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Swiss National Science Foundation

Maximal entanglement and symmetries in 2HDM

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07.04.2025

Overview

- Exploring relations between symmetries and entanglement: **could entanglement underlie the emergence of symmetries?**

- Consider a Two Higgs Doublet Model and compute the $2 \to 2$ scatterings of the scalar doublets viewed as qubits, with the scattering matrix understood as a quantum gate

- Maximal entanglement leads to a $U(2) \times U(2)$ symmetry on the scalar potential and relations on the mass spectrum!

- These symmetries can also be gauged without reducing the amount of entanglement and yields an **exact copy of the SM electroweak gauge group**

Qubits in 2HDM

[2307.08112: Carena, Low, Wagner, Xiao]

- 2HDM consists of the SM and an additional complex $SU(2)$ scalar doublet

$$\Phi_1 = \begin{bmatrix} \Phi_1^+ \\ \Phi_1^0 \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} \Phi_2^+ \\ \Phi_2^0 \end{bmatrix}$$

- Consider $2 \to 2$ scatterings of the $SU(2)$ **components** of the scalar doublets

$$\Phi_a^+ \Phi_b^0 \to \Phi_c^+ \Phi_d^0$$

As an example, we will consider charged-neutral scattering

- Computational basis is encoded in the **(Higgs) flavor** of the scalar doublets

$$\{|0\rangle_A, |1\rangle_A\} = \{\Phi_1^+, \Phi_2^+\}$$
$$\{|0\rangle_B, |1\rangle_B\} = \{\Phi_1^0, \Phi_2^0\}$$

Concurrence

[2307.08112: Carena, Low, Wagner, Xiao]

- Two **distinguishable** qubits

$$\{|0\rangle_I, |1\rangle_I\}, \; I = A, B$$

- Define the quantum state

$$|\psi\rangle = \sum_{i,j=1}^{2} c_{ij} \, |i\rangle_A \, |j\rangle_B$$

- Concurrence as entanglement measure

$$\Delta(|\psi\rangle) = \frac{2|c_{11}c_{22} - c_{12}c_{21}|}{|c_{11}|^2 + |c_{12}|^2 + |c_{21}|^2 + |c_{22}|^2}$$

Separable

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

$$0 \leq \Delta \leq 1$$

[2104.10835: Low, Mehen]

Maximally entangled

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

Entanglement

[2307.08112: Carena, Low, Wagner, Xiao]

- Max entanglement is determined by requiring that the outgoing state is

$$T\,|\Phi_a^+\Phi_b^0\rangle = \underbrace{|\text{flavor}\rangle \otimes |\text{kinematics}\rangle}_{\text{Product state in (Higgs) flavor and momentum}}\ , \quad \underbrace{\Delta(|\text{flavor}\rangle) = 1}_{\text{Wave function in the flavor subspace is maximally entangled}}$$

Product state in (Higgs) flavor and momentum

Wave function in the flavor subspace is maximally entangled

Entanglement

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Product state in (Higgs) flavor and momentum

Wave function in the flavor subspace is maximally entangled

- As we want to **maximize entanglement** in 2 → 2 scalar scattering, we consider only the T matrix

$$S = 1 + i\,T$$

Identity is the "nothing happened" gate, and therefore can only reduce the entanglement

T is not unitary at tree level (not a proper quantum gate) but valid in the perturbative expansion!

Scalar scattering

[2404.13743: Kowalska, Sessolo]

- We thus focus on the following **scattering amplitude**

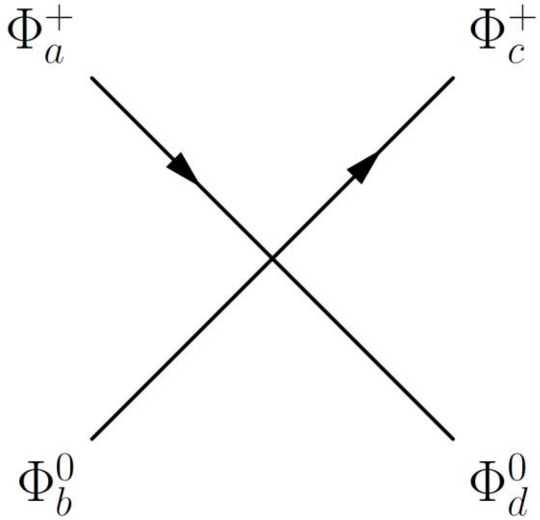
$$\langle \Phi_c \Phi'_d | \, iT \, | \Phi_a \Phi'_b \rangle = i(2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_c) M_{ab,cd}$$

- The scattering amplitude depends on the scalar couplings and the potential in the **CP conserving** limit of the 2HDM is

$$\begin{aligned} V = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \left\{ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} \end{aligned}$$

Symmetric phase

- Start in the symmetric phase, namely considering only **quartic interactions** for the scattering amplitude



As an example, focus on charged-neutral scattering

$$i\mathcal{M}_0^{(2)}(\Phi_a^+\Phi_b^0 \to \Phi_c^+\Phi_d^0) = -i\begin{bmatrix} \lambda_1 & \lambda_6 & \lambda_6 & \lambda_5 \\ \lambda_6 & \lambda_3 & \lambda_4 & \lambda_7 \\ \lambda_6 & \lambda_3 & \lambda_4 & \lambda_7 \\ \lambda_5 & \lambda_7 & \lambda_7 & \lambda_2 \end{bmatrix}$$

Maximizing entanglement

- We want to maximize the concurrence on **each element of the computational basis**

$$\Delta(\mathcal{M}_0^{(2)}|\Phi_1^+\Phi_1^0\rangle) = \frac{\left|2(\lambda_1\lambda_5 - \lambda_6^2)\right|}{2\lambda_6^2 + \lambda_1^2 + \lambda_5^2} ,$$

$$\Delta(\mathcal{M}_0^{(2)}|\Phi_1^+\Phi_2^0\rangle) = \Delta(\mathcal{M}_0^{(2)}|\Phi_2^+\Phi_1^0\rangle) = \frac{|2(\lambda_6\lambda_7 - \lambda_3\lambda_4)|}{\lambda_7^2 + \lambda_6^2 + \lambda_3^2 + \lambda_4^2} ,$$

$$\Delta(\mathcal{M}_0^{(2)}|\Phi_2^+\Phi_2^0\rangle) = \frac{\left|2(\lambda_2\lambda_5 - \lambda_7^2)\right|}{2\lambda_7^2 + \lambda_2^2 + \lambda_5^2}$$

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- Max entanglement on **each basis element** yields

$$(1): \quad \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \pm\lambda_5 \neq 0 \ , \quad \lambda_6 = \lambda_7 = 0$$

$$(2): \quad \lambda_1 = \lambda_2 = -\lambda_3 = -\lambda_4 = \pm\lambda_5 \neq 0 \ , \quad \lambda_6 = \lambda_7 = 0$$

$$(3): \quad \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0 \ , \quad \lambda_6 = \pm\lambda_7 \neq 0$$

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Potential not bounded from below!

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We focus on these solutions

- Max entanglement on **each basis element** yields

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Potential not bounded from below! (applies to (2) and (3))

Entanglement and symmetries

- Max entanglement on the computation basis yields a **symmetry on the potential**, explicit when arranging the scalar doublets in the fundamental representation of $U(4)$: $\Phi = (\Phi_1, \Phi_2)^T$

$\Delta = 1$	Potential	Generators
$\lambda = \lambda_i = \lambda_5$	$V_+^{(4)} = \frac{\lambda}{2}\left[(\Phi^\dagger\Phi)^2 + (\Phi^\dagger\tau_1\Phi)^2\right]$	$\{\mathbb{1}\otimes\sigma^\mu, \tau^1\otimes\sigma^\mu\}$
$\lambda = \lambda_i = -\lambda_5$	$V_-^{(4)} = \frac{\lambda}{2}\left[(\Phi^\dagger\Phi)^2 + (\Phi^\dagger\tau_2\Phi)^2\right]$	$\{\mathbb{1}\otimes\sigma^\mu, \tau^2\otimes\sigma^\mu\}$

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- Under these $U(2)\times U(2)$ symmetries ($\lambda_i = \pm\lambda_5$), the charged neutral scattering amplitude is

$$i\mathcal{M}_0^{(2)}(\Phi^+\Phi^0 \to \Phi^+\Phi^0) = -i\lambda \begin{bmatrix} 1 & 0 & 0 & \pm 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \pm 1 & 0 & 0 & 1 \end{bmatrix}$$

Symmetries

- The symmetries **factorize** into two **orthogonal** $U(2)$ copies

$$V_+^{(4)}: \qquad U(2)_S \to T_S^\mu = \frac{\mathbb{1}+\tau^1}{2} \otimes \sigma^\mu \,, \quad U(2)_A \to T_A^\mu = \frac{\mathbb{1}-\tau^1}{2} \otimes \sigma^\mu$$

$$V_-^{(4)}: \qquad U(2)_L \to T_L^\mu = \frac{\mathbb{1}+\tau^2}{2} \otimes \sigma^\mu \,, \quad U(2)_R \to T_R^\mu = \frac{\mathbb{1}-\tau^2}{2} \otimes \sigma^\mu$$

- This is transparent when rotating to a **different flavor basis** for the scalar doublets (qubits)

Basis	Potential	Symmetry
$\Phi_{\mathsf{S,A}} = \frac{1}{\sqrt{2}}(\Phi_1 \pm \Phi_2)$	$V_+^{(4)} = \lambda \left[(\Phi_{\mathsf{S}}^\dagger \Phi_{\mathsf{S}})^2 + (\Phi_{\mathsf{A}}^\dagger \Phi_{\mathsf{A}})^2 \right]$	$U(2)_{\mathsf{S}} \times U(2)_{\mathsf{A}}$
$\Phi_{\mathsf{L,R}} = \frac{1}{\sqrt{2}}(\Phi_1 \mp i\Phi_2)$	$V_-^{(4)} = \lambda \left[(\Phi_{\mathsf{L}}^\dagger \Phi_{\mathsf{L}})^2 + (\Phi_{\mathsf{R}}^\dagger \Phi_{\mathsf{R}})^2 \right]$	$U(2)_{\mathsf{L}} \times U(2)_{\mathsf{R}}$

General results

- Since the two $U(2)$ are orthogonal, the amplitudes must be the **sum of these two sectors!** For instance, for the $U(2)_S \times U(2)_A$ symmetry ($\lambda_i = \lambda_5$), **all scattering amplitudes** are

$$\mathcal{M}_0^{(I)} \propto \underbrace{\frac{\mathbb{1} + \tau^1_{(\mathrm{a})}}{2} \otimes \frac{\mathbb{1} + \tau^1_{(\mathrm{b})}}{2}}_{\text{Projector on } U(2)_S} + \underbrace{\frac{\mathbb{1} - \tau^1_{(\mathrm{a})}}{2} \otimes \frac{\mathbb{1} - \tau^1_{(\mathrm{b})}}{2}}_{\text{Projector on } U(2)_A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

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Transforming computational basis in Bell states!

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Transforming computational basis in Bell states!

- **However: scattering does not depend on mass terms which indeed explicitly broken these symmetries!**

$$V_2 = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \mathrm{h.c.}]$$

Broken phase

- Now turn to the broken phase (after spontaneous symmetry breaking of the electroweak gauge group), and compute the scattering amplitudes **including trilinear vertices**

- To study the alignment limit (and much more) it is convenient to work in the so-called Higgs basis

- Exploit a residual global $U(2)$ symmetry of the scalar potential to rotate the scalar doublets such that

$$\langle \Phi_1 \rangle = v_1, \langle \Phi_2 \rangle = v_2 \longrightarrow \langle \Phi_1 \rangle = v, \langle \Phi_2 \rangle = 0$$

- In this basis we relabel the couplings as

$$\lambda_{1,2,3,4,5,6,7} \Longleftrightarrow Z_{1,2,3,4,5,7,6} \; ; \qquad m^2_{11,22,12} \Longleftrightarrow Y_{1,2,3}$$

Alignment

- After spontaneous symmetry breaking, we end up with 2 charged, 2 neutral CP-even and 2 neutral CP-odd scalars

- Scalar masses depend on the scalar couplings and therefore **symmetries of the potential impact the mass spectrum**

$$M_{\pm}^2 = \begin{bmatrix} 0 & 0 \\ 0 & Y_2 + Z_3 v^2/2 \end{bmatrix}$$

$$M_{\mathrm{CP-even}}^2 = \begin{bmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & Y_2 + (Z_3 + Z_4 + Z_5) v^2/2 \end{bmatrix}$$

$$M_{\mathrm{CP-odd}}^2 = \begin{bmatrix} 0 & 0 \\ 0 & Y_2 + (Z_3 + Z_4 - Z_5) v^2/2 \end{bmatrix}$$

Alignment

- After spontaneous symmetry breaking, we end up with 2 charged, 2 neutral CP-even and 2 neutral CP-odd scalars

- Scalar masses depend on the scalar couplings and therefore **symmetries of the potential impact the mass spectrum**

- Under the symmetries derived from the quartic interactions ($Z_i = Z_5$) the mass matrix becomes

$$M^2_{\pm} = \begin{bmatrix} 0 & 0 \\ 0 & Y_2 + Z_3 v^2/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & Y_2 + Z v^2/2 \end{bmatrix}$$

$$M^2_{\mathrm{CP-even}} = \begin{bmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & Y_2 + (Z_3 + Z_4 + Z_5) v^2/2 \end{bmatrix} = \begin{bmatrix} Z v^2 & 0 \\ 0 & Y_2 + (2Z + Z_5) v^2/2 \end{bmatrix}$$

$$M^2_{\mathrm{CP-odd}} = \begin{bmatrix} 0 & 0 \\ 0 & Y_2 + (Z_3 + Z_4 - Z_5) v^2/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & Y_2 + (2Z - Z_5) v^2/2 \end{bmatrix}$$

Alignment

- After spontaneous symmetry breaking, we end up with 2 charged, 2 neutral CP-even and 2 neutral CP-odd scalars

- Scalar masses depend on the scalar couplings and therefore **symmetries of the potential impact the mass spectrum**

- Under the symmetries derived from the quartic interactions ($Z_i = Z_5$) the mass matrix becomes

Max entanglement of the quartic interactions ($Z_6 = 0$) implies alignment limit!

$$M^2_{\pm} = \begin{bmatrix} 0 & 0 \\ 0 & Y_2 + Z_3 v^2/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & Y_2 + Z v^2/2 \end{bmatrix}$$

$$M^2_{\mathrm{CP-even}} = \begin{bmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & Y_2 + (Z_3 + Z_4 + Z_5) v^2/2 \end{bmatrix} = \begin{bmatrix} Z v^2 & 0 \\ 0 & Y_2 + (2Z + Z_5) v^2/2 \end{bmatrix}$$

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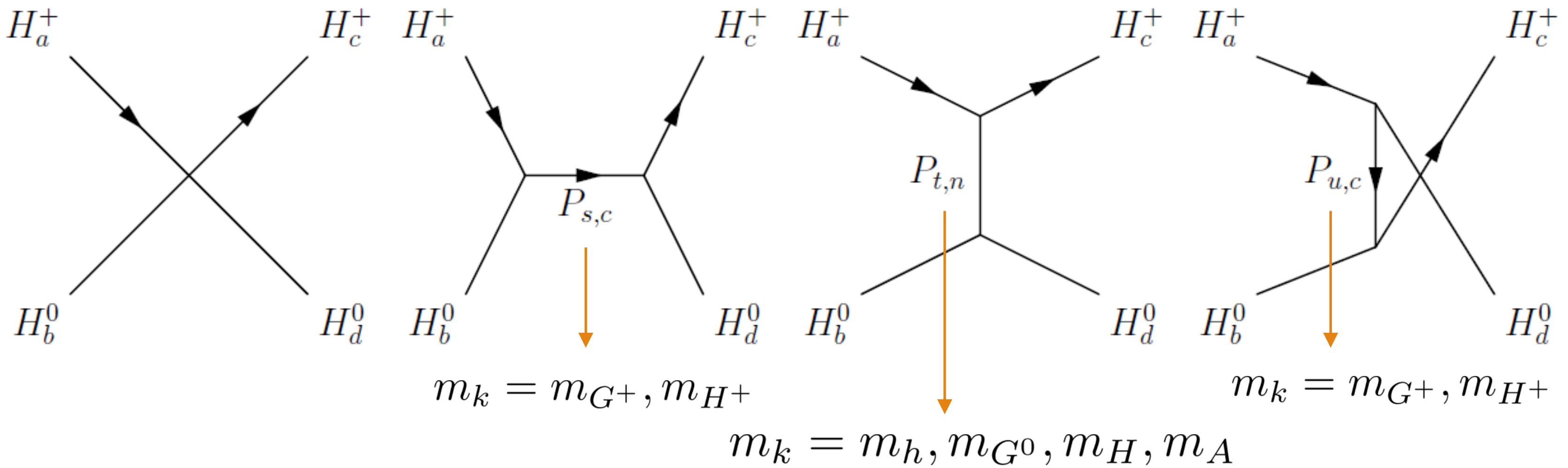
Scattering amplitudes

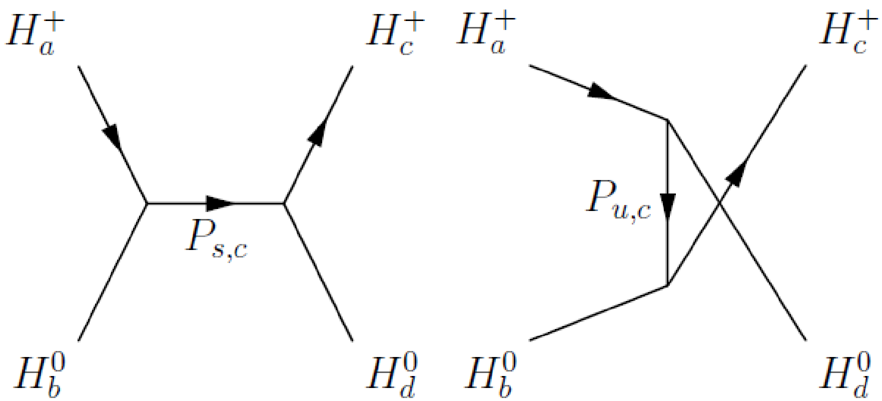
- Compute the scattering amplitudes including trilinear vertices of the scalar potential, paying attention to the **different kinematic structure of each channel**

$$i\mathcal{M} = i\mathcal{M}_0 - \frac{v^2}{2}\sum_k \sum_{r=s,t,u} \mathcal{M}_{r,k}\; P_{r,k}$$

$$P_{r,k} = i/(r - m_k^2), \quad r = s, t, u$$



Charged-mediated (s, u)



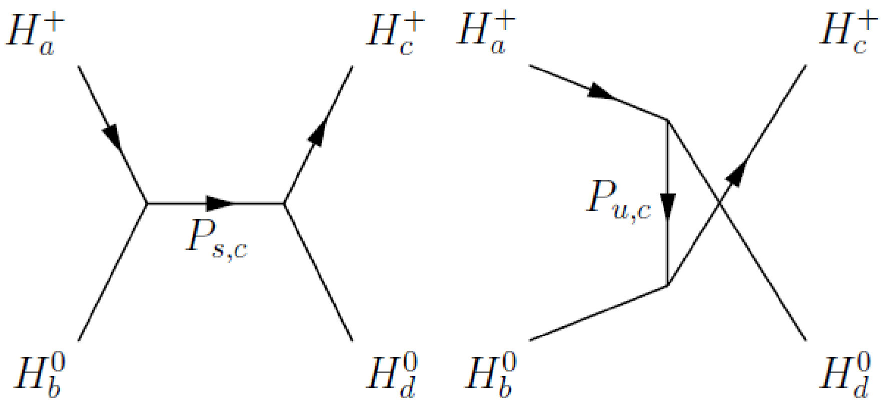
$$\frac{i}{(s/u - m_k^2)}$$

$$m_k = m_{G^+}, m_{H^+}$$

$$\mathcal{M}_{s,G^\pm} = \begin{bmatrix} Z_1^2 & 0 & 0 & Z_5 Z_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Z_5 Z_1 & 0 & 0 & Z_5^2 \end{bmatrix}, \quad \mathcal{M}_{s,H^\pm} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & Z_4^2 & Z_3 Z_4 & 0 \\ 0 & Z_3 Z_4 & Z_3^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{M}_{u,G^\pm} = \begin{bmatrix} Z_1^2 & 0 & 0 & 0 \\ 0 & 0 & Z_1 Z_4 & 0 \\ 0 & Z_1 Z_4 & 0 & 0 \\ 0 & 0 & 0 & Z_4^2 \end{bmatrix}, \quad \mathcal{M}_{u,H^\pm} = \begin{bmatrix} 0 & 0 & 0 & Z_5 Z_3 \\ 0 & Z_5^2 & 0 & 0 \\ 0 & 0 & Z_3^2 & 0 \\ Z_5 Z_3 & 0 & 0 & 0 \end{bmatrix}$$

Charged-mediated (s, u)



$$\frac{i}{(s/u - m_k^2)}$$

$$m_k = m_{G^+}, m_{H^+}$$

$$\mathcal{M}_{s,G^\pm} = Z^2 \begin{bmatrix} 1 & 0 & 0 & \pm1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm1 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{M}_{s,H^\pm} = Z^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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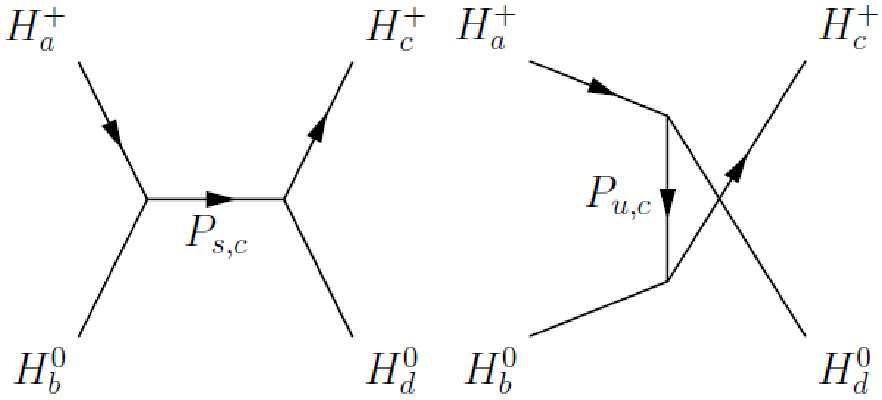
Charged-mediated (s, u)



$$\frac{i}{(s/u - m_k^2)}$$

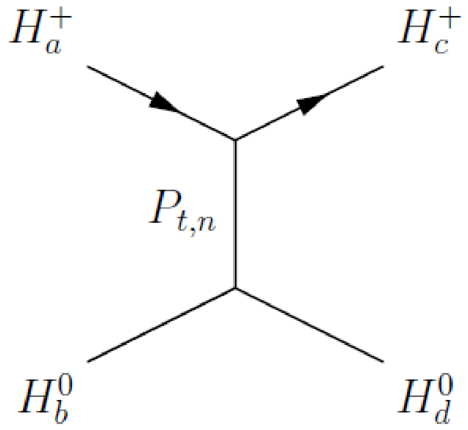
$$m_k = m_{G^+}, m_{H^+}$$

$$m_{H^\pm} = m_{G^\pm} = 0$$

$$\mathcal{M}_{s,G^\pm} = Z^2 \begin{bmatrix} 1 & 0 & 0 & \pm 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{M}_{s,H^\pm} = Z^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{M}_{u,G^\pm} = Z^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{M}_{u,H^\pm} = Z^2 \begin{bmatrix} 0 & 0 & 0 & \pm 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \pm 1 & 0 & 0 & 0 \end{bmatrix}$$

Neutral-mediated (t)



$$\frac{i}{(s/u - m_k^2)}$$

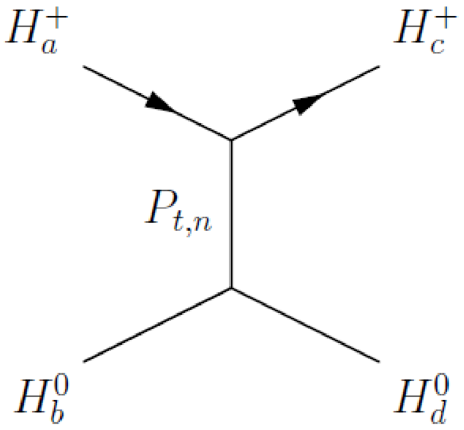
$$m_k = m_h, m_{G^0}, m_H, m_A$$

$$Z_{345} = 2Z_5 \pm (Z_3 + Z_4)$$

$$\mathcal{M}_{t,h} = \begin{bmatrix} Z_1^2 & 0 & 0 & 0 \\ 0 & \frac{1}{2}Z_1(Z_3+Z_4) & 0 & 0 \\ 0 & 0 & Z_1 Z_3 & 0 \\ 0 & 0 & 0 & \frac{1}{2}Z_3(Z_3+Z_4) \end{bmatrix}, \qquad \mathcal{M}_{t,G^0} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$\mathcal{M}_{t,H/A} = \begin{bmatrix} 0 & 0 & 0 & \pm\frac{1}{8}(Z_5 \pm Z_4)Z_{345} \\ 0 & 0 & \frac{1}{8}(Z_5 \pm Z_4)Z_{345} & 0 \\ 0 & \frac{1}{8}(Z_5 \pm Z_4)Z_{345} & 0 & 0 \\ \pm\frac{1}{8}(Z_5 \pm Z_4)Z_{345} & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{M}_{t,A/H} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Neutral-mediated (t)



$$\frac{i}{(s/u - m_k^2)}$$

$$m_k = m_h, m_{G^0}, m_H, m_A$$

$$\mathcal{M}_{t,h} = Z^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{M}_{t,G^0} = Z^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

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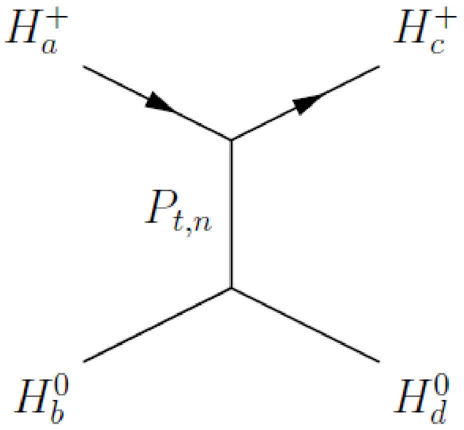

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$$m_k = m_h, m_{G^0}, m_H, m_A$$

$$Z_i = \pm Z_5$$

$$m_h = m_{H/A}$$

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Maximizing entanglement

- Max entanglement on the computational basis elements **for each channel** including trilinear vertices implies:

  1. **The same** symmetries as in the symmetric phase ($Z_i = \pm Z_5$)
  2. **Equal masses** for the states in each channel so that the flavor sub-amplitudes can be added with equal weights

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- Sum of flavor sub-amplitudes with same mass for propagators in each channel reproduces the pattern of the quartic interactions **for any scattering**

$$\sum_{k=G^\pm,H^\pm} \mathcal{M}_{s,m_k} \propto \sum_{k=G^\pm,H^\pm} \mathcal{M}_{u,m_k} \propto \sum_{k=h,G^0,H,A} \mathcal{M}_{t,m_k} \quad \propto \quad \mathcal{M}_0 = \lambda \begin{bmatrix} 1 & 0 & 0 & \pm 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \pm 1 & 0 & 0 & 1 \end{bmatrix}$$

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Transforming the computational basis in Bell states!

Exact symmetry!

- The requirement of $m_{H^\pm} = m_{G^\pm}$ arises from the s/u-channel leading to **the mass terms**

$$Y_1 = -\frac{1}{2} Z v^2 \qquad \rightarrow \text{Minimization of potential}$$

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and the **full potential** is $U(2)_{S/L} \times U(2)_{A/R}$ symmetric!

$$V = \sum_{\Gamma = \mathsf{S},\mathsf{A} \text{ or } \mathsf{L},\mathsf{R}} Y\, \Phi^\dagger_\Gamma \Phi_\Gamma + Z \left(\Phi^\dagger_\Gamma \Phi_\Gamma \right)^2$$

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$$V = \sum_{\Gamma = \text{S,A or L,R}} Y\, \Phi_\Gamma^\dagger \Phi_\Gamma + Z \left(\Phi_\Gamma^\dagger \Phi_\Gamma \right)^2$$

- Notice that in the Higgs basis we have $Y_3 = Z_6 = Z_7 = 0$ and $U(2)_S \times U(2)_A$ is equivalent to $U(2)_L \times U(2)_R$ via a rephasing of $H_2 \rightarrow i\, H_2$ which indeed flips the sign of Z_5

Gauge interactions

- Spontaneous symmetry breaking yields **6 massless Goldstone** and **2 scalars with equal mass**

$$U(2)_{S/L} \times U(2)_{A/R} \xrightarrow{\langle \Phi_1 \rangle = v} U(1)_{S/L} \times U(1)_{A/R}$$

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- For simplicity, we gauge $SU(2)_{S/L} \times SU(2)_{A/R}$

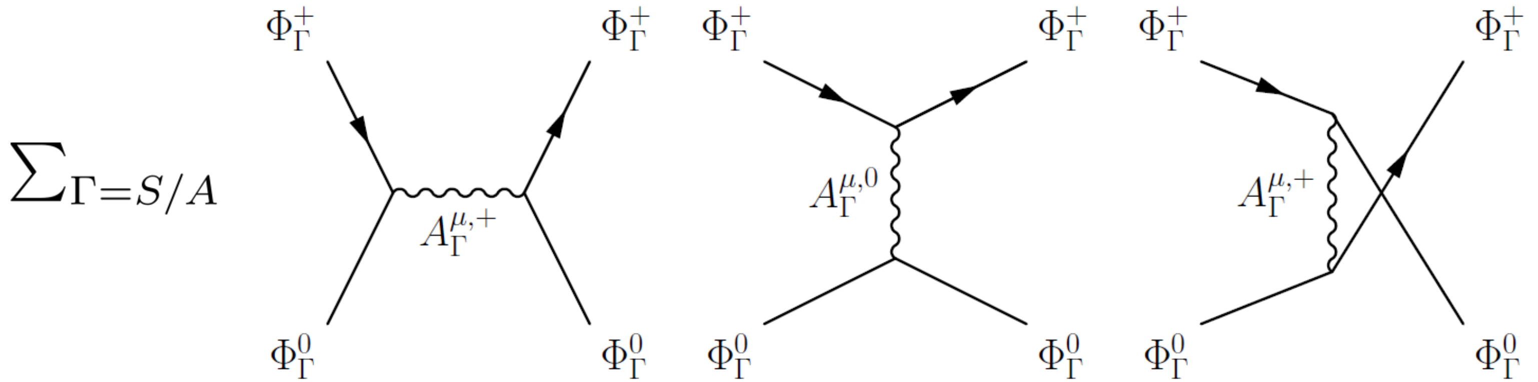
$$T^a_{\mathsf{S/L}} = \frac{\mathbb{1} + \tau^i}{2} \otimes \frac{\sigma^2}{2}$$
$$T^a_{\mathsf{A/R}} = \frac{\mathbb{1} - \tau^i}{2} \otimes \frac{\sigma^2}{2} \,, \quad i = 1, 2$$

- Recall that the $SU(2)$ copies are **orthogonal**

$$F^{\mu\nu}_{\Gamma} = \partial^\mu A^\nu_\Gamma - \partial^\nu A^\mu_\Gamma + i g_\Gamma [A^\mu_\Gamma, A^\nu_\Gamma]$$
$$\mathcal{L}_{\mathrm{YM}} = \sum_{\substack{\Gamma = \mathsf{S,A} \\ \text{or } \mathsf{L,R}}} -\frac{1}{4} \left(F_{\Gamma\,\mu\nu} F^{\mu\nu}_{\Gamma} \right)$$

Gauge amplitudes

- We work in the Lorentz gauge $\xi = 0$ and consider $Z_i = Z_5$
- $SU(2)_{\Gamma=S/A}$ **only** mediates scattering between $\Phi_{\Gamma=S/A}$
- Each channel is the **sum of two independent sectors**



$$\mathcal{M}_{r,A} = -P_r^S f_r^S \begin{bmatrix} g_S^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - P_r^A f_r^A \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_A^2 \end{bmatrix}$$

(P_r: propagator, f_r: kinematic factors, $r = s, t, u$)

Maximal entanglement

- Max entanglement requires **same gauge** coupling for the two $SU(2)$ symmetries, namely **equal mass** for all gauge bosons
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- In the **computational basis**, we reproduce the pattern for the quartic scalar interactions ($Z_i = Z_5$, r=$s/t/u$)

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3. Gauge bosons:
   - **Equal gauge couplings** of the $U(2)$ copies
   - **Equal gauge bosons masses**
   - Emergence of a discrete $\mathbb{Z}_2$ symmetry!

THANK YOU FOR YOUR ATTENTION!

Back-up slides

$a.\quad H^0H^0 \to H^0H^0$

$$\mathcal{M}_0^{(1)} = 2\,Z \begin{bmatrix} 1 & 0 & 0 & \pm1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \pm1 & 0 & 0 & 1 \end{bmatrix} ; \tag{A1}$$

$$\mathcal{M}_{s,h/G^0}^{(1)} = 2\,Z^2 \begin{bmatrix} 1 & 0 & 0 & \pm1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm1 & 0 & 0 & 1 \end{bmatrix} , \qquad \mathcal{M}_{s,H/A}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \tag{A2}$$

$$\mathcal{M}_{u,h}^{(1)} = 2\,Z^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \qquad \mathcal{M}_{u,G^0}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \tag{A3}$$

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$$\mathcal{M}_{t,h}^{(1)} = 2\,Z^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \qquad \mathcal{M}_{t,G^0}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \tag{A5}$$

$$\mathcal{M}_{t,H/A}^{(1)} = 2\,Z^2 \begin{bmatrix} 0 & 0 & 0 & \pm1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \pm1 & 0 & 0 & 0 \end{bmatrix} , \qquad \mathcal{M}_{t,A/H}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \tag{A6}$$

$b.\quad H^+\tilde{H}^0 \to H^+\tilde{H}^0$

$$\mathcal{M}_0^{(3)} = Z \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \pm 1 & 0 \\ 0 & \pm 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} ; \tag{A7}$$

$$\mathcal{M}_{s,G^\pm}^{(3)} = Z^2 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} , \qquad \mathcal{M}_{s,H^\pm}^{(3)} = Z^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm 1 & 0 \\ 0 & \pm 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \tag{A8}$$

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$c.\quad H^+H^- \to H^+H^-$

$$\mathcal{M}_0^{(4)} = 2\,Z \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \pm 1 & 0 \\ 0 & \pm 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} ; \tag{A12}$$

$$\mathcal{M}_{s,h}^{(4)} = \frac{Z^2}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} , \qquad \mathcal{M}_{s,G^0}^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \tag{A13}$$

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$$\mathcal{M}_{t,h}^{(4)} = \frac{Z^2}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \qquad \mathcal{M}_{t,G^0}^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \tag{A15}$$

$$\mathcal{M}_{t,H/A}^{(4)} = \frac{Z^2}{2} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \pm 1 & 0 \\ 0 & \pm 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} , \qquad \mathcal{M}_{t,A/H}^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \tag{A16}$$

d. $H^0\tilde{H}^0 \to H^0\tilde{H}^0$

$$\mathcal{M}_0^{(4)} = 2\,Z \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \pm 1 & 0 \\ 0 & \pm 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \; ; \tag{A17}$$

$$\mathcal{M}_{s,h}^{(4)} = 2\,Z^2 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \; , \qquad \mathcal{M}_{s,G^0}^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \; ; \tag{A18}$$

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$$\mathcal{M}_{u,h/G^0}^{(4)} = 2\,Z^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 \\ 0 & \pm 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \; , \qquad \mathcal{M}_{u,H/A}^{(4)} = 2\,Z^2 \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \; ; \tag{A20}$$

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e. $H^0\tilde{H}^0 \to H^+H^- \;/\; H^+H^- \to H^0\tilde{H}^0$

$$\mathcal{M}_0^{(5)} = Z \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \pm 1 & 0 \\ 0 & \pm 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} ; \tag{A23}$$

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