Entanglement and Bell Nonlocality with Bottom-Quark Pairs at Hadron Colliders

David Uzan

Based on work done in collaboration with:

Y. Afik, Y. Kats, J. R. Muñoz de Nova, A. Soffer

arXiv:2406.04402

Motivation

- Entanglement between spins was measured in $pp \to t\bar{t}$ samples in both ATLAS and CMS.

ATLAS Collaboration, Nature 633 (2024) 542

CMS Collaboration, ROPP 87 (2024) 117801

CMS Collaboration, PRD 110 (2024) 112016

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CMS Collaboration, ROPP 87 (2024) 117801
CMS Collaboration, PRD 110 (2024) 112016

- Can similar measurements be conducted in $pp \to b\bar{b}$ samples?
  - The system is hadronizing.
  - Ultra-relativistic regime at low invariant mass $M_{b\bar{b}}$.
  - b-jets have high tagging efficiency.
  - Spin correlation is expected to be measurable in b-quark pairs at the LHC.

Y. Kats, D. Uzan, JHEP 03 (2024) 063

$b\bar{b}$ Spin Correlations Predictions

To measure entanglement and Bell nonlocality we need spin correlations.

- We give predictions for the spin correlations.
  - Same as for $t\bar{t}$ predictions, with $m_t \to m_b$.
  - Some predictions are shown on the right.

C_{kk}	0.96
C_{rr}	0.63
C_{nn}	-0.62

- Cross sections and expected number of events are calculated using NLO QCD simulations.

Polarization in Hadronization

- Polarization largely retained through hadronization:

$$\mu_b \propto \frac{1}{m_b},\ m_b \gg \Lambda_{QCD}$$

Falk and Peskin, PRD 49, 3320 (1994)

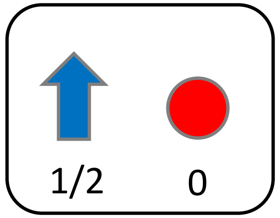
In the heavy quark limit, gluon radiation is not expected to affect the quark's spin.

Matthias Neubert, arXiv:hep-ph/9610266

Estimated to affect the b quark polarization by about $\sim 3\%$.

Körner et al., Z.Phys. C63 (1994) 575-579

Polarization in Hadronization



$\boldsymbol{\Lambda_b} = b \quad qq$

The measurement can be performed on $\boldsymbol{b \to \Lambda_b}$:

- Most common b-baryon $f(b \to \Lambda_b) \approx 7\%$.
- ud form a spin and isospin singlet
  - b-quark carries the baryon spin.

Some polarization loss due to $\Sigma_b^{(*)} \to \Lambda_b \pi$, soft π.

Retention Factor

- Can be quantified through the retention factors r_T and r_L:

$$r_{T/L} = \frac{\mathcal{P}_{T/L}(\Lambda_q)}{\mathcal{P}_{T/L}(q)}$$

$$0.4 \leq r_L \leq 0.8, \qquad 0.5 \leq r_T \leq 0.8$$

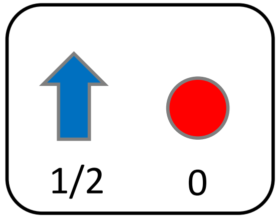
Falk and Peskin, PRD 49, 3320 (1994)
Galanti et al., JHEP 11 (2015) 067

- Measured in LEP in Z boson decay $r_L = 0.47 \pm 0.14$.

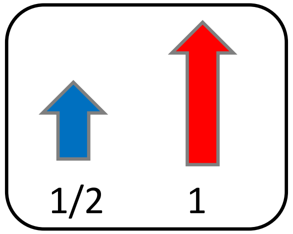
ALEPH Collaboration, PLB 365, 437 (1996)
OPAL Collaboration, PLB 444, 539 (1998)
DELPHI Collaboration, PLB 474, 205 (2000)

- r_L also measurable in ATLAS/CMS $t\bar{t}$ samples, even in Run 2 data.

Galanti, Giammanco, Grossman, Kats, Stamou, Zupan, JHEP 11 (2015) 067



$\boldsymbol{\Lambda_b} = \boldsymbol{b} \quad \boldsymbol{qq}$



$\boldsymbol{\Sigma_b^{(*)}} = \boldsymbol{b} \quad \boldsymbol{qq}$

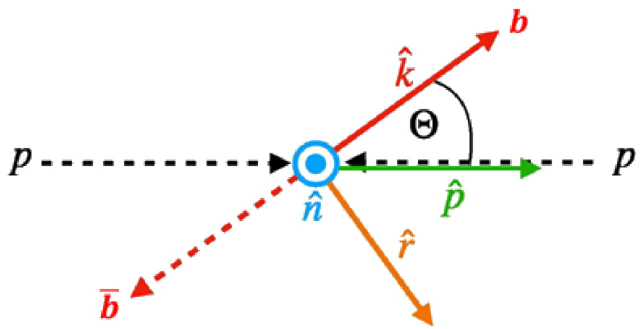
Measuring Polarization and Spin Correlations

We use the spin density matrix

$$\rho = \frac{I_4 + \sum_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

where $i, j = \{\hat{k}, \hat{n}, \hat{r}\}$.

- $\hat{p}$ - the proton beam axis
- $\hat{k}$ - the direction of b in the $b\bar{b}$ rest frame
- $\hat{r} = \frac{\hat{p} - \cos\Theta \hat{k}}{\sin\Theta}, \cos\Theta = \hat{k} \cdot \hat{p}$
- $\hat{n} = \hat{r} \times \hat{k}$



$\boldsymbol{B}^{\pm}$ are spin polarizations and $\boldsymbol{C}$ is the spin correlation matrix.

Baryon Decay Angular Distributions

Using θ_i, the decay product angle in relation to one of the axes of the helicity basis $\{\hat{k}, \hat{n}, \hat{r}\}$:

$$\frac{1}{\sigma}\frac{d\sigma}{d\left(\cos\theta_i^+ \cos\theta_j^-\right)} = \frac{1}{2}\left(1 + c_{ij}\cos\theta_i^+ \cos\theta_j^-\right)\ln\left(\frac{1}{\left|\cos\theta_i^+ \cos\theta_j^-\right|}\right)$$

$$c_{ij} = \alpha_+ \alpha_- r_i r_j f C_{ij}$$

Spin analyzing power

Polarization retention factor (r_T or r_L)

$$f = \frac{N_{sig}}{N_{sig} + N_{bg}}$$

Sample Purity

Methods for Measurements

- We use the concurrence $\mathcal{C}[\rho]$: quantitative measurement of entanglement.

- $0 \leq \mathcal{C}[\rho] \leq 1$, where $\mathcal{C}[\rho] \neq 0$ iff the state is entangled.

Afik, Muñoz de Nova, EPJP 136 (2021) 907

Methods for Measurements

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- $0 \leq \mathcal{C}[\rho] \leq 1$, where $\mathcal{C}[\rho] \neq 0$ iff the state is entangled.

- At LO QCD: $\mathcal{C}[\rho] = \max(\Delta, 0)$, where:

$$\Delta = \frac{-C_{nn} + |C_{rr} + C_{kk}| - 1}{2}.$$

- $\Delta > 0$ is always a sufficient condition for entanglement.

Afik, Muñoz de Nova, EPJP 136 (2021) 907

Methods for Measurements

- To measure Bell Nonlocality, we look for CHSH violation:

$$\sqrt{\mu_1 + \mu_2} \geq 1$$

where μ_i are the two maximal eigenvalues of $\boldsymbol{C}^T\boldsymbol{C}$.

Clauser et al., PRL 23 (1969) 23

Horodecki, PLA 200 (1995) 340

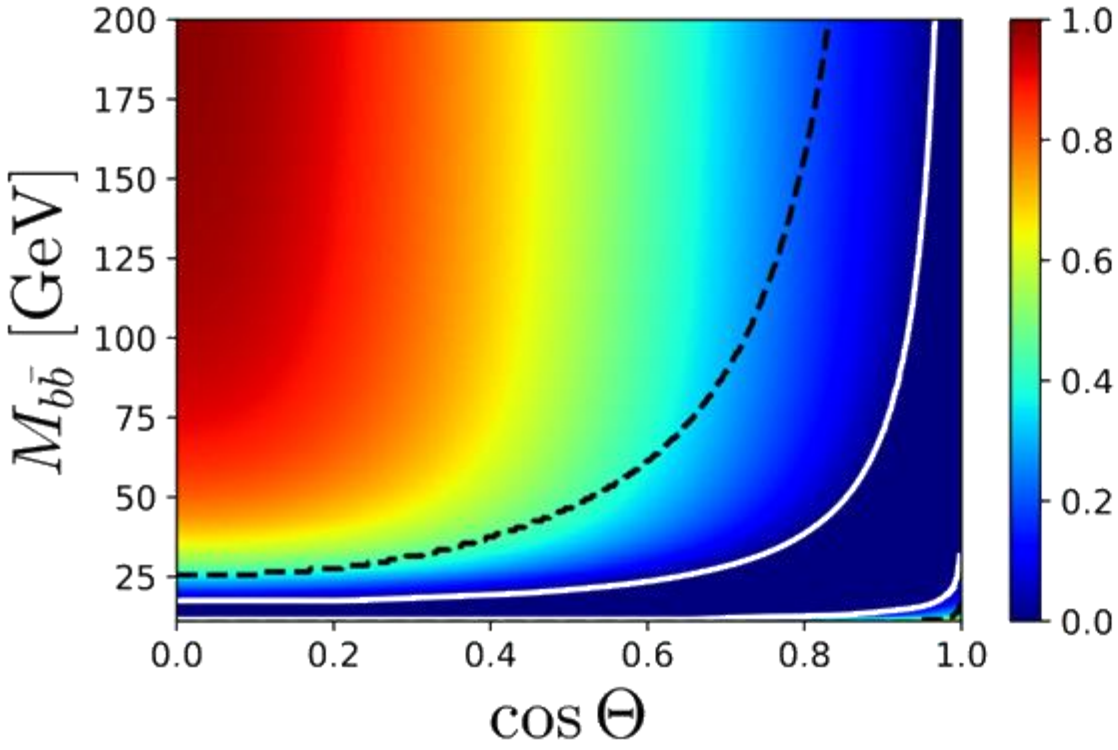
- An indicator for Bell nonlocality:

$$\mathcal{V} = C_{kk}^2 + C_{rr}^2 - 1 \leq \mu_1 + \mu_2 - 1.$$

- $\mathcal{V} > 0$ is a sufficient condition to indicate the Bell nonlocality.
- In ultra-relativistic regime $\boldsymbol{C}$ is diagonal with $C_{kk}^2, C_{rr}^2 > C_{nn}^2$.

Concurrence in Phase Space

- Solid white line is $\Delta > 0$.
- Dashed black line $\mathcal{V} > 0$.
- Two regions with strong correlations:
  - $M_{bb} \simeq 2m_b$, spin-singlet.
  - Ultra-relativistic $M_{bb} \gg 2m_b$, spin-triplet.



Decay Channel

- We will use $\boldsymbol{\Lambda_b \to X_c \ell^- \bar{\nu}_\ell}$.
  - ℓ is a μ in our case, $\mathrm{BR}\left(\Lambda_b \to X_c \mu^- \bar{\nu}_\mu\right) \approx 11\%$.
  - X_c is set of particles with at least one charmed hadron, usually Λ_c^+.
  - We require a reconstructed Λ_c^+ on one side, through fully charged hadronic decays, $\mathrm{BR}(\Lambda_c^+ \to \mathrm{reco.}) \approx 18\%$.
  - $\alpha_\nu \simeq 1,\ \alpha_\mu \simeq -0.26$.

Galanti et al., JHEP 11 (2015) 067
Y. Kats, D. Uzan, JHEP 03 (2024) 063

Feasibility Study

- **ATLAS Selection:**
  - **Dimuon Trigger:** $p_T^\mu > 15$ GeV and $|\eta| < 2.4$ without isolation for both.
  - $p_T^\mu / p_T^{\text{jet}} > 0.2$ for at least one muon.
  - At least one b tagged jet.

$\rightarrow$ planned to be updated in HL-LHC $p_T^\mu > 10$ GeV and $|\eta| < 2.5$.

ATLAS Collaboration, JINST 15 (2020), P09015

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ATLAS Collaboration, JINST 15 (2020), P09015

- **LHCb Selection:**
  - **Trigger:** $p_T^\mu > 1.8$ GeV, with $2 < \eta < 5$,
    2-4 SV displaced tracks,
    charged particle with $p_T > 1.6$ GeV, not from PV.
  - An additional muon with $p_T^\mu > 0.5$ GeV and $2 < \eta < 5$.
  - At least one b tagged jet.
  - $M_{b\bar{b}} > 20$ GeV.

LHCb Collaboration, JHEP 08 (2017), 055

Feasibility Study

➢ CMS B Parking Selection:

- Large amount of data ($\sim 42\ \mathrm{fb}^{-1}$) recorded by CMS during Run 2, low trigger threshold, high statistics.
- Was processed when sufficient computational power became available.

Trigger:

CMS Collaboration, arXiv:2403.16134

- Single muon trigger.
- Lower specialized trigger p_T cuts, between 7 and 12 GeV with $|\eta| < 1.5$.
- Transverse impact parameter significance cut.

Additional cuts:

- We apply $p_T^{\mu} > 5$ GeV and $|\eta| < 2.4$ on an additional muon.

Feasibility Study

- **For ATLAS and LHCb:**

$$N = 2\sigma\epsilon_{\mu\mu}f^2(b \to \Lambda_b)BR^2\left(\Lambda_b \to X_c\mu^-\bar{\nu}_\mu\right)$$
$$\times BR(\Lambda_c^+ \to \mathrm{reco.})\epsilon_{\mathrm{reco.}}\epsilon_{b,2}$$

$\epsilon_{\mathrm{reco.}} \approx 50\%$ the average Λ_c^+ decay reconstruction efficiency.
$\epsilon_{b,2}$ at least one jet to be b-tagged.

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➢ **For CMS B parking:**

$$N = 2f^2(b \to \Lambda_b)BR\left(\Lambda_b \to X_c\mu^-\bar{\nu}_\mu\right)\epsilon_{\mu,2}$$
$$\times BR(\Lambda_c^+ \to \text{reco.})\epsilon_{\text{reco.}}N_0$$

$N_0 \approx 10^{10}$, total number of $b\bar{b}$ events is CMS B parking dataset.
$\epsilon_{\mu,2} \approx 36\%$ selecting a muon on the non-triggering side.

Results

	$\sigma\epsilon_{\mu\mu}$ [pb]	$\mathcal{L}$ [fb^{-1}]	N	C_{kk}	C_{rr}	C_{nn}	Δ	$\mathcal{V}$	r_L	$\sigma_\Delta^{\rm stat}$	$\sigma_\mathcal{V}^{\rm stat}$	$\frac{\Delta}{\sigma_\Delta^{\rm stat}}$	$\frac{\mathcal{V}}{\sigma_\mathcal{V}^{\rm stat}}$	$\frac{\Delta}{\sigma_\Delta^{\rm tot}}$	$\frac{\mathcal{V}}{\sigma_\mathcal{V}^{\rm tot}}$
	Run 2, $\sqrt{s} = 13$ TeV														
ATLAS	1.9×10^4	140	2.7×10^4	0.94	0.57	-0.56	0.54	0.21	0.75	0.14	0.33	3.9	0.6	3.1	0.6
									0.45	0.23	0.78	2.3	0.3	2.1	0.3
LHCb, $\Delta > 0.2$	3.9×10^6	5.7	1.8×10^4	0.55	0.67	-0.56	0.39	-0.24	0.75	0.17	0.34	2.2	-0.7	2.0	-0.7
									0.45	0.29	0.62	1.3	-0.4	1.3	-0.4
CMS B parking, $\Delta > 0.2$	7.9×10^5	41.6	1.8×10^5	0.76	0.63	-0.59	0.49	-0.03	0.75	0.055	0.120	8.9	-0.3	4.4	-0.3
									0.45	0.092	0.256	5.3	-0.1	3.6	-0.1
	HL-LHC, $\sqrt{s} = 14$ TeV														
ATLAS, $\mathcal{V} > 0.3$	9.9×10^4	3000	1.0×10^6	0.91	0.85	-0.83	0.79	0.55	0.75	0.02	0.06	> 10	8.7	4.9	4.3
									0.45	0.04	0.13	> 10	4.3	4.9	3.3
LHCb, $\mathcal{V} > 0.3$	4.3×10^6	300	8.2×10^4	0.79	0.88	-0.81	0.74	0.43	0.75	0.080	0.215	9.2	2.0	4.4	1.8
									0.45	0.135	0.406	5.5	1.0	3.7	1.0
CMS B parking, $\mathcal{V} > 0.2$	8.4×10^5	800	1.2×10^6	0.83	0.82	-0.78	0.71	0.35	0.75	0.021	0.055	> 10	6.4	4.9	3.9
									0.45	0.036	0.110	> 10	3.2	4.9	2.7

We fix $r_T = 0.7$.

On the right, we assumed 20% systematic uncertainty.

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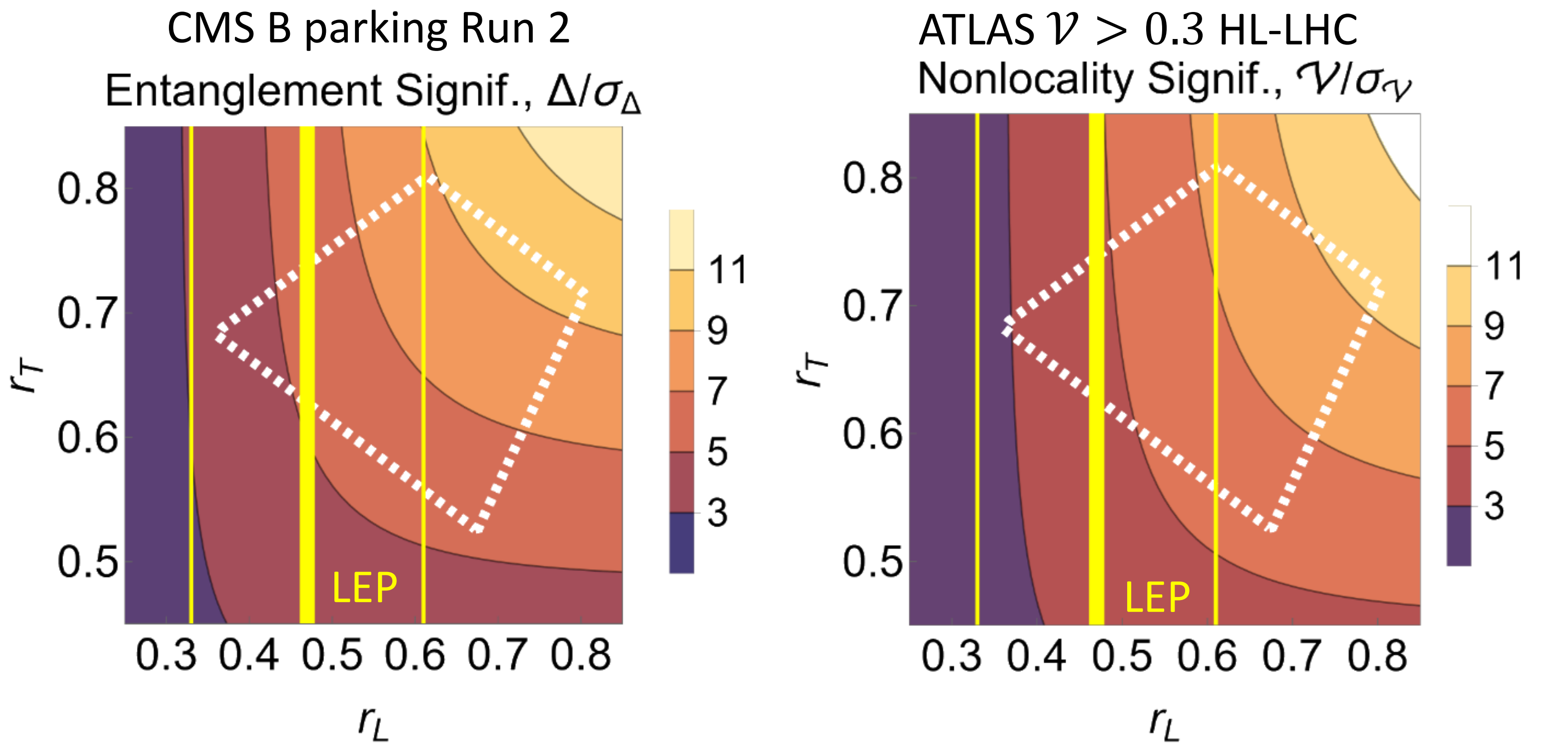
We fix $r_T = 0.7$.
On the right, we assumed 20% systematic uncertainty.

Significance as a Function of Unknown r_T and r_L

- White dashed line: possible range of r_T and r_L.

We can use the non-entangled area in phase space to measure $r_{L/T}$ or use $t\bar{t}$ Run 2 samples for r_L.



Conclusions

- We show that Entanglement and Bell nonlocality are possible to measure in $b\bar{b}$ pairs.

- A hadronizing system (almost explicitly rejected in previous works) and highly boosted at low invariant mass.
- Some new papers using QI methods to research hadronization.

- The most promising channel, with currently available Run 2 data, is CMS B parking.

- Some experimental challenges:
  - Reconstruction of a difficult decay channel for Λ_b, especially the unmeasured neutrino.
  - Limited statistics due to FF, BR and efficiencies.

Thank you!

Λ_c^+ Reconstructible Channels

Relevant Decay Modes	Branching Ratio
$\Lambda_c^+ \to pK^-\pi^+$	6.3%
$\Lambda_c^+ \to \Lambda\pi^+ \to p\pi^-\pi^+$	0.8%
$\Lambda_c^+ \to pK_S \to p\pi^-\pi^+$	1.1%
$\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^- \to p\pi^+\pi^+\pi^-\pi^-$	2.3%
$\Lambda_c^+ \to pK_S\pi^+\pi^- \to p\pi^+\pi^+\pi^-\pi^-$	1.1%
$\Lambda_c^+ \to \Sigma^+\pi^+\pi^-$	4.5%
$\Lambda_c^+ \to \Sigma^-\pi^+\pi^+$	1.9%
total	18%

Spin Retention

Spin is retained during hadronization:

$$B \propto \Lambda_{QCD}^2, \qquad t_{QCD} \propto \frac{1}{\Lambda_{QCD}}$$

Similar to electrodynamics:

$$\frac{d\vec{\mu}_q}{dt} = \gamma \vec{\mu}_q \times \vec{B}; \;\; \vec{\mu} = \gamma \vec{S}, \gamma \propto \frac{g_s}{m_q}$$

$$\Delta S \propto \mu_q B t_{QCD} \propto \frac{\Lambda_{QCD}}{m_q} \ll 1$$

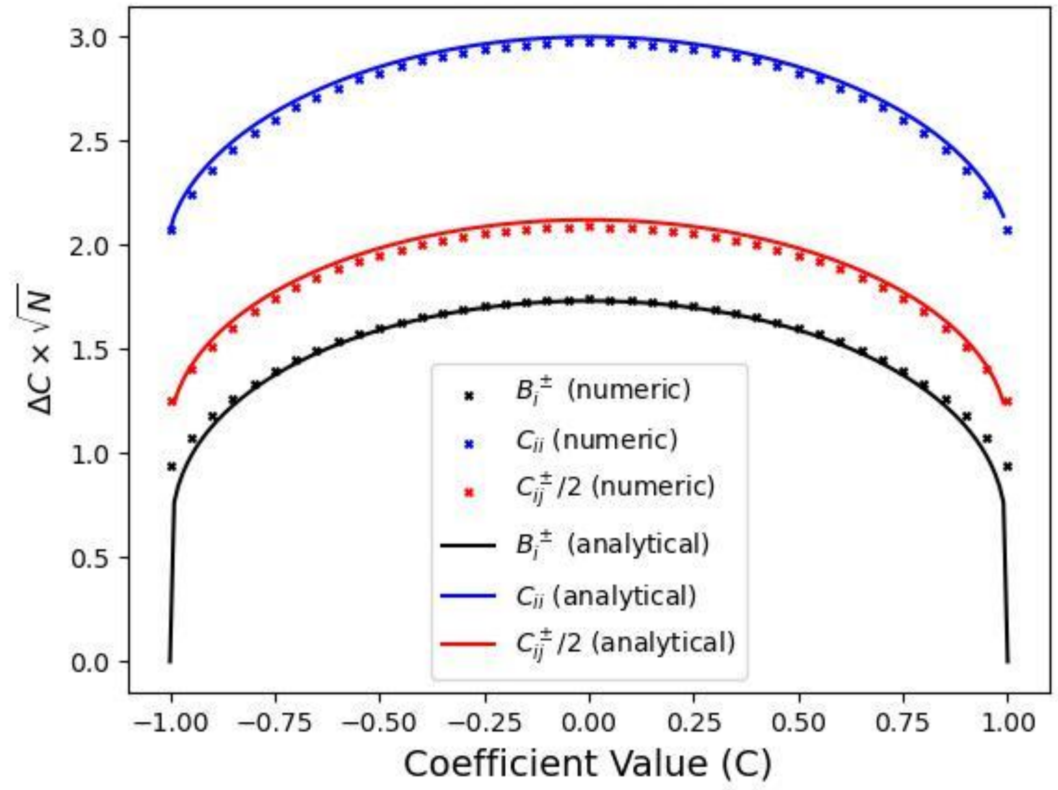
Due to $m_b \gg \Lambda_{QCD} \approx 0.2$ GeV.

Statistical Uncertainty of Polarization and Spin Correlations

$$\Delta B_i = \frac{A_B(B_i)}{r_i \alpha \sqrt{f N_{sig}}}, \; A_B(0) = \sqrt{3}$$

$$\Delta C_{ii} = \frac{A_{C_{ii}}(C_{ii})}{r_i^2 \alpha^2 \sqrt{f N_{sig}}}, \; A_{C_{ii}}(0) = 3$$

$$\Delta C_{k(ij)} = \frac{A_{C_{ij}}(C_{ij})}{r_i r_j \alpha^2 \sqrt{f N_{sig}}}, \; A_{C_{ij}}(0) = \frac{3\sqrt{2}}{2}$$



Spin Measurement Through Mesons

Mesons can't be used for spin measurements.

$$|\downarrow\rangle_b |\downarrow\rangle_{\bar{q}}, \qquad |\downarrow\rangle_b |\uparrow\rangle_{\bar{q}}$$

For B^* meson, where $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1 = \mathrm{B} \oplus \mathrm{B}^*$:

$$P\big(\bar{B}^*(-1)\big) = \frac{1}{2}, \qquad P\big(\bar{B}^*(0)\big) = \frac{1}{4}, \qquad P\big(\bar{B}^*(1)\big) = 0, \qquad P\big(B(0)\big) = \frac{1}{4}$$

$$B^* \to B\gamma$$

We get uniform angular distribution, no information of the spin of the b quark.

Falk and Peskin, PRD 49, 3320 (1994)

Spin Measurement through Baryons

For the baryons the spin is retained:

$$|\downarrow\rangle_b|\downarrow\rangle_q|\downarrow\rangle_{q'}, \qquad |\downarrow\rangle_b|\downarrow\rangle_q|\uparrow\rangle_{q'}, \qquad |\downarrow\rangle_b|\uparrow\rangle_q|\downarrow\rangle_{q'}, \qquad |\downarrow\rangle_b|\uparrow\rangle_q|\uparrow\rangle_{q'}$$

$$\left(\frac{1}{2}\right)_b \otimes \left(\frac{1}{2}\right)_q \otimes \left(\frac{1}{2}\right)_{q'} = \frac{1}{2} \otimes (1 \oplus 0) = \underset{\boldsymbol{\Lambda_b}}{\frac{1}{2}} \oplus \underset{\boldsymbol{\Sigma_b}}{\frac{1}{2}} \oplus \underset{\boldsymbol{\Sigma_b^*}}{\frac{3}{2}}$$

We lose polarization information in hadronization through indirect Λ_b production:

$$b \to \Sigma_b, \Sigma_b^*, \Lambda_b, \qquad \Sigma_b^{(*)} \to \Lambda_b \pi$$

Falk and Peskin, PRD 49, 3320 (1994)

Energy and Momentum Reconstruction

Energy and momentum reconstruction:

$$E_{\Lambda_b} = \langle z \rangle E_b = E_{X_c\mu} + E_\nu$$

$$E_{X_c\mu} = \frac{3\langle z \rangle E'_{jet} - (1 - \langle z \rangle)E_\nu}{1 + 2\langle z \rangle} \approx \frac{3\langle z \rangle}{2\langle z \rangle + 1} E'_{jet}$$

The term E'_{jet} is the jet energy with subtraction from all tracks originating in the primary vertex.

Energy and Momentum Reconstruction

We want to also approximate $\vec{p}_{X_c\mu}$:

- The momentum $\vec{p}_\mu$ is easily measured.

- We estimate the direction of $\vec{p}_{X_c}$ with all the tracks not from PV.

- Assuming $m_{X_c} \approx m_{\Lambda_c^+}$, and using $E_{X_c} = E_{X_c\mu} - E_\mu$:

$$\left|\vec{p}_{X_c\mu}\right| = \sqrt{E_{X_c}^2 - m_{X_c}^2}$$

Energy and Momentum Reconstruction

We can get the momentum of the neutrino through:

$$P_\nu^\perp = -P_\perp, \qquad P_\nu^\parallel = -a \pm \sqrt{b}$$

relative to the vector connecting PV and SV, where:

$$a = \frac{\left(m_{\Lambda_b}^2 - m^2 - 2P_\perp^2\right)P_\parallel}{2(P_\parallel^2 - E^2)}$$

$$b = \frac{\left(m_{\Lambda_b}^2 - m^2 - 2P_\perp^2\right)E^2}{4(P_\parallel^2 - E^2)} + \frac{E^2 P_\perp^2}{P_\parallel^2 - E^2}$$

Previous Use of Semi-leptonic Reconstruction

The reconstruction algorithm was used in LHCb to measure:

- $|V_{ub}|$ and $|V_{cb}|$ **LHCb Collaboration, Nature Physics 10, (2015) 1038**
- $B_s^0 \to K^-\mu^+\nu_\mu$ and the ratio $|V_{ub}|/|V_{cb}|$ **LHCb Collaboration, PRL 126, 081804 (2021)**

Distinguishing between the two ν solution via linear regression using:

$$\frac{1}{\sin\theta}, |F|$$

where θ is the polar angle of the flight vector and $|F|$ its magnitude.

Ciezarek et al., JHEP 02 (2017) 021