The Page-Wootters scheme of time emerging from quantum entanglement

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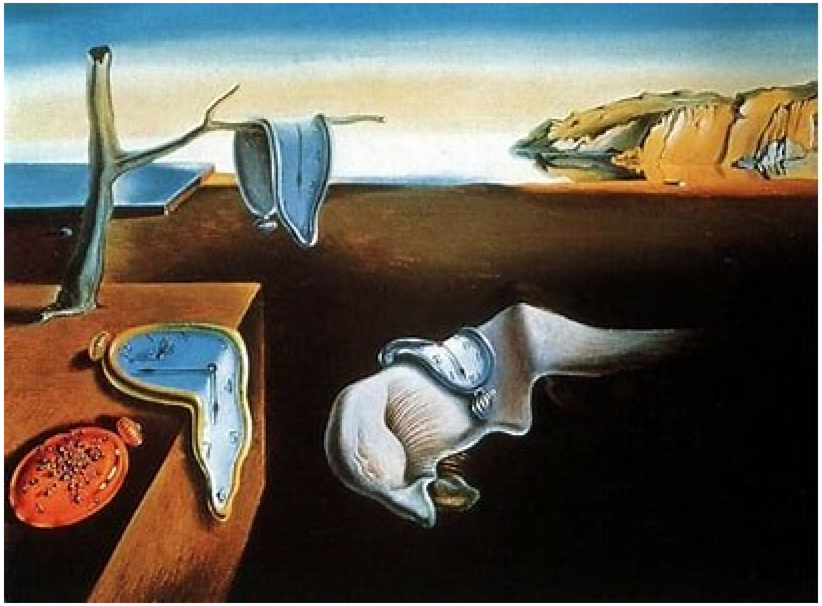
πάντα ῥεῖ ὡς ποταμός **[Eraclitos]**

"It is what passes (*shi* 逝) like that, indeed, not ceasing day or night." **[Confucius, The Analects, 2491]**

Quid est ergo tempus? si nemo ex me quaerat, scio; si quaerenti explicare velim, nescio.

Aurelius Augustinus Hipponensis, Confessiones, XI, 14



Tempus item per se non est, sed rebus ab ipsis consequitur sensus, transactum quid sit in aevo, tum quae res instet, quid porro deinde sequatur

Titus Lucretius Carus, De Rerum Natura

[Time also exists not by itself, but simply from the things which happen the sense apprehends what has been done in time past, as well as what is present and what is to follow after.]

- Earlier ideas of **Parmenides of Elea** (Παρμενίδης ὁ Ἐλεάτης): the phenomena of movement and change are simply appearances of a static, eternal reality.

Time in modern philosophy

[Del Santo, Gisen arXiv 2404.06566
Le Bihan Qu'est ce que le temps?]

- **A Theory:** monadic attribution. An event is present, past or future **(flux of time)**

- **Presentism** **only present objects exist, present becomes past, future becomes present**
- **Dynamical Eternalism**

Moving spotlight: events to be set on a fixed manifold, the present being singled out as it was lit by a light moving along the manifold

But if only present is real, what about propositions about past?
How to build «verificators» about these propositions?
Verificators about past can only exist in present for a presentist
What about trans-temporal relations?

Deflationist Presentism : propositions about past are false or undefined :
In what future differs from past? No knowledge of past?
Reductionist Presentism: past as a new methaphysic category
No clear intuition, Ockham's rasor
…

Hybrid-Theories

- **Growing block**: events come into existence at present, but persist in past

Future is not existing

Then a proposition not true with the present bloc (now + past) will become true.
At the end this requires two time : one, **objective time**, describing the evolution of the bloc past-present and an **indexical time** corresponding to perception of time inside the bloc: proliferation of entities (Ockham's razor)

Problems with the inacessability of objective time and on the definition of «status» of past

In different reference frames different simultaneity hyperplanes: how do I define the «present»?

- **B-Theory**: dyadic relation **(time exists but doesn't flow).** Given two events E_1 E_2

E_1 before E_2 E_1 after E_2 or E_1 simultaneous to E_2

- **Eternalism** Bloc universe. Only temporal relations exist. Past, Present and Future are all ontologicallly real

largely anti-intuitive: future contingency?

A-Theories

Presentist

Presentisme deflationist (propositions about past are false o undertimed), **presentism reductionist minimal** (propositions about past are true but without past verificators)**, presentism reductionist inflationist** (propositions about past are true but with past verificators but outside nature laws)**, presentism** ersatzist (universe at certain time is an abstract object, a set of consistent propositions), ..

Eternalist

Moving light spot, …

Hybrid Theories

Non-futurism (block theory, but with undetermined future), **non-futurism à la Forest** (only persons living **now** are conscious et live really)**, growing block (**past and present are a growing block), **dynamic tree universe** (Branching of possible futures with a fixed past)**, …**

B-Theories

Eternalist

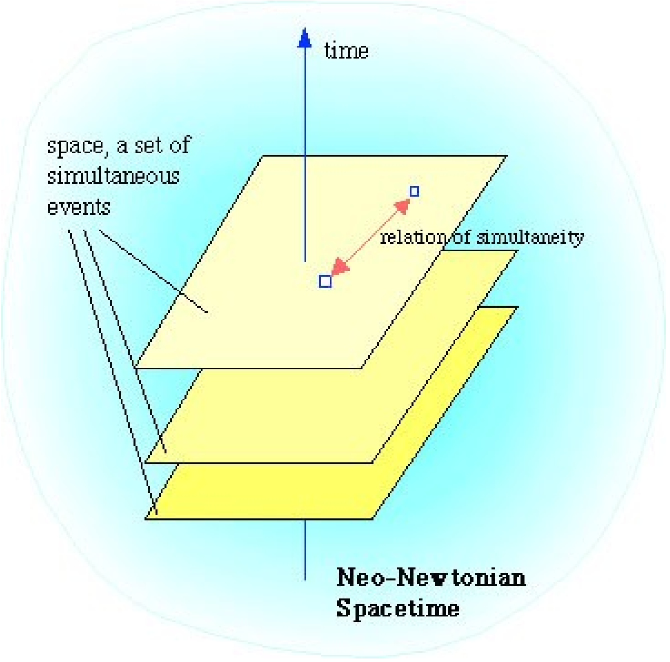
Multiple futures, modal realism (every possibility of existence of the universe it exists a universe) **, counterparts theory** (different modalities are individuals retaining a relation of similarity), **static tree universe** («fixed» branching tree of events, past and future) **(same ontological meaning of futures, «red line theory»)**, **determinations cloud, modal conventionalism** (future closed by «linguistic conventions», open by reality structure), **…**

Time in Physics

- Classical Mechanics Absolute Time (non relativistic QM): a fixed background parameter
- SR Time: a proper time for each observer, time as 4^{th} coordinate (set of privileged inertial frames)
- GR Time: not at all an absolute time. Time is a general spacetime coordinate (time non-orientability, closed timelike curves…)

Classical (deterministic) mechanics
Absolute Time
a fixed background parameter
(B-theory)

Absolute, true, and mathematical **time**, from its own nature, passes equably without relation to anything external, and thus without reference to any change or way of measuring of time



Non relativistic quantum mechanics
Absolute Time

again a fixed background parameter, but
Interpretation of time "interpretation dependent"

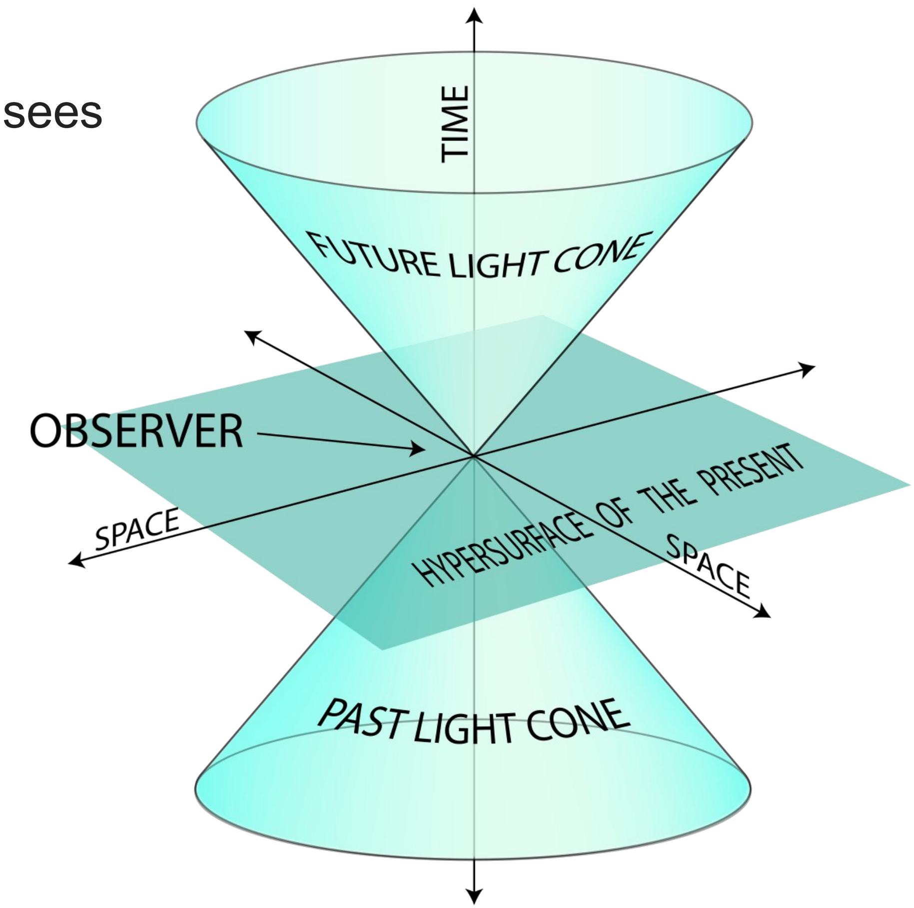
Properties emerge by quantum measurement
e.g. Copenhagen interpretation
Multiple futures possible, the collapse of wave function determines one
A theory (with eternalism)

But not for multi-universe (B theory, static tree universe, …)

SR Time: a proper time for each observer, time as 4th coordinate (set of privileged inertial frames)

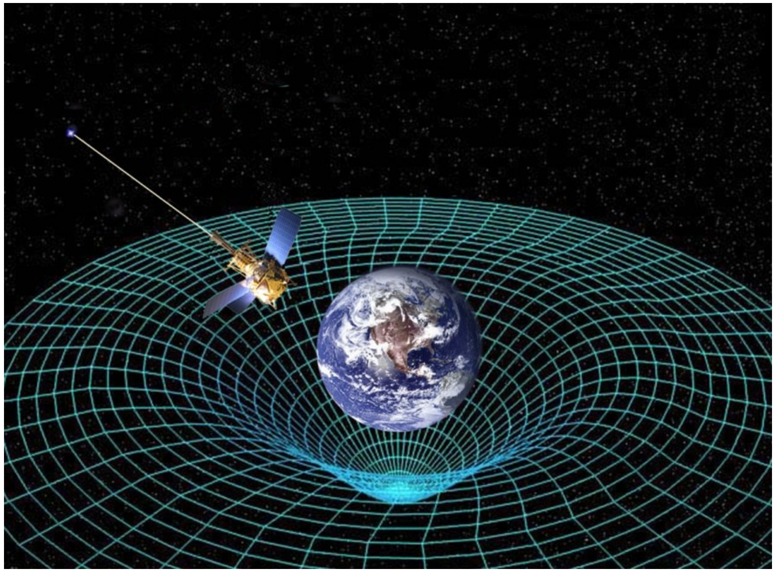
proper time : time that an observer sees on a clock at rest

$$\begin{aligned}\Delta\tau &= \int_P \frac{1}{c}\sqrt{\eta_{\mu\nu}\,dx^\mu\,dx^\nu} \\ &= \int_P \sqrt{dt^2 - \frac{dx^2}{c^2} - \frac{dy^2}{c^2} - \frac{dz^2}{c^2}} \\ &= \int \sqrt{1 - \frac{1}{c^2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]}\,dt \\ &= \int \sqrt{1 - \frac{v(t)^2}{c^2}}\,dt = \int \frac{dt}{\gamma(t)},\end{aligned}$$



GR Time: not at all an absolute time. Time is a general spacetime coordinate

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{\mu\nu}\, dx^\mu\, dx^\nu}$$

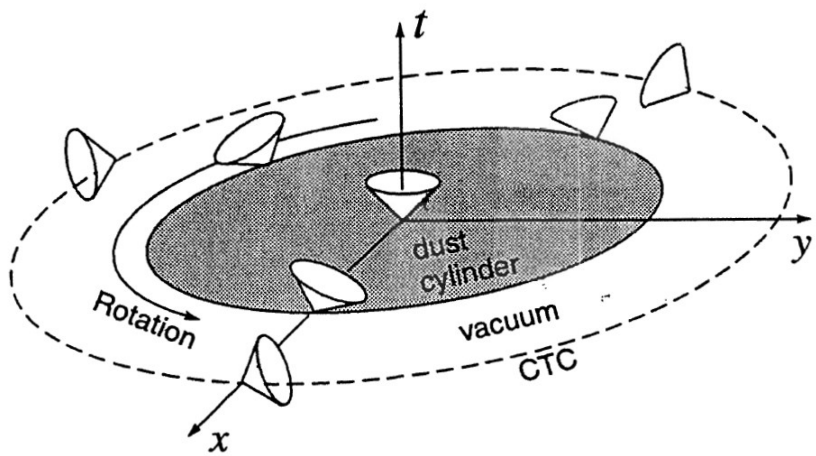


Problems in cosmology

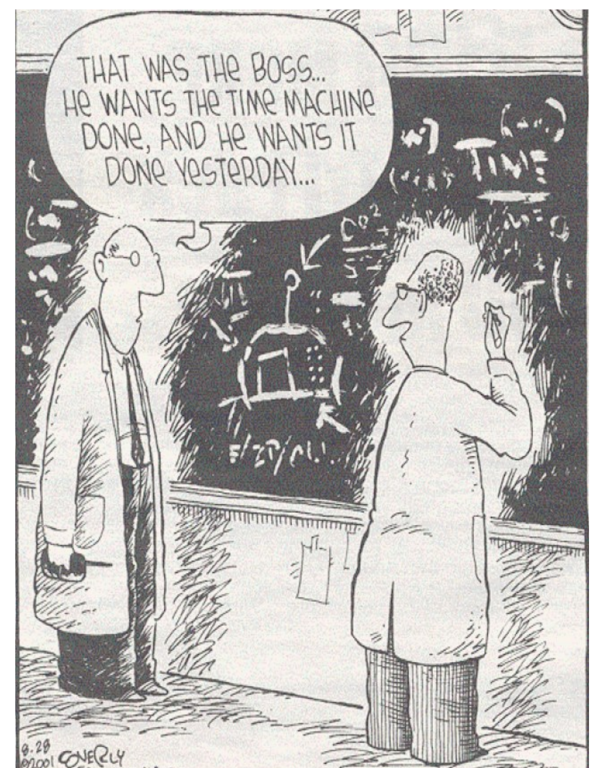
If it exists a «cosmic time»: against B theories in favour of a theory A eternalist such as «moving spotlight»

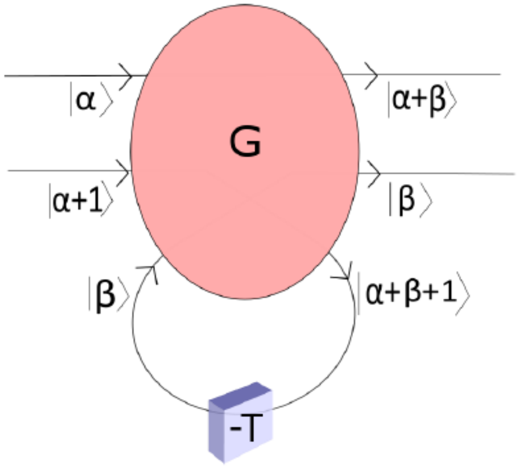
Time non-orientability: closed timelike curves…: no A theory…

Closed timelike curves



Grand father paradox





α (β) = 0, 1 (presence, lack in the path)

Problem of consistency in classical case

Interaction

$$|\alpha\rangle\,|\alpha+1\rangle\,|\beta\rangle \;\to |\alpha+\beta\rangle\,|\alpha+\beta+1\rangle\,|\beta\rangle$$

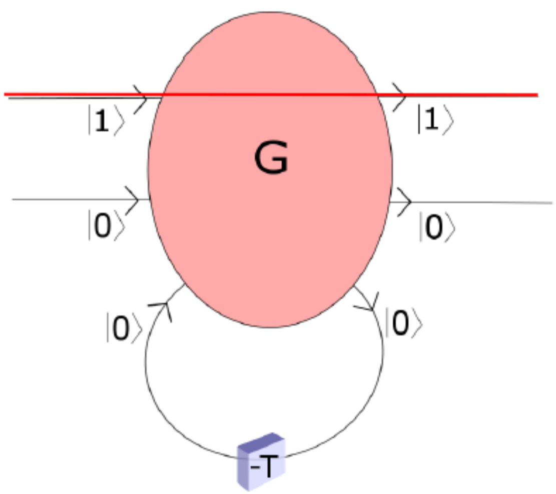
Consistency condition

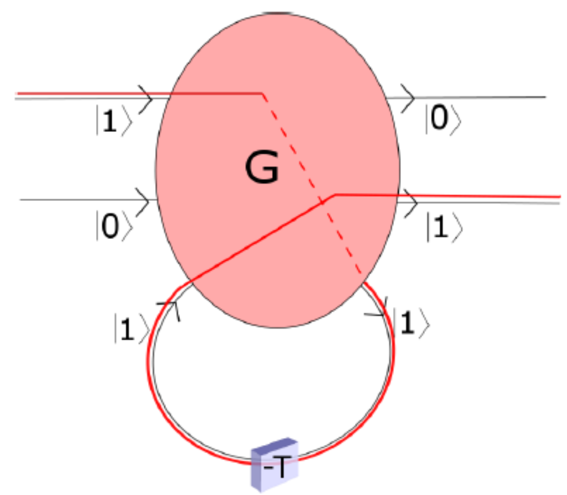
Two paths, lower one going to CTC

$$|\beta\rangle = |\alpha+\beta+1\rangle$$

α = 0 no consitent solution

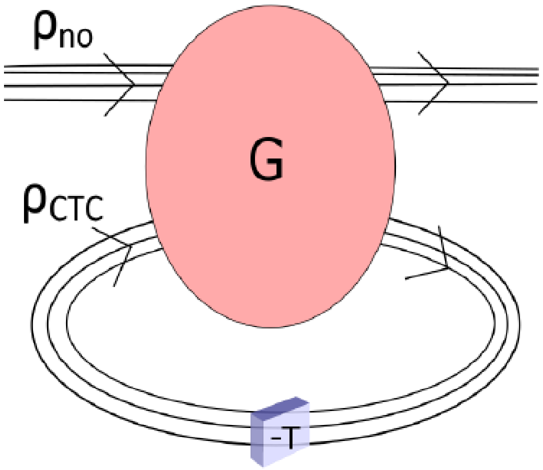
α =1 possible consistency





Solution thanks to quantum superpositions

[Deutsch Phys. Rev. D 44, 10 (1991)]



$$Tr_{no}\left(U(\rho_{no}\otimes\rho_{CTC})U^{\dagger}\right)=\rho_{CTC}.$$

Tracing out the degrees of freedom corresponding to the system not crossing CTC one must find ρ(CTC)

But: non-linearity due to CTC

- perfect discrimination of non-orthogonal states
- violation of the uncertainty principle
- violation of no-cloning theorem
- ....

Problems even in an open time-like curve (OTC), when the qubit does not interact with its past copy, but it is initially entangled with another, chronology-respecting, qubit.

To avoid **violating entanglement monogamy**, one has to postulate a non-linear evolution

Alternative approach to OTCs, to preserve linearity and avoid all other drastic consequences → qubit state in the OTC described by pseudo-density operator

[C.Marletto, et al., Nature communications 10 (2019) 182

Problems with closed systems: the whole universe

In canonical quantum gravity general relativistic evolution and constraints →
Wheeler – De Witt equation for “the universe wave function” **ψ**

$$\mathbf{H_{tot}\,\psi = 0}$$

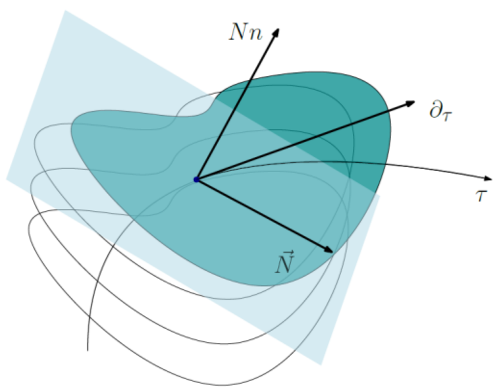
Stationary equation!!

[E.Anderson, Ann.Phys. 524 (2012) 757;R.Sorkin, Int J.Th.Phys 85 (1994) 523; W.Unruh and R.Wald,PRD 40 (1989) 2598;…]

B-theory- Bloc universe eternism

Sketch of derivation:

- Hamiltonian and Lagrangian formalism should be equivalent, but there may be issues on the inversion of Legendre transform (e.g. singularity Hessian matrix $\partial L / \partial(\partial_0 q_i)\partial(\partial_0 q_j)$ → constraints
- ADM formalism for GR: foliate Lorentzian space-time in space like manifold (timelike) Σ



N (shift vector)
Nn (lapse vector)

- Einstein – Hilbert Action $S = \int d^4x \, 1/(16\pi) \sqrt{g} \, R$ is rewritten in terms of the metric h on Σ and the «extrinsic curvature» K where $R^m{}_{ijk} = {}^3R^m{}_{ijk} + K_{jk}K_i{}^m - K_{ik}K_j{}^m$

$$\mathcal{L}_{ADM} = \frac{1}{16\pi} N\sqrt{h}({}^3R + K_{ij}K^{ij} - K^2)$$

- Due to the independence of K on derivatives of N (shift vector) and Nn (lapse vector) → constraints

$$\Pi := \frac{\partial \mathcal{L}_{ADM}}{\partial(\partial_\tau N)} \approx 0 \qquad \Pi_k := \frac{\partial \mathcal{L}_{ADM}}{\partial(\partial_\tau N^k)} \approx 0$$

- Defined

$$\Pi^{ij} := \frac{\partial \mathcal{L}_{ADM}}{\partial(\partial_\tau h_{ij})} \qquad G_{ijrs} = \frac{1}{2\sqrt{h}}(h_{ir}h_{js} + h_{is}h_{jr} - h_{ij}h_{rs})$$

$$\mathcal{H} := 16\pi G_{ijrs}\Pi^{rs}\Pi^{ij} - \frac{1}{16\pi}\sqrt{h}\,{}^3R \qquad \mathcal{H}_k := -2h_{kj}\,{}^3\nabla_i\Pi^{ij}$$

$$H_{ADM} = \lambda\Pi + \lambda^i\Pi_i + N^k\mathcal{H}_k + N\mathcal{H} + 2\,{}^3\nabla_i(\Pi^{ij}N^k k_{kj})$$

- The form of H_{ADM} leads to

$$0 \approx \{\Pi, H_{ADM}\} \qquad 0 \approx \{\Pi_k, H_{ADM}\}$$

- Quantizing Poisson brackets

$$[\widehat{h}_{ij}(x^0,x), \widehat{\Pi}^{k\ell}(x^0,y)] = \frac{i}{2}(\delta_i{}^k\delta_j{}^\ell + \delta_i{}^\ell\delta_j{}^k)\delta^{(3)}(x-y)$$

$$[\widehat{N}(x^0,x), \widehat{\Pi}(x^0,y)] = i\delta^{(3)}(x-y)$$

$$[\widehat{N}^i(x^0,x), \widehat{\Pi}_j(x^0,y)] = i\delta^i{}_j\delta^{(3)}(x-y),$$

- Constraints $\Pi \approx \Pi_k \approx 0$ lead to $-i\frac{\partial \Psi}{\partial N} = -i\frac{\partial \Psi}{\partial N^k} = 0$

for a generic functional $\Psi[h_{ij}, N^i, N]$

- And finally to Wheeler - De Witt equation

$$\widehat{\mathcal{H}}\Psi = -\left[: 16\pi \widehat{G}_{ijk\ell}(x) \frac{\partial^2}{\partial h_{ij}(x)\partial h_{k\ell}(x)} : + \frac{1}{16\pi}\sqrt{h}^{\,3}\widehat{R}(x)\right]\Psi[h_{ij}] = 0$$

A POSSIBLE WAY OUT

[D.Page and W.Wootters,PRD 27 (1983) 2885; W.Wootters, IJTP 23 (1984) 701]

It may exist states of a system composed by **entangled subsystems** that are **stationary**, but **subsystems** can be interpreted as **evolving**

Stationary state ρ **$[H, \rho] = 0$**

Let decompose the system in "a clock" with Hamiltonian H
The rest, with Hamiltonian H'

Let choose a special state of the clock to mark 0 time $| \psi_c (0) >$, $H=H_c \otimes I_r + H' \otimes I_c$

For each state

$| \psi_c (t) > = \exp(- i H_c t) | \psi_c (0) >$

one associates time t

For any stationary observable A of the rest of the system the conditional expectation value E(A,t) when the clock reads t is

$$\mathbf{P_t = | \psi_c(t) > < \psi_c(t) | \otimes I_r}$$

$$\mathbf{E(A,t) = Tr(A P_t \rho) / Tr(P_t \rho) = Tr(A \exp[-i(H_c \otimes I_r)t] P_0 \exp[i(H_c \otimes I_r)t] \rho) / Tr(P_t \rho) =}$$

$$\mathbf{Tr(\exp[i(I_c \otimes H')t] A \exp[-i(I_c \otimes H')t] P_0 \rho) / Tr(P_t \rho)}$$

Where one used [H, ρ] = 0

Thus one recovers Heisenberg's evolution

$$\mathbf{E(A,t) = Tr[\exp[i(H')t] A \exp[-i(H')t] \rho_r]}$$

$$\mathbf{\rho_r = Tr_c(P_0 \rho) / Tr(P_t \rho)}$$

Gambini Pullin scheme [PRD 79 (2009) 041501]

Extension for multi-time measurements

The conditional probability of obtaining an outcome d for an observable at «time» t

$$p(d|t) = \frac{\int dT \ \mathrm{Tr}[P_{d,t}(T)\rho]}{\int dT \ \mathrm{Tr}[P_t(T)\rho]}$$

is extended to multi-time measurements

$$p(d = d'|t_f, d_i, t_i) = \frac{\int dT \ \int dT' \ \mathrm{Tr}[P_{d',t_f}(T) P_{d_i,t_i}(T') \ \rho \ P_{d_i,t_i}(T')]}{\int dT \ \int dT' \ \mathrm{Tr}[P_{t_f}(T) P_{d_i,t_i}(T') \ \rho \ P_{d_i,t_i}(T')]}$$

Further developments

- - Compatibility of canonical typicality (i.e. for all pure states in which Universe can be, after tracing over the environment a small subsytem is described by canonical distribution) [Favalli, Smerzi PRD 105 (2022) 023525]
- Time emerging from interation with Environment [Gemsheim, Rost PRL 131 (2023) 140202]
- Attempts toward extensions to emergent space-time [de la Hamette et al., arXiv:2110.13824; Ahmad et al, Phys. Rev. Lett. 128, 170401 (2022); Hoehn, et al . Math. Phys. 63, 112207 (2022).Favalli & Smerzi, AVS Quantum Sci. 4, 044403 (2022)]
- …

OTHER (EVENTUALLY RELATED) PROPOSALS

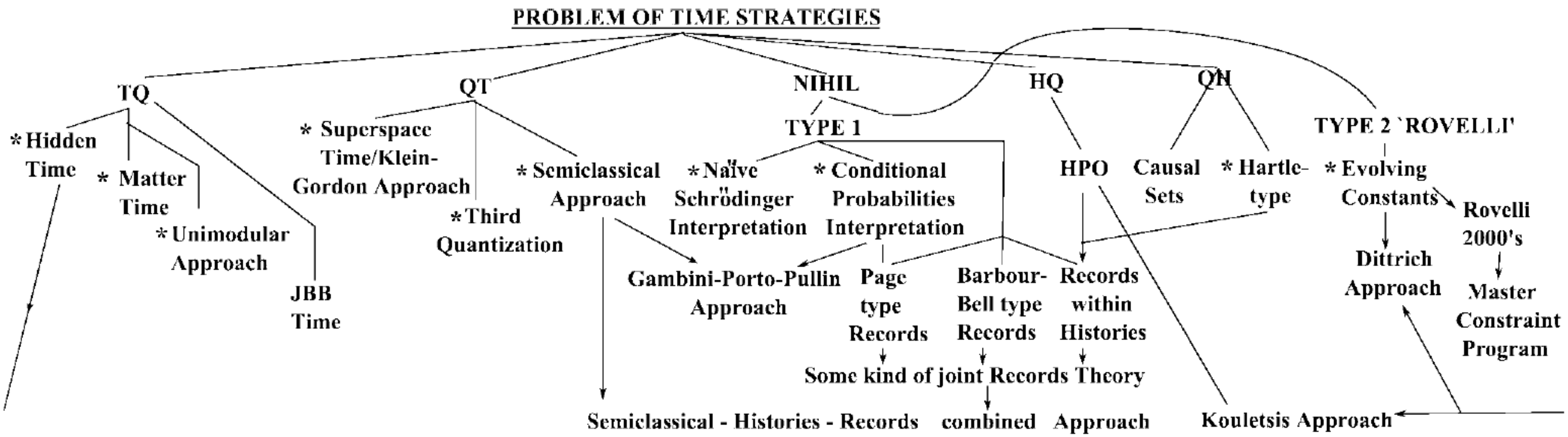
-**Problem of Time in quantum Gravity [E.Anderson, Ann.Phys. 12 (2012) 757]**

-***There is a fundamental time:*** **apparent frozenness is a formalism dependent statement [Kuchar, …], Reference matter time [Anderson,…], cosmological constant as a reference fluid [Unruh Wald,…],…**

- ***Time emerging in quantum regime:*** **semiclassical approaches [Haliwell & Hawking,..]**

- ***"Timeless approaches "*** **[Barbour, Rovelli], …**

- ***Quantum hystories*** **[Gell-Mann & Hartle, …]**



E.Anderson

Ann. Phys. (Berlin) 524, No. 12, 757–786 (2012) / DOI 10.1002/andp.201200147

AN EXPERIMENTAL APPROACH

E.Moreva,M.Gramegna,G.Brida,V.Giovannetti,L.Maccone,M.Genovese ***Phys. Rev. A 89(2014) 052122***

We want to show that a static state can have subsystems that evolve and one subsytem can act as clock for the others

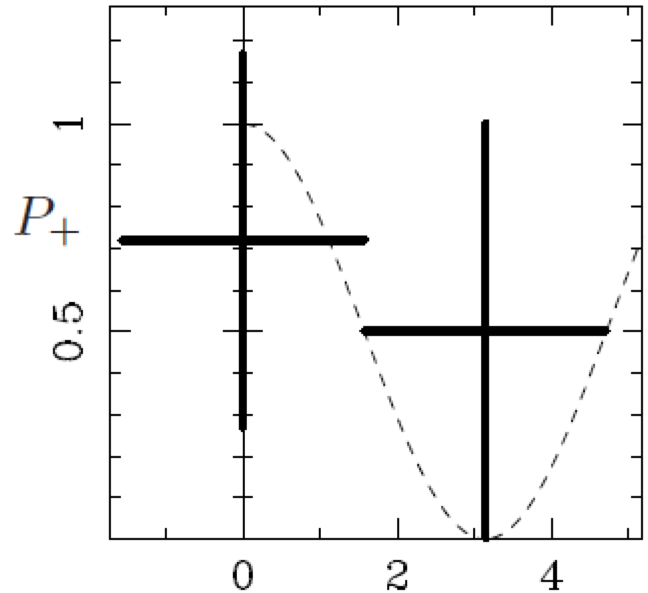
For this purpose we consider

$$|\psi^{-}\rangle = \frac{(|H\rangle|V\rangle - |V\rangle|H\rangle)}{\sqrt{(2)}}$$

The polarisation of one photon is the “clock”.

The clock is initialised in | + > and then evolves as

$$|+\rangle = |0\rangle + |1\rangle \rightarrow |0\rangle + e^{i\delta}|1\rangle$$

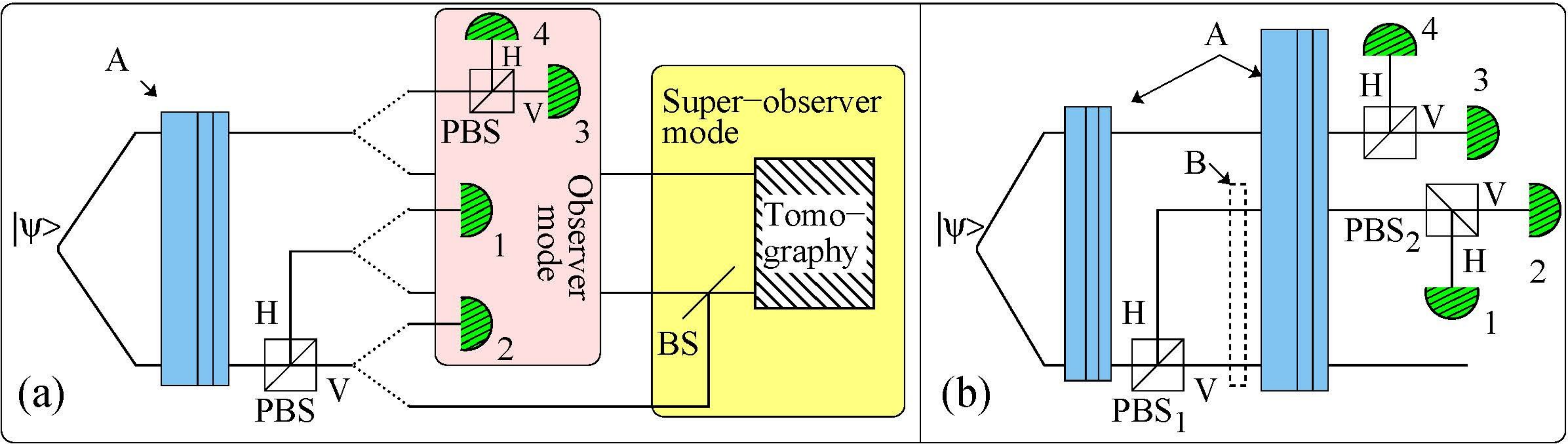


clock readout measurement in the basis

|+> t= 0

|-> t = π/(2 ω)

Once time has been measured from clock photon we use it to prove that the other photon's polarisation evolves: making many measurements for several birefringent material thickness we have a set of couples (0,+), (π/(2 ω),-), (π/(2 ω),+), …

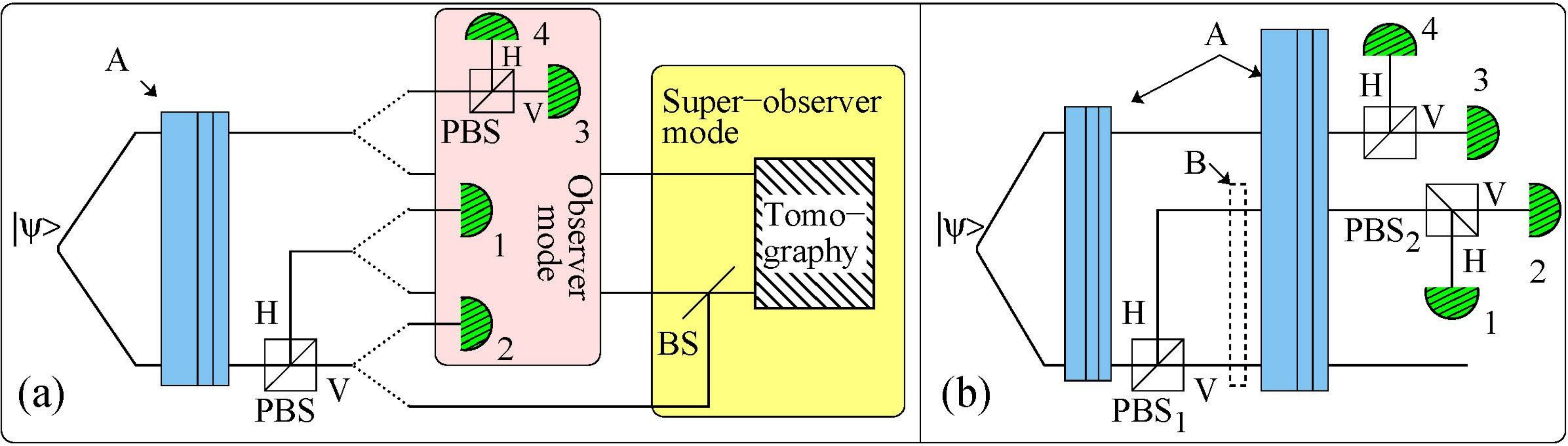


Page - Wootters scheme

In **observer mode** (a, pink box) ***the clock is the polarization*** of a photon.
It is an extremely simple clock: it has a dial with only two values, either |H> (detector 1 clicked) corresponding to time t = t1, or |V> (detector 2 clicked) time t=t2.

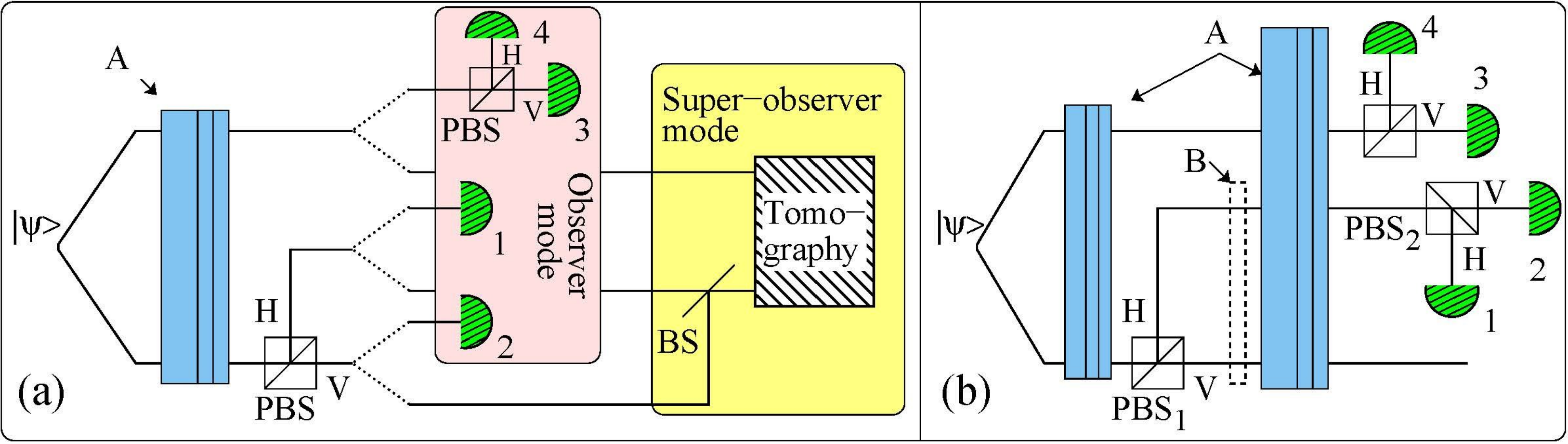
The experimenter also measures the polarization of the first photon with detectors 3 and 4.

This last measurement can be expressed as a function of time (t1,t2: he has access to time only through the clock photon) by considering the correlations between the results from the two photons



In **super-observer** mode (a, yellow box) the experimenter takes the place of a hypothetical observer external to the universe that has access to the abstract coordinate time and tests the global state of the universe.
He must perform a quantum interference experiment that tests the coherence between the different histories (wavefunction branches) corresponding to the different measurement outcomes of the internal observers.

In our setup, this interference is implemented by the BS: a quantum erasure experiment



Gambini Pullin (GPPT) scheme [R. Gambini et.al., PRD79 041501(2009)] (b).

Two times measurements implemented by the two PBS.
PBS1: initial time measurement (non-demolition measurement obtained by coupling the photon
polarization to its propagation direction; initialization of the system state implemented through entanglement).
PBS2: final time measurement (together with detectors 1 and 2)

Between these two time measurements both the **system and the clock evolve freely** (birefringent plates A).

The abstract coordinate time (the thickness of the quartz plates A) is inaccessible
As before, the time dependent probability of finding the system photon vertically polarized is p(t1)=P{3|1} and p(t2)=P{3|2}.

THE EXPERIMENTAL SET-UP

- The experimenter does not have access to external clocks (no knowledge of thickness)

- He considers correlation between detectors (coincidences P_{14} (+,0) … for several thicknesses)

showing the evolution of the second photon for the "two times measured by the clock"

-He demonstrates that the total state does not evolve by erasing "the clock photon" qubit with a second beam splitter

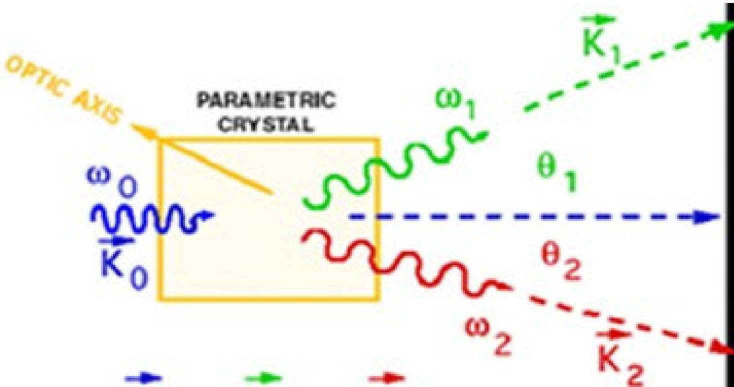
If | + > (upper arm) = |1000> (upper arm)
| + > (lower arm) = |0010> (upper arm)

|- > (upper arm) = |0100 >
|- > (lower arm) = |0001 >

The effect of a balanced BS

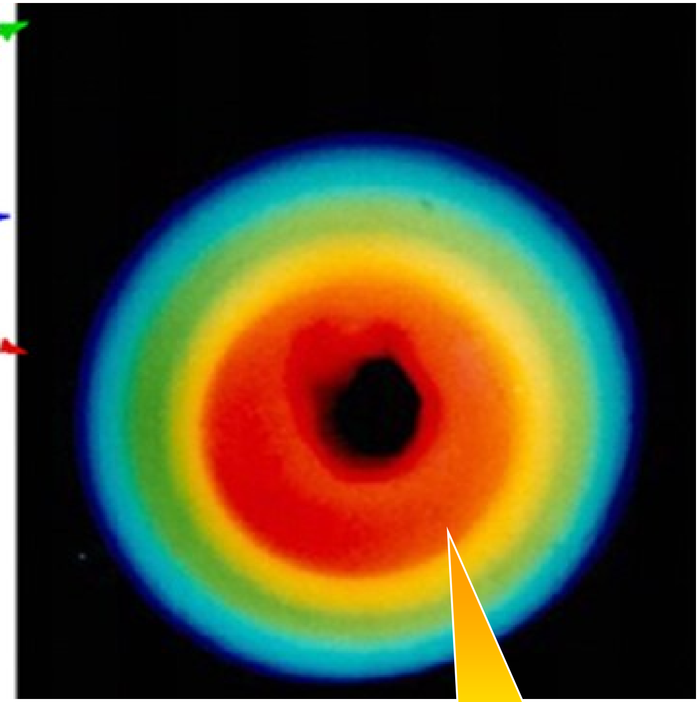
$$
\begin{aligned}
|1000\rangle &\rightarrow (|1000\rangle - |0010\rangle)/\sqrt{2} \\
|0100\rangle &\rightarrow (|0100\rangle - |0001\rangle)/\sqrt{2} \\
|0010\rangle &\rightarrow (|1000\rangle + |0010\rangle)/\sqrt{2} \\
|0001\rangle &\rightarrow (|0100\rangle + |0001\rangle)/\sqrt{2}\ .
\end{aligned}
$$

PDC: a brief summary

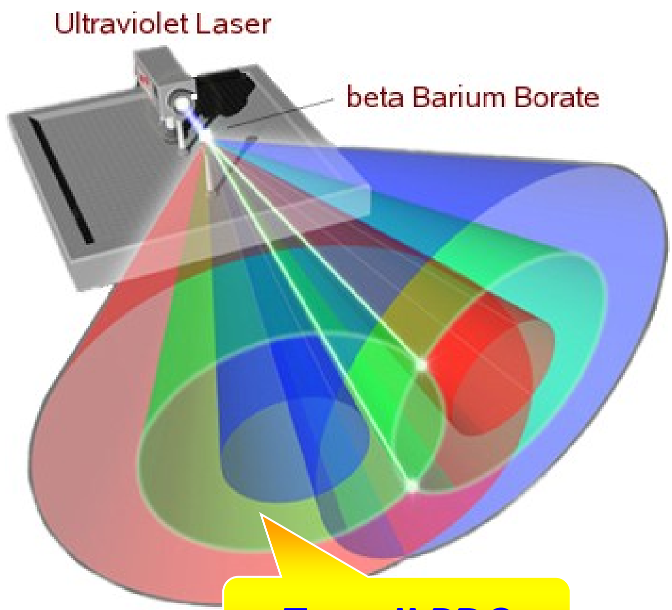


$$\vec{K}_0 = \vec{K}_1 + \vec{K}_2$$

$$\omega_0 = \omega_1 + \omega_2$$



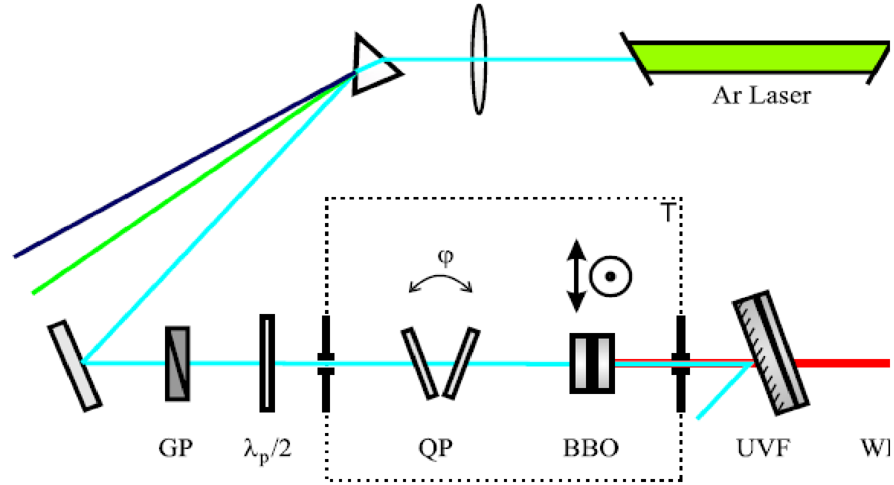
Type-I PDC



Type-II PDC

Our source

$$\frac{[(\,|H\rangle|V\rangle - |V\rangle|H\rangle\,)]}{\sqrt{(2)}}$$



The produced state can be reconstructed by tomography and compared with ψ^- state: F = 99%

Measurements

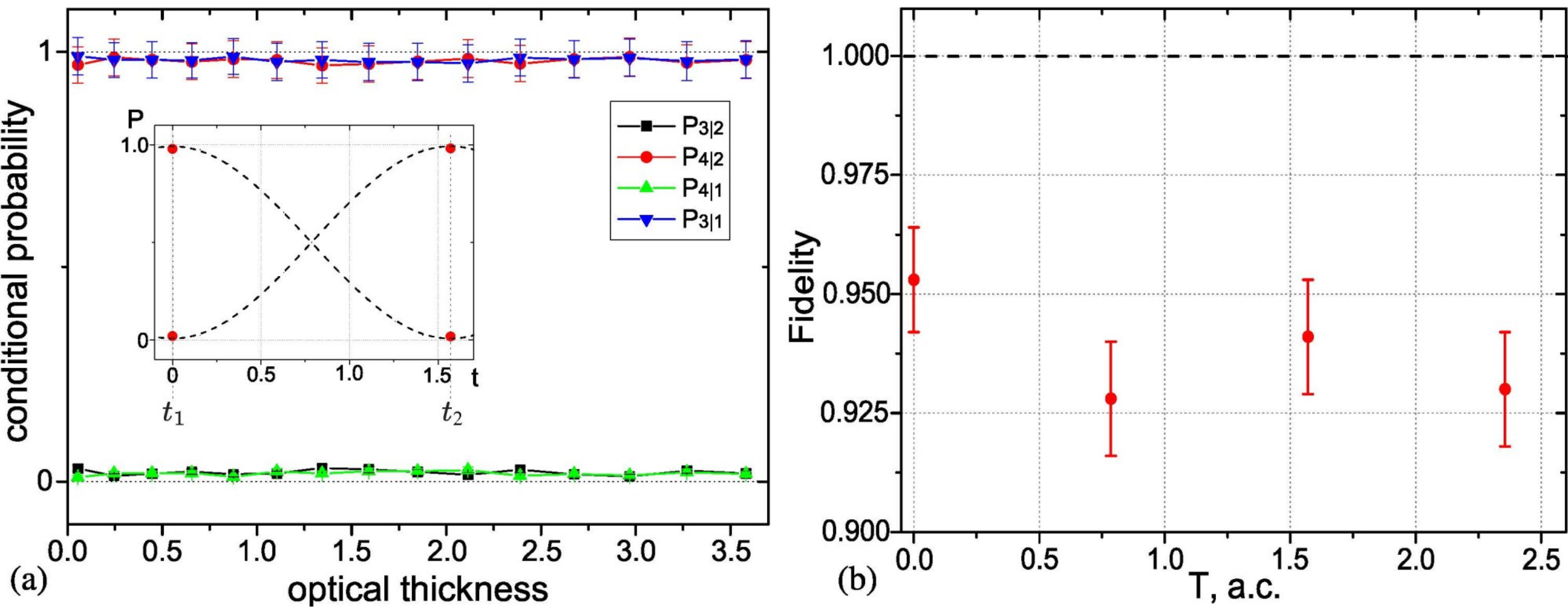
One photon of the photons of ψ^- state is then addressed to interferometer (clock state)

The photons are measured by Si SPAD detectors

We evaluate several coincidences: R_{++}, R_{+-}, R_{-+}, R_{--}

From which probabilities are calculated

PaW RESULTS

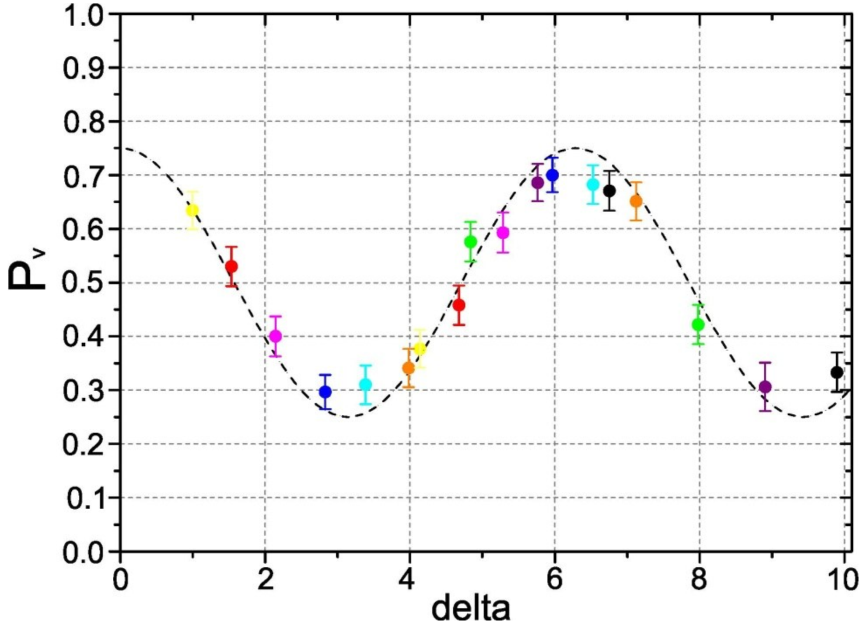


<u>Observer mode</u>: plot of the **clock-time dependent probabilities** of measurement outcomes as a function of the of the plate thickness (abstract coordinate time T); circles and squares represent p(t1)=P{3|1} and p(t2)=P{3|2}. As expected from the PaW mechanism: independent of T.

The inset shows the graph that the observer himself would plot as a function of clock-time: circles representing the probabilities of finding the system photon V at the two times t1, t2, the triangles of finding it H.

<u>Super-observer mode</u>: plot of the conditional fidelity between the tomographic reconstructed state and the theoretical initial state **|ψ⁻ >**

Gambini Pullin Results



<u>GPPT experimental results compared with theoretical curve</u>.
Probability p(t) that the upper photon is V as a function of the time t recovered from the lower photon.
The points with matching colours represent p(t1+**δ**i) and p(t2+ **δ i**): additional birefringent elements (time delays in clock photon)

System with a memory [Giovannetti,Lloyd, Maccone PRD 92 (2015) 045033]

The initial memory state |r> evolves when interacting with the clock/system state

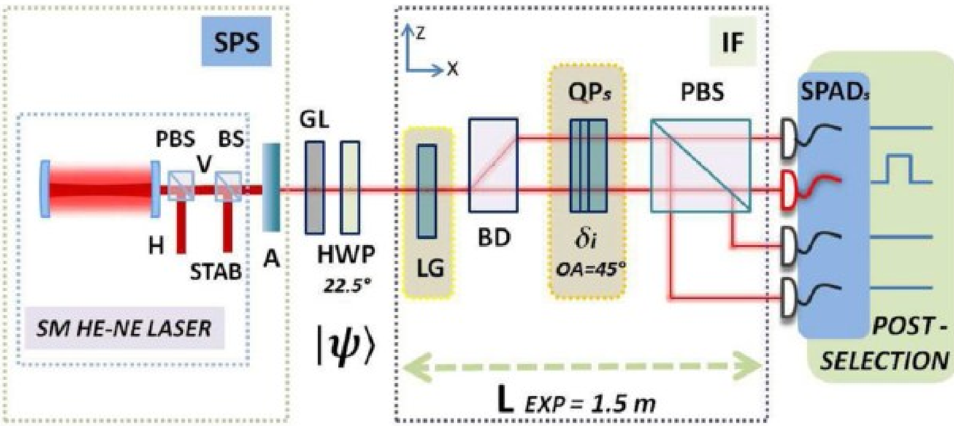
Single measurement

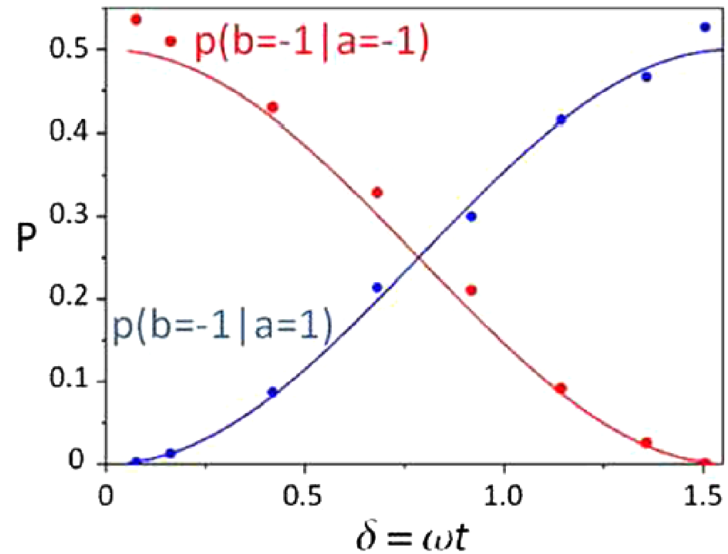
$$|\Psi\rangle\rangle = \int_{-\infty}^{t_a} dt|t\rangle_c U_t|\psi(0)\rangle_s|r\rangle_m + \int_{t_a}^{+\infty} dt|t\rangle_c \sum_a U_{t-t_a}|a\rangle_s\langle a|\psi(t_a)\rangle_s|a\rangle_m$$

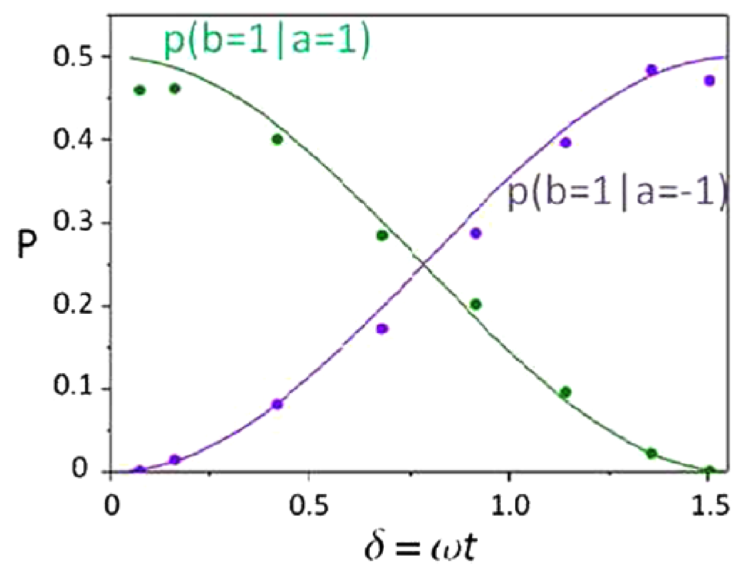
Two measurements

$$\begin{aligned}|\Psi\rangle\rangle = &\int_{-\infty}^{t_a} dt|t\rangle_c U_t|\psi(0)\rangle_s|r\rangle_{m_1}|r\rangle_{m_2}\\ &+ \int_{t_a}^{t_b} dt|t\rangle_c \sum_a \langle a|\psi(t_a)\rangle_s U_{t-t_a}|a\rangle_s|a\rangle_{m_1}|r\rangle_{m_2}\\ &+ \int_{t_b}^{+\infty} dt|t\rangle_c \sum_{ab}\langle b|U(t_b-t_a)|a\rangle_s\langle a|\psi(t_a)\rangle_s\\ &\times U_{t-t_b}|b\rangle_s|a\rangle_{m_1}|b\rangle_{m_2},\end{aligned}$$

[E.Moreva,M.Gramegna,G.Brida,L.Maccone, M.Genovese, PRD 96 (2017) 102005]







Sigle photon

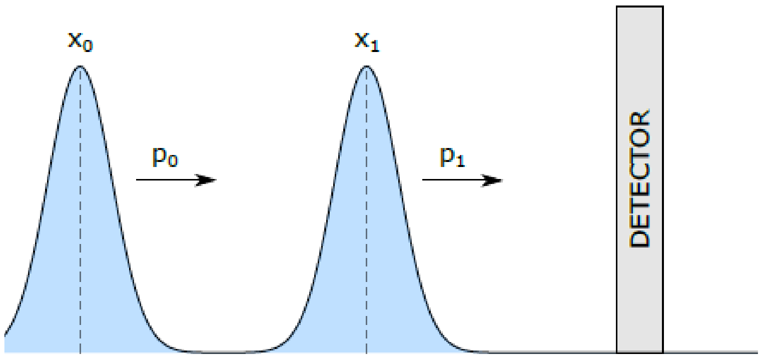
- Position along x : the clock
- Polarisation: the system
- Position along z: memories

$$p(b|a) = \begin{cases} \cos^2[\omega(t_b - t_a)]/2 & \text{for } a = b \\ \sin^2[\omega(t_b - t_a)]/2 & \text{for } a \neq b \end{cases}$$

$L_{coherence} > 100$ m

Further experimental ideas: time of arrival

The definition of time of arrival in QM is «model dependent»
[Roncallo, Sacha, Maccone, Quantum 7 (2023) 968]



Kijowski proposal [Kijowski Rep Math Phys 6 (1974) 361]

Time of arrival probability

$$\Pi_{\pm}(t) = \frac{1}{4\pi^2 m\sqrt{\hbar}} \left| \int_0^{\pm\infty} dp \; \widetilde{\Psi}(p,t)\sqrt{|p|} \right|^2$$

Quantum flux proposal [Delgado & Muda PRA 56 (1997) 3425]

Expectation value of the Schrödinger current

$$\hat{J} = \frac{\hat{p}}{2m} \left| x \right\rangle \left\langle x \right| + \frac{1}{2m} \left| x \right\rangle \left\langle x \right| \hat{p}$$

$$\Pi_F(x,t) = \frac{\hbar}{N_F m} \mathrm{Im}[\Psi^*(x,t)\partial_x \Psi(x,t)]$$

Semiclassical proposal [Vona & DÜrr]]

L distance source-detector, in terms of FT of ψ

$$\Pi_{sc}(x,t) = \frac{mL}{t^2}\left|\tilde{\Psi}\left(\frac{mL}{t}\right)\right|^2$$

Quantum Clock proposal

[Maccone & Sacha PRL 124 (2022) 11040 following Giovannetti et al PRD 92 (2015) 045033]

PVOM describing a joint measurement on particle and «clock»

$$|\Psi\rangle = \frac{1}{\sqrt{T}}\int_T dt|t\rangle|\psi(t)\rangle$$

Described by a POVM

$$\forall\, t:\ \Pi_t \equiv |t\rangle\langle t| \otimes P_d; \qquad \Pi_{na} = \mathbb{1} - \int dt \Pi_t \qquad\qquad P_d = \int_D dx|x\rangle\langle x|$$

(x can be the position, but also other observables, e.g. polarization, …)

That leads to the joint probability that the particle arrived and the time t

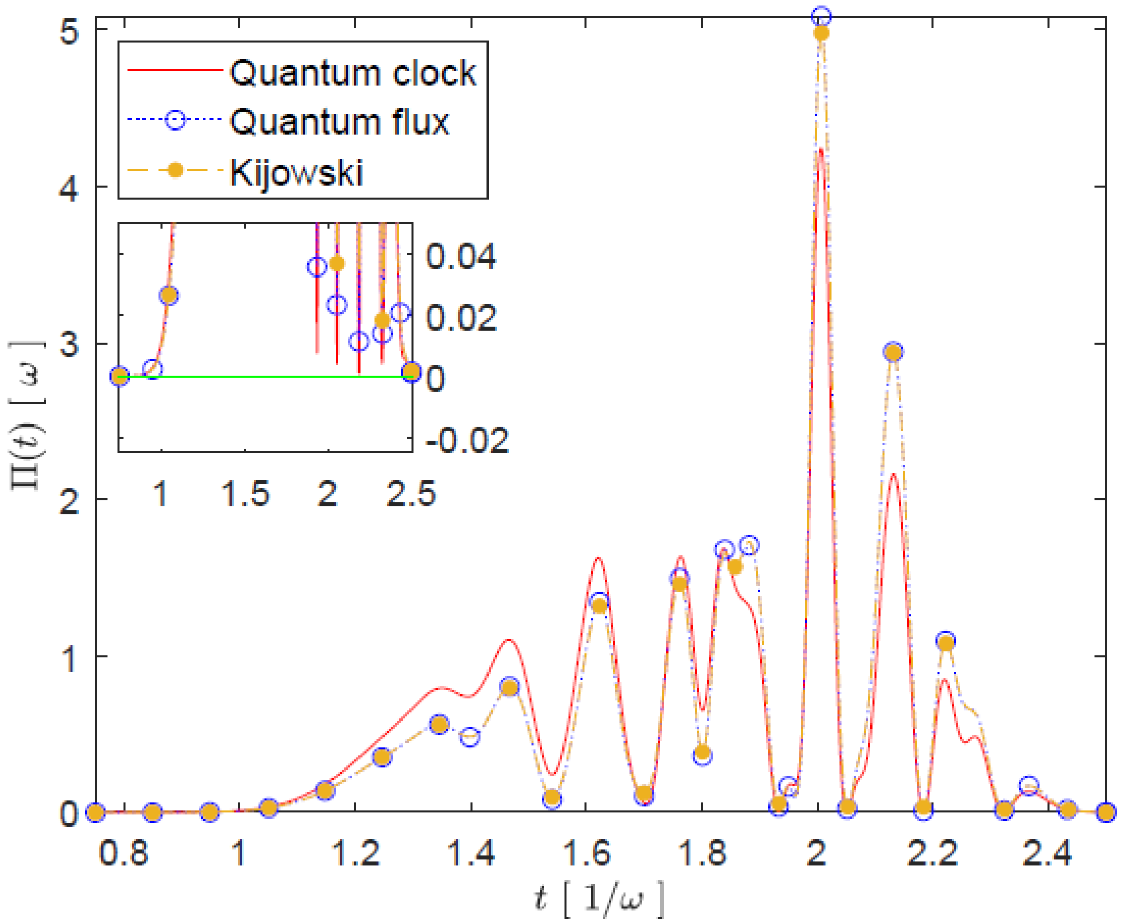
$$p(t, x \in D) = \mathrm{Tr}[|\Psi\rangle\langle\Psi|\Pi_t] = \frac{1}{T}\int_{x\in D} dx|\psi(x|t)|^2$$

and then to the time of arrival probability

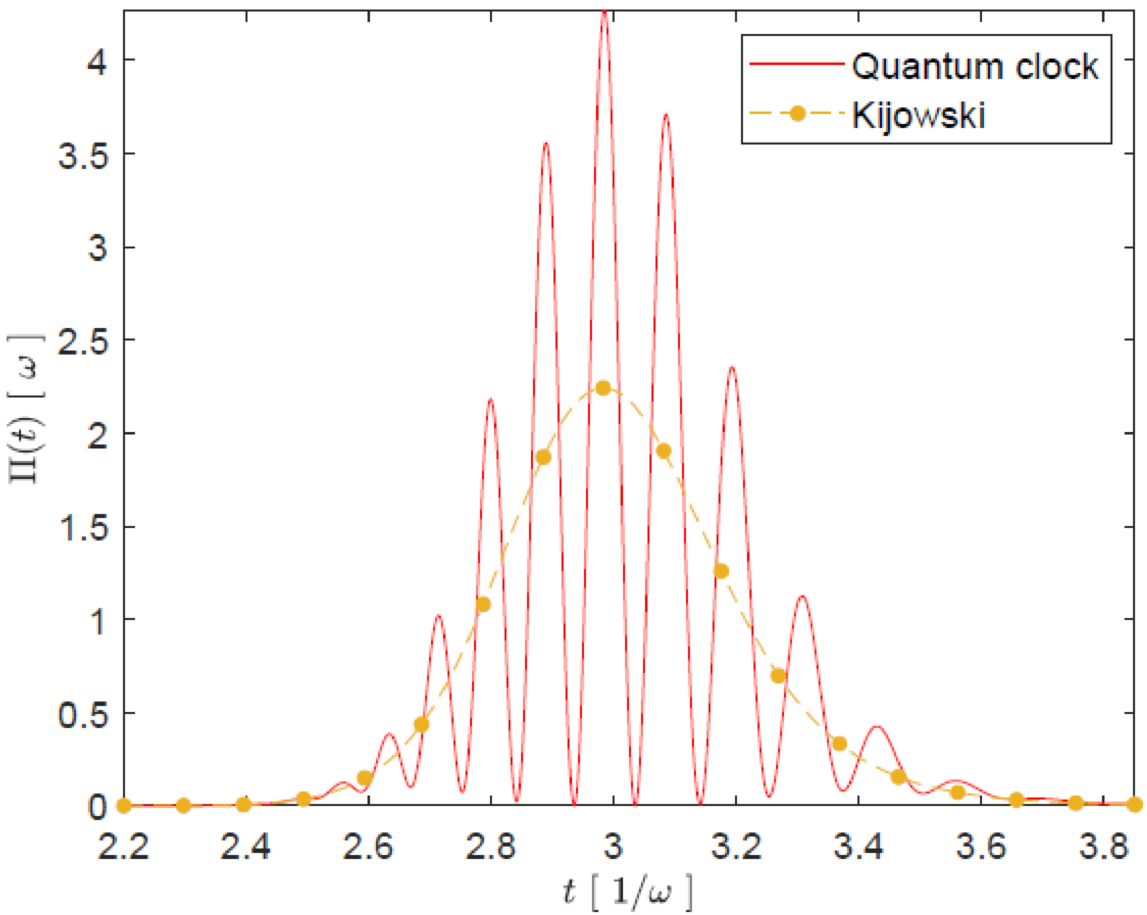
$$p(t|x \in D) = \int_{x\in D} dx|\psi(x|t)|^2 \Big/ \int_T dt \int_{x\in D} dx|\psi(x|t)|^2$$

Time of arrival interference (n= 4 gaussian wave packets)

$$|\Psi(t)\rangle = \frac{1}{\sqrt{n}} \sum_{k=0}^{n-1} |\Psi_k(t)\rangle$$



N=4



N=2 counterpropagating

How to realise it?

Neutrons?

Photons?

Photons acquire an effective mass in wave guides [Raudorf Am. J.Phys. 46 (1978) 35]

In a rectangular wave guide

$$k_c = (k_1^2 + k_2^2)^{1/2} = [(\pi m/a)^2 + (\pi n/b)^2]^{1/2};$$

One introduces an effective mass from

$$E^2 = m_0^2 c^4 + p^2 c^2 \qquad (\hbar\omega)^2 = m_0^2 c^4 + (\hbar k_3 c)^2$$

with

$$m_0 = \hbar\omega_c / c^2$$

CONCLUSIONS

Page Wootters scheme can provide a solution of the problem of the definition of time (Eternalist B theory)

We have realised **two experiments addressed to show Wootters and Page idea at work**:

-a globally stationary state respect to evolution (polarisation rotations) is created

- we show how the two subsystems evolve and that the evolution of one subsystem can be used as clock for the other

- we demonstrate that the global wave function remains stationary while the two subsystems evolve

- the GPPT scheme and memories are also “realised”

-New ideas under examination

[V.Giovannetti,S.Lloyd,L.Maccone PRD 2015; Roncallo, Sacha, Maccone, Quantum 7 (2023) 968]