Prospects for Quantum Process Tomography at high energies

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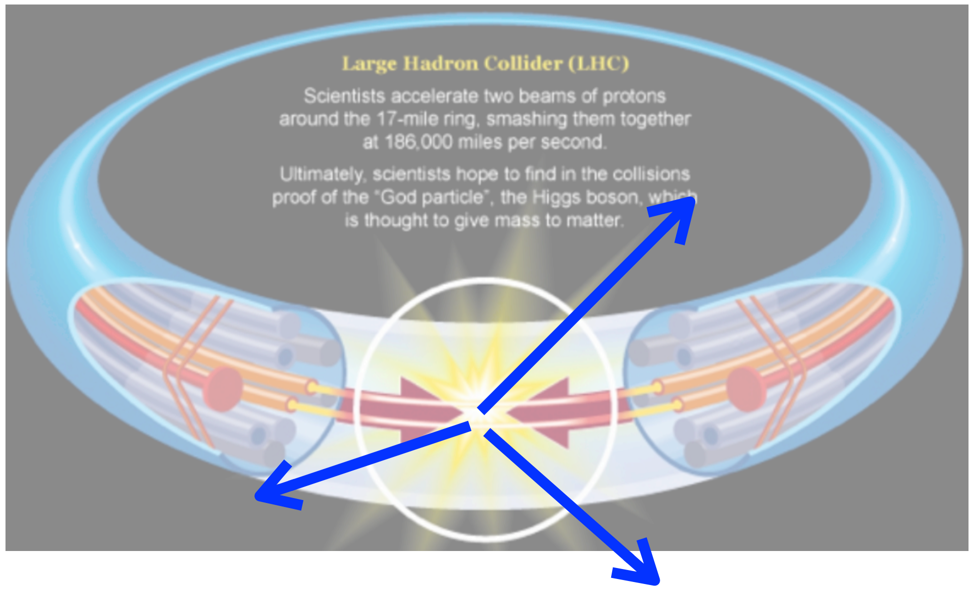
09/04/2025 Quantum Observables for Collider Physics 2025 @ GGI, Florence

Particle Collider = Quantum Computer



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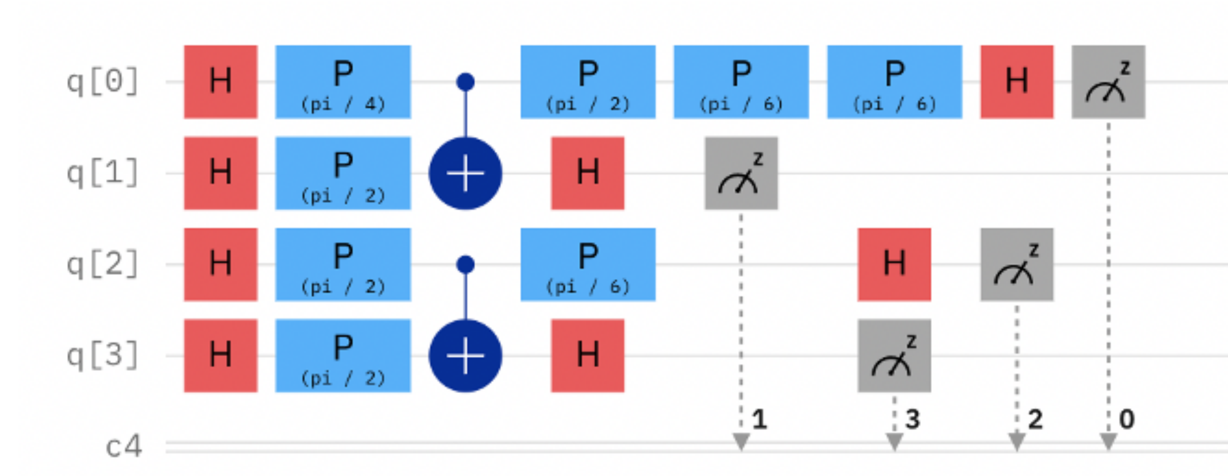
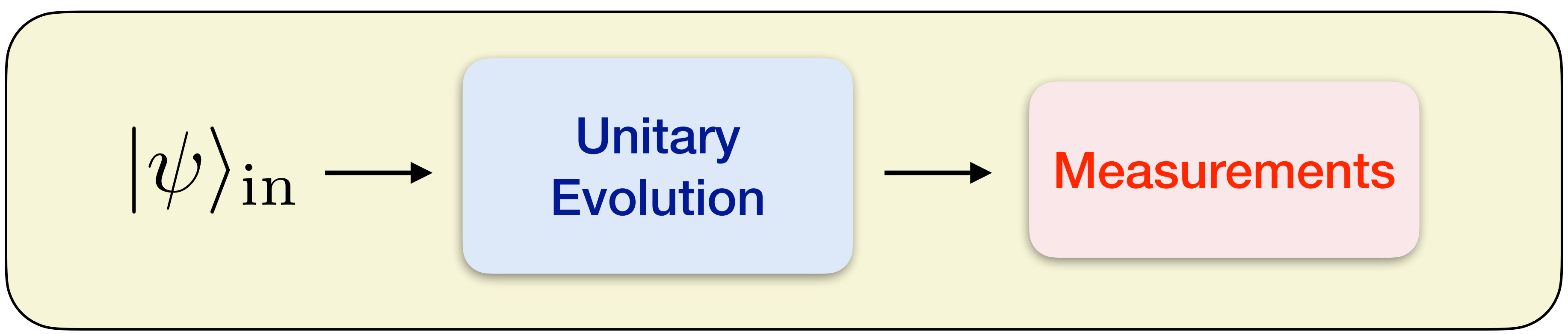
C. Altomonte, A.Barr [2312.02242]

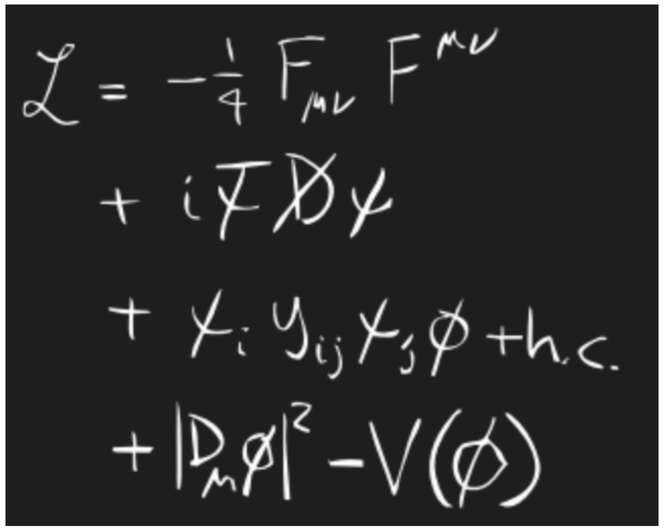




To investigate the **computational property** of colliders, we need to **control input (spin) states**.

➡ Polarised lepton colliders

Particle Theory = Quantum Computer



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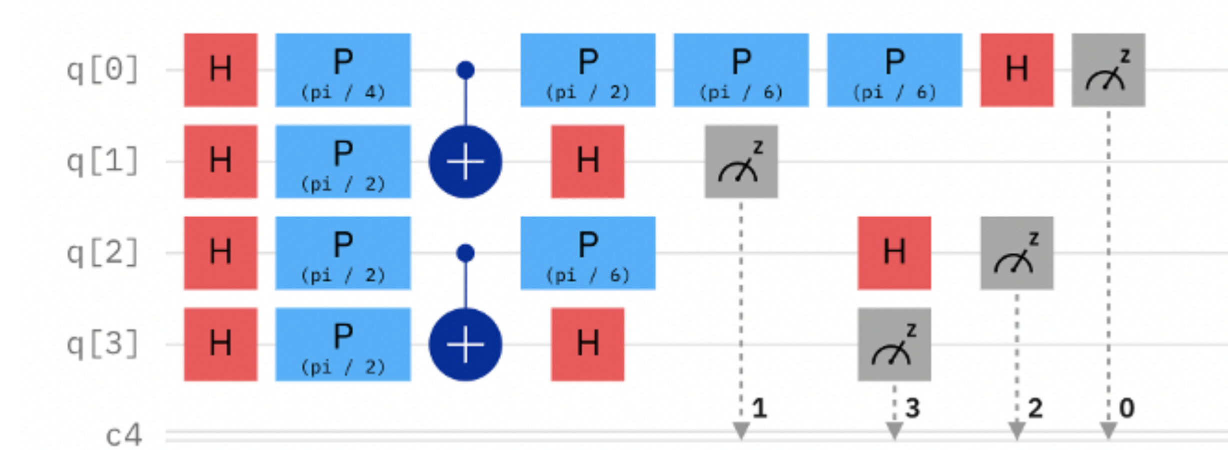

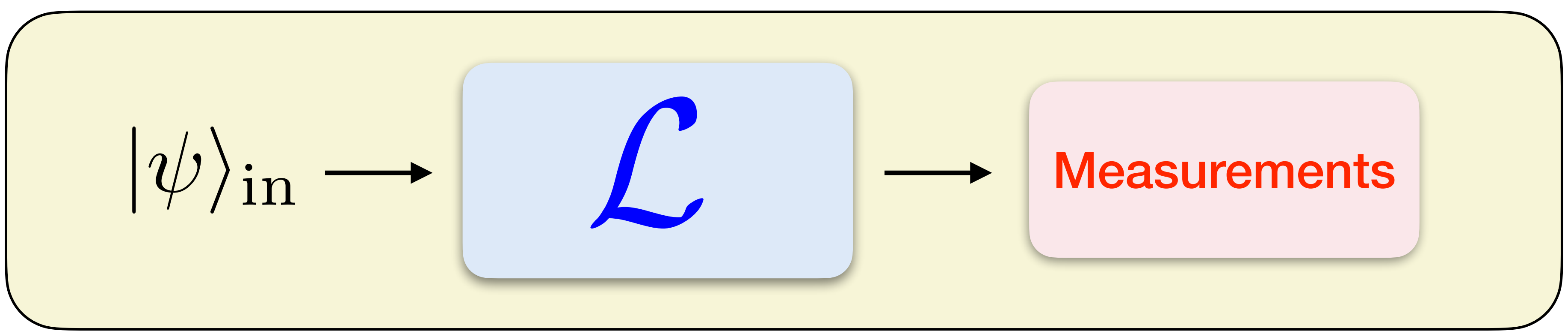

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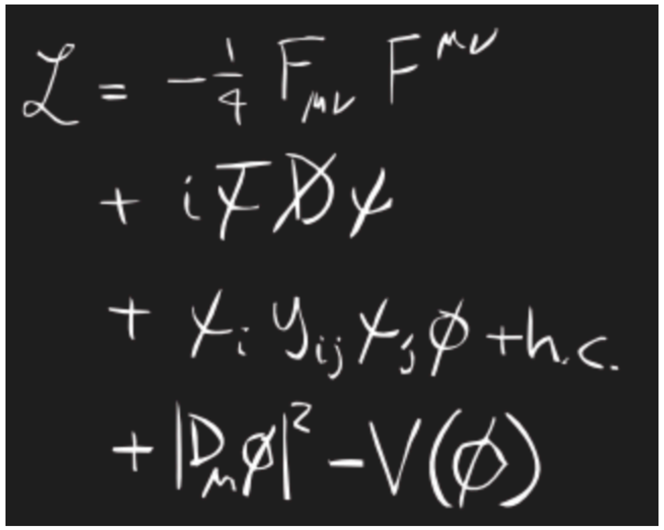

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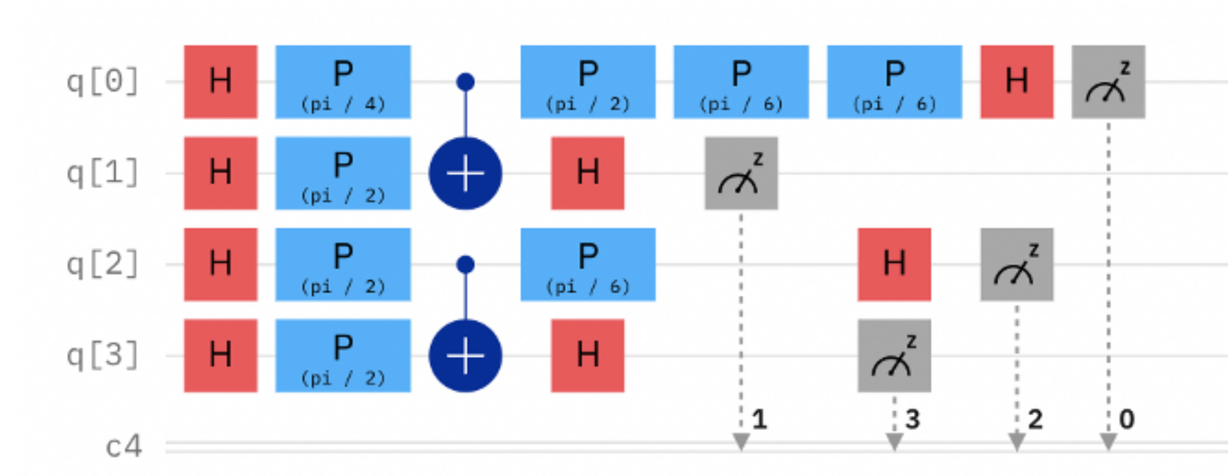
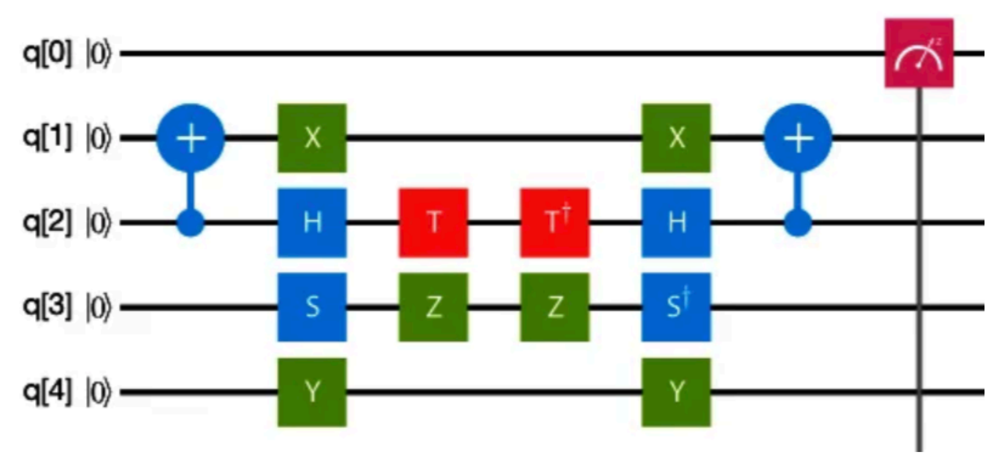


$$\mathcal{L}_{\mathrm{BSM}} =$$



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Particle Theory = Quantum Computer

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \psi_i y_{ij}\psi_j\phi + h.c. + |D_\mu\phi|^2 - V(\phi)$$

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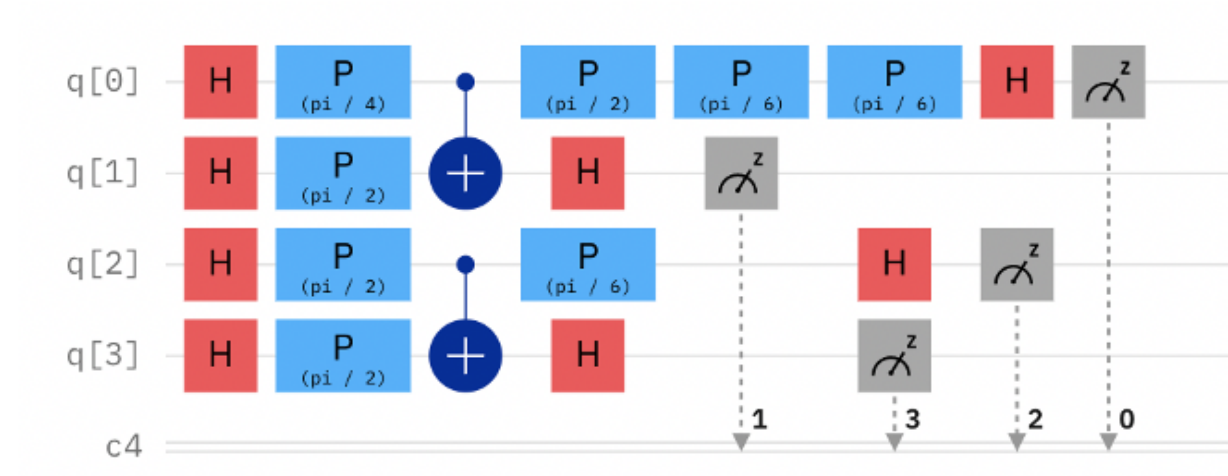
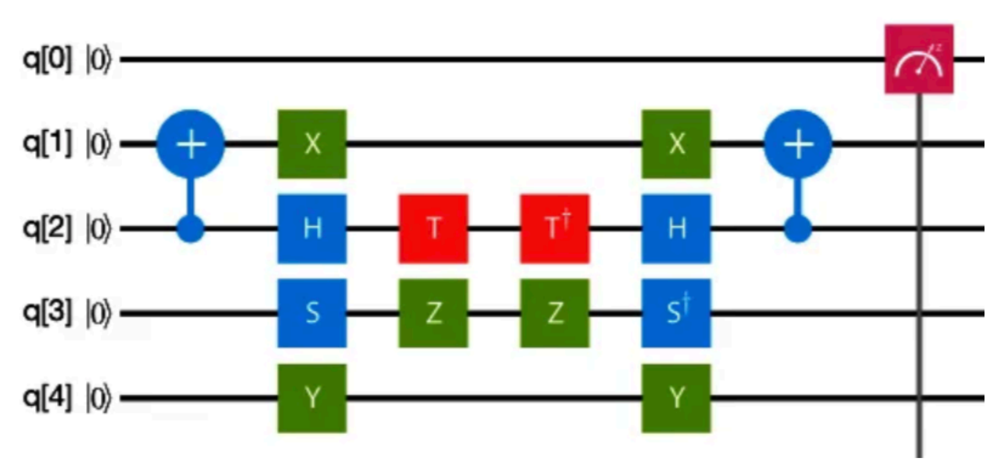


- New precision test of SM
- Foundational test of QM

$$\mathcal{L}_{\mathrm{BSM}} =$$



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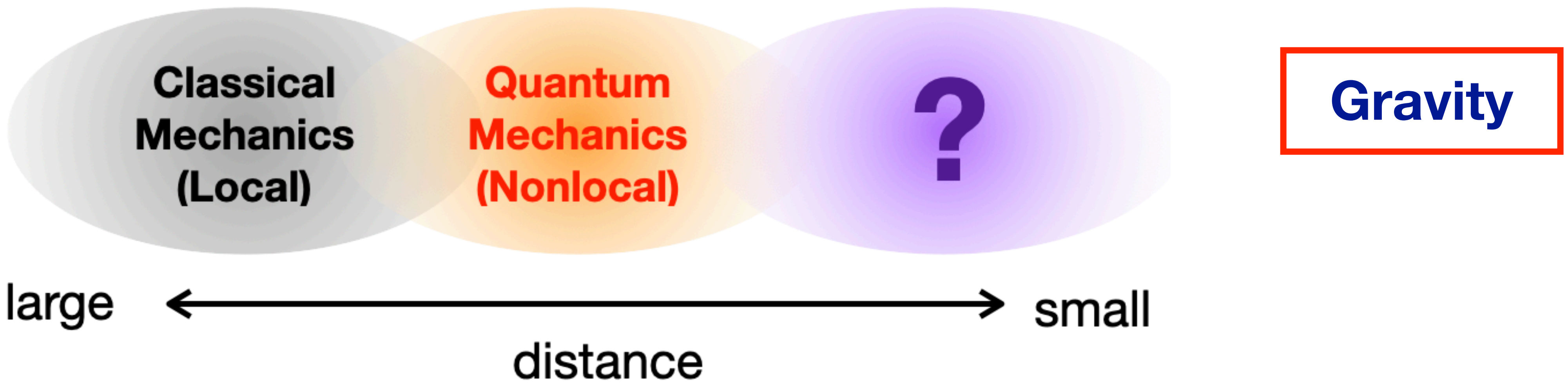
High Energy Test of Quantum Mechanics



Motivation:

- QM might be a low energy effective theory of more fundamental short-distance theory
- QM might be modified at shorter distances to be married with gravitation

Currently, **no LHC analysis** can distinguish between anomalies from **QFT-based BSM** and those from **beyond-QFT**

Possible Modifications of QM

Static:

Dynamical:

Possible Modifications of QM

Static:

Particles may exhibit a strong correlation that cannot be explained within QM

Bell test

$$\mathcal{B} \equiv (A + A')B + (A - A')B'$$

➡ $\langle \mathcal{B} \rangle_{\rm QM} \leq 2\sqrt{2}$ [Tsirelson '87]

QM will be excluded if this bound is violated!

➡
- violation of Born rule
- state can't be written by ρ

Candidate:

- No-signalling theory [Popescu, Rohrlich(1994)]

$$\langle \mathcal{B} \rangle_{\rm NS}^{\rm max} = 4$$

Dynamical:

Possible Modifications of QM

Static:

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Candidate:

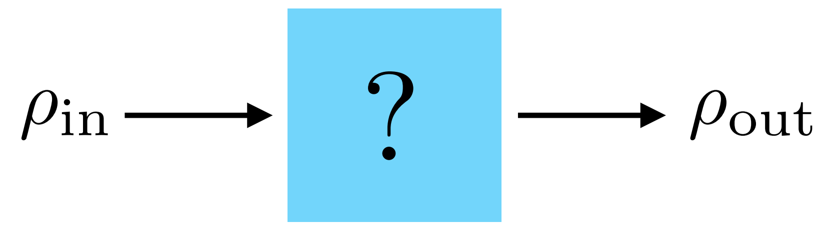
- **No-signalling theory** [Popescu, Rohrlich(1994)]

$$\langle \mathcal{B} \rangle_{\text{NS}}^{\max} = 4$$

Dynamical:

Quantum dynamics may be modified.

- Schrodinger evolution
- Wave function collapse



Candidate:

- Non-linear extensions of QM:

[Weinberg (1989), Polchinski (1991), D.E.Kaplan, Rajendran (2021)]

$$i\partial_t|\chi\rangle = \int d^3x \left[\hat{\mathcal{H}}(x) + \langle\chi|\hat{\mathcal{O}}_1(x)|\chi\rangle\hat{\mathcal{O}}_2(x)\right]|\chi\rangle$$

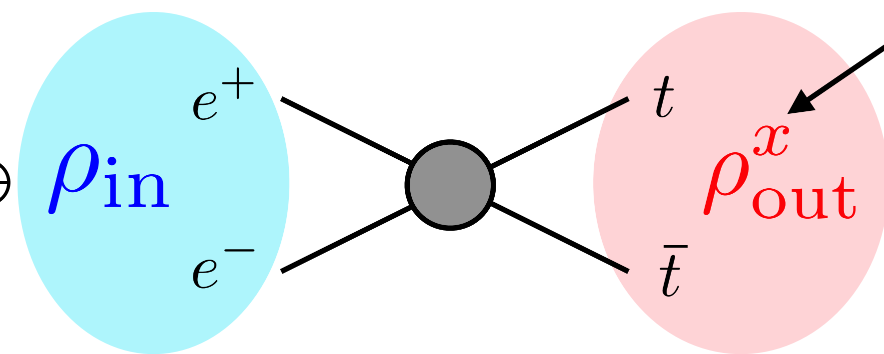
non-linear state-dependent term

Proposed method **can detect** modifications of **quantum dynamics**

Spin-Spin transition as an **open quantum system**

x: momentum label for $t\bar{t}$

$$\mathcal{H}_{\rm in} = \mathbb{C}^2_{e^-} \otimes \mathbb{C}^2_{e^+} \ni \rho_{\rm in} \quad \rho^x_{\rm out} \in \mathbb{C}^2_t \otimes \mathbb{C}^2_{\bar{t}} = \mathcal{H}_{\rm out}$$



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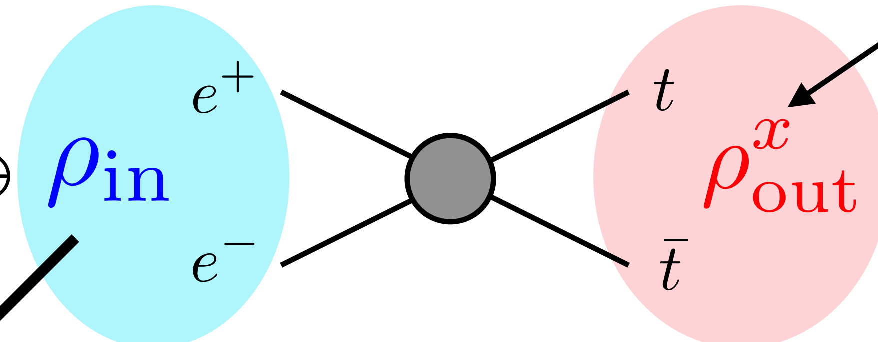

$$\rho^{\rm total}_{\rm in} = \rho_{\rm in} \otimes \underbrace{\rho_{\vec{p}_{e^+}\vec{p}_{e^-}}}_{e^+e^-\text{momentum}}$$

e^+e^-momentum

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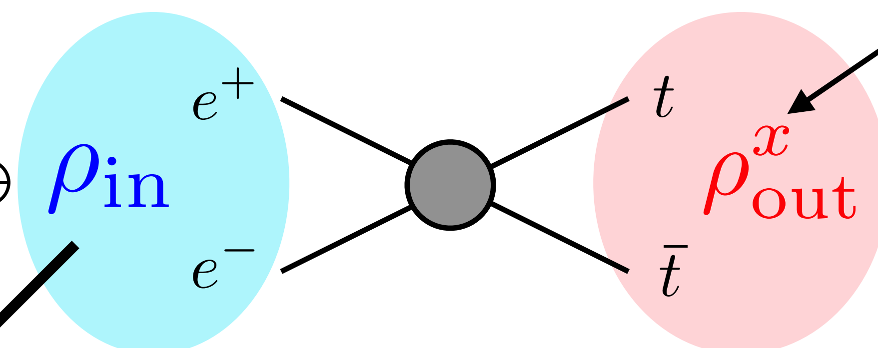

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$$\rho^{\rm total}_{\rm in} = \rho_{\rm in} \otimes \underline{\rho_{\vec{p}_{e^+}\vec{p}_{e^-}}}$$

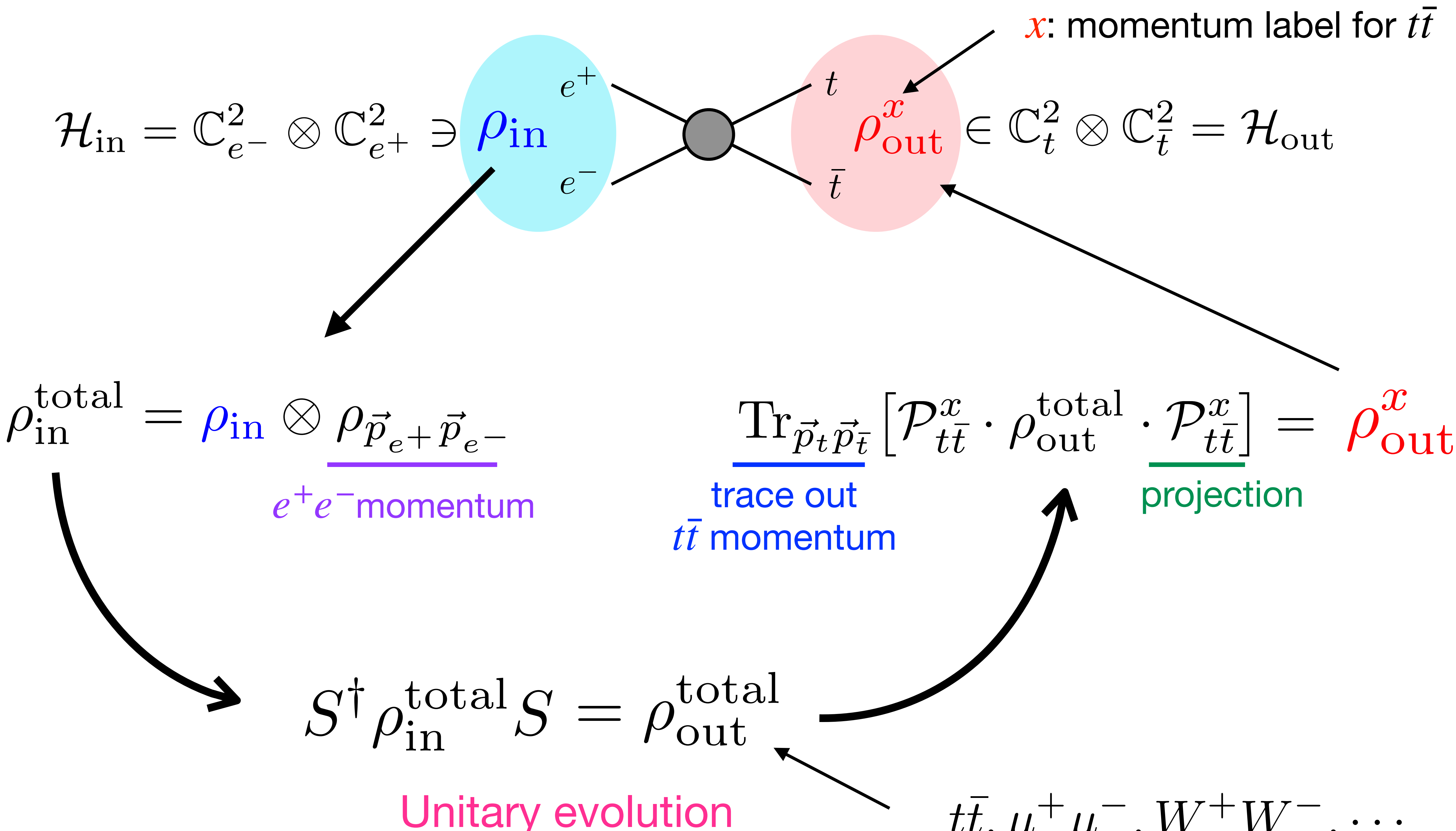
e^+e^-momentum

$$S^\dagger \rho^{\rm total}_{\rm in} S = \rho^{\rm total}_{\rm out}$$

Unitary evolution

$t\bar{t}, \mu^+\mu^-, W^+W^-, \cdots$

Spin-Spin transition as an **open quantum system**



Quantum Instrument

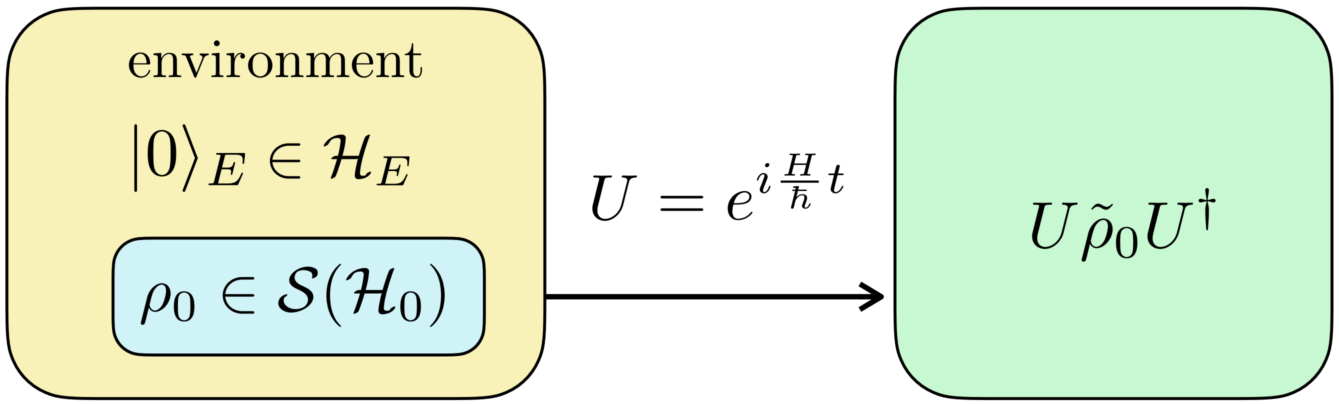
environment

$|0\rangle_E \in \mathcal{H}_E$

$\rho_0 \in \mathcal{S}(\mathcal{H}_0)$

$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0$$

Quantum Instrument



$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0 \xrightarrow{\quad U \quad} U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger$$

Quantum Instrument

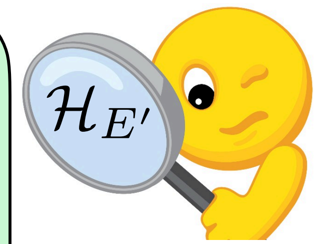
environment

$|0\rangle_E \in \mathcal{H}_E$

$\rho_0 \in \mathcal{S}(\mathcal{H}_0)$

$U = e^{i\frac{H}{\hbar}t}$

$U\tilde{\rho}_0 U^\dagger$



$$\mathcal{P}_{x} = \sum_{k \in x} |e_k\rangle\langle e_k|_{E'}$$

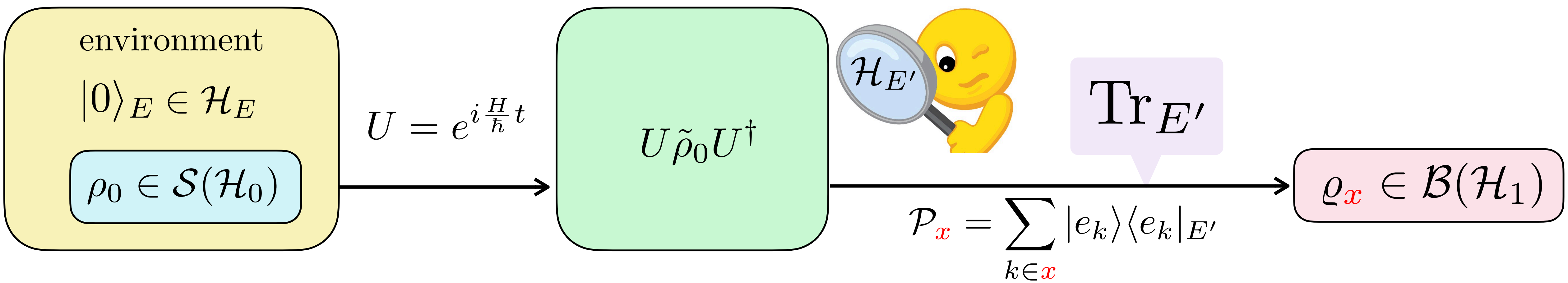
measurement outcome lies in x

$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0 \xrightarrow{U} U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger$$

measuring E'

$$\mathcal{P}_x U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger \mathcal{P}_x$$

Quantum Instrument



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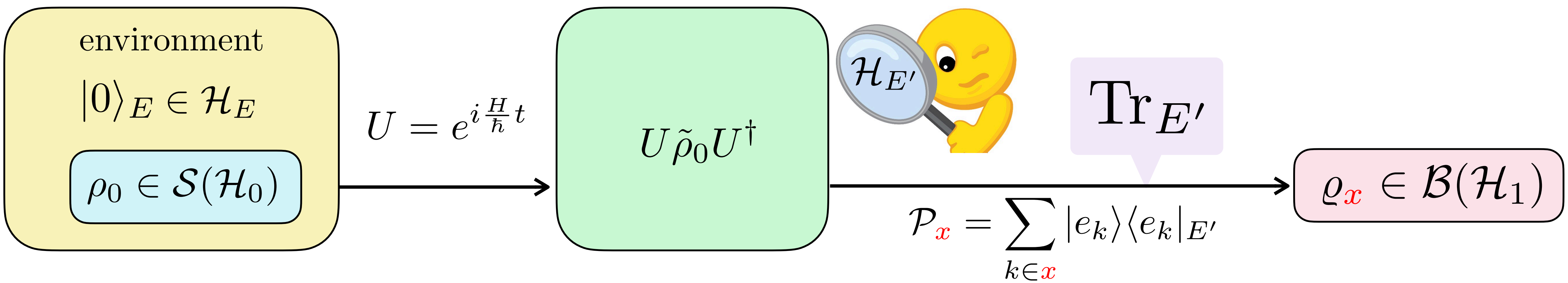
↓ measuring E'

$$\mathcal{P}_x U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger \mathcal{P}_x$$

↓ tracing out E'

$$\varrho_x = \mathrm{Tr}_{E'}\Big[\mathcal{P}_x U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger \mathcal{P}_x\Big]$$

Quantum Instrument



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$$= \sum_{k\in x} \langle e'_k|U|0\rangle\langle 0|_E \otimes \rho_0 U^\dagger|e'_k\rangle$$

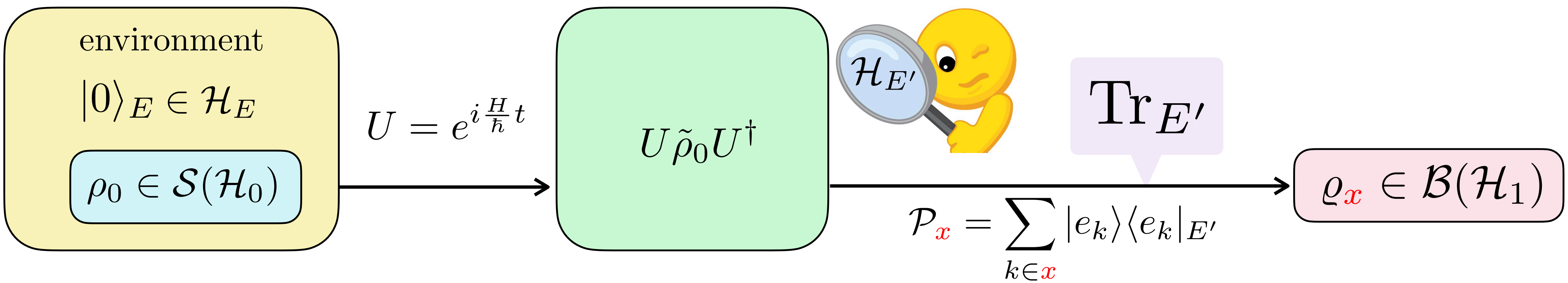
$$= \sum_{k\in x} E_k \rho_0 E_k^\dagger$$

Kraus operators:

$$E_k = \langle e'_k|U|0\rangle_E$$

$$E_k : \mathcal{H}_0 \to \mathcal{H}_1$$

Quantum Instrument



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Quantum Instrument I_x

Kraus operators:

$$E_k = \langle e_k'|U|0\rangle_E$$

$$E_k : \mathcal{H}_0 \to \mathcal{H}_1$$

$$\mathcal{I}_x : \mathcal{S}(\mathcal{H}_0) \to \mathcal{B}(\mathcal{H}_1)$$

$$\mathcal{I}_x(\rho_0) = \varrho_x$$

Q. What is the general property of the Q-Instrument map?

$$\mathcal{I}_x(\rho_0) = \varrho_x = \sum_{k\in x} E_k \rho_0 E_k^\dagger$$

Kraus operators:

$$E_k = \langle e'_k|U|0\rangle_E$$

$$E_k : \mathcal{H}_0 \to \mathcal{H}_1$$

A.

- Linear
- *Completely* Positive

$$\varrho_x = \sum_{k\in x} (E_k\sqrt{\rho_0})(\sqrt{\rho_0}E_k^\dagger)$$

Completely positive map

$$\mathbf{1}_R \otimes E_k$$

$$R\otimes \mathcal{S}(\mathcal{H}_0) \longrightarrow R\otimes \mathcal{B}(\mathcal{H}_1)$$

is also positive

Outcome of Q-Instrument map ϱ_x is ***not*** normalised:

post-QI (normalised) state

$$\rho_x = \frac{\varrho_x}{\mathrm{Tr}\,\varrho_{\mathrm{x}}}$$

← state-to-(normalised)state map is ***NOT*** linear

Q-Instrument map is completely specified by the **Choi matrix**

$$\rho_{\rm in} \to \varrho_{\rm out}^{x}$$

$$|I, J\rangle = |I\rangle_{e^-} \otimes |J\rangle_{e^+}$$

$$I, J, K, L \in \{+, -\}$$

$$\hat{\rho}_{\rm in} = \sum_{[I,J],[K,L]} \rho_{[I,J],[K,K]} |I, J\rangle\langle K, L|$$

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$$\downarrow \mathcal{I}_x$$

$$\hat{\varrho}_{\text{out}}^{x} = \mathcal{I}_x(\hat{\varrho}_{\text{in}}) = \sum_{[I,J],[K,L]} \rho_{[I,J],[K,K]} \mathcal{I}_x\big(|I,J\rangle\langle K,L|\big)$$

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$$I, J, K, L \in \{+, -\}$$

$$|A, B\rangle = |A\rangle_t \otimes |B\rangle_{\bar{t}}$$

$$A, B, C, D \in \{0, 1\}$$

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$\downarrow \mathcal{I}_x$

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$\downarrow$ matrix rep.

Choi matrix

$$\left[\varrho^x_{\rm out}\right]_{[A,B],[C,D]} = \sum_{[I,J],[K,L]} \rho_{[I,J],[K,K]} \langle A, B|\mathcal{I}_x\big(|I, J\rangle\langle K, L|\big)|C, D\rangle$$

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$\mathcal{I}_x$ ↓

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$$\widetilde{\mathcal{I}}_x(|I,J\rangle\langle K,L|)_{[A,B],[C,D]} = \begin{pmatrix} \mathcal{I}_x(|++\rangle\langle ++|)_{[A,B],[C,D]} & \mathcal{I}_x(|++\rangle\langle +-|)_{[A,B],[C,D]} & \cdots \\ \mathcal{I}_x(|-+\rangle\langle ++|)_{[A,B],[C,D]} & \cdots & \\ \mathcal{I}_x(|+-\rangle\langle ++|)_{[A,B],[C,D]} & \cdots & \\ \mathcal{I}_x(|--\rangle\langle ++|)_{[A,B],[C,D]} & \cdots & \mathcal{I}_x(|--\rangle\langle --|)_{[A,B],[C,D]} \end{pmatrix}$$

16 x 16 matrix

- From given input state, the output state can be immediately from the Choi matrix.
- **Different theories** predicting different spin-to-spin transition necessarily gives **different Choi matrices**.
- **Choi matrix** = **Theory**
- Complete positivity of QI-map => Choi matrix is **positive-semidefinite**.

Choi matrix

$$\left[\varrho^x_{\mathrm{out}}\right]_{[A,B],[C,D]} = \sum_{[I,J],[K,L]} \rho_{[I,J],[K,K]} \langle A,B|\mathcal{I}_x\big(|I,J\rangle\langle K,L|\big)|C,D\rangle$$

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16 x 16 matrix

QFT calculation

$$e^- e^+ \to t\bar{t}$$

$$\rho_0 \in \mathcal{S}(\mathcal{H}_0) = \mathbb{C}^2_{e^-} \otimes \mathbb{C}^2_{e^+} \longrightarrow \varrho_x \in \mathcal{B}(\mathcal{H}_1) = \mathbb{C}^2_t \otimes \mathbb{C}^2_{\bar{t}}$$

$$\rho_0 = \sum_{s_e} q_{s_e} |s_e\rangle\langle s_e| \qquad s_e = \{++, +-, -+, --\}$$

QFT calculation

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1) $\rho_0 \to |p_{\rm in}\rangle\langle p_{\rm in}| \otimes \rho_0 = \tilde{\rho}_0 \qquad |p_{\rm in}\rangle = |p_{e^-}, p_{e^+}\rangle$

QFT calculation

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2) $\tilde{\rho}_0 \to S\tilde{\rho}_0 S^\dagger = \tilde{\rho}_1$

3) $\tilde{\rho}_1 \to {\rm Tr}_{p_t, p_{\bar{t}}} [\mathcal{P}_x \tilde{\rho}_1 \mathcal{P}_x] = \varrho'_x$

$t\bar{t}$ phase-space selection

$$\mathcal{P}_x = \sum_{s_t} \int_x d\Pi_{t\bar{t}} |p_t, s_t\rangle\langle p_t, s_t|$$

$$\hat{\mathbf{1}} = \sum_f \left[\left(\prod_{i \in f} \int d\Pi_i \right) |f\rangle\langle f| \right], \qquad d\Pi_i = \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i}$$

QFT calculation

$$e^- e^+ \to t\bar{t}$$

$$\rho_0 \in \mathcal{S}(\mathcal{H}_0) = \mathbb{C}^2_{e^-} \otimes \mathbb{C}^2_{e^+} \longrightarrow \varrho_x \in \mathcal{B}(\mathcal{H}_1) = \mathbb{C}^2_t \otimes \mathbb{C}^2_{\bar{t}}$$

$$\rho_0 = \sum_{s_e} q_{s_e} |s_e\rangle\langle s_e| \qquad s_e = \{++, +-, -+, --\}$$

1) $\rho_0 \to |p_{\rm in}\rangle\langle p_{\rm in}| \otimes \rho_0 = \tilde{\rho}_0 \qquad |p_{\rm in}\rangle = |p_{e^-}, p_{e^+}\rangle$

2) $\tilde{\rho}_0 \to S\tilde{\rho}_0 S^\dagger = \tilde{\rho}_1$

3) $\tilde{\rho}_1 \to {\rm Tr}_{p_t, p_{\bar{t}}} [\mathcal{P}_x \tilde{\rho}_1 \mathcal{P}_x] = \varrho'_x$

$t\bar{t}$ phase-space selection

$$\mathcal{P}_x = \sum_{s_t} \int_x d\Pi_{t\bar{t}} |p_t, s_t\rangle\langle p_t, s_t|$$

$$\hat{\mathbf{1}} = \sum_f \left[\left(\prod_{i\in f} \int d\Pi_i \right) |f\rangle\langle f| \right], \qquad d\Pi_i = \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}$$

$$\varrho'_x = \sum_{s_e, s_t, s'_t} q_{s_e} \int_x d\Pi_{t\bar{t}} \langle p_t, s_t | S | p_{\rm in}, s_e\rangle \langle p_{\rm in}, s_e | S^\dagger | p_{t\bar{t}}, s'_t\rangle \, |s_t\rangle\langle s'_t|$$

$$\propto \frac{1}{\sigma_\mathcal{N}} \sum_{s_e, s_t, s'_t} q_{s_e} \left[\frac{1}{2s} \int_x d\Pi^{t\bar{t}}_{\rm LIPS} \mathcal{M}^{p_{\rm in}, s_e}_{p_t, s_t} [\mathcal{M}^{p_{\rm in}, s_e}_{p_t, s'_t}]^* \right] |s_t\rangle\langle s'_t|$$

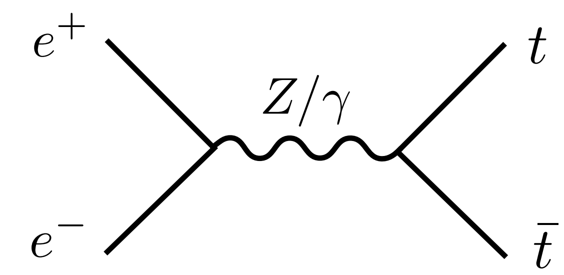
$$\rho_0^{\rm mix} = \frac{1}{4} \sum_{s_e} |s_e\rangle\langle s_e|$$

$$\sigma_\mathcal{N} = \sigma[e^- e^+ (\rho_0^{\rm mix}) \to t\bar{t}]$$

Theoretical Prediction

$$d\widetilde{\mathcal{I}}(|I,J\rangle\langle K,L|)_{(A,B),(C,D)} = \frac{1}{\sigma_{\mathcal{N}}}\frac{1}{2s}\int_x d\Pi\, \mathcal{M}^{I,J}_{A,B}\left(\mathcal{M}^{K,L}_{C,D}\right)^*$$

$$\mathcal{L} \ni \sum_i \frac{1}{\Lambda_i^2}[\bar{\psi}_e\gamma_\mu(c_L^i P_L + c_R^i P_R)\psi_e][\bar{\psi}_t\gamma^\mu(d_L^i P_L + d_R^i P_R)\psi_t]$$



i	Λ_i^2	c_L^i	c_R^i	d_L^i	d_R^i
A	s	$-e$	$-e$	$\frac{2}{3}e$	$\frac{2}{3}e$
Z	$s-m_Z^2$	$g_Z\left(-\frac{1}{2}+\sin^2\theta_w\right)$	$g_Z\sin^2\theta_w$	$g_Z\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_w\right)$	$g_Z\left(-\frac{2}{3}\sin^2\theta_w\right)$

e^-e^+ (helicities, upper index) ; $t\bar{t}$ (helicities, lower index)

$$\mathcal{M}^{++}_{00} = \mathcal{M}^{++}_{11} = e^{i\phi}\sum_i \frac{s}{2\Lambda_i^2}\gamma^{-1}c_R^i\sin\theta(d_L^i+d_R^i)\,,$$

$$\mathcal{M}^{++}_{01} = -e^{i\phi}\sum_i \frac{s}{2\Lambda_i^2}c_R^i(1+\cos\theta)[d_L^i(1-\beta)+d_R^i(1+\beta)]\,,$$

$$\mathcal{M}^{++}_{10} = -e^{i\phi}\sum_i \frac{s}{2\Lambda_i^2}c_R^i(1-\cos\theta)[d_L^i(1+\beta)+d_R^i(1-\beta)]\,,$$

$$\mathcal{M}^{--}_{00} = \mathcal{M}^{--}_{11} = e^{-i\phi}\sum_i \frac{s}{2\Lambda_i^2}\gamma^{-1}c_L^i\sin\theta(d_L^i+d_R^i)\,,$$

$$\mathcal{M}^{--}_{01} = -e^{-i\phi}\sum_i \frac{s}{2\Lambda_i^2}c_L^i(1-\cos\theta)[d_L^i(1-\beta)+d_R^i(1+\beta)]\,,$$

$$\mathcal{M}^{--}_{10} = -e^{-i\phi}\sum_i \frac{s}{2\Lambda_i^2}c_L^i(1+\cos\theta)[d_L^i(1+\beta)+d_R^i(1-\beta)]$$

$$\mathcal{M}^{+-}_{A,B} = \mathcal{M}^{-+}_{A,B} = 0$$

$$\sigma_{\mathcal{N}} = \frac{1}{8\pi}\frac{q}{s\sqrt{s}}\int\frac{1}{4}\sum_{I,J,A,B}\left|\mathcal{M}^{I,J}_{A,B}\right|^2\frac{d\Omega}{4\pi}\,.$$

$$\widetilde{\mathcal{I}}_x = \begin{pmatrix} I_x^{(++,++)} & 0 & 0 & I_x^{(++,--)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(--,++)} & 0 & 0 & I_x^{(--,--)} \end{pmatrix} \qquad I_x^{(--,++)} = [I_x^{(++,--)}]^\dagger$$

$$\frac{dI^{(++,++)}}{d\cos\theta d\phi} = \begin{pmatrix} a_{11}^{(+)} s_\theta^2 & a_{12}^{(+)} s_\theta(1+c_\theta) & a_{13}^{(+)} s_\theta(1-c_\theta) & a_{14}^{(+)} s_\theta^2 \\ a_{21}^{(+)} s_\theta(1+c_\theta) & a_{22}^{(+)}(1+c_\theta)^2 & a_{23}^{(+)}(1-c_\theta^2) & a_{24}^{(+)} s_\theta(1+c_\theta) \\ a_{31}^{(+)} s_\theta(1-c_\theta) & a_{32}^{(+)}(1-c_\theta^2) & a_{33}^{(+)}(1-c_\theta)^2 & a_{34}^{(+)} s_\theta(1-c_\theta) \\ a_{41}^{(+)} s_\theta^2 & a_{42}^{(+)} s_\theta(1+c_\theta) & a_{43}^{(+)} s_\theta(1-c_\theta) & a_{44}^{(+)} s_\theta^2 \end{pmatrix},$$

$$\frac{dI^{(++,--)}}{d\cos\theta d\phi} = e^{i2\phi}\begin{pmatrix} a_{11}^{(+-)} s_\theta^2 & a_{12}^{(+-)} s_\theta(1-c_\theta) & a_{13}^{(+-)} s_\theta(1+c_\theta) & a_{14}^{(+-)} s_\theta^2 \\ a_{21}^{(+-)} s_\theta(1+c_\theta) & a_{22}^{(+-)}(1-c_\theta^2) & a_{23}^{(+-)}(1+c_\theta)^2 & a_{24}^{(+-)} s_\theta(1+c_\theta) \\ a_{31}^{(+-)} s_\theta(1-c_\theta) & a_{32}^{(+-)}(1-c_\theta)^2 & a_{33}^{(+-)}(1-c_\theta^2) & a_{34}^{(+-)} s_\theta(1-c_\theta) \\ a_{41}^{(+-)} s_\theta^2 & a_{42}^{(+-)} s_\theta(1-c_\theta) & a_{43}^{(+-)} s_\theta(1+c_\theta) & a_{44}^{(+-)} s_\theta^2 \end{pmatrix}$$

$$\frac{dI^{(--,--)}}{d\cos\theta d\phi} = \begin{pmatrix} a_{11}^{(-)} s_\theta^2 & a_{12}^{(-)} s_\theta(1-c_\theta) & a_{13}^{(-)} s_\theta(1+c_\theta) & a_{14}^{(-)} s_\theta^2 \\ a_{21}^{(-)} s_\theta(1-c_\theta) & a_{22}^{(-)}(1-c_\theta)^2 & a_{23}^{(-)}(1-c_\theta^2) & a_{24}^{(-)} s_\theta(1-c_\theta) \\ a_{31}^{(-)} s_\theta(1+c_\theta) & a_{32}^{(-)}(1-c_\theta^2) & a_{33}^{(-)}(1+c_\theta)^2 & a_{34}^{(-)} s_\theta(1+c_\theta) \\ a_{41}^{(-)} s_\theta^2 & a_{42}^{(-)} s_\theta(1-c_\theta) & a_{43}^{(-)} s_\theta(1+c_\theta) & a_{44}^{(-)} s_\theta^2 \end{pmatrix}.$$

$$a^{(+)}\big|_{\sqrt{s}=370\,\mathrm{GeV}} = \begin{pmatrix} 0.503 & -1.442 & 0.364 & 0.503 \\ -1.442 & 4.137 & -1.043 & -1.442 \\ 0.364 & -1.043 & 0.263 & 0.364 \\ 0.503 & -1.442 & 0.364 & 0.503 \end{pmatrix}\cdot 10^{-2}, \qquad a^{(+)}\big|_{\sqrt{s}=1\,\mathrm{TeV}} = \begin{pmatrix} 0.219 & -0.777 & -0.494 & 0.219 \\ -0.777 & 2.755 & 1.751 & -0.777 \\ -0.494 & 1.751 & 1.113 & -0.494 \\ 0.219 & -0.777 & -0.494 & 0.219 \end{pmatrix}\cdot 10^{-2},$$

$$a^{(+-)}\big|_{\sqrt{s}=370\,\mathrm{GeV}} = \begin{pmatrix} 0.800 & 0.264 & -1.979 & 0.800 \\ -2.293 & -0.758 & 5.676 & -2.293 \\ 0.578 & 0.191 & -1.431 & 0.578 \\ 0.800 & 0.264 & -1.980 & 0.800 \end{pmatrix}\cdot 10^{-2}, \qquad a^{(+-)}\big|_{\sqrt{s}=1\,\mathrm{TeV}} = \begin{pmatrix} 0.340 & -0.810 & -1.162 & 0.340 \\ -1.205 & 2.870 & 4.117 & -1.205 \\ -0.766 & 1.824 & 2.617 & -0.766 \\ 0.340 & -0.810 & -1.162 & 0.340 \end{pmatrix}\cdot 10^{-2},$$

$$a^{(-)}\big|_{\sqrt{s}=370\,\mathrm{GeV}} = \begin{pmatrix} 1.271 & 0.420 & -3.146 & 1.271 \\ 0.420 & 0.139 & -1.040 & 0.420 \\ -3.146 & -1.040 & 7.788 & -3.146 \\ 1.271 & 0.420 & -3.146 & 1.271 \end{pmatrix}\cdot 10^{-2}, \qquad a^{(-)}\big|_{\sqrt{s}=1\,\mathrm{TeV}} = \begin{pmatrix} 0.527 & -1.256 & -1.802 & 0.527 \\ -1.256 & 2.990 & 4.290 & -1.256 \\ -1.802 & 4.290 & 6.154 & -1.802 \\ 0.527 & -1.256 & -1.802 & 0.527 \end{pmatrix}\cdot 10^{-2},$$

Two theories giving different spin transitions necessarily give different Choi matrices

➡ **Sensitive to BSM extension!**

Reconstruction of Choi matrix (= Theory)

= Quantum Process Tomography

- Reconstruction of the diagonal part:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

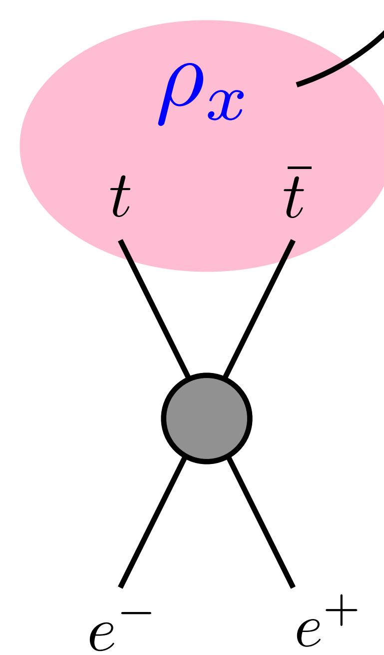
- Consider 4 purely polarised beam settings:

$$\{|i\rangle\} = \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\} \qquad \rho_0^i = |i\rangle\langle i|$$

$$\mathcal{I}_x(|i\rangle\langle i|) = \varrho_x^i = \mathrm{Tr}\,\varrho_x^i \cdot \frac{\varrho_x^i}{\mathrm{Tr}\,\varrho_x^i}$$

measurable

$$\frac{\sigma_x[e^-e^+(\rho_0^i) \to t\bar{t}]}{\sigma[e^-e^+(\rho_0^{\rm mix}) \to t\bar{t}]}$$



Reconstruction of spin-density matrix is possible

Quantum State Tomography

Y.Afik, J.Nova [2003.02280], Ashby-Pickering, Barr, Wierzchucka [2209.13990]

- Reconstruction of **off**-diagonal elements:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

- Consider polarisations ***NOT*** in the direction of the beam:

$$|\mathbf{m}\rangle = \alpha\,|+\rangle + \beta\,|-\rangle\,, \quad |-\mathbf{m}\rangle = \bar{\alpha}\,|+\rangle + \bar{\beta}\,|-\rangle\,, \qquad |\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1$$

$$|\mathbf{n}\rangle = \gamma\,|+\rangle + \delta\,|-\rangle\,, \quad |-\mathbf{n}\rangle = \bar{\gamma}\,|+\rangle + \bar{\delta}\,|-\rangle\,. \qquad \alpha\bar{\alpha}^* + \beta\bar{\beta}^* = \gamma\bar{\gamma} + \delta\bar{\delta}^* = 0$$

- Consider the beam setting $(e^-, e^+) = (+, \mathbf{m})$

$$\begin{aligned} \rho_0^{(+,\mathbf{m})} &= |+\rangle\langle+| \otimes |\mathbf{m}\rangle\langle\mathbf{m}| \\ &= |\alpha|^2|++\rangle\langle++| + \alpha\beta^*|++\rangle\langle+-| + \alpha^*\beta|+-\rangle\langle++| + |\beta|^2|--\rangle\langle--| \end{aligned}$$

- Reconstruction of **off**-diagonal elements:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

- Consider polarisations ***NOT*** in the direction of the beam:

$$|\mathbf{m}\rangle = \alpha\,|+\rangle + \beta\,|-\rangle\,, \quad |-\mathbf{m}\rangle = \bar{\alpha}\,|+\rangle + \bar{\beta}\,|-\rangle\,, \qquad |\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1$$

$$|\mathbf{n}\rangle = \gamma\,|+\rangle + \delta\,|-\rangle\,, \quad |-\mathbf{n}\rangle = \bar{\gamma}\,|+\rangle + \bar{\delta}\,|-\rangle\,. \qquad \alpha\bar{\alpha}^* + \beta\bar{\beta}^* = \gamma\bar{\gamma} + \delta\bar{\delta}^* = 0$$

- Consider the beam setting $(e^-, e^+) = (+, \mathbf{m})$

$$\rho_0^{(+,\mathbf{m})} = |+\rangle\langle+| \otimes |\mathbf{m}\rangle\langle\mathbf{m}|$$

$$= |\alpha|^2|++\rangle\langle++| + \alpha\beta^*|++\rangle\langle+-| + \alpha^*\beta|+-\rangle\langle++| + |\beta|^2|--\rangle\langle--|$$

$\mathcal{I}_x$

- $\varrho_x^{(+,\mathbf{m})}$ — measure this time
- $\varrho_x^{(+,+)}$ — already reconstructed
- $\mathcal{I}_x(|++\rangle\langle+-|)$ — target
- $\mathcal{I}_x(|+-\rangle\langle++|)$ — target
- $\varrho_x^{(-,-)}$ — already reconstructed

- Reconstruction of **off**-diagonal elements:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

- Consider polarisations ***NOT*** in the direction of the beam:

$$|\mathbf{m}\rangle = \alpha|+\rangle + \beta|-\rangle\,, \quad |-\mathbf{m}\rangle = \bar{\alpha}|+\rangle + \bar{\beta}|-\rangle\,, \qquad |\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1$$

$$|\mathbf{n}\rangle = \gamma|+\rangle + \delta|-\rangle\,, \quad |-\mathbf{n}\rangle = \bar{\gamma}|+\rangle + \bar{\delta}|-\rangle\,. \qquad \alpha\bar{\alpha}^* + \beta\bar{\beta}^* = \gamma\bar{\gamma} + \delta\bar{\delta}^* = 0$$

- Consider the beam setting $(e^-, e^+) = (+, \mathbf{m})$

$$\rho_0^{(+,\mathbf{m})} = |+\rangle\langle+| \otimes |\mathbf{m}\rangle\langle\mathbf{m}|$$

$$= |\alpha|^2|++\rangle\langle++| + \alpha\beta^*|++\rangle\langle+-| + \alpha^*\beta|+-\rangle\langle++| + |\beta|^2|--\rangle\langle--|$$

$\mathcal{I}_x$ ↓

$\varrho_x^{(+,\mathbf{m})}$ — measure this time

$\varrho_x^{(+,+)}$ — already reconstructed

$\mathcal{I}_x(|++\rangle\langle+-|)$ — target

$\mathcal{I}_x(|+-\rangle\langle++|)$ — target

$\varrho_x^{(-,-)}$ — already reconstructed

- With another beam setting $(e^-, e^+) = (+, \mathbf{n})$

$$\begin{pmatrix} \varrho_x^{(+,\mathbf{m})} - |\alpha|^2\varrho_x^{(+,+)} - |\beta|^2\varrho_x^{(+,-)} \\ \varrho_x^{(+,\mathbf{n})} - |\gamma|^2\varrho_x^{(+,+)} - |\delta|^2\varrho_x^{(+,-)} \end{pmatrix} = \begin{pmatrix} \alpha\beta^* & \alpha^*\beta \\ \gamma\delta^* & \gamma^*\delta \end{pmatrix}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle+-|) \\ \mathcal{I}_x(|+-\rangle\langle++|) \end{pmatrix}$$

12 polarisation settings

$$(+,\mathbf{m}),(+,\mathbf{n}),(-,\mathbf{m}),(-,\mathbf{n})$$
$$(\mathbf{m},+),(\mathbf{n},+),(\mathbf{m},-),(\mathbf{n},-)$$
$$(\mathbf{m},-\mathbf{m}),(\mathbf{m},\mathbf{n}),(\mathbf{n},-\mathbf{m}),(\mathbf{n},-\mathbf{n})$$

All 12 off-diagonal elements can be reconstructed

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

12 polarisation settings

$$(+,\mathbf{m}),(+,\mathbf{n}),(-,\mathbf{m}),(-,\mathbf{n})$$
$$(\mathbf{m},+),(\mathbf{n},+),(\mathbf{m},-),(\mathbf{n},-)$$
$$(\mathbf{m},-\mathbf{m}),(\mathbf{m},\mathbf{n}),(\mathbf{n},-\mathbf{m}),(\mathbf{n},-\mathbf{n})$$

All 12 off-diagonal elements can be reconstructed

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

Powerful probe of BSM

confront with the SM prediction

$$\widetilde{\mathcal{I}}_x = \begin{pmatrix} I_x^{(++,++)} & 0 & 0 & I_x^{(++,--)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(--,++)} & 0 & 0 & I_x^{(--,--)} \end{pmatrix}$$

$$\frac{dI^{(++,++)}}{d\Omega} = \begin{pmatrix} a_{11}^{(+)} s_\theta^2 & a_{12}^{(+)} s_\theta(1+c_\theta) & a_{13}^{(+)} s_\theta(1-c_\theta) & a_{14}^{(+)} s_\theta^2 \\ a_{21}^{(+)} s_\theta(1+c_\theta) & a_{22}^{(+)}(1+c_\theta)^2 & a_{23}^{(+)} s_\theta^2 & a_{24}^{(+)} s_\theta(1+c_\theta) \\ a_{31}^{(+)} s_\theta(1-c_\theta) & a_{32}^{(+)} s_\theta^2 & a_{33}^{(+)}(1-c_\theta)^2 & a_{34}^{(+)} s_\theta(1-c_\theta) \\ a_{41}^{(+)} s_\theta^2 & a_{42}^{(+)} s_\theta(1+c_\theta) & a_{43}^{(+)} s_\theta(1-c_\theta) & a_{44}^{(+)} s_\theta^2 \end{pmatrix}$$

$$a^{(+)}\big|_{\sqrt{s}=370\,\mathrm{GeV}} = \begin{pmatrix} 0.503 & -1.442 & 0.364 & 0.503 \\ -1.442 & 4.137 & -1.043 & -1.442 \\ 0.364 & -1.043 & 0.263 & 0.364 \\ 0.503 & -1.442 & 0.364 & 0.503 \end{pmatrix} \cdot 10^{-2}$$

12 polarisation settings

$$(+,\mathbf{m}),(+,\mathbf{n}),(-,\mathbf{m}),(-,\mathbf{n})$$
$$(\mathbf{m},+),(\mathbf{n},+),(\mathbf{m},-),(\mathbf{n},-)$$
$$(\mathbf{m},-\mathbf{m}),(\mathbf{m},\mathbf{n}),(\mathbf{n},-\mathbf{m}),(\mathbf{n},-\mathbf{n})$$

All 12 off-diagonal elements can be reconstructed

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

Powerful probe of BSM

Foundational test of QM

confront with the SM prediction

$$\widetilde{\mathcal{I}}_x = \begin{pmatrix} I_x^{(++,++)} & 0 & 0 & I_x^{(++,--)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(--,++)} & 0 & 0 & I_x^{(--,--)} \end{pmatrix}$$

$$\frac{dI^{(++,++)}}{d\Omega} = \begin{pmatrix} a_{11}^{(+)} s_\theta^2 & a_{12}^{(+)} s_\theta(1+c_\theta) & a_{13}^{(+)} s_\theta(1-c_\theta) & a_{14}^{(+)} s_\theta^2 \\ a_{21}^{(+)} s_\theta(1+c_\theta) & a_{22}^{(+)}(1+c_\theta)^2 & a_{23}^{(+)} s_\theta^2 & a_{24}^{(+)} s_\theta(1+c_\theta) \\ a_{31}^{(+)} s_\theta(1-c_\theta) & a_{32}^{(+)} s_\theta^2 & a_{33}^{(+)}(1-c_\theta)^2 & a_{34}^{(+)} s_\theta(1-c_\theta) \\ a_{41}^{(+)} s_\theta^2 & a_{42}^{(+)} s_\theta(1+c_\theta) & a_{43}^{(+)} s_\theta(1-c_\theta) & a_{44}^{(+)} s_\theta^2 \end{pmatrix}$$

$$a^{(+)}\big|_{\sqrt{s}=370\,\mathrm{GeV}} = \begin{pmatrix} 0.503 & -1.442 & 0.364 & 0.503 \\ -1.442 & 4.137 & -1.043 & -1.442 \\ 0.364 & -1.043 & 0.263 & 0.364 \\ 0.503 & -1.442 & 0.364 & 0.503 \end{pmatrix} \cdot 10^{-2}$$

- Check if all eigenvalues are non-negative

  → **Positivity Test**

- Confront the prediction with the experimental outcome

$$\rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j| \rightarrow \varrho_x = \sum_{i,j} \rho_{ij} \mathcal{I}_x(|i\rangle\langle j|)$$

$\updownarrow$ confront with experiment

→ **Linearity Test**

12 polarisation settings

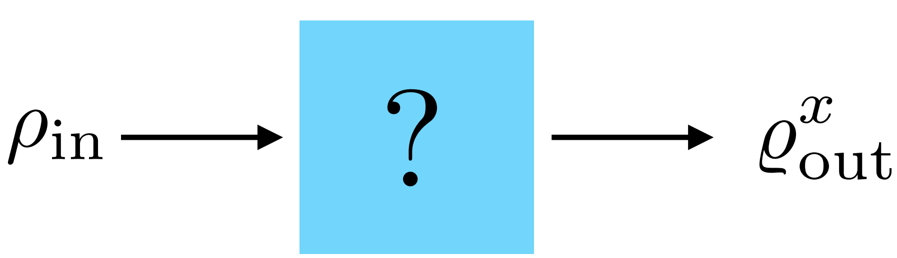
$$(+,\mathbf{m}),(+,\mathbf{n}),(-,\mathbf{m}),(-,\mathbf{n})$$
$$(\mathbf{m},+),(\mathbf{n},+),(\mathbf{m},-),(\mathbf{n},-)$$
$$(\mathbf{m},-\mathbf{m}),(\mathbf{m},\mathbf{n}),(\mathbf{n},-\mathbf{m}),(\mathbf{n},-\mathbf{n})$$

All 12 off-diagonal elements can be reconstructed

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

Foundational test of QM

Test of quantum dynamics



complimentary to Bell tests

- Check if all eigenvalues are non-negative

  → **Positivity Test**

- Confront the prediction with the experimental outcome

$$\rho_0 = \sum_{i,j} \rho_{ij}|i\rangle\langle j| \rightarrow \varrho_x = \sum_{i,j}\rho_{ij}\mathcal{I}_x(|i\rangle\langle j|)$$

↕ confront with experiment

→ **Linearity Test**

Summary

- We formulated **spin-to-spin transitions** in particle collisions as **Quantum Instrument** — an quantum evolution of subsystem in an environment (environment = momentum Hilbert space).

- The state-to-state map can be described by Choi matrix, enabling us to write a theory in a matrix form. 16 x 16 matrix for 2-qubit -> 2-qubit

- Experimental reconstruction of the Choi matrix (Quantum Process Tomography) offers :

  ▶ A powerful probe of BSM physics

  Two theories that predicting different spin-spin transition necessarily give different Choi matrices

  ▶ Foundational tests of Quantum Mechanics

  - **Linearity test:** confront the Choi matrix prediction with the measurement

  - **Positivity test:** eigenvalues of the Choi matrix must be non-negative

If one of these tests fails, QM will be experimentally falsified!

Thank you for listening!

Quantum State Tomography

More generally,

$$\frac{d\Gamma}{d\Omega} = \frac{1 + \alpha_x \cdot (\vec{x} \cdot \mathbf{s})}{2}$$

$\alpha_x \in [-1, +1]$: spin analyzing power

- tau decay

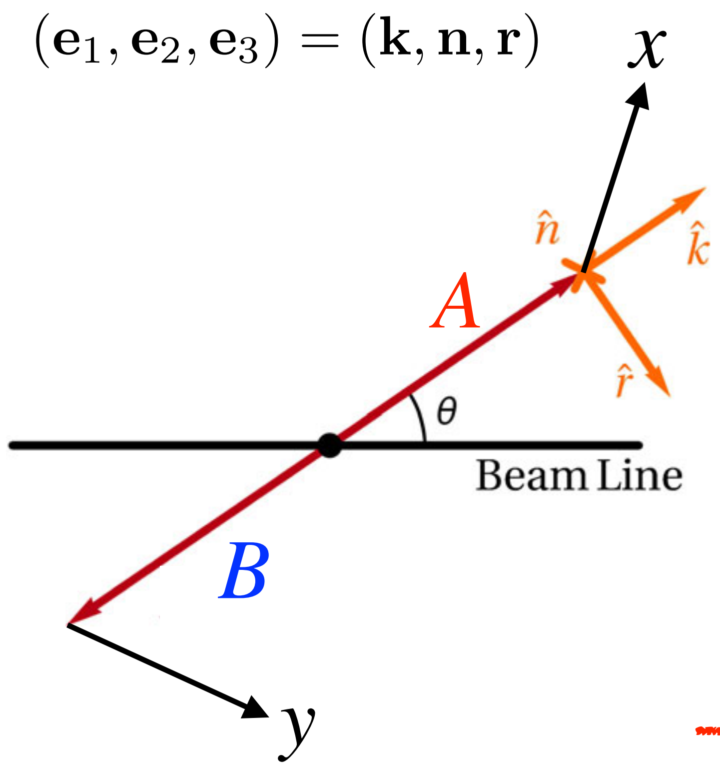
$\alpha_x = 1$ for ($x = \pi^-$ in $\tau^- \to \pi^- \nu$)

- top decay

decay product x	α_x
b	$-0.3925(6)$
W^+	$0.3925(6)$
ℓ^+ (from a W^+)	$0.999(1)$
$\bar{d}$, $\bar{s}$ (from a W^+)	$0.9664(7)$
u, c (from a W^+)	$-0.3167(6)$

helicity basis

$(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3) = (\mathbf{k}, \mathbf{n}, \mathbf{r})$



$$B_i = \langle \mathbf{s}^A \cdot \mathbf{e}_i \rangle = \frac{3}{\alpha_x} \langle \mathbf{x} \cdot \mathbf{e}_i \rangle$$

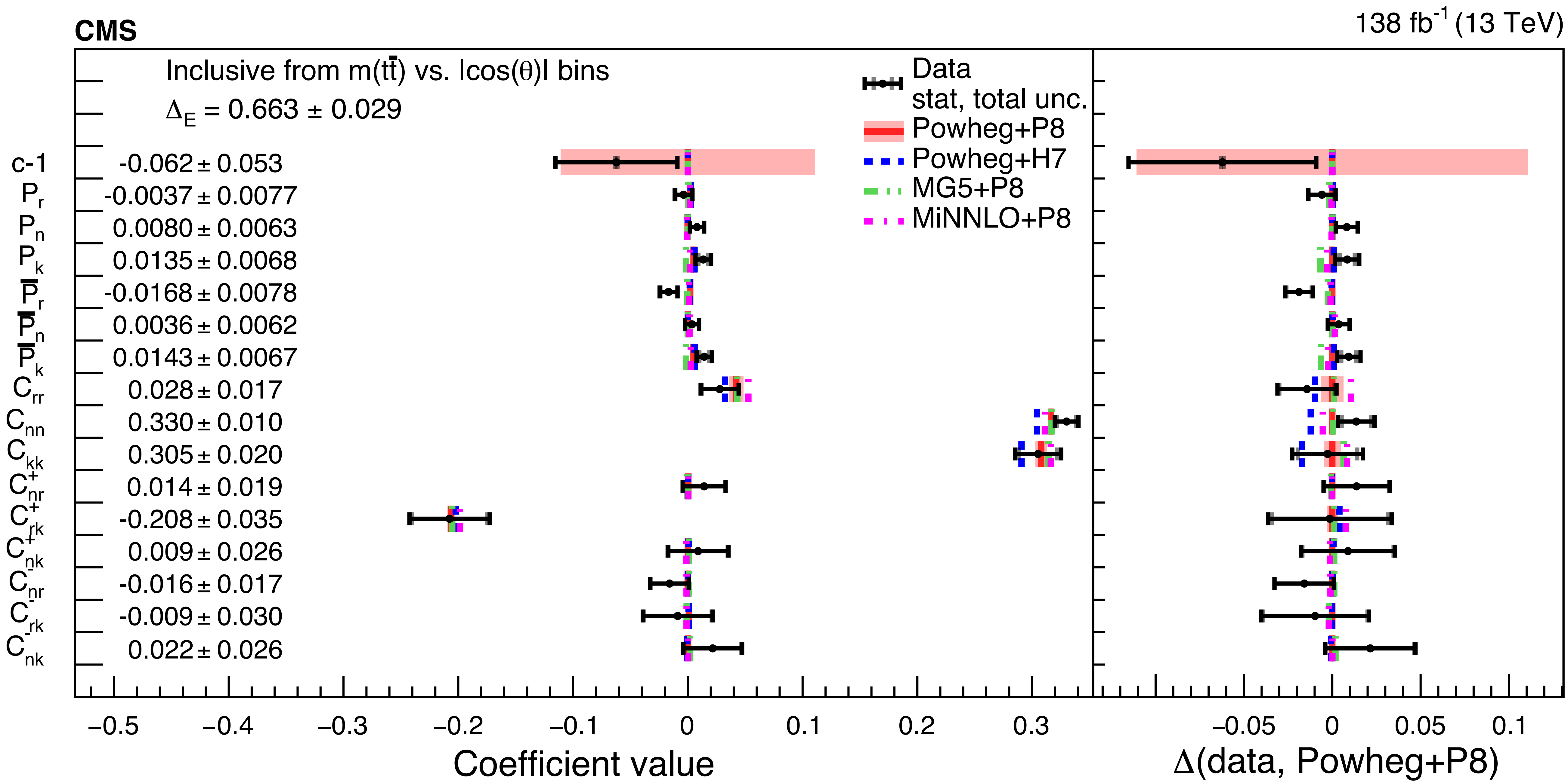
$$\overline{B}_i = \langle \mathbf{s}^B \cdot \mathbf{e}_i \rangle = \frac{3}{\alpha_y} \langle \mathbf{y} \cdot \mathbf{e}_i \rangle$$

$$C_{ij} = \langle (\mathbf{s}^A \cdot \mathbf{e}_i)(\mathbf{s}^B \cdot \mathbf{e}_j) \rangle = \frac{9}{\alpha_x \alpha_y} \langle (\mathbf{x} \cdot \mathbf{e}_i)(\mathbf{y} \cdot \mathbf{e}_j) \rangle$$

observable

$$\rho = \frac{1}{4}\Big(\mathbf{1}_{AB} + B_i \cdot [\sigma^i \otimes \mathbf{1}_B] + \overline{B}_i \cdot [\mathbf{1}_A \otimes \sigma^i] + C_{ij} \cdot [\sigma^i \otimes \sigma^j] \Big)$$

[2409.11067]



CMS
138 fb^{-1} (13 TeV)
Inclusive from m(tt̄) vs. |cos(θ)| bins
Δ_E = 0.663 ± 0.029
Data
stat, total unc.
Powheg+P8
Powheg+H7
MG5+P8
MiNNLO+P8
c-1 -0.062 ± 0.053
P_r -0.0037 ± 0.0077
P_n 0.0080 ± 0.0063
P_k 0.0135 ± 0.0068
$\bar{P}_r$ -0.0168 ± 0.0078
$\bar{P}_n$ 0.0036 ± 0.0062
$\bar{P}_k$ 0.0143 ± 0.0067
C_{rr} 0.028 ± 0.017
C_{nn} 0.330 ± 0.010
C_{kk} 0.305 ± 0.020
C^+_{nr} 0.014 ± 0.019
C^+_{rk} -0.208 ± 0.035
C^+_{nk} 0.009 ± 0.026
C^-_{nr} -0.016 ± 0.017
C^-_{rk} -0.009 ± 0.030
C^-_{nk} 0.022 ± 0.026
−0.5 −0.4 −0.3 −0.2 −0.1 0 0.1 0.2 0.3
Coefficient value
−0.05 0 0.05 0.1
Δ(data, Powheg+P8)

Nonlocal correlations beyond quantum mechanics

Quantum bound on CHSH correlations [Tsirelson (1980)]

$$S = C_{\mathrm{QM}}(x,y) + C_{\mathrm{QM}}(x,y') + C_{\mathrm{QM}}(x',y) - C_{\mathrm{QM}}(x',y') \leq 2\sqrt{2} < 4$$

Could we have $S = 4$ *assuming* free choice and no-signalling? Yes, we can!

No-signalling boxes [Popescu, Rohrlich (1994)]

$$P(a,b\,|\,x,y) = \begin{cases} \frac{1}{2}, & \text{if } a \oplus b = xy, \\ 0, & \text{otherwise,} \end{cases} \qquad S_{\mathrm{PR}} = 4.$$

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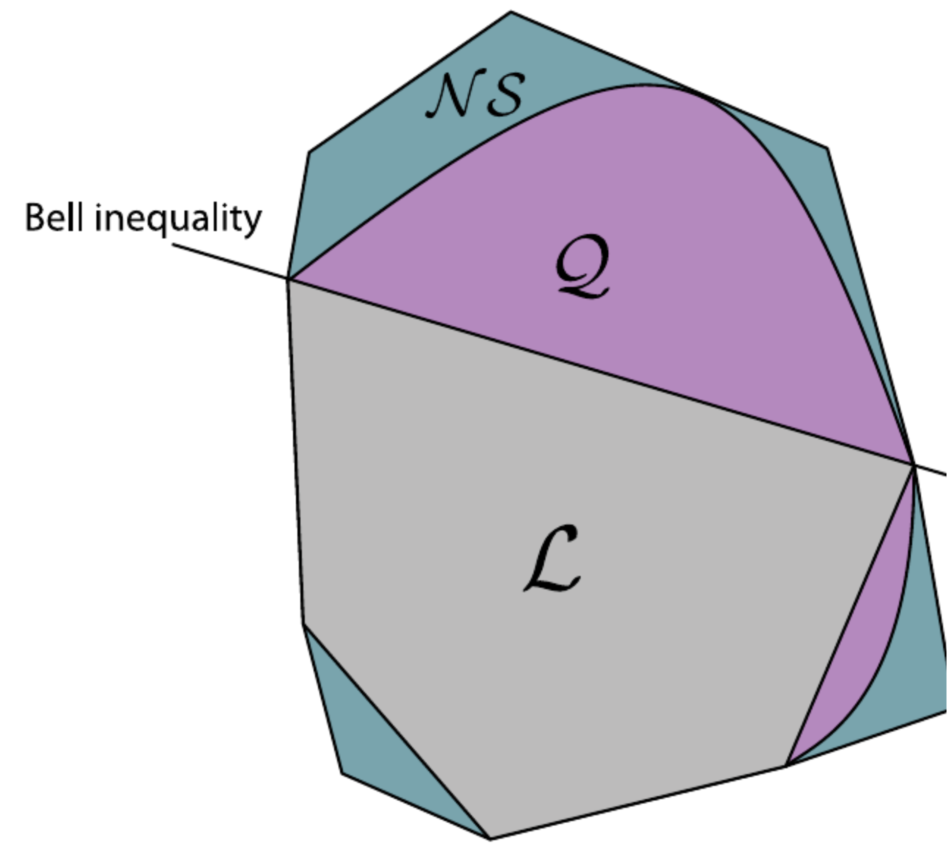
Nonlocality beyond quantum mechanics

Sandu Popescu

- No-signalling principle admits correlations that are **stronger than entanglement**.



[N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, *Rev. Mod. Phys.* **86**, 419 (2014)]

- For not perfectly polarised beams,

$$\rho_{e^-}^{(\omega^-)} = \frac{1}{2}(1+\omega^- q)|+\rangle\langle+| + \frac{1}{2}(1-\omega^- q)|-\rangle\langle-|$$

$$\rho_{e^+}^{(\omega^+)} = \frac{1}{2}(1+\omega^+ \bar{q})|+\rangle\langle+| + \frac{1}{2}(1-\omega^+ \bar{q})|-\rangle\langle-|$$

$$0 < q, \bar{q} < 1$$

$$(\omega^-, \omega^+) = \{(+,+), (+,-), (-,+), (-,-)\}$$

4 beam settings

$$\rho_0^{(\omega^-,\omega^+)} = \rho_{e^-}^{(\omega^-)} \otimes \rho_{e^+}^{(\omega^+)} = \frac{1}{4}\Big[(1+\omega^- q)(1+\omega^+ \bar{q})|++\rangle\langle++| + (1+\omega^- q)(1-\omega^+ \bar{q})|+-\rangle\langle+-|$$
$$+(1-\omega^- q)(1+\omega^+ \bar{q})|-+\rangle\langle-+| + (1-\omega^- q)(1-\omega^+ \bar{q})|--\rangle\langle--|\Big]$$

input beam state

convex linear sum of four pure states

- For not perfectly polarised beams,

$$\rho_{e^-}^{(\omega^-)} = \frac{1}{2}(1+\omega^- q)|+\rangle\langle+| + \frac{1}{2}(1-\omega^- q)|-\rangle\langle-|$$

$$\rho_{e^+}^{(\omega^+)} = \frac{1}{2}(1+\omega^+ \bar{q})|+\rangle\langle+| + \frac{1}{2}(1-\omega^+ \bar{q})|-\rangle\langle-|$$

$$0 < q, \bar{q} < 1$$

$$(\omega^-, \omega^+) = \underbrace{\{(+,+),(+,-),(-,+),(-,-)\}}_{\text{4 beam settings}}$$

$$\rho_0^{(\omega^-,\omega^+)} = \rho_{e^-}^{(\omega^-)} \otimes \rho_{e^+}^{(\omega^+)} = \frac{1}{4}\Big[(1+\omega^- q)(1+\omega^+\bar{q})|++\rangle\langle++| + (1+\omega^- q)(1-\omega^+\bar{q})|+-\rangle\langle+-|$$
$$+(1-\omega^- q)(1+\omega^+\bar{q})|-+\rangle\langle-+| + (1-\omega^- q)(1-\omega^+\bar{q})|--\rangle\langle--|\Big]$$

input beam state

convex linear sum of four pure states

$$\begin{pmatrix} \rho_0^{(+,+)} \\ \rho_0^{(+,-)} \\ \rho_0^{(-,+)} \\ \rho_0^{(-,-)} \end{pmatrix} = \frac{1}{4}\begin{pmatrix} (1+q)(1+\bar{q}) & (1+q)(1-\bar{q}) & (1-q)(1+\bar{q}) & (1-q)(1-\bar{q}) \\ (1+q)(1-\bar{q}) & (1+q)(1+\bar{q}) & (1-q)(1-\bar{q}) & (1-q)(1+\bar{q}) \\ (1-q)(1+\bar{q}) & (1-q)(1-\bar{q}) & (1+q)(1+\bar{q}) & (1+q)(1-\bar{q}) \\ (1-q)(1-\bar{q}) & (1-q)(1+\bar{q}) & (1+q)(1-\bar{q}) & (1+q)(1+\bar{q}) \end{pmatrix} \begin{pmatrix} |++\rangle\langle++| \\ |+-\rangle\langle+-| \\ |-+\rangle\langle-+| \\ |--\rangle\langle--| \end{pmatrix}$$

- For not perfectly polarised beams,

$$\rho_{e^-}^{(\omega^-)} = \frac{1}{2}(1+\omega^- q)|+\rangle\langle+| + \frac{1}{2}(1-\omega^- q)|-\rangle\langle-|$$

$$\rho_{e^+}^{(\omega^+)} = \frac{1}{2}(1+\omega^+ \bar{q})|+\rangle\langle+| + \frac{1}{2}(1-\omega^+ \bar{q})|-\rangle\langle-|$$

$$0 < q, \bar{q} < 1$$

$$(\omega^-, \omega^+) = \{(+,+), (+,-), (-,+), (-,-)\}$$

4 beam settings

$$\rho_0^{(\omega^-,\omega^+)} = \rho_{e^-}^{(\omega^-)} \otimes \rho_{e^+}^{(\omega^+)} = \frac{1}{4}\Big[(1+\omega^- q)(1+\omega^+\bar{q})|++\rangle\langle++| + (1+\omega^- q)(1-\omega^+\bar{q})|+-\rangle\langle+-|$$

$$+(1-\omega^- q)(1+\omega^+\bar{q})|-+\rangle\langle-+| + (1-\omega^- q)(1-\omega^+\bar{q})|--\rangle\langle--|\Big]$$

input beam state

convex linear sum of four pure states

$$\mathcal{I}_x\left[\begin{pmatrix}\rho_0^{(+,+)}\\ \rho_0^{(+,-)}\\ \rho_0^{(-,+)}\\ \rho_0^{(-,-)}\end{pmatrix} = \frac{1}{4}\begin{pmatrix}(1+q)(1+\bar{q}) & (1+q)(1-\bar{q}) & (1-q)(1+\bar{q}) & (1-q)(1-\bar{q})\\ (1+q)(1-\bar{q}) & (1+q)(1+\bar{q}) & (1-q)(1-\bar{q}) & (1-q)(1+\bar{q})\\ (1-q)(1+\bar{q}) & (1-q)(1-\bar{q}) & (1+q)(1+\bar{q}) & (1+q)(1-\bar{q})\\ (1-q)(1-\bar{q}) & (1-q)(1+\bar{q}) & (1+q)(1-\bar{q}) & (1+q)(1+\bar{q})\end{pmatrix}\begin{pmatrix}|++\rangle\langle++|\\ |+-\rangle\langle+-|\\ |-+\rangle\langle-+|\\ |--\rangle\langle--|\end{pmatrix}\right]$$

diag. entries of Choi matrix

$$\begin{pmatrix}\mathcal{I}_x(|++\rangle\langle++|)\\ \mathcal{I}_x(|+-\rangle\langle+-|)\\ \mathcal{I}_x(|-+\rangle\langle-+|)\\ \mathcal{I}_x(|--\rangle\langle--|)\end{pmatrix} = \Big(\qquad\Big)^{-1}\begin{pmatrix}\mathcal{I}_x(\rho_0^{(+,+)})\\ \mathcal{I}_x(\rho_0^{(+,-)})\\ \mathcal{I}_x(\rho_0^{(-,+)})\\ \mathcal{I}_x(\rho_0^{(-,-)})\end{pmatrix}$$

measurable