Magic and BSM physics in the top sector

Martin White

Based on arXiv:2406.07321 with Chris White
+ new results with Rafael Aoude, Hannah Banks (arXiv:tbc)

Quantum Observables for Collider Physics 2025

Motivation

- Much recent work has looked at **entanglement** of top quarks at the LHC (Afik, de Nova; Dong, Gonçalves, Kong, Navarro; Fabbrichesi, Floreanini, Panizzo; Aoude, Madge, Maltoni, Mantani, Severi, Boschi, Sioli; Aguilar-Saavedra, Casas).
- There are lots of other concepts from Quantum Computation / Information theory that can be explored at colliders
- I will talk about *magic* (or *non-stabiliserness*) – has already become the subject of a growing literature (White, White; Liu, Low, Yin; Fabbrichesi, Low, Marzola; CMS PAS TOP-25-001)
- Will also briefly mention future lepton colliders

A bit of quantum computing

- In quantum computers, classical bits (with values {0,1}) are replaced by *qubits*:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Orthogonal
basis
states

where the complex coefficients satisfy $|\alpha|^2 + |\beta|^2 = 1$.

- A spin-1/2 particle is a single "qubit", where the above states are spin states.

- For multi-qubit systems, a choice of basis states is

$$|\psi_1\psi_2\ldots\psi_n\rangle \equiv |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_n\rangle$$

Why use quantum computers?

- Quantum computers don't always outperform classical computers
- To see why, we need the concept of a *stabiliser state.*
- These are states that give a simple spectrum for *Pauli string* operators:

$$\mathcal{P}_n = P_1 \otimes P_2 \otimes \ldots \otimes P_N\,, \quad P_a \in \{\sigma_1^{(a)}, \sigma_2^{(a)}, \sigma_3^{(a)}, I^{(a)}\}$$

Pauli matrix
acting on qubit *a*

Identity matrix
acting on qubit *a*

- Given a state $|\psi\rangle$, we can consider the *Pauli spectrum*

$$\mathrm{spec}(|\psi\rangle) = \{\langle\psi|P|\psi\rangle, \quad P \in \mathcal{P}_n\}$$

- Stabiliser states have 2^n values +1 or -1, and the rest zero.

Why stabiliser states matter

For every quantum computer containing stabiliser states only, there is a classical computer that is just as efficient! 😅

- Stabiliser states include certain maximally entangled states

- *Non-stabliserness* (or *magic*) is a measure of quantum advantage
- Different definitions exist. We use *Stabilizer Rényi Entropies*: (Leone, Oliviero, Hamma)

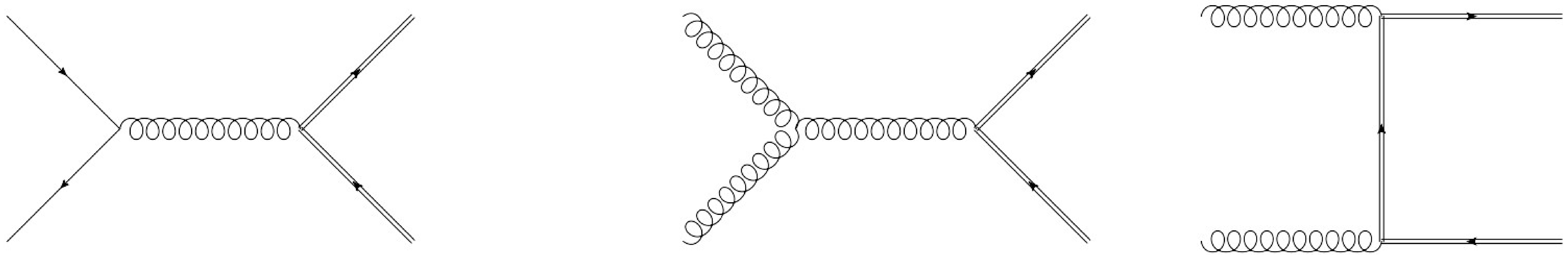
$$M_q = \frac{1}{1-q}\log_2\left(\zeta_q\right), \quad \zeta_q \equiv \sum_{P\in\mathcal{P}_n} \frac{\langle\psi|P|\psi\rangle^{2q}}{2^n}$$

- In what follows, examining *q*=2 is enough: the *Second Stabilizer Rényi Entropy (SSRE).*

- The SSRE **vanishes** for stabiliser states

Are top quarks magic?

- Consider top quark pair production at the LHC



- …such that the final state is a two-qubit system
- However, the final state is a *mixed state* (superposition of many different *pure states*), where the SM tells us what this is in principle.
- Mixed states can be described in terms of their *density matrix*:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Probability of being in state I

Top quark spin density matrix

- On general grounds, the top quark spin density matrix has decomposition:

$$\rho^I \sim \tilde{A}^I I_4 + \sum_i \left(\tilde{B}_i^{I+} \sigma_i \otimes I_2 + \tilde{B}_i^{I-} I_2 \otimes \sigma_i + \sum_{i,j} \tilde{C}_{ij}^I \sigma_i \otimes \sigma_j \right)$$

Contribution from partonic channel I

Identity matrix

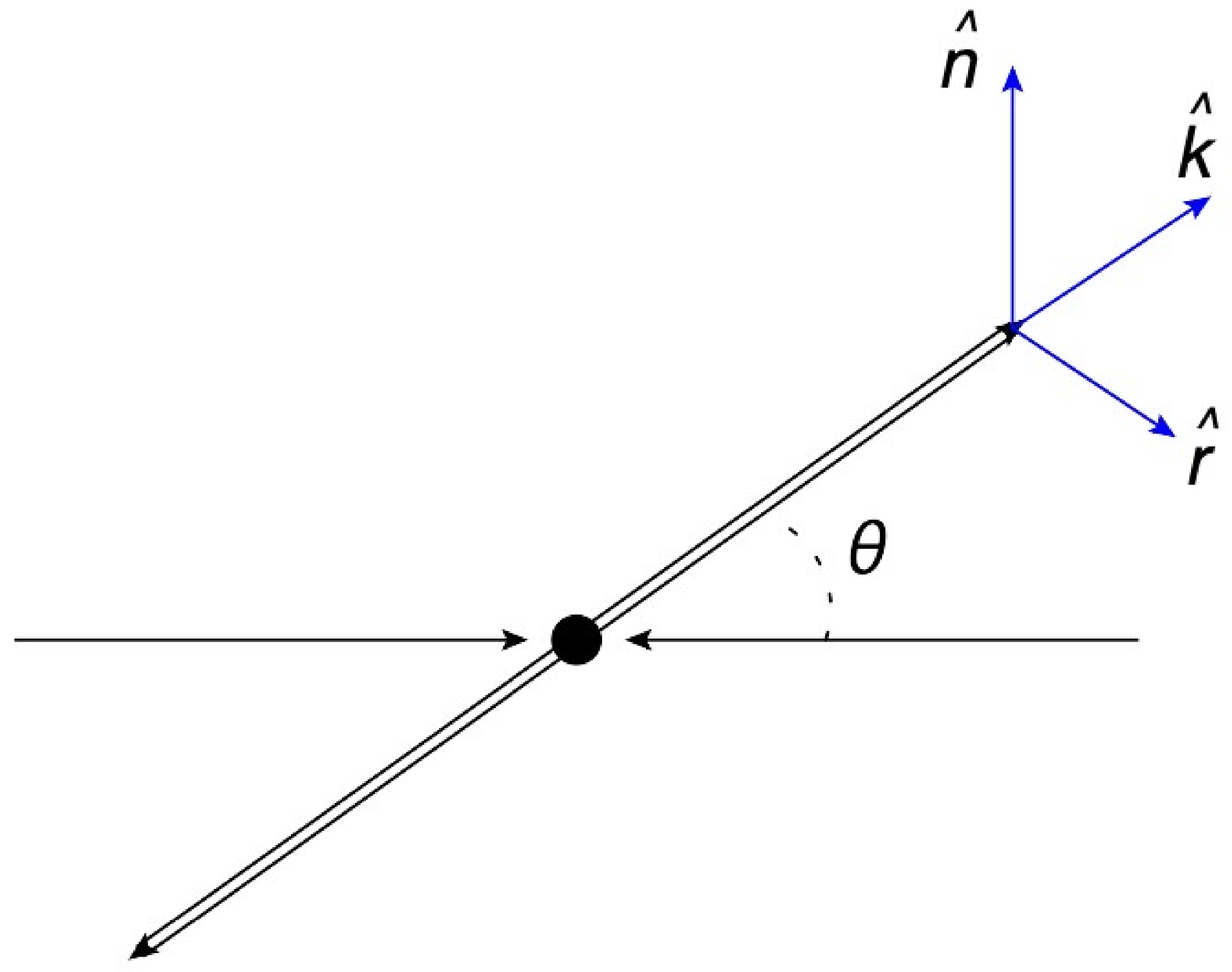
Identity matrix

Pauli matrix

- The *Fano coefficients* $\{\tilde{A}^I, \tilde{B}_i^{I\pm}, \tilde{C}_{ij}^I\}$ depend on the top quark kinematics…
- …as well as the basis relating spin directions (1,2,3) to physical space.
- A common choice is the *helicity basis*.

The helicity basis

- Helicity basis: choose an axis parallel to the top quark direction and two transverse directions (Baumgart, Tweedie).



- Each Fano coefficient is then a function of

$$z = \cos\theta, \quad \beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}.$$

- At LO in the SM, one has:

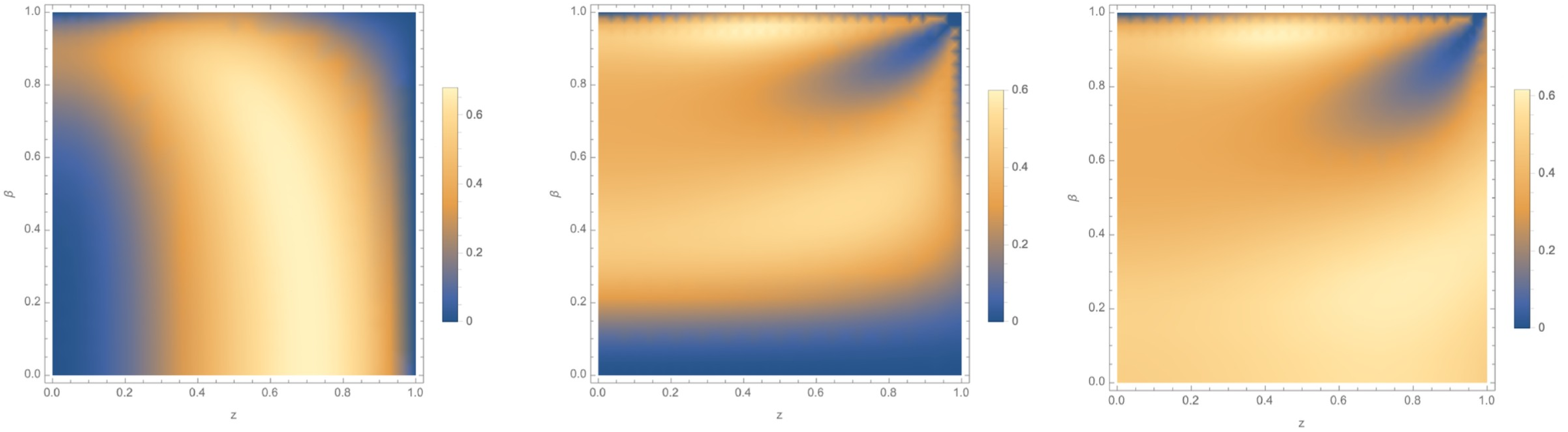
$$\tilde{B}_i^{I+} = \tilde{B}_i^{I-} = \tilde{C}_{nr}^I = \tilde{C}_{nk}^I = 0, \quad \tilde{C}_{ij}^I = \tilde{C}_{ji}^I$$

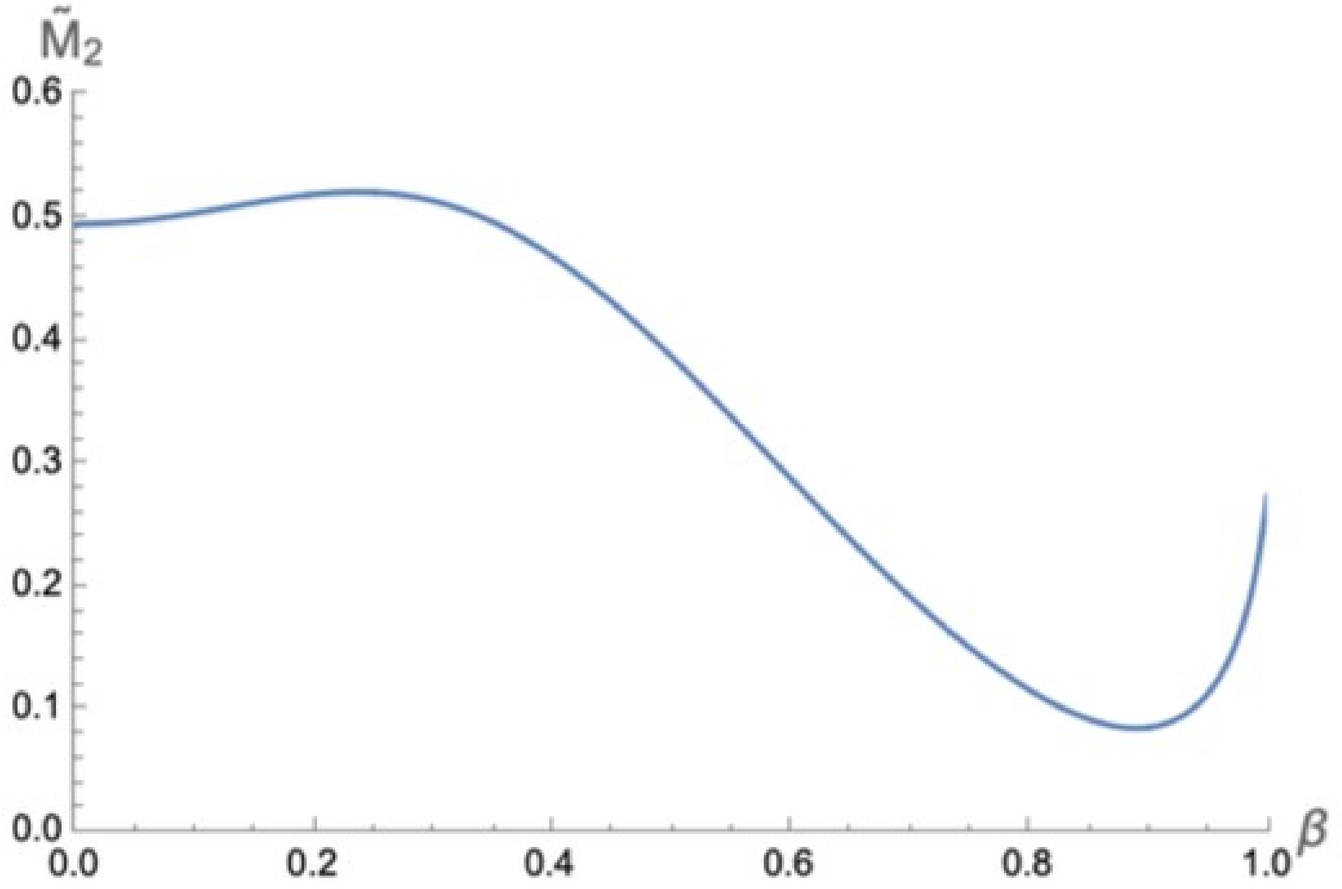
- The SSRE can be corrected for mixed states (Leone, Oliviero, Hamma), and yields

$$\tilde{M}_2(\rho^I) = -\log_2\left(\frac{(\tilde{A}^I)^4 + (\tilde{C}_{nn}^I)^4 + (\tilde{C}_{kk}^I)^4 + (\tilde{C}_{rr}^I)^4 + 2(\tilde{C}_{rk}^I)^4}{(\tilde{A}^I)^2[(\tilde{A}^I)^2 + (\tilde{C}_{nn}^I)^2 + (\tilde{C}_{kk}^I)^2 + (\tilde{C}_{rr}^I)^2 + 2(\tilde{C}_{rk}^I)^2]}\right)$$

Results

$q\bar{q}$ gg pp

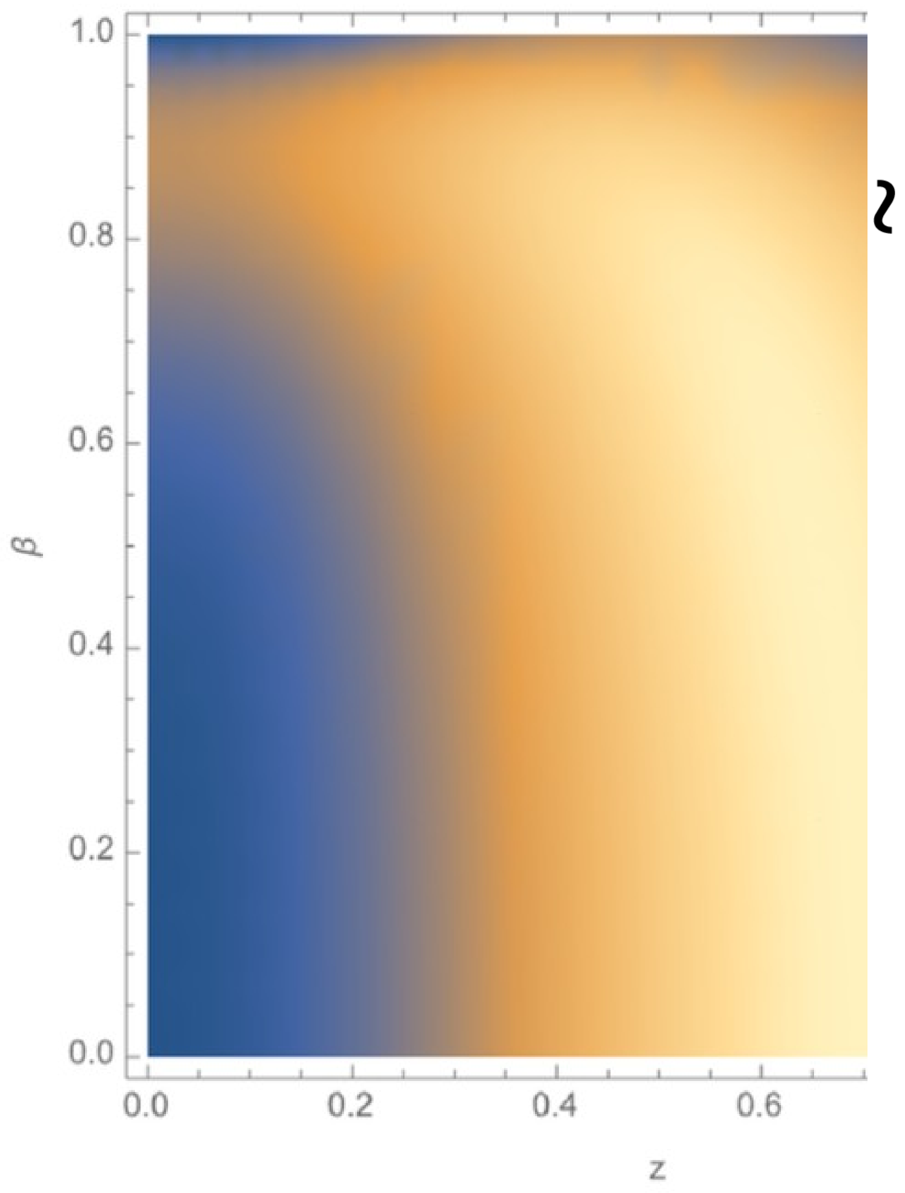


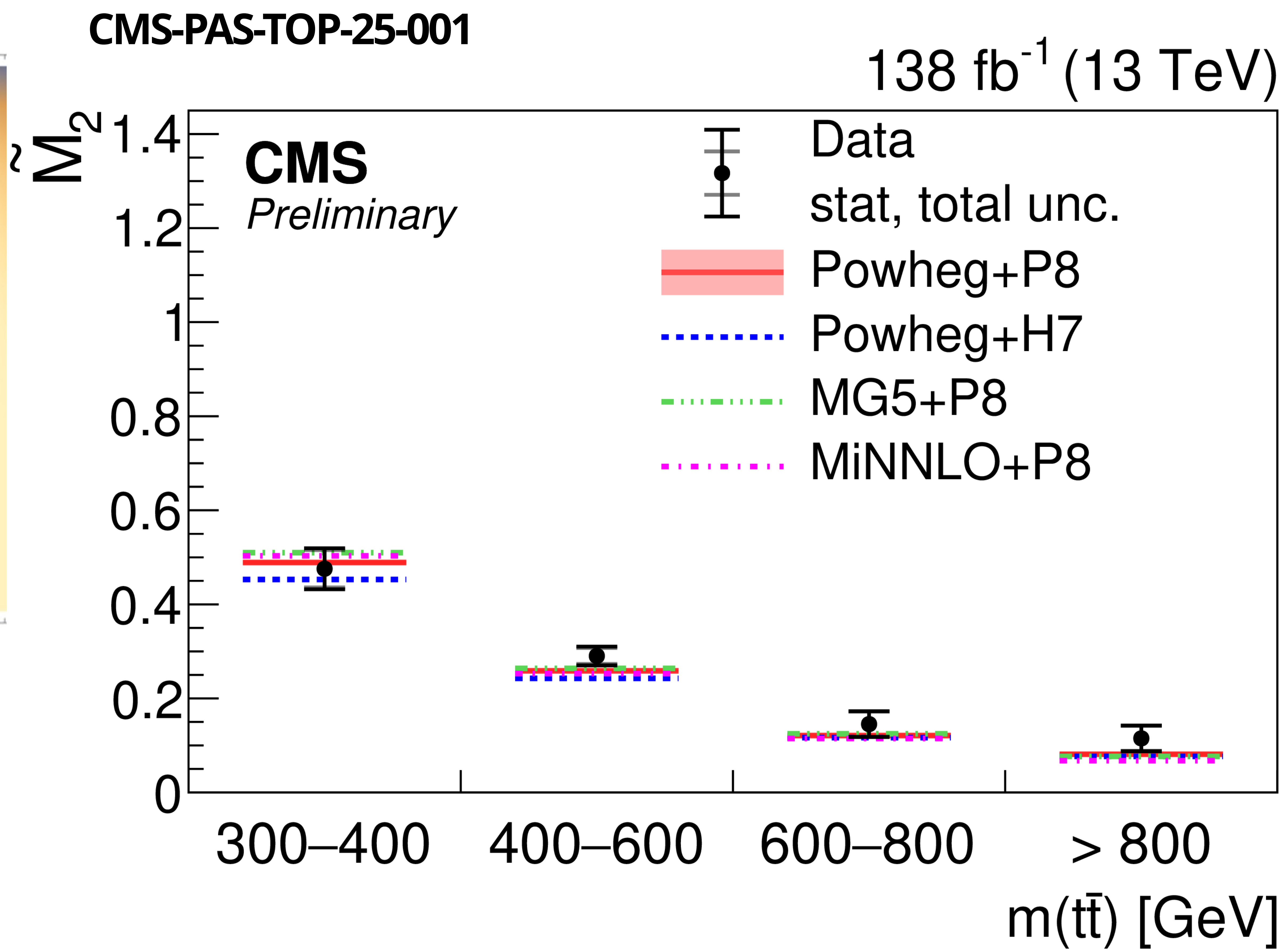


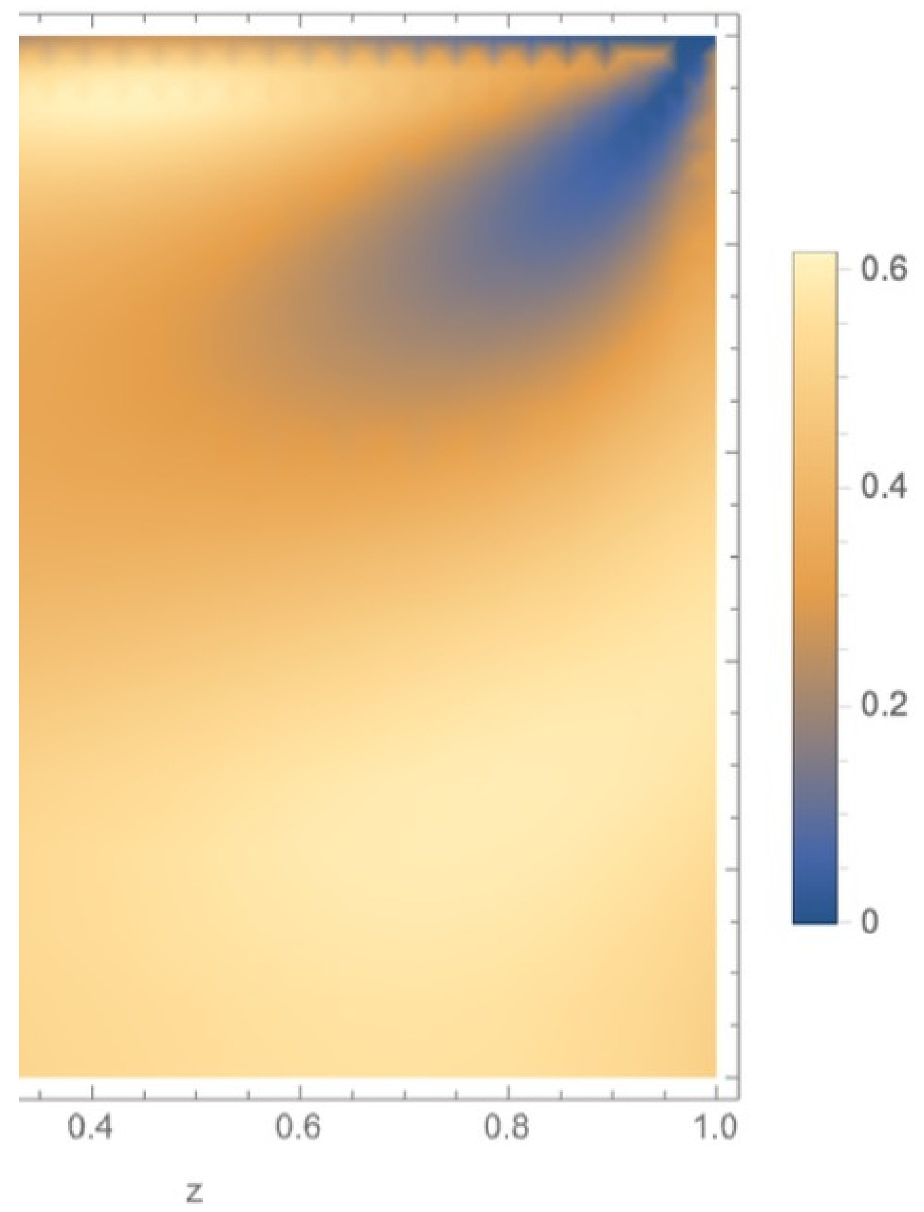
- Regions of zero magic in individual partonic channels correspond to known entanglement behaviour
- Averaging (PDF plus angular) leads to more mixed states → increases magic!

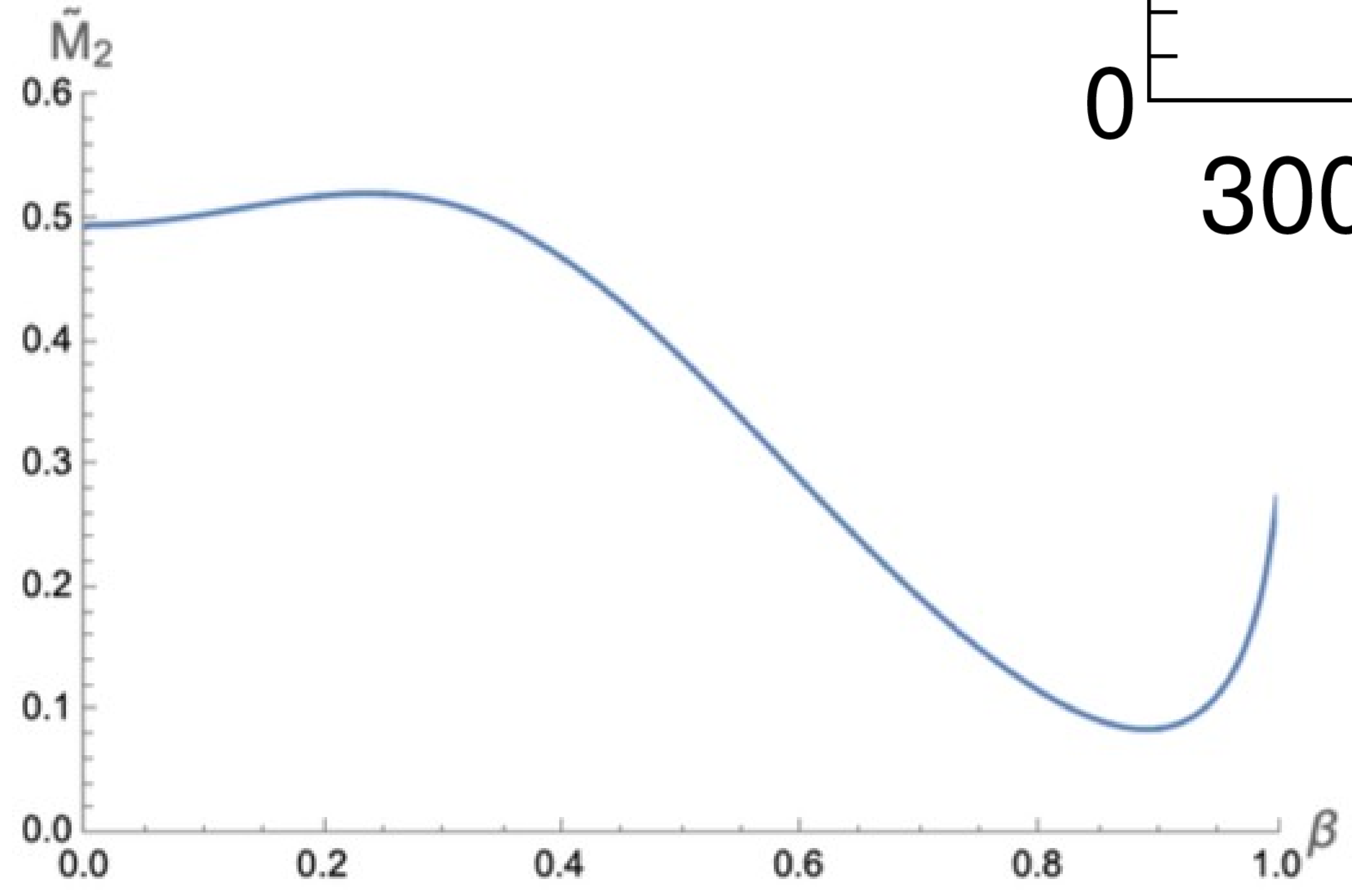
Results

$q\bar{q}$ gg pp







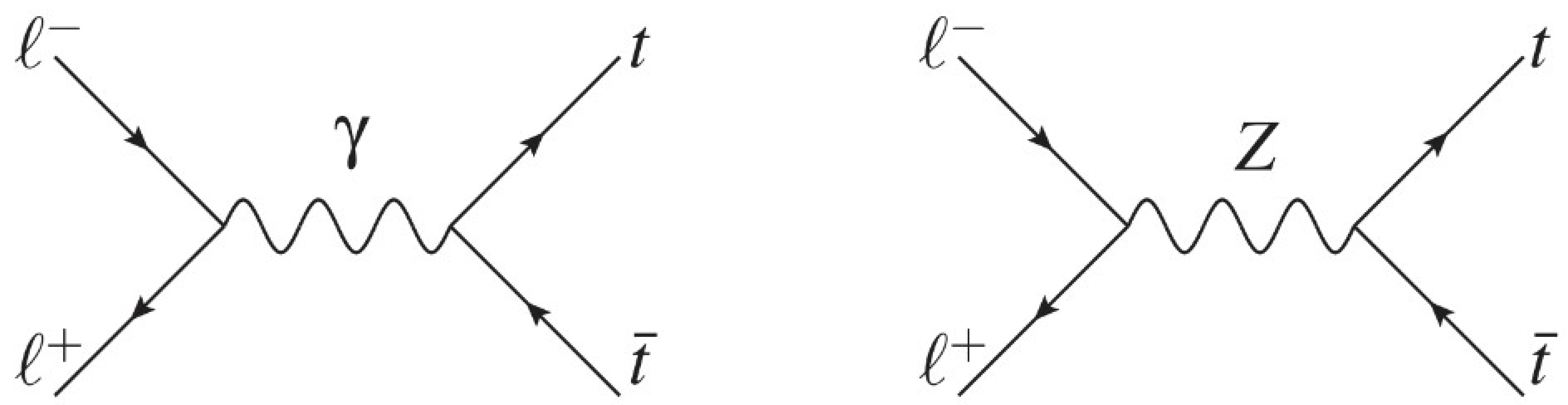


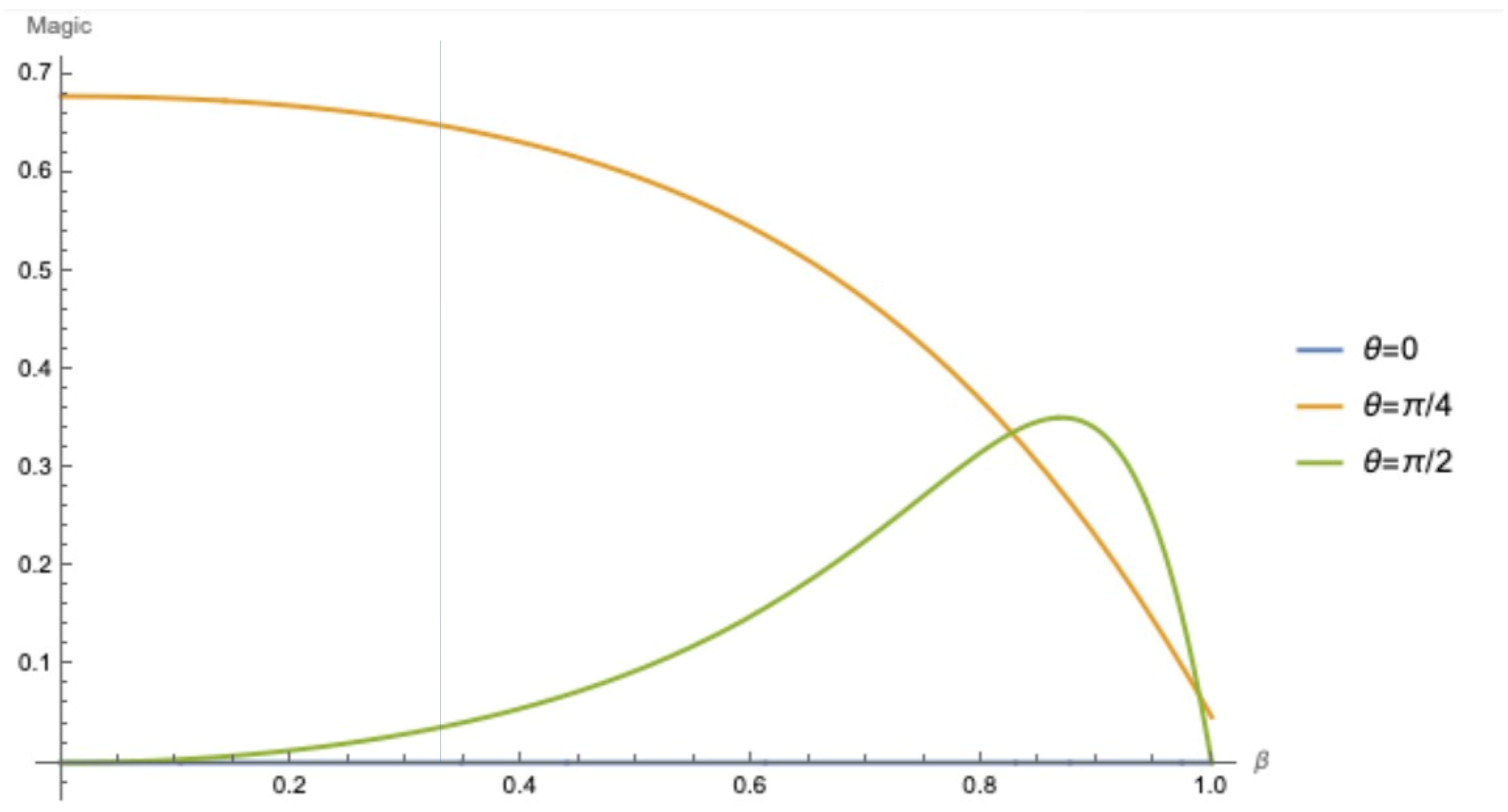
ividual partonic
n entanglement

- Averaging (PDF plus angular) leads to more mixed states → increases magic!

Future lepton colliders

- Entanglement in top quark production at future lepton colliders was previously studied in (Maltoni, Severi, Tentori, Vryonidou)





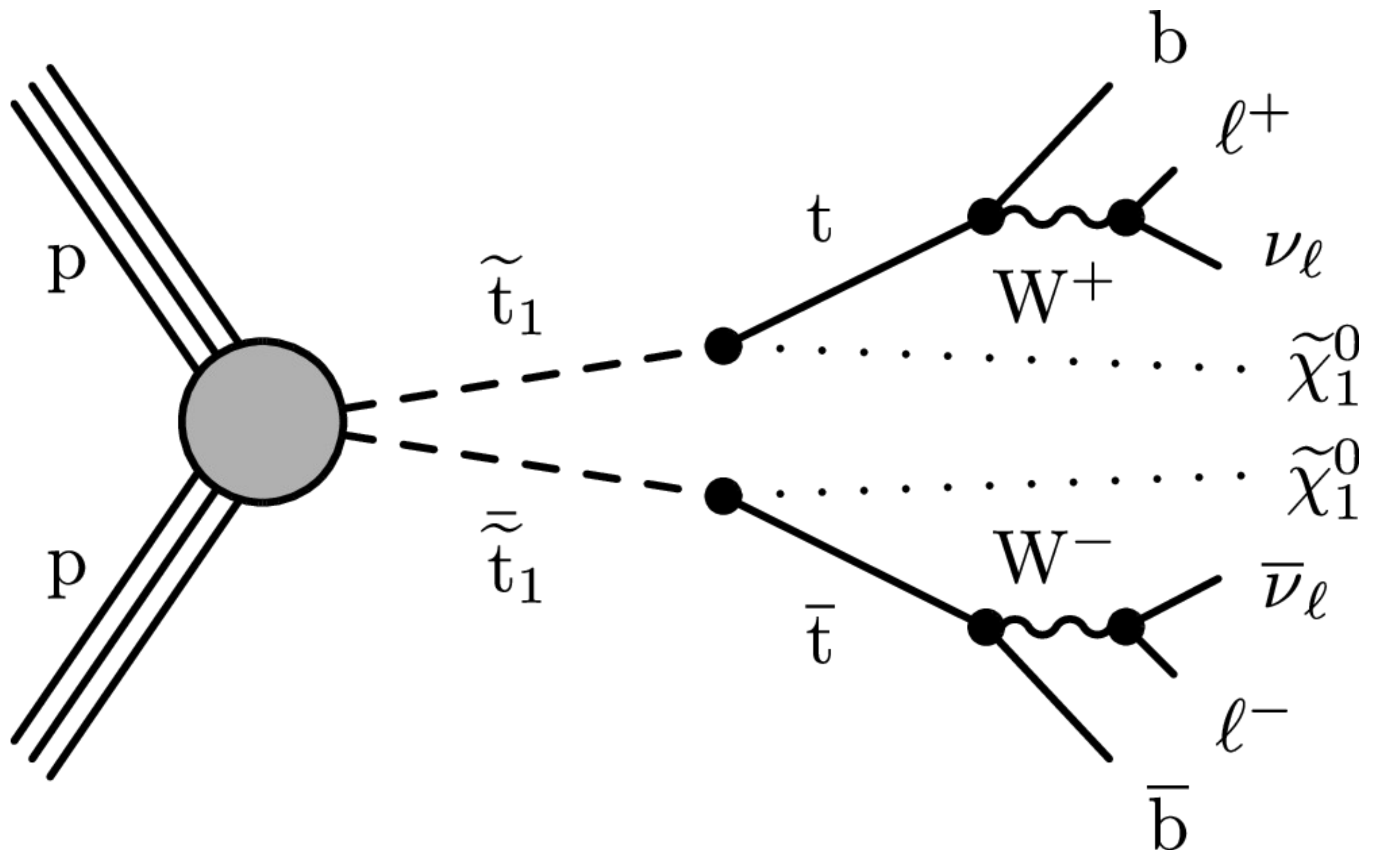
- Top quark pairs are now produced via EW processes → different quantum states are probed, complementary to LHC measurements
- Possible to observe non-zero magic over a wide range of centre of mass energies (dependent on θ)

Aoude, Banks, White[2]

PRELIMINARY

Beyond Standard Model physics

- The relationship between the spins of the two top quarks produced in a collider depends on the production density matrix
- If we go beyond the Standard Model, the production density matrix changes, and so *any* function of it changes
- Classic example: supersymmetry



- Tops produced via scalars → no spin correlations! The tops also acquire large individual polarisations (Maltoni, Severi, Tentori, Vryonidou)
- Take home message: quantum information observables *may* offer sensitivity to BSM physics (see also: Fabbrichesi, Low, Marzola)

SMEFT

- A standard “agnostic” way to parameterise beyond-Standard Model physics is to use Standard Model Effective Field Theory (SMEFT)

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_{n=1}^{\infty} \frac{1}{\Lambda^n} \sum_i c_i^{(n)} \mathcal{O}_i^{(n)}.$$

- First operators relevant for top pair production occur at *n* = 2 (mass dimension six)

- Assuming a unit CKM matrix and retaining only third generation Yukawa couplings, get these operators affecting gluon-induced top production

$$\mathcal{O}_G = g_s f^{ABC} G_\nu^{A,\mu}\, G_\rho^{B,\nu}\, G_\mu^{C,\rho}, \quad \mathcal{O}_{\phi G} = \left(\phi^\dagger \phi - \frac{v^2}{2}\right) G_A^{\mu\nu}\, G_{\mu\nu}^A,$$
$$\mathcal{O}_{tG} = g_s \left(\bar{Q}\sigma^{\mu\nu} T^A t\right) \tilde{\phi}\, G_{\mu\nu}^A + \mathrm{h.c.}$$

- For quark-induced production also have:

$$\mathcal{O}_{Qq}^{(8,1)} = \left(\bar{Q}_L \gamma_\mu T^A Q_L\right)\left(\bar{q}_L \gamma^\mu T^A Q_L\right), \quad \mathcal{O}_{Qq}^{(8,3)} = \left(\bar{Q}_L \gamma_\mu T^A \tau^a Q_L\right)\left(\bar{q}_L \gamma^\mu T^A \tau^a q_L\right),$$
$$\mathcal{O}_{tu}^{(8)} = \left(\bar{t}_R \gamma_\mu T^A t_R\right)\left(\bar{u}_R \gamma^\mu T^A u_R\right), \quad \mathcal{O}_{td}^{(8)} = \left(\bar{t}_R \gamma_\mu T^A t_R\right)\left(\bar{d}_R \gamma^\mu T^A d_R\right),$$
$$\mathcal{O}_{Qu}^{(8)} = \left(\bar{Q}_L \gamma_\mu T^A Q_L\right)\left(\bar{u}_R \gamma^\mu T^A u_R\right), \quad \mathcal{O}_{Qd}^{(8)} = \left(\bar{Q}_L \gamma_\mu T^A Q_L\right)\left(\bar{d}_R \gamma^\mu T^A d_R\right),$$
$$\mathcal{O}_{tq}^{(8)} = \left(\bar{t}_R \gamma_\mu T^A t_R\right)\left(\bar{q}_L \gamma^\mu T^A q_L\right), \quad \{\mathcal{O}_{Qq}^{(1,1)}, \mathcal{O}_{Qq}^{(1,3)}, \mathcal{O}_{ij}^{(1)}\}$$

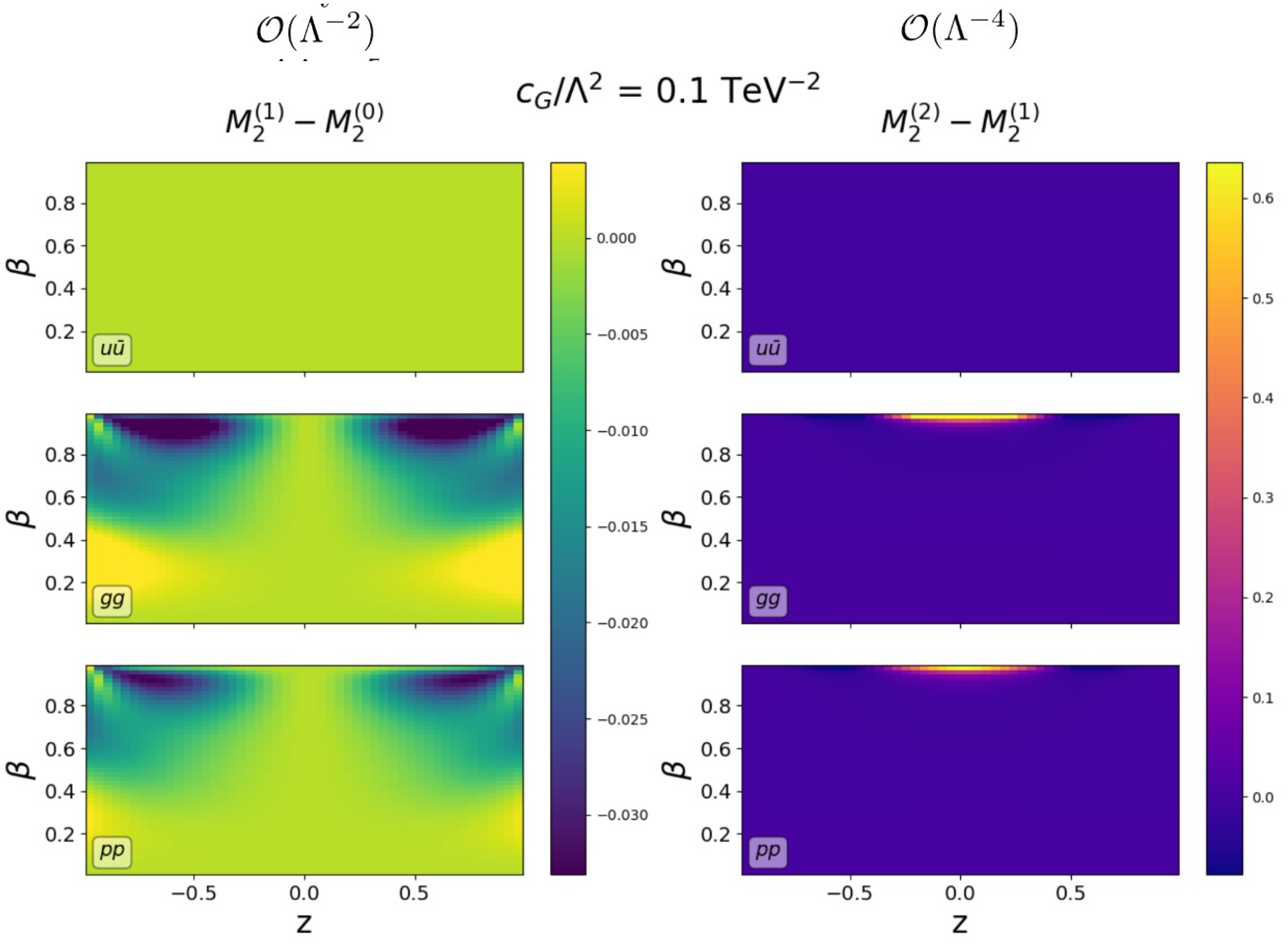
SMEFT and magic

- It is useful to separate out the SM and BSM contributions when calculating the normalised production density matrix

$$\rho^I = \frac{R^{I,SM} + R^{I,EFT}}{\mathrm{Tr}(R^{I,SM}) + \mathrm{Tr}(R^{I,EFT})}$$

- Can then expand the magic (or other observables) in the Wilson coefficients
- We do this at linear $(\mathcal{O}(\Lambda^{-2}))$ and quadratic $(\mathcal{O}(\Lambda^{-4}))$ order
- Use Fano coefficients previously presented in (Aoude, Madge, Tentori, Vryonidou)
- At linear order, possible to understand changes to the magic analytically
- At quadratic order, things get *very complicated very quickly!*
- Also take care at quadratic order: dimension 8 linear interference and double-EFT insertions contribute at the same order but are not yet included

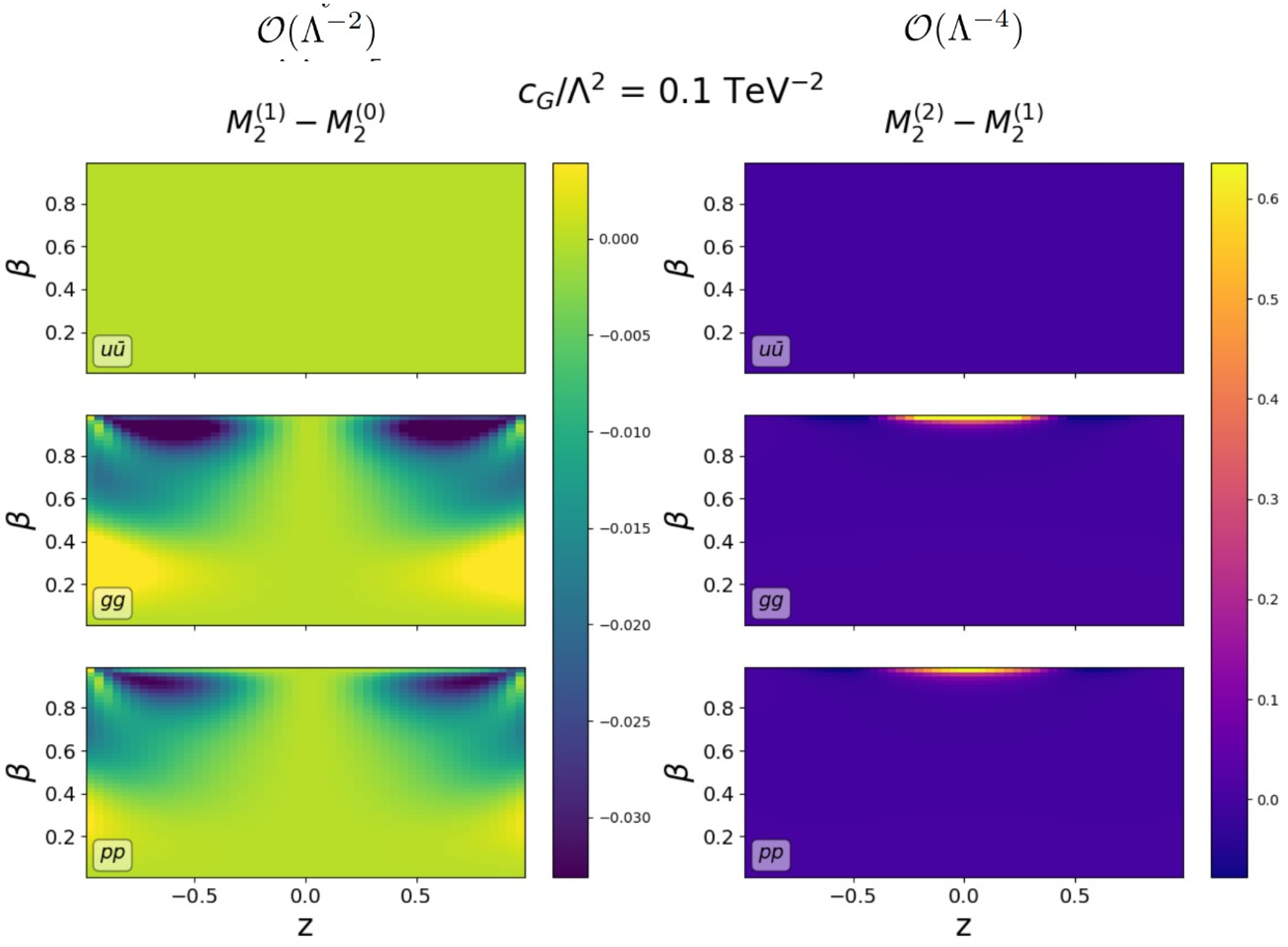
How does SMEFT change the magic?



- Example 1: For c_G (and $c_{\varphi G}$) no difference to the magic in the quark channel, since the corresponding operators only contribute to the *gg* channel
- In the *gg* channel, there are regions where the magic does not change
- e.g. when $\beta \to 0$, contributions to concurrence vanish at linear order
- Tops remain in a stabiliser state → magic does not change

Aoude, Banks, White[2]

How does SMEFT change the magic?

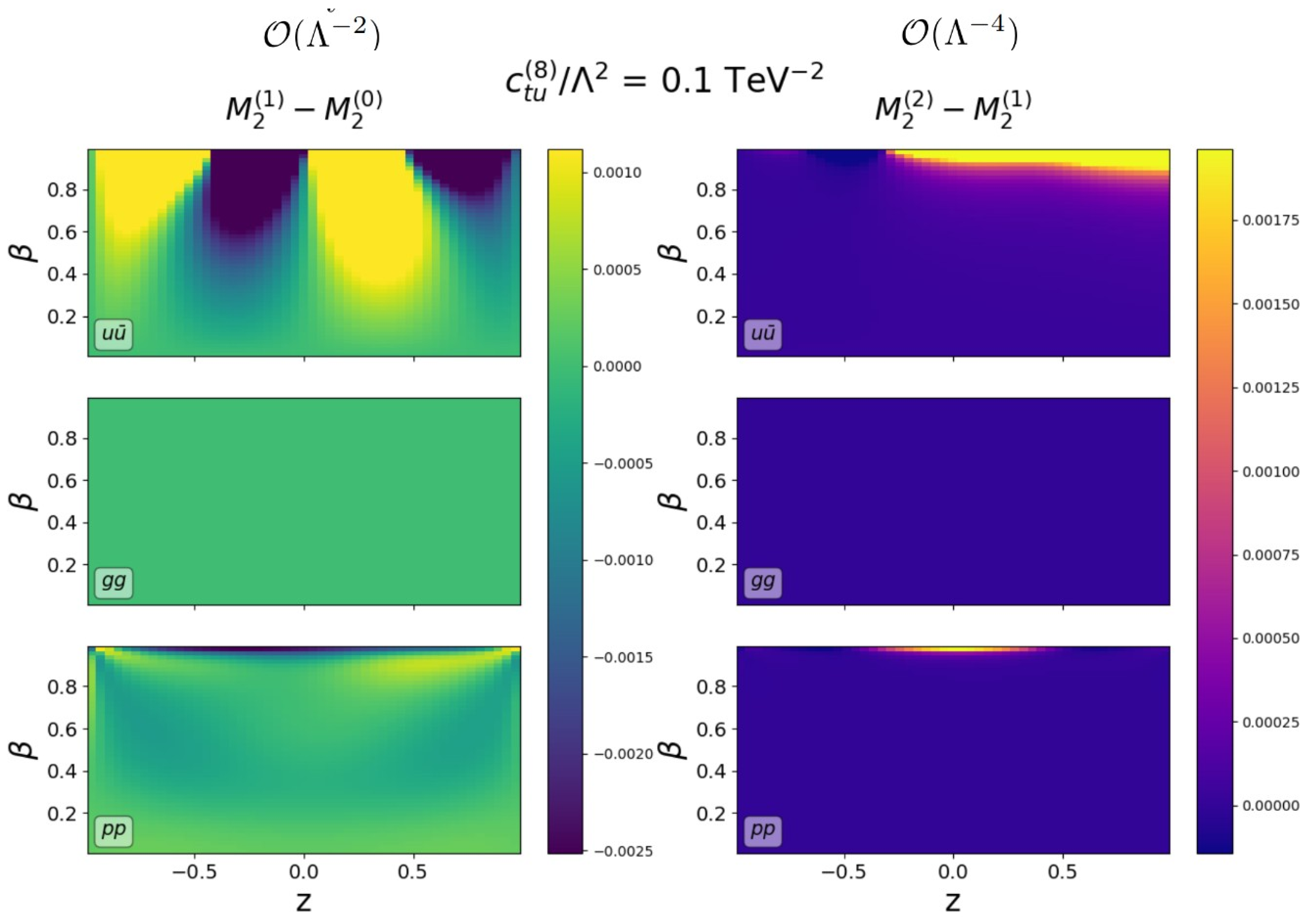


- Very small change in magic at threshold in the *gg* channel at quadratic order due to creation of triplet state on top of SM singlet state
- Largest changes at high β, but care must be taken with EFT validity (results are ok for $\beta < 0.96$)
- Quadratic corrections induce mixed states in the central region at high β (in SM the state is purely entangled)

Aoude, Banks, White[2]

PRELIMINARY

Example of four fermion operators



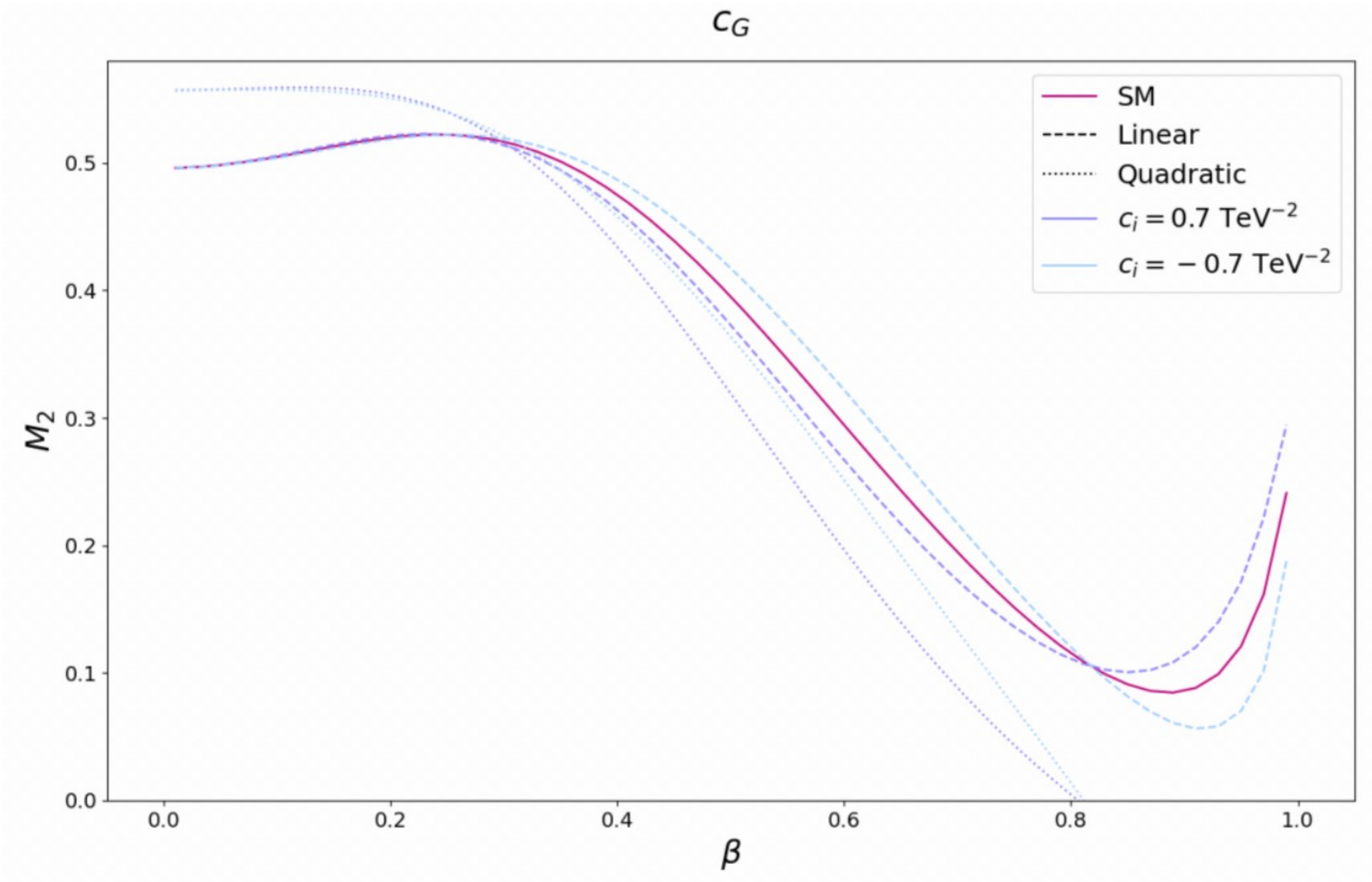
- No contribution to the *gg* channel as expected
- In quark channel, state becomes a stabiliser state at threshold in the SM, c_{tu} only provides a small correction
- Away from threshold, get a very interesting pattern of corrections that change sign depending on the scattering angle → pp is dominated by *gg* however
- In general: can get non-trivial cancellations once all operators are turned on

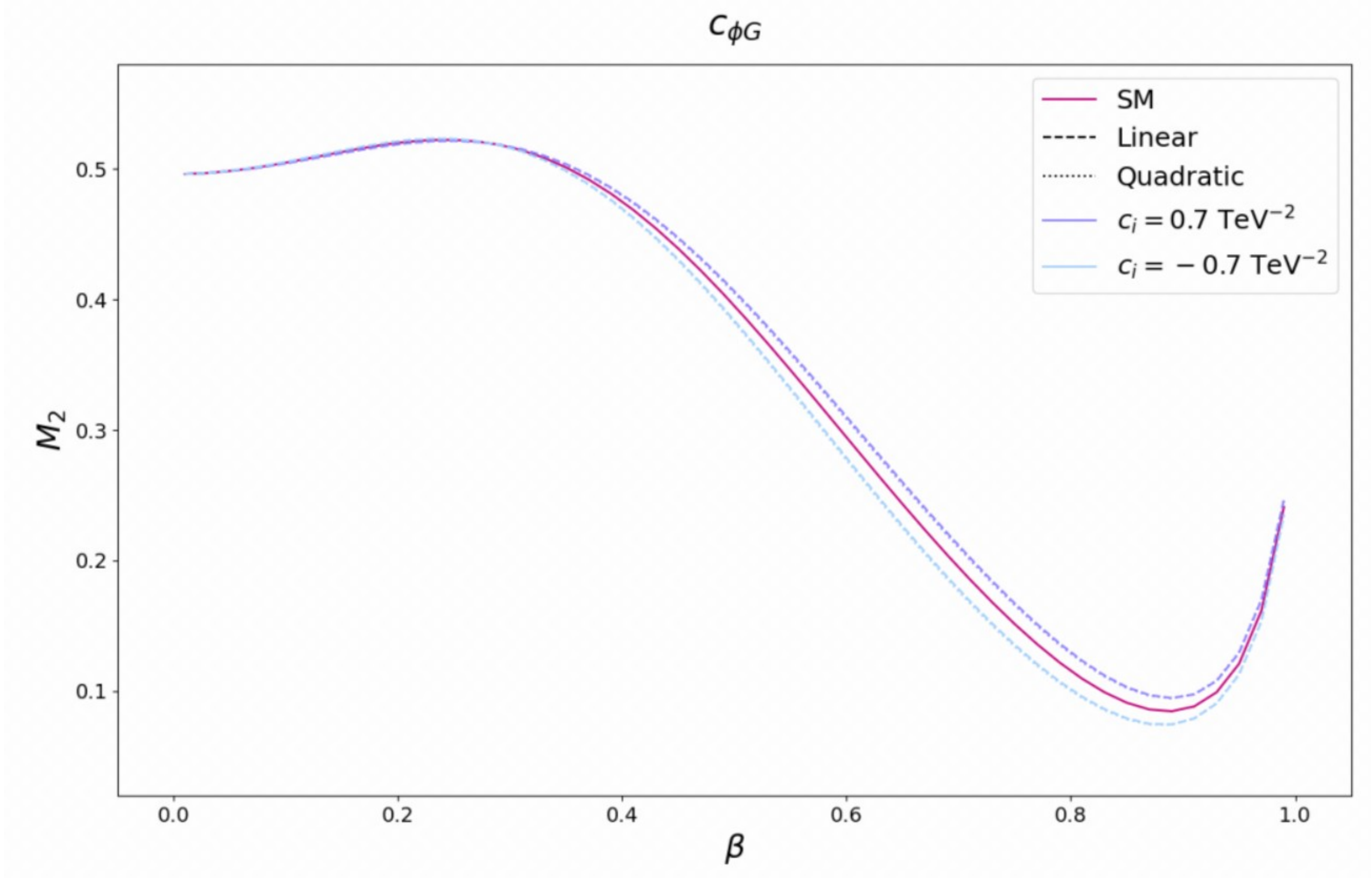
Aoude, Banks, White[2]

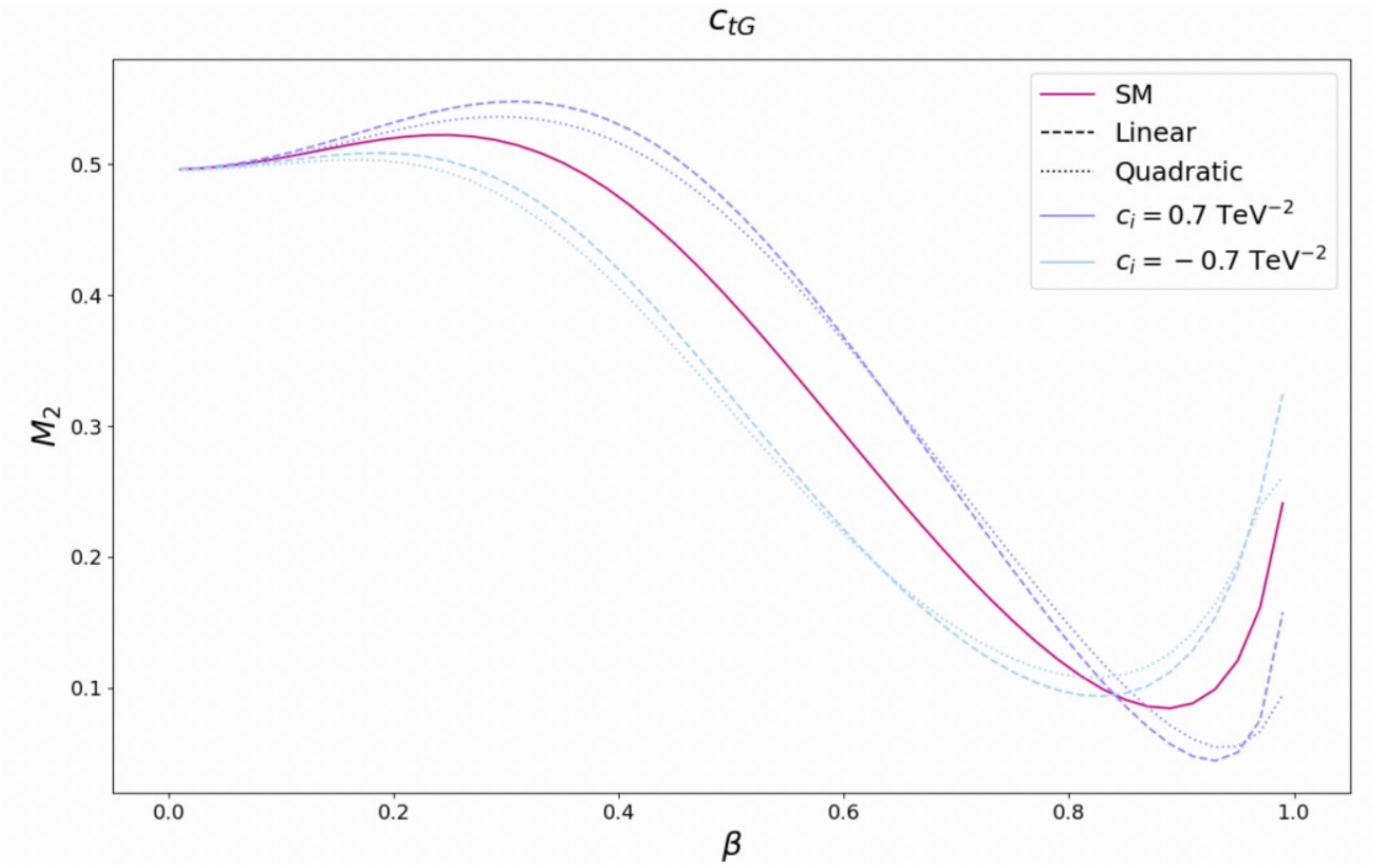
PRELIMINARY

Angular-averaged results

- Confirm that largest contributions arise from gluon operators







Thoughts/future directions

- We're still studying the efficacy of magic for *actually* gaining sensitivity to BSM physics
- We're also comparing magic to other quantum information observables (and canonical spin-sensitive variables)
- Remains to be seen if QI gives us something we don't get by other means in the hunt for BSM physics – it will be model-dependent so we're also checking specific UV models along with SMEFT operators
- Lots to think about/work on!

Conclusions

- Magic is a property of quantum states that distinguishes computational advantage over classical computers.
- We have shown that top quark pairs are naturally produced in magic states at the LHC (and CMS have *actually* shown it – see next talk!)
- Preliminary results show how the magic changes in the presence of BSM physics – offers a potential new window to BSM physics effects
- Can also obtain complementary measurements of magic in the top pair system at future lepton colliders