Quantum tops at lepton colliders

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Quantum Observables in Collider Physics
GGI, Florence

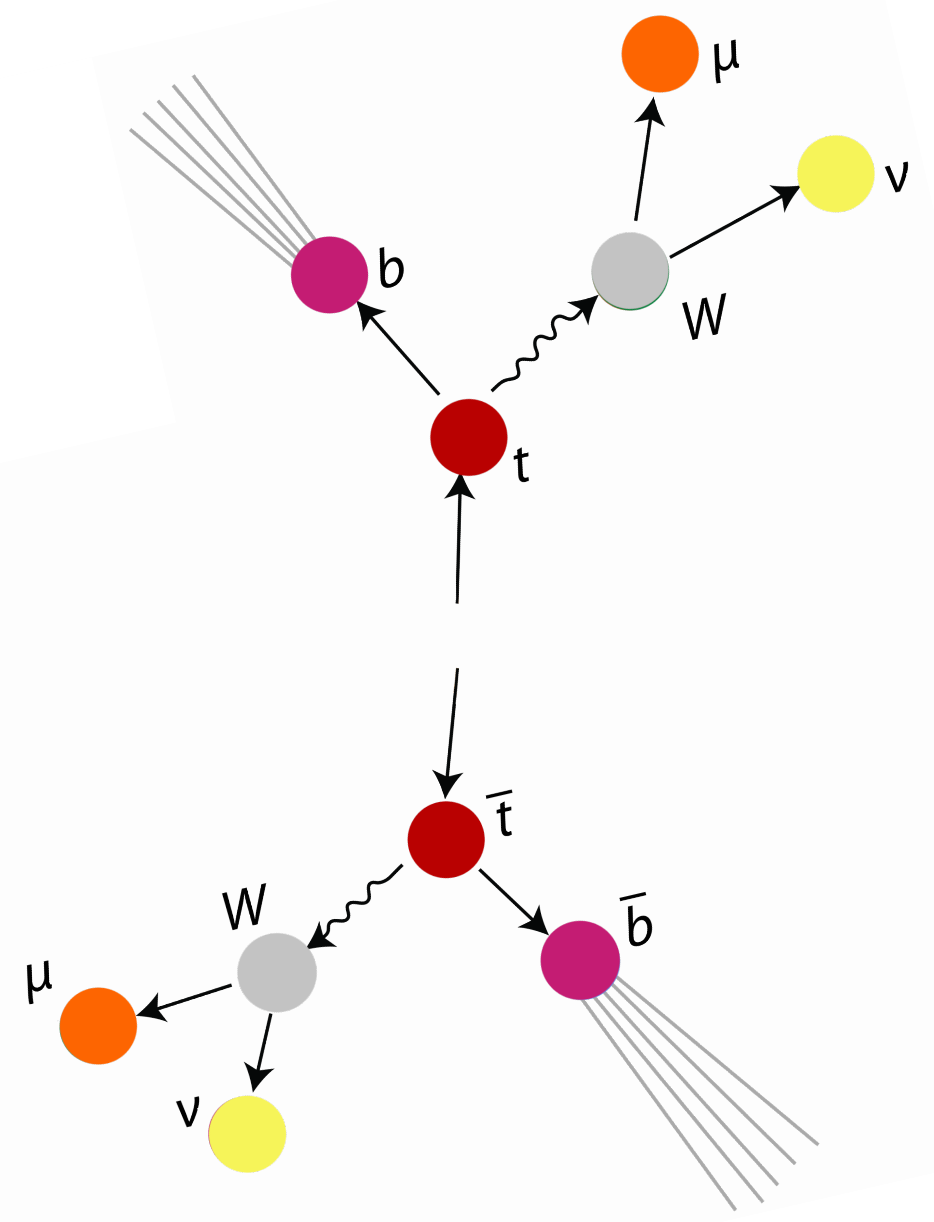
9/4/25

Spin density matrix

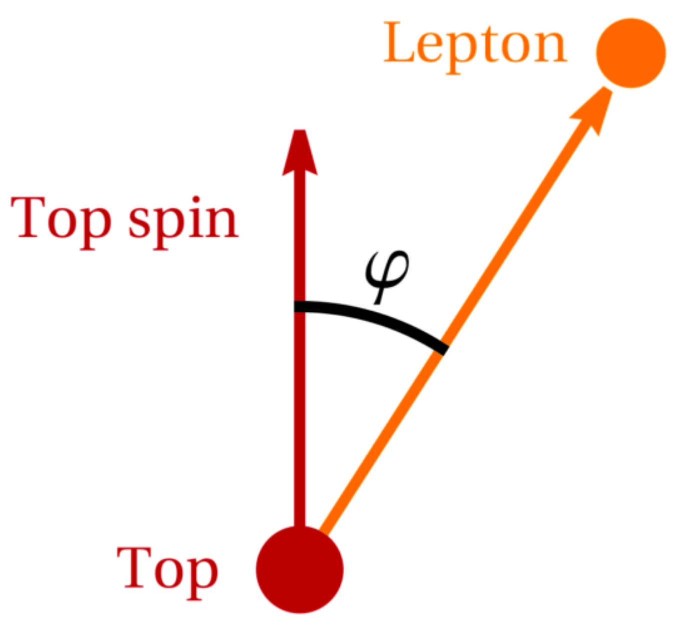
Tops produced in pairs have their spins S_i, S_j correlated: a two-qubit system

Spin density matrix:

$$\boxed{\rho = \frac{1}{4}\left(\mathbb{1}\otimes\mathbb{1} + \sum_{i=1}^{3} B_i\,\sigma_i\otimes\mathbb{1} + \sum_{i=j}^{3} \bar{B}_j\,\mathbb{1}\otimes\sigma_j + \sum_{i=1}^{3}\sum_{j=1}^{3} C_{ij}\,\sigma_i\otimes\sigma_j\right)}$$



15 parameters describe the quantum state of the top pair

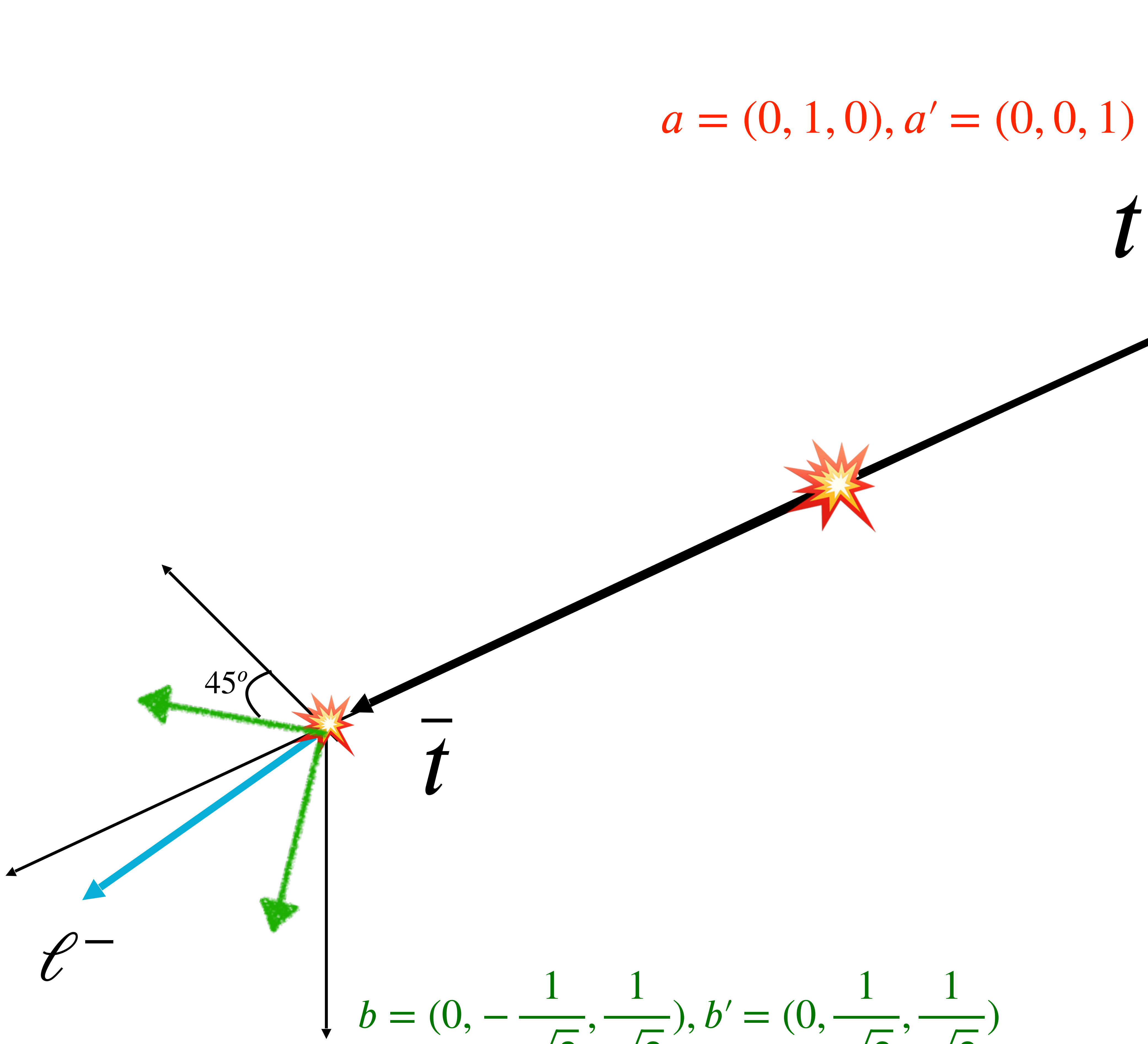


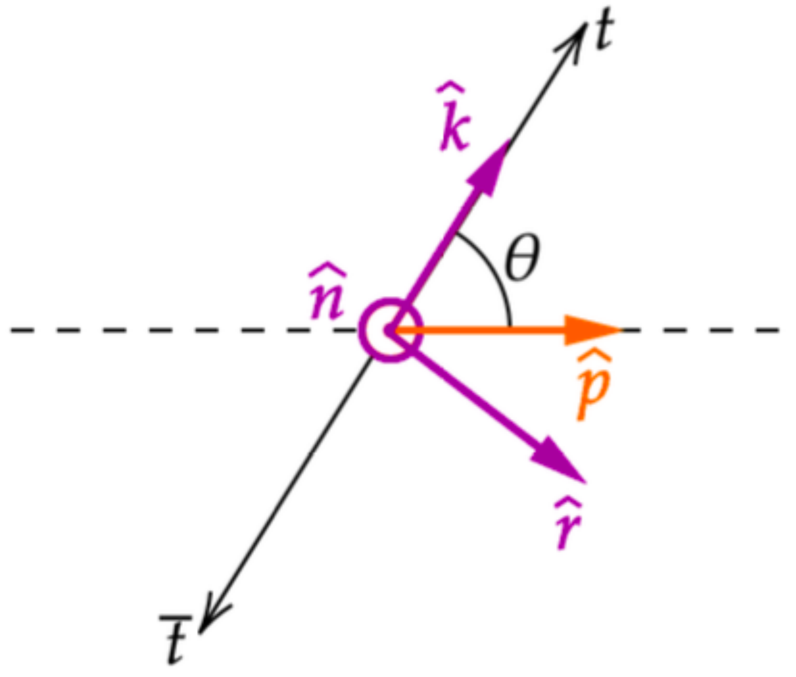
$$\langle S_i\rangle = B_i, \quad \langle \bar{S}_i\rangle = \bar{B}_j, \quad \langle S_i\bar{S}_j\rangle = C_{ij}$$

Extracted by measuring angular distributions of decay products

Quantum tomography is measurement of 15 parameters: 6 polarisations and 9 correlations

Kinematics





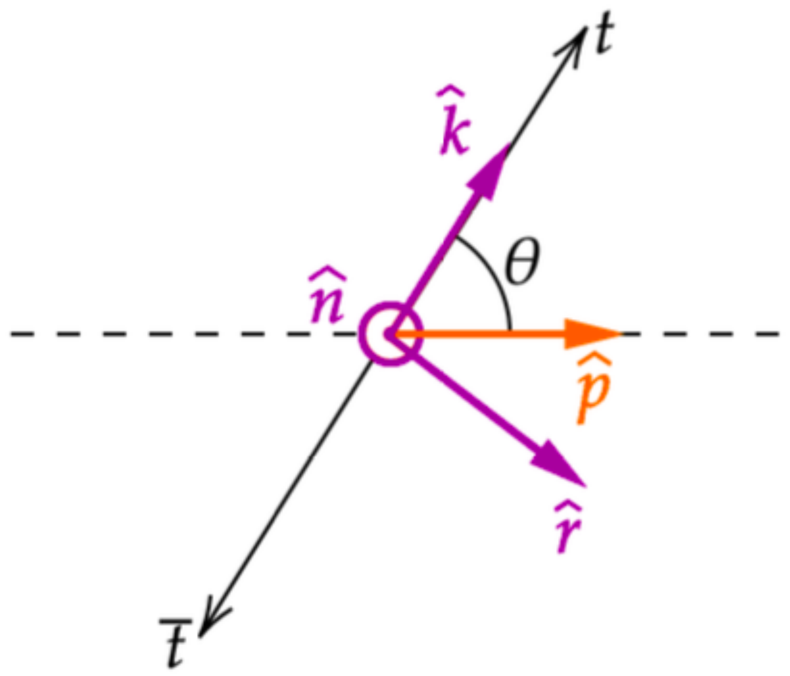
$$\hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p} \times \hat{k}}{\sin\theta}$$

Helicity basis

$$C_{ij}^{[i_1 i_2]}(m_{t\bar{t}}, \theta) = \frac{9/\alpha_a\alpha_b \int \cos\theta_{ai} \cos\theta_{bj} \, |\mathcal{M}_{i_1 i_2 \to t\bar{t} \to a b X}|^2 \, d\pi}{\int |\mathcal{M}_{i_1 i_2 \to t\bar{t} \to a b X}|^2 \, d\pi}$$

Spin correlation coefficients are averages of angles

From spin correlations to entanglement



$$\hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p} \times \hat{k}}{\sin\theta}$$

$$\begin{aligned}
D^{(1)} &= 1/3(+C_{kk} + C_{rr} + C_{nn}), \\
D^{(k)} &= 1/3(+C_{kk} - C_{rr} - C_{nn}), \\
D^{(r)} &= 1/3(-C_{kk} + C_{rr} - C_{nn}), \\
D^{(n)} &= 1/3(-C_{kk} - C_{rr} + C_{nn}).
\end{aligned}$$

$$D_{\min} \equiv \min\{D^{(1)}, D^{(k)}, D^{(r)}, D^{(n)}\}$$

Entanglement markers, from the Peres-Horodecki criterion

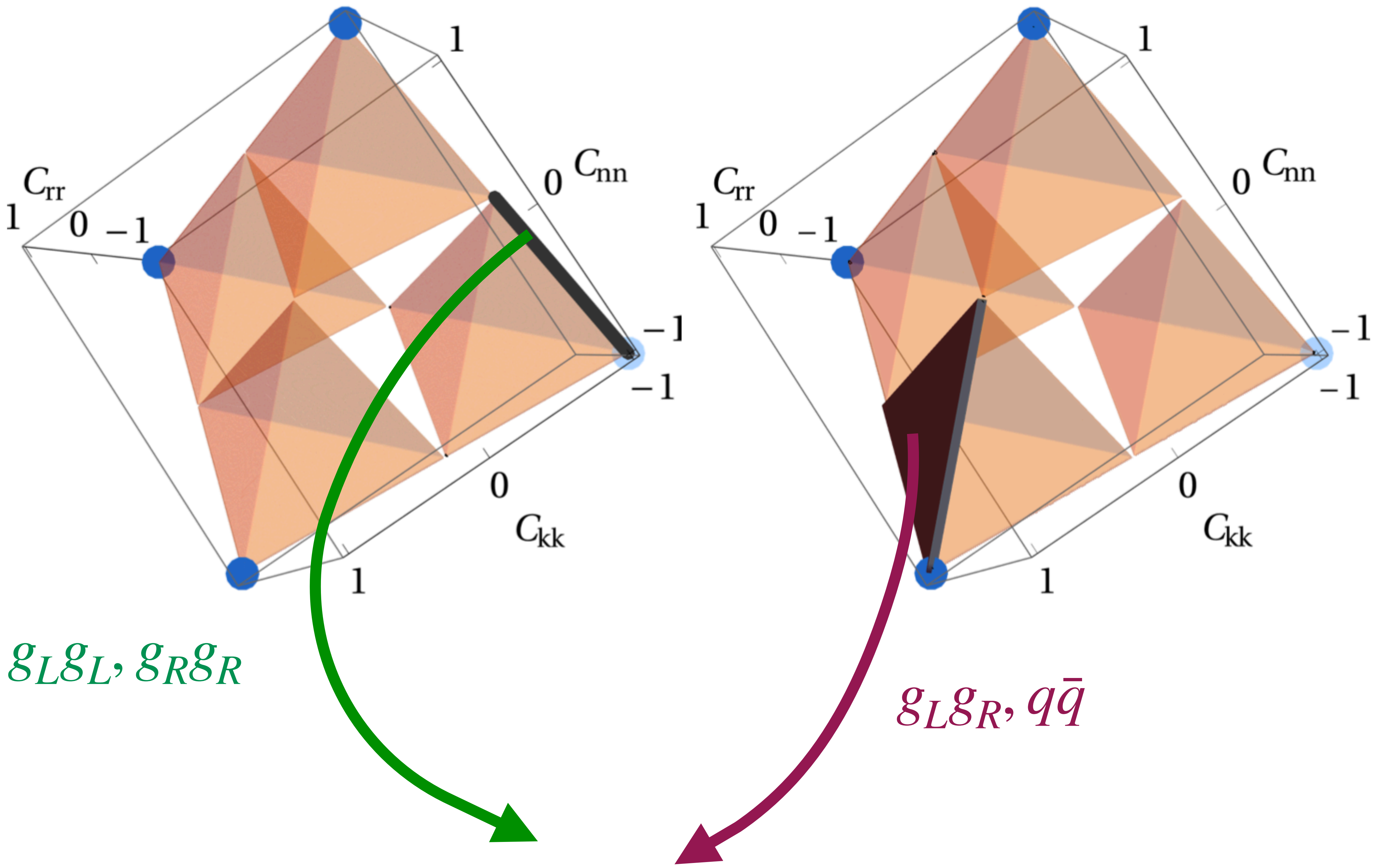
$$D_{\min} < -1/3$$

for a proof see arXiv:2003.02280

Necessary and sufficient condition for entanglement

$$C = \frac{1}{2}\max\left(0, -1 - 3D_{\min}\right) > 0$$

When are tops entangled?



reachable entangled states

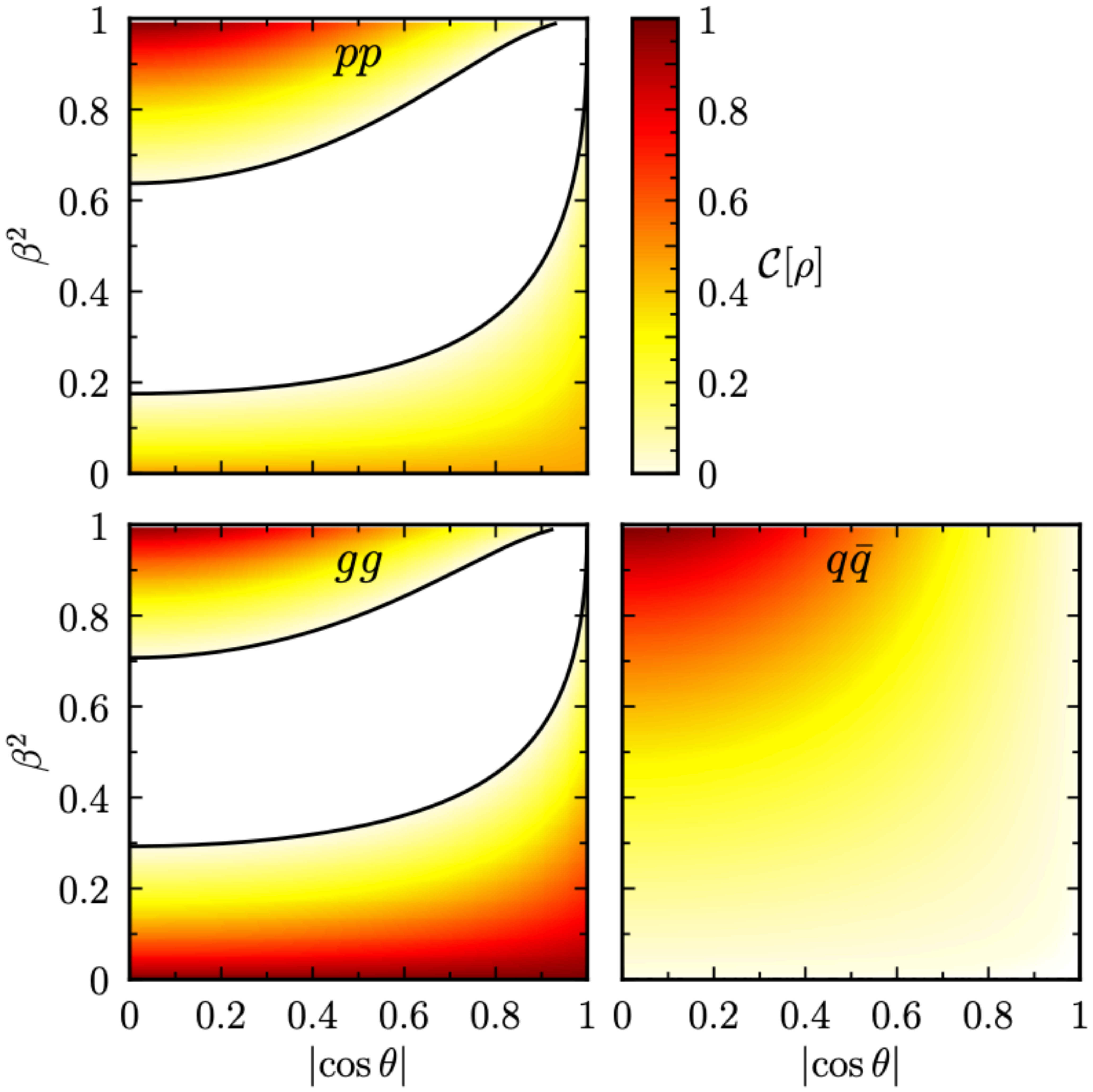
Consider top pair production in pp collisions
Which spin states can be reached?

Threshold:
- entangled singlet state
- from same helicity gluons

Boosted:
- entangled triplet state
- for qqbar pairs and opposite helicity gluons

C. Severi, F.Maltoni, S. Tentori, EV: 2404.08049

Entanglement in the SM



Concurrence: $C = \frac{1}{2}\max\left(0, -1 - 3D_{\text{min}}\right)$

White regions: no entanglement (C<0)

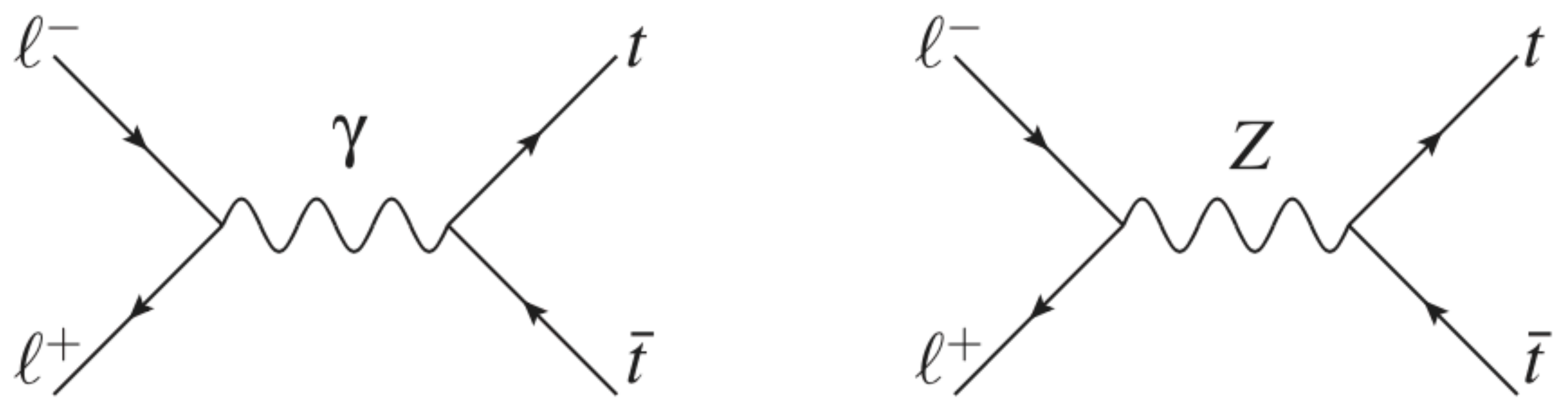
Maximal entanglement regions

At threshold: $\beta^2 = 0, \forall\theta$

High-Energy: $\beta^2 \to 1, \cos\theta = 0$

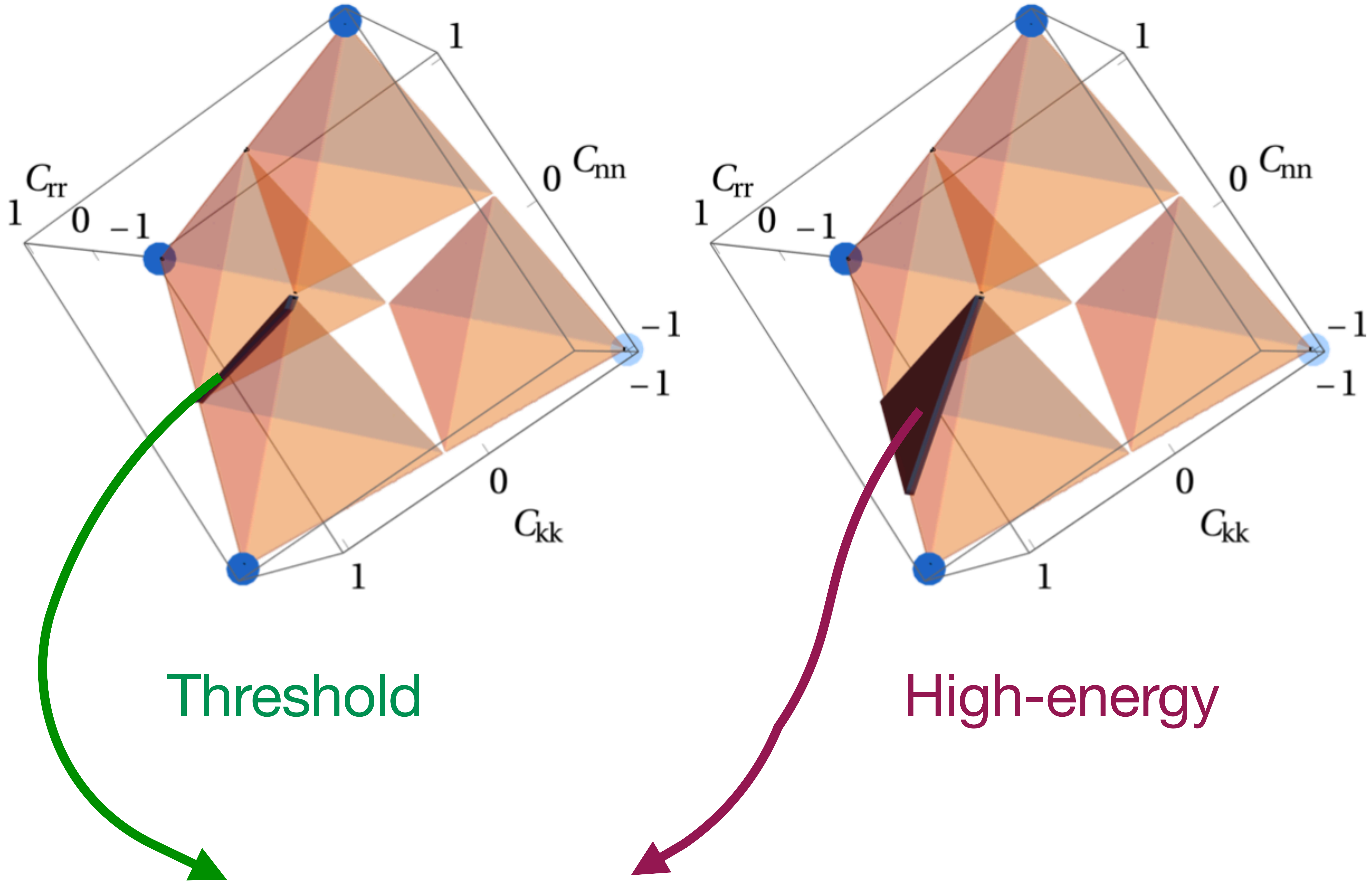
C. Severi, C. Boschi, F. Maltoni, M. Sioli : 2110.10112

Tops in lepton colliders



$$1/3\,\mathrm{Tr}\,[\mathcal{C}] = D^{(1)} = +\frac{1}{3},$$

Spin-1 exchange
Spin triplet state



reachable entangled states

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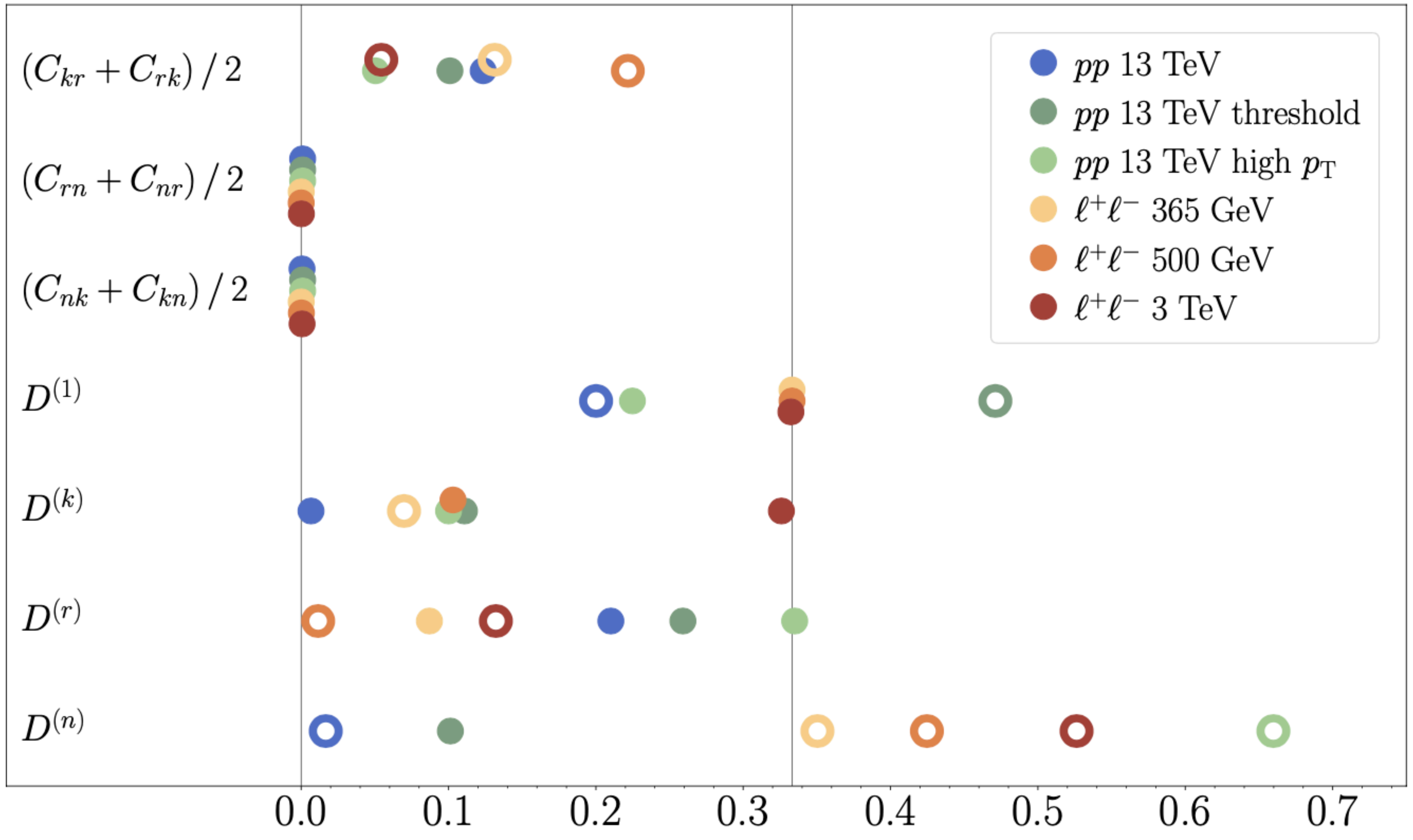
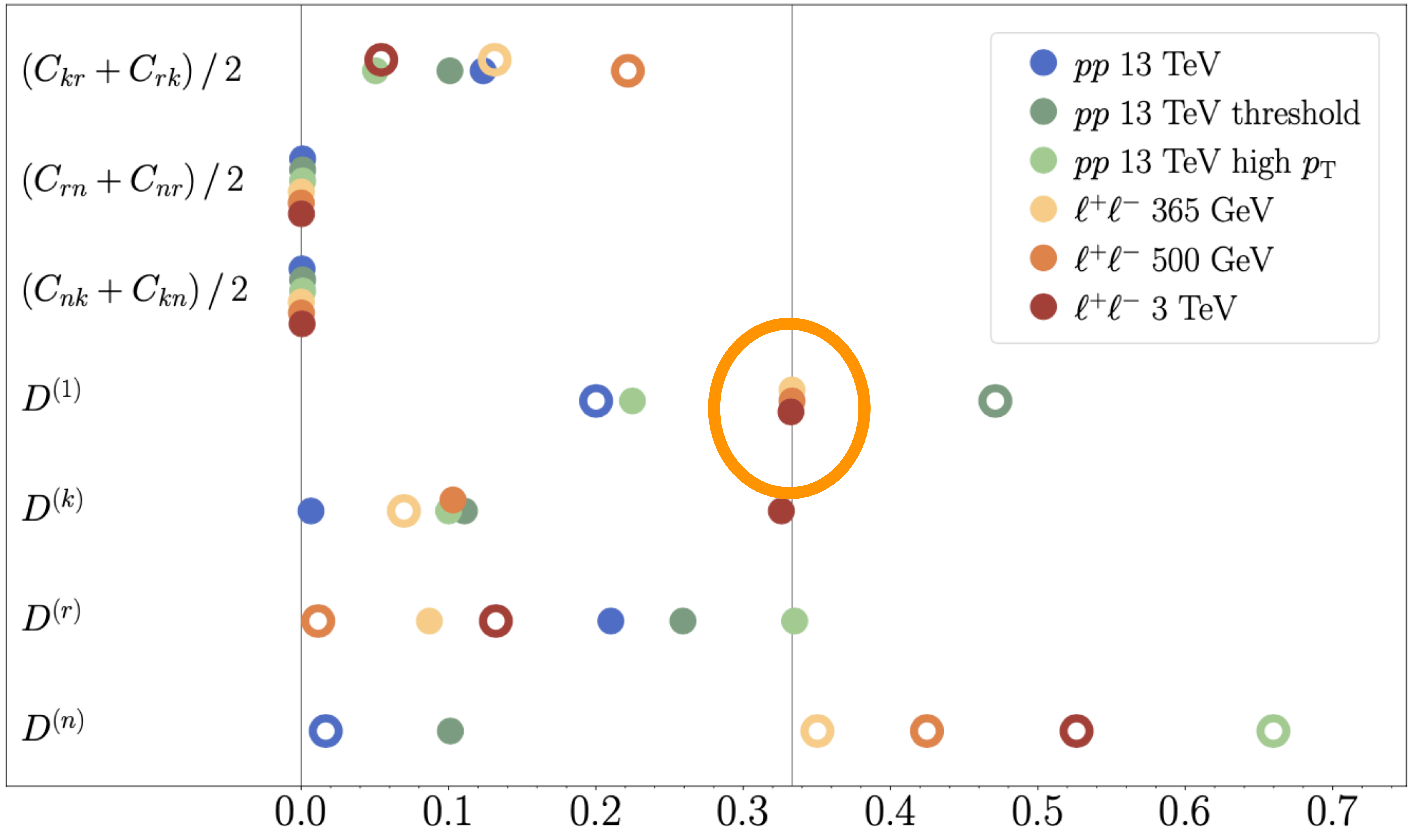
Lepton vs pp collisions



C. Severi, F. Maltoni, S. Tentori, EV: 2404.08049

- Spin Triplet state $D^{(1)} = +1/3$
- Entanglement through $D^{(n)}$ for lepton colliders
- Entanglement through $D^{(1)}$ for LHC at threshold
- Entanglement through $D^{(n)}$ for LHC at high transverse momentum

Lepton vs pp collisions



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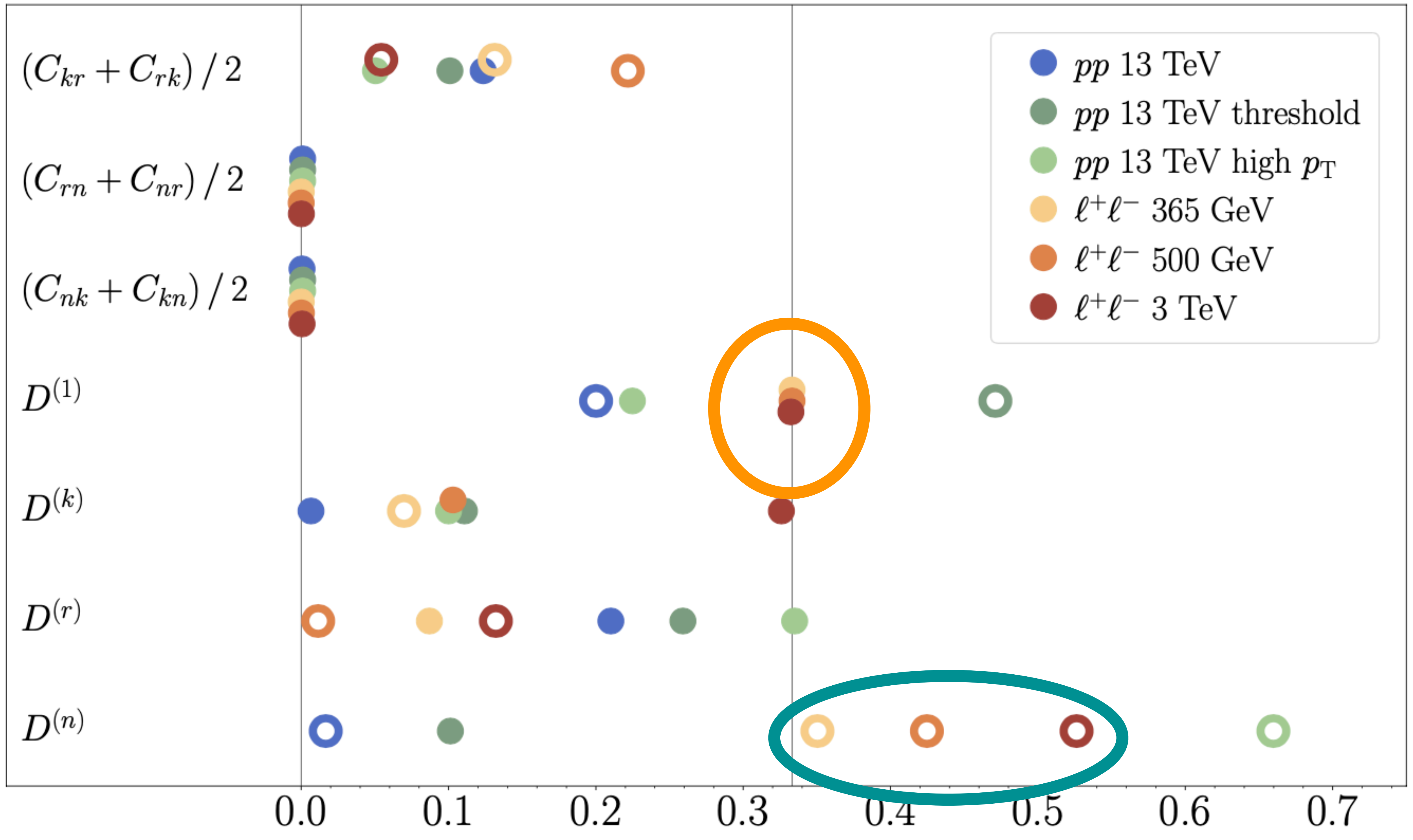

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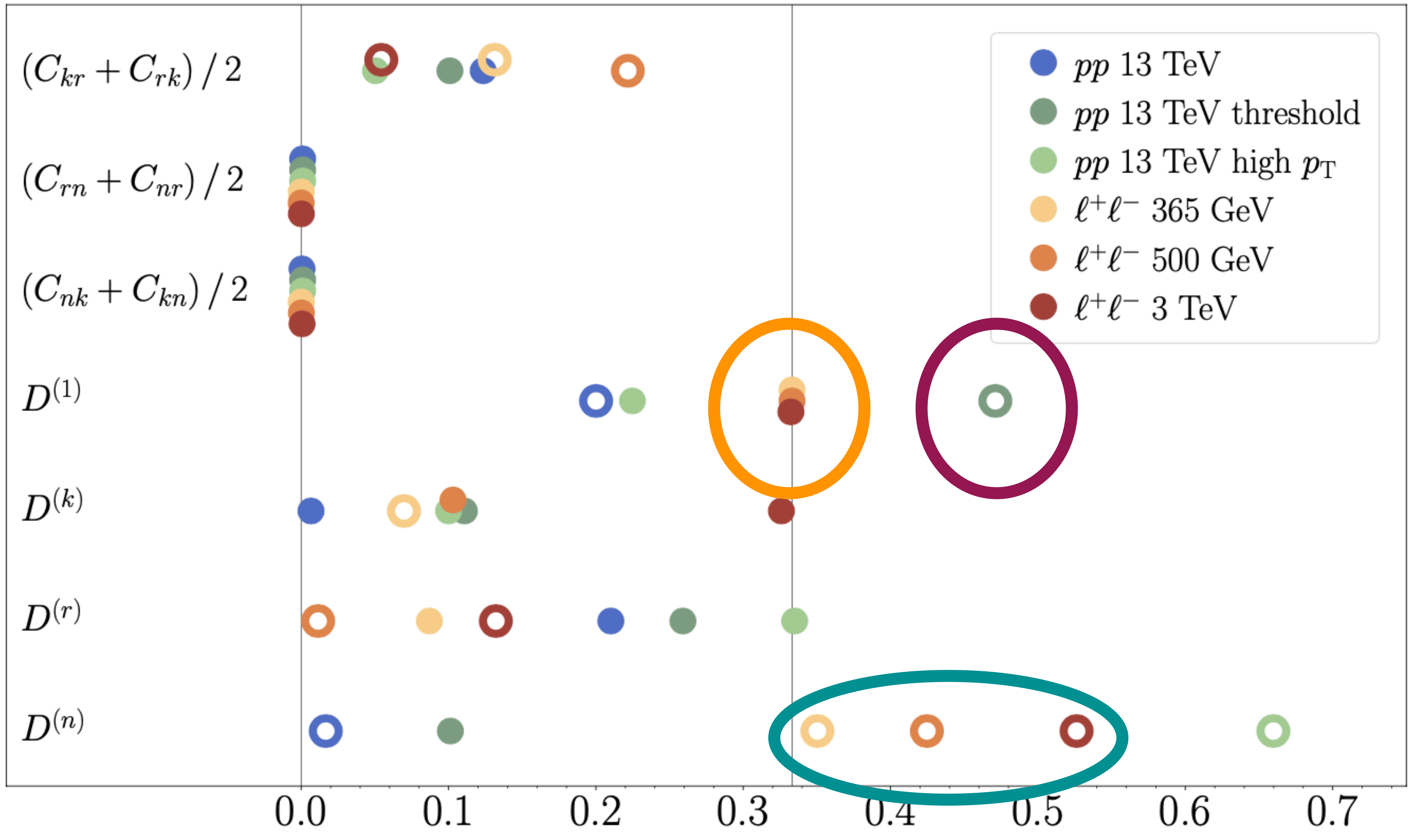

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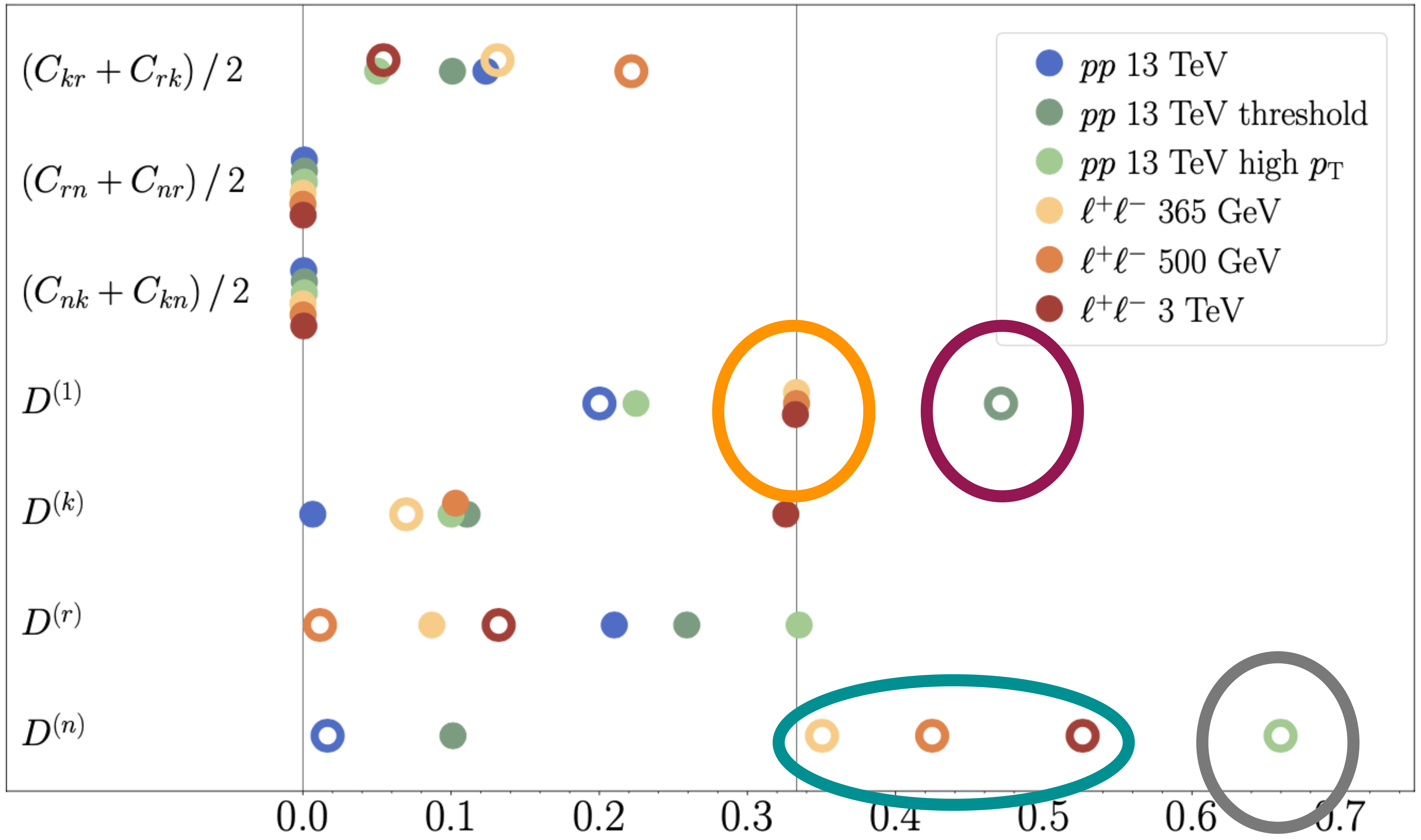

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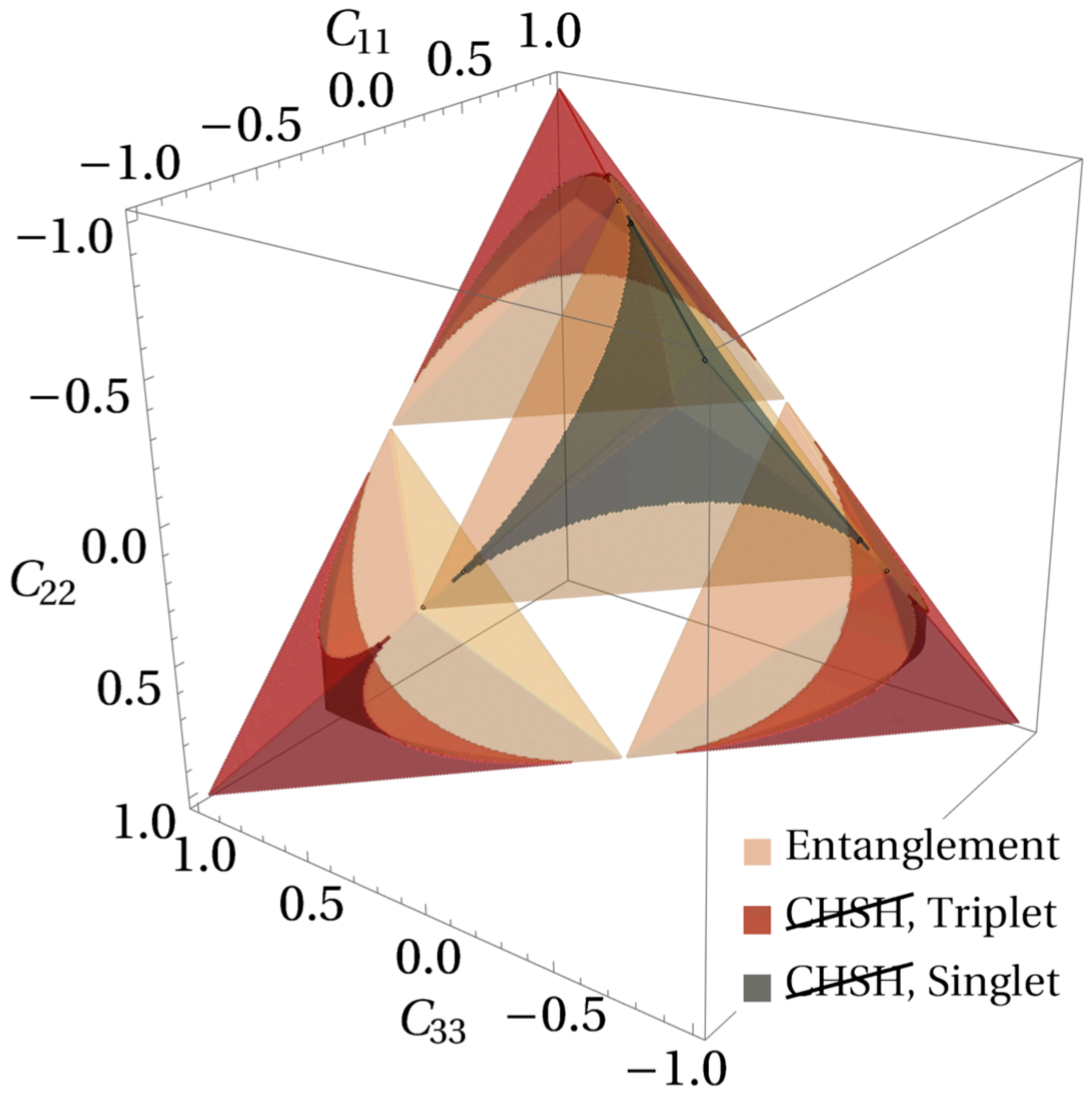
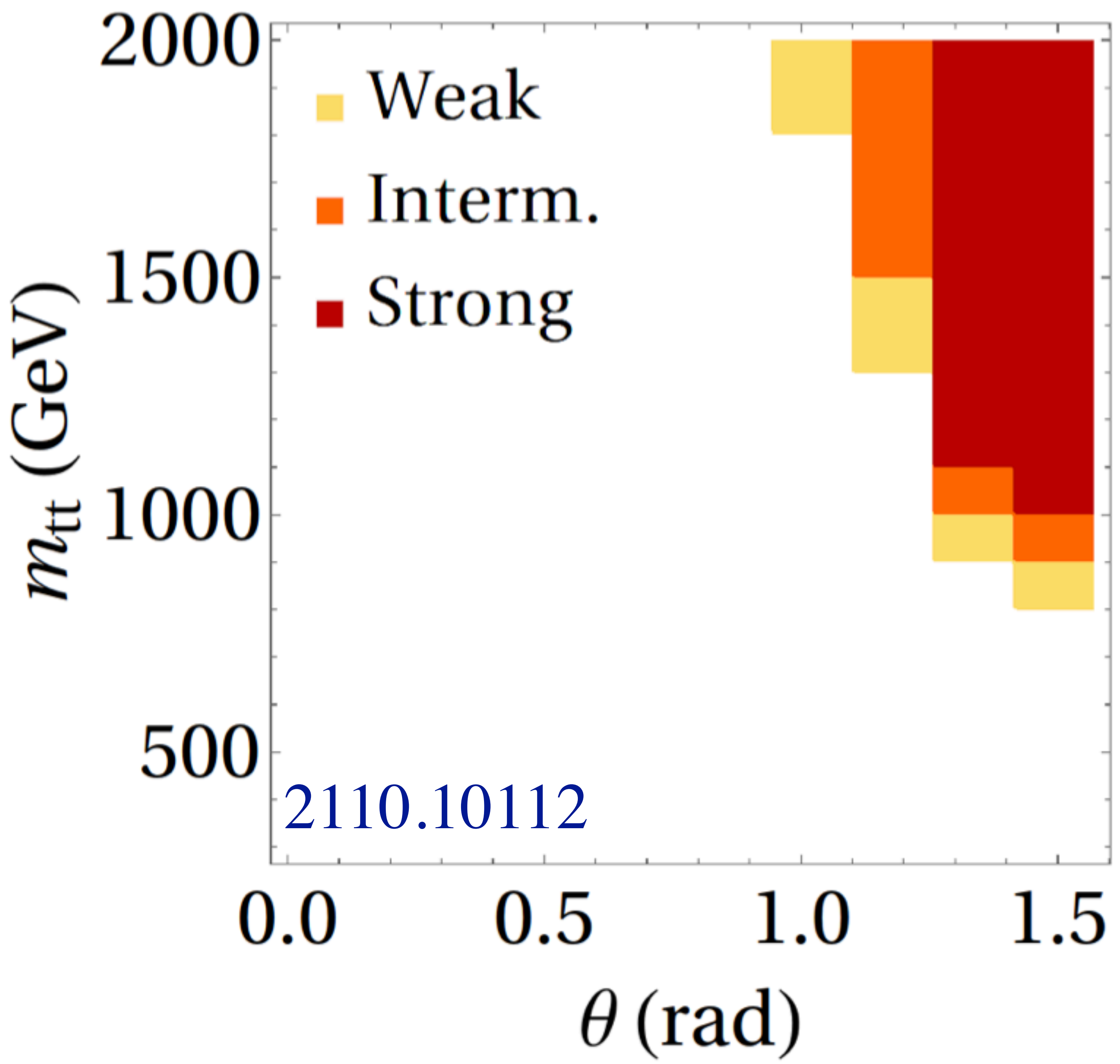
How about Bell inequalities?





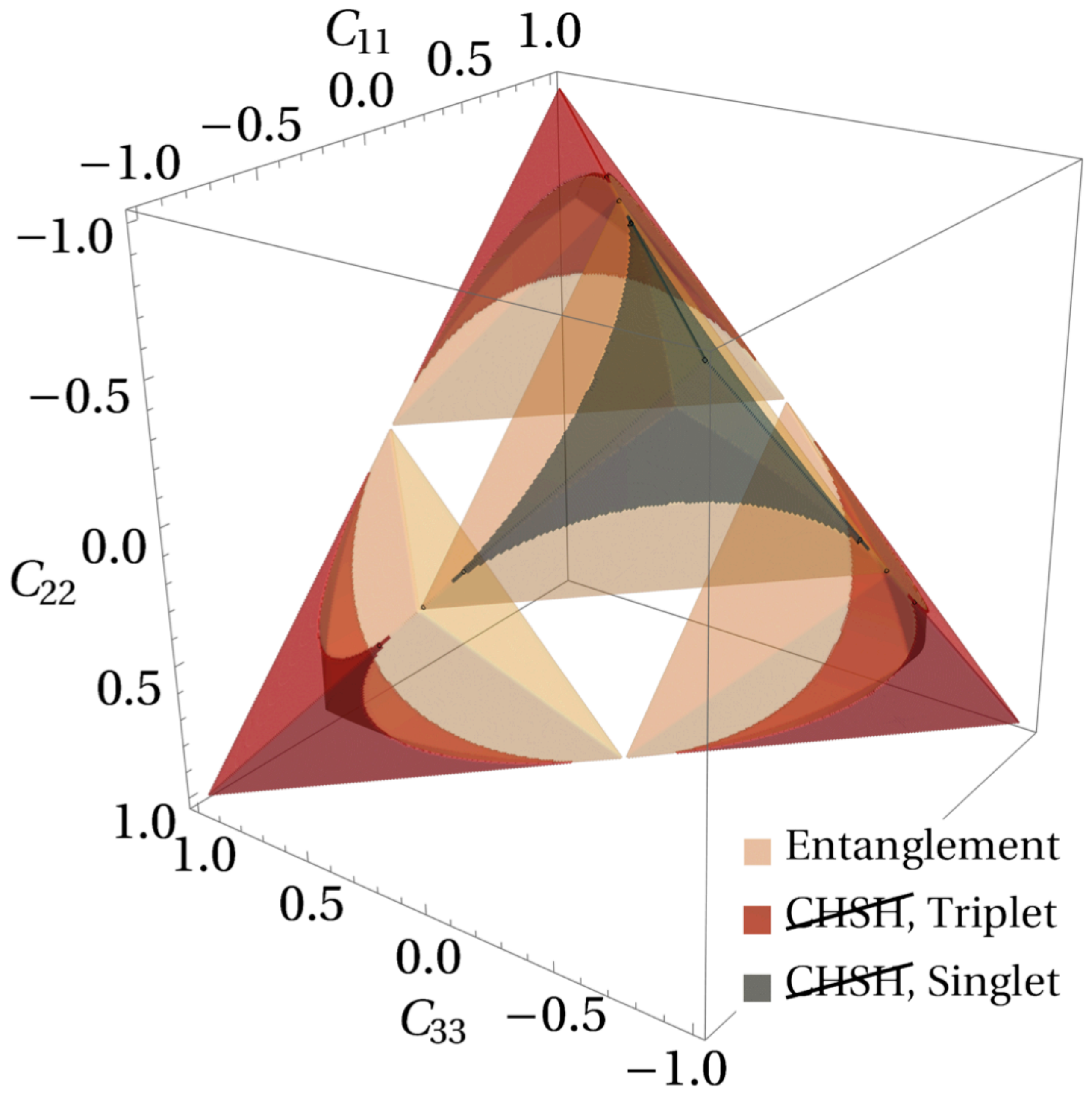
CHSH $\left|\sum_{ij} C_{ij}\left(a_i b_j - a_i b'_j + a'_i b_j + a'_i b'_j\right)\right| \leq 2$

$$\max_{a\,a'\,b\,b'} \left|\sum_{ij} C_{ij}\left(a_i b_j - a_i b'_j + a'_i b_j + a'_i b'_j\right)\right| = 2\sqrt{\lambda + \lambda'},$$

two largest eigenvalues of $C^T C$

Much harder to see Bell inequalities violation at the LHC

How about Bell inequalities?



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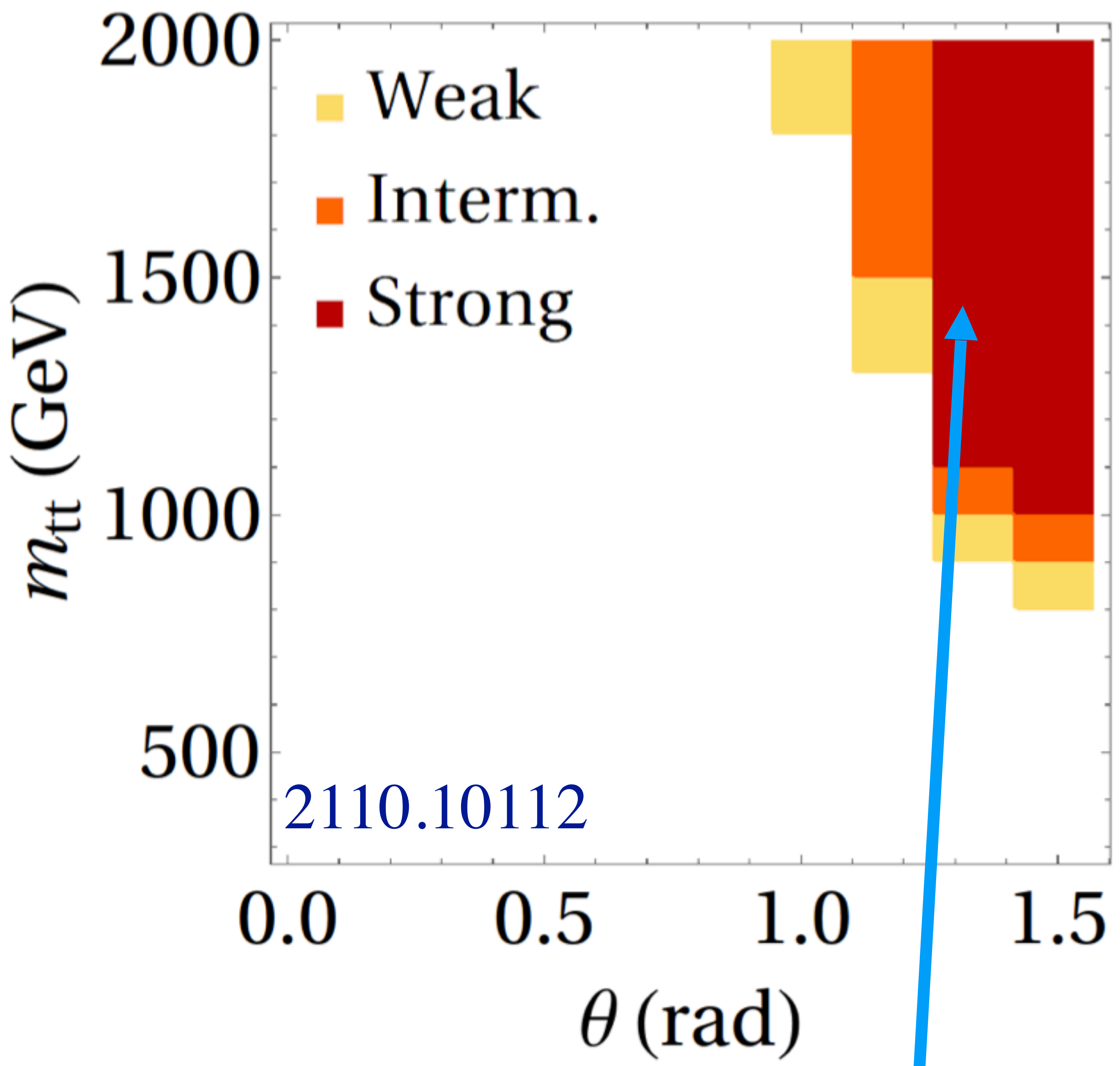
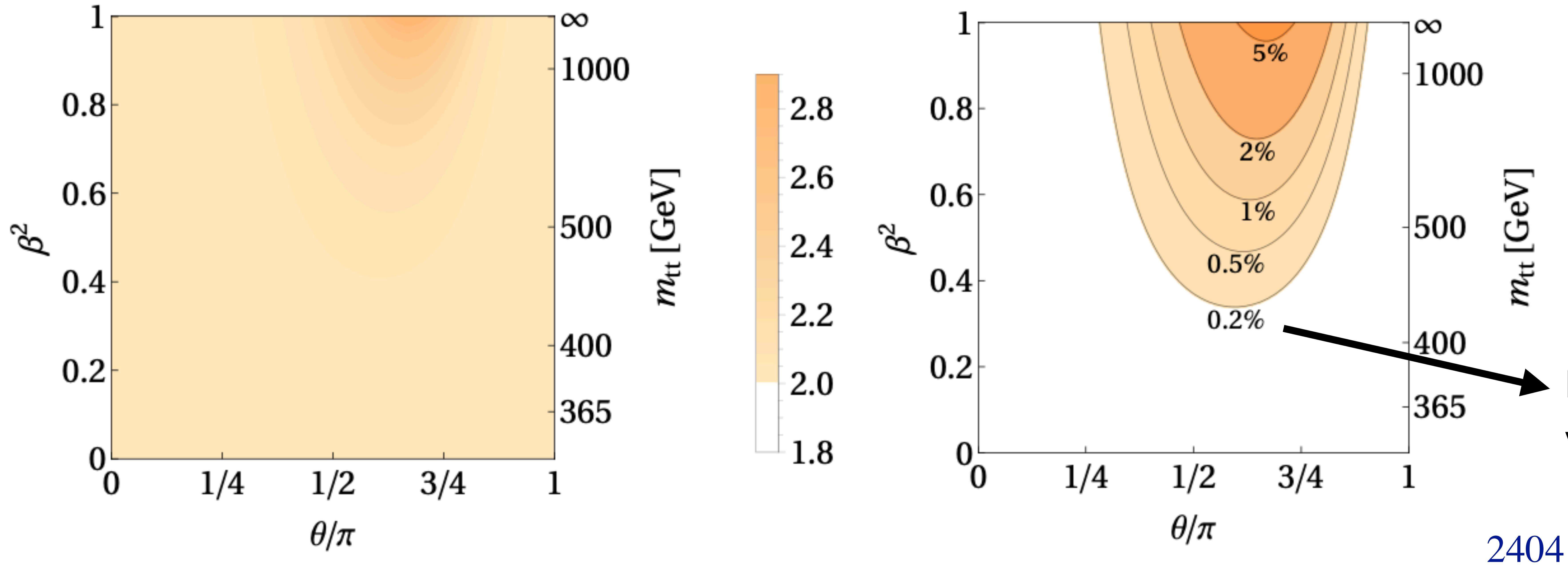


$\sqrt{2}\,\left|-C_{rr} + C_{nn}\right| > 2$

Much harder to see Bell inequalities violation at the LHC

Bell inequalities at lepton colliders



Experimental accuracy needed to establish Bell violation

2404.08049

$$\langle a\,b + a\,b' + a'\,b - a'\,b' \rangle \equiv \langle\, \mathcal{B}(a, a', b, b')\,\rangle > 2, \quad \Longrightarrow \quad \text{Bell violation.}$$

Bell violation everywhere, but B~2

Better prospects of Bell violation at higher energy lepton colliders (extremely hard at 365 GeV)

Entanglement and parity

EW interactions do not conserve parity

Parity would be conserved for $\boxed{\text{purely axial vector } Q_t = 0 \text{ and } g_{Vt} = 0}$ or $\boxed{\text{purely vector } g_{At} = 0}$

In the purely vectorial case EW interactions are like QCD

Maximally-entangled pure triplet case

$$\mathcal{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \theta = \pi/2, \;\; \beta \to 1$$

In the purely axial vector case:

$$\mathcal{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A mixture spoils purity

Entanglement and parity

$g_{At} = g_{Vt}$ or $g_{At} = -g_{Vt}$

$\implies$ maximal violation of P.

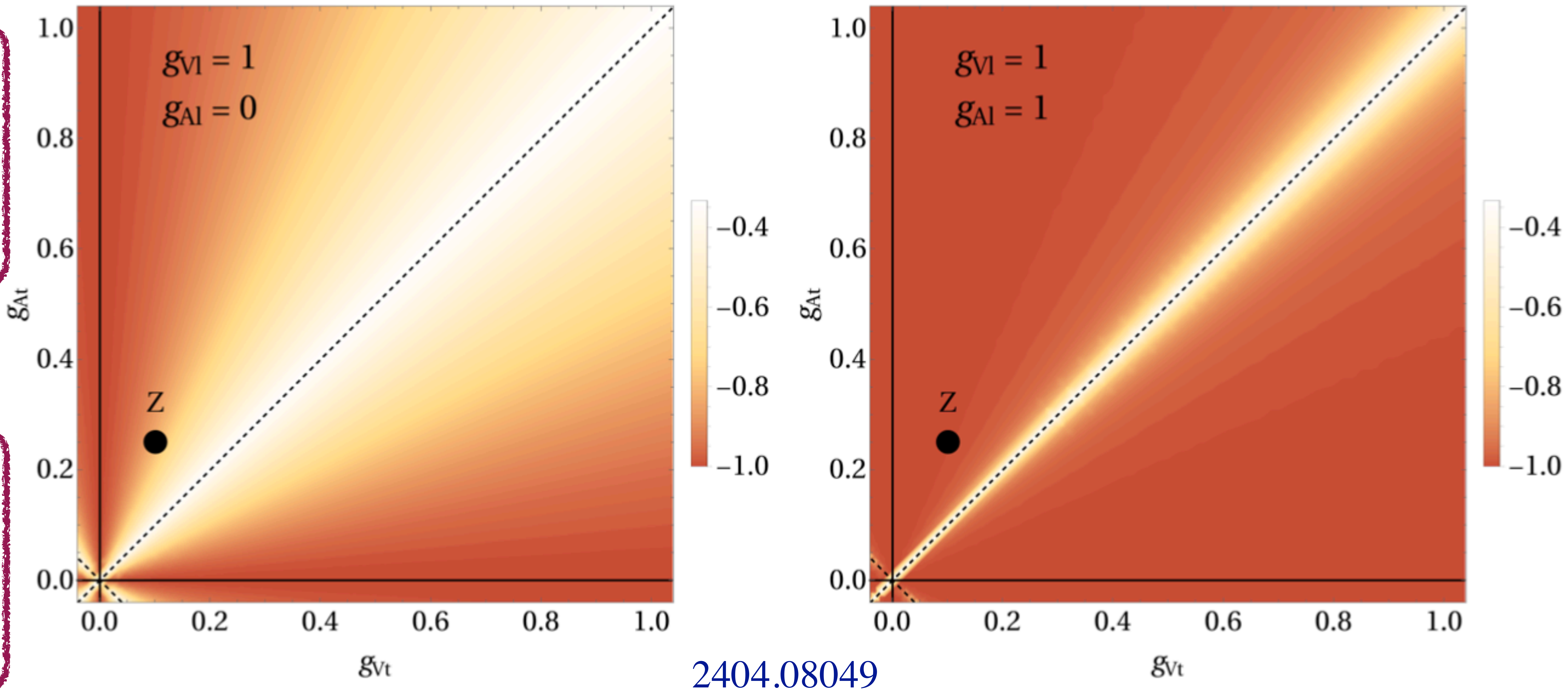
Reduced entanglement

$g_{At} = 0$ or $g_{Vt} = 0$

$\implies$ conservation of P,

Maximal entanglement

D_{min}



2404.08049

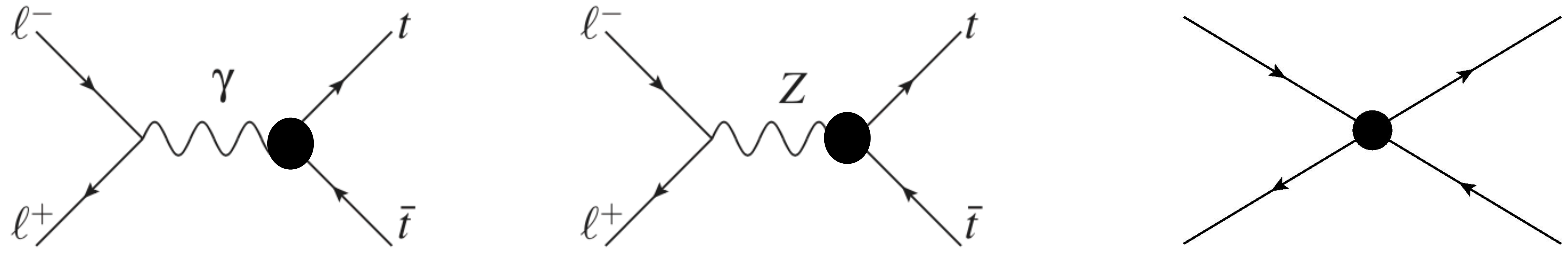
$$\mathcal{L} \supset \frac{e}{s_W c_W}\left(Z_\mu \bar{t}\gamma^\mu (g_{Vt} + \gamma^5 g_{At})\, t + Z_\mu \bar{\ell}\gamma^\mu (g_{V\ell} + \gamma^5 g_{A\ell})\, \ell \right)$$

Using QI for new physics

Can they tell us anything interesting/new?

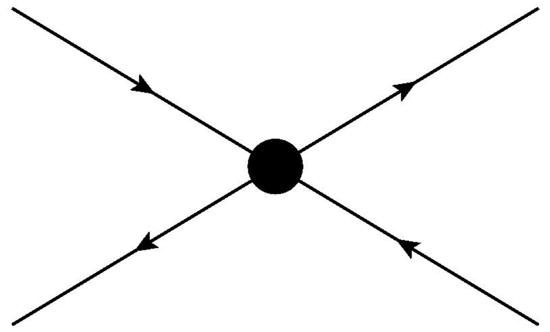
- SMEFT → New Interactions of SM particles

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \mathcal{O}(\Lambda^{-4})$$



SMEFT in lepton colliders

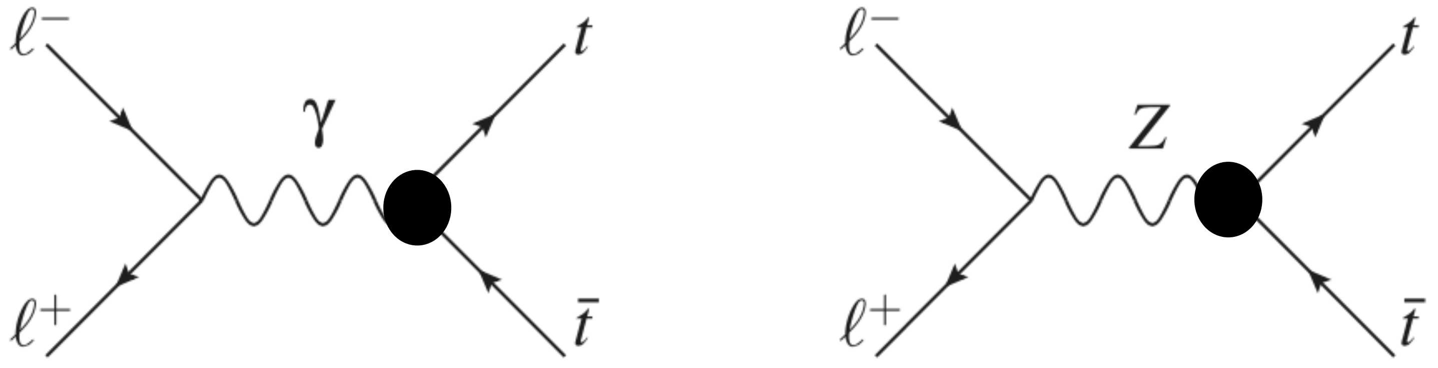
$$\mathcal{O}_{Q\ell}^{(1)} = (\bar{Q}_L\gamma^\mu Q_L)(\bar{\ell}_L\gamma_\mu\ell_L),$$
$$\mathcal{O}_{Q\ell}^{(3)} = (\bar{Q}_L\gamma^\mu\sigma_I Q_L)(\bar{\ell}_L\gamma_\mu\sigma^I\ell_L),$$
$$\mathcal{O}_{Qe} = (\bar{Q}_L\gamma^\mu Q_L)(\bar{\ell}_R\gamma_\mu\ell_R),$$
$$\mathcal{O}_{t\ell} = (\bar{t}_R\gamma^\mu t_R)(\bar{\ell}_L\gamma_\mu\ell_L),$$
$$\mathcal{O}_{te} = (\bar{t}_R\gamma^\mu t_R)(\bar{\ell}_R\gamma_\mu\ell_R).$$



4-fermion operators

$$\mathcal{O}_{\phi Q}^{(1)} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{Q}_L\gamma^\mu Q_L),$$
$$\mathcal{O}_{\phi Q}^{(3)} = i(\phi^\dagger \overleftrightarrow{D}_{\mu I} \phi)(\bar{Q}_L\gamma^\mu\sigma^I Q_L),$$
$$\mathcal{O}_{\phi t} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{t}_R\gamma^\mu t_R),$$

current operators



$$\mathcal{O}_{tW} = (\bar{Q}_L\gamma^{\mu\nu}\sigma_I t_R)\,\tilde{\phi}\,W^I_{\mu\nu},$$
$$\mathcal{O}_{tB} = (\bar{Q}_L\gamma^{\mu\nu} t_R)\,\tilde{\phi}\,B_{\mu\nu}.$$

dipole operators

Degrees of freedom

$$c_{Q\ell}^{(3)} + c_{Q\ell}^{(1)},$$
$$c_{\mathrm{VV}} = \frac{1}{4}\big(c_{Q\ell}^{(1)} - c_{Q\ell}^{(3)} + c_{te} + c_{t\ell} + c_{Qe}\big),$$
$$c_{\mathrm{AV}} = \frac{1}{4}\big(-c_{Q\ell}^{(1)} + c_{Q\ell}^{(3)} + c_{te} + c_{t\ell} - c_{Qe}\big),$$
$$c_{\mathrm{VA}} = \frac{1}{4}\big(-c_{Q\ell}^{(1)} + c_{Q\ell}^{(3)} + c_{te} - c_{t\ell} + c_{Qe}\big),$$
$$c_{\mathrm{AA}} = \frac{1}{4}\big(c_{Q\ell}^{(1)} - c_{Q\ell}^{(3)} + c_{te} - c_{t\ell} - c_{Qe}\big).$$

$$c_{\phi Q}^{(3)} + c_{\phi Q}^{(1)},$$
$$c_{\phi\mathrm{V}} = \frac{1}{2}\big(c_{\phi t} + c_{\phi Q}^{(1)} - c_{\phi Q}^{(3)}\big),$$
$$c_{\phi\mathrm{A}} = \frac{1}{2}\big(c_{\phi t} - c_{\phi Q}^{(1)} + c_{\phi Q}^{(3)}\big).$$

$$c_{tZ} = c_{\mathrm{W}}\, c_{tW} - s_{\mathrm{W}}\, c_{tB},$$
$$c_{t\gamma} = s_{\mathrm{W}} c_{tW} + c_{\mathrm{W}} c_{tB},$$

Structure of spin correlations within SMEFT

Degeneracy between possible structures arising from SM and EFT

$$\left.\begin{aligned} A^{[0]} &= F^{[0]}\left(\beta^2 c_\theta^2 - \beta^2 + 2\right) \\ A^{[1]} &= 2\,F^{[1]}\,c_\theta \\ A^{[2]} &= F^{[2]}\,\left(1 + c_\theta^2\right) \end{aligned}\right\}\text{SM}$$

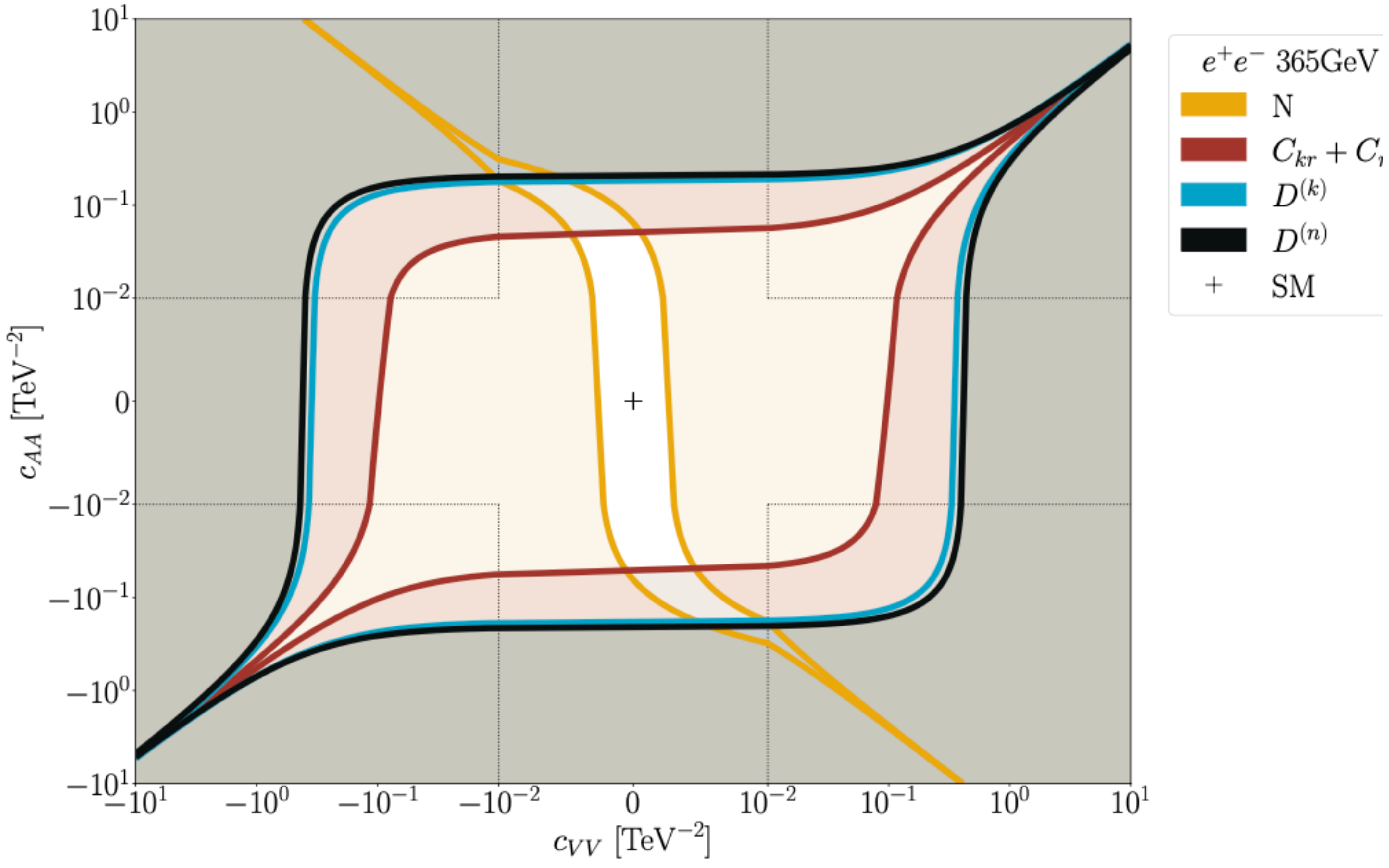
$$\left.\begin{aligned} A^{[6,0,\mathrm{D}]} &= F^{[6,0,\mathrm{D}]} \\ A^{[6,1,\mathrm{D}]} &= F^{[6,1,\mathrm{D}]}\,c_\theta \\ A^{[8,\mathrm{DD}]} &= F^{[8,\mathrm{DD}]}\left(-\beta^2 c_\theta^2 - \beta^2 + 2\right) \end{aligned}\right\}\text{BSM}$$

		$\mathcal{M}_1$		
		Q_t, g_{Vt}, c_{VV}, c_{VA}, $c_{\phi\mathrm{V}}$	g_{At}, c_{AV}, c_{AA}, $c_{\phi\mathrm{A}}$	$c_{\mathrm{t}Z}$, $c_{\mathrm{t}\gamma}$
$\mathcal{M}_2$	Q_t, g_{Vt} c_{VV}, c_{VA}, $c_{\phi\mathrm{V}}$	$A^{[0]}$	$A^{[1]}$	$A^{[6,0,D]}$
	g_{At} c_{AV}, c_{AA}, $c_{\phi\mathrm{A}}$	$A^{[1]}$	$A^{[2]}$	$A^{[6,1,D]}$
	$c_{\mathrm{t}Z}$, $c_{\mathrm{t}\gamma}$	$A^{[6,0,D]}$	$A^{[6,1,D]}$	$A^{[8,\mathrm{DD}]}$

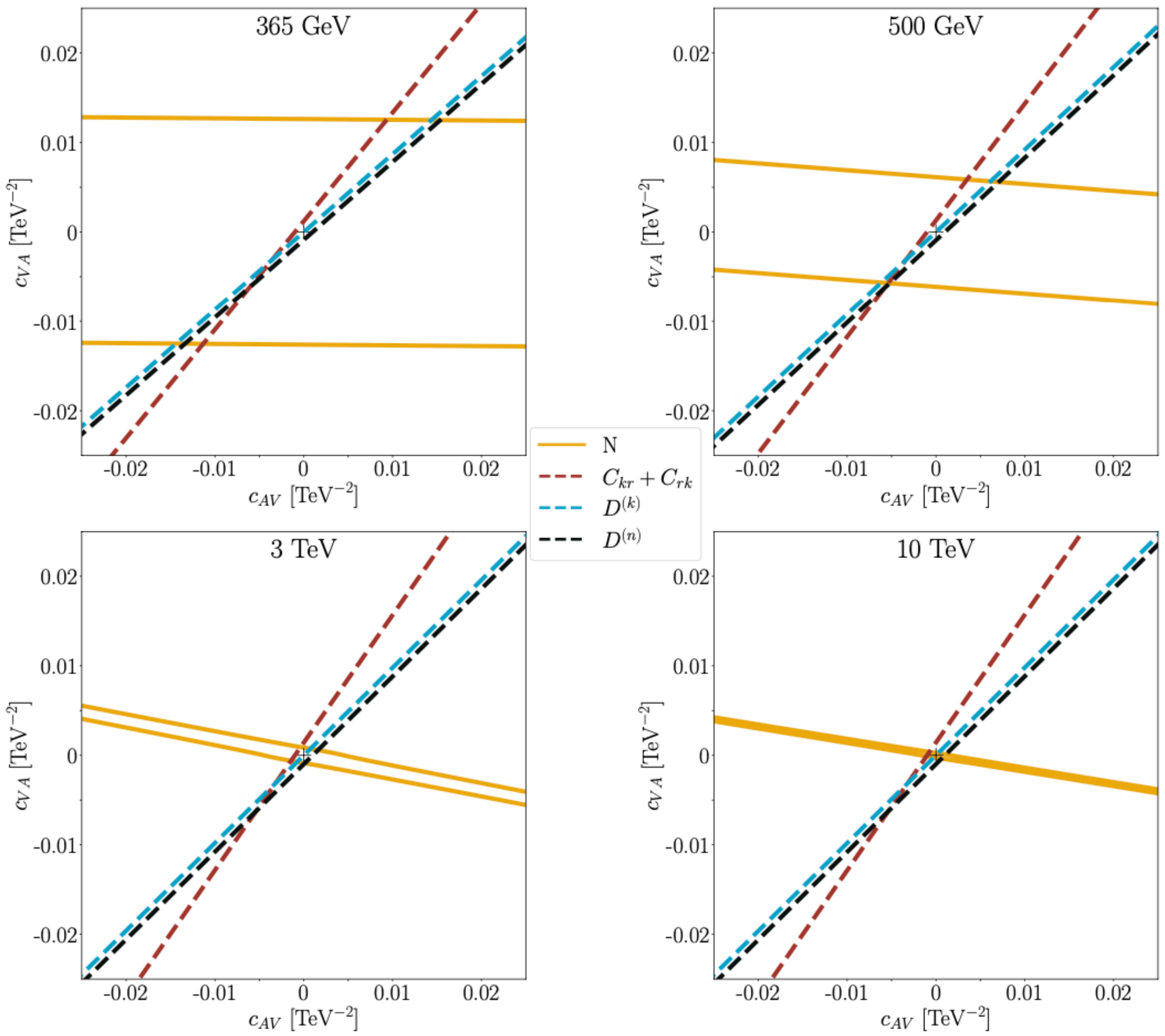
New structures related to dipole operators, the rest gives linear combinations of pre-existing structures

C. Severi, F. Maltoni, S. Tentori, EV: 2404.08049

Breaking degeneracies with Quantum Obs



C. Severi, F. Maltoni, S. Tentori, EV: 2404.08049[hep-ph]

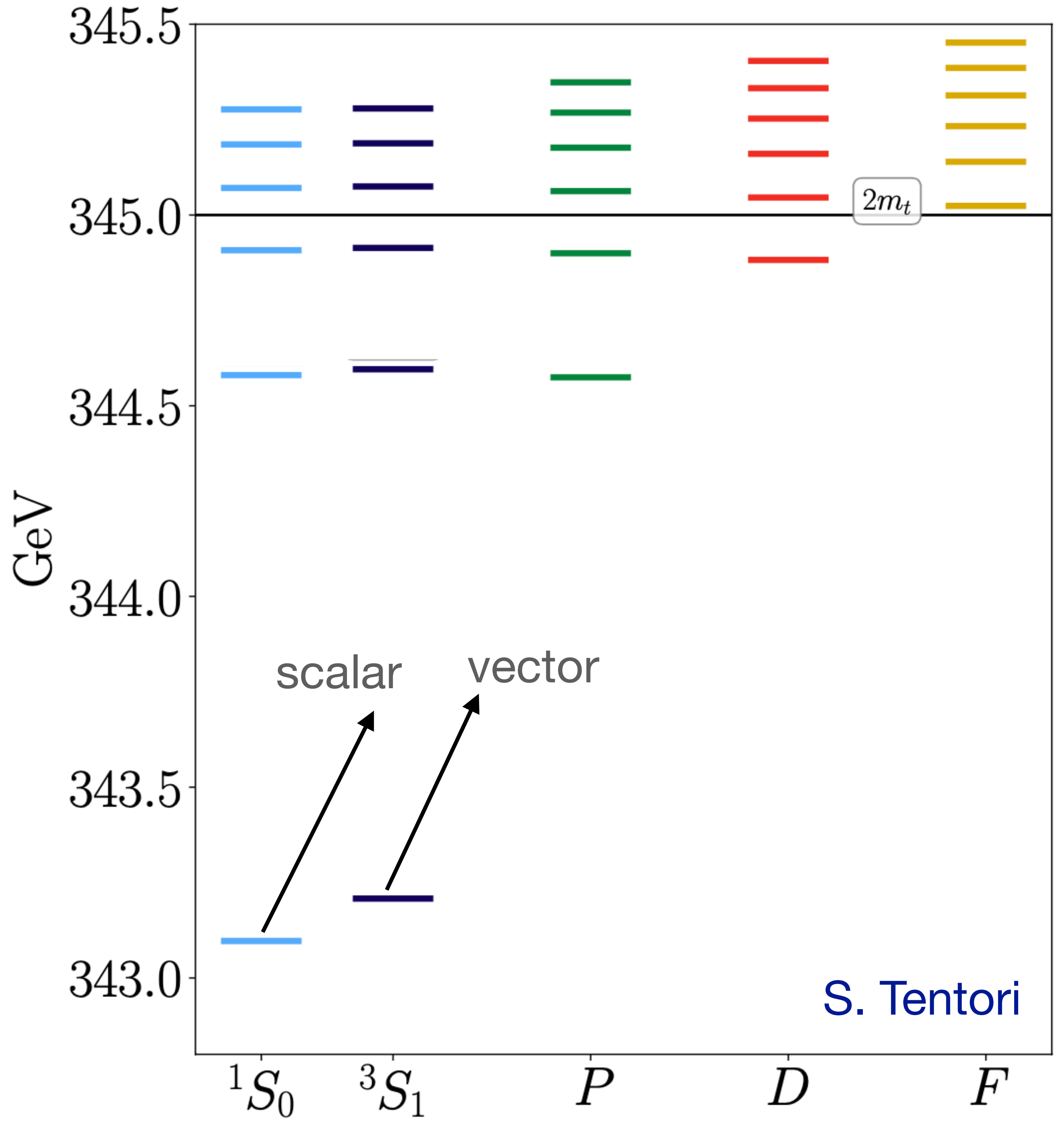


Spin correlation observables probe different linear combinations of Wilson coefficients

Breaking degeneracies

Old New Physics: Threshold effects

- Quasi-Bound State of top and antitop
- Energy states obtained by solving Schrödinger equation with QCD potential
- Described by NRQCD
- Ground state n=1 S-wave
- Spin-singlet vs spin-triplet depending on production mode
    - spin singlet for pp and spin triplet for e^+e^-
- See morning talks



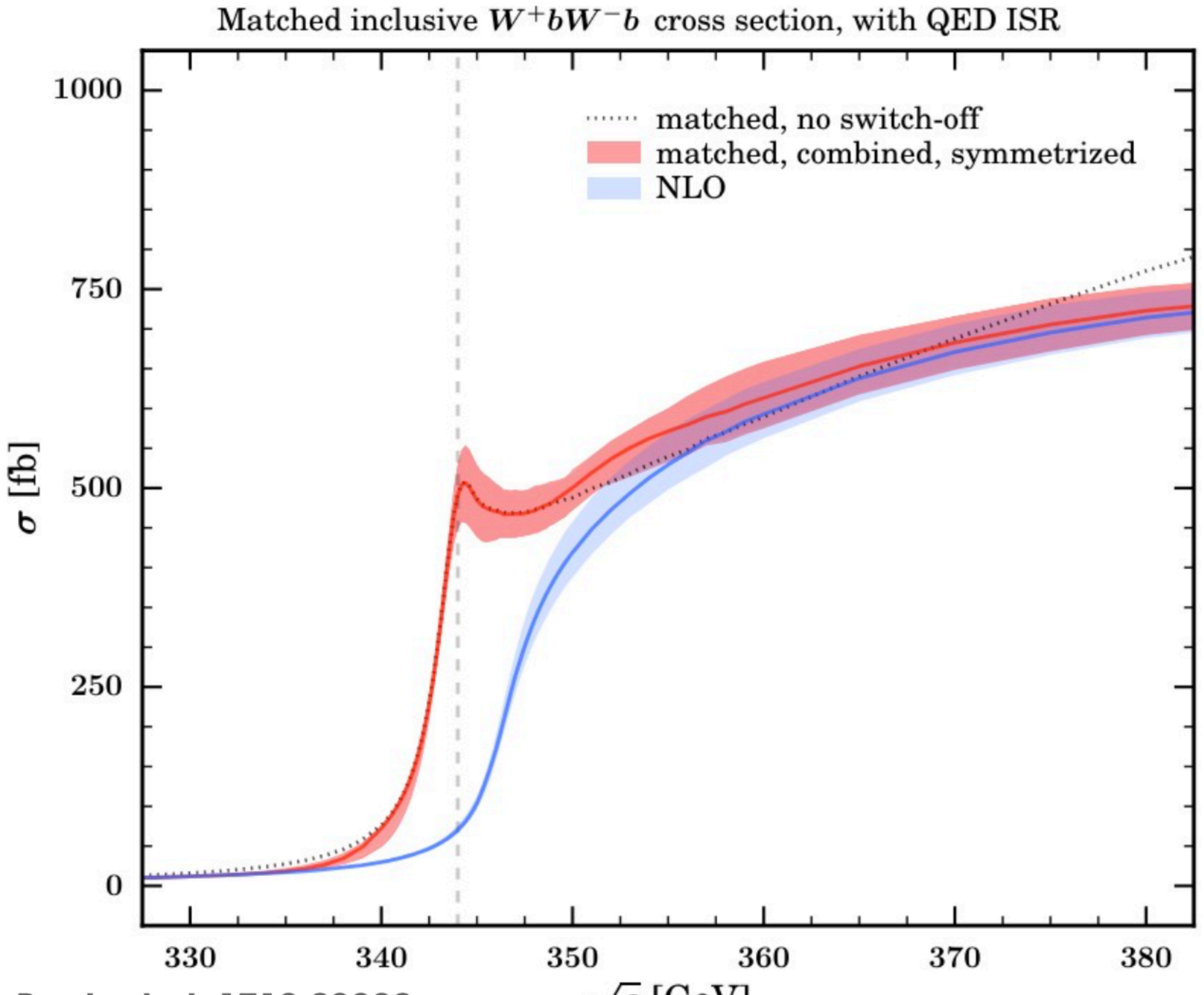
$$\left[(E + i\Gamma_t) - \left(\frac{\nabla^2}{m_t} + V(\boldsymbol{r})\right)\right] G(\boldsymbol{r}, E + i\Gamma_t) = \delta^{(3)}(\boldsymbol{r})$$

$$V_{\rm QCD}(r,\mu_B) = C^{[\rm col]}\,\frac{\alpha_s(\mu_B)}{r}\left[1 + \frac{\alpha_s}{4\pi}\left(2\,\beta_0 \log(e^{\gamma}\mu_B\, r) + \frac{31}{9}C_A - \frac{10}{9}n_f\right) + \mathcal{O}(\alpha_s^2)\right]$$

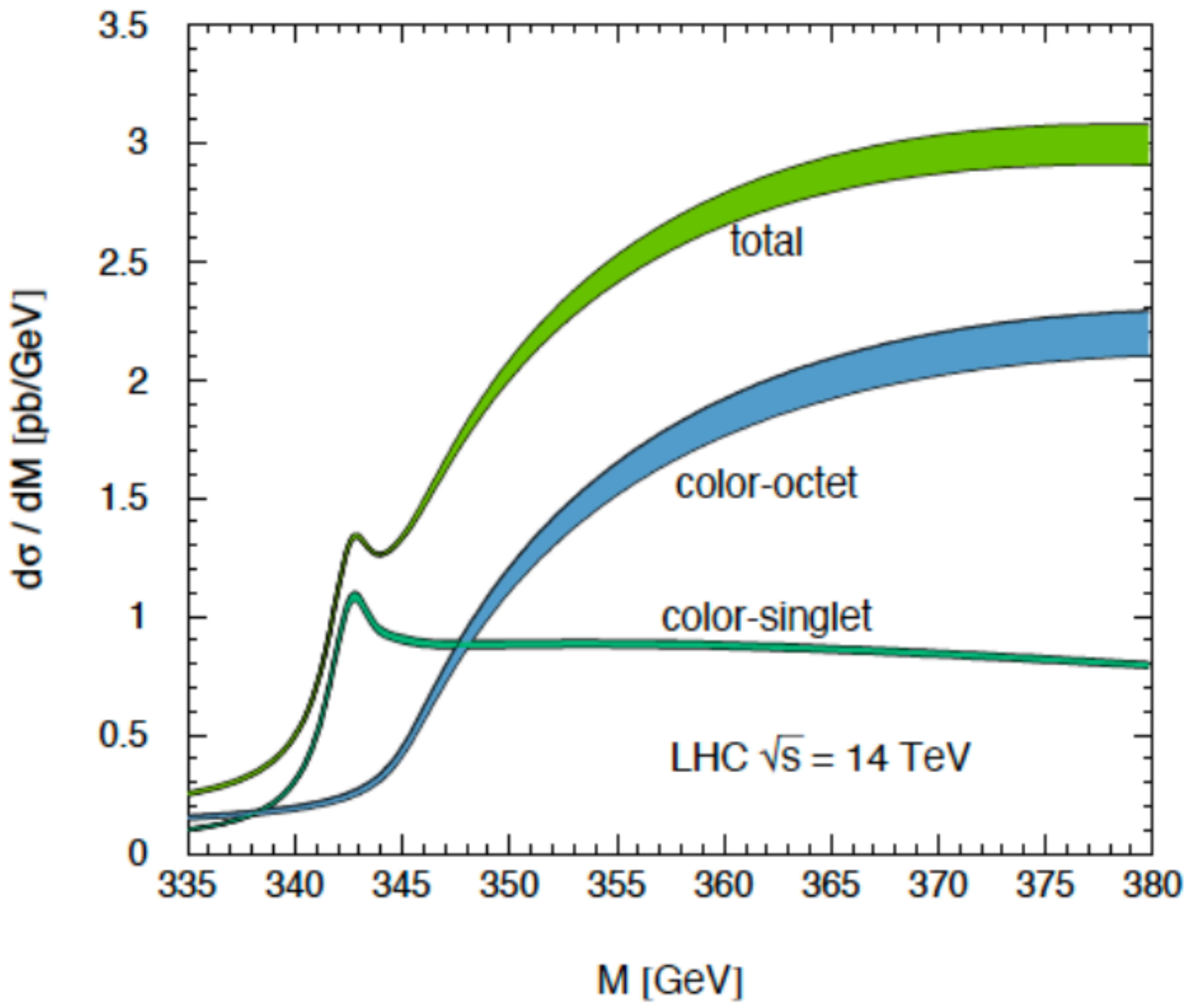
What do we know about toponium?

e^+e^-

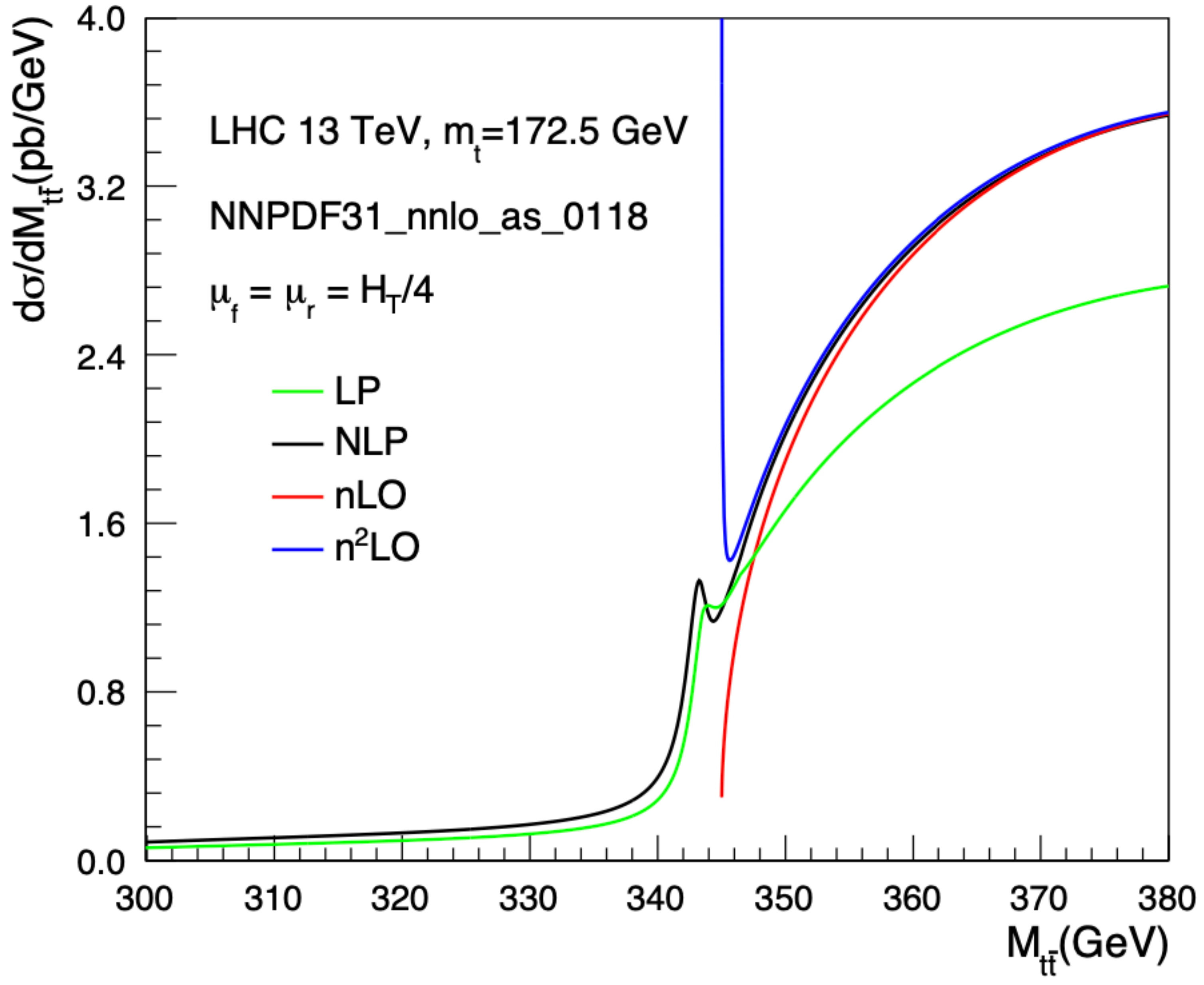
LHC results



Bach et al. 1712.02220



Kiyo, Kühn, Steinhauser, Moch, Uwer arXiv: 0812.0919



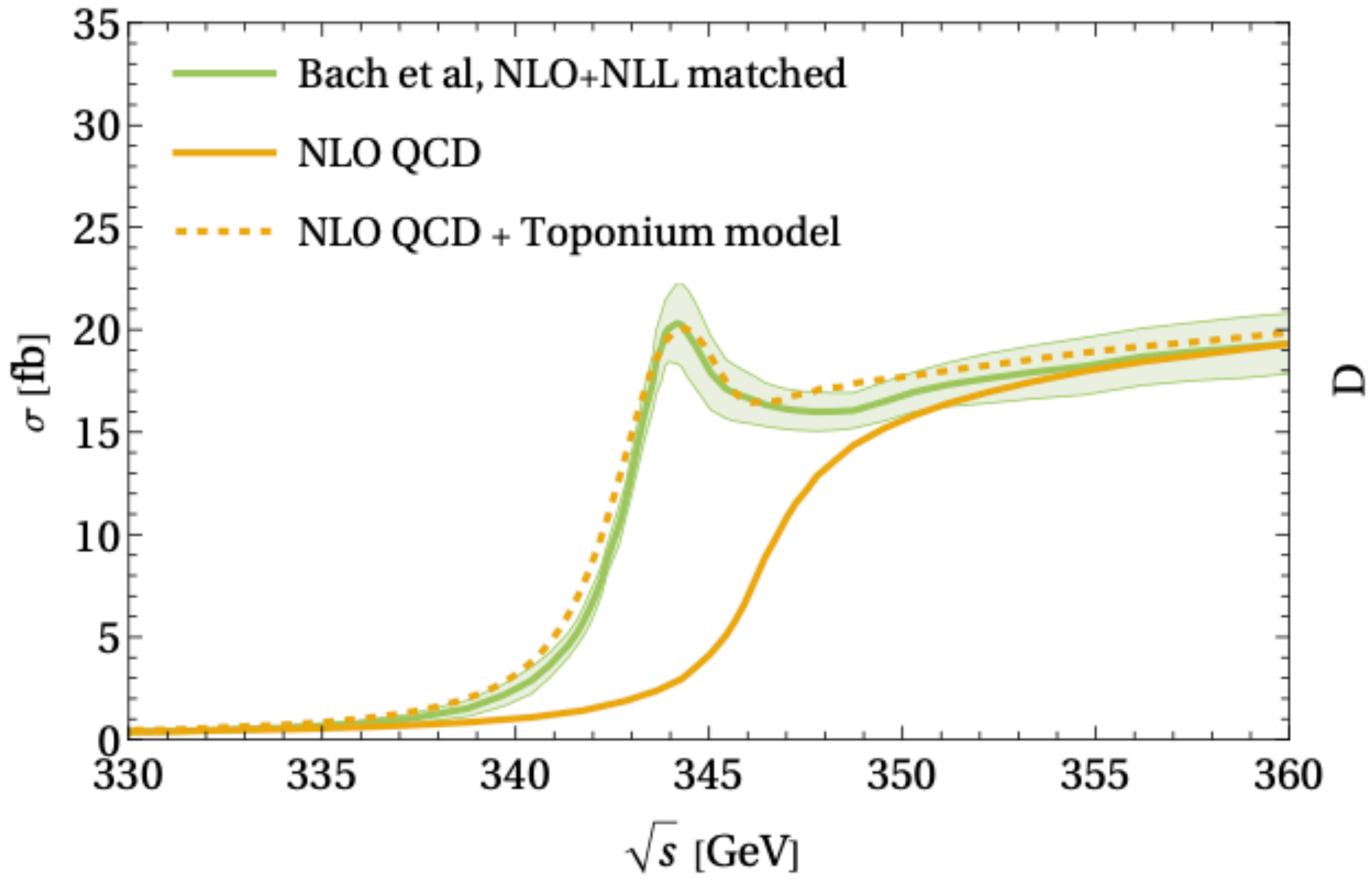
Ju, Wang, Wang, Xu, Xu and Li Lin Yang arXiv:2004.03088

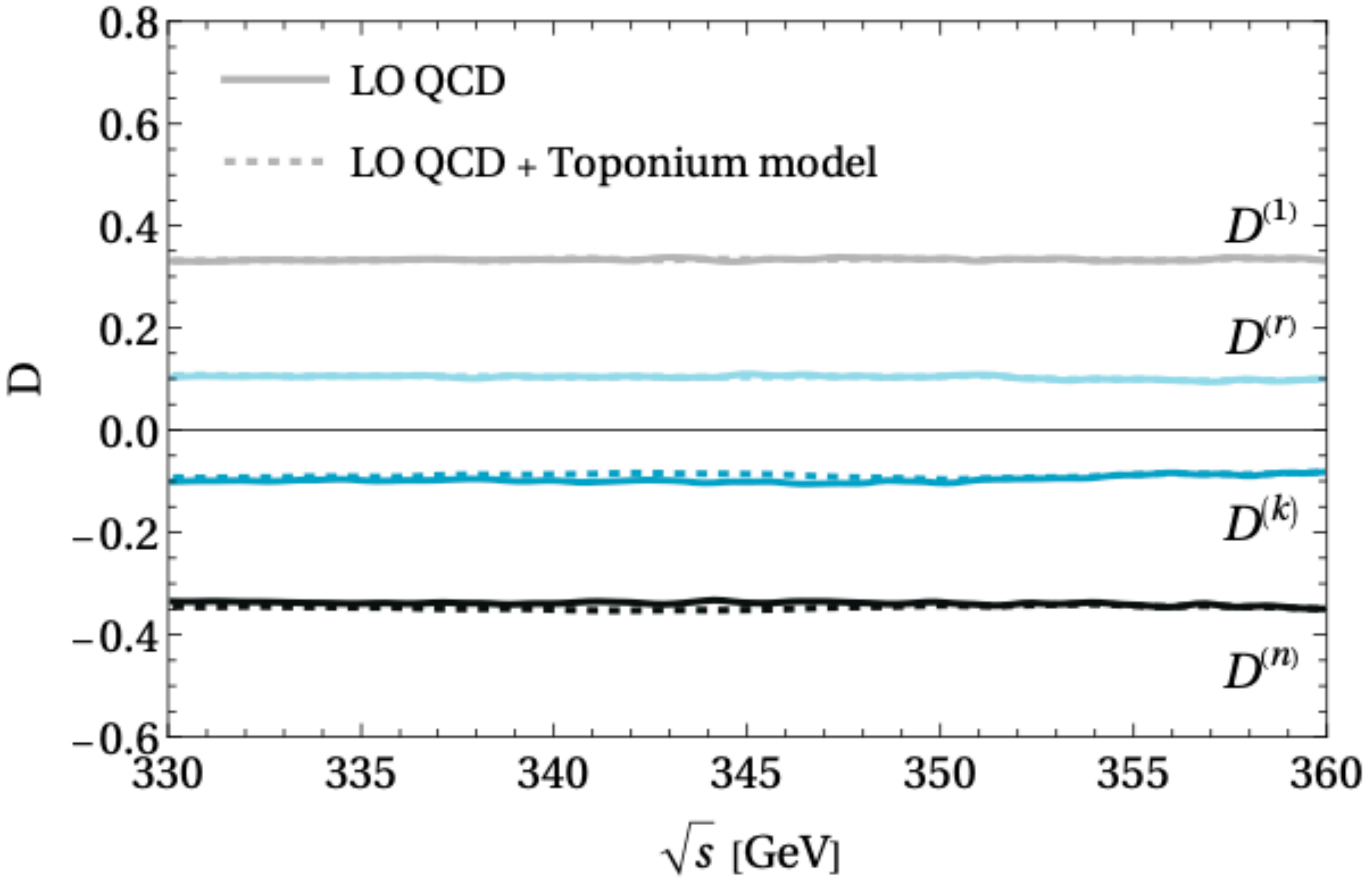
Fully differential NLO+LL, Coulomb Resummation

Coulomb Resummation

Any computation needs matching between below threshold, toponium region, continuum

Toponium in e^+e^-





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Bound state effects have an impact on the lineshape (increase of cross-section)

No impact on entanglement markers (unlike the LHC)

Vector resonance leads to the same spin correlations as the EW Standard Model

Conclusions

- Top pairs an ideal testing ground, different degrees of correlations can be observed
- Different patterns for lepton and hadron colliders
- SMEFT introduces new structures, thus probing new linear combinations between coefficients
- QI observables can break degeneracies between operators when combined with standard observables

Thank you for your attention