Measurement of magic and other quantum information inspired observables in $t\bar{t}$ pairs at CMS

Otto Hindrichs
on behalf of the CMS Collaboration

University of Rochester

Quantum Observables for Collider Physics 2025, GGI, Florence

08.04.2025

- Important test of standard model; allows for testing quantum properties of the $\mathrm{t\bar{t}}$ system such as entanglement, magic, and violation of Bell-Inequality (in the future?)
- Spin observables can improve sensitivity of searches: high mass $\mathrm{t\bar{t}}$ resonances [*CMS-HIG-22-013*], toponium measurements [*TOP-24-007*]

- Typical spin decorrelation time is $m_{\mathrm{t}}/\Lambda^2_{\mathrm{QCD}} \approx 10^{-21}\,\mathrm{s} >$ top quark lifetime $5 \cdot 10^{-25}\,\mathrm{s}$ $\rightarrow$spin correlation accessible from decay products.
- In the helicity-frame the differential cross section of the top quark decay products can be parameterized by the polarization vector $\boldsymbol{P}$ and the spin correlation matrix C:

$$\frac{d^4\sigma}{d\boldsymbol{\Omega} d\bar{\boldsymbol{\Omega}}} \propto 1 + \kappa \boldsymbol{P} \cdot \boldsymbol{\Omega} + \bar{\kappa} \bar{\boldsymbol{P}} \cdot \bar{\boldsymbol{\Omega}} + \kappa\bar{\kappa} \boldsymbol{\Omega} \cdot C \bar{\boldsymbol{\Omega}}$$

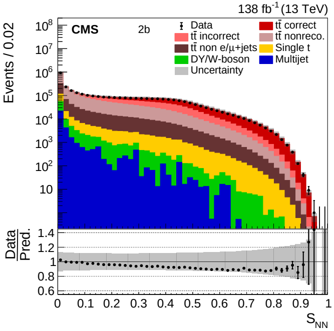
- $\boldsymbol{\Omega}$ is direction of top decay product
- Including an overall normalization, there are 16 parameters
- Spin analyzing power κ depends on decay particle. Best sensitivity ($\kappa \approx 1$) for charged leptons and down-type quarks from W decays.

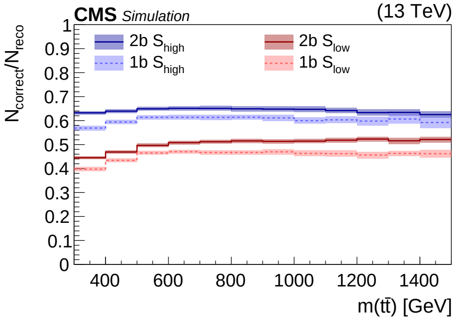
138 fb^{-1} (13 TeV) CMS: *Phys. Rev. D 110 (2024) 112016*

Observables in helicity-frame require full reconstruction of the top quark and anti-quark
(Axes defined by top quark direction; boost into their rest-frames.)

- challenging identification of down-type quark in W decay

Use dense neural network for identification of the top decay products (7 layers, 220 nodes)
– inputs: [ℓ, p_T^{miss}, b_ℓ, b_h, j_{down}, j_{up}, additional jets]; momentum and b-tagging information for jets
– present all permutations for the jets from $t\bar{t}$ to NN and train for high score if the 4 jets are at the correct positions
half of the time there is a c-jet in the W decay; in average, down-type jets are softer (65% correctly identified)

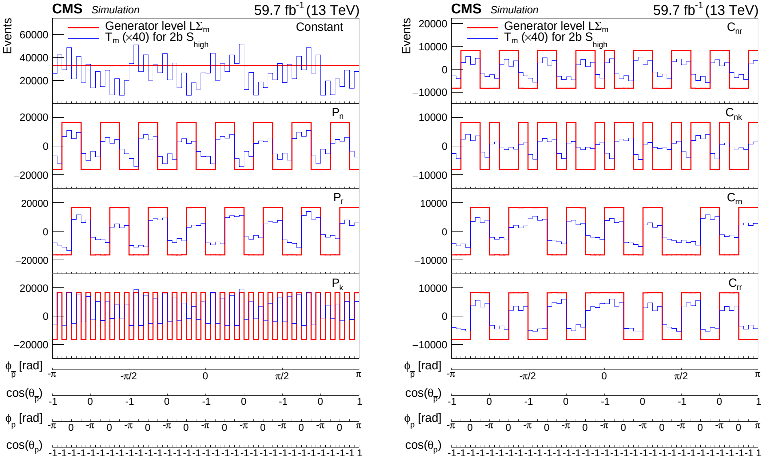




S_{low}: 0.1–0.36, S_{high}: > 0.36

Extraction of the parameters

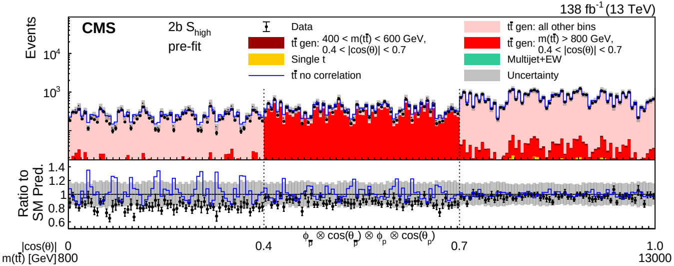
According to cross section formula, each coefficient is proportional to a function (red) of the angles $\cos(\theta_{p/\bar{p}})$ and $\phi_{p/\bar{p}}$:

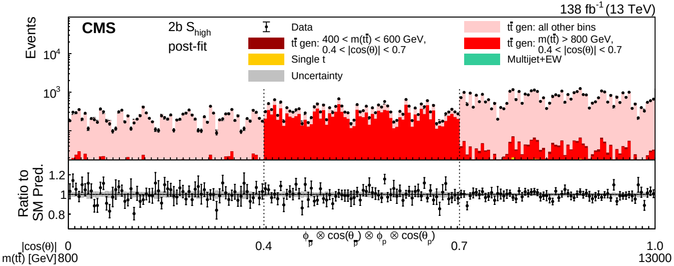


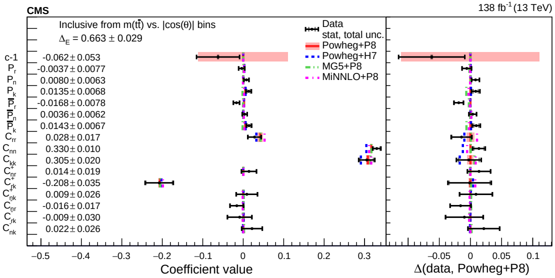
Blue lines include the detector effect (acceptance, resolution, efficiencies...)

→fit linear combination of the detector-level templates to the data.

Due to variation of detector effects, templates depends on the kinematic region ($m(t\bar{t})$, $\cos(\theta_t)$, ...)
$\rightarrow$bias avoided by fitting in bins of $m(t\bar{t})$ vs $\cos(\theta_t)$ or $p_T(t)$ vs $\cos(\theta_t)$
such that the templates are approximately constant in each bin.

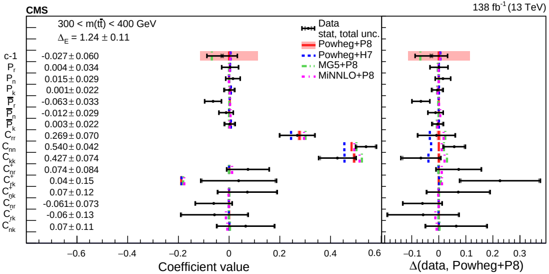


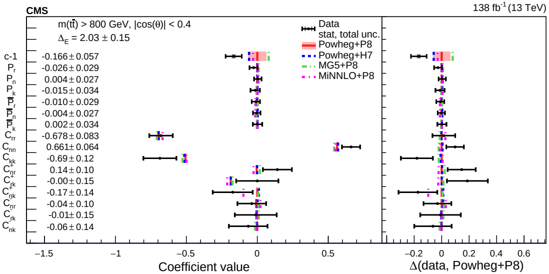




- Only C^+_{rk} is invariant under P and C transformation →only non-zero off-diagonal element
- Diagonal elements indicate the transition from a dominant spin-singlet state at low to a triplet-state at high $m(t\bar{t})$
- All coefficients in good agreement with SM values
- Access to full density matrix :

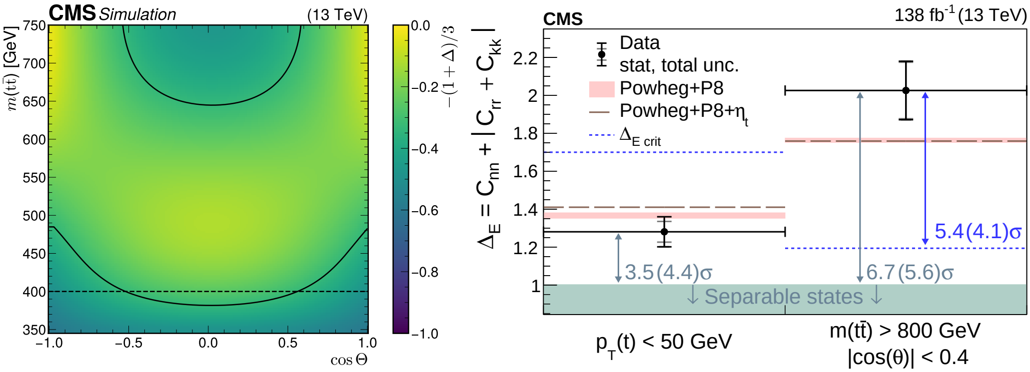
$$\rho = \frac{1}{4}\left(\mathbb{1}_4 + \sum_i P_i \sigma_i \otimes \mathbb{1}_2 + \sum_j \bar{P}_j \mathbb{1}_2 \otimes \sigma_j + \sum_{ij} C_{ij} \sigma_i \otimes \sigma_j\right)$$





Off-diagonal redefined $C^{\pm}_{ij} = C_{ij} \pm C_{ij}$, all results are available on HEP data

Quantum Entanglement



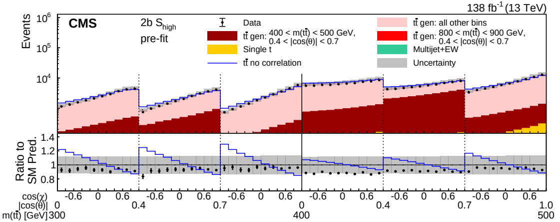
- At the threshold and at high $m(t\bar{t})$ with low $\cos(\theta_t)$ $t\bar{t}$ is expected to be produced in entangled states
- Criterion for entanglement (based on Peres-Horodecki criterion): $\Delta E = C_{nn} + |C_{rr} + C_{kk}| > 1$

Assuming that the $t\bar{t}$ system is described by QM, this is the first observation of an entangled quantum state at high $m(t\bar{t})$

Alternative methods with single parameter fits:

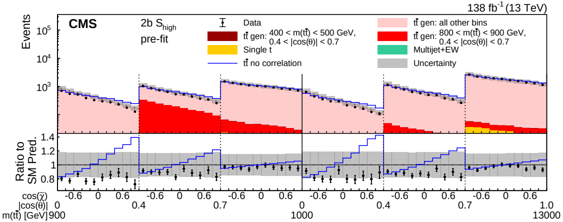
– low $m(\mathrm{t\bar{t}})$: $\frac{d\sigma}{d\cos(\chi)} \propto 1 - D\cos(\chi)$

with $D = -\frac{1}{3}(C_{nn} + C_{rr} + C_{kk})$ and $\cos(\tilde{\chi}) = \Omega_n\bar{\Omega}_n + \Omega_r\bar{\Omega}_r + \Omega_k\bar{\Omega}_k$ (angle between lepton and down-type quark)

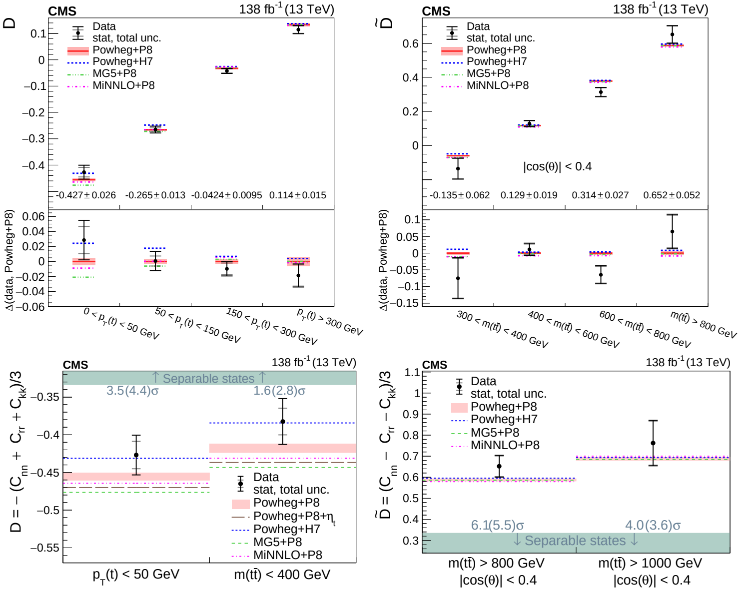


– high $m(\mathrm{t\bar{t}})$: $\frac{d\sigma}{d\cos(\tilde{\chi})} \propto 1 - \tilde{D}\cos(\tilde{\chi})$

with $\tilde{D} = \frac{1}{3}(-C_{nn} + C_{rr} + C_{kk})$ and $\cos(\tilde{\chi}) = -\Omega_n\bar{\Omega}_n + \Omega_r\bar{\Omega}_r + \Omega_k\bar{\Omega}_k$ (similar to χ but with C_{nn} sign flipped)



- method is technically easier: lower number of bins and fit parameters required
- results for entanglement observables consistent

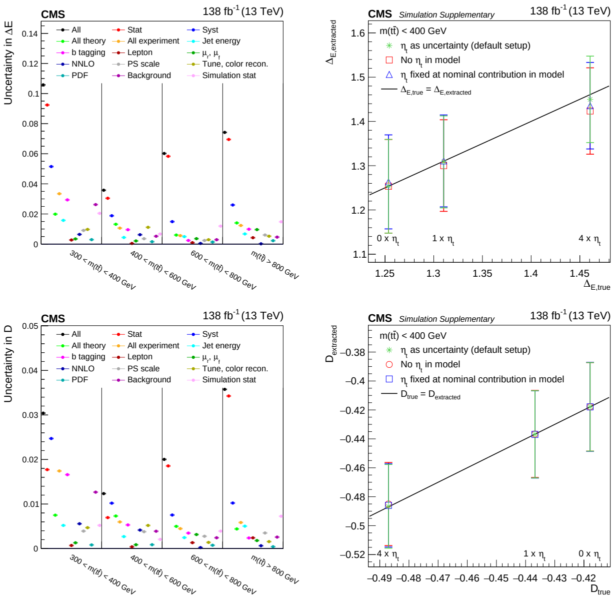


Uncertainties and tests

- measurements are mostly statistically limited
  systematic uncertainties in D are larger; more assumption about modeling and detector effects made
- toponium signal injection tests show that the correct values can be obtained.
  however, the signal is within uncertainties

toponium simulated as a pseudo-scalar particle with mass 343 GeV, $\Gamma = 2m_t$, and production cross section of 6.4 pb

- tests with altered injected coefficients successfully performed in many regions of phase space



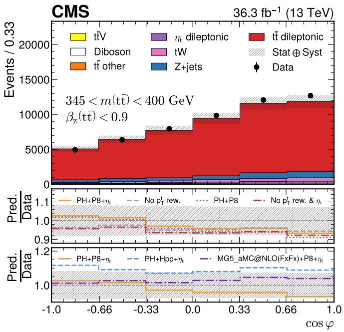
36 fb^{-1} (13 TeV) CMS: *Rep. Prog. Phys. 87 (2024) 117801*

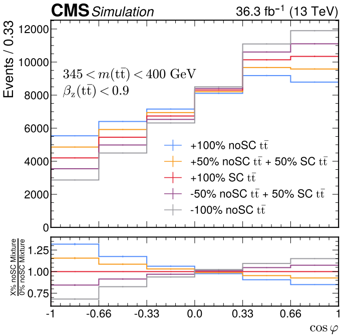
At the $t\bar{t}$ production threshold, where all diagonal elements of C are positive $D = -\frac{1}{3}\Delta E$

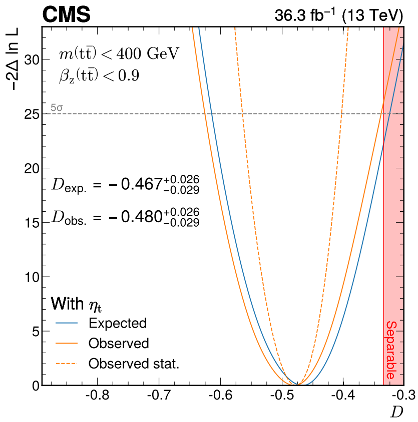
$$\frac{d\sigma}{d\cos(\phi)} \propto 1 - D\cos(\phi)$$

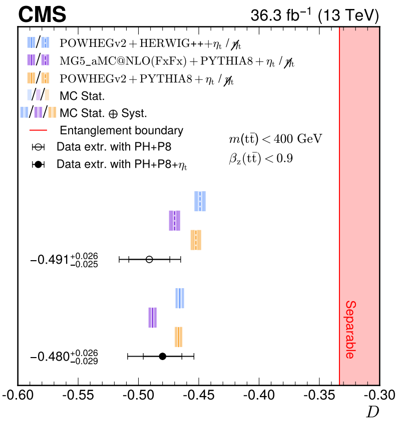
with ϕ the angle between the two charged leptons in the helicity-frame

- Reconstruction of top quarks based on analytical calculation of neutrino momenta using m_W, m_t, and p_T^{miss} balance
- Extract D using template fit of SM template and template without any spin correlation









- Observation of entanglement at the $t\bar{t}$ production threshold $> 5\sigma$ (also observed by ATLAS [*Nat. 633 (2024) 542*)])
- Dilepton channel profits from direct identification of charged leptons with higher acceptance
- CMS: analysis performed with and without taking into account a $t\bar{t}$ "bound"-state (toponium).
  →analysis not sensitive enough to observe the differences, but lower D preferred by data.

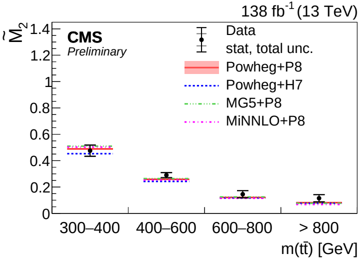
138 fb^{-1} (13 TeV) CMS: *CMS-TOP-25-001 NEW!*

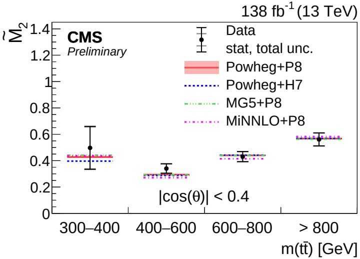
- In quantum information science pure eigen-states of unitary operators (mostly tensor products of Pauli matrices) are called stabilizer states. For these magic is zero.
- Non-stabilizer states have enhanced properties for quantum computing [*D. Gottesman*]

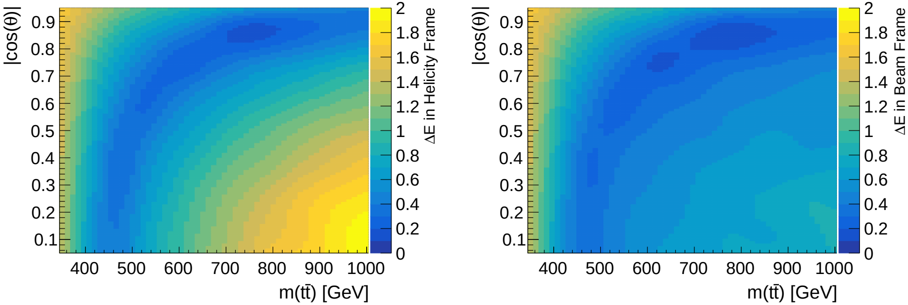
A generalized definition of magic for mixed states [*C. White*]:

$$\tilde{M}_2 = -\log_2 \left(\frac{1 + \sum_{i \in n,k,r} [(P_i^4 + \bar{P}_i^4)] + \sum_{i,j \in n,k,r} C_{ij}^4}{1 + \sum_{i \in n,k,r} [(P_i^2 + \bar{P}_i^2)] + \sum_{i,j \in n,k,r} C_{ij}^2} \right)$$

This can be calculated from the measured spin correlations:

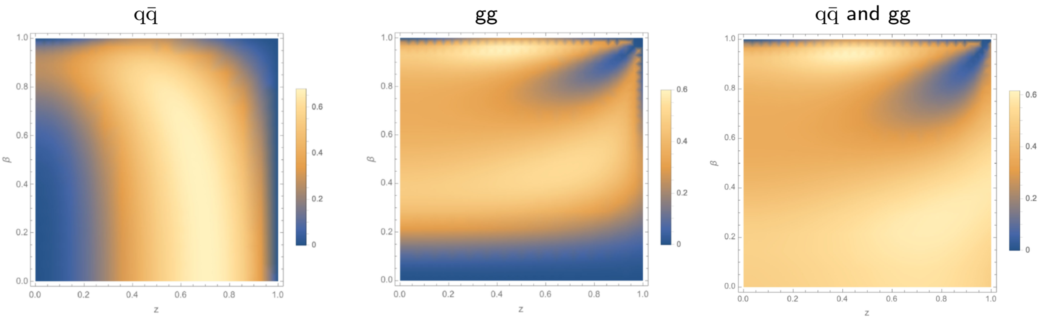






- Observation of high mass entanglement depends on selection of coordinate system (no direct transformation of C between the two results)
- Entanglement at the threshold is similar in both systems; spin zero state is rotational invariant (measured using the rotational invariant angle χ between the two decay products)
- This is not true for the spin 1 state at high $m(\mathrm{t\bar{t}})$ ($\tilde{\chi}$ is not rotational invariant)
- In any case, averaged spin/polarization coefficients over all contributing states measured. Since the averaged ΔE cannot exceed the maximum value, there must be a contribution of states with $\Delta E \geq$ the observed value.
$\rightarrow$there are entangled states (*helicity basis results in better suited observables*).

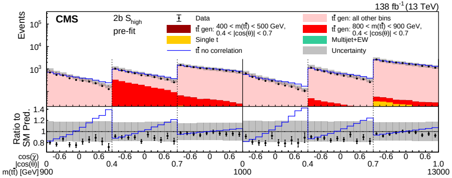
- measured spin correlation coefficients averaged over states and kinematic regions (binned) in contrast to ΔE, magic is not a linear in the measured coefficients $\rightarrow$do not obtain the average $\tilde{M}_2$
- Alone the combination of different processes gg $\rightarrow \mathrm{t}\bar{\mathrm{t}}$, $q\bar{q}$ $\rightarrow \mathrm{t}\bar{\mathrm{t}}$ makes an interpretation hard. *Zero $\tilde{M}_2$ at low $m(\mathrm{t}\bar{\mathrm{t}})$ and z disappears in combination.* [*C. White*]

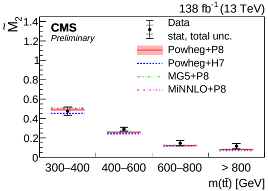


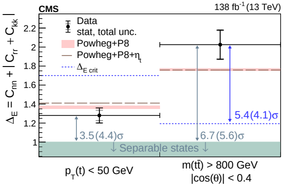
$\rightarrow$non-zero magic of the mixed states does not imply non-zero magic for the individual quantum states

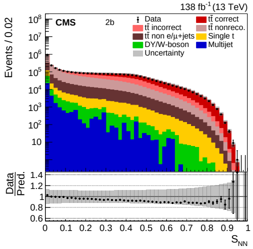
- Similar statements are true for quantum discord [T. Han]

$\rightarrow$values of magic, discord etc... also depend on the coordinate systems. This is not a problem, but their interpretation is not as straight forward as for the linear observables (entanglement: ΔE; Bell-Ineq.: $C_{rr} + C_{kk} > \sqrt{2}$)









- Measurements of spin density matrix in various phase space regions
- Observation of entanglement at low and high $m(t\bar{t})$
- Tests of quantum observables: magic...

BACK UP

140 fb^{-1}, 13 TeV, ATLAS: *Nat. 633 (2024) 542*

ATLAS analysis calculates D based on average $\cos(\phi)$:

$$D = -\frac{1}{3} < \cos(\phi) >$$

Calibration Detector-level $\leftrightarrow$ Particle-level from reweighted MC:
Particle-level top quarks are constructed using generator-level leptons and b-jets using m_W and m_t criteria.