# Tripartite entanglement and Bell non-locality in loop-induced Higgs boson decays

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# Outline

- Introduction
- Three-body Higgs boson decays $H \to \gamma l \bar{l}$
- Entanglement quantifiers for 3-qubit states
- Related bipartite Higgs decays
- Numerical results
- Summary and perspectives

# Introduction

**High-energy colliders as "quantum information laboratories"**

Significant progress mainly focuses on quantum properties in bipartite systems: top-pair, diboson production, $H \to VV^*$, tau-pair, VBS, single-top, neutrinos, $B$, $K$, ...

BSM (SMEFT, anomalous couplings, ...), Fundamental Principles (MaxEnt, Area Law, ...)

> ATLAS and CMS measurements for $t\bar{t}$ production
>
> [Nature 633 (2024) 542; Rept.Prog.Phys. 87 (2024) 117801; PRD 110 (2024) 11, 112016]

Deeper understanding of multipartite correlations is needed.
Crucial not only for make progress fundamental physics but also for developing emerging quantum technologies.

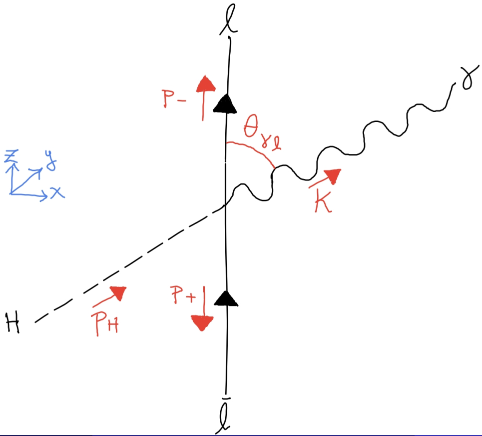
Systematic investigations of three-particle entanglement and non-locality in particle physics is underexplored [Acin et al., Sakurai et al.; Subba et al.; Fonseca et al.; Quinta et al.; Blasone et al.; Konwar et al.; Aguilar-Saavedra; Bernal et al.; RAM]

# Extending to multi-particle states

Three-body Higgs decay $H(p_H) \to \gamma(k) l(p_-) \bar{l}(p_+)$ as a case study

Proposals to study CP properties of the Higgs boson and BSM theories.
Related to bipartite Higgs decaying into $\gamma\gamma$ and $\gamma Z$.

13 TeV data: ATLAS evidence below 30 GeV and CMS $\sigma$ bounds at $Z$-peak.
[PLB 819, 136412 (2021)] [JHEP 11, 152 (2018)]



# Goals and approach

**Detailed theoretical predictions for quantum properties of this exotic Higgs decay**

- Identify relevant kinematical regions with high degree of entanglement.
- Quantify deviations from local realistic predictions in the phase space.

**Caveat**

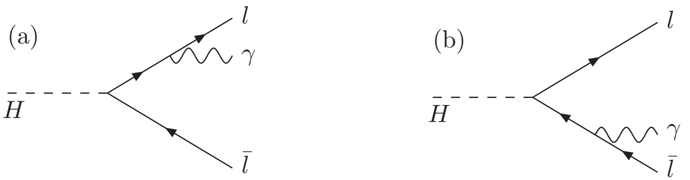
Very low statistics and lacking of techniques to measure spin of stable particles.

**Plan**

Compute the helicity amplitudes $\mathcal{M}_{s_1 s_2 s_3}$ of three-body Higgs decays.
Arrive to the full knowledge of $\rho$.
Define quantifiers related to entanglement detection and test Bell non-locality.

# Three-body Higgs boson decay at tree level

SM modified by $\tilde{\mathcal{L}}_{Hll} = -\frac{Y_l}{\sqrt{2}}\kappa^l_{\rm CP} H\bar{\psi}_l(\cos\delta^l_{\rm CP} + i\gamma^5 \sin\delta^l_{\rm CP})\psi_l \quad (l = e, \mu, \tau)$



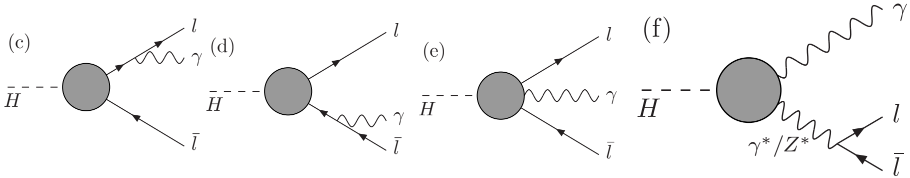

$$\mathcal{M}^{\rm Tree}_{s_1 s_2 s_3} = e\frac{\mathbf{Y_l}}{\sqrt{2}}\kappa^l_{\rm CP}\bar{u}_{s_2}\left(\frac{(\not\varepsilon^*_{s_1}\not k + 2\varepsilon^*_{s_1}\cdot p_-)(\cos\delta^l_{\rm CP} + i\gamma^5\sin\delta^l_{\rm CP})}{2k\cdot p_-} - \frac{(\cos\delta^l_{\rm CP} + i\gamma^5\sin\delta^l_{\rm CP})(\not k\not\varepsilon^*_{s_1} + 2\varepsilon^*_{s_1}\cdot p_+)}{2k\cdot p_+}\right)v_{s_3}$$

Two relevant kinematical variables in the dilepton rest frame:

dilepton invariant mass ($m_{l\bar{l}} = \sqrt{s}$) and the polar angle between photon and lepton ($\theta_{\gamma l}$)

**CP-effects here and Yukawa suppressed amplitude**

# Three-body loop-induced Higgs boson decay




$$\mathcal{M}^{1-\text{loop}}_{s_1 s_2 s_3} = \bar{u}_{s_2}\Big( (\varepsilon^*_{s_1}\cdot p_- \not{k} - k\cdot p_- \not{\varepsilon}^*_{s_1})(a_1 P_R + b_1 P_L) + (\varepsilon^*_{s_1}\cdot p_+ \not{k} - k\cdot p_+ \not{\varepsilon}^*_{s_1})(a_2 P_R + b_2 P_L)\Big) v_{s_3}$$

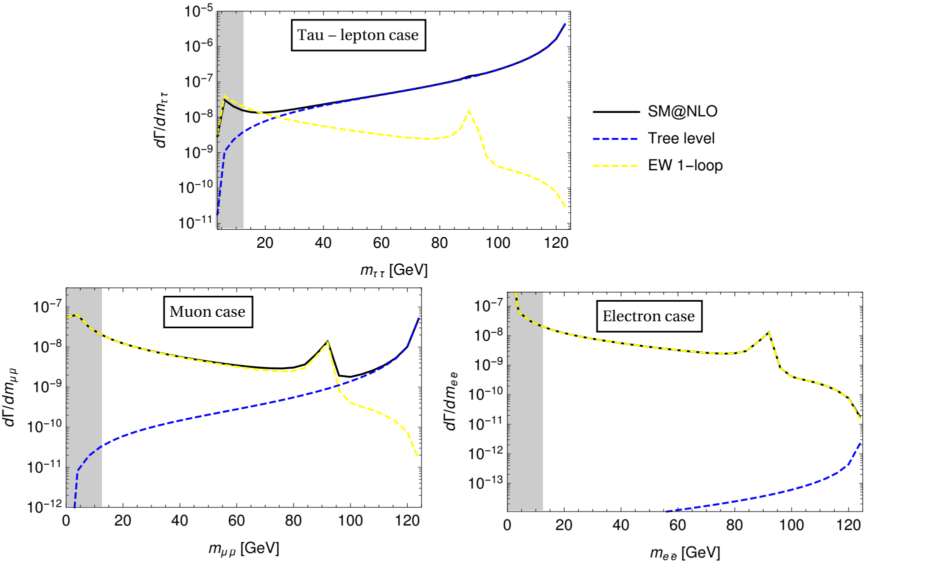
SM form factors from Kachanovich et al. [PRD 101, 073003 (2020)]

No Yukawa suppression nor CP-effects here (in contrast to tree-level)

**Unique opportunity to examine quantum correlations from radiative corrections**

[see also Severi et al.; Grossi et al.]

# Tree vs Loop contributions




$$\frac{d\Gamma_{H\to\gamma l\bar{l}}}{dm_{l\bar{l}}\, d\cos(\theta_{\gamma l})} = \frac{\sqrt{s-4m_l^2}(m_{\rm h}^2 - s)}{256\pi^3 m_{\rm h}^3} \sum_{s_1,s_2,s_3} |\mathcal{M}_{s_1 s_2 s_3}|^2 \quad \text{where} \quad (0.1 m_{\rm h})^2 \le s = m_{l\bar{l}}^2 \le s_{\rm cut} = m_{\rm h}^2 - 2m_{\rm h} E_{cut}^{\gamma}$$

# 3-qubit formalism

$8\times8$ density matrix $\rho$ in terms of the helicity amplitudes

$$\langle s_1\, s_2\, s_3 \underbrace{|\psi\rangle\langle\psi|}_{\rho} \tilde{s}_1\, \tilde{s}_2\, \tilde{s}_3\rangle = \left(\sum_{s_1,s_2,s_3} |\mathcal{M}_{s_1 s_2 s_3}|^2\right)^{-1} \mathcal{M}_{s_1 s_2 s_3} \mathcal{M}^{\dagger}_{\tilde{s}_1 \tilde{s}_2 \tilde{s}_3}$$

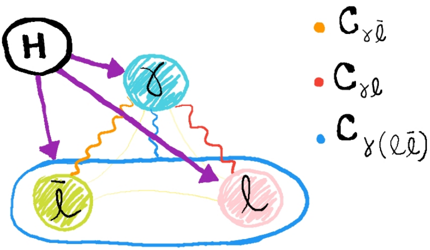
All information of the pure $2\otimes2\otimes2$ quantum final state in

$$\begin{aligned}
|\psi\rangle \;\simeq\; \frac{-i}{N}\Bigg( & 8e\frac{Y_l}{\sqrt{2}}\kappa^l_{\mathrm{CP}}e^{-i\delta^l_{\mathrm{CP}}}\frac{s}{(m_{\mathrm{h}}^2-s)\sin(\theta_{\gamma l})}|+++\rangle + a_2(m_{\mathrm{h}}^2-s)\sqrt{s}(1+\cos(\theta_{\gamma l}))|++-\rangle \\
& + b_1(m_{\mathrm{h}}^2-s)\sqrt{s}(1-\cos(\theta_{\gamma l}))|+-+\rangle + 8e\frac{Y_l}{\sqrt{2}}\kappa^l_{\mathrm{CP}}e^{i\delta^l_{\mathrm{CP}}}\frac{m_{\mathrm{h}}^2}{(m_{\mathrm{h}}^2-s)\sin(\theta_{\gamma l})}|+--\rangle \\
& + 8e\frac{Y_l}{\sqrt{2}}\kappa^l_{\mathrm{CP}}e^{-i\delta^l_{\mathrm{CP}}}\frac{m_{\mathrm{h}}^2}{(m_{\mathrm{h}}^2-s)\sin(\theta_{\gamma l})}|-++\rangle - a_1(m_{\mathrm{h}}^2-s)\sqrt{s}(1-\cos(\theta_{\gamma l}))|-+-\rangle \\
& - b_2(m_{\mathrm{h}}^2-s)\sqrt{s}(1+\cos(\theta_{\gamma l}))|--+\rangle + 8e\frac{Y_l}{\sqrt{2}}\kappa^l_{\mathrm{CP}}e^{i\delta^l_{\mathrm{CP}}}\frac{s}{(m_{\mathrm{h}}^2-s)\sin(\theta_{\gamma l})}|---\rangle\Bigg)
\end{aligned}$$

Multipartite systems have richer structure than bipartite ones.

# 3-qubit concurrences

fully-separable, bi-separable
or entangled tripartite state?




**One-to-other** concurrences

$$\mathcal{C}_{i(jk)} = \mathcal{C}_{jk|i} = \sqrt{2(1 - \mathrm{Tr}[\rho_{jk}^2])}$$

where $\rho_{jk}$ is the reduced density matrix of subsystem $jk$ by tracing over particle $i$ ($\rho_{jk} = \mathrm{Tr}_i[\rho]$)

**One-to-one** concurrences

$$\mathcal{C}_{jk} = \mathrm{Max}\{0, \eta_1^{jk} - \eta_2^{jk} - \eta_3^{jk} - \eta_4^{jk}\}$$

from eigenvalues of $R_{jk} = \sqrt{\sqrt{\rho_{jk}}(\sigma_2 \otimes \sigma_2)\rho_{jk}^*(\sigma_2 \otimes \sigma_2)\sqrt{\rho_{jk}}}$

Coffman-Kundu-Wootters (CKW) *monogamy inequality* $0 \leq t_3 = \mathcal{C}_{jk|i}^2 - \mathcal{C}_{ij}^2 - \mathcal{C}_{ik}^2$

**Area** of concurrence triangle is a measure of the *genuine* entanglement

$$\mathcal{F}_3 = \sqrt{\frac{16}{3} S(S - \mathcal{C}_{23|1})(S - \mathcal{C}_{31|2})(S - \mathcal{C}_{12|3})}$$

# 3-qubit Bell non-locality

LRHV or QM (or '*beyond QM*')?

Different notions of non-locality arising as extensions of the bipartite definition.

**Mermin operator**

$$\mathcal{M}_3 = \hat{a}_1 \otimes \hat{b}_1 \otimes \hat{c}_2 + \hat{a}_1 \otimes \hat{b}_2 \otimes \hat{c}_1 + \hat{a}_2 \otimes \hat{b}_1 \otimes \hat{c}_1 - \hat{a}_2 \otimes \hat{b}_2 \otimes \hat{c}_2$$

to test *fully local-real theories* (no non-local correlation among any pair $jk$ nor $i(jk)$)

It achieves maximum values $\langle \mathcal{M}_3 \rangle_{\mathrm{LRHV}} = 2$, $\langle \mathcal{M}_3 \rangle_{\mathrm{QM}} = 4$ and $\langle \mathcal{M}_3 \rangle_{\mathrm{alg}} = 4$.

**Svetlichny operator**

$$\begin{aligned}\mathcal{S}_3 = {} & \hat{a}_1 \otimes \hat{b}_1 \otimes \hat{c}_1 + \hat{a}_1 \otimes \hat{b}_1 \otimes \hat{c}_2 + \hat{a}_1 \otimes \hat{b}_2 \otimes \hat{c}_1 + \hat{a}_2 \otimes \hat{b}_1 \otimes \hat{c}_1 \\ & - \hat{a}_2 \otimes \hat{b}_2 \otimes \hat{c}_2 - \hat{a}_2 \otimes \hat{b}_2 \otimes \hat{c}_1 - \hat{a}_2 \otimes \hat{b}_1 \otimes \hat{c}_2 - \hat{a}_1 \otimes \hat{b}_2 \otimes \hat{c}_2\end{aligned}$$

to test *bipartite local-real theories* ($jk$ non-locally correlated but separated from $i$)

It achieves maximum values $\langle \mathcal{S}_3 \rangle_{\mathrm{LRHV}} = 4$, $\langle \mathcal{S}_3 \rangle_{\mathrm{QM}} = 4\sqrt{2}$ and $\langle \mathcal{S}_3 \rangle_{\mathrm{alg}} = 8$.

$H \to \gamma l \bar{l}$ **essentially results in the same behaviour for both operators**

(some final state configurations compatible with LRHV using $\mathcal{S}_3$ instead of $\mathcal{M}_3$)

# Dilepton subsytem and CP-effects

$H \to \gamma l\bar{l}$ as NLO correction of the observable dilepton decay $H \to l\bar{l}$.

The reduced density matrix $\rho_{l\bar{l}}$, neglecting the lepton mass, has vanishing polarizations $A$ and $B$ and the correlation matrix is independent of $\kappa^l_{\mathrm{CP}}$

$$C = \begin{pmatrix} \cos(2\delta^l_{\mathrm{CP}})\frac{2m_{\mathrm{h}}^2 s}{m_{\mathrm{h}}^4+s^2} & \sin(2\delta^l_{\mathrm{CP}})\frac{2m_{\mathrm{h}}^2 s}{m_{\mathrm{h}}^4+s^2} & 0 \\ \sin(2\delta^l_{\mathrm{CP}})\frac{2m_{\mathrm{h}}^2 s}{m_{\mathrm{h}}^4+s^2} & -\cos(2\delta^l_{\mathrm{CP}})\frac{2m_{\mathrm{h}}^2 s}{m_{\mathrm{h}}^4+s^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The resulting concurrence is $\mathcal{C}_{l\bar{l}} = \frac{2m_{\mathrm{h}}^2 s}{m_{\mathrm{h}}^4+s^2}$.

NO dependence on the CP-phase in this entanglement measure.
Sensitivity to $\delta^l_{\mathrm{CP}}$ by fitting entries of the correlation matrix, as $H \to \tau\bar{\tau}$.
[Fabbrichesi et al.; Altakach et al.]

**Concurrences are not sensitive to CP-effects**

$\Rightarrow$ SM computation from now on

# $H \to \gamma Z$ post-decay entanglement and *MaxEnt* Principle

Diphoton decay is maximally entangled and saturates the CHSH operator.

[Fabbrichesi et al.]

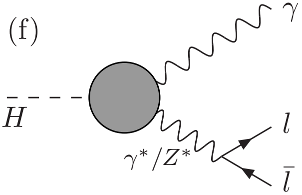
$2 \otimes 3$ system from $H \to \gamma Z$ decay achieves maximal concurrence $=1$ but NO generalized-CHSH violation.

[Caban et al.; Barr; see also Bernal et al.]

$$\rho_{H\to\gamma Z} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

For $\sqrt{s} \sim m_{\mathrm{Z}}$ (using NWA):

$$\mathcal{M}^{s_1 s_2 s_3}_{(\mathrm{f})}|_{\mathrm{Z(OS)}} \propto \sum_{\bar{s}_1} \mathcal{M}^{s_1 \bar{s}_1}_{H\to\gamma Z} \mathcal{M}^{\bar{s}_1 s_2 s_3}_{Z\to l\bar{l}}$$




Post-decay phenomenon [Aguilar-Saavedra et al.]

$$\mathcal{C}^{(\mathrm{f})}_{\gamma l|\bar{l}} = \mathcal{C}^{(\mathrm{f})}_{\gamma \bar{l}|l} = \frac{4s_{\mathrm{w}}^2(1-2s_{\mathrm{w}}^2)}{1-4s_{\mathrm{w}}^2+8s_{\mathrm{w}}^4} \approx \mathbf{0.976}$$

very close to the corresponding one of the $\gamma Z$ state!

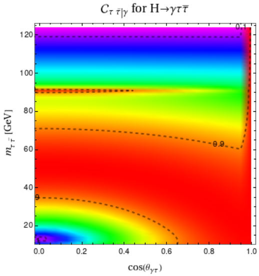
Imposing '*MaxEnt* Principle', weak angle should satisfy $\mathbf{s}_{\mathrm{w}}^{\mathbf{2}} = \mathbf{0.25}$

[Latorre et al.; Low et al.; Carena et al.; Thaller et al.]

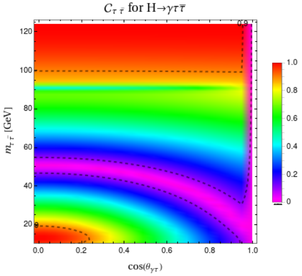
# Tau case results

Tree level dominates above 30 GeV.

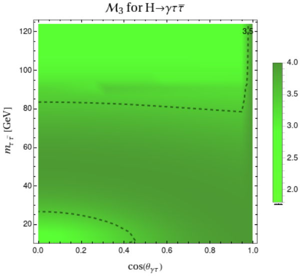
$\mathcal{C}_{\tau\bar{\tau}|\gamma}$




$\mathcal{C}_{\tau\bar{\tau}}$




$\mathcal{M}_3$




$\mathcal{C}_{\gamma\bar{\tau}|\tau}$ and $\mathcal{C}_{\gamma\tau|\bar{\tau}}$ very close to 1.

$\mathcal{F}_3$ nearly same distribution as $\mathcal{C}_{\tau\bar{\tau}|\gamma}$.

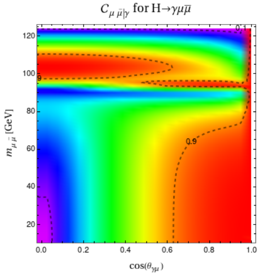
$\mathcal{C}_{\gamma\tau}$ and $\mathcal{C}_{\gamma\bar{\tau}}$ close to 0.

$\mathcal{M}_3 > 2.8$ and close to 4 when soft photons are collinear with leptons.

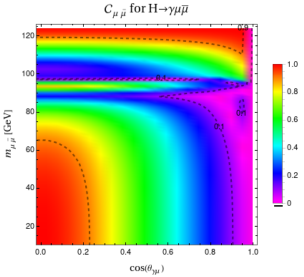
# Muon case results

Tree level dominates above 100 GeV.
Distributions change respect to the tau case.

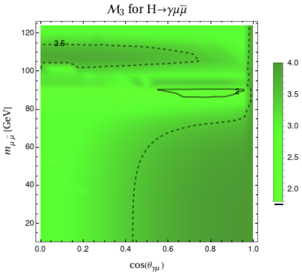
$\mathcal{C}_{\mu\bar{\mu}|\gamma}$




$\mathcal{C}_{\mu\bar{\mu}}$




$\mathcal{M}_3$




$\mathcal{C}_{\gamma\bar{\mu}|\mu}$ and $\mathcal{C}_{\gamma\mu|\bar{\mu}}$ close to 1 but have minimum 0.6 around $Z$-pole.

$\mathcal{F}_3$ almost same distribution as $\mathcal{C}_{\mu\bar{\mu}|\gamma}$ (tiny variations around $Z$-pole).

$\mathcal{C}_{\gamma\mu}$ and $\mathcal{C}_{\gamma\bar{\mu}}$ close to 0.

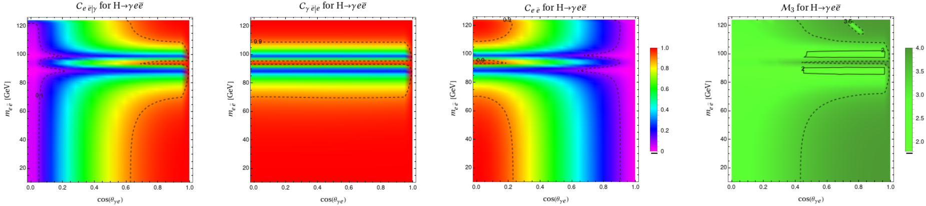
Mermin operator lower than tau case with small region compatible with LR.

# Electron case results

Tree level is negligible.

Distributions change respect to the tau and muon cases.

$\mathcal{C}_{e\bar{e}|\gamma}$ $\quad$ $\mathcal{C}_{\gamma\bar{e}|e} = \mathcal{C}_{\gamma e|\bar{e}}$ $\quad$ $\mathcal{C}_{e\bar{e}}$ $\quad$ $\mathcal{M}_3$




$\mathcal{F}_3$ small variations w.r.t. $\mathcal{C}_{e\bar{e}|\gamma}$

$\mathcal{C}_{\gamma e}$ and $\mathcal{C}_{\gamma\bar{e}}$ close to 0.

Mermin operator similar behaviour as the muon case.

# Available experimental data for tripartite systems?

Future avenues and potential collaborations:

**Novel entanglement quantifiers accessible with current HEP data?**
Other relevant tripartite decays ($B \to K^* l\bar{l}$).
Actual limitation in spin measurements: incomplete quantum state tomography
$\Rightarrow$ Witness involving just partial information of the density matrix.

*Preliminary:* $\mathcal{C}_{Kl|l}$ just depend on $|\mathcal{M}_{s_1 s_2 s_3}|$ (no relative complex phases)

**Non-locality test for $2 \otimes 2 \otimes 3$ systems?**
such as $t\bar{t}Z$ production or $B \to K^* l\bar{l}$ decay
$\Rightarrow$ *Inductive* Mermin/Svetlichny construction together with
optimal Bell inequalities of $2 \otimes d$ systems [Bernal et al.]

# Summary

**$H \to \gamma l \bar{l}$ analysis from a new perspective:**
Quantify degree of entanglement and non-local correlations over the phase space of this three-body decay (3-qubit)

- Yukawa suppressed at tree-level $\Rightarrow$ examine quantum correlations in EW 1-loop corrections.
- In general, $\mathcal{C}_{\gamma\bar{l}|l}$ and $\mathcal{C}_{\gamma l|\bar{l}}$ close to 1 and $\mathcal{C}_{\gamma l}$ and $\mathcal{C}_{\gamma \bar{l}}$ close to 0
- Mermin operator compatible with LR just in tiny regions for $e$ and $\mu$. Quantum bound almost saturated for all families.
- CP-effects have negligible impact in this kind of observables
- Post-decay phenomenon at the $Z$ resonance and '*MaxEnt* Principle' favors weak angle close to SM value.

# Backup slides

# Kinematics for $H(p_H) \to \gamma(k) l(p_-) \bar{l}(p_+)$ decay

**Rest frame of the lepton-pair**: $z$-axis along the direction of the lepton, $y$-axis perpendicular to the decay plane and photon momentum has positive $x$-component.

$$p_H = \left( \sqrt{m_{\rm h}^2 + |\vec{k}|^2}, |\vec{k}| \sin(\theta_{\gamma l}), 0, |\vec{k}| \cos(\theta_{\gamma l}) \right) , \quad k = \left( |\vec{k}|, |\vec{k}| \sin(\theta_{\gamma l}), 0, |\vec{k}| \cos(\theta_{\gamma l}) \right) ,$$

$$p_- = \left( \sqrt{s}/2, 0, 0, \sqrt{s}\beta_l/2 \right) , \quad p_+ = \left( \sqrt{s}/2, 0, 0, -\sqrt{s}\beta_l/2 \right) \text{ with } |\vec{k}| = \frac{m_{\rm h}^2 - s}{2\sqrt{s}}$$

and $\beta_l = \sqrt{1 - 4m_l^2/s}$ is the lepton velocity in this frame.
The two transverse polarization vectors of the photon are

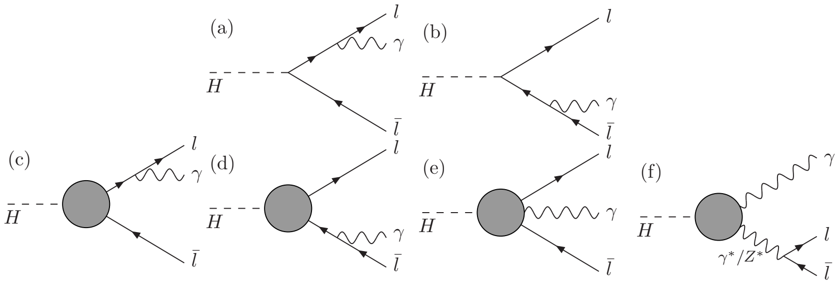
$$\varepsilon_\pm(k) = \frac{1}{\sqrt{2}} \left( 0, -i\cos(\theta_{\gamma l}), \mp 1, i\sin(\theta_{\gamma l}) \right)$$

Of course, we can chose the Higgs rest frame by performing the Lorentz transformation

$$L^{(H)} = \begin{pmatrix} \sqrt{m_{\rm h}^2 + |\vec{k}|^2}/m_{\rm h} & -|\vec{k}|\sin(\theta_{\gamma l})/m_{\rm h} & 0 & -|\vec{k}|\cos(\theta_{\gamma l})/m_{\rm h} \\ 0 & \cos(\theta_{\gamma l}) & 0 & -\sin(\theta_{\gamma l}) \\ 0 & 0 & 1 & 0 \\ -|\vec{k}|/m_{\rm h} & \sqrt{m_{\rm h}^2 + |\vec{k}|^2}\sin(\theta_{\gamma l})/m_{\rm h} & 0 & \sqrt{m_{\rm h}^2 + |\vec{k}|^2}\cos(\theta_{\gamma l})/m_{\rm h} \end{pmatrix}$$

In that frame, the photon momentum is $k^{(H)} = \left( \frac{m_{\rm h}^2 - s}{2m_{\rm h}}, 0, 0, \frac{m_{\rm h}^2 - s}{2m_{\rm h}} \right)$ and a lower cut $E_{cut}^\gamma$ to the photon energy is imposed in order to avoid IR divergences in the tree level contribution. Then the upper bound on the dilepton invariant mass is obtained.

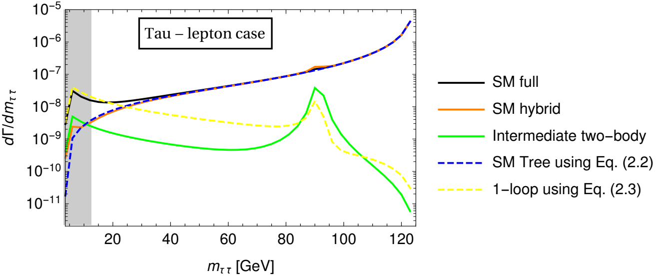
# Three-body Higgs boson decays computation

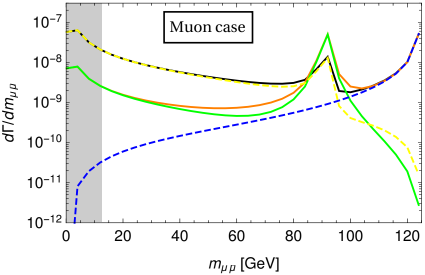


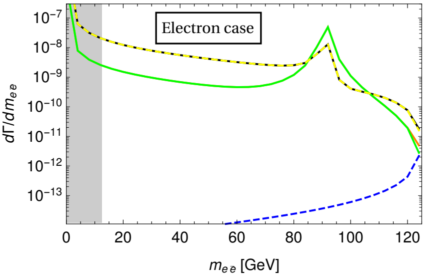

Interesting combinations

$$\begin{aligned}
\mathcal{M}_{\rm full} &= \mathcal{M}_{\rm (a)} + \ldots + \mathcal{M}_{\rm (f)} \quad = \mathcal{M}_{\rm Tree} + \mathcal{M}_{\rm 1-loop}\,,\\
\mathcal{M}_{\rm hybrid} &= \mathcal{M}_{\rm (a)} + \mathcal{M}_{\rm (b)} + \mathcal{M}_{\rm (f)} = \mathcal{M}_{\rm Tree} + \mathcal{M}_{\rm (f)}\,,\\
\mathcal{M}_{\rm two-body} &= \mathcal{M}_{\rm (f)}
\end{aligned}$$

# Full, hybrid and two-body contributions




$$\frac{d\Gamma_{H\to\gamma l\bar{l}}}{dm_{l\bar{l}}\, d\cos(\theta_{\gamma l})} = \frac{\sqrt{s-4m_l^2}\,(m_{\rm h}^2 - s)}{256\pi^3 m_{\rm h}^3} \sum_{s_1,s_2,s_3} |\mathcal{M}_{s_1 s_2 s_3}|^2 \quad \text{where} \quad (0.1 m_{\rm h})^2 \le s = m_{l\bar{l}}^2 \le s_{\rm cut} = m_{\rm h}^2 - 2m_{\rm h} E_{cut}^{\gamma}$$

# Tree level one-to-other concurrences

$$
\mathcal{C}^{\mathrm{Tree}}_{\gamma\bar{l}|l} = \mathcal{C}^{\mathrm{Tree}}_{\gamma l|\bar{l}} = 1\,,
$$

$$
\begin{aligned}
\mathcal{C}^{\mathrm{Tree}}_{l\bar{l}|\gamma} = {} & \frac{\left(m_{\mathrm{h}}^2 - s\right)\left(s - c_{\gamma l}^2\left(s - 4m_l^2\right)\right)^{1/2}}{c_{\gamma l}^2\left(4m_l^2 - s\right)\left(s\left(s - 8m_l^2 c_{\mathrm{CP}}^2\right) + m_{\mathrm{h}}^4\right) + s\left(8m_l^2 c_{\mathrm{CP}}^2\left(4m_l^2 - s\right) + m_{\mathrm{h}}^4 - 8m_{\mathrm{h}}^2 m_l^2 + s^2\right)} \times \\
& \times \left(c_{\gamma l}^2\left(4m_l^2 - s\right)\left(s\left(s - 16m_l^2 c_{\mathrm{CP}}^2\right) + m_{\mathrm{h}}^4 + 2m_{\mathrm{h}}^2 s\right)\right. \\
& \left. + s\left(16m_l^2 c_{\mathrm{CP}}^2\left(4m_l^2 - s\right) + m_{\mathrm{h}}^4 + 2m_{\mathrm{h}}^2\left(s - 8m_l^2\right) + s^2\right)\right)^{1/2}
\end{aligned}
$$

In the vanishing lepton mass limit,

$$
\mathcal{C}^{\mathrm{Tree}}_{l\bar{l}|\gamma}\big|_{m_l \ll \sqrt{s}} = \frac{m_{\mathrm{h}}^4 - s^2}{m_{\mathrm{h}}^4 + s^2}
$$

which is very close to zero in the cut energy $\sqrt{s}_{\mathrm{cut}}$ and we almost have the biseparable state $\sim (|+\rangle + |-\rangle) \otimes (|++\rangle + e^{2i\delta^l_{\mathrm{CP}}}|--\rangle)$, where the normalization factor is omitted.

On the contrary, if the photon is collinear with lepton or antilepton, $\mathcal{C}^{\mathrm{Tree}}_{l\bar{l}|\gamma}$ reaches the maximal value 1.

# 1-loop one-to-other concurrences

neglecting lepton mass terms

$$\mathcal{C}^{1-\mathrm{loop}}_{\gamma\bar{l}|l} = \mathcal{C}^{1-\mathrm{loop}}_{\gamma l|\bar{l}} =$$

$$= \frac{2\left(\left(|a_1|^2(1-c_{\gamma l})^2 + |a_2|^2(1+c_{\gamma l})^2\right)\left(|b_1|^2(1-c_{\gamma l})^2 + |b_2|^2(1+c_{\gamma l})^2\right)\right)^{1/2}}{(|a_1|^2+|b_1|^2)(1-c_{\gamma l})^2 + (|a_2|^2+|b_2|^2)(1+c_{\gamma l})^2},$$

$$\mathcal{C}^{1-\mathrm{loop}}_{l\bar{l}|\gamma} = \frac{2\left|a_1 b_1(1-c_{\gamma l})^2 - a_2 b_2(1+c_{\gamma l})^2\right|}{(|a_1|^2+|b_1|^2)(1-c_{\gamma l})^2 + (|a_2|^2+|b_2|^2)(1+c_{\gamma l})^2}$$

# Genuine entanglement

Concurrence vector formalism to this 3-qubit system [Bernal et al.] $\{q_0, q_1, q_2\} = \{0, 0, 0\}$, negelcting lepton mass

$$
\begin{aligned}
q_0 &= \frac{1}{N^2} A_{\rm Tree} \kappa^l_{\rm CP} e^{-i\delta^l_{\rm CP}} \sqrt{\frac{s}{1 - c^2_{\gamma l}}} \left( b_1 (1 - c_{\gamma l}) m_{\rm h}^2 + b_2 (1 + c_{\gamma l}) s \right) \,, \\
q_1 &= -\frac{1}{N^2} A_{\rm Tree} \kappa^l_{\rm CP} e^{i\delta^l_{\rm CP}} \sqrt{\frac{s}{1 - c^2_{\gamma l}}} \left( a_1 (1 - c_{\gamma l}) m_{\rm h}^2 + a_2 (1 + c_{\gamma l}) s \right) \,, \\
q_2 &= \frac{1}{8N^2} \Bigg( 64 A^2_{\rm Tree} (\kappa^l_{\rm CP})^2 \frac{m_{\rm h}^2 + s}{(m_{\rm h}^2 - s)(1 - c^2_{\gamma l})} \\
&\qquad - (m_{\rm h}^2 - s)^2 s \left( a_1 b_1 (1 - c_{\gamma l})^2 - a_2 b_2 (1 + c_{\gamma l})^2 \right) \Bigg)
\end{aligned}
$$

Of course, our previous findings for either tree level or 1-loop contribution are recovered from these conditions. The $q_0 = 0$ and $q_1 = 0$ equations relate the form factors to each other, and $q_2 = 0$ establish a relation between them and the tree level factors. For each kind of computation (full or two-body intermediate decay), we have non-trivial dependence of the form factors with $s$ and $\cos(\theta_{\gamma l})$, and very particular kinematical configurations could correspond to biseparable states .

# One-to-one concurrences

Neglecting lepton mass terms, both photon-to-lepton $\mathcal{C}_{\gamma l}$ and photon-to-antilepton $\mathcal{C}_{\gamma \bar{l}}$ exactly vanish for both tree level and 1-loop contributions (then lepton mass terms in the spinors will be relevant in the numerical analysis).
The lepton-to-antilepton $\mathcal{C}_{l\bar{l}}$ for the tree level is very compact

$$\mathcal{C}_{\gamma l}^{\mathrm{Tree},1-\mathrm{loop}}|_{m_l \ll \sqrt{s}} = \mathcal{C}_{\gamma \bar{l}}^{\mathrm{Tree},1-\mathrm{loop}}|_{m_l \ll \sqrt{s}} = 0$$

$$\mathcal{C}_{l\bar{l}}^{\mathrm{Tree}}|_{m_l \ll \sqrt{s}} = \frac{2m_{\mathrm{h}}^2 s}{m_{\mathrm{h}}^4 + s^2}$$

Observe that $\mathcal{C}_{l\bar{l}}^{\mathrm{Tree}}$ never vanishes and is very close to 1 in the cut energy $\sqrt{s}_{\mathrm{cut}}$.

# Helicity amplitudes

$$
\begin{aligned}
\mathcal{M}_{+++} &= \frac{-i\sqrt{2}A_{\rm Tree}\kappa^{l}_{\rm CP}s_{\gamma l}}{(m_{\rm h}^2-s)(1-c_{\gamma l}^2\beta_l^2)}\left(c_{\rm CP}(m_{\rm h}^2(1-\beta_l)+s(1+\beta_l)-8m_l^2)+is_{\rm CP}(m_{\rm h}^2(1-\beta_l)-s(1+\beta_l))\right)\\
&\quad-\frac{im_l(m_{\rm h}^2-s)s_{\gamma l}}{4\sqrt{2}}\left(a_1(1-\beta_l)+a_2(1+\beta_l)+b_1(1+\beta_l)+b_2(1-\beta_l)\right)\ ,\\
\mathcal{M}_{++-} &= \frac{-i2\sqrt{2}A_{\rm Tree}\kappa^{l}_{\rm CP}e^{i\delta^{l}_{\rm CP}}(1+c_{\gamma l})m_l}{\sqrt{s}(1-c_{\gamma l}^2\beta_l^2)}\\
&\quad-\frac{i(m_{\rm h}^2-s)(1+c_{\gamma l})}{4\sqrt{2}\sqrt{s}}\left(2a_1m_l^2+a_2(s(1+\beta_l)-2m_l^2)+b_1(s(1-\beta_l)-2m_l^2)+2b_2m_l^2\right)\ ,\\
\mathcal{M}_{+-+} &= \frac{-i2\sqrt{2}A_{\rm Tree}\kappa^{l}_{\rm CP}e^{i\delta^{l}_{\rm CP}}(1-c_{\gamma l})m_l}{\sqrt{s}(1-c_{\gamma l}^2\beta_l^2)}\\
&\quad-\frac{i(m_{\rm h}^2-s)(1-c_{\gamma l})}{4\sqrt{2}\sqrt{s}}\left(2a_1m_l^2+a_2(s(1-\beta_l)-2m_l^2)+b_1(s(1+\beta_l)-2m_l^2)+2b_2m_l^2\right)\ ,\\
\mathcal{M}_{+--} &= \frac{-i\sqrt{2}A_{\rm Tree}\kappa^{l}_{\rm CP}s_{\gamma l}}{(m_{\rm h}^2-s)(1-c_{\gamma l}^2\beta_l^2)}\left(c_{\rm CP}(m_{\rm h}^2(1+\beta_l)+s(1-\beta_l)-8m_l^2)+is_{\rm CP}(m_{\rm h}^2(1+\beta_l)-s(1-\beta_l))\right)\\
&\quad-\frac{im_l(m_{\rm h}^2-s)s_{\gamma l}}{4\sqrt{2}}\left(a_1(1+\beta_l)+a_2(1-\beta_l)+b_1(1-\beta_l)+b_2(1+\beta_l)\right)
\end{aligned}
$$

# Helicity amplitudes

$$\begin{aligned}
\mathcal{M}_{-++} &= \frac{-i\sqrt{2}A_{\mathrm{Tree}}\kappa^{l}_{\mathrm{CP}}s_{\gamma l}}{(m_{\mathrm{h}}^2-s)(1-c_{\gamma l}^2\beta_l^2)}\left(c_{\mathrm{CP}}(m_{\mathrm{h}}^2(1+\beta_l)+s(1-\beta_l)-8m_l^2)-is_{\mathrm{CP}}(m_{\mathrm{h}}^2(1+\beta_l)-s(1-\beta_l))\right)\\
&-\frac{im_l(m_{\mathrm{h}}^2-s)s_{\gamma l}}{4\sqrt{2}}\left(a_1(1-\beta_l)+a_2(1+\beta_l)+b_1(1+\beta_l)+b_2(1-\beta_l)\right)\ ,\\
\mathcal{M}_{-+-} &= \frac{i2\sqrt{2}A_{\mathrm{Tree}}\kappa^{l}_{\mathrm{CP}}e^{-i\delta^{l}_{\mathrm{CP}}}(1-c_{\gamma l})m_l}{\sqrt{s}(1-c_{\gamma l}^2\beta_l^2)}\\
&+\frac{i(m_{\mathrm{h}}^2-s)(1-c_{\gamma l})}{4\sqrt{2}\sqrt{s}}\left(a_1(s(1+\beta_l)-2m_l^2)+2a_2m_l^2+2b_1m_l^2+b_2(s(1-\beta_l)-2m_l^2)\right)\ ,\\
\mathcal{M}_{--+} &= \frac{i2\sqrt{2}A_{\mathrm{Tree}}\kappa^{l}_{\mathrm{CP}}e^{-i\delta^{l}_{\mathrm{CP}}}(1+c_{\gamma l})m_l}{\sqrt{s}(1-c_{\gamma l}^2\beta_l^2)}\\
&+\frac{i(m_{\mathrm{h}}^2-s)(1+c_{\gamma l})}{4\sqrt{2}\sqrt{s}}\left(a_1(s(1-\beta_l)-2m_l^2)+2a_2m_l^2+2b_1m_l^2+b_2(s(1+\beta_l)-2m_l^2)\right)\ ,\\
\mathcal{M}_{---} &= \frac{-i\sqrt{2}A_{\mathrm{Tree}}\kappa^{l}_{\mathrm{CP}}s_{\gamma l}}{(m_{\mathrm{h}}^2-s)(1-c_{\gamma l}^2\beta_l^2)}\left(c_{\mathrm{CP}}(m_{\mathrm{h}}^2(1-\beta_l)+s(1+\beta_l)-8m_l^2)-is_{\mathrm{CP}}(m_{\mathrm{h}}^2(1-\beta_l)-s(1+\beta_l))\right)\\
&-\frac{im_l(m_{\mathrm{h}}^2-s)s_{\gamma l}}{4\sqrt{2}}\left(a_1(1+\beta_l)+a_2(1-\beta_l)+b_1(1-\beta_l)+b_2(1+\beta_l)\right)
\end{aligned}$$

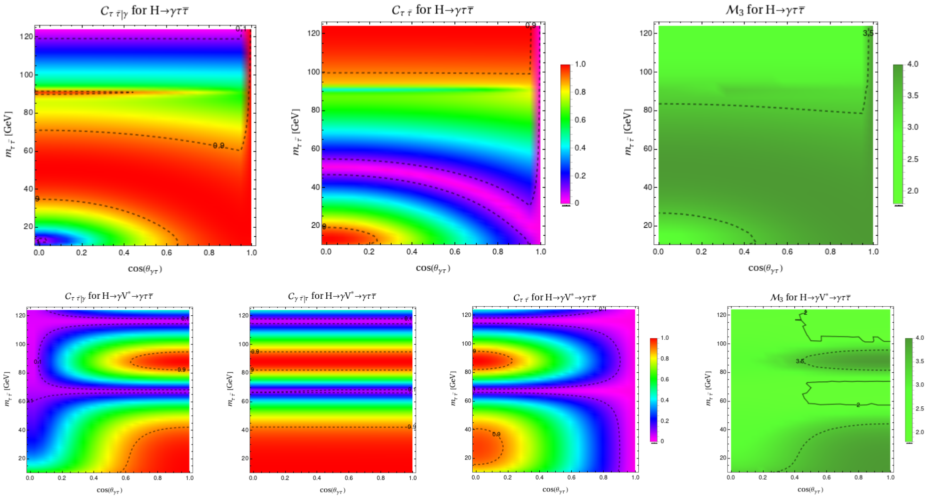
# Interesting kinematical limits

Vanishing photon energy ($s = m_{\rm h}^2$), the 1-loop contribution vanishes for all helicity amplitudes and the tree level yields to IR divergences for $\{+++, +--, -+++, ---\}$ amplitudes (which are avoided by the lower cut $E^{\gamma}_{cut}$).
When photon collinear with lepton ($\theta_{\gamma l} = 0$) or with antilepton ($\theta_{\gamma l} = \pi$), just two helicity amplitudes are non-vanishing for each case:

$$\begin{aligned}
\mathcal{M}_{++-}|_{\theta_{\gamma l}=0} = & -\frac{i\sqrt{2}A_{\rm Tree}\kappa^l_{\rm CP}e^{i\delta^l_{\rm CP}}\sqrt{s}}{m_l} \\
& -\frac{i(m_{\rm h}^2 - s)}{2\sqrt{2}\sqrt{s}}\left(2a_1 m_l^2 + a_2(s(1+\beta_l) - 2m_l^2) + b_1(s(1-\beta_l) - 2m_l^2) + 2b_2 m_l^2\right) , \\
\mathcal{M}_{--+}|_{\theta_{\gamma l}=0} = & \frac{i\sqrt{2}A_{\rm Tree}\kappa^l_{\rm CP}e^{-i\delta^l_{\rm CP}}\sqrt{s}}{m_l} \\
& +\frac{i(m_{\rm h}^2 - s)}{2\sqrt{2}\sqrt{s}}\left(a_1(s(1-\beta_l) - 2m_l^2) + 2a_2 m_l^2 + 2b_1 m_l^2 + b_2(s(1+\beta_l) - 2m_l^2)\right)
\end{aligned}$$

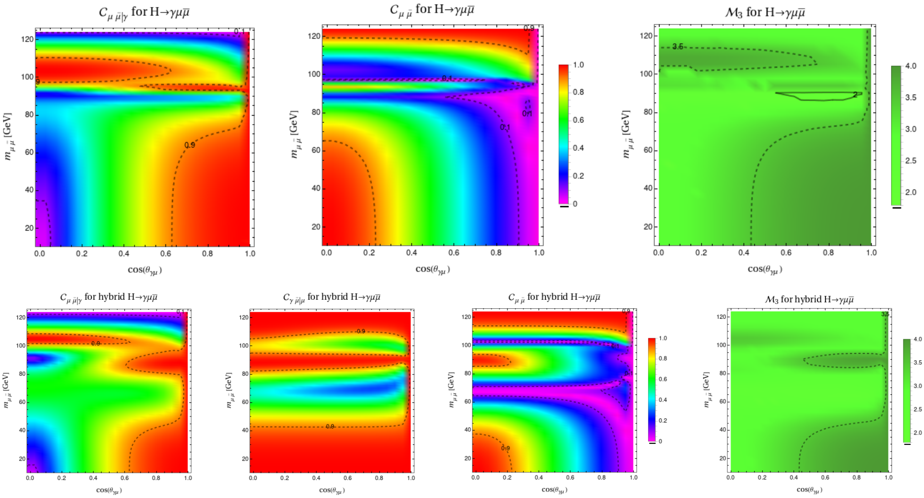
Massless lepton case ($m_l = 0$, $\beta_l = 1$) has IR divergences when photon is collinear with lepton and antilepton.

# Tau case results for two-body computation



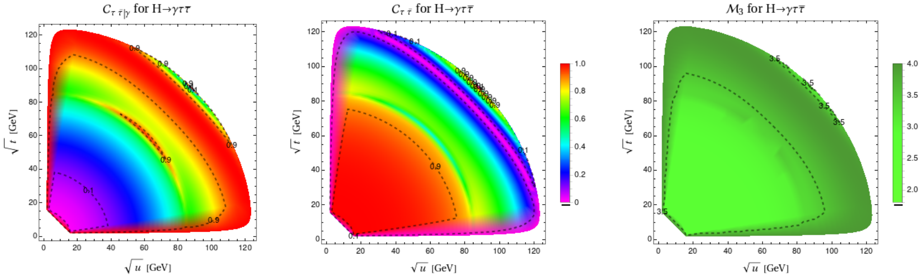

Full predictions similar to hybrid computation having broader regions of high entanglement than two-body case.

# Muon case results for hybrid computation

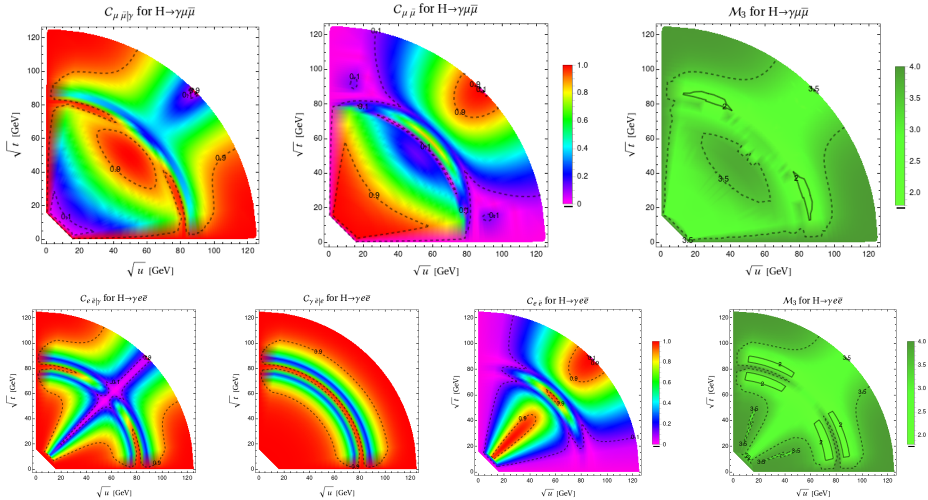



Hybrid predictions as a sort of transition between full and two-body computations reducing entanglement.

Change of variables $\cos(\theta_{\gamma l}) = \frac{u-t}{(m_{\mathrm{h}}^2-s)\sqrt{1-4m_l^2/s}} \Rightarrow$
frame independent representation using Mandelstam variables
$s = (p_- + p_+)^2$, $t = (k + p_-)^2$ and $u = (k + p_+)^2$



# Dalitz plots representation for $e$ and $\mu$ cases



# Testing non-locality at colliders

**Using Bell inequalities at colliders to test non-locality is challenging** due to the difficulty in measuring the final state's non-commuting observables.

Issues to conduct a loophole-free test of local realism at conventional collider experiments [Abel et al.; Horodecki et al.]:

*Detection loop-hole*: high-energy collider experiments have imperfect acceptance and detection efficiency

*Freedom-of-choice loophole*: we do not have the freedom to choose the spin measurement axes, as the spin measurement is constituted indirectly and statistically by analysing the momentum distributions.

*There is always some hidden variable theory that can explain the observed momentum data* since momenta of observed particles are essentially commuting observables.

**The interest of this work is not a high-energy test of local realism!!!**

# Three-body $B$ meson decays

Processes $B \to K^{*0} l \bar{l}$ and $\bar{B} \to \bar{K}^{*0} l \bar{l}$ with final state in the helicity amplitude basis $\{+, 0, -\} \otimes \{+, -\} \otimes \{+, -\}$:

$|\psi\rangle = \mathcal{H}_{++-}|++-\rangle + \mathcal{H}_{+-+}|+-+\rangle + \mathcal{H}_{0+-}|0+-\rangle + \mathcal{H}_{0-+}|0-+\rangle + \mathcal{H}_{-+-}|-+-\rangle + \mathcal{H}_{--+}|--+\rangle$

We found one-to-other concurrences (NOT accessible with current data)

$$
\begin{aligned}
&\mathcal{C}_{\bar{K}^* l^+|l^-} = \mathcal{C}_{\bar{K}^* l^-|l^+} = \\
&= 2\left(\frac{d\Gamma}{dq^2}\right)^{-1} \left(|\mathcal{A}_0^L|^2 + |\mathcal{A}_\perp^L|^2 + |\mathcal{A}_\parallel^L|^2\right)^{1/2} \left(|\mathcal{A}_0^R|^2 + |\mathcal{A}_\perp^R|^2 + |\mathcal{A}_\parallel^R|^2\right)^{1/2}
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{C}_{l^- l^+|\bar{K}^*} = \\
&= 2\left(\frac{d\Gamma}{dq^2}\right)^{-1} \Big(|\mathcal{A}_0^L|^2|\mathcal{A}_\perp^R|^2 + |\mathcal{A}_0^R|^2|\mathcal{A}_\perp^L|^2 - 2|\mathcal{A}_0^L||\mathcal{A}_0^R||\mathcal{A}_\perp^L||\mathcal{A}_\perp^R|\cos\left(\phi_0^L - \phi_0^R - \phi_\perp^L + \phi_\perp^R\right) \\
&\quad + |\mathcal{A}_0^L|^2|\mathcal{A}_\parallel^R|^2 + |\mathcal{A}_0^R|^2|\mathcal{A}_\parallel^L|^2 - 2|\mathcal{A}_0^L||\mathcal{A}_0^R||\mathcal{A}_\parallel^L||\mathcal{A}_\parallel^R|\cos\left(\phi_\parallel^L - \phi_\parallel^R - \phi_0^L + \phi_0^R\right) \\
&\quad + |\mathcal{A}_\perp^L|^2|\mathcal{A}_\parallel^R|^2 + |\mathcal{A}_\perp^R|^2|\mathcal{A}_\parallel^L|^2 - 2|\mathcal{A}_\perp^L||\mathcal{A}_\perp^R||\mathcal{A}_\parallel^L||\mathcal{A}_\parallel^R|\cos\left(\phi_\perp^L - \phi_\perp^R - \phi_\parallel^L + \phi_\parallel^R\right)\Big)^{1/2}
\end{aligned}
$$

where $\mathcal{A}_\perp^{L(R)} = \frac{\mathcal{H}_{+-+(++-)} - \mathcal{H}_{--+(-+-)}}{\sqrt{2}}$, $\mathcal{A}_\parallel^{L(R)} = \frac{\mathcal{H}_{+-+(++-)} + \mathcal{H}_{--+(-+-)}}{\sqrt{2}}$, $\mathcal{A}_0^{L(R)} = \mathcal{H}_{0-+(0+-)}$