Quantum Entanglement in $H \to ZZ$ Process at Lepton Colliders

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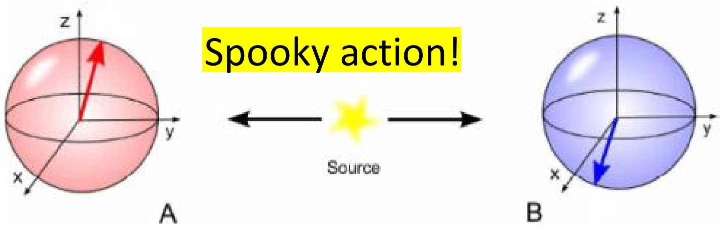
Entanglement measurement history

- **Einestein-Polosky-Rosen Paradox (1935):**
  A and B are produced in a way that their *physical property* like momentum and position are *entangled*: measuring p_A will determine p_B, and simultaneously x_B can also be determined! Incompatible with uncertainty principle!



"Can Quantum Mechanical Description of Physical Reality Be Considered Complete?"

A. Einstein B. Podolski N. Rosen

Quantum Mechanical Description of physics reality described wave function is not complete without a "hidden-variable theory"

Physical reality must be local! - Podolsky

EPR Paradox

Phys. Rev. 47, 777 (1935)

Entanglement measurement history

- **Wu-Shaknov Experiment (1950)**

One first touch to free lepton quantum entanglement!

1912~1997
Chien-Shiung Wu

pair annihilation

$$e^+ + e^- \to \gamma + \gamma$$

Compton scat.

$$\gamma \to e + \gamma'$$

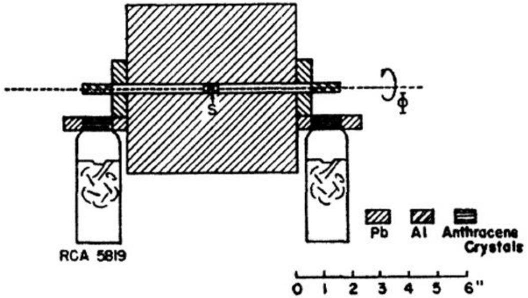


Figura 1: Aparato do experimento WS. Fonte: Wu e Shaknov (1950, p. 1).

$$\frac{\text{Coincidence counting rate } (\perp)}{\text{Coincidence counting rate } (\|)} = 2.04 \pm 0.08,$$

Phys. Rev. 77, 136 (1950)

The Little-Known Origin Story behind the 2022 Nobel Prize in Physics

In 1949 physicist Chien-Shiung Wu devised an experiment that documented evidence of entanglement. Her findings have been hidden in plain sight for more than 70 years

Sientific American

Entanglement measurement history

◆ Formulation of Bell Inequalities

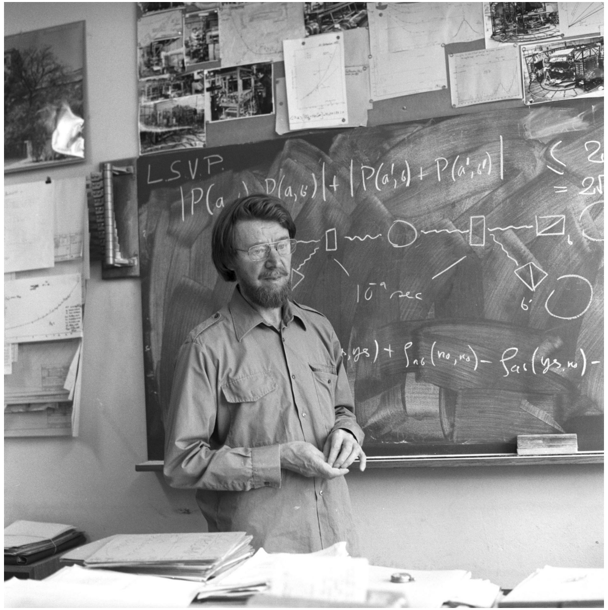


Figure source (1982@CERN)

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 1 + P(\vec{b},\vec{c}).$$

[Physics Physique Fizika 1, 195 (1964)]

$$|E(a,b) - E(a,b')| \leq 2 - |E(a',b') + E(a',b)|$$

[Philosophical Reflections and Syntheses 199–217 (1971)]

[Phys. Rev. D 10, 526 (1974)]

$$I_d \equiv \sum_{k=0}^{[d/2]-1} \left(1 - \frac{2k}{d-1}\right)\{+[P(A_1 = B_1 + k) + P(B_1 = A_2 + k + 1) + P(A_2 = B_2 + k) + P(B_2 = A_1 + k)] - [P(A_1 = B_1 - k - 1) + P(B_1 = A_2 - k) + P(A_2 = B_2 - k - 1) + P(B_2 = A_1 - k - 1)]\}. \quad (6)$$

[Phys. Rev. Lett. 88, 040404 (2022)]

Entanglement measurement history

1947~ Alain Aspect 2022 | 1942~ John F. Clauser 2022 | 1945~ Anton Zeilinger 2022

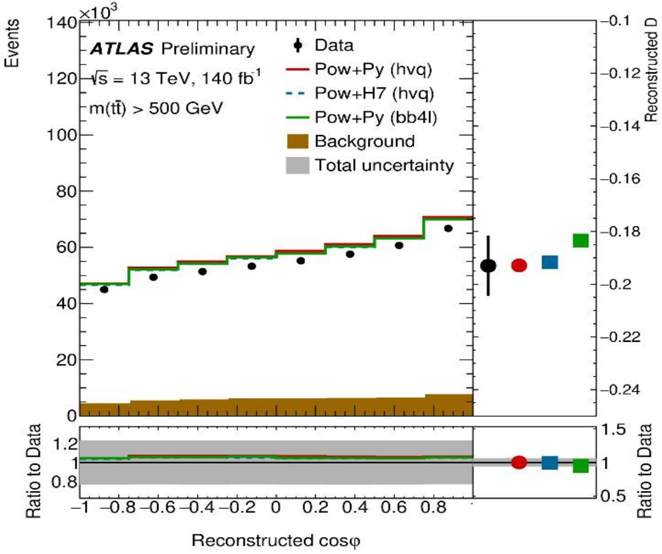
Quantum Mechanical description of physical reality is incompatible with HVT
Quantum Mechanics is a non-local theory!

Entanglement measurement history

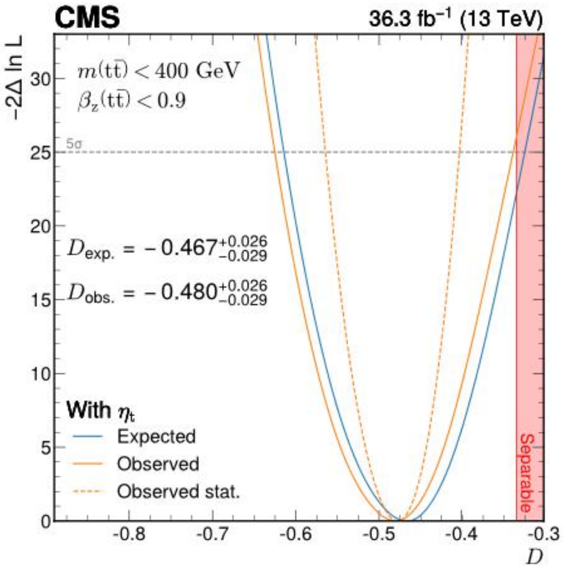
◆ $t\bar{t}$ QE discovery @LHC

$$D = \frac{tr[C]}{3} = -3\,\langle cos\varphi \rangle$$

$$D \leq -\frac{1}{3}$$ Entanglement condition



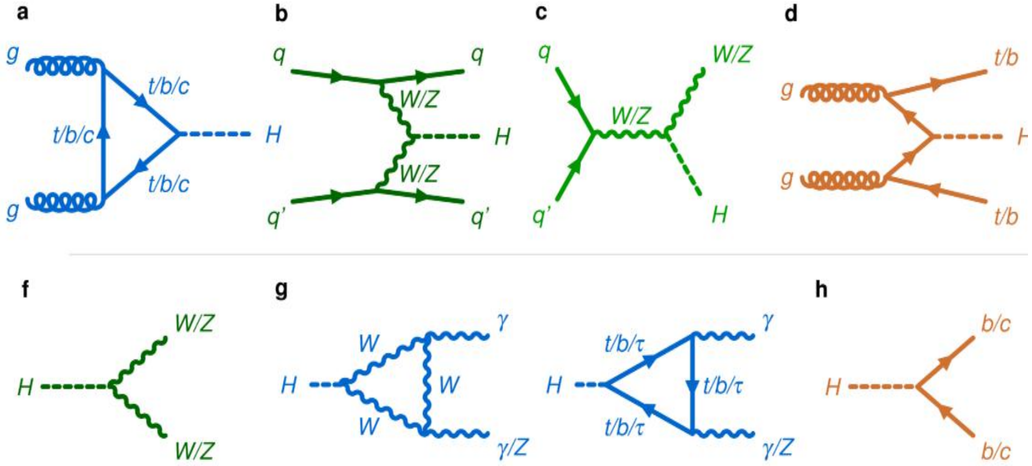
[Nature 633, 542–547 (2024)]

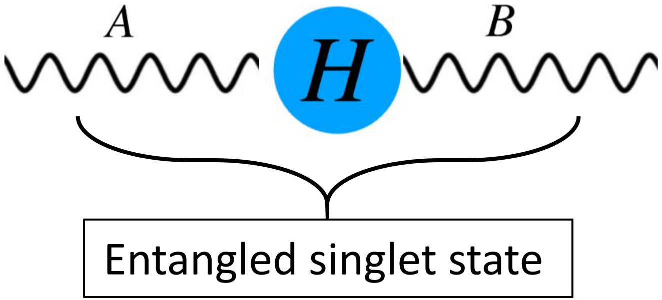


[Rep. Prog. Phys. 87 117801 (2024)]

Higgs boson in entanglement generation

Higgs decay: generator of entangled particles

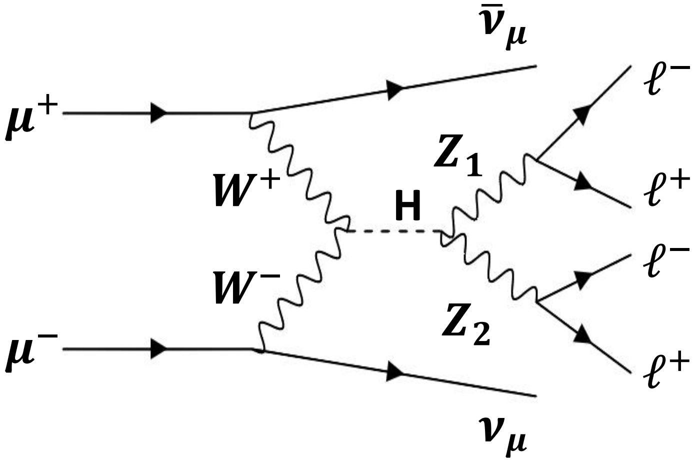


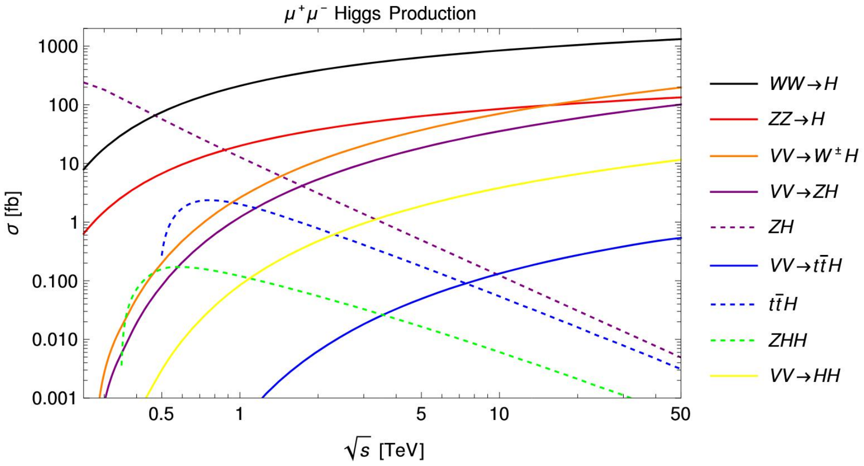


Plenty of two-qubit and two-qutrit systems!

Higgs production at muon colliders

- **The vector boson fusion (VBF) process dominants**





Entanglement observables

1903-1957
John von Neuman

$$S = -k\,Tr(\rho \ln \rho)$$

ρ = density matrix of a quantum state

Phys. Rev. D 34, 373 (1986)

- Negativity: the absolute sum of the negative eigenvalues of a *partially transposed density matrix*

$$\mathcal{N}(\rho) = \sum_k \frac{|\lambda_k| - \lambda_k}{2}$$

- Concurrence: an entanglement witness used generally

$$\mathscr{C}[|\psi\rangle] \equiv \sqrt{2\left(1 - \mathrm{Tr}[(\rho_A)^2]\right)} = \sqrt{2\left(1 - \mathrm{Tr}[(\rho_B)^2]\right)}.$$

- Expectation value of Bell operator

$$\langle \hat{B} \rangle = \mathrm{Tr}[\rho B]$$

Derived via density matrix!

$H \to ZZ$: A spin-1 biparticle system

◆ The CGLMP inequality for spin-1 biparticle systems

$$\begin{aligned} I_3 \;=\;& P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \le 2 \end{aligned}$$

Local theory predicts: $I_3 = \langle \mathcal{O}_{\text{Bell}} \rangle \;= \text{Tr}\{\rho\, \mathcal{O}_{\text{Bell}}\} \le 2$

◆ Spin density matrix (SDM)

Phys. Rev. D 107, 016012 (2023)

JHEP 10 211 (2024)

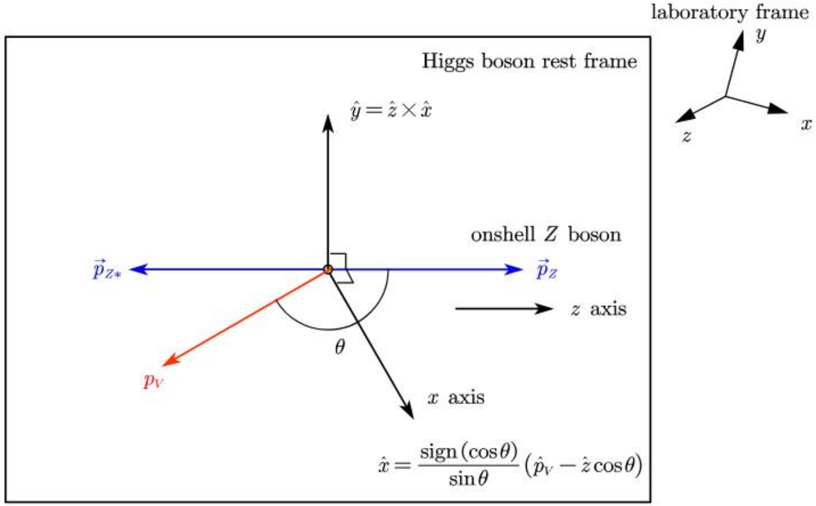
➢ **Gell-Mann Parameterization**

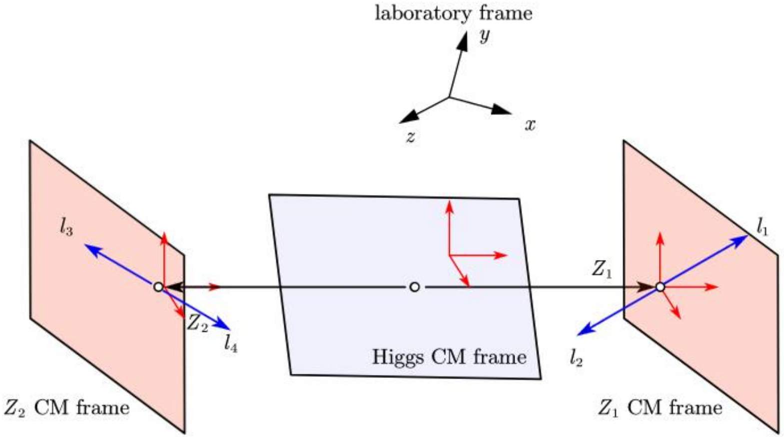
$$\rho(\lambda_1, \lambda_1', \lambda_2, \lambda_2') = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [T^a \otimes \mathbb{1}] + \sum_a g_a [\mathbb{1} \otimes T^a] + \sum_{ab} h_{ab} \left[T^a \otimes T^b \right] \right)_{\lambda_1 \lambda_1' \lambda_2 \lambda_2'}$$

➢ **Irreducible Tensor Parameterization**

$$\rho = \frac{1}{9} \left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM}\, T^L_M \otimes \mathbb{1}_3 + A^2_{LM}\, \mathbb{1}_3 \otimes T^L_M + C_{L_1 M_1 L_2 M_2}\, T^{L_1}_{M_1} \otimes T^{L_2}_{M_2} \right]$$

$H \to ZZ$: Coordinations





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$H \to ZZ$: Spin density matrix reconstruction

- **Differential Cross section: $ZZ \to \ell_1^+\ell_1^-\ell_2^+\ell_2^-$ (parent particle's rest frame)**

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}[1 + A^1_{LM}Y^M_L(\theta_1,\phi_1) + A^2_{LM}B_LY^M_L(\theta_2,\phi_2) + C_{L_1M_1L_2M_2}B_{L_1}B_{L_2}Y^{M_1}_{L_1}(\theta_1,\phi_1)Y^{M_2}_{L_2}(\theta_2,\phi_2)],$$

with $B_1 = -\sqrt{2\pi}\eta_\ell$, and $B_2 = \sqrt{2\pi/5}$.

- **Statistical average over events gives the polarization and correlation coefficient**

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^M_L(\Omega_j)d\Omega_j = \frac{B_L}{4\pi}A^j_{LM}, \qquad j = 1,2.$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1)Y^{M_2}_{L_2}(\Omega_2)d\Omega_1 d\Omega_2 = \frac{B_{L_1}B_{L_2}}{(4\pi)^2}C_{L_1M_1L_2M_2},$$

Phys. Rev. D 107, 016012 (2023)

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$$\eta_\ell = \frac{1 - 4s_W^2}{1 - 4s_W^2 + 8s_W^4} \simeq 0.13$$

$H \to ZZ$: Simulation results for muon colliders

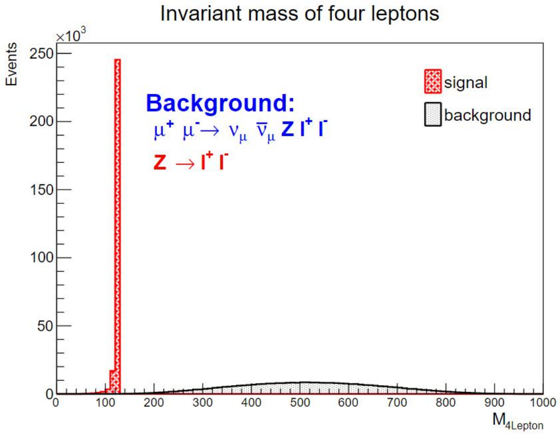
◆ $\mu^+\mu^-\text{VBS} \to H \to 4\ell$ Events

MG5@LO

$\sqrt{s_\mu}$[TeV]	σ[fb]	Luminosity	Events
1	1.51×10^{-2}	30 ab^{-1}	455
3	3.56×10^{-2}	30 ab^{-1}	1089
10	6.06×10^{-2}	30 ab^{-1}	1890

◆ On-shell and off-shell bosons are identified based on the invariant mass of the lepton pairs.

◆ Almost background free (right figure).



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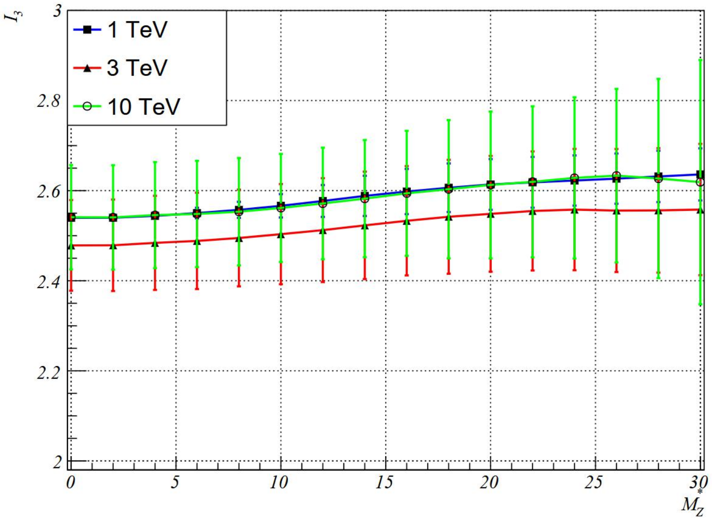
$H \to ZZ$: Simulation results for muon colliders

- $\mu^+\mu^-\text{VBS} \to H \to 4\ell$ Events



Towards large M_Z, fewer events, thus large uncertainty.

The expectation value of I_3 as a function of the off-shell Z mass M_Z^*

JHEP 10 211 (2024)

$H \to ZZ$: Simulation results for muon colliders

- Entanglement criteria:

$\sqrt{s} = 1$ TeV

M_{Z_2} (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.563 ± 0.325	-0.928 ± 0.216	0.527 ± 0.164
10.000	2.596 ± 0.335	-0.943 ± 0.220	0.553 ± 0.179
20.000	2.654 ± 0.373	-0.977 ± 0.248	0.574 ± 0.192
30.000	2.663 ± 0.508	-0.979 ± 0.334	0.589 ± 0.248

$\sqrt{s} = 3$ TeV

M_{Z_2} (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.467 ± 0.217	-0.871 ± 0.121	0.493 ± 0.377
10.000	2.499 ± 0.225	-0.891 ± 0.135	0.502 ± 0.390
20.000	2.538 ± 0.254	-0.908 ± 0.163	0.536 ± 0.365
30.000	2.543 ± 0.342	-0.890 ± 0.216	0.606 ± 0.423

$\sqrt{s} = 10$ TeV

M_{Z_2} (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.539 ± 0.312	-0.930 ± 0.196	0.466 ± 0.232
10.000	2.569 ± 0.295	-0.946 ± 0.194	0.482 ± 0.217
20.000	2.616 ± 0.321	-0.969 ± 0.218	0.514 ± 0.219
30.000	2.644 ± 0.517	-0.943 ± 0.334	0.527 ± 0.280

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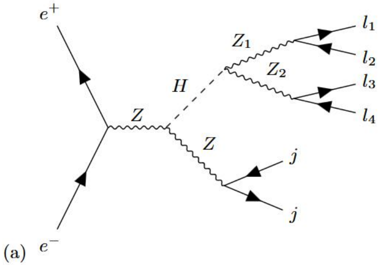
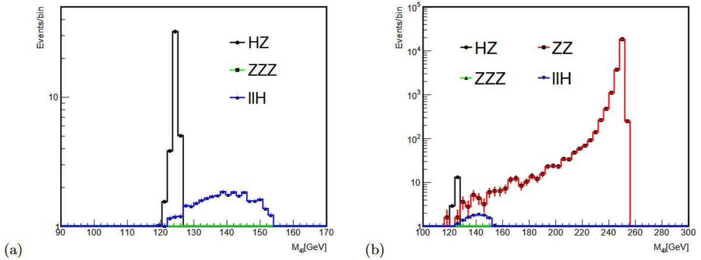
$H \to ZZ$: Simulation results for CEPC





M_z^* [GeV]	$\mathcal{I}_3$	C_{212-1}	C_{222-2}
0	$2.823 \pm 0.640(1.29\sigma)$	$-1.080 \pm 0.420(2.57\sigma)$	$0.637 \pm 0.559(1.14\sigma)$
10	$2.913 \pm 0.692(1.32\sigma)$	$-1.126 \pm 0.451(2.50\sigma)$	$0.677 \pm 0.598(1.13\sigma)$
20	$3.092 \pm 0.800(1.37\sigma)$	$-1.225 \pm 0.514(2.38\sigma)$	$0.761 \pm 0.734(1.04\sigma)$
30	$3.048 \pm 1.816(0.58\sigma)$	$-1.160 \pm 1.192(0.97\sigma)$	$0.875 \pm 1.338(0.65\sigma)$

Phys. Rev. D 111, 036008 (2025)

Summary

- Quantum mechanics is non-local
- No local hidden variable theory is compatible with quantum theory
- Higgs decay can provide maximally entangled bipartite system
- Quantum entanglement is still present at extremely relativistic environment

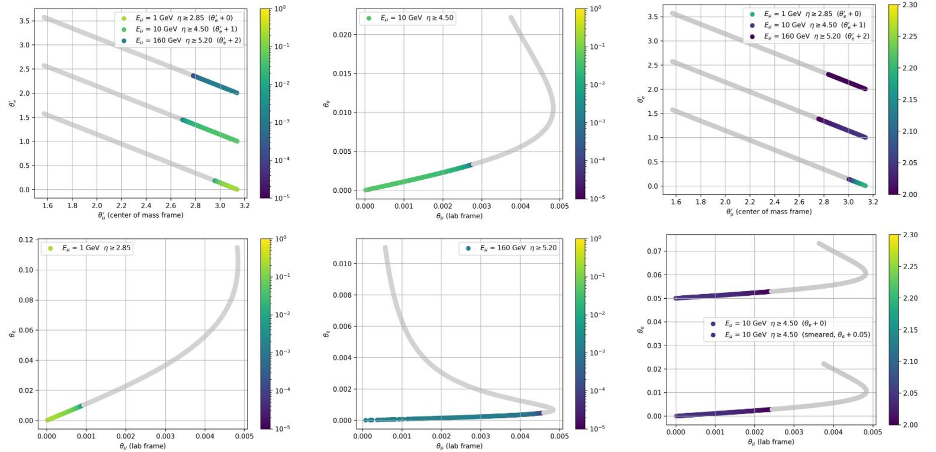
Thanks for your attention!

A quick glace: poster on free lepton entanglement

$e\mu$ QE sources via muon targeting

Concurrence $\mathcal{C}(\rho_f)$:

Optimal I_2:



- The light gray regions depict $\mathcal{C}(\rho_f) = 0$ or $I_2 \leq 2$
- Assuming a 1-day run with a 10 GeV muon beam of flux $10^5/s$ on 10 cm Al targets, the expected number of events with $\mathcal{C}(\rho_f) > 0$ is 2.6×10^4

A quick glace: poster on free lepton entanglement

A first electron-positron beam correlation measurement proposal

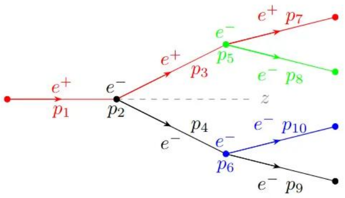


Fig. 3: The proposed cascade experiment

Simulation setup:

- 0.05 rad $\leq \theta_3 \leq$ 0.1 rad in a 1 GeV positron on-target experiment
- The spins of target electrons 5 and 6 are aligned with the beam direction
- Consider the main component of the primary state, $(LL + RR)/\sqrt{2}$*

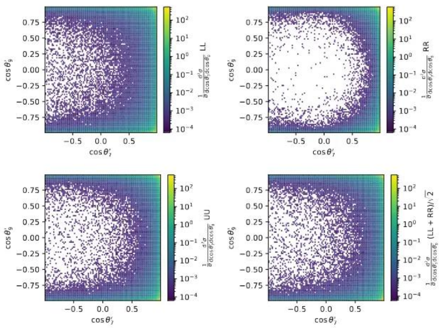
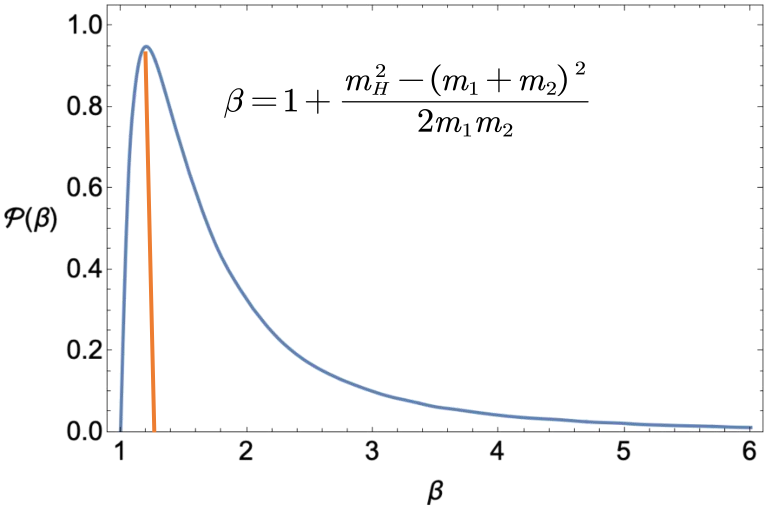


Fig. 4: Joint distributions of the secondary scatterings

- Assuming the two secondary targets are 10 cm thick iron, the event rate in $\cos\theta'_7 \leq 0.5 \wedge -0.75 \leq \theta'_9 \leq 0.75$ is 1.4×10^2/s for the state $(LL + RR)/\sqrt{2}$
- The optimized ratio of the yields of $(LL + RR)/\sqrt{2}$ to UU is 1.29 ± 0.03(MC), corresponding to 4.4×10^3 post-optimization efficient signal event counts and an expected signal yield over a 27-second run; 0.78 ± 0.02(MC) for $(LR + RL)/\sqrt{2}$ in comparison
- For the 20% polarized targets, the ratios are 1.010 ± 0.009 and 0.986 ± 0.009, corresponding to 2.5×10^4 efficient event counts accumulated in 680 seconds

$H \to ZZ$: Spin density matrix

$$|\psi_{ZZ}\rangle = \frac{1}{\sqrt{2+\beta^2}}\left(|+-\rangle - \beta|00\rangle + |-+\rangle\right)$$



Only 9 entries are non-zero!

$$\rho = \begin{bmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&1&0&-\beta&0&1&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&-\beta&0&\beta^2&0&-\beta&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&1&0&-\beta&0&1&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$H \to ZZ$: Spin density matrix

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}\left(1-\sqrt{2}A^1_{2,0}\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_{2,2,2,-2} = \frac{1}{\sqrt{2}}A^1_{2,0}.$$

$$\mathcal{B} = \left[\frac{2}{3\sqrt{3}}\left(T^1_1\otimes T^1_1 - T^1_0\otimes T^1_0 + T^1_1\otimes T^1_{-1}\right) + \frac{1}{12}\left(T^2_2\otimes T^2_2 + T^2_2\otimes T^2_{-2}\right) + \frac{1}{2\sqrt{6}}\left(T^2_2\otimes T^2_0 + T^2_0\otimes T^2_2\right) - \frac{1}{3}\left(T^2_1\otimes T^2_1 + T^2_1\otimes T^2_{-1}\right) + \frac{1}{4}T^2_0\otimes T^2_0\right] + \text{h.c.}.$$

$$I_3 = \frac{1}{36}\left(18+16\sqrt{3}-\sqrt{2}\left(9-8\sqrt{3}\right)A^1_{2,0} - 8\left(3+2\sqrt{3}\right)C_{2,1,2,-1} + 6\,C_{2,2,2,-2}\right)$$

$$I_3 = \text{Tr}\left\{\rho\,\mathcal{O}_{\text{Bell}}\right\}$$

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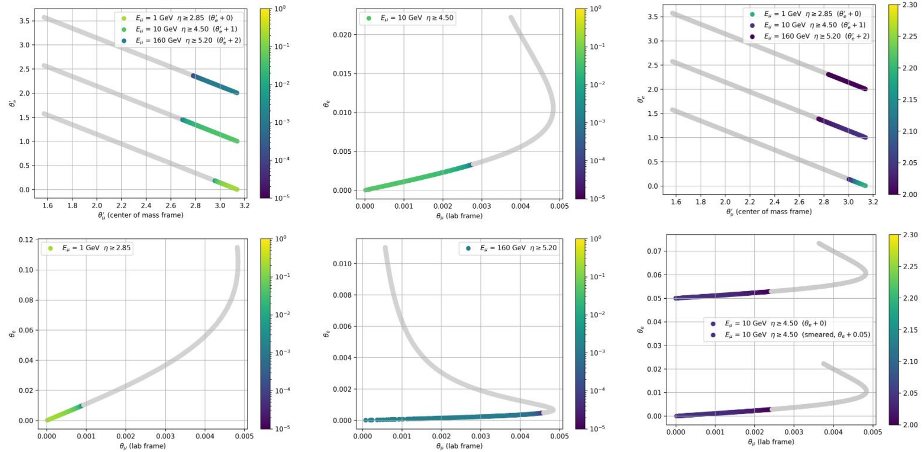
$e\mu$ QE sources via muon targeting

Concurrence $\mathcal{C}(\rho_f)$: Optimal I_2:



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