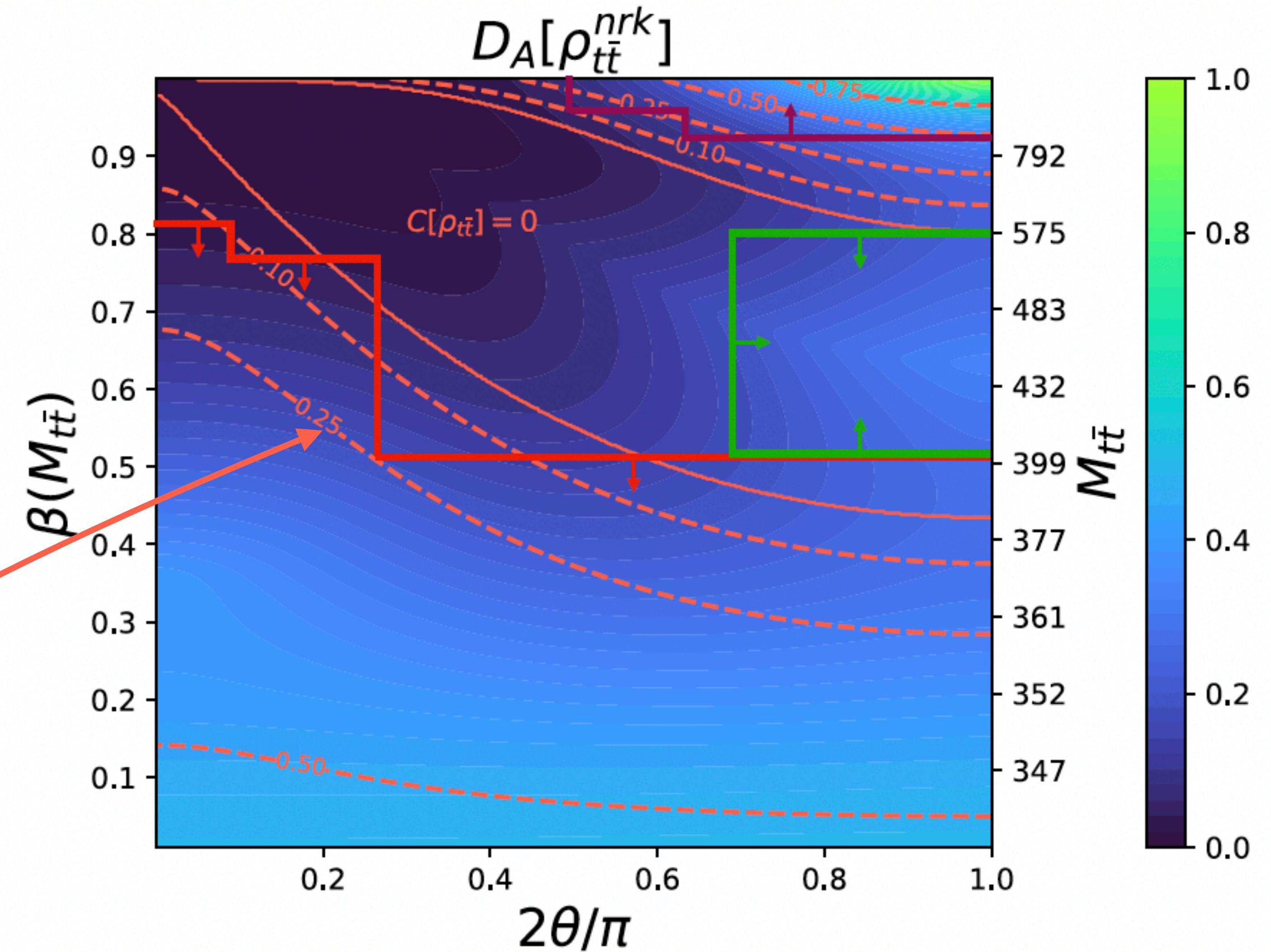


Discord at colliders

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Contours of concurrence



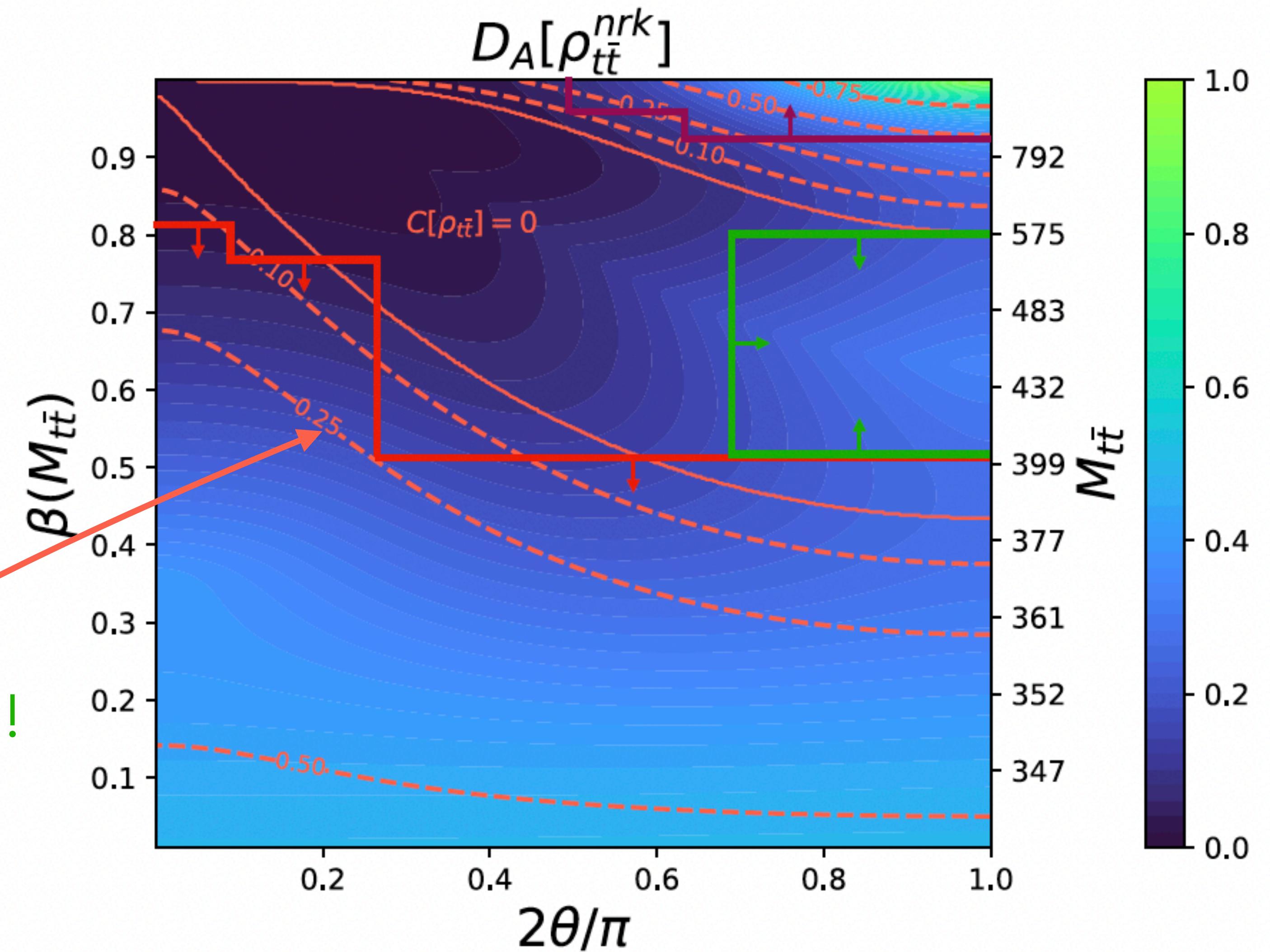
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Separable
states w
genuine
quantum
correlations!

Contours of concurrence



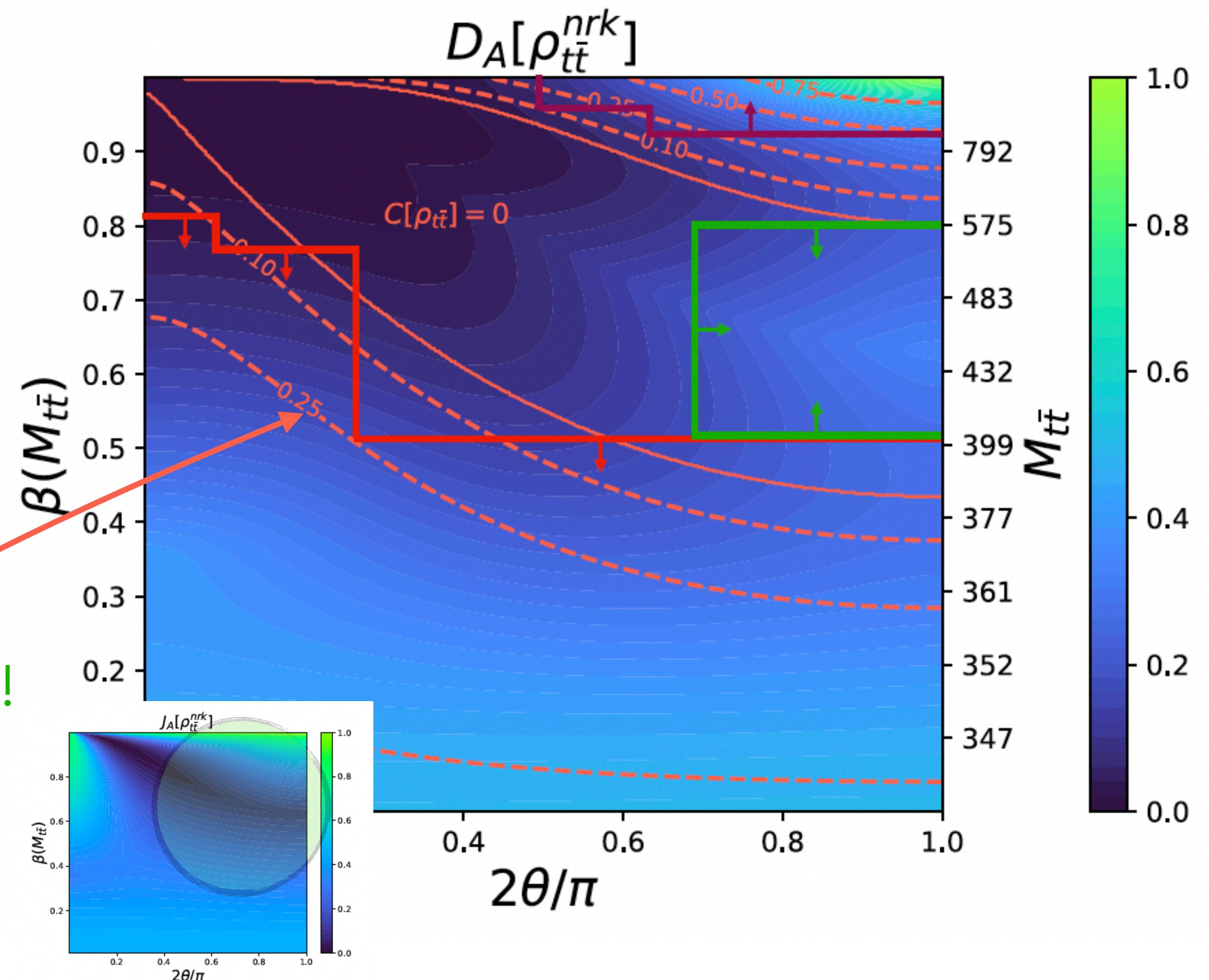
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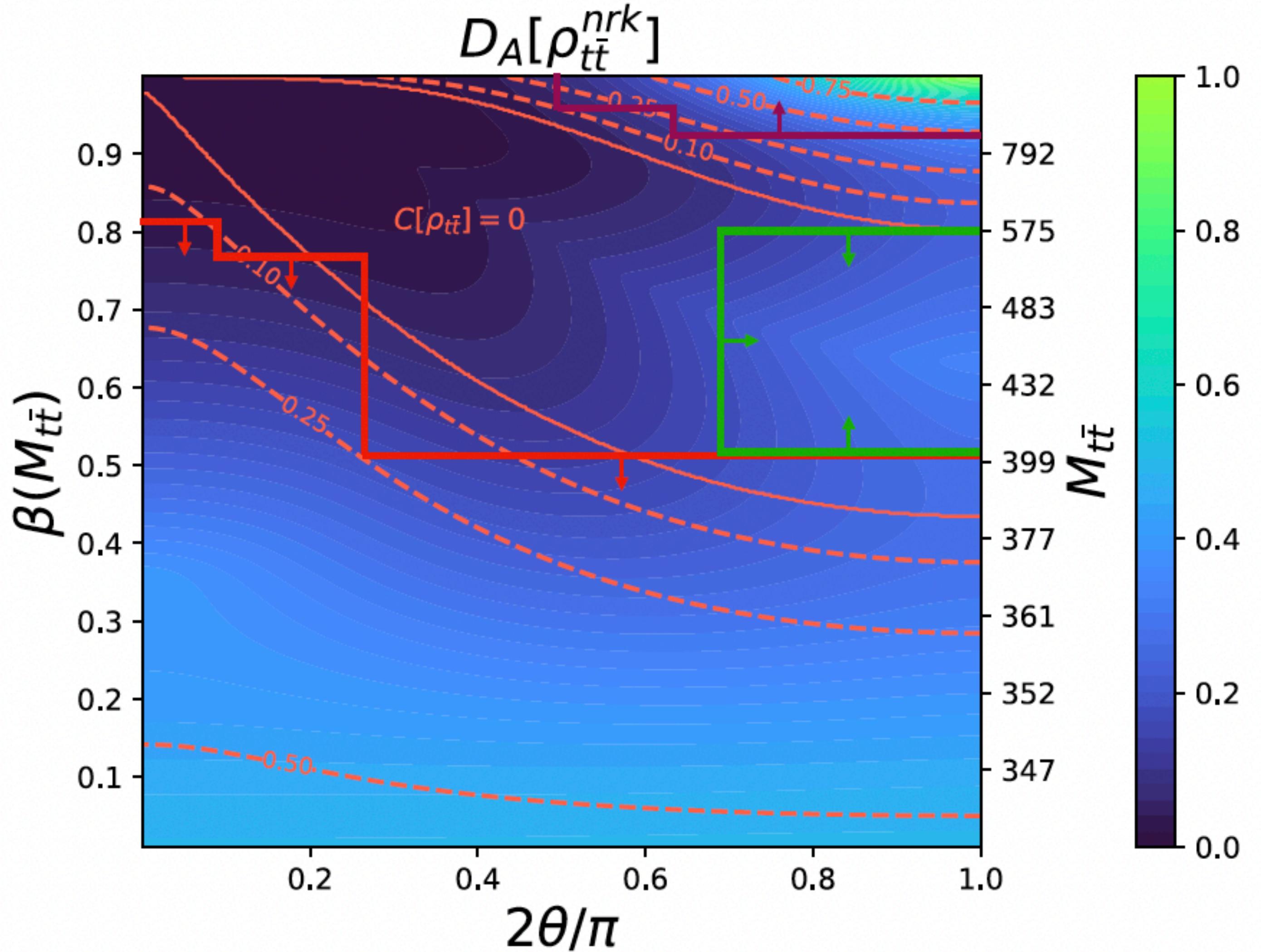
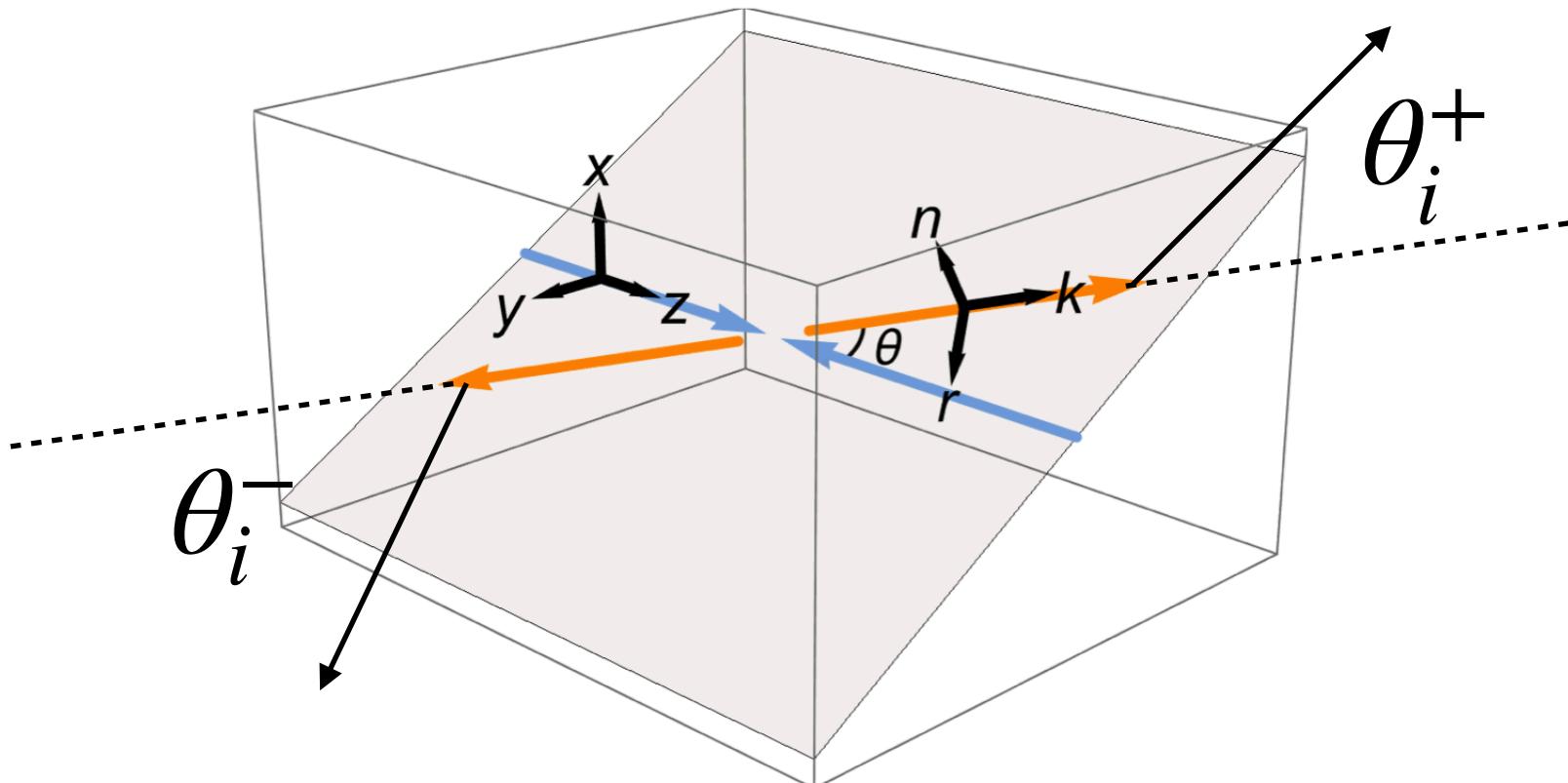
Separable
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Collider Analysis

$$pp \rightarrow t\bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + \cancel{E}_T.$$

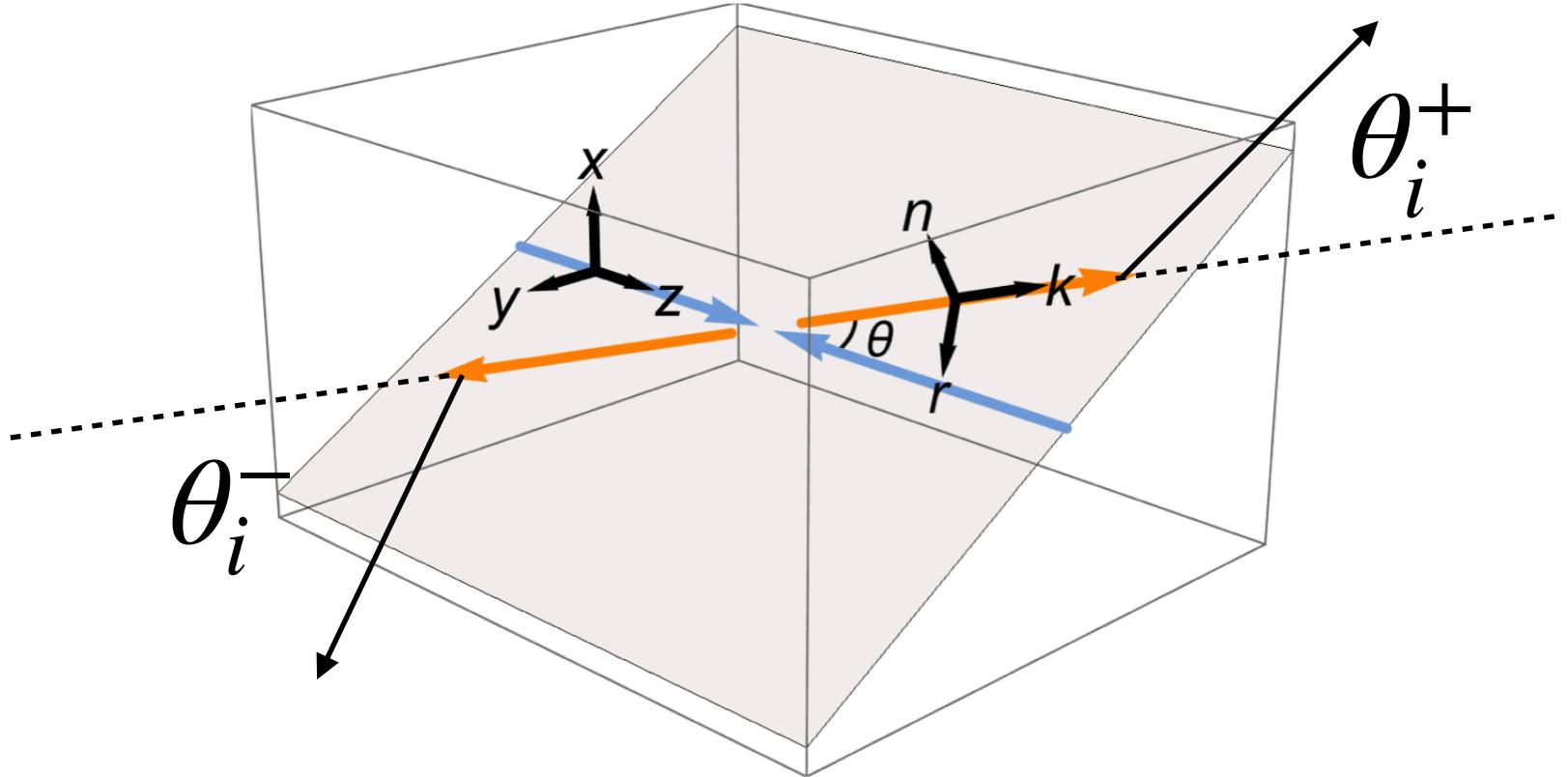
- Decay method



Collider Analysis

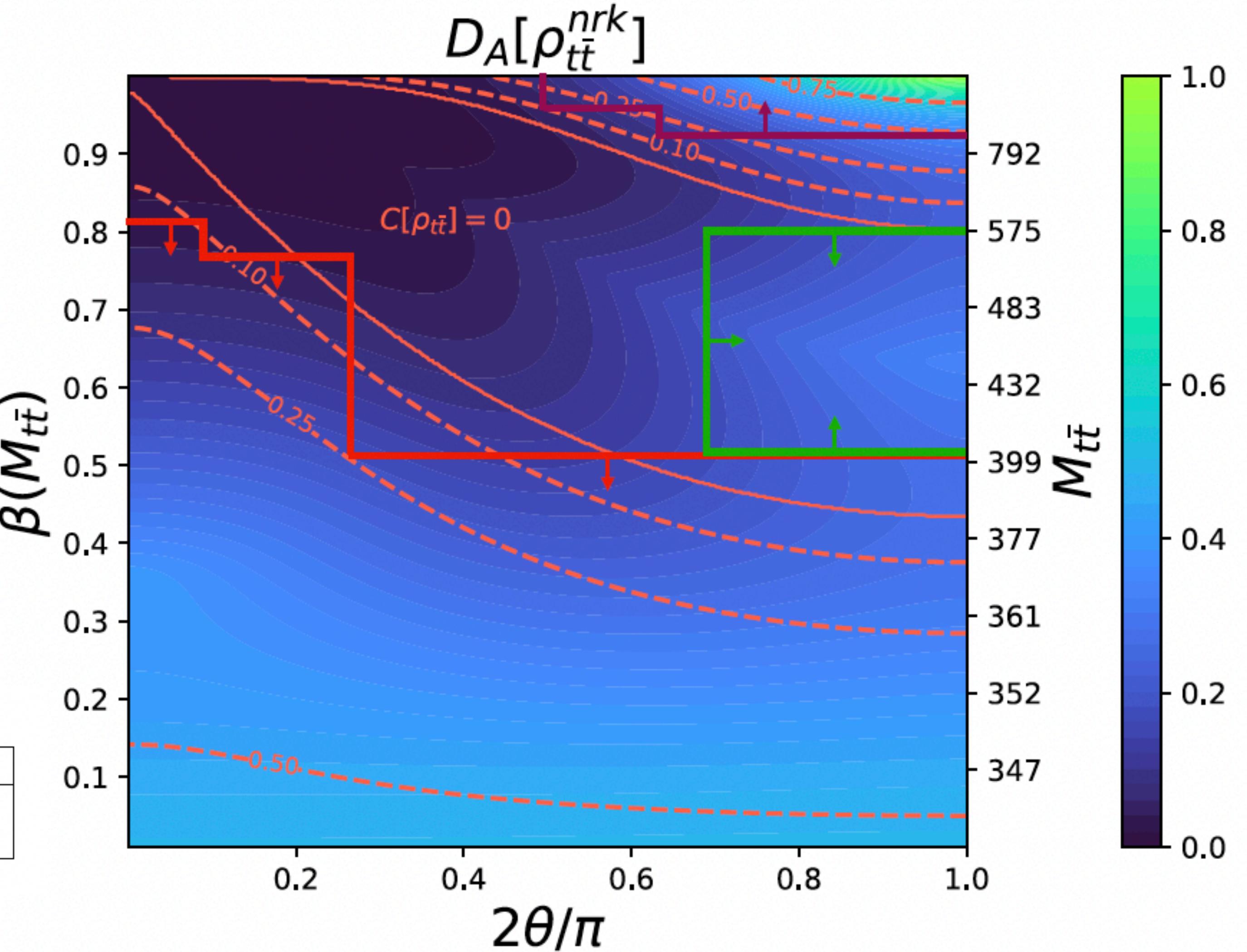
$$pp \rightarrow t\bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + \cancel{E}_T.$$

- Decay method



	Threshold Region		Separable Region		Boosted Region	
	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}})$	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}})$	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}})$
Parton						
Reconstructed	0.10	0.200 ± 0.003	0.28	0.255 ± 0.008	0.08	0.197 ± 0.003
		0.23 ± 0.04		0.18 ± 0.05		0.20 ± 0.05

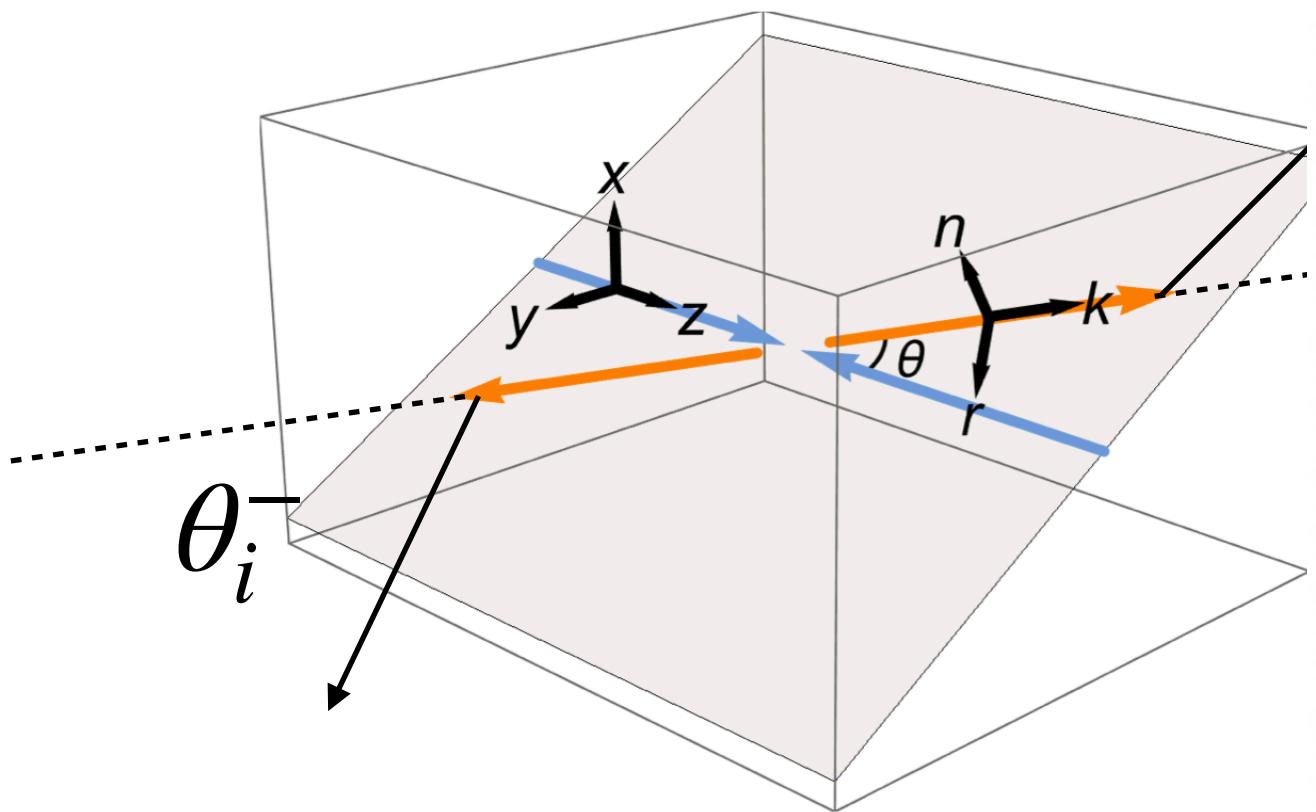
Uncertainties: statistical $\mathcal{L} = 139 \text{ fb}^{-1}$ + syst. (detector efficiencies + unfolding)



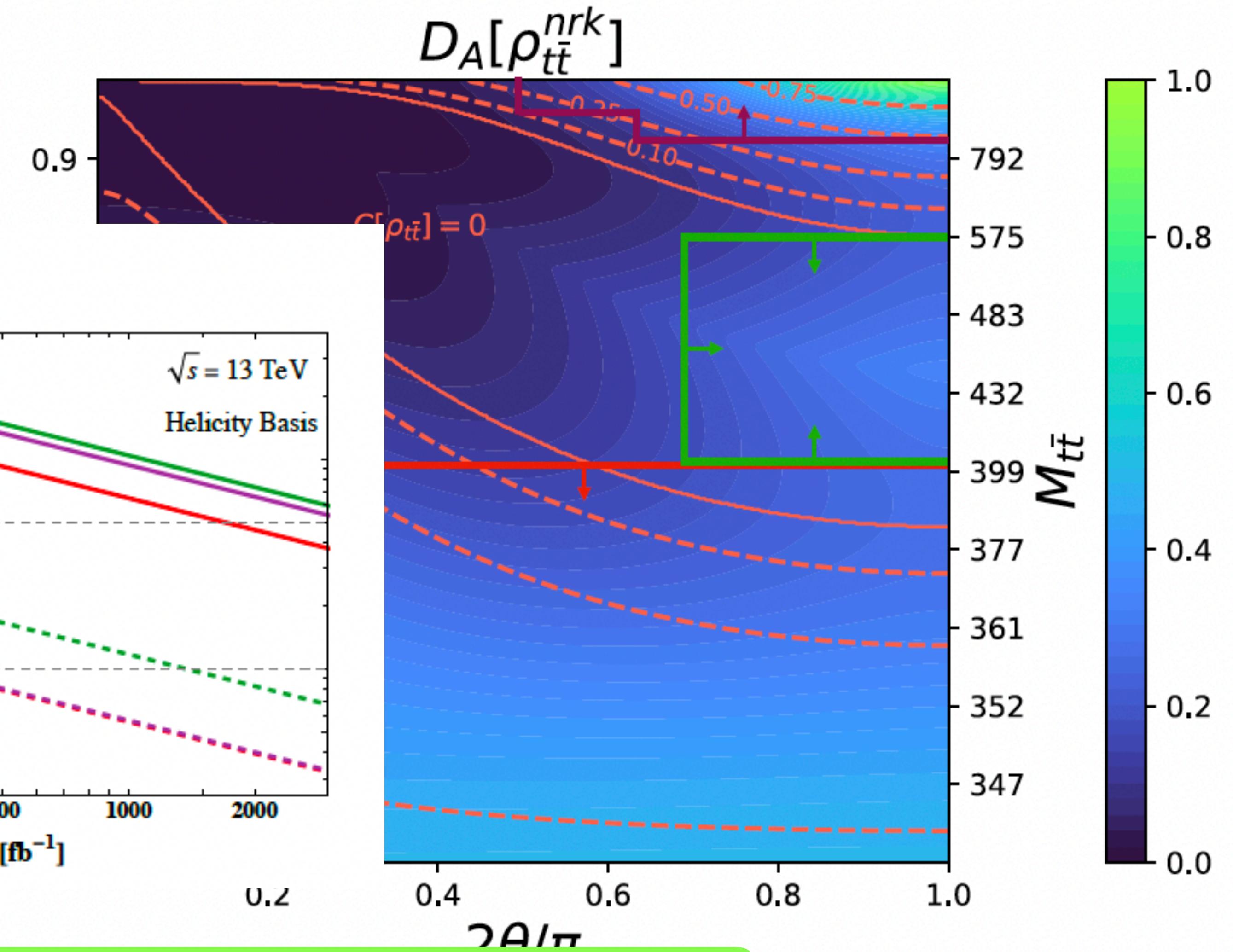
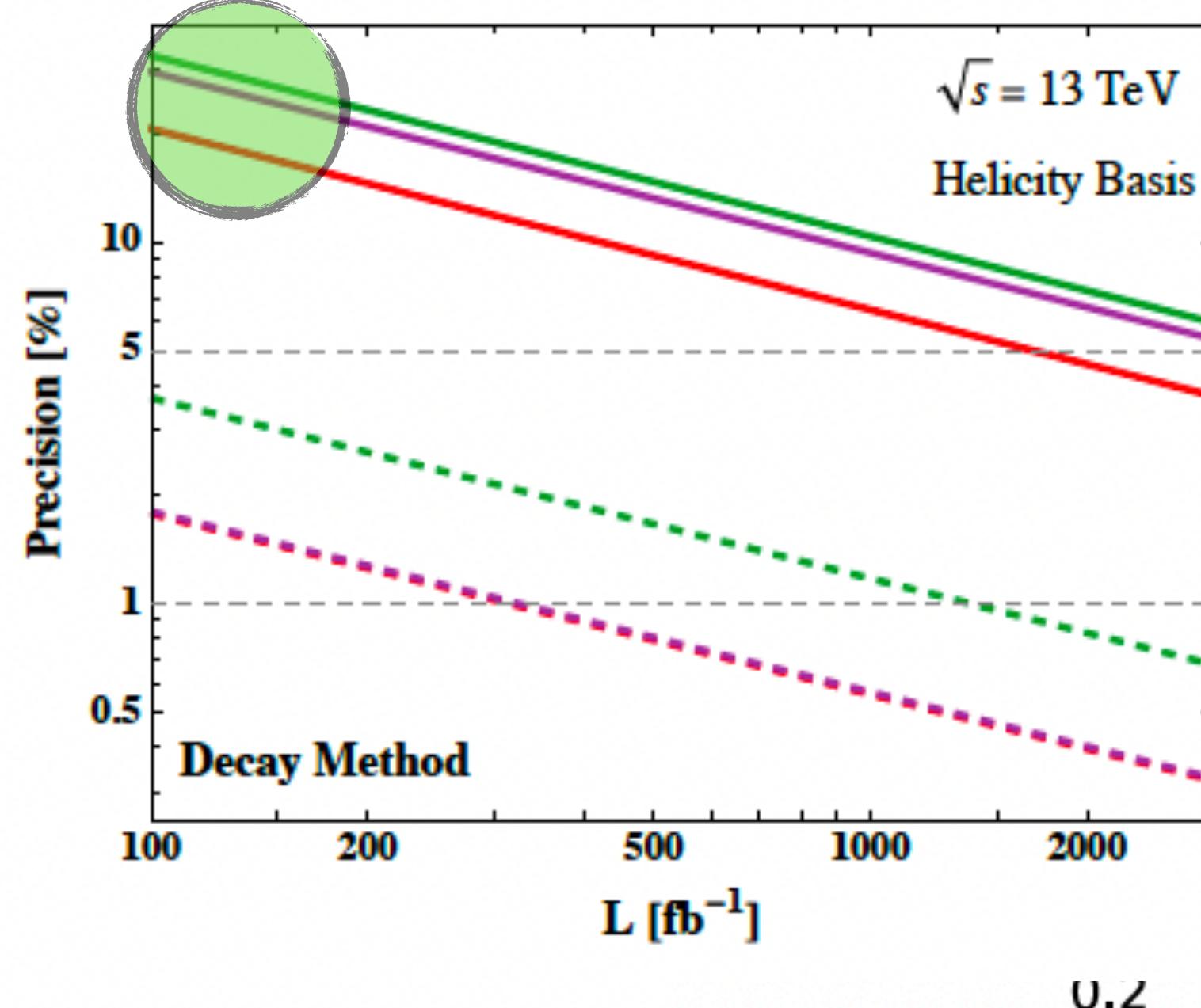
Collider Analysis

$$pp \rightarrow t\bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + \cancel{E}_T.$$

- Decay method



	Threshold Region		Separable Re	
	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}})$	$\langle \epsilon_{rec} \rangle$	D_A
Parton	0.10	0.200 ± 0.003	0.28	0.255
Reconstructed	0.10	0.23 ± 0.04	0.28	0.18



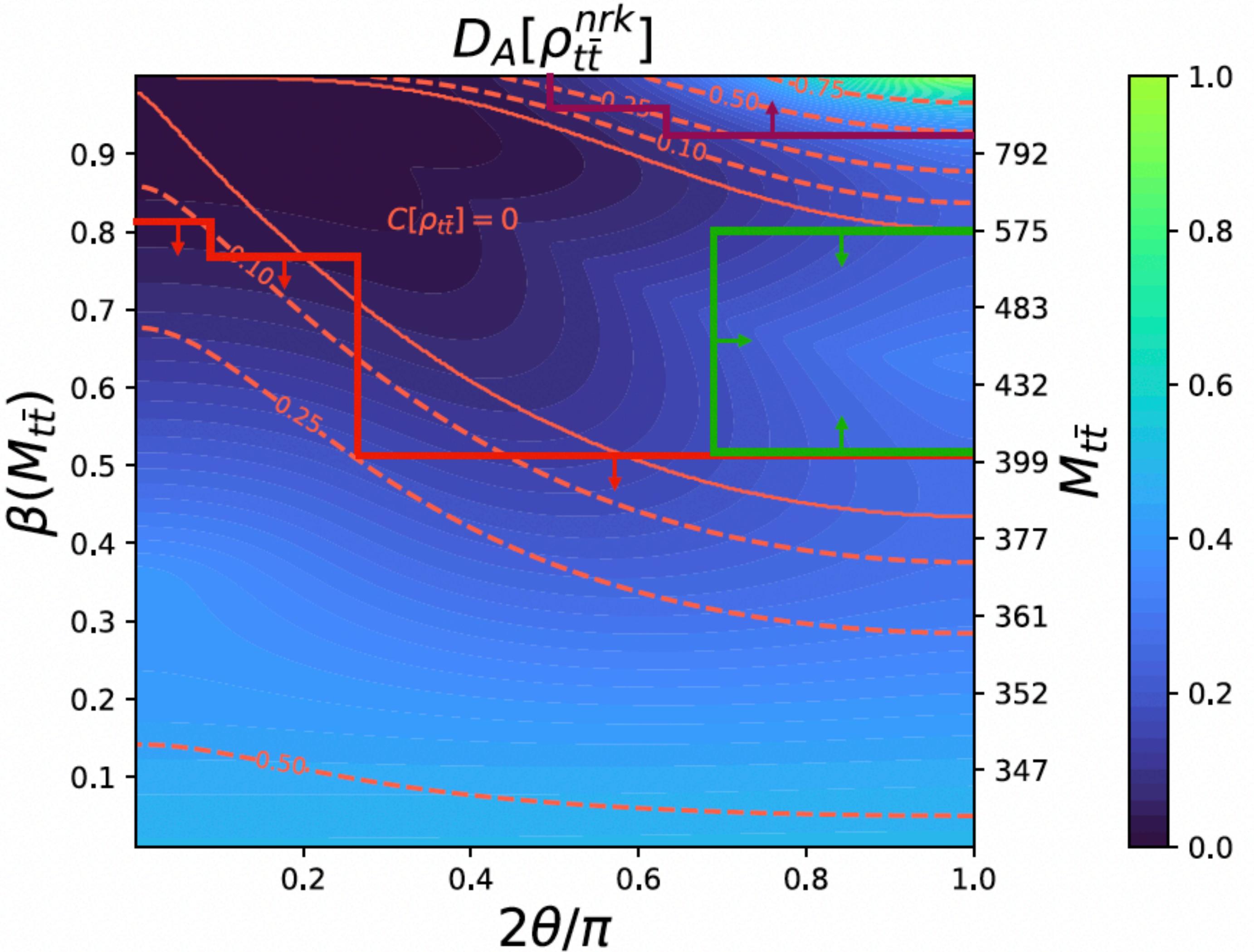
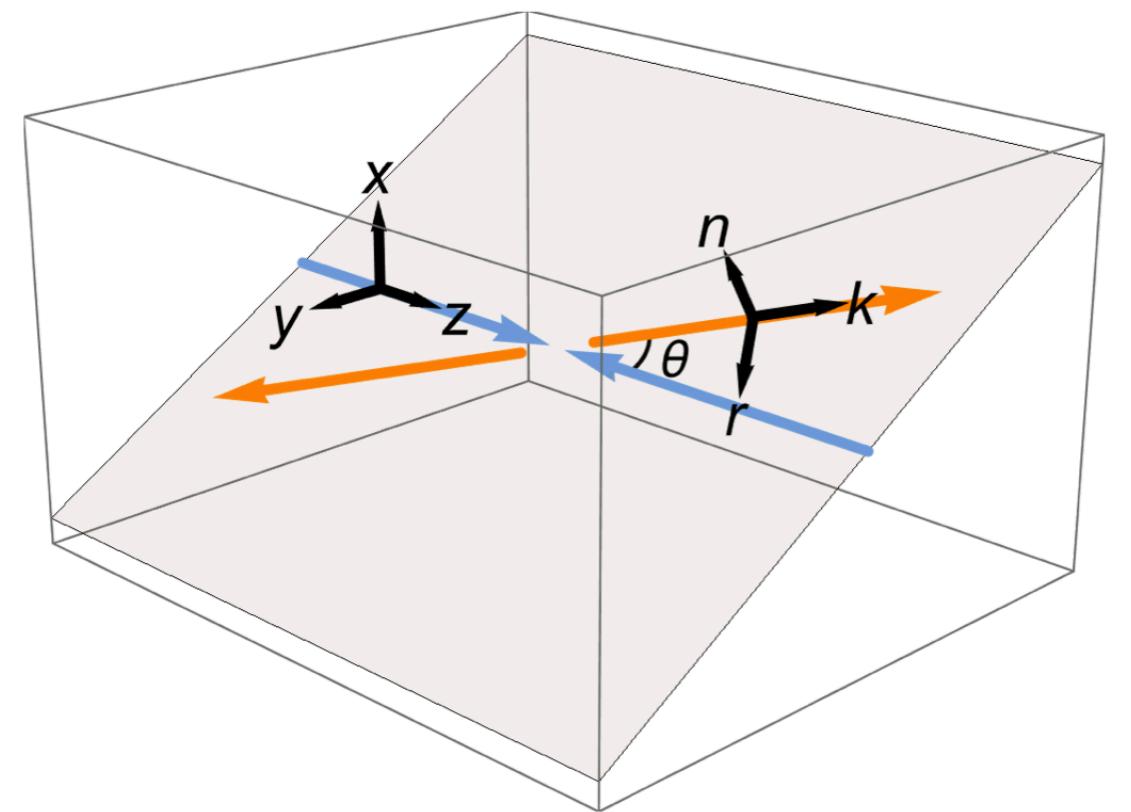
Uncertainties: statistical $\mathcal{L} = 139 \text{ fb}^{-1}$ + syst. (detector efficiencies + unfolding)

Est. precision w/ current data: ~20%

Collider Analysis

$$pp \rightarrow t\bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + \cancel{E}_T.$$

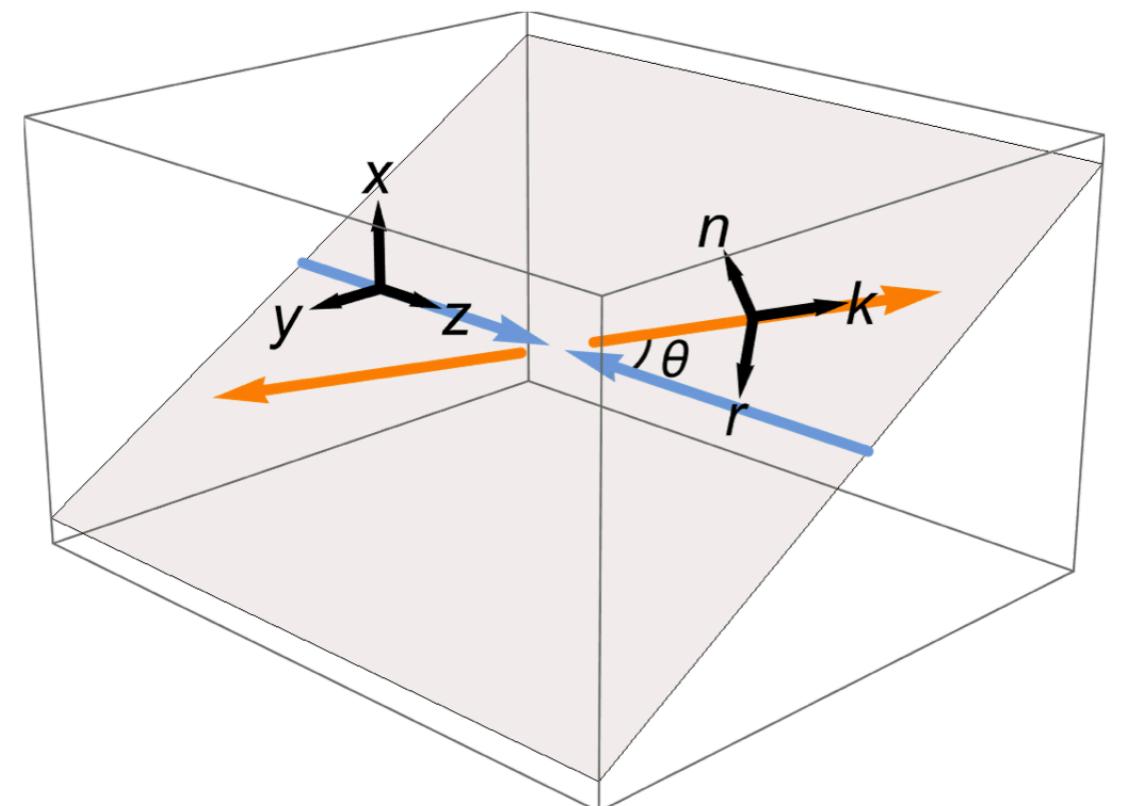
- Kinematic method



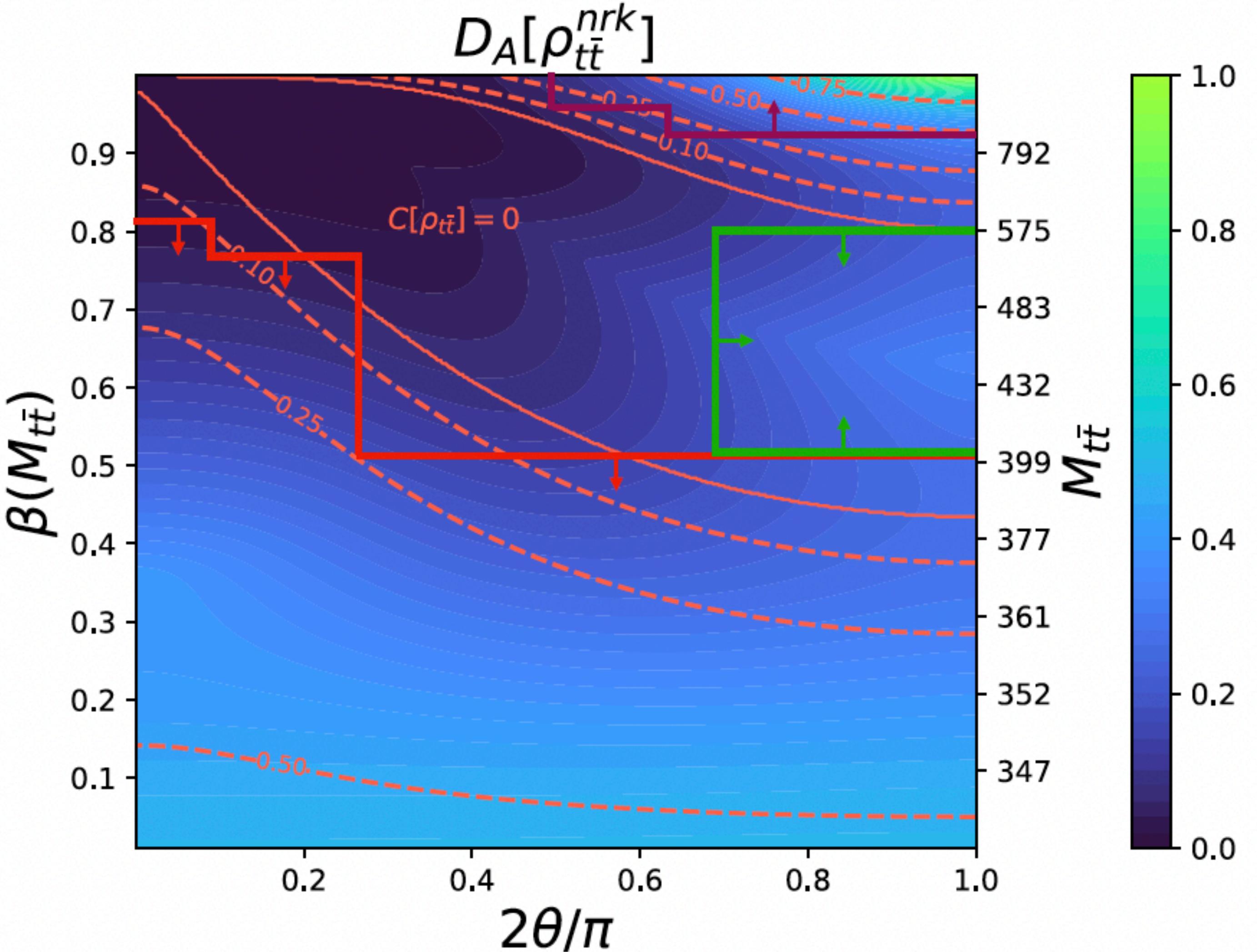
Collider Analysis

$$pp \rightarrow t\bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + \cancel{E}_T.$$

- Kinematic method



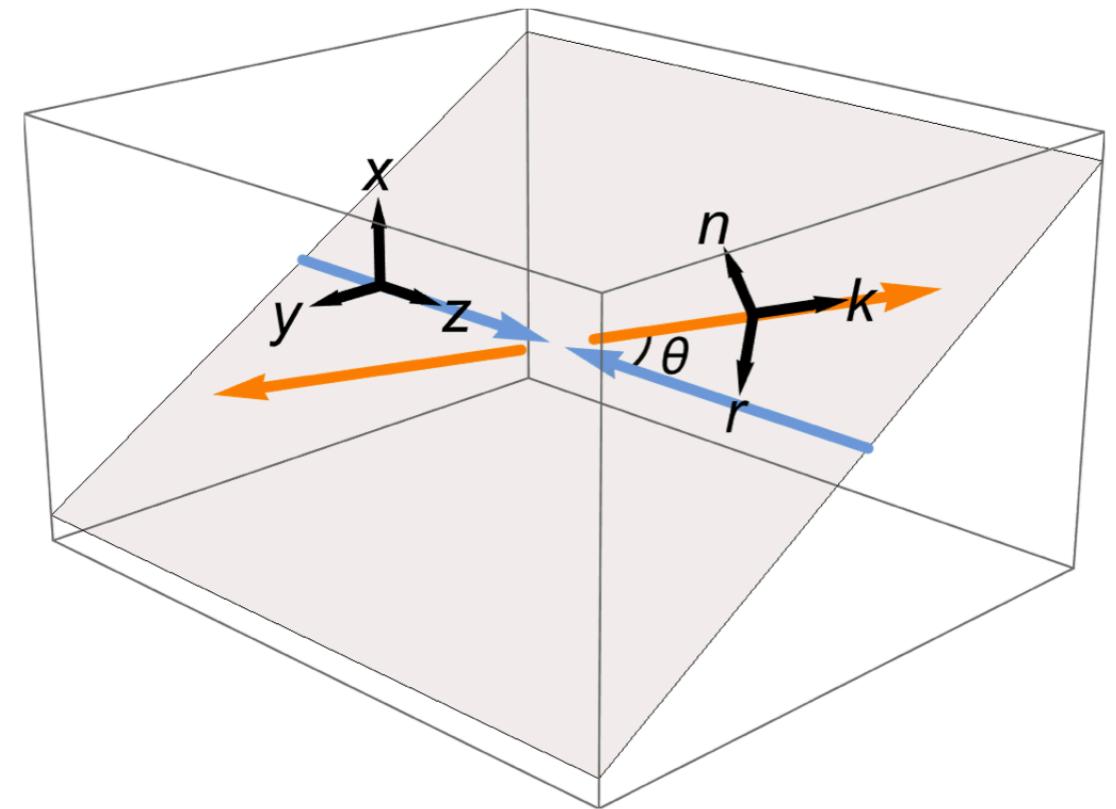
$$C_{ij}^{\text{helicity}} = \begin{pmatrix} C_k(\theta, M_{t\bar{t}}) & C_{kr}(\theta, M_{t\bar{t}}) & 0 \\ C_{kr}(\theta, M_{t\bar{t}}) & C_r(\theta, M_{t\bar{t}}) & 0 \\ 0 & 0 & C_n(\theta, M_{t\bar{t}}) \end{pmatrix},$$



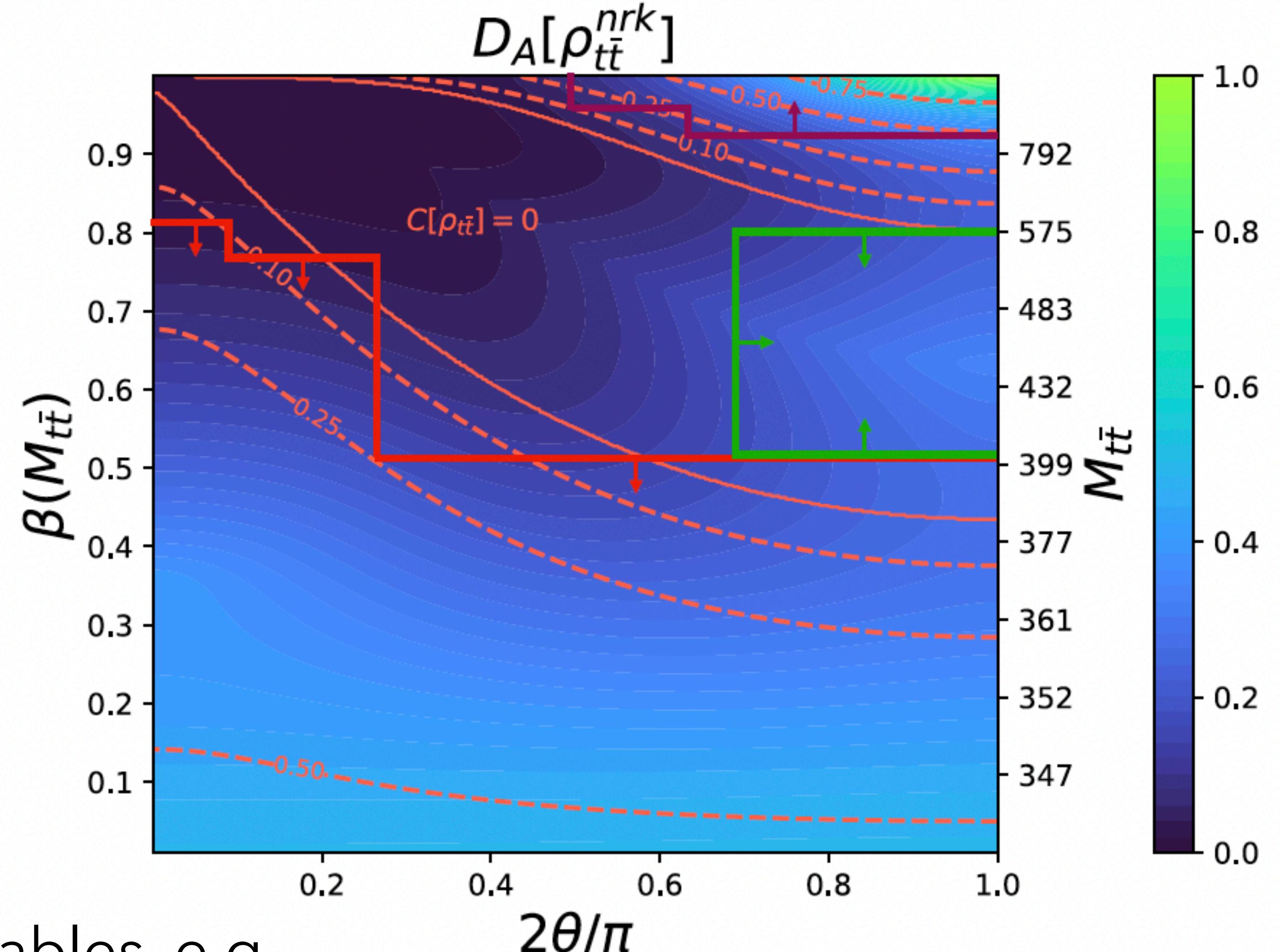
Collider Analysis

$$pp \rightarrow t\bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + \cancel{E}_T.$$

- Kinematic method



$$C_{ij}^{\text{helicity}} = \begin{pmatrix} C_k(\theta, M_{t\bar{t}}) & C_{kr}(\theta, M_{t\bar{t}}) & 0 \\ C_{kr}(\theta, M_{t\bar{t}}) & C_r(\theta, M_{t\bar{t}}) & 0 \\ 0 & 0 & C_n(\theta, M_{t\bar{t}}) \end{pmatrix},$$



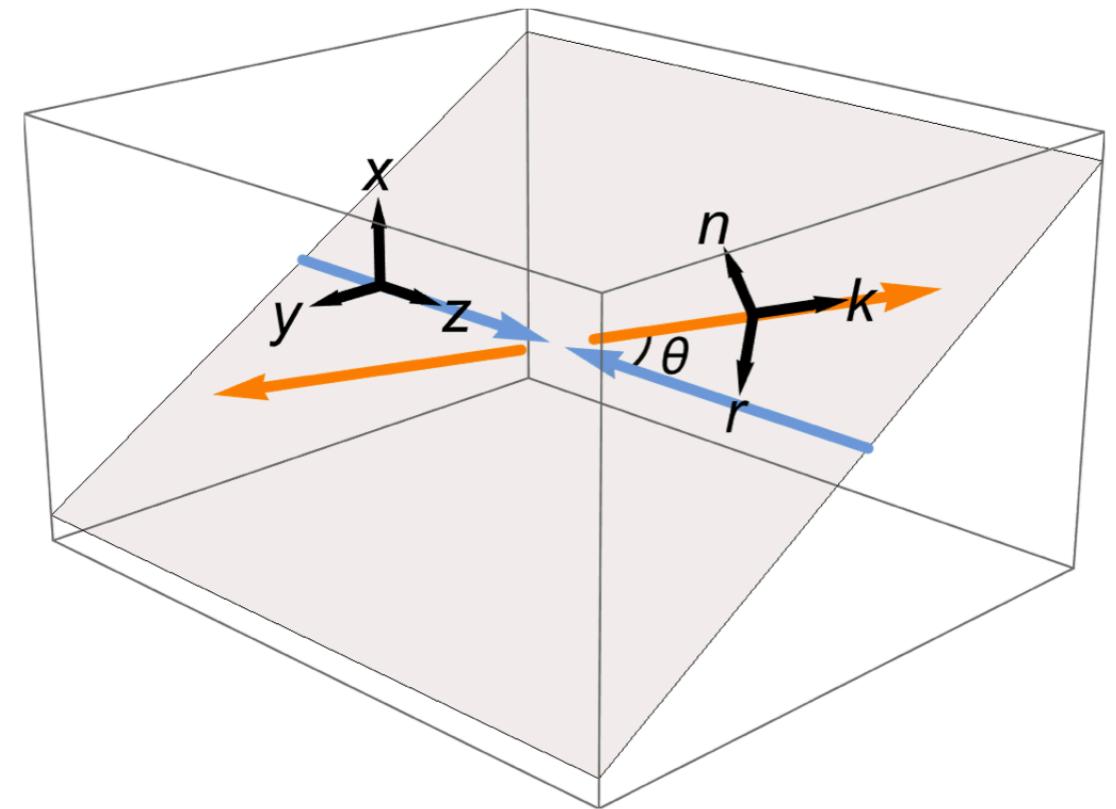
Subtlety: $\mathcal{O}(C_{ij})$ for non-linear observables, e.g.

Discord, VN entropy, etc. (for Bell can do $\mathcal{B}(\theta, M_{t\bar{t}})$)

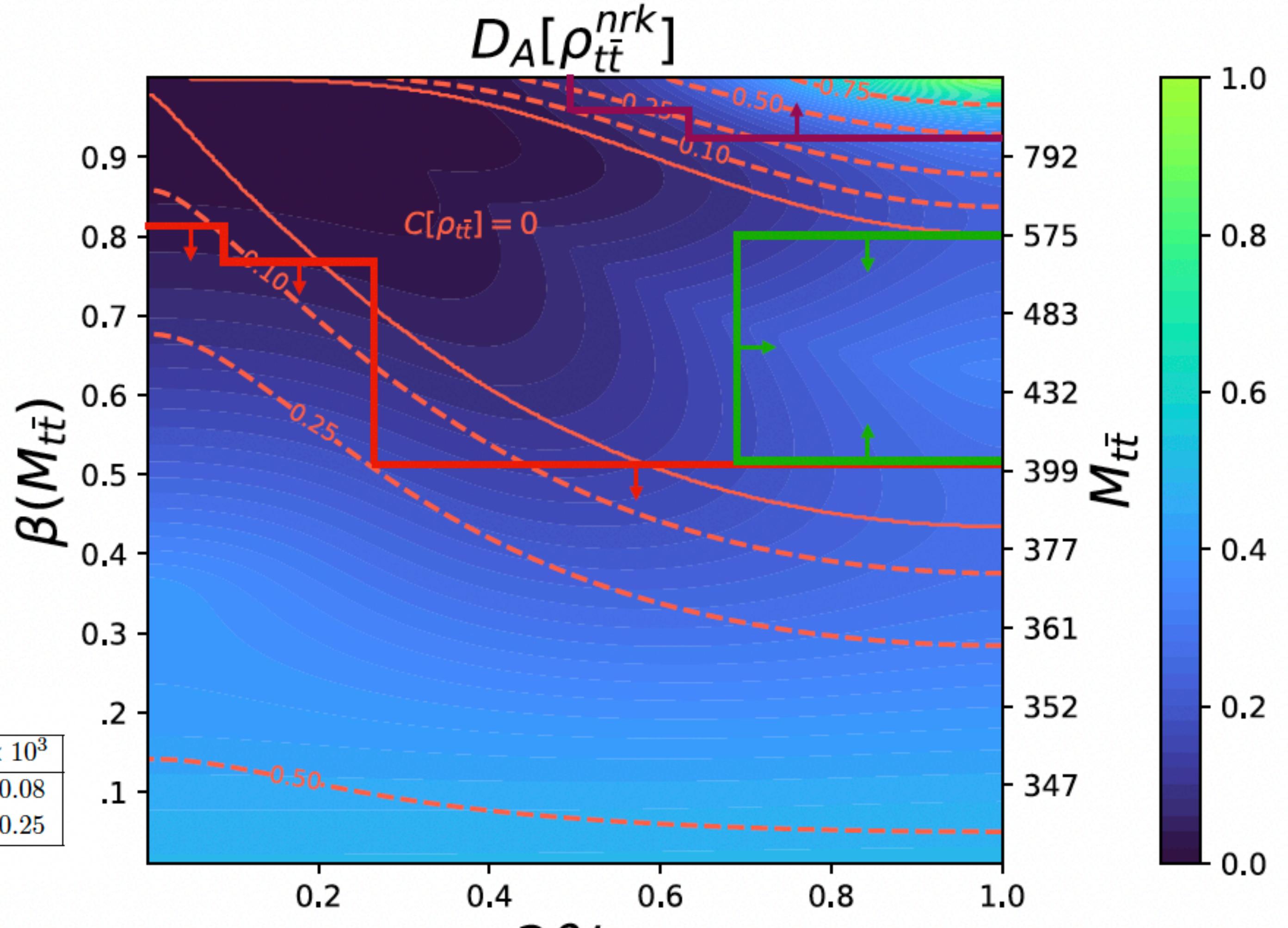
Collider Analysis

$$pp \rightarrow t\bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + \cancel{E}_T.$$

- Kinematic method



	Threshold Region		Separable Region		Boosted Region	
	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}}) \times 10^3$	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}}) \times 10^3$	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}}) \times 10^3$
Parton						
Reconstructed	0.10	173.42 ± 0.07	0.28	249.72 ± 0.24	0.08	200.81 ± 0.08
		147.10 ± 0.24		232.54 ± 0.47		188.49 ± 0.25



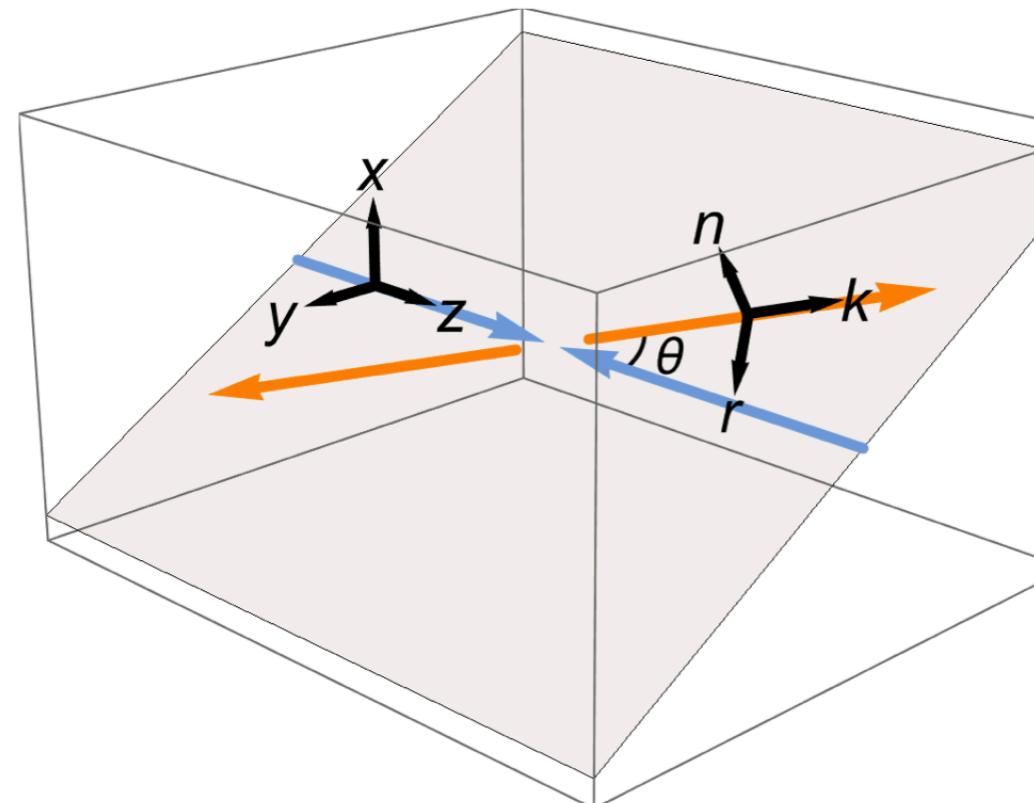
Subtlety: $\mathcal{O}(\bar{C}_{ij})$ for non-linear observables, e.g.

Discord, VN entropy, etc. (for Bell can do $\bar{\mathcal{B}}(\theta, M_{t\bar{t}})$)

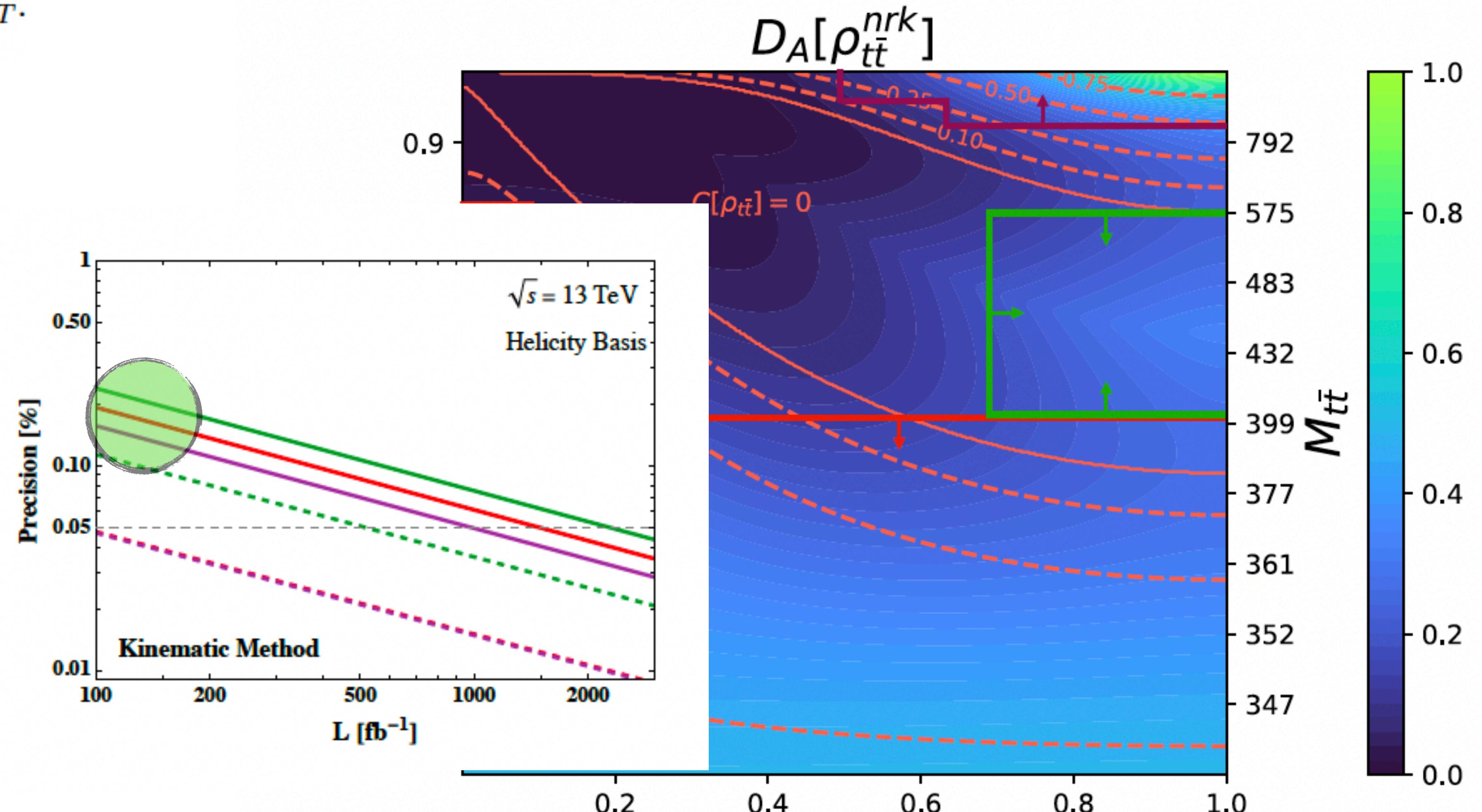
Collider Analysis

$$pp \rightarrow t\bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + \cancel{E}_T.$$

- Kinematic method



	Threshold Region	Separabl		
	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}}) \times 10^3$	$\langle \epsilon_{rec} \rangle$	I
Parton				
Reconstructed	0.10	173.42 ± 0.07	0.28	2
		147.10 ± 0.24		



Subtlety: $\mathcal{O}(\bar{C}_{ij})$ for
Discord, VN entropy, ...

Est. precision w/ current data: ~0.3%

Collider Analysis

Looking at the CMS	
Metric	Measured
$D=Tr[C]/3$	-0.221 ± 0.010
<i>Magic</i>	0.238 ± 0.014
<i>Discord</i>	0.073 ± 0.010
<i>LQU</i>	0.051 ± 0.007

Obtained from the **inclusive**
(100k toys) → **clear observa**
But how do we compare with experimental

From yesterday's talk:

Est. precision w/ actual data: ~14%

(Using full spin-density matrix)

Summary

- Discord is a clear next step in the exploration of QI at colliders
 - Computable analytically for $t\bar{t}$ (at LO) in terms of “collider friendly” observables
 - Can be framed in robust way in terms of existing analysis strategies (dilepton, lepton + jets, etc.), clear advantages for kinematic method
- Important subtleties uncovered (convexity, linearity) which must be considered for other observables
- Future directions:
 - Higher order effects
 - CP-odd corrections (EW, NP)