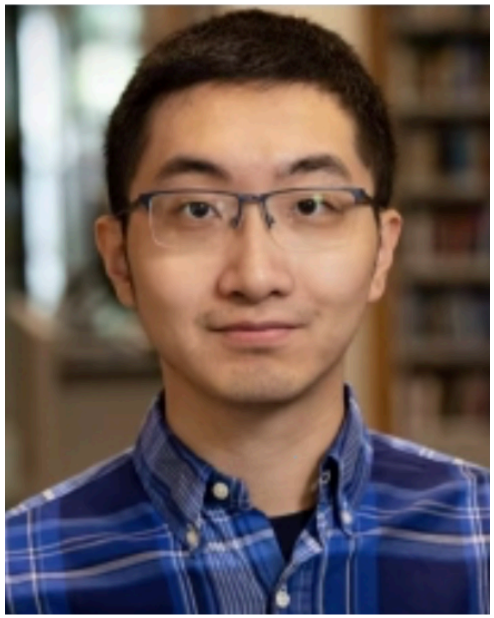
QUANTUM MAGIC

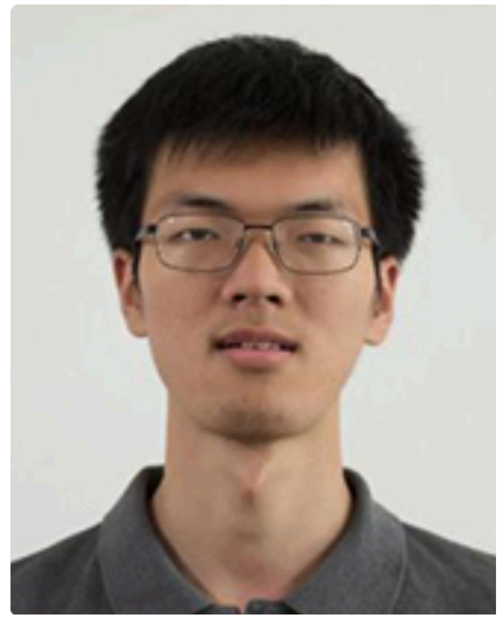
IN QUANTUM ELECTRODYNAMICS

- Ian Low
- Argonne/Northwestern

- GGI Workshop on Quantum Observables for Collider Physics
- April 8, 2025

Based on works collaborated with:





Zhewei Yin

(moving across the pond to Louvain in the Fall)

Qiaofeng Liu

in arXiv: 2502.17550; 2503.03098.

- Identifying and characterizing quantum resources are among the most important topics in the quantum era:

  Entanglement is the most prominent example.

- Not all quantum resources provide computational advantages over classical algorithms (Gottesman-Knill theorem):

  Need a second layer of Quantumness– the magic (non-stabilizerness)

- Magic is an essential ingredient for universal quantum computation. (Bravyi and Kitaev: quant-ph/0403025)

What is the Question?

- Basic forces in nature are known to generate quantum resources such as entanglement easily and abundantly.

- What about computational advantages? How well can fundamental interactions generate quantum advantages?

- Is the Quantum Advantage built into the fundamental physics in the UV or is it an emergent phenomenon in the IR?

What is the Question?

- Basic forces in nature are known to generate quantum resources such as entanglement easily and abundantly.

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As a starting point, we consider the ability of QED to generate magic states in 2-to-2 scatterings of electrons and muons, starting from an initial state with zero magic.

The Stabilizer Formalism:

- The Pauli gropu G_n for n-qubit:

$$G_n = \{\phi \, P_1 \otimes P_2 \otimes \cdots \otimes P_n \mid P_i \in \{I, X, Y, Z\} \text{ and } \phi \in \{\pm 1, \pm i\} \ \}$$

$$I = \sigma^0, \ X = \sigma^1, \ Y = \sigma^2 \text{ and } Z = \sigma^3$$

- A "Stabilizer" state is an eigenstate of some elements of G_n :

$$g|\psi\rangle = |\psi\rangle \ , \qquad g \in G_n$$

Such *g's* form an abelian subgroup called the "Stabilizer Group."

The maximal stabilizer group *S* of each stabilizer state is unique!

- For n-qubit, the maximal stabilizer group *S* has 2^n elements but only *n* generators, whose products generate *S*.

- A unitary operation *U* on a stabilizer state is another stabilizer state:

$$U|\psi\rangle = Ug|\psi\rangle = UgU^\dagger U|\psi\rangle$$

  whose stabilizer group is $USU^\dagger$.

- Instead of specify 2^n amplitudes of $U|\psi\rangle$, one can simply specify the *n* generators of $USU^\dagger$.

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This is the essence of Gottesman-Knill theorem and why the stabilizer states can be simulated efficiently using classical algorithms!

- The stabilizer formalism is particularly powerful when applying to the "Clifford gates," which consists of the Hadamard gate, the Phase gate and the Controlled-Not gate:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

These are the generators of the Clifford Group, which is the normalizer of the Pauli group:

$$N(G_n) = \{U \mid UG_nU^\dagger = G_n\}$$

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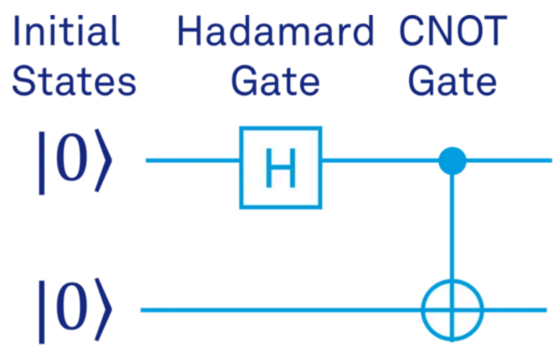
Gottesman-Knill theorem states that quantum circuits involving Clifford gates and stabilizer states can be simulated efficiently using classical computers.

- Clifford gates and stabilizer states are heavily utilized in quantum computing, because they are sufficient for generating highly entangled states such as the Bell states:



$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

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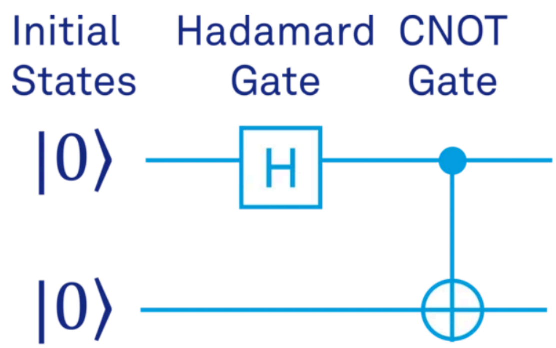

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

- Fundamental interactions are known the generate maximal entanglement abundantly in 2-to-2 scatterings:

Maximal Entanglement in High Energy Physics 1703.02989

Alba Cervera-Lierta,[1] José I. Latorre,[1,2] Juan Rojo,[3] and Luca Rottoli[4]
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[2] *Center for Quantum Technologies, National University of Singapore.*
[3] *Department of Physics and Astronomy, VU University, De Boelelaan 1081, 1081HV Amsterdam, and Nikhef, Science Park 105, NL-1098 XG Amsterdam, The Netherlands*
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(Dated: November 27, 2017)

- However, Clifford gates and stabilizer states are NOT universal – they are not able to approximate all unitary transformations.

Clifford gate + magic states are universal

- However, Clifford gates and stabilizer states are NOT universal – they are not able to approximate all unitary transformations.

Clifford gate + magic states are universal

- Stabilizer states by definition have zero magic. For 2-q system, there are 60 stabilizer states:

$$|\psi\rangle = c_1|\uparrow\uparrow\rangle + c_2|\uparrow\downarrow\rangle + c_3|\downarrow\uparrow\rangle + c_4|\downarrow\downarrow\rangle.$$

Order #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c_1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
c_2	0	0	1	0	1	-1	-1	1	i	$-i$	$-i$	i	i	-1	$-i$	1	$-i$	-1	i	1
c_3	0	0	0	1	1	-1	1	-1	i	$-i$	i	$-i$	1	i	-1	$-i$	1	$-i$	-1	i
c_4	0	1	0	0	1	1	-1	-1	-1	-1	1	1	i	$-i$	i	$-i$	$-i$	i	$-i$	i
Order #	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c_1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1	1	1
c_2	$-i$	1	0	0	i	1	0	0	1	1	0	0	-1	1	0	0	0	0	0	0
c_3	0	0	1	i	0	0	1	$-i$	0	0	1	-1	0	0	1	1	0	0	0	0
c_4	0	$-i$	i	0	0	i	$-i$	0	0	1	-1	0	0	-1	1	0	1	-1	i	$-i$
Order #	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c_1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
c_2	1	1	1	1	-1	-1	1	1	$-i$	i	-1	$-i$	-1	i	$-i$	i	1	1	$-i$	i
c_3	1	-1	i	$-i$	-1	1	-1	1	$-i$	i	$-i$	-1	i	-1	1	1	$-i$	i	i	$-i$
c_4	0	0	0	0	-1	1	1	-1	1	1	$-i$	$-i$	i	i	i	$-i$	i	$-i$	-1	-1

- Among them 24 states are maximally entangled!
- Entanglement does not imply computational advantage!

- There are several quantitative measures of non-stabilizerness – the magic – and we will adopt the Stabilizer Renyi Entropy (SRE):

$$\mathcal{P}_n = \{P_1 \otimes P_2 \otimes \cdots \otimes P_n\}\,, \quad P_i \in \{I, X, Y, Z\}$$

$$\Xi_P(|\psi\rangle) = \frac{1}{d}\,\langle\psi|P|\psi\rangle^2$$

$$\begin{aligned} M_\alpha\left(|\psi\rangle\right) &= \frac{1}{1-\alpha}\log\sum_{P\in\mathcal{P}_n}\Xi_P^\alpha(|\psi\rangle) - \log d \\ &= \frac{1}{1-\alpha}\log\sum_{P\in\mathcal{P}_n}\frac{1}{d}\,\langle\psi|P|\psi\rangle^{2\alpha} \ . \end{aligned}$$

- For a stabilizer state, SRE vanishes because $\Xi_P(|\psi\rangle) = \pm 1$

- We will use the 2nd order SRE:

$$M_2(|\psi\rangle) = -\log \Xi_2(|\psi\rangle)\ , \qquad \Xi_2(|\psi\rangle) \equiv \sum_{P\in\mathcal{P}_n} \frac{\langle\psi|P|\psi\rangle^4}{4}$$

- For 2-q states, the maximal SRE is

$$M_2 \leq \log\frac{16}{7} \approx 0.827$$

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- For 2-q states, the maximal SRE is

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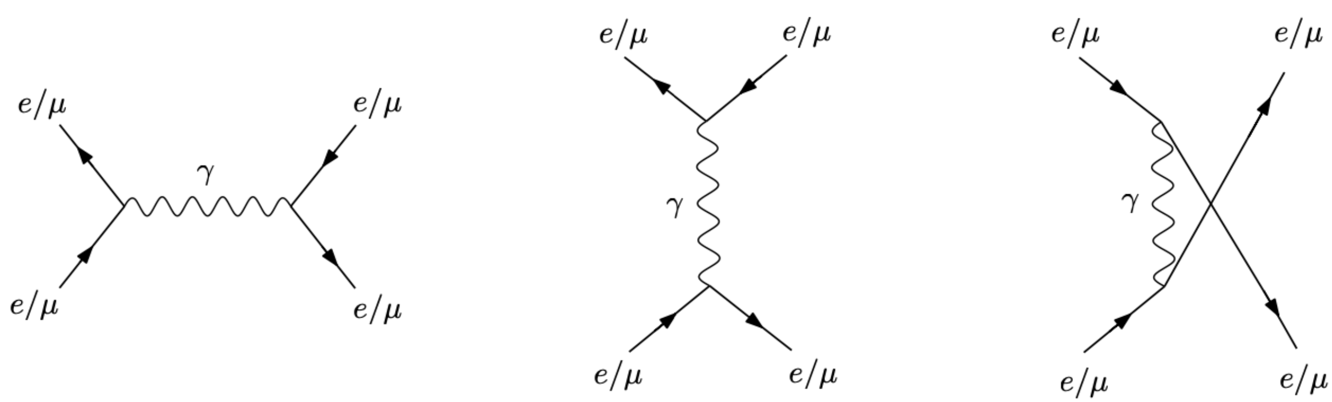
- Our goal is to start from a stabilizer state and compute the final state SRE for QED processes:



(a) (b) (c)

The Setup:

- SRE involves computing the expectation values of Pauli matrices; they are not rotationally invariant and require a choice of basis.

- We choose to project the spin along the beam axis in the CM frame for both initial and final states:

$$\vec{p}_1 + \vec{p}_2 \to \vec{k}_1 + \vec{k}_2$$

$$\hat{z} = \hat{p}_1, \qquad \hat{y} = \frac{\vec{k}_1 \times \vec{p}_1}{|\vec{k}_1 \times \vec{p}_1|}, \qquad \hat{x} = \hat{z} \times \hat{y}$$

- We do not use the helicity basis, which projects the spin along the direction of motion, since we want to compare the magic in initial and final states.

- We consider the following scattering processes, in both the non-relativistic and ultra-relativistic limits:

$$e^- e^+ \to \mu^- \mu^+$$

$$\textbf{Møller scattering } e^- e^- \to e^- e^-$$

$$\textbf{Bhabha scattering } e^- e^+ \to e^- e^+$$

$$e^- \mu^- \to e^- \mu^-$$

$$\mu^- \mu^+ \to e^- e^+$$

- We include all 60 stabilizer states as the initial states and compute the final state magic as a function of the scattering angle θ.

Low Energy Limit: $e^- e^+ \to \mu^- \mu^+$

- Near the kinematic threshold $\sqrt{s} \geq 2m_\mu$ the amplitude only depends on

$$\lambda = \frac{m_e}{m_\mu}\ , \qquad \lambda \approx 0.005$$ in real world

- We compute the magic as a function of λ :

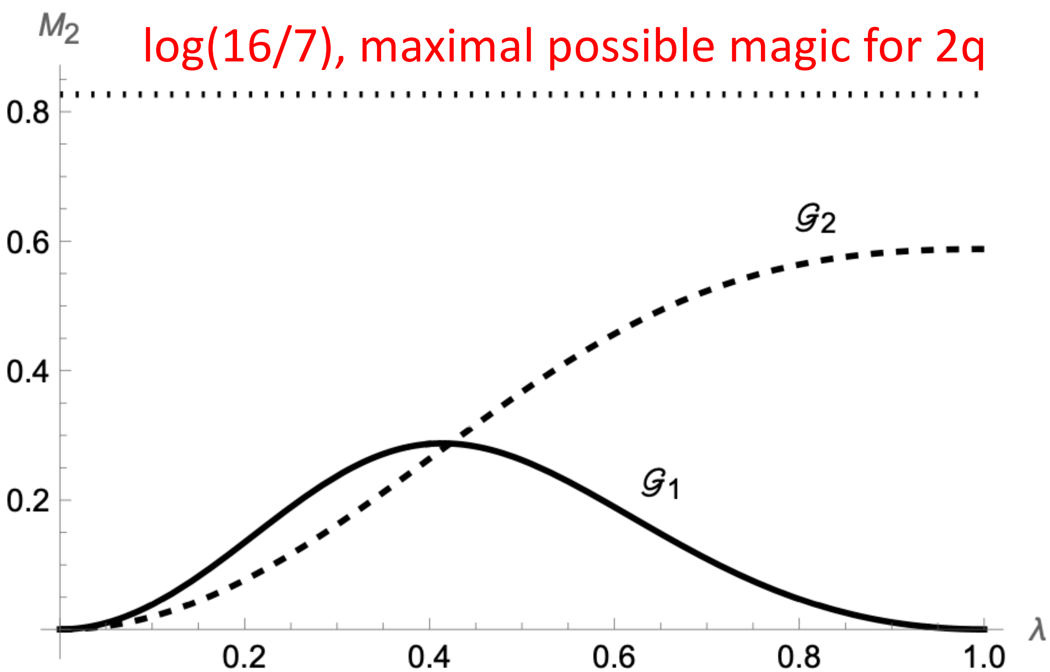
Stabilizer States	Ξ_2
$1, 2, 3, 4, 5, 6, 9, 10, 37, 38, 39, 40, 42, 43, 44, 45, 48, 49, 50$	$\mathcal{F}_1$
$7, 8, 11, 12, 46, 47, 59, 60$	$\mathcal{G}_1$
$13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28,$ $29, 30, 31, 32, 33, 34, 35, 36, 51, 52, 53, 54, 55, 56, 57, 58$	$\mathcal{G}_2$
41	—

$$\mathcal{F}_1 = 1, \qquad \mathcal{G}_1 = \frac{\lambda^8 + 14\lambda^4 + 1}{(\lambda^2+1)^4}, \qquad \mathcal{G}_2 = \frac{\lambda^8 + 28\lambda^4 + 16}{(\lambda^2+2)^4}$$

$$M_2 = \begin{cases} -\log \mathcal{F}_1 = 0 \\ -\log \mathcal{G}_1 \sim 10^{-5} \\ -\log \mathcal{G}_2 \sim 10^{-5} \end{cases}$$

- Using the real world value, the magic produced is practically zero.
- Among the 60 stabilizer initial states, only three different magic is produced.

- We can plot the magic production as a function of λ:



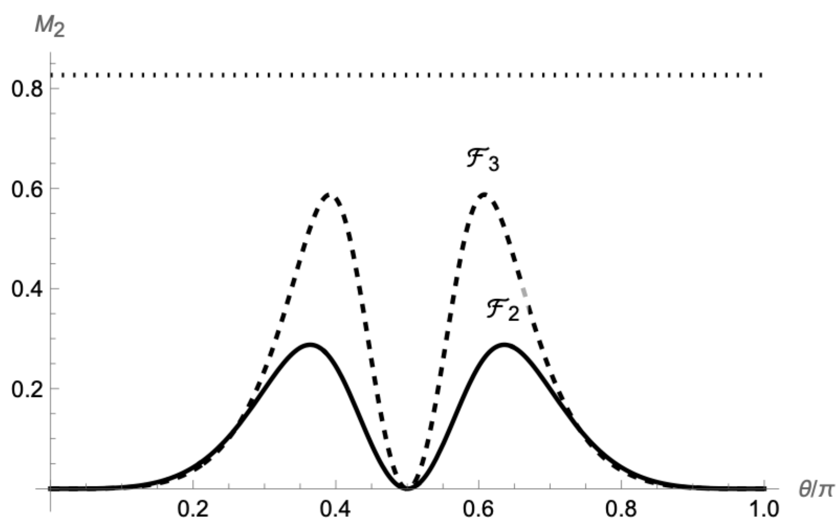
Ξ_2	$(M_2)_{\max}$	$\lambda_{\max}$
$\mathcal{F}_1$	—	—
$\mathcal{G}_1$	$\log(4/3)$	$\sqrt{2}-1$
$\mathcal{G}_2$	$\log(9/5)$	1

- Even if we allow λ to vary, the largest magic produced is significantly less than the maximum value.
- **These observations persist in most other channels:**

Møller scattering $e^-e^- \to e^-e^-$

Stabilizer States	Ξ_2	$(M_2)_{\max}$	$\theta_{\max}$
$1, 2, 5, 6, 9, 10, 37, 38, 39, 40, 41, 42, 45, 48, 49, 50$	$\mathcal{F}_1$	0	Arbitrary
$3, 4, 7, 8, 11, 12, 43, 44, 46, 47, 59, 60$	$\mathcal{F}_2$	$\log(4/3)$	$2\arctan\sqrt{\sqrt{2}-1}$, $\pi - 2\arctan\sqrt{\sqrt{2}-1}$
$13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 51, 52, 53, 54, 55, 56, 57, 58$	$\mathcal{F}_3$	$\log(9/5)$	$\arctan 2\sqrt{2}$, $\pi - \arctan 2\sqrt{2}$

Again only three different magic is produced among the 60 initial states. And we never reach the maximal magic for 2q.



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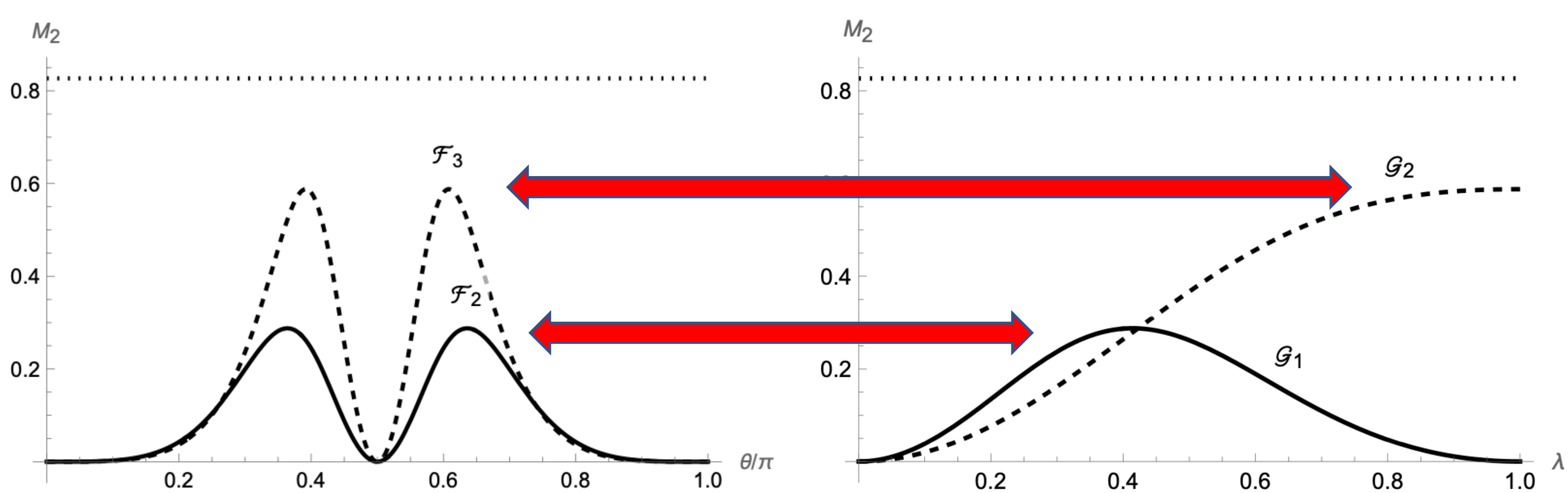
$13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 51, 52, 53, 54, 55, 56, 57, 58$	$\mathcal{F}_3$	$\log(9/5)$	$\arctan 2\sqrt{2}$, $\pi - \arctan 2\sqrt{2}$

Curiously, the largest magic produced are the same as in $ee \to \mu\mu$.



- The magic produced in the low-energy limit vanishes identically for

$$\textbf{Bhabha scattering } e^-e^+ \to e^-e^+$$

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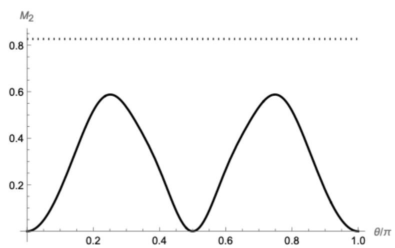
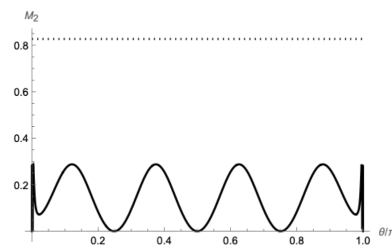
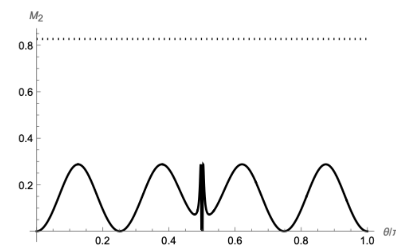
- The most interesting channel is $\mu^-\mu^+ \to e^-e^+$, which has a much richer structure and the magic production is governed by 8 different patterns:



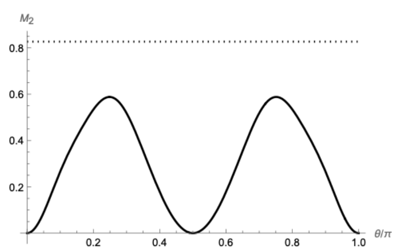
(a) $\mathcal{G}_3$



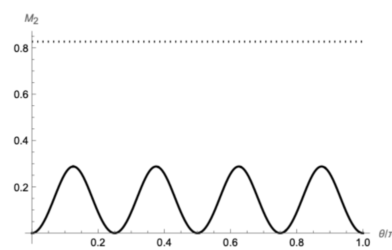
(b) $\mathcal{G}_4$



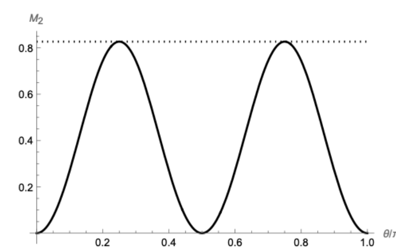
(c) $\mathcal{G}_5$



(d) $\mathcal{G}_6$

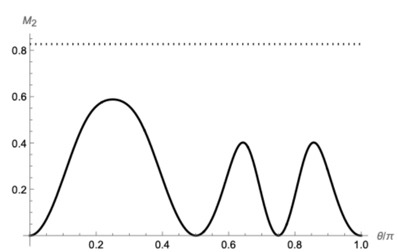


(e) $\mathcal{G}_7$

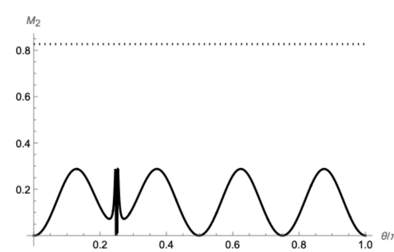


(f) $\mathcal{G}_8$

Plotted for real world λ



(g) $\mathcal{G}_9$



(h) $\mathcal{G}_{10}$

- The magic produced in the low-energy limit vanishes identically for

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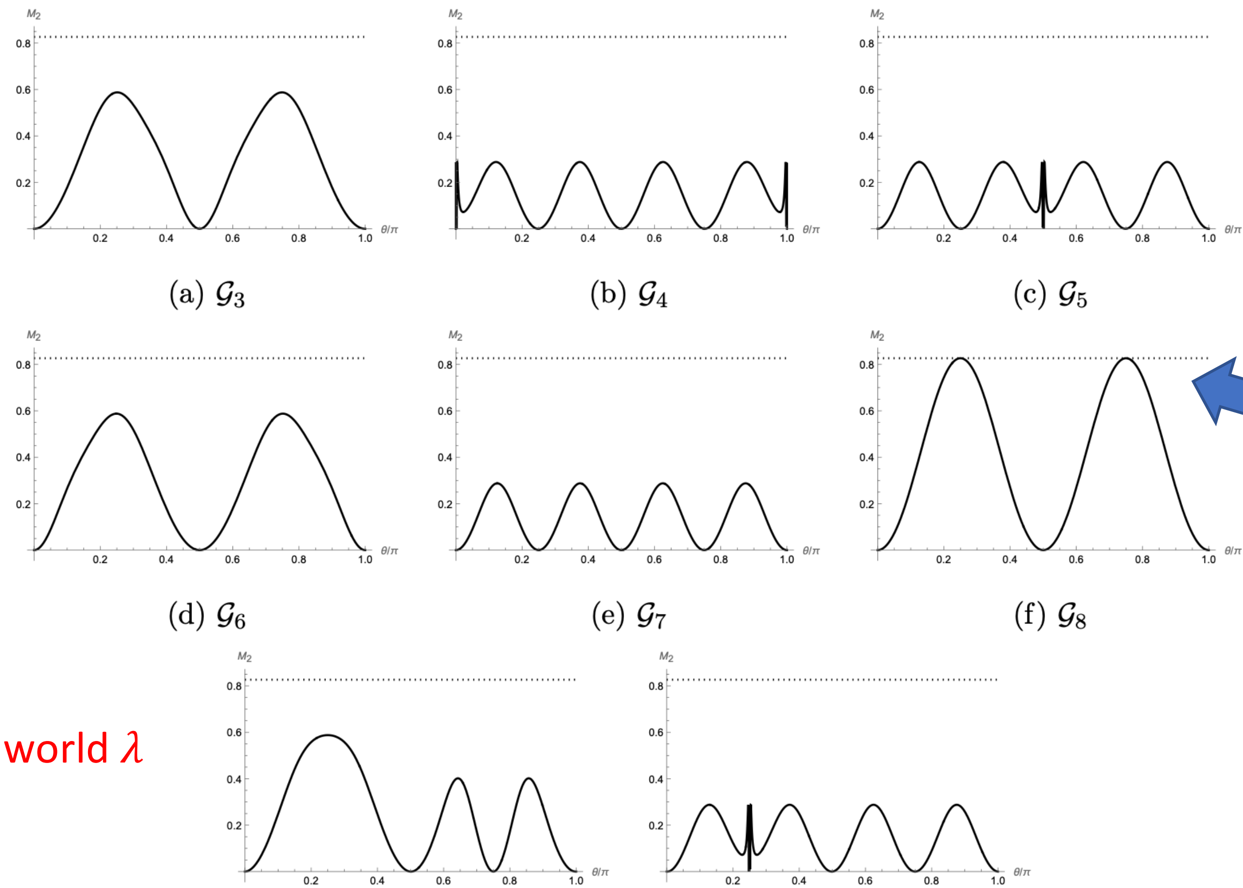

(a) $\mathcal{G}_3$ (b) $\mathcal{G}_4$ (c) $\mathcal{G}_5$

(d) $\mathcal{G}_6$ (e) $\mathcal{G}_7$ (f) $\mathcal{G}_8$

(g) $\mathcal{G}_9$ (h) $\mathcal{G}_{10}$

It appears this may reach the maximal magic!

Plotted for real world λ

Let's take a closer look and plot it as a function of λ:

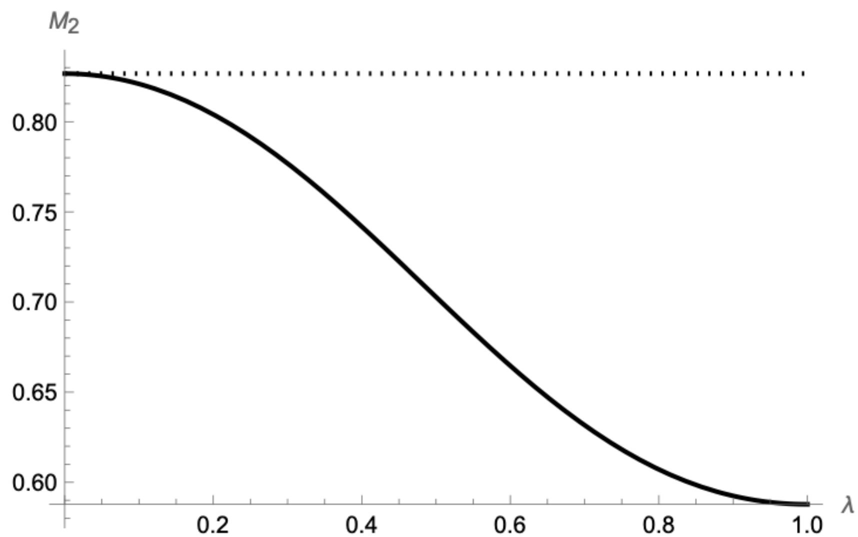

FIG. 5: The maximal magic achieved from initial states $|\psi_s\rangle_i$, $i = 13, 14, \cdots, 28$, in the low energy scattering of $\mu^-\mu^+ \to e^-e^+$. These maximal values are attained at $\theta = \pi/4$ or $3\pi/4$.

It reaches log(16/7) for $\lambda = 0$. The real world value of 0.005 gets us very close to the maximal possible magic.

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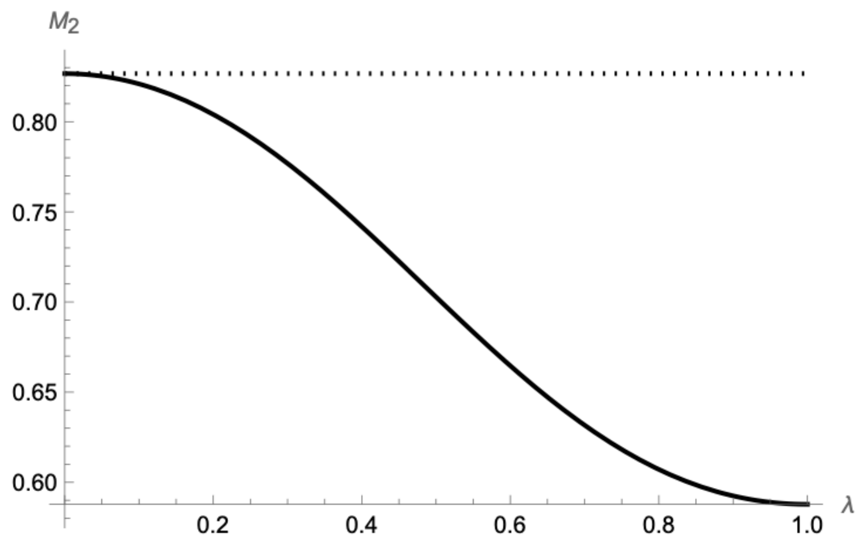


FIG. 5: The maximal magic achieved from initial states $|\psi_s\rangle_i$, $i = 13, 14, \cdots, 28$, in the low energy scattering of $\mu^-\mu^+ \to e^-e^+$. These maximal values are attained at $\theta = \pi/4$ or $3\pi/4$.

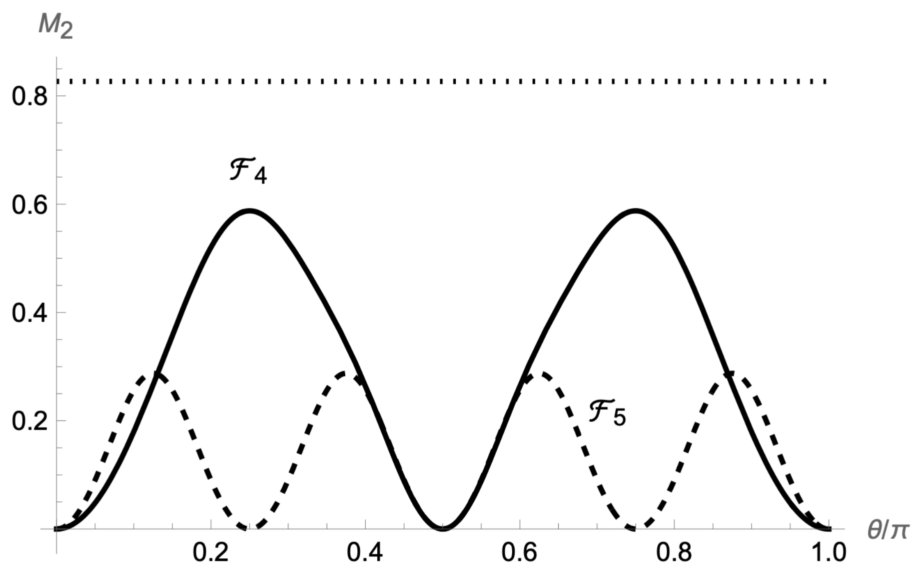
It reaches log(16/7) for λ = 0. The real world value of 0.005 gets us very close to the maximal possible magic.

Among the processes we studied, this is the only instance where maximal magic is achieved.

High Energy Limit: $e^- e^+ \to \mu^- \mu^+$

- The ultra-relativistic limit is given by $\mu = \dfrac{\vec{p}}{m_e} \to \infty$

Stabilizer States	Ξ_2	$(M_2)_{\max}$	$\theta_{\max}$
$1, 2, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 51, 52, 53, 54, 55, 56, 57, 58$	$\mathcal{F}_4$	$\log(9/5)$	$\pi/4,\ 3\pi/4$
$3, 4, 5, 6, 11, 12, 37, 42, 43, 44, 46, 47, 49, 50$	$\mathcal{F}_5$	$\log(4/3)$	$\pi/8,\ 3\pi/8,\ 5\pi/8,\ 7\pi/8$
$7, 8, 9, 10, 38, 45, 48, 59, 60$	$\mathcal{F}_1$	0	Arbitrary
41	—	—	—



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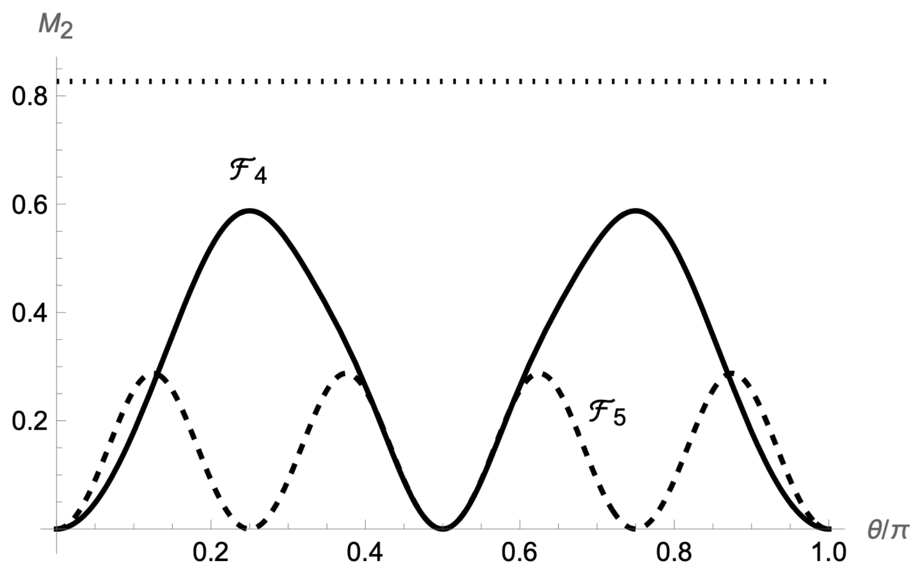


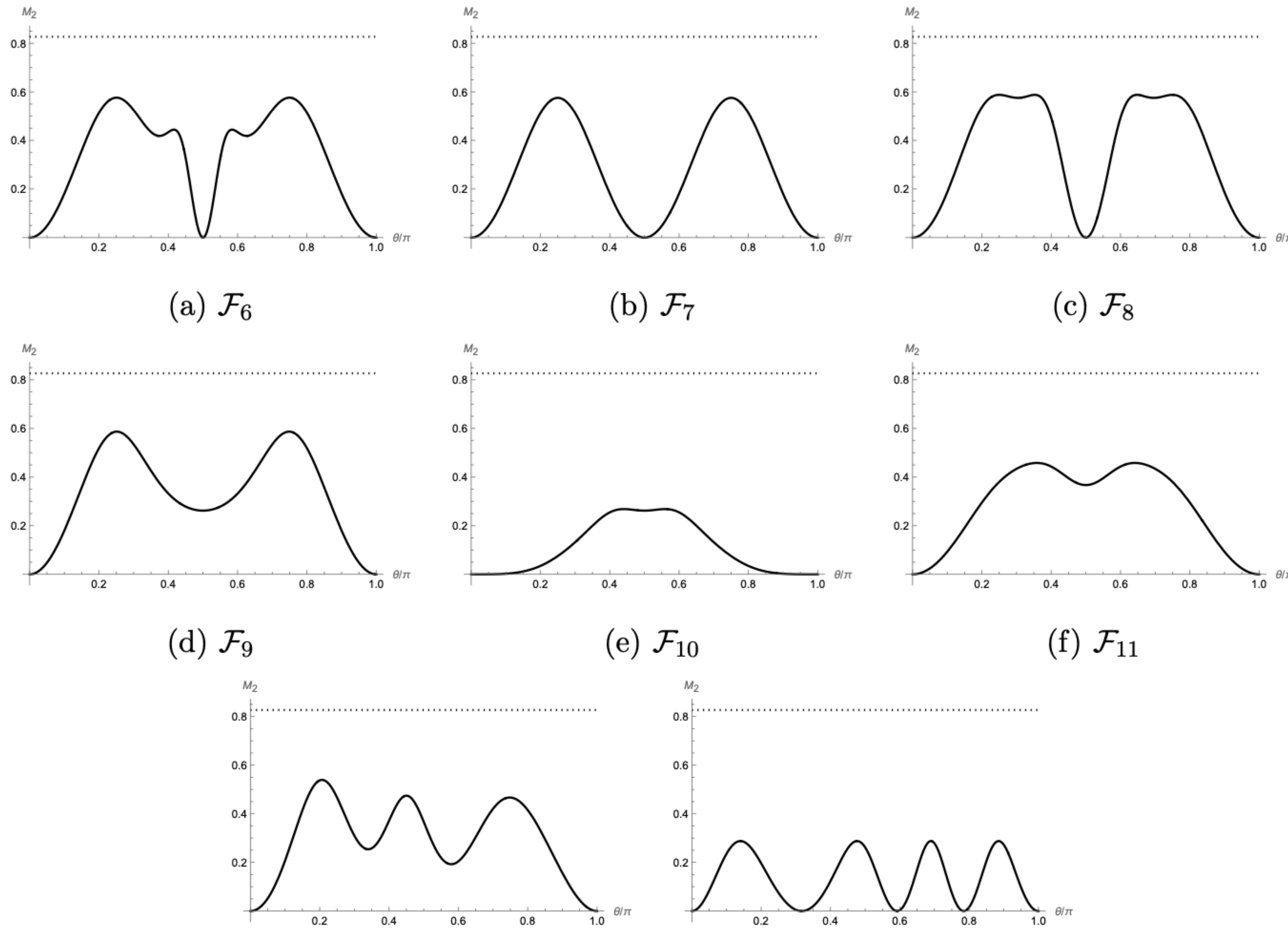
These values appeared previously!
Do they carry special meanings?

Møller scattering $e^- e^- \to e^- e^-$ has a rich structure in the high energy:

Stabilizer States	Ξ_2	$(M_2)_{\max}$	$\theta_{\max}$
$1, 2, 39, 40$	$\mathcal{F}_6$	$0.576\cdots$	$(\pi/2) \pm 0.783\cdots$
$3, 4, 43, 44$	$\mathcal{F}_7$	$\log(16/9)$	$\pi/4,\ 3\pi/4$
$5, 6, 49, 50$	$\mathcal{F}_8$	$\log(9/5)$	$\pi/4,\ 3\pi/4,\ (\pi/2) \pm \mathrm{arccot}\sqrt{2}$
$7, 8, 59, 60$	$\mathcal{F}_9$	$0.586\cdots$	$(\pi/2) \pm 0.781\cdots$
$9, 10$	$\mathcal{F}_{10}$	$0.268\cdots$	$(\pi/2) \pm 0.186\cdots$
$11, 12, 46, 47$	$\mathcal{F}_2$	$\log(4/3)$	$2\arctan\sqrt{\sqrt{2}-1}$, $\pi - 2\arctan\sqrt{\sqrt{2}-1}$
$13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28$	$\mathcal{F}_{11}$	$0.458\cdots$	$(\pi/2) \pm 0.444\cdots$
$29, 31, 34, 36, 55, 56, 57, 58$	$\mathcal{F}_{12}$	$0.539\cdots$	$0.649\cdots$
$30, 32, 33, 35, 51, 52, 53, 54$	$\tilde{\mathcal{F}}_{12}$	$0.539\cdots$	$\pi - 0.649\cdots$
$37, 42$	$\mathcal{F}_1$	0	Arbitrary
$38, 41$	$\mathcal{F}_5$	$\log(4/3)$	$\pi/8,\ 3\pi/8,\ 5\pi/8,\ 7\pi/8$
45	$\mathcal{F}_{13}$	$\log(4/3)$	$0.440\cdots,\ 1.49\cdots,$ $2.16\cdots,\ 2.78\cdots$
48	$\tilde{\mathcal{F}}_{13}$	$\log(4/3)$	$\pi - 0.440\cdots,\ \pi - 1.49\cdots,$ $\pi - 2.16\cdots,\ \pi - 2.78\cdots$

Curiously, **Bhabha scattering** $e^- e^+ \to e^- e^+$ in the high energy limit involves the same 13 functions!

Møller scattering $e^-e^- \to e^-e^-$ has a rich structure in the high energy:



(a) $\mathcal{F}_6$ (b) $\mathcal{F}_7$ (c) $\mathcal{F}_8$

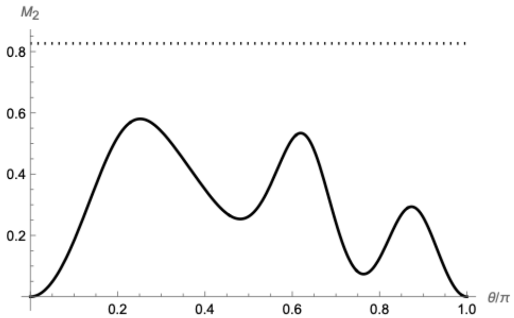
(d) $\mathcal{F}_9$ (e) $\mathcal{F}_{10}$ (f) $\mathcal{F}_{11}$

(g) $\mathcal{F}_{12}$ (h) $\mathcal{F}_{13}$

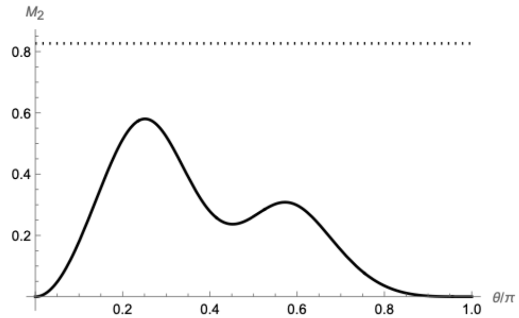
$e^- \mu^- \to e^- \mu^-$

Stabilizer States	Ξ_2	$(M_2)_{\max}$	$\theta_{\max}$
$1, 2, 3, 4, 39, 40, 43, 44$	$\mathcal{F}_7$	$\log(16/9)$	$\pi/4,\ 3\pi/4$
$5, 6, 49, 50$	$\mathcal{F}_{14}$	$0.580\cdots$	$0.790\cdots$
$7, 8, 59, 60$	$\mathcal{F}_{15}$	$0.580\cdots$	$0.789\cdots$
$9, 10$	$\mathcal{F}_{16}$	$0.405\cdots$	$1.95\cdots$
$11, 12, 46, 47$	$\mathcal{F}_{17}$	$\log(4/3)$	$2\arctan 2^{1/4}$
$13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28$	$\mathcal{F}_{18}$	$0.628\cdots$	$2.31\cdots$
$29, 31, 34, 36, 55, 56, 57, 58$	$\mathcal{F}_{19}$	$0.569\cdots$	$0.710\cdots$
$30, 32, 33, 35, 51, 52, 53, 54$	$\mathcal{F}_{20}$	$0.550\cdots$	$0.849\cdots$
$37, 42$	$\mathcal{F}_1$	0	Arbitrary
$38, 41$	$\mathcal{F}_5$	$\log(4/3)$	$\pi/8,\ 3\pi/8,$ $5\pi/8,\ 7\pi/8$
45	$\mathcal{F}_{21}$	$\log(4/3)$	$0.414\cdots,\ 1.45\cdots,\ 2.70\cdots$
48	$\mathcal{F}_{22}$	$\log(4/3)$	$0.375\cdots,\ 1.03\cdots,\ 1.62\cdots$ $2.17\cdots,\ 2.78\cdots$

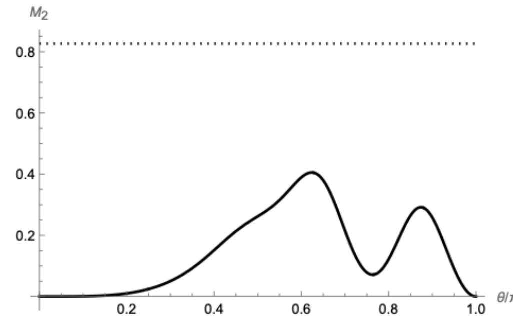
$e^-\mu^- \to e^-\mu^-$



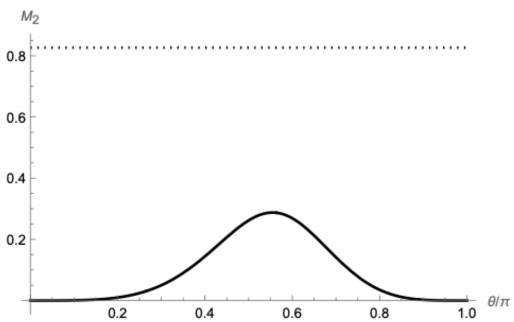
(a) $\mathcal{F}_{14}$



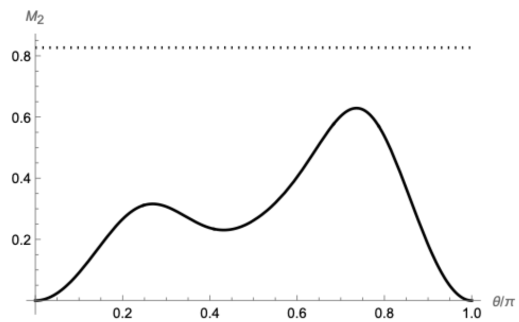
(b) $\mathcal{F}_5$



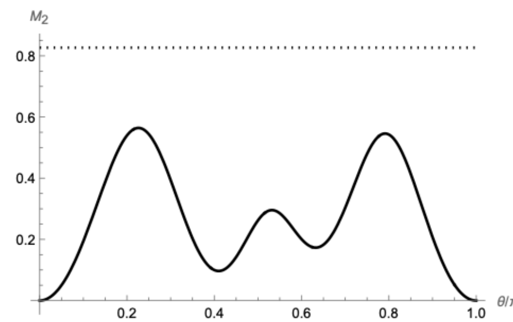
(c) $\mathcal{F}_{16}$



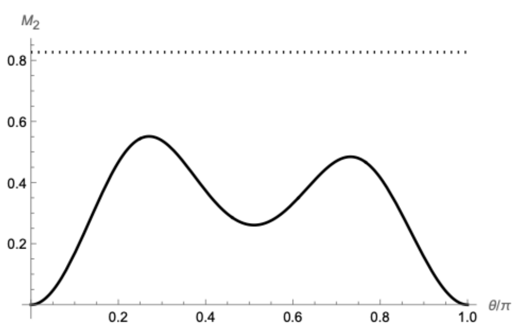
(d) $\mathcal{F}_{17}$



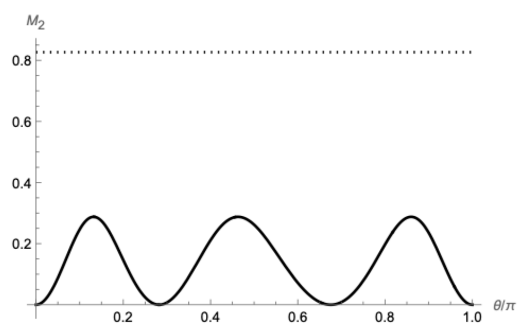
(e) $\mathcal{F}_{18}$



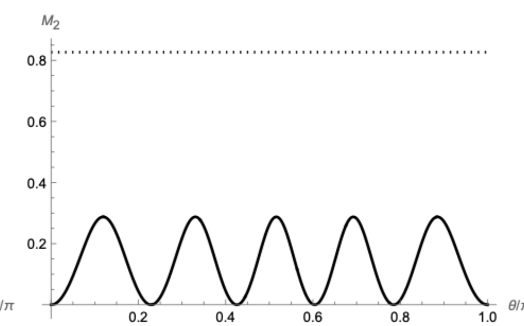
(f) $\mathcal{F}_{19}$



(g) $\mathcal{F}_{20}$



(h) $\mathcal{F}_{21}$



(i) $\mathcal{F}_{22}$

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- QED doesn’t seem to be too magical. What interactions can give rise to maximal quantum advantage computationally?