Entanglement and Bell Nonlocality in $\tau^+\tau^-$ at the LHC

using Machine Learning for Neutrino Reconstruction

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2504.01496

Apr 7, 2025

Quantum Observables for Collider Physics

$t\bar{t}$ vs. $\tau^+\tau^-$

- What **qubit** measurements can we **currently** make at the LHC?

$$pp \to t\bar{t}$$

Afik, de Nova
Fabbrichesi, Floreanini, Panizzo
Severi, Boschi, Maltoni, Sioli
Aoude, Madge, Maltoni, Mantani
Afik, de Nova
Aguilar-Saavedra, Casas
Fabbrichesi, Floreanini, Gabrielli
Afik, de Nova
Ashby-Pickering, Barr, Wierzchucka
Severi, Vryonidou
Mantani
Dong, Gonçalves, Kong, Navarro
Aguilar-Saavedra
Han, ML, Wu
Cheng, Han, ML
Maltoni, Severi, Tentori, Vryonidou
Aguilar-Saavedra
Barr, Fabbrichesi, Floreanini, Gabrielli, Marzola
Aguilar-Saavedra
White, White
Cheng, Han, ML
Demina, Landi
Han, ML, McGinnis, Su

Entanglement, Quantum Discord, Magic, …

$$pp \to \tau^+\tau^-$$

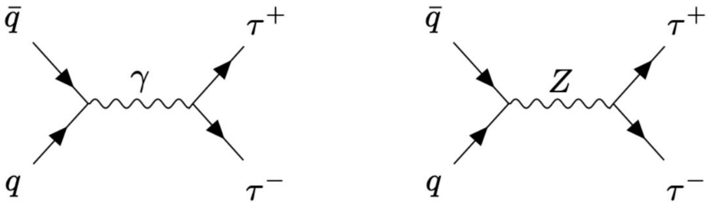
Fabbrichesi, Floreanini, Gabrielli
Barr, Fabbrichesi, Floreanini, Gabrielli, Marzola
Lo Chiatto

Entanglement, Bell Nonlocality, …

tt vs. $\tau^+\tau^-$

Fabbrichesi, Floreanini, Gabrielli 2208.11723

- Electroweak production in the **s-channel**



- When the **photon** dominates

$$\rho_{\tau\bar{\tau}} = \underbrace{\lambda\rho^{(+)}}_{\text{pure, entangled}} + (1-\lambda)\underbrace{\rho^{(1)}_{\text{mix}}}_{\text{mixed, separable}} \qquad \lambda = \frac{\beta_\tau^2}{2-\beta_\tau^2} \in [0,1]$$

- When the **Z** dominates

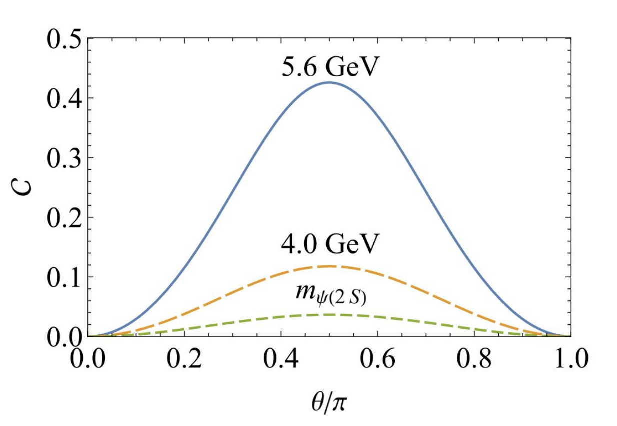
$$\rho_{\tau\bar{\tau}} = \underbrace{\lambda\tilde{\rho}^{(+)}}_{\text{pure, entangled}} + (1-\lambda)\underbrace{\tilde{\rho}^{(2)}_{\text{mix}}}_{\text{mixed, separable}} \qquad \lambda = \frac{(g_A^\tau)^2-(g_V^\tau)^2}{(g_A^\tau)^2+(g_V^\tau)^2}$$

tt vs. τ⁺τ⁻

- When the **photon** dominates

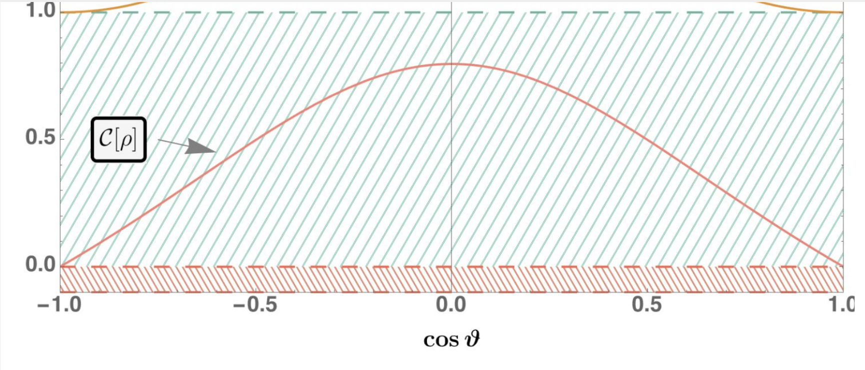
$$\underbrace{\rho_{\tau\bar{\tau}} = \lambda\rho^{(+)}}_{\text{pure, entangled}} + \underbrace{(1-\lambda)\rho^{(1)}_{\text{mix}}}_{\text{mixed, separable}} \qquad \lambda = \frac{\beta_\tau^2}{2-\beta_\tau^2} \in [0,1]$$

- ***Side remark***: in $e^+ e^- \to \tau^+ \tau^-$
  - Challenging, but **doable** near threshold



Han, ML, Su 2501.04801

  - Signal already **sizable** at CM of 10.6 GeV



Ehatäht, Fabbrichesi, Marzola, Velkeen 2311.17555

$t\bar{t}$ vs. $\tau^+\tau^-$

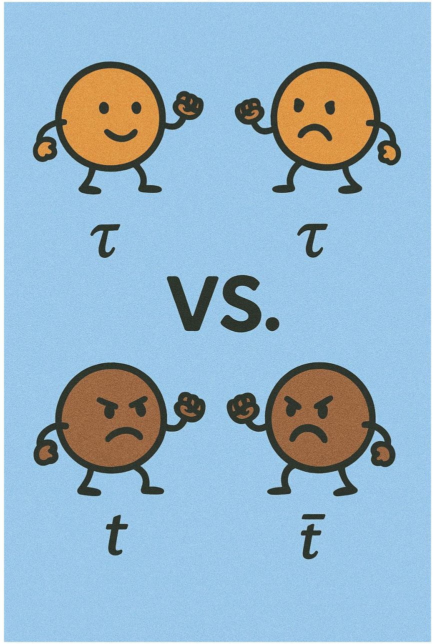
- Brief **comparison** between these final states

$$pp \to t\bar{t}$$
$$\sigma_{\text{tot}} = 834 \text{ pb}$$
$$\text{BR(leptonic)} = 5\%$$
$$\kappa_\ell = 1.0$$
$$\sigma_{\text{boosted}} = 20 \text{ fb}$$

$$pp \to Z \to \tau^+\tau^-$$
$$\sigma_{\text{tot}} = 1848 \text{ pb}$$
$$\text{BR}(\pi \text{ or } \rho) = 13\%$$
$$\kappa_\pi = 1.0$$

$$C_{ij} = \begin{pmatrix} 0.5 & 0.1 & 0 \\ 0.1 & 0.6 & 0 \\ 0 & 0 & -0.5 \end{pmatrix}$$
$$\vec{B}^+ = \vec{B}^- = (0,0,0)$$

$$C_{ij} = \begin{pmatrix} 1.0 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & -0.8 \end{pmatrix}$$
$$\vec{B}^+ = \vec{B}^- = (-0.2,0,0)$$



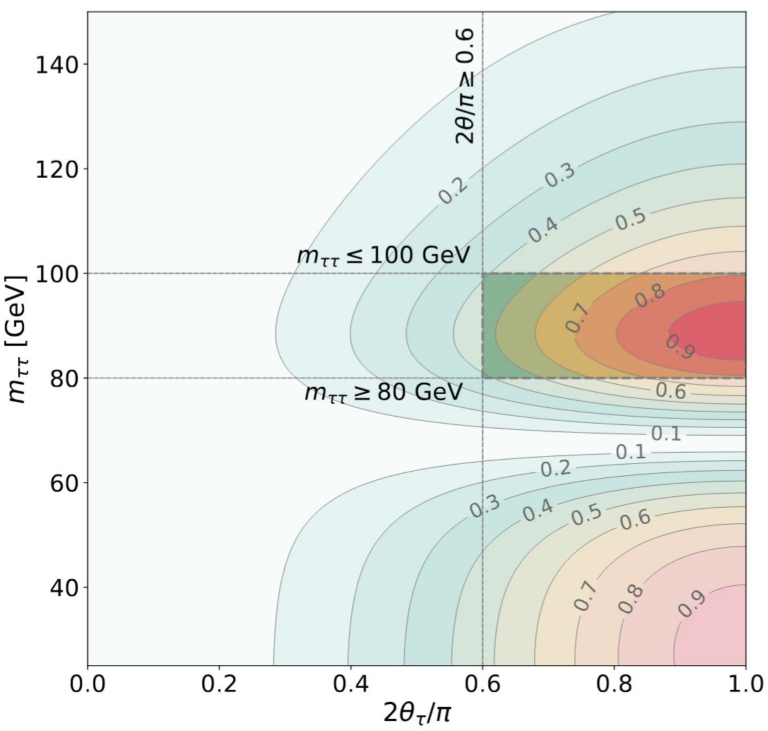
- Both final states achieve nearly **maximal** entanglement and Bell nonlocality
- **Non-zero polarization** has implications for quantum discord

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i B_i^+ \sigma_i \otimes \mathbb{I}_2 + \sum_j B_j^- \mathbb{I}_2 \otimes \sigma_j + \sum_{ij} C_{ij}\sigma_i \otimes \sigma_j\right)$$

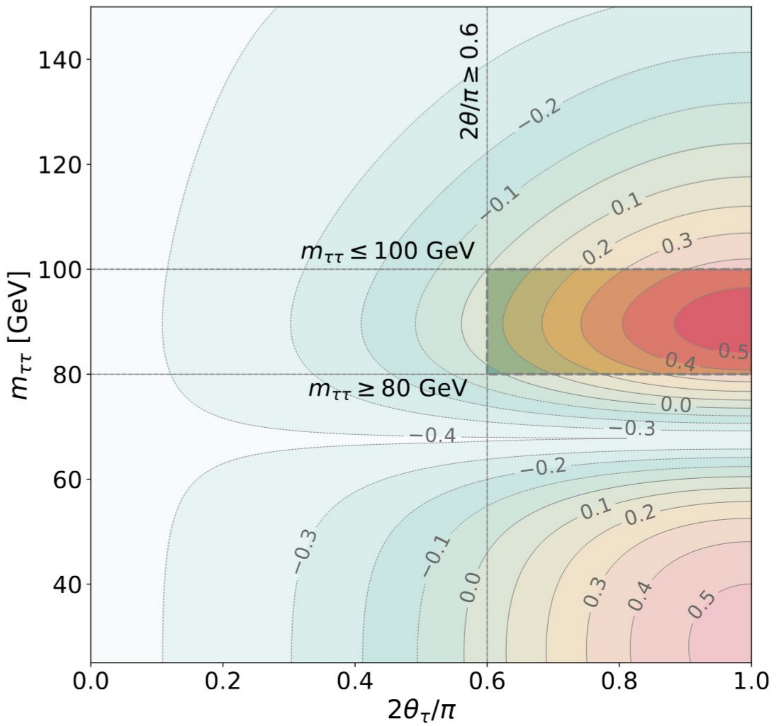
C_{ij}	spin correlations
B_i^+	qubit 1 polarization
B_j^-	qubit 2 polarization

τ Leptons

Concurrence (C>0)



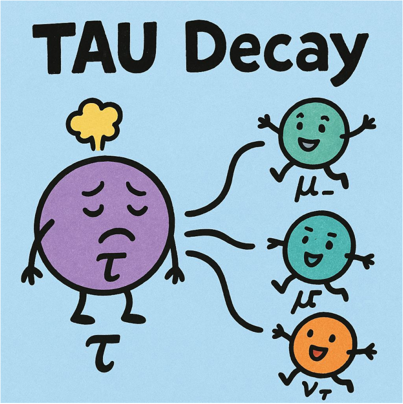
Bell Nonlocality (B>0)



τ Leptons

- Decays of **single τ**

Decay	Spin Analyzing Power	Branching Ratio
$\pi\nu_\tau$	1.00	10.8%
$\rho(\pi\pi^0)\nu_\tau$	0.41	25.5%
$e\nu_e\nu_\tau$	−0.33	17.8%
$\mu\nu_\mu\nu_\tau$	−0.34	17.4%

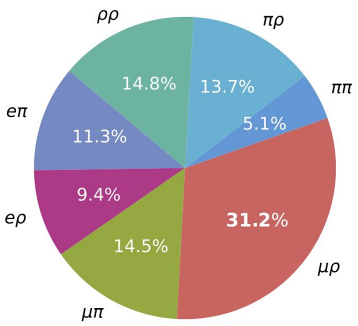


- Decays of **τ⁺τ⁻**

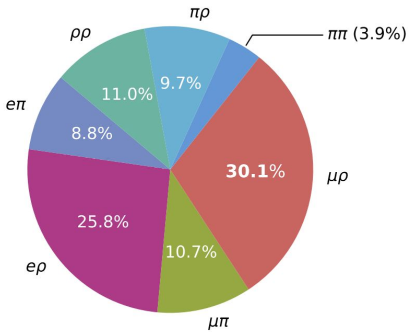
ππ	1%
πρ	6%
ρρ	7%
eπ	4%
eρ	9%
μπ	4%
μρ	9%

- Overall results benefits from **all channels**

Concurrence Impact

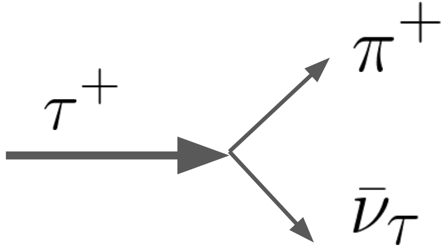


Bell Impact



τ Lepton Reconstruction

- Can we **reconstruct** both τ⁺ and τ⁻



8 unknowns, **6 constraints** (π, Hadron Collider)

$$p_{\nu_1}^2 = 0 \qquad p_{\nu_2}^2 = 0$$
$$(p_{\pi_1} + p_{\nu_1})^2 = m_\tau^2 \qquad (p_{\pi_2} + p_{\nu_2})^2 = m_\tau^2$$
$$E_y^{\text{miss}} = p_{y,\nu_1} + p_{y,\nu_2} \qquad E_x^{\text{miss}} = p_{x,\nu_1} + p_{x,\nu_2}$$

8 unknowns, **8 constraints** (π, Lepton Collider)

8 unknowns, **4 constraints** (e, Hadron Collider)

- System is **underconstrained**, need estimation technique

- Missing Mass Calculator — Elagin, Murat, Pranko, Safonov 1012.4686
  - Originally developed by the CDF collaboration at Tevatron → adopted by the ATLAS experiment.
  - Accounts for the kinematic constraints while considering the variation of energy and position of the particles in the decay cascades over the allowed phase space.
  - The solution with the highest likelihood and largest weight is set as a final estimator of m_H.

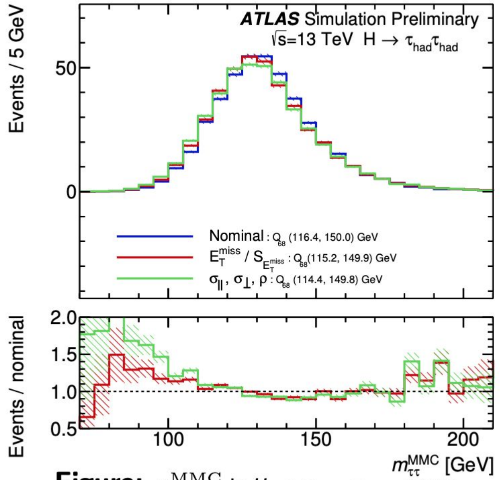
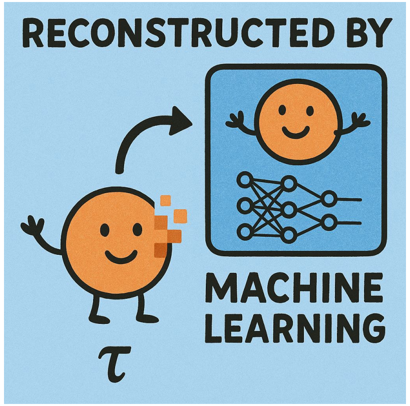


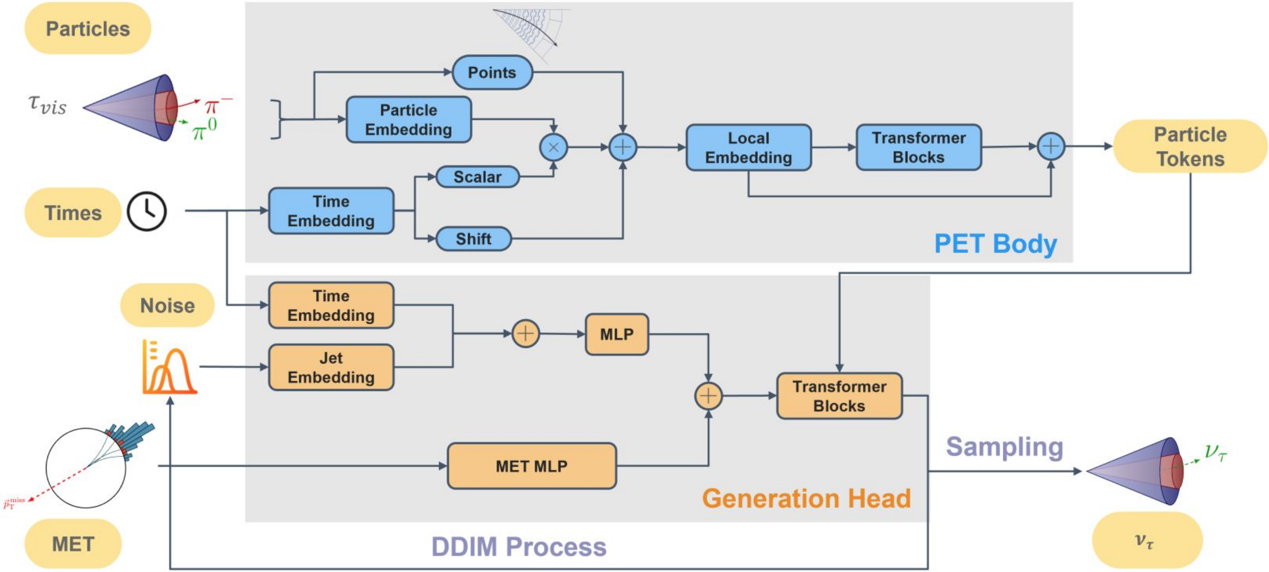
Figure: $m_{\tau\tau}^{\text{MMC}}$ in $H \to \tau_{\text{had}}\tau_{\text{had}}$ events.

$H \to \tau^+ \tau^-$ Petukhova

τ Lepton Reconstruction

- We train a **generative network** to infer **neutrino momenta** directly from event-level observables
  - **Diffusion** models learn the underlying probability distribution
  - We use the **Point-Edge Transformer** (PET) architecture





τ Lepton Reconstruction

- We train a **generative network** to infer **neutrino momenta** directly from event-level observables
  - Train each **decay channel** separately
  - Generate 10 million for each channel (80% training, 20% validation)

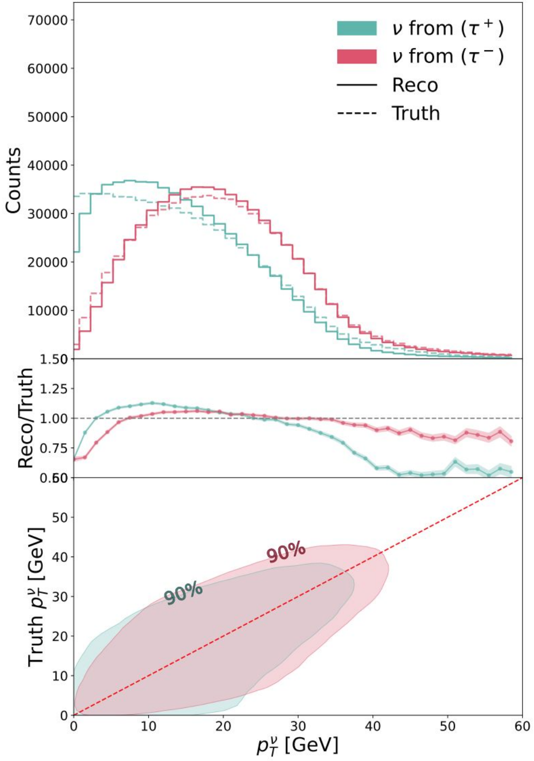
Input Features

Category	Variables	Description
E_T^{miss}	$(p_T^{\text{miss}}, \phi^{\text{miss}})$	Missing transverse momentum vector
τ Visible Components	(p_T, η, ϕ, E)	Four-momentum
	Charge	Electric charge of τ-visible parts
	PID	Electron, muon, or pion identification
Small-R Jets	(p_T, η, ϕ, E)	Four-momentum
	Charge	Electric charge of the jet
	PID	Particle identification

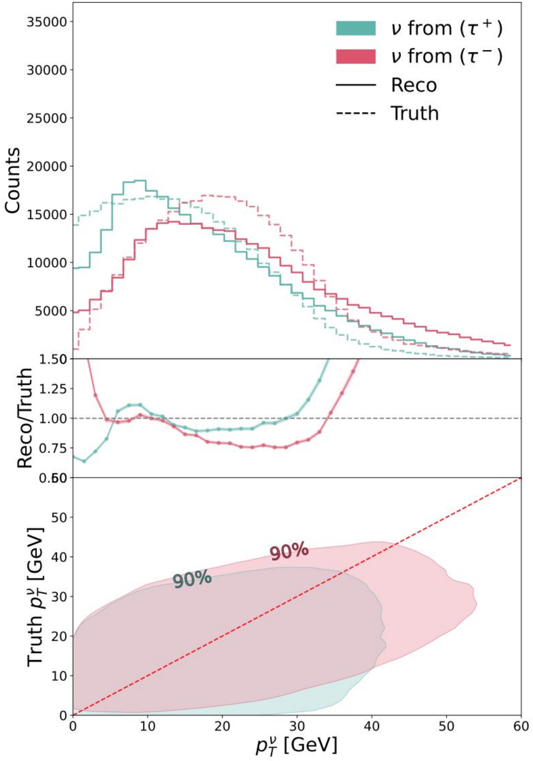
τ Lepton Reconstruction

- Network performance for **neutrino kinematics**

Our PET network



MMC



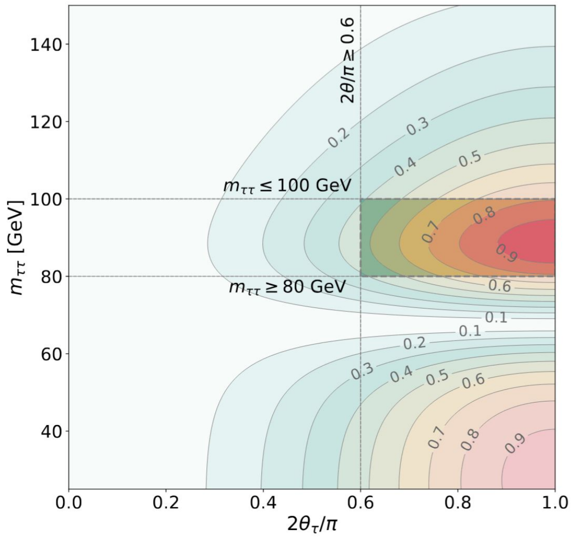
	$\pi\pi$ (%)	
	ML	MMC
$\Delta p^x_{\tau+}$	**18.97**	25.99
$\Delta p^y_{\tau+}$	**19.01**	26.02
$\Delta p^z_{\tau+}$	**19.47**	25.48
$\Delta p^x_{\tau-}$	**18.77**	25.78
$\Delta p^y_{\tau-}$	**18.71**	25.96
$\Delta p^z_{\tau-}$	**19.69**	25.43
$\Delta m_{\tau\tau}$	**7.94**	23.27

Half-width at half-maximum of **resolution**

The $\tau^+\tau^-$ Final State

- **Signal region** around the Z

$$80 \text{ GeV} < m_{\tau\tau} < 100 \text{ GeV}$$
$$0.6(\pi/2) < \theta_\tau < \pi/2$$



- **Triggers**
  - Di-Tau (ππ, πρ, ρρ)
    - Leading $p_T(\tau) > 35$ GeV
    - Sub-leading $p_T(\tau) > 25$ GeV
  - Tau + Muon (μπ, μρ)
    - $p_T(\tau) > 25$ GeV
    - $p_T(\mu) > 14$ GeV
  - Tau + Electron (eπ, eρ)
    - $p_T(\tau) > 25$ GeV
    - $p_T(e) > 17$ GeV

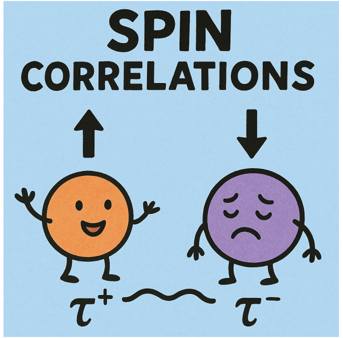
The $\tau^+\tau^-$ Final State

- We **reconstruct** the density matrix both ways

- **Decay Approach**

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$$

  - Measure **angles** in τ^+ and τ^- rest frames
  - Use **template fit** to extract C_{ij} components



- **Kinematic Approach** Cheng, Han, ML 2410.08303
  - **Parametrize** spin correlation matrix by θ and β
  - θ = scattering angle, β = speed of τ
  - *Example*: at the Z-pole

$$C_{ij} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sin^2\theta}{1+\cos^2\theta} & 0 \\ 0 & 0 & -\frac{\sin^2\theta}{1+\cos^2\theta} \end{pmatrix}$$

  - Measure of **θ** to get C_{ij}
  - Corrections (due to smearing) **not needed**
  - **Full formula** used for results

The $\tau^+\tau^-$ Final State

- **Background** events per 1 fb^{-1}

Subchannel	$W \to \ell\nu$	$W \to \tau\nu$	$Z \to \ell\ell$	$t\bar{t}$	QCD	Total
SR & di-τ Trigger ($p_T^{\tau_1} > 35$ GeV & $p_T^{\tau_2} > 25$ GeV)						
$\pi\pi$	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01
$\pi\rho$	< 0.01	< 0.01	< 0.01	0.05 ± 0.05	< 0.01	0.05 ± 0.05
$\rho\rho$	< 0.01	< 0.01	< 0.01	0.10 ± 0.07	< 0.01	0.10 ± 0.07
SR & $e + \tau$ Trigger ($p_T^e > 14$ GeV & $p_T^\tau > 25$ GeV) or single-e Trigger ($p_T^e > 26$ GeV)						
$e\pi$	< 0.01	< 0.01	< 0.01	2.87 ± 0.38	< 0.01	2.87 ± 0.38
$e\rho$	1.93 ± 0.86	0.62 ± 0.36	< 0.01	11.19 ± 0.75	< 0.01	13.74 ± 1.20
SR & $\mu + \tau$ Trigger ($p_T^\mu > 17$ GeV & $p_T^\tau > 25$ GeV) or single-μ Trigger ($p_T^\mu > 26$ GeV)						
$\mu\pi$	< 0.01	0.41 ± 0.29	< 0.01	4.13 ± 0.46	< 0.01	4.55 ± 0.54
$\mu\rho$	1.16 ± 0.67	0.62 ± 0.36	< 0.01	< 0.01	< 0.01	1.78 ± 0.76

Reconstructed channels

The $\tau^+\tau^-$ Final State

- **Signal** events per 1 fb^{-1} by channel

Subchannel	Prongness	$\lvert\kappa_A \times \kappa_B\rvert$	$\tau\tau \to \ell\pi$	$\tau\tau \to \ell\rho$	$\tau\tau \to \pi\pi$	$\tau\tau \to \pi\rho$	$\tau\tau \to \rho\rho$
SR & di-τ Trigger ($p_T^{\tau_1} > 35$ GeV & $p_T^{\tau_2} > 25$ GeV)							
$\pi\pi$			3.92 ± 0.18	0.09 ± 0.04	$\boxed{89.43 \pm 0.48}$	2.90 ± 0.13	0.14 ± 0.04
$\pi\rho$			0.12 ± 0.03	22.71 ± 0.65	1.61 ± 0.06	$\boxed{206.39 \pm 1.09}$	6.29 ± 0.28
$\rho\rho$			< 0.01	0.56 ± 0.10	0.06 ± 0.01	4.51 ± 0.16	$\boxed{629.99 \pm 2.83}$
SR & $e + \tau$ Trigger ($p_T^e > 14$ GeV & $p_T^\tau > 25$ GeV) or single-e Trigger ($p_T^e > 26$ GeV)							
$e\pi$			$\boxed{378.90 \pm 1.79}$	17.52 ± 0.57	< 0.01	0.01 ± 0.01	< 0.01
$e\rho$			8.33 ± 0.27	$\boxed{1233.90 \pm 4.82}$	< 0.01	0.03 ± 0.01	0.15 ± 0.04
SR & $\mu + \tau$ Trigger ($p_T^\mu > 17$ GeV & $p_T^\tau > 25$ GeV) or single-μ Trigger ($p_T^\mu > 26$ GeV)							
$\mu\pi$			$\boxed{565.94 \pm 2.19}$	25.21 ± 0.69	< 0.01	< 0.01	< 0.01
$\mu\rho$			12.63 ± 0.33	$\boxed{1862.06 \pm 5.92}$	< 0.01	< 0.01	0.04 ± 0.02

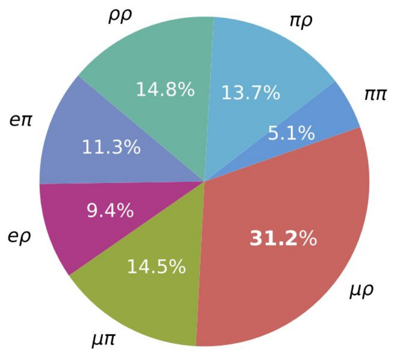
Reconstructed channels

The $\tau^+\tau^-$ Final State

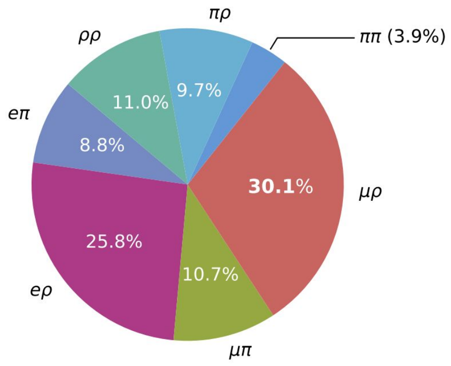
- **Systematic uncertainties**
  - **Systematics** we consider are listed below
  - Pie charts show each channel weighted by **inverse variance**

	SR Only			SR & Trigger		
	$\rho\rho$	$\mu\rho$	Combined	$\rho\rho$	$\mu\rho$	Combined
All Systematics	**29.81%**	**29.76%**	**31.29%**	**31.00%**	**29.82%**	**31.35%**
MC Statistics	29.31%	29.56%	30.05%	28.93%	29.55%	28.66%
Luminosity	0.12%	0.08%	0.90%	7.73%	0.74%	6.10%
Background Cross-Section	3.39%	0.07%	2.01%	1.51%	0.05%	1.06%
Signal Cross-Section	0.23%	0.23%	1.71%	3.12%	0.52%	2.41%
Tau Energy Scale	1.47%	2.50%	2.12%	1.20%	0.89%	1.47%
Jet Enery Scale	1.67%	1.50%	4.49%	2.41%	1.72%	8.05%
Soft MET (p_x, p_y)	3.66%	1.90%	6.57%	6.68%	3.42%	7.11%
ν Sampling	0.02%	0.02%	0.03%	0.06%	0.03%	0.07%

Concurrence Impact

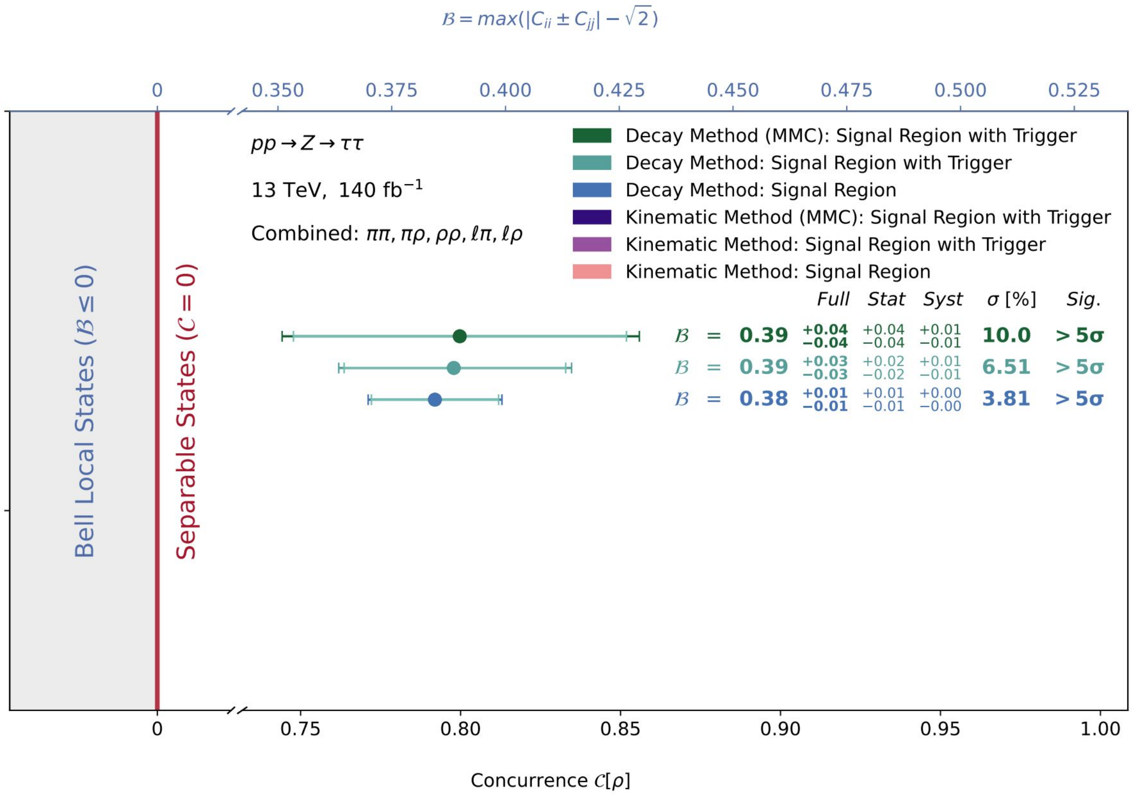


Bell Impact



The $\tau^+\tau^-$ Final State

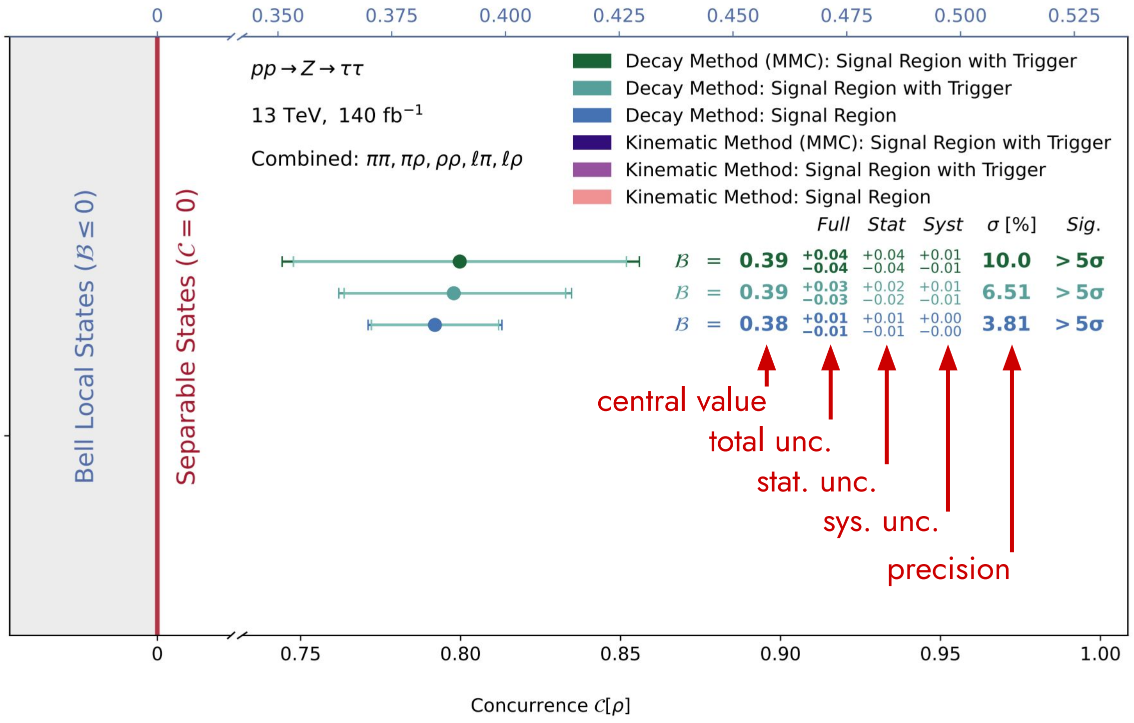
- **Results**: >5σ for **Bell nonlocality**



The $\tau^+\tau^-$ Final State

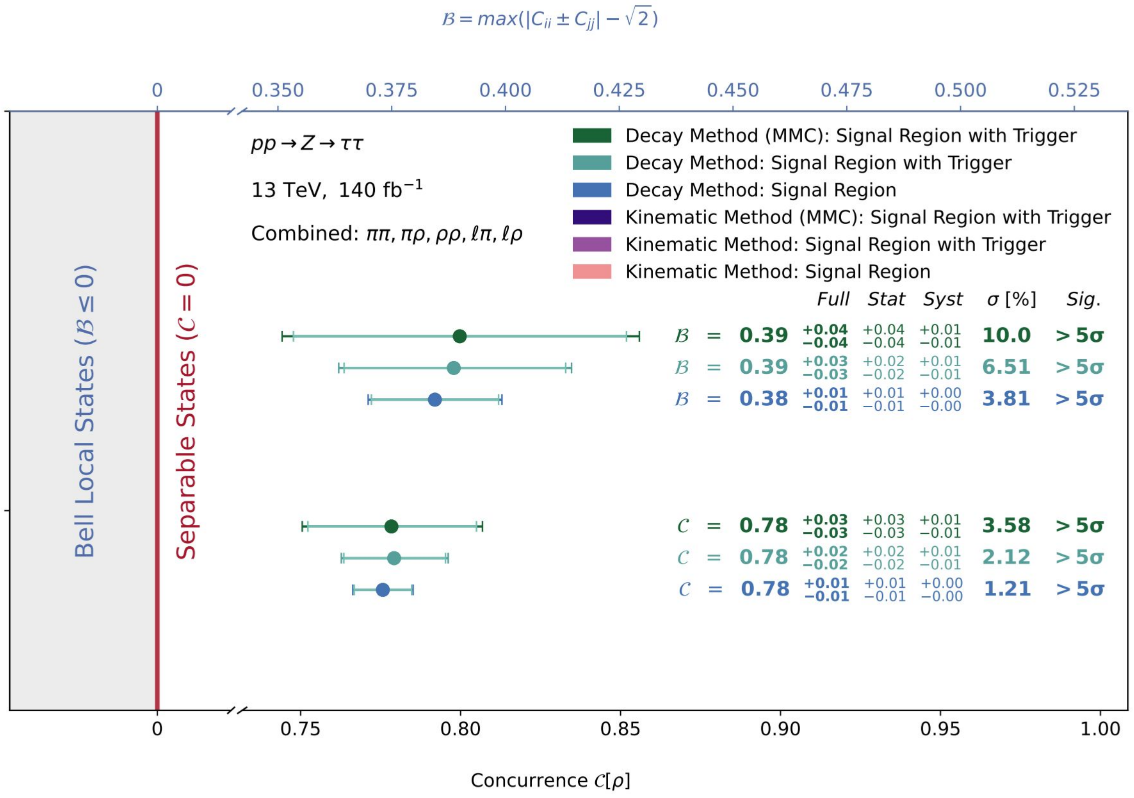
- **Results**: >5σ for **Bell nonlocality**

$\mathcal{B} = max(|C_{ii} \pm C_{jj}| - \sqrt{2})$ (Bell states have B = 0.586)



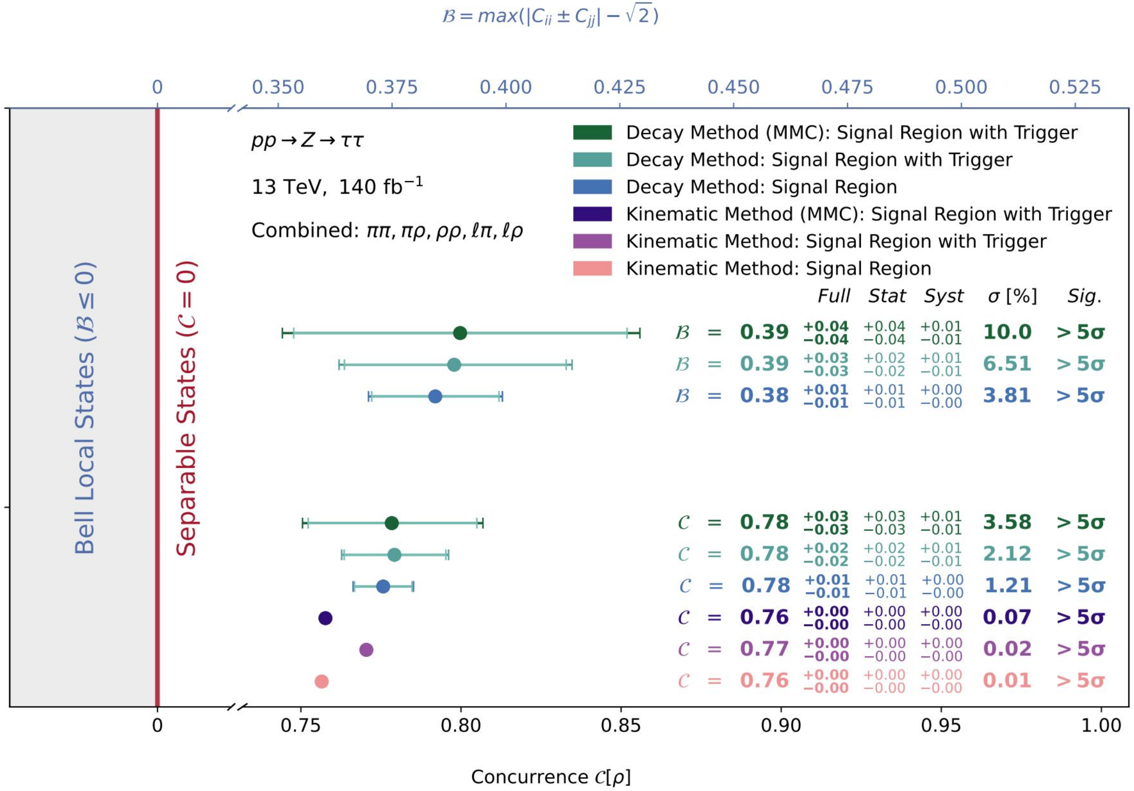
The τ⁺τ⁻ Final State

- **Results:** >5σ for **concurrence**



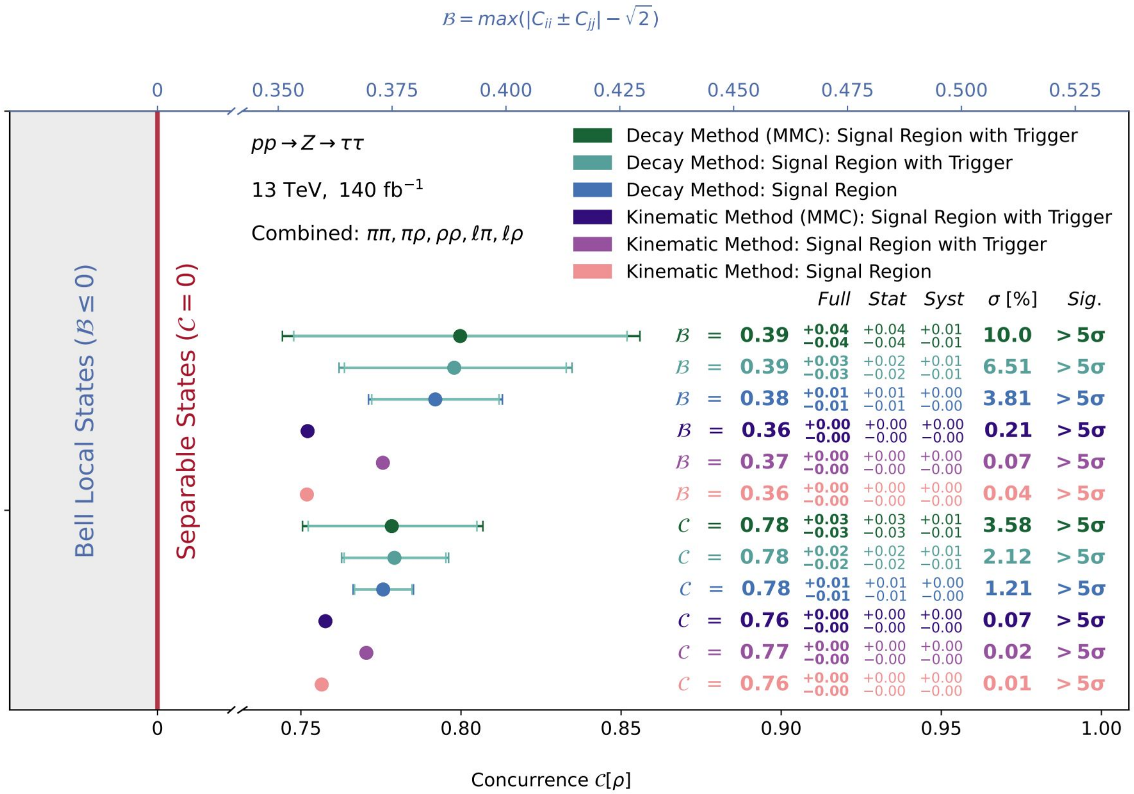
The $\tau^+\tau^-$ Final State

- **Results**: >5σ **concurrence** (with the **kinematic approach**)

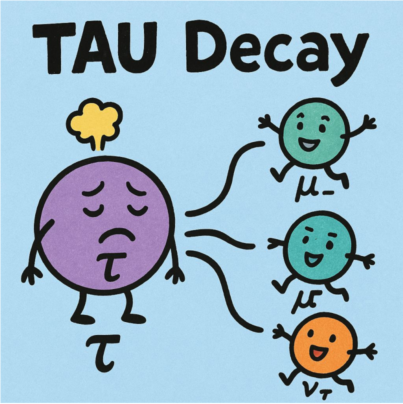


The τ⁺τ⁻ Final State

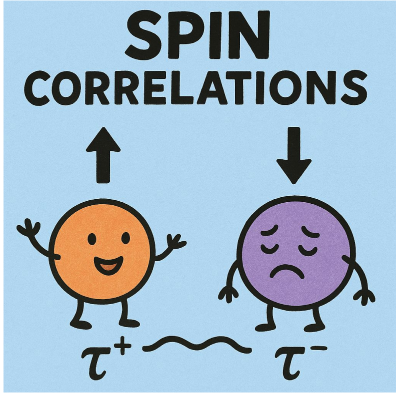
- **Results**: >5σ for **Bell nonlocality** (with the **kinematic approach**)

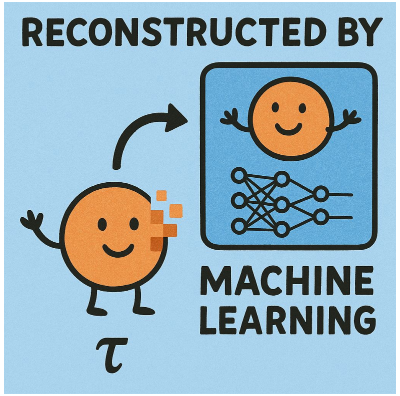


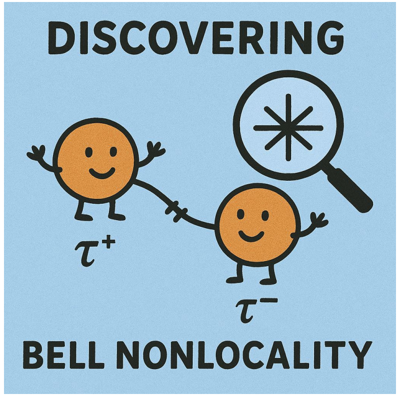
Conclusions



- τ+τ- an **excellent** channel
  - **More** events than tt (strong kinematic cuts not needed)
  - **Neutrino reconstruction** solved by Point-Edge Transformer
  - **>5σ** for entanglement and Bell nonlocality with current data
- Can **other** quantum correlations be measured?







Bell Variable

- We use the atypical normalization of the CHSH inequality

$$\mathcal{B}(\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2) = \frac{1}{\sqrt{2}} |\langle \vec{a}_1 \cdot \vec{\sigma} \otimes \vec{b}_1 \cdot \vec{\sigma} \rangle - \langle \vec{a}_1 \cdot \vec{\sigma} \otimes \vec{b}_2 \cdot \vec{\sigma} \rangle + \langle \vec{a}_2 \cdot \vec{\sigma} \otimes \vec{b}_1 \cdot \vec{\sigma} \rangle + \langle \vec{a}_2 \cdot \vec{\sigma} \otimes \vec{b}_2 \cdot \vec{\sigma} \rangle| - \sqrt{2}.$$

- We use the approximate maximization

$$\mathcal{B} = \max_{ij} |C_{ii} \pm C_{jj}| - \sqrt{2}.$$

- This corresponds to the regions

$$\begin{cases} -\sqrt{2} \leq \mathcal{B} \leq 0 & \text{Bell local,} \\ 0 < \mathcal{B} \leq 2 - \sqrt{2} & \text{Bell nonlocal,} \end{cases}$$