Quantum uncertainties with top quarks

GGI Quantum Observables for Collider Physics workshop, 09/04/2025

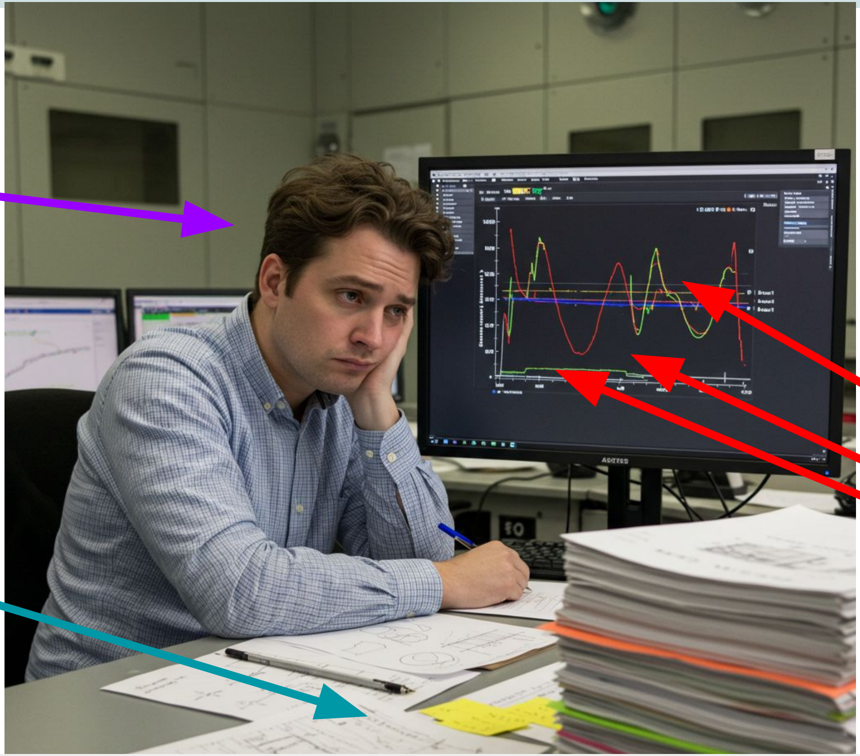
Baptiste Ravina (CERN)

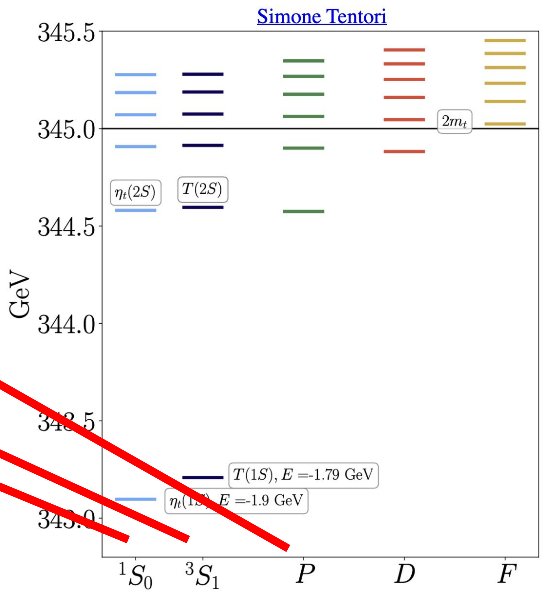
Foreword (Gemini's hallucination)

Me

(but less French)

QIT literature dive?





Toponia?

Can you generate a picture of an ATLAS physicist being sad they can't work on their pet project as much as they'd like, because they have to work full-time to replicate a result from the CMS collaboration that discovered a new resonance?

Quantumness of correlations

Classical correlations arise from common causes, shared history, or direct interactions: fully captured by probability distributions and mutual information + systems maintain individual, well-defined properties

Quantum mechanics presents phenomena that have no classical analog (e.g. entanglement) → **subsystems can no longer be described independently**

$$\rho = \sum_n p_n \rho_n^a \otimes \rho_n^b, \quad \sum_n p_n = 1, \; p_n \geq 0$$ [separability]

How do we **quantify the quantumness of correlations**, i.e. the transition between purely classical and maximally entangled states?

[entangled = non-separable BUT separable != classical]

Will focus on **bipartite qubit systems**:

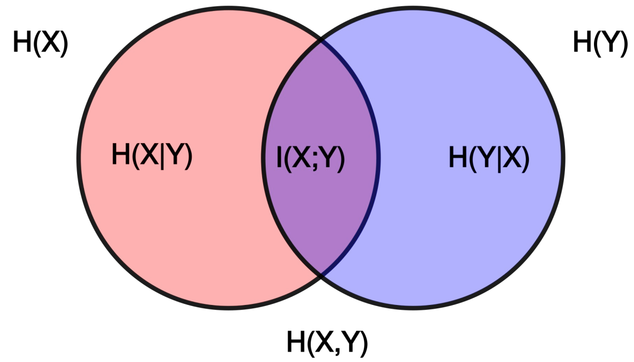
$$\rho = \frac{\mathbb{1} + \sum_i \left(B_i^+ \sigma^i \otimes \mathbb{1} + B_i^- \mathbb{1} \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

Quantum Discord

Y. Afik & JRMdN (2023), T. Han & M. Low & N. McGinnis & S. Su (2024)
Talk by Navin McGinnis tomorrow

Captures the ***non-classicality of correlations*** by measuring differences in the **total mutual information** [Shannon entropy H → von Neumann entropy S]

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$
$$J(X;Y) = H(X) - H(X|Y)$$
$$I(X;Y) \stackrel{?}{=} J(X;Y)$$ [classically they are the same]



Discord → $\mathcal{D}_A = S(\rho_B) - S(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} S(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} S(\rho_{-\hat{\mathbf{n}}})$

- In general: **$0 \le D_A \le 1$** and **D_A != D_B**
- **Experimentally very challenging!** Requires a ***minimisation*** over projective measurements.
- **Analytical results exist** for special classes of states (e.g. Bell-diagonal states, T states)

$$S(\rho) = -\mathrm{Tr}\rho \log_2 \rho \qquad \rho_{\hat{\mathbf{n}}} = \frac{1 + \mathbf{B}^+_{\hat{\mathbf{n}}} \cdot \sigma}{2}, \; \mathbf{B}^+_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^+ + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}, \; p_{\hat{\mathbf{n}}} = \frac{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}{2}$$

Local Quantum Uncertainty

D. Girolami et al. (2013)

Alternative [with clearer interpretability?] based on **quantum uncertainty under local measurements:** even when a system is *not entangled*, a local measurement on one part can disturb the global state *if there are* quantum correlations.

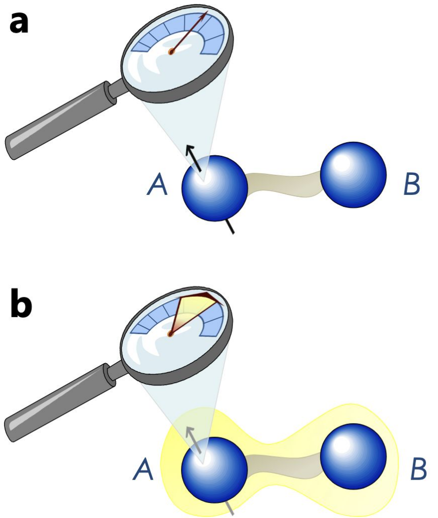
Wigner-Yanase skew information: $\mathcal{I}(\rho, K_A \otimes \mathbb{I}_B) = -\frac{1}{2}\mathrm{Tr}([\sqrt{\rho}, K_A \otimes \mathbb{I}_B]^2)$

quantifies the non-commutativity between state ρ and observable K
→ the part that is not just "classical ignorance" [=quantum-certain]

Therefore if the **minimal value of I** achievable on a single local measurement is **non-zero**, we are dealing with a **discordant state**

$$\boxed{\mathcal{U}(\rho) \equiv \min_{K_A} \mathcal{I}(\rho, K_A \otimes \mathbb{I}_B)}$$

the Local Quantum Uncertainty (LQU)



Closed-form formulae when A is a qubit

T. Han & M. Low & N. McGinnis & S. Su (2024)

$$\mathcal{D}_A = S(\rho_B) - S(\rho) + \boxed{\min_{\hat{\mathbf{n}}}} p_{\hat{\mathbf{n}}} S(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} S(\rho_{-\hat{\mathbf{n}}})$$

$$\mathcal{U}(\rho) \equiv \boxed{\min_{K_A}} \mathcal{I}(\rho, K_A \otimes \mathbb{I}_B)$$

Our **alternative measure of discord** suffers from the **same issue**: we need to perform a **minimisation** over possible measurements…

- Discord: get around this **only for X-states** [this is the case for tt̄ at LO in QCD]
- **LQU: closed-form formula valid for generic 2 x d systems!** [qubit-qudit]

Pick non-degenerate observables K_A on the qubit A such that $K_A = \vec{n} \cdot \vec{\sigma}, \ |\vec{n}| = 1$

Then compute the eigenvalues (w_1,w_2,w_3) of the matrix

$$(\mathcal{W})_{ij} \equiv \mathrm{Tr}\left\{\sqrt{\rho}\left(\sigma_{Ai} \otimes \mathbb{I}_2\right)\sqrt{\rho}\left(\sigma_{Aj} \otimes \mathbb{I}_2\right)\right\}$$

The LQU simplifies to

$$\boxed{\mathcal{U}(\rho) = 1 - \max(\omega_1, \omega_2, \omega_3)}$$

[we only need the SDM!]

Closed-form formulae beyond qubit-qudits

A. Sen et al. (2015)

For the qudit-qudit case d_1 x d_2, *things are a little bit more complicated…*

The generators of SU(d) are

$$\lambda_j = \begin{cases} \sqrt{\frac{2}{j(j+1)}}\left(\sum_{k=1}^{j} |k\rangle\langle k| - j|j+1\rangle\langle j+1|\right), j = 1, \ldots, d-1 \\ |k\rangle\langle m| + |m\rangle\langle k| (1 \le k < m \le d), j = d, \ldots, \frac{d(d+1)}{2} - 1 \\ \mathrm{i}(|k\rangle\langle m| - |m\rangle\langle k|)(1 \le k < m \le d), j = \frac{d(d+1)}{2}, \ldots, d^2 - 1 \end{cases}$$

satisfying

$$\lambda_i\lambda_j = \mathrm{i}\sum_k f_{ijk}\lambda_k + \sum_k g_{ijk}\lambda_k + \frac{2}{d}\delta_{ij}\mathbb{I}_d \qquad f_{ijk} = \frac{1}{4\mathrm{i}}\mathrm{Tr}([\lambda_i, \lambda_j]\lambda_k), g_{ijk} = \frac{1}{4}\mathrm{Tr}(\{\lambda_i, \lambda_j\}\lambda_k)$$

and we instead end up looking at the matrix

$$\boxed{\mathcal{W}_{ij} = \mathrm{Tr}\{\sqrt{\rho}(\lambda_i \otimes \mathbb{I}_{d_2})\sqrt{\rho}(\lambda_j \otimes \mathbb{I}_{d_2})\} - G_{ij}L}$$

$$G_{ij} = (g_{ij1}, \cdots, g_{ijk}, \cdots, g_{ijd_1^2-1}),$$
$$L = (\mathrm{Tr}(\rho\lambda_1 \otimes \mathbb{I}_{d_2}), \cdots, \mathrm{Tr}(\rho\lambda_k \otimes \mathbb{I}_{d_2}), \cdots, \mathrm{Tr}(\rho\lambda_{d_1^2-1} \otimes \mathbb{I}_{d_2}))^T$$

- For **d_1=2** we have G_{ij}=0 and we recover the result on the previous slide
- For **d_1>2** we have a closed-form lower bound when L=0 ⇔ $\mathrm{Tr}[\rho\lambda_i \otimes 1_n] = 0$
- Unfortunately for **H→ZZ** we are not in this configuration as soon as the Z's are not at rest 😓

A note on the validity of the closed-form formulae

The derivation **assumes** we can **pick non-degenerate observables K_A.**

Maybe there are better ways of checking that, but one possibility is:

- apply the formula, compute W, and decompose it as $\mathcal{W} = UDU^{-1}$
- get the row of U^{-1} that corresponds to the maximum eigenvalue of W
- substitute that row into $K_A = \vec{n} \cdot \vec{\sigma}$ and check it is not degenerate

→ Seems to work fine for qubit-qubit systems 👍

Predictions for SM tt̄ at LO QCD

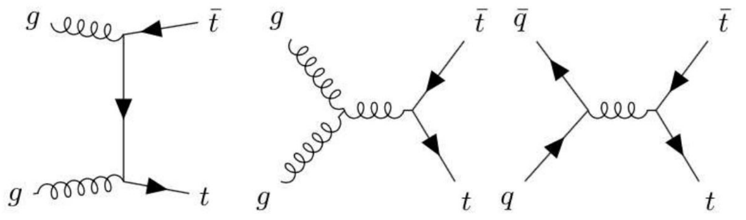
Y. Afik & JRMdN (2021)
R. Aoude & E. Madge & F. Maltoni & L. Mantani (2022)

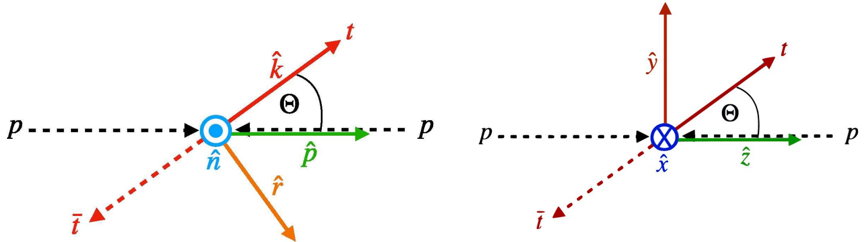
$$\rho = \frac{\mathbb{1} + \sum_i \left(B_i^+ \sigma^i \otimes \mathbb{1} + B_i^- \mathbb{1} \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

⇒ **Bipartite qubit system** described in terms of the top and anti-top quark spin polarisations B_i and spin correlations C_{ij}

In a given basis, **Fano coefficients** computed in terms of the top velocity (β) and scattering angle (cosΘ).

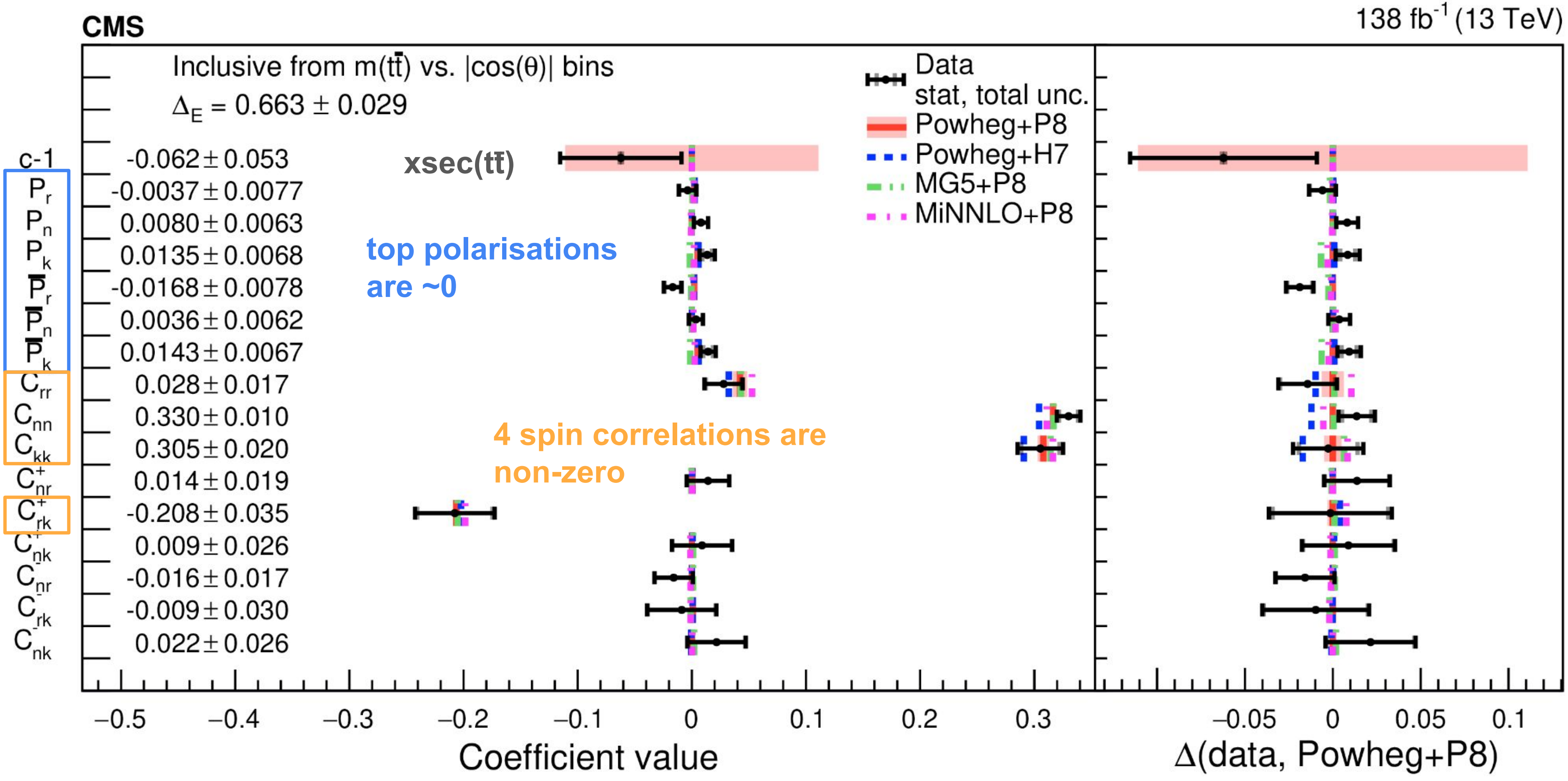
At LO: B_i=0 and C_{ij}~diagonal → **tt̄ in an X-state** → *many things simplify!*



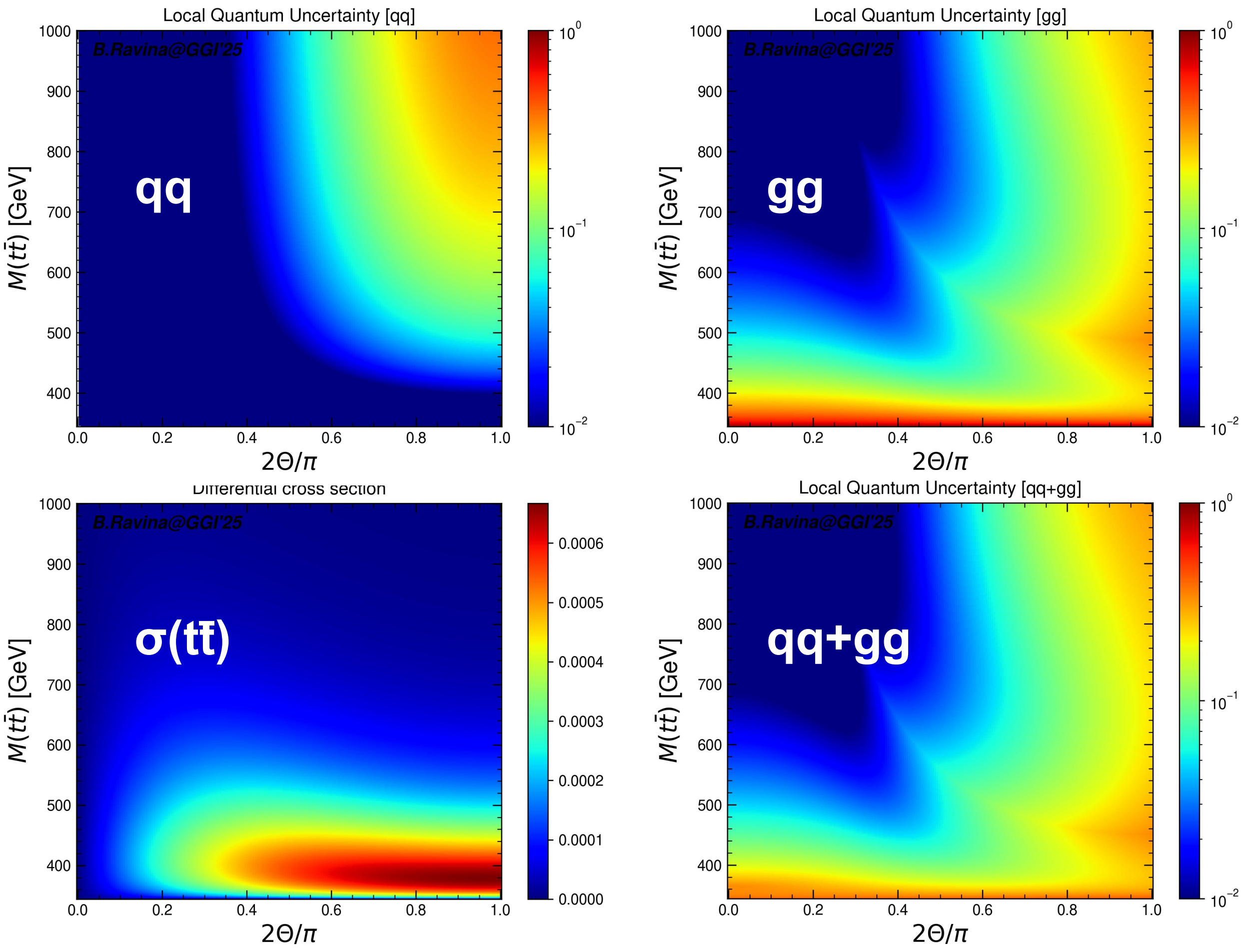


The CMS lepton+jets measurement

CMS-TOP-23-007
Talk by Otto Hindrichs yesterday



LQU per channel: SM $t\bar{t}$ at LO QCD, in the helicity basis

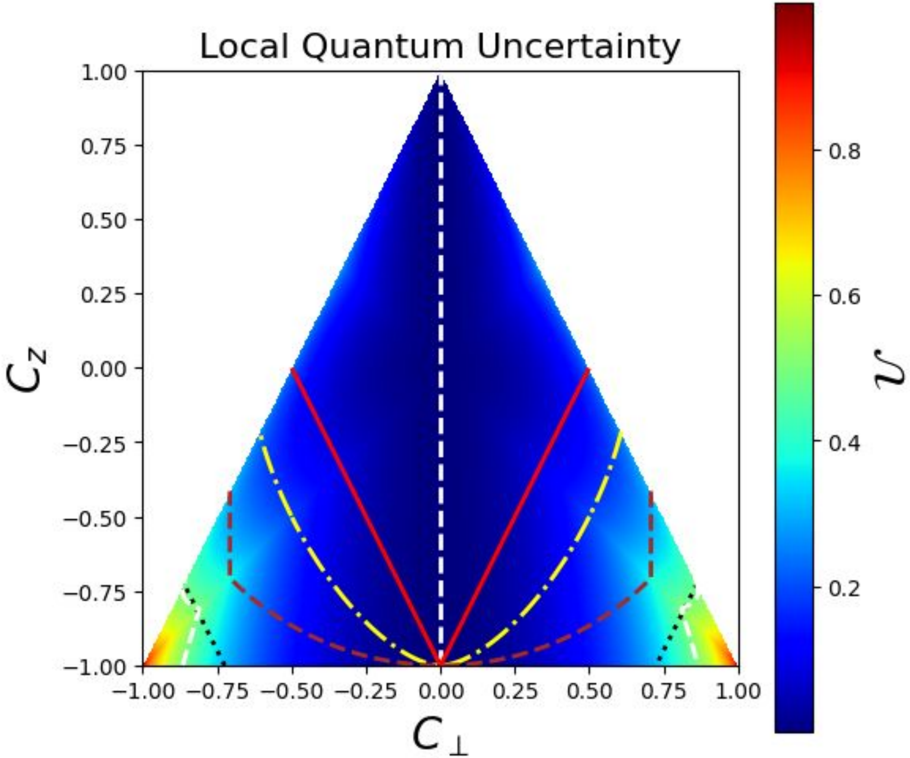


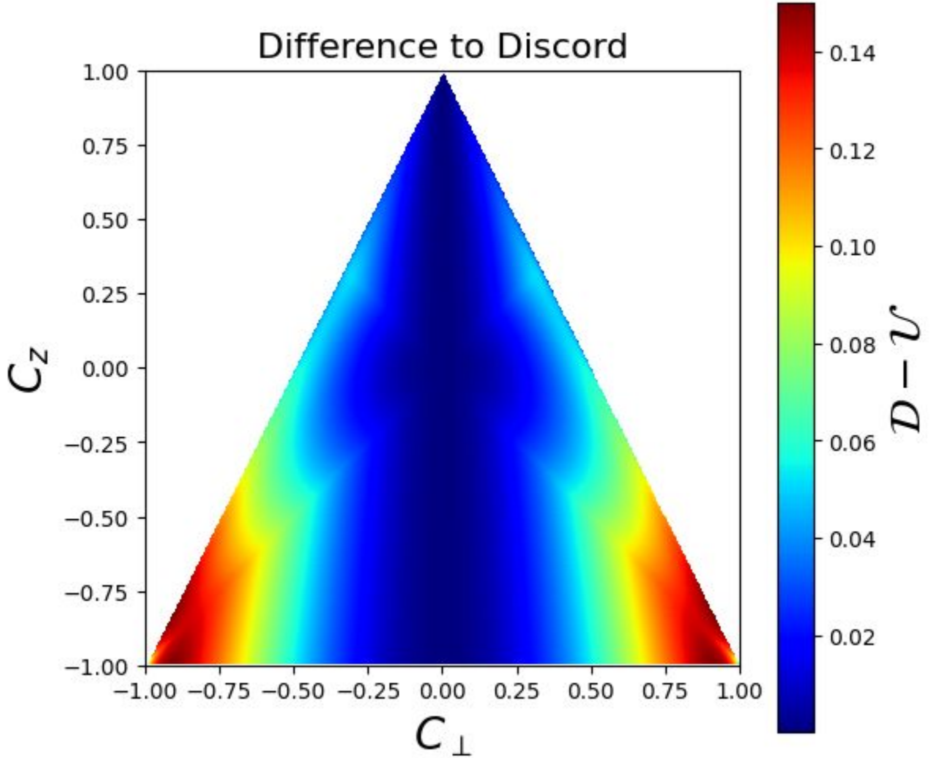
Note that the color map is in log scale!

⇒ we're looking at a small effect, localised near maximally entangled regions

As expected, we have **0<U≤1**, with near saturation at threshold in the gg-channel

LQU vs Discord: SM $t\bar{t}$ at LO QCD, in the beam basis





White: pure classical states
Red: separability
Brown: Bell non-locality

LQU provides a lower bound on Discord:

0<U<D≤1 everywhere at the LHC

[need to find a way to prove it… 🤔]

Beyond SM $t\bar{t}$ at LO QCD

F. Maltoni & C. Severi & S. Tentori & E. Vryonidou (2024)
R. Aoude & E. Madge & F. Maltoni & L. Mantani (2022)

"Higher-order SM effects only have a small impact on the spin coefficients", **but…**

1) these might be exacerbated by looking at **specific regions of phase-space**
2) **QI observables** may also accidentally enhance them
3) **Monte Carlo simulations** only include/approximate some of them

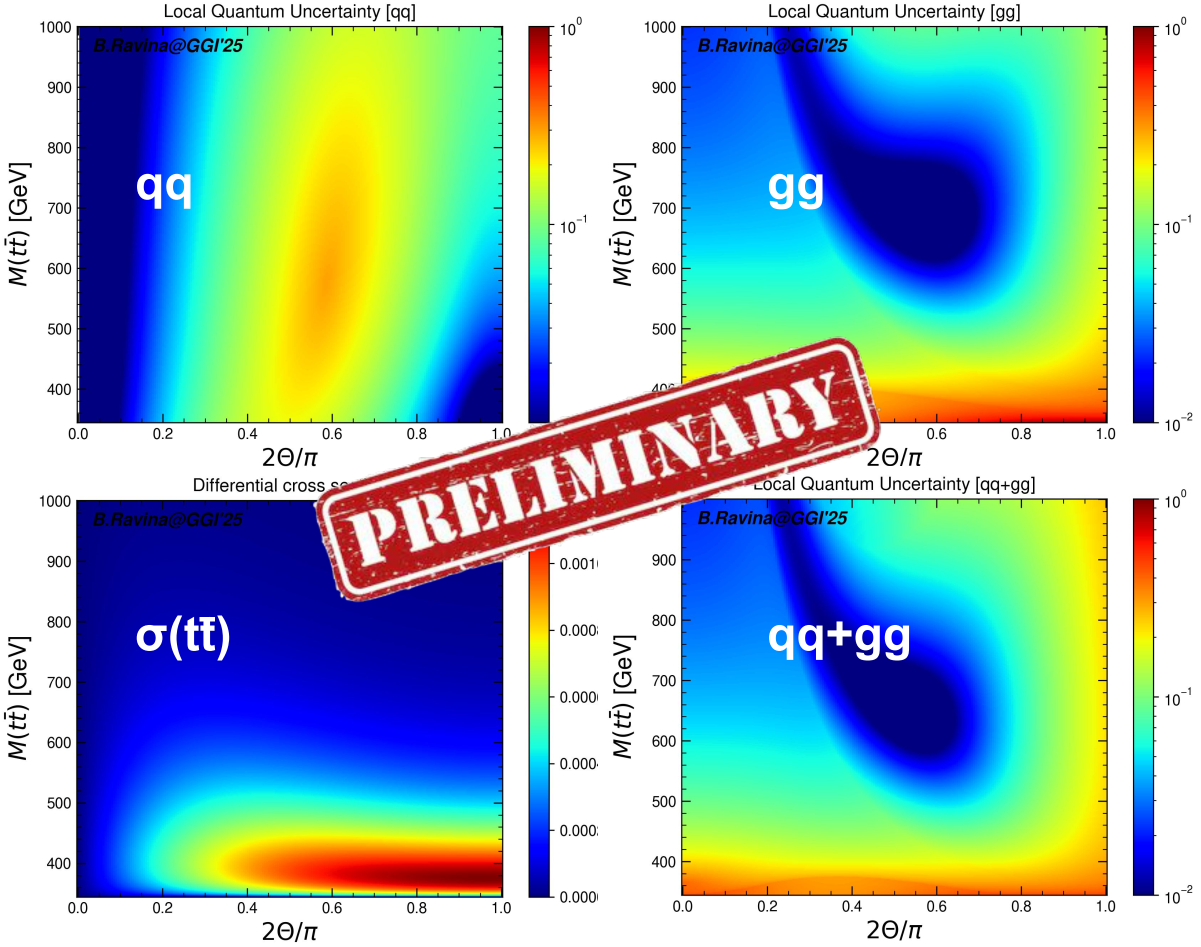
Can further consider 2 **extensions of the SM** to guide the BSM phenomenology:

- **pseudo-scalar** A→$t\bar{t}$ near threshold ~ *toponium*
- **dimension-6 SMEFT**

Talk by Benjamin Fuks this morning

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{n=1}^{\infty} \frac{1}{\Lambda^n} \sum_i c_i^{(n)} \mathcal{O}_i^{(n)} \qquad \mathcal{O}_{tG} = g_s \left(\bar{Q}\sigma^{\mu\nu} T^A t\right) \tilde{\phi}\, G^A_{\mu\nu} + \text{h.c.}$$

LQU per channel: SM + EFT $t\bar{t}$ in the helicity basis

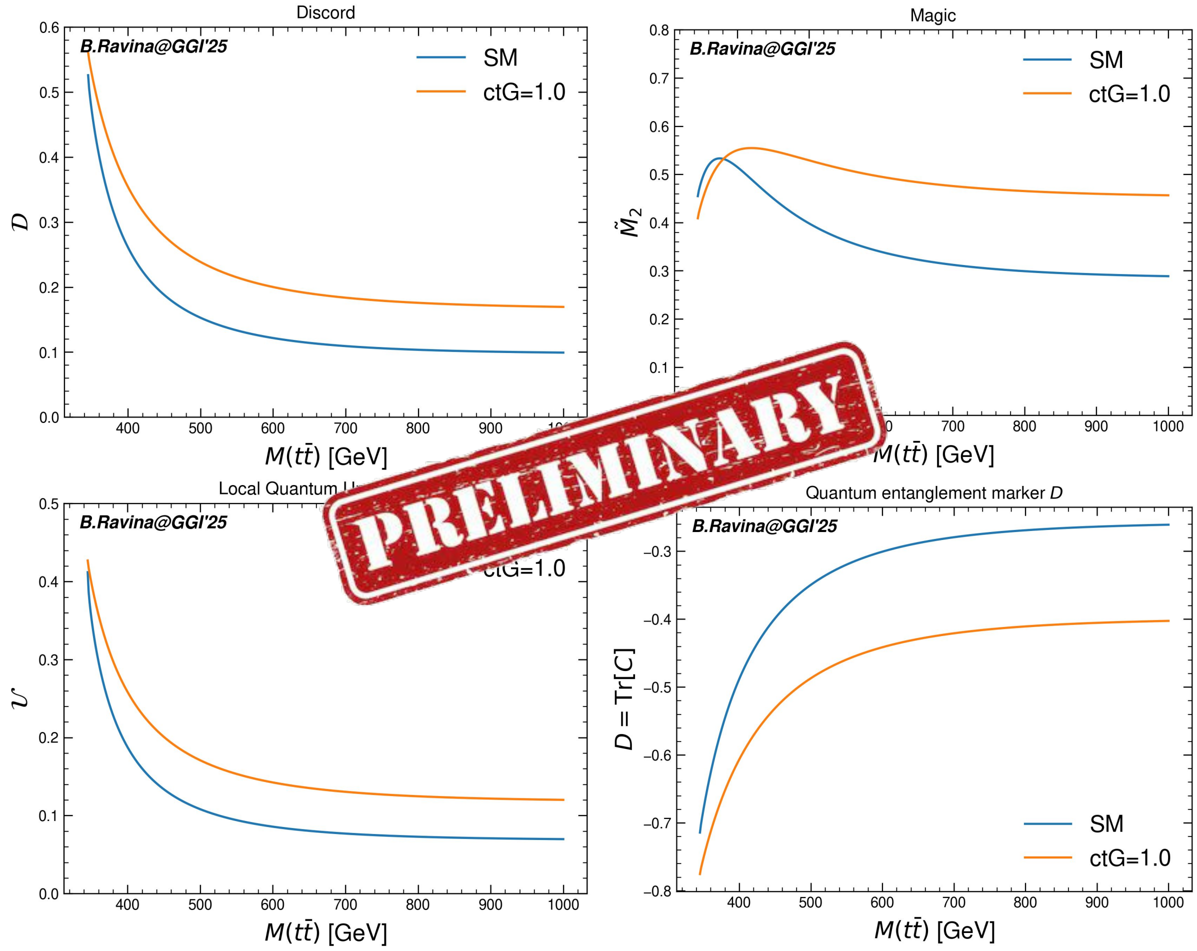


Note that the color map is in log scale!

Here consider a **large** BSM effect [ctG=1 TeV^{-2}] for visualisation purposes.

Picture changes quite a lot for the qq-channel, *to be checked…*

Comparison of different quantum correlation metrics



SM and **EFT** predictions as a function of the integration interval [$2m_t$, $M(t\bar{t})$] for **Discord**, **Magic**, **LQU** and **D**

⇒ all of them are sensitive to BSM, but ***which is "best"?***

[see also M. Fabbrichesi & M. Low & L. Marzola (2025)]

Y. Afik & JRMdN (2023), T. Han & M. Low & N. McGinnis & S. Su (2024)
Talk by Navin McGinnis tomorrow
Y. Afik & JRMdN (2021)
R. Aoude & E. Madge & F. Maltoni & L. Mantani (2022)
C. White & M. White (2024)
Talk by Martin White yesterday

Looking at the CMS lepton+jets measurement

CMS-TOP-23-007

Metric		**LO QCD**	**NLO Powheg +Pythia8**			
D=Tr[C]/3		-0.257	-0.222			
Magic		0.293	0.232			
Discord		0.103	0.072			
LQU		0.073	0.051			

Obtained from the **inclusive measurement by CMS**, using the **full SDM** and its **covariance** (100k toys)

Looking at the CMS lepton+jets measurement

Metric		LO QCD	NLO Powheg +Pythia8	NLO Powheg +Herwig7	NLO MG5_aMC +Pythia8	NNLO MiNNLOps +Pythia8
D=Tr[C]/3		-0.257	-0.222	-0.209	-0.227	-0.225
Magic		0.293	0.232	0.214	0.235	0.236
Discord		0.103	0.072	0.065	0.073	0.075
LQU		0.073	0.051	0.045	0.051	0.053

Obtained from the **inclusive measurement by CMS**, using the **full SDM** and its **covariance** (100k toys)

See the talk by Ethan Simpson this morning for the differences in parton shower

Looking at the CMS lepton+jets measurement

Metric	Measured	LO QCD	NLO Powheg +Pythia8	NLO Powheg +Herwig7	NLO MG5_aMC +Pythia8	NNLO MiNNLOps +Pythia8
D=Tr[C]/3	-0.221 ± 0.010	-0.257	-0.222	-0.209	-0.227	-0.225
Magic		0.293	0.232	0.214	0.235	0.236
Discord		0.103	0.072	0.065	0.073	0.075
LQU		0.073	0.051	0.045	0.051	0.053

Obtained from the **inclusive measurement by CMS**, using the **full SDM** and its **covariance** (100k toys)

Looking at the CMS lepton+jets measurement

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D=Tr[C]/3	-0.221 ± 0.010	-0.257	-0.222	-0.209	-0.227	-0.225
Magic	0.238 ± 0.014	0.293	0.232	0.214	0.235	0.236
Discord		0.103	0.072	0.065	0.073	0.075
LQU		0.073	0.051	0.045	0.051	0.053

Obtained from the **inclusive measurement by CMS**, using the **full SDM** and its **covariance** (100k toys) → **clear observation of Magic** [beyond 5σ]

See the talk by Otto Hindrichs yesterday for the full differential results!

Looking at the CMS lepton+jets measurement

CMS-TOP-23-007

Metric	Measured	LO QCD	NLO Powheg +Pythia8	NLO Powheg +Herwig7	NLO MG5_aMC +Pythia8	NNLO MiNNLOps +Pythia8
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LQU	0.051 ± 0.007	0.073	0.051	0.045	0.051	0.053

Obtained from the **inclusive measurement by CMS**, using the **full SDM** and its **covariance** (100k toys) → **clear observation of Magic & Discord & LQU** [beyond 5σ]

But how do we interpret those given the mixed states, angular averaging and experimental bin averaging?.. [need to look differentially at entangled regions]

Next steps...

- Check the relevance of **theory uncertainties** [scales & PDF]
- Check the behaviour of **further EFT operators** and **pseudo-scalar states**
- Exploit the full potential of the CMS results → **differential in M(tt̄) and cosϴ**
- *This would then allow one to check the* ***Quantum Fisher Information*** *of the system*
    - get the "score" of the tt̄ SDM with respect to SMEFT operators in the vicinity of the SM
    - **Cramér-Rao bound:** ***QFI determines the absolute best sensitivity*** one could hope to extract from such a measurement
    - can we **optimise the phase-space cuts** in this way?
    - how does it **compare to e.g. trace distance** / fidelity? [QFI ~ Bures]
- Explore the possibility of defining a lower bound for discord in **H→VV***

LQU gives an exactly computable lower bound on Discord for tt̄ at the LHC

Observation of LQU > 0 from the recent CMS measurement CMS-TOP-23-007

Preprint to appear on arXiv hopefully soon… [or viXra? 👀]