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Quantum Tomography in Flavor Oscillations

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Based on work in preparation
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Quantum correlations in flavor space

- Particles can be entangled in flavor space, as informative as spin
  - E.g., meson pair decayed from $\Upsilon(J^{PC} = 1^{--})$

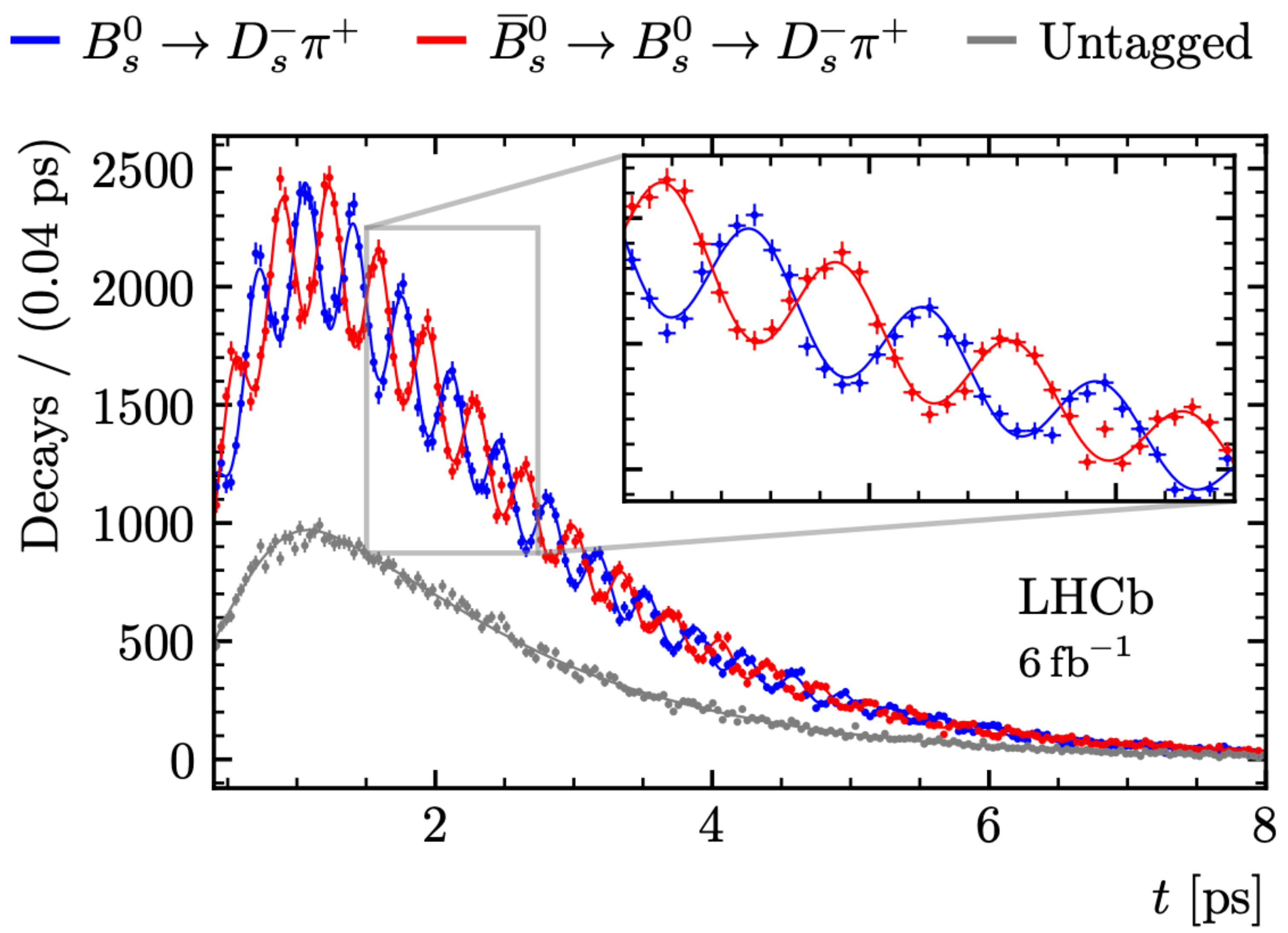
$$\frac{1}{\sqrt{2}}(|B^0\bar{B}^0\rangle - |\bar{B}^0 B^0\rangle)$$

  - Vast number of events

$$\text{e.g., } B_s\bar{B}_s\text{: } 7\times 10^6 \text{ at Belle, } 8\times 10^8 \text{ at LHCb}$$

$$K_0\bar{K}_0\text{: } 8\times 10^9 \text{ at KLOE and KLOE-2, etc}$$

  - Precise measurement of flavor oscillation and correlation.



Decay time can be constructed very well!
[LHCb, 2104.04421]

- Quantum correlation, Bell inequality, quantum decoherence, etc have been studied in BB at Belle, KK at KLOE… [Belle, hep-ph/0702267; KLOE, hep-ph/0607027]

- Reconstruct the complete **flavor density matrix**?

$$\rho_{MM} = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}}\sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}}\mathbb{1}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

Flavor density matrix of MM pair when they are produced, i.e., at $t = 0$

Introduction: Quantum tomography

— from a complementary set of measurements

- One qubit: 3 operators

$$\rho = \frac{\mathbb{1}_2 + R_i\sigma_i}{2}, \quad R_i = \langle\sigma_i\rangle$$

- Two qubit: 3+3+9 operators

$$\rho = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}}\sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}}\mathbb{1}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

$$R_i^{\mathcal{A}} = \langle\sigma_i \otimes \mathbb{1}_2\rangle$$
$$R_i^{\mathcal{B}} = \langle\mathbb{1}_2 \otimes \sigma_i\rangle$$
$$C_{ij} = \langle\sigma_i \otimes \sigma_j\rangle$$

- Spin-1/2 particle as qubit:
  - $\vec{\sigma}$ is embeded in 3d spatial space.
  - Different direction of $\vec{\sigma}$ $\implies$ complementary.
  - Collider envoronments: infer $\langle\sigma_i\rangle$ from decay
- **What about qubit in flavor space**?
  - An obvious challenge: cannot choose“*direction*” of measurement freely?
  - e.g. flavor tagging, only σ_z: $\sigma_z\,|B^0\rangle = |B^0\rangle\,,\ \ \sigma_z\,|\bar{B}^0\rangle = -\,|\bar{B}^0\rangle$

Introduction: Flavor/Mass eigenstate

- Hamitonian in flavor eigenstate: $|M\rangle, |\bar{M}\rangle$: $H = \mathbf{M} - i\mathbf{\Gamma}/2 = \begin{pmatrix} m - i\frac{\Gamma}{2} & P^2 \\ Q^2 & m - i\frac{\Gamma}{2} \end{pmatrix},$
- Mass eigenstate is

$$|M_1\rangle = p|M\rangle + q|\bar{M}\rangle, \qquad (m_1, \Gamma_1)$$
$$|M_2\rangle = p|M\rangle - q|\bar{M}\rangle, \qquad (m_2, \Gamma_2)$$

- CP conserving case: $|M_{1/2}\rangle \propto |M\rangle \pm |\bar{M}\rangle$

difference between flavor and mass eigenstate leads to flavor oscillation

- $\rho_M = \frac{\mathbb{1}_2 + R_i \sigma_i}{2}$, R_i still have physical meaning; conversions are:
  - z-direction $\Longrightarrow$ flavor eigenstate, (oscillates with y) $\quad |M\rangle, R_z = 1$; $|\bar{M}\rangle, R_z = -1$
  - x-direction $\Longrightarrow$ mass eigenstates (and CP eigenstates) $\quad |M_1\rangle, R_x = 1$; $|M_2\rangle, R_x = -1$

State evolution in Bloch-vector space

$$\rho(t) \propto U(t)\rho U(t)^\dagger \qquad U = \begin{pmatrix} \frac{1}{2}(e^{-\Gamma_1 t/2 - im_1 t} + e^{-\Gamma_2 t/2 - im_2 t}) & \frac{q}{2p}(e^{-\Gamma_1 t/2 - im_1 t} - e^{-\Gamma_2 t/2 - im_2 t}) \\ \frac{p}{2q}(e^{-\Gamma_1 t/2 - im_1 t} - e^{-\Gamma_2 t/2 - im_2 t}) & \frac{1}{2}(e^{-\Gamma_1 t/2 - im_1 t} + e^{-\Gamma_2 t/2 - im_2 t}) \end{pmatrix}$$

- State evolution (Operator evolution)
  - Neglect CPV and decay ($\Gamma \ll \Delta m$):

Schrödinger picture

density matrix $\rho = \frac{I_2 + \vec{R}\cdot\vec{\sigma}}{2}$

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] = \left(\vec{X} \times \vec{R}(t)\right) \cdot \vec{\sigma},$$
$$\frac{d\vec{R}(t)}{dt} = \vec{X} \times \vec{R}(t).$$

Heisenberg picture

operator $A = \vec{a} \cdot \vec{\sigma}$

$$\frac{dA(t)}{dt} = i[H, A(t)] = -\left(\vec{X} \times \vec{a}(t)\right) \cdot \vec{\sigma},$$
$$\frac{d\vec{a}(t)}{dt} = -\vec{X} \times \vec{a}(t).$$

  - Precession around $\vec{X} = (\Delta m, 0, 0)$
- Example operator $\sigma_z |M\rangle = +|M\rangle, \quad \sigma_z |\bar{M}\rangle = -|\bar{M}\rangle$
  - Flavor tagging at different times $\Longrightarrow$ both σ_z and σ_y

Collapse the superposition

$$\rho_M = \frac{\mathbb{1}_2 + R_i\sigma_i}{2}$$

- Two kinds of decay final states, CP eigenstate or not
  - $M \to f$ with $CP|f\rangle = |\bar{f}\rangle \neq |f\rangle$: project to $|M\rangle, |\bar{M}\rangle$ with $P_{M/\bar{M}} = \frac{1 \pm R_z}{2}$
  - $M \to f_\eta$ with $CP|f_\eta\rangle = \eta|f_\eta\rangle$, $\eta = \pm 1$: project to $|M_1\rangle, |M_2\rangle$ with $P_{M_1/M_2} = \frac{1 \pm R_x}{2}$

$\mathrm{Br}(M \to f_{\eta_{CP}})$		$\mathrm{Br}(M \to f)$	
$B_d^0 \to J/\psi K_S$	$(8.91 \pm 0.21) \times 10^{-4}$	$B_d^0 \to \ell^+\nu_\ell X^-$	$(20.66 \pm 0.56)\%$
$B_s^0 \to J/\psi\eta$	$(3.9 \pm 0.7) \times 10^{-4}$	$B_s^0 \to \ell^+\nu_\ell X^-$	$(19.2 \pm 1.6)\%$

Collapse the superposition

- Two kinds of decay final states, CP eigenstate or not
  - $M \to f$ with $CP|f\rangle = |\bar{f}\rangle \neq |f\rangle$: project to $|M\rangle, |\bar{M}\rangle$ with $P_{M/\bar{M}} = \frac{1 \pm R_z}{2}$
- Decay rate asymmetry to $f, \bar{f}$:

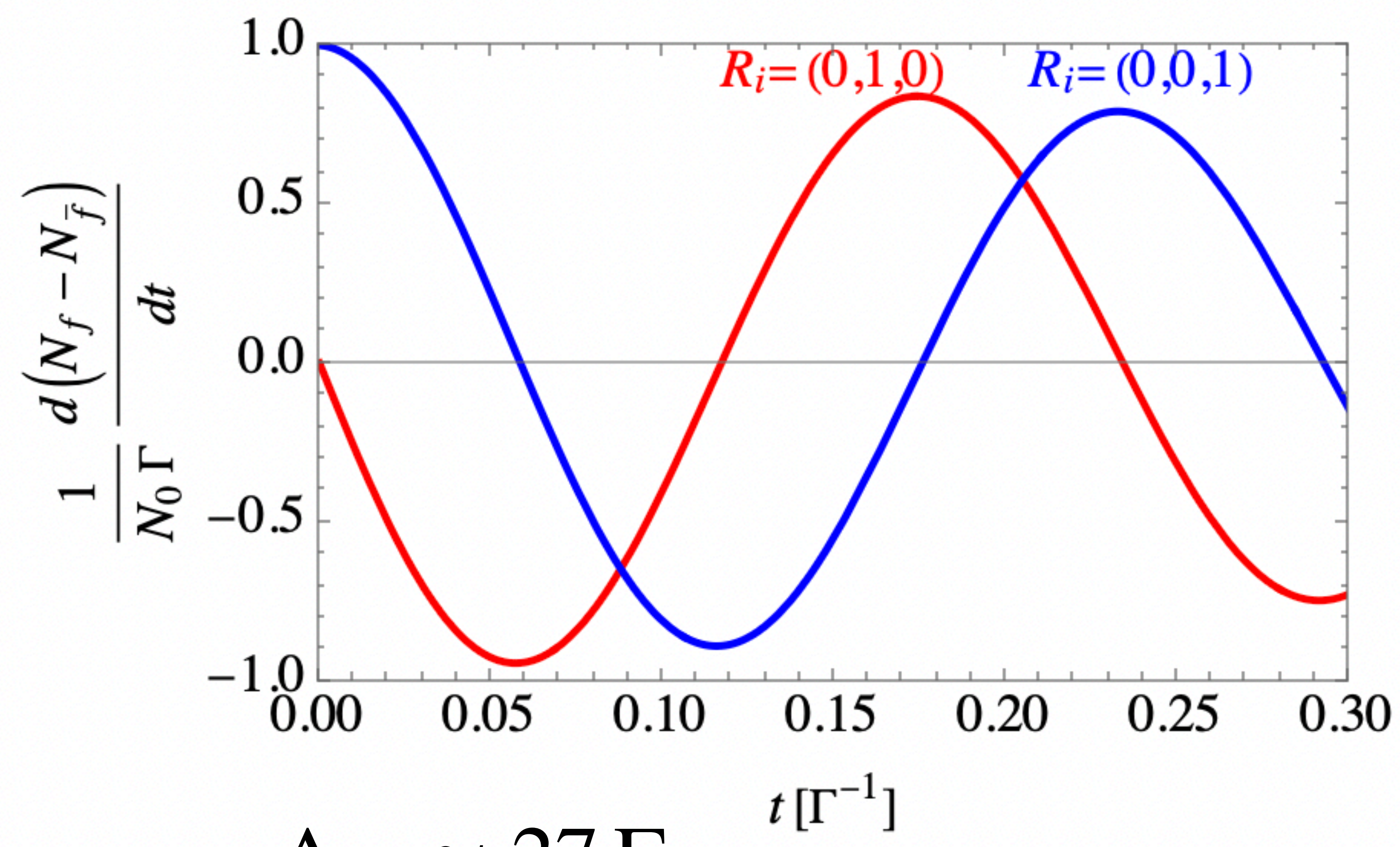
$$\Gamma_{M(t)\to f/\bar{f}} = \frac{1 \pm R_z(t)}{2}\Gamma_{M\to f} \qquad (N_f - N_{\bar{f}}) \sim \langle \sigma_z \rangle = R_z$$

- Observable:

$$\frac{1}{N_0}\frac{d(N_f - N_{\bar{f}})}{dt} = e^{-\Gamma t}\left(R_z \cos(\Delta m t) - R_y \sin(\Delta m t)\right)\Gamma_{M\to f}$$

Meson flavor state when it is produced, $t = 0$

- Both R_y and R_z are obtained as they oscillated into each other.



$\Delta m \approx 27\,\Gamma$,
slightly damped oscillation

Collapse the superposition

— x direction in the Bloch vector space

- Conside a meson that only decay to flavor eigenstate $|f\rangle$ (such as semileptonic) or CP-even eigenstate $|f_+\rangle$

$$\Gamma_{M(t)\to f_+} = \frac{1+R_x(t)}{2}\Gamma_{M_1\to f_+} = (1+R_x(t))\Gamma_{M\to f_+}$$

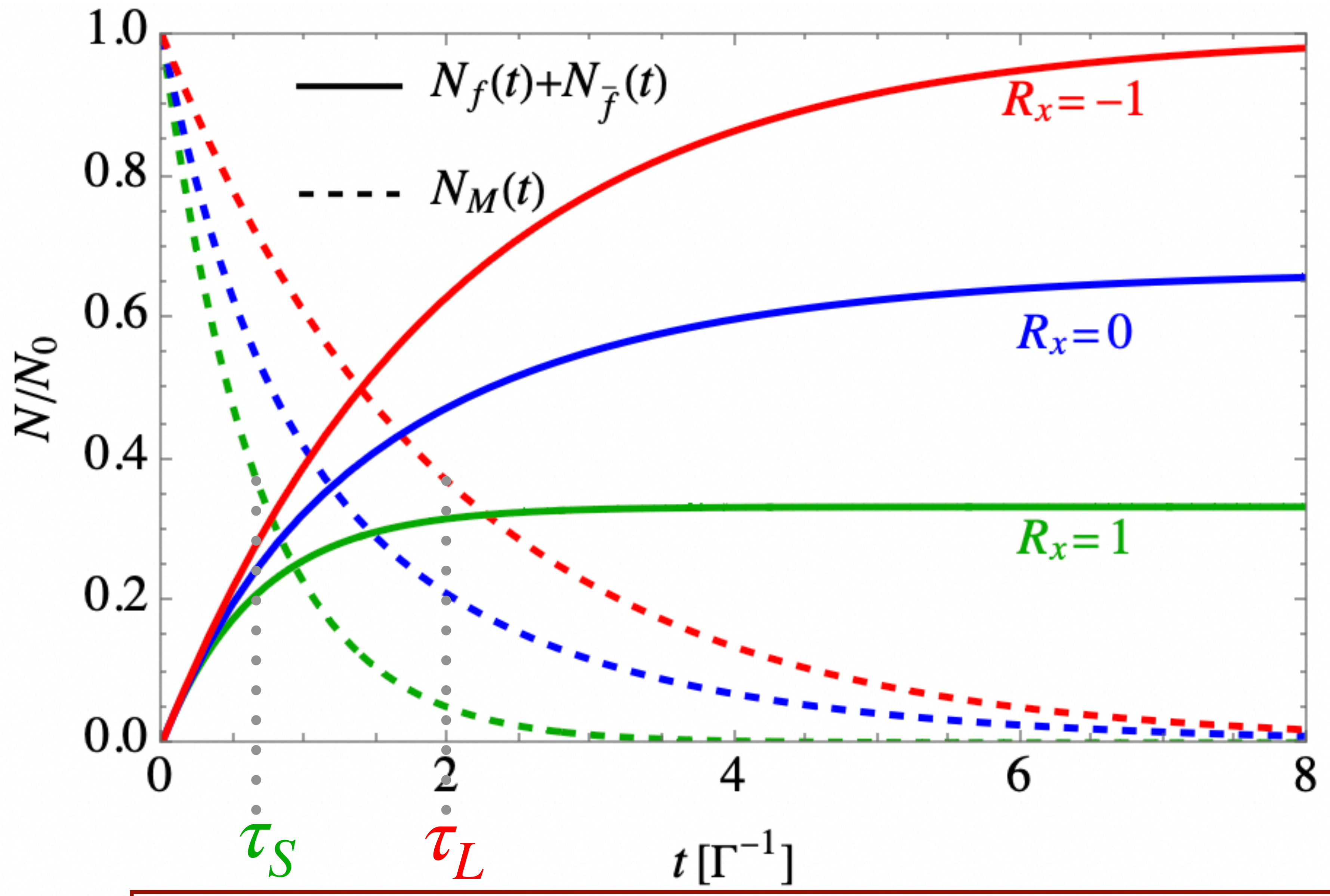
- Both $M(t)\to f_+$ and $M(t)\to f/\bar{f}$ depend on R_x

affected by branching fraction

  - $R_x = 1$, M_1 can decay to f_+
  - $R_x = -1$, M_2 can't decay to f_+, more f

$$\frac{1}{N_0}\frac{d(N_f+N_{\bar{f}})}{dt} = e^{-\Gamma t}(\cosh(\Delta\Gamma t/2) - R_x\sinh(\Delta\Gamma t/2))\Gamma_{M\to f}$$

- Semi-leptonic channel is enough. e.g. $B_s \to \ell^+\nu_\ell X + h.c.$



difference between each lines $\propto$ $\Delta\Gamma$
i.e., decay to CP eigenstates.

Observables in semileptonic decay channel

- Reconstruct ρ_{MM} at $t = 0$.

$$\rho_{MM} = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}} \sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}} \mathbb{1}_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

- One meson:
  - $N_f - N_{\bar{f}} \Longrightarrow R_y, R_z$
  - $N_f + N_{\bar{f}} \Longrightarrow R_x$

$$\frac{1}{N_0} \frac{d(N_f - N_{\bar{f}})}{dt} = e^{-\Gamma t} \left(R_z \cos(\Delta m t) - R_y \sin(\Delta m t) \right) \Gamma_{M \to f}$$

$$\frac{1}{N_0} \frac{d(N_f + N_{\bar{f}})}{dt} = e^{-\Gamma t} \left(\cosh(\Delta \Gamma t/2) - R_x \sinh(\Delta \Gamma t/2) \right) \Gamma_{M \to f}$$

- Meson pair:
  - Four observables from the correlation between the above two

$\mathscr{H}_A \otimes \mathscr{H}_B$	$N_{f\bar{f}}$: meson A decay to f **and** meson B decay to $\bar{f}$
$I_2 \otimes I_2$ $\longrightarrow$	$N_{\text{tot}} = N_{ff} + N_{\bar{f}f} + N_{f\bar{f}} + N_{\bar{f}\bar{f}}$
$\sigma_z \otimes \sigma_z$ $\longrightarrow$	$A_{ff} = N_{ff} - N_{\bar{f}f} - N_{f\bar{f}} + N_{\bar{f}\bar{f}} = N_{\text{like}} - N_{\text{unlike}}$
$\sigma_z \otimes I_2$ $\longrightarrow$	$A_f^{\mathcal{A}} = N_{ff} + N_{f\bar{f}} - N_{\bar{f}f} - N_{\bar{f}\bar{f}}$
$I_2 \otimes \sigma_z$ $\longrightarrow$	$A_f^{\mathcal{B}} = N_{ff} - N_{f\bar{f}} + N_{\bar{f}f} - N_{\bar{f}\bar{f}}$

Observables in semileptonic decay channel

- Reconstruct ρ_{MM} at $t = 0$.

$$\rho_{MM} = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}}\sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}}\mathbb{1}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

- One meson:
  - $N_f - N_{\bar{f}} \Longrightarrow R_y, R_z$
  - $N_f + N_{\bar{f}} \Longrightarrow R_x$

$$\frac{1}{N_0}\frac{d(N_f - N_{\bar{f}})}{dt} = e^{-\Gamma t}\left(R_z \cos(\Delta m t) - R_y \sin(\Delta m t)\right)\Gamma_{M\to f}$$

$$\frac{1}{N_0}\frac{d(N_f + N_{\bar{f}})}{dt} = e^{-\Gamma t}(\cosh(\Delta\Gamma t/2) - R_x \sinh(\Delta\Gamma t/2))\Gamma_{M\to f}$$

- Meson pair:
  - Four observables from the correlation between the above two

$$\frac{\mathrm{d}N_{\mathrm{tot}}}{\mathrm{d}t_1\mathrm{d}t_2} = N_0\Gamma^2_{B_0\to f}\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_1}\mathrm{ch}_{t_2} - \mathrm{ch}_{t_1}\mathrm{sh}_{t_2} R_x^{\mathcal{A}} - \mathrm{sh}_{t_1}\mathrm{ch}_{t_2} R_x^{\mathcal{B}} + C_{xx}\mathrm{sh}_{t_1}\mathrm{sh}_{t_2}\right) + \mathcal{O}(\epsilon)$$

$$\frac{\mathrm{d}A_{ff}}{\mathrm{d}t_1\mathrm{d}t_2} = N_0\Gamma^2_{B_0\to f}\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{s}_{t_1}\mathrm{s}_{t_2} C_{yy} - \mathrm{c}_{t_1}\mathrm{s}_{t_2} C_{zy} - \mathrm{s}_{t_1}\mathrm{c}_{t_2} C_{yz} + \mathrm{c}_{t_1}\mathrm{c}_{t_2} C_{zz}\right) + \mathcal{O}(\epsilon)$$

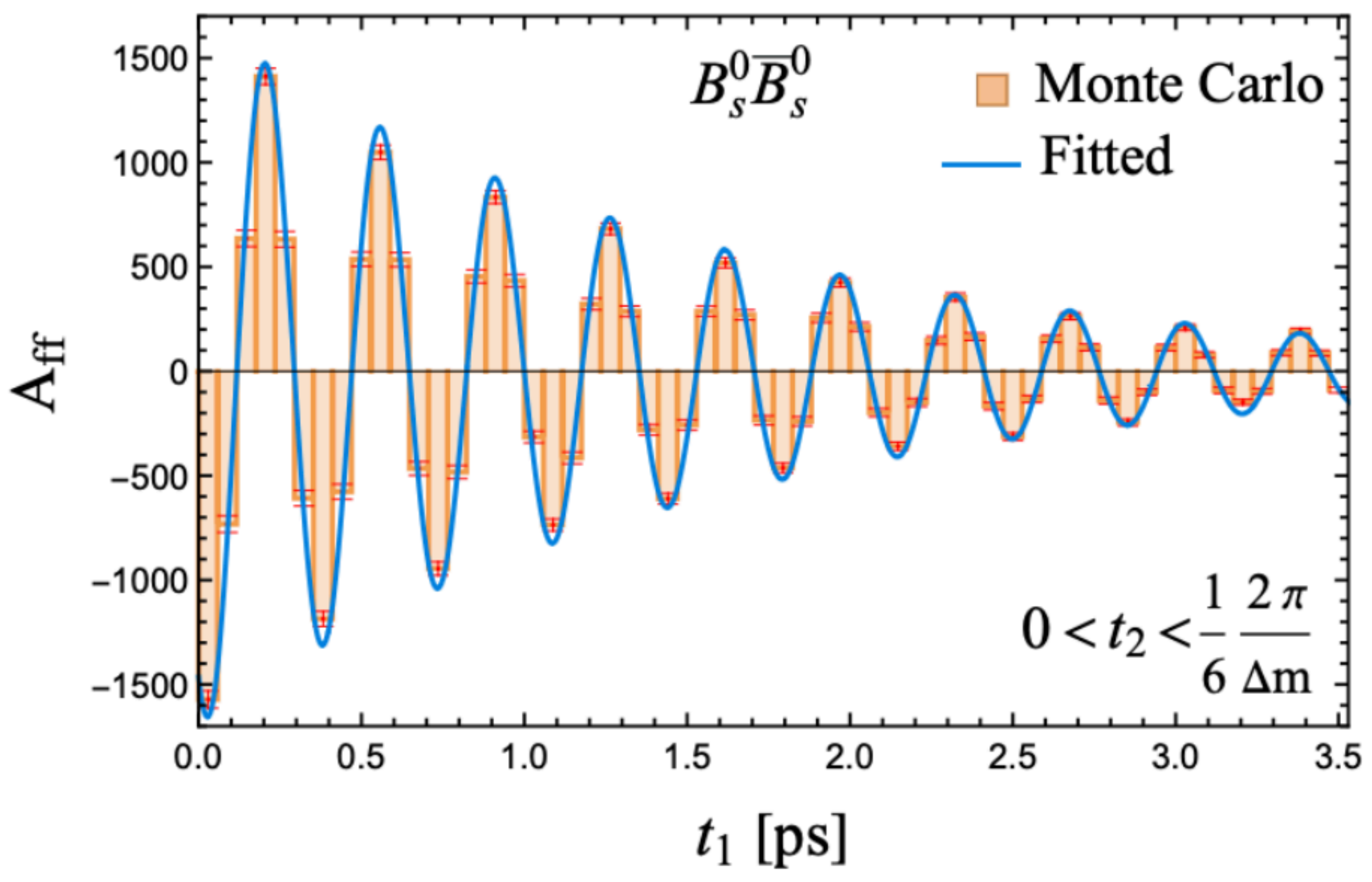
$$\frac{\mathrm{d}A_f^{\mathcal{A}}}{\mathrm{d}t_1\mathrm{d}t_2} = N_0\Gamma^2_{B_0\to f}\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_2}(\mathrm{c}_{t_1} R_z^{\mathcal{A}} - \mathrm{s}_{t_1} R_y^{\mathcal{A}}) - \mathrm{sh}_{t_2}(\mathrm{c}_{t_1} C_{zx} - \mathrm{s}_{t_1} C_{yx})\right) + \mathcal{O}(\epsilon)$$

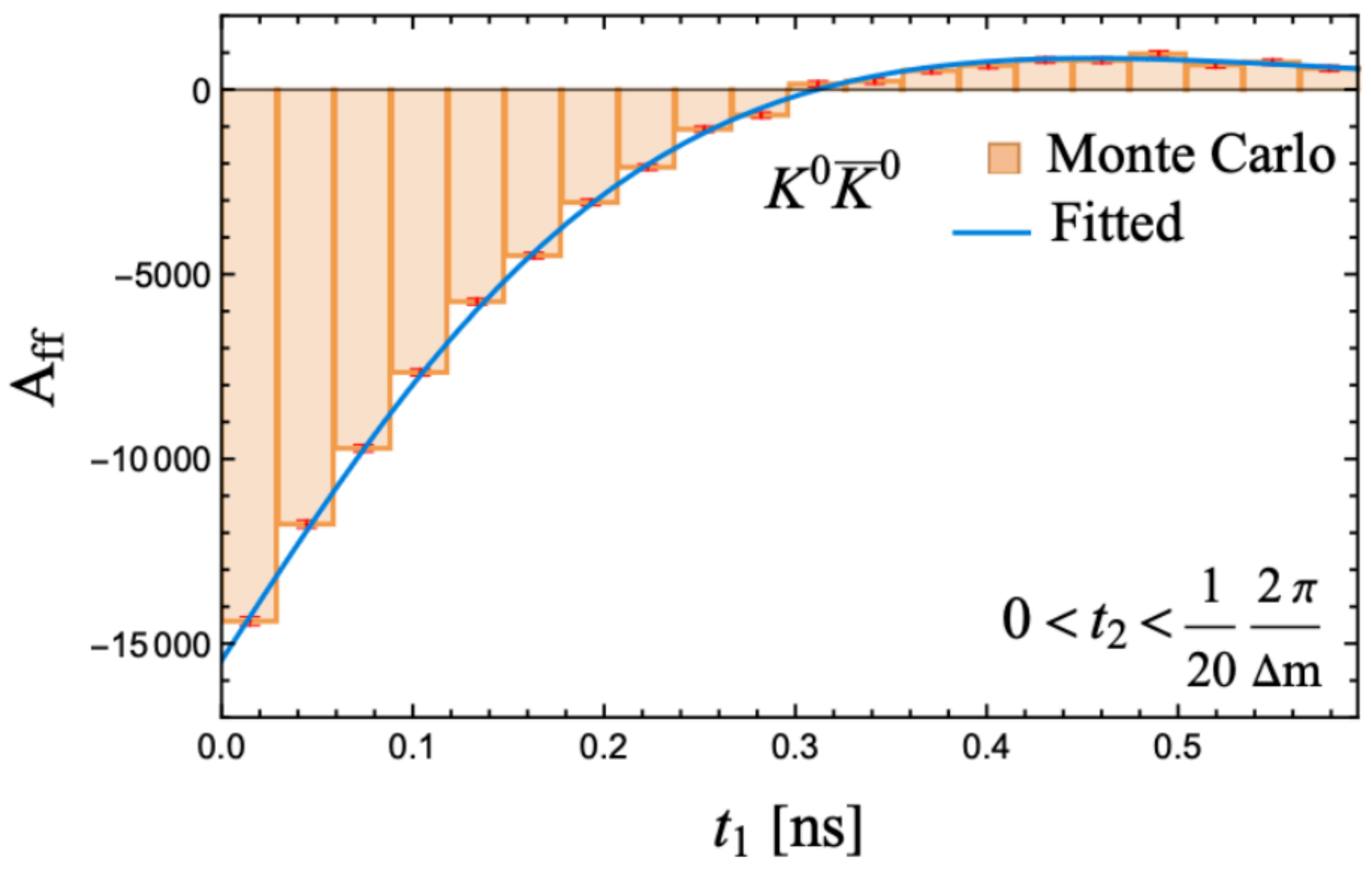
$$\frac{\mathrm{d}A_f^{\mathcal{B}}}{\mathrm{d}t_1\mathrm{d}t_2} = N_0\Gamma^2_{B_0\to f}\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_1}(\mathrm{c}_{t_2} R_z^{\mathcal{B}} - \mathrm{s}_{t_2} R_y^{\mathcal{B}}) - \mathrm{sh}_{t_1}(\mathrm{c}_{t_2} C_{xz} - \mathrm{s}_{t_2} C_{xy})\right) + \mathcal{O}(\epsilon)$$

$\epsilon \sim 10^{-3} - 10^{-5}$

Sensitivity estimation

- We consider a C-odd flavor state $(|M\bar{M}\rangle - |\bar{M}M\rangle)/\sqrt{2}$
  - Analogy to spin singlet $R_i = 0, \quad C_{ij} = -\delta_{ij}$
- B_s pair: x-direction is hard to measure, y and z are the same
- K_0 pair: y-direction slightly worse than z





	$B_s^0\bar{B}_s^0$ fitted	$K^0\bar{K}^0$ fitted	Obs.
$R_x^{\mathcal{A}}$	-0.01 ± 0.06	-0.002 ± 0.006	$N_{\rm tot}$
$R_x^{\mathcal{B}}$	-0.01 ± 0.06	-0.003 ± 0.006	
$R_y^{\mathcal{A}}$	0.000 ± 0.003	0.005 ± 0.006	$A_f^{\mathcal{A}}$
$R_z^{\mathcal{A}}$	0.000 ± 0.003	0.003 ± 0.005	
$R_y^{\mathcal{B}}$	0.000 ± 0.003	0.005 ± 0.006	$A_f^{\mathcal{B}}$
$R_z^{\mathcal{B}}$	0.001 ± 0.003	0.002 ± 0.004	
C_{xx}	-1.2 ± 1.0	-1.005 ± 0.012	$N_{\rm tot}$
C_{yx}	0.00 ± 0.06	0.005 ± 0.008	$A_f^{\mathcal{A}}$
C_{zx}	0.00 ± 0.05	0.006 ± 0.006	
C_{xy}	0.00 ± 0.05	0.006 ± 0.007	$A_f^{\mathcal{B}}$
C_{xz}	0.00 ± 0.05	0.004 ± 0.006	
C_{yy}	-1.001 ± 0.004	-1.003 ± 0.008	A_{ff}
C_{yz}	0.001 ± 0.003	0.000 ± 0.007	
C_{zy}	0.000 ± 0.003	0.000 ± 0.006	
C_{zz}	-1.000 ± 0.003	-1.001 ± 0.003	
Concurrence	1.1 ± 0.5	1.005 ± 0.007	

Tab. Statistical uncertainty with 10^6 events

- B_sB_s: $10^5 \sim 10^6$ at LHCb (HL-LHC)
- KK: 8×10^9 at KLOE and KLOE-2, a fraction of 10^{-5} decay to lepton in the first period we consider.

Conclusion

- The complete flavor density matrix of meson pair can be reconstructed!
  - We can ask about concurrence, Bell, discord and magic, etc.
  - Larger $\Delta m/\Gamma$, better sensitvity on y, z components.
  - Larger $\Delta\Gamma/\Gamma$, better senstivity on x components

	B_s^0	B_d^0	D^0	K^0
$\Delta m/\text{ps}^{-1}$	17.76	0.506	9.2×10^{-3}	5.29×10^{-3}
Γ/ps^{-1}	0.662	0.658	2.44	5.59×10^{-3}
$\Delta\Gamma/\text{ps}^{-1}$	0.082	2.6×10^{-3}	0.030	0.0111

- Discussion

[LHCb plots]

  - Belle ⟷ (complementary) LHCb
    - LHCb: focuses on one meson, but a large fraction of $b\bar{b}$ are both in detectable region
    - Belle: C-odd state of meson pair
    - LHCb: not very clear about the flavor state, so interesting to measure
  - B_s at Belle, decay life time can also be constructed now (see poster of Timothy Mahood)
  - B_d: nothing new to do for now, but we would have much more events at Belle-II (aim to collect 5×10^{10}).
  - CP violation, more information, more precision, combine different channel, etc.