SSM Scuola Superiore Meridionale

Graviton/Axion-Photon conversion in the Stochastic magnetic field

Based on: arXiv:2401.15965, 2411.09042

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5/Feb/2025 in COST WG3

Outline

- GW-photon mixing
  - Stochastic resonance
  - Primordial magnetic field
  - Compare with Domain-like model
- Axion-photon mixing
  - Radio excess
  - 21cm absorption line

GW&EM

- GW channel: BH/NS merger, Stochastic GW background (Inflation, First-order phase transition)
- Electromagnetic channel: supernova, BH image, CMB, primordial magnetic field
- Two-channel Interaction:

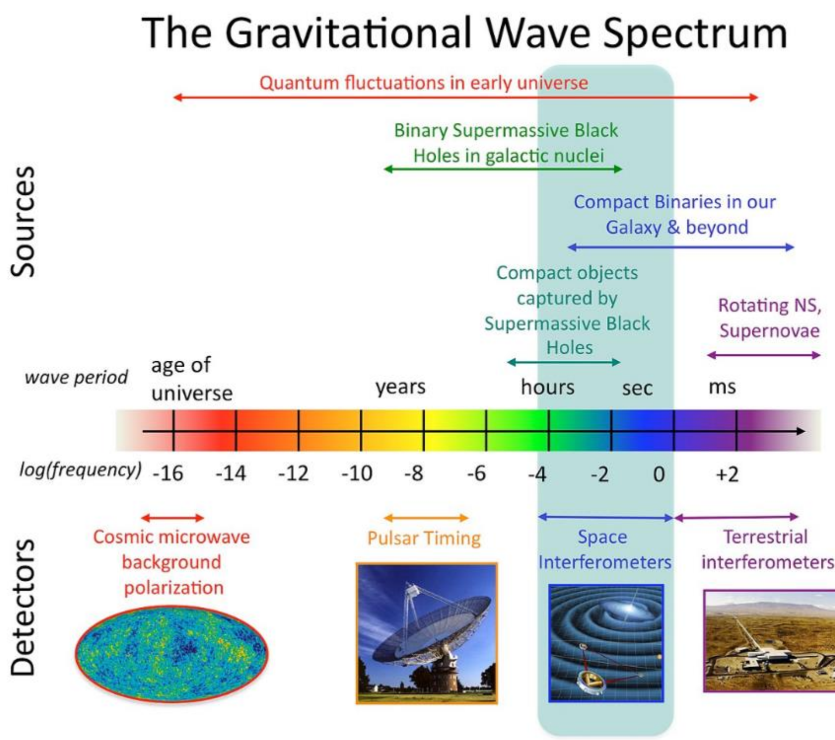
  Speed of GW

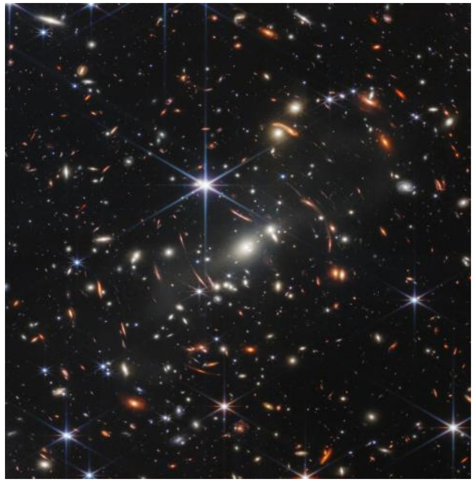
  pulsar timing arrays

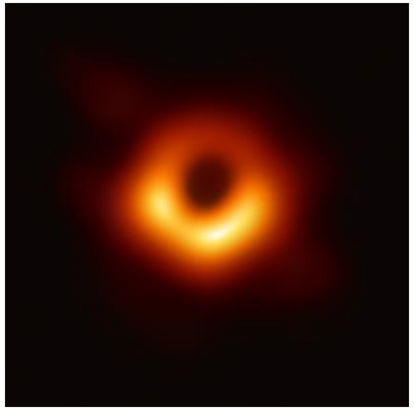
  E/B-modes

  Graviton-photon mixing?

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\mathcal{R}+\mathcal{F}^2\right)$$





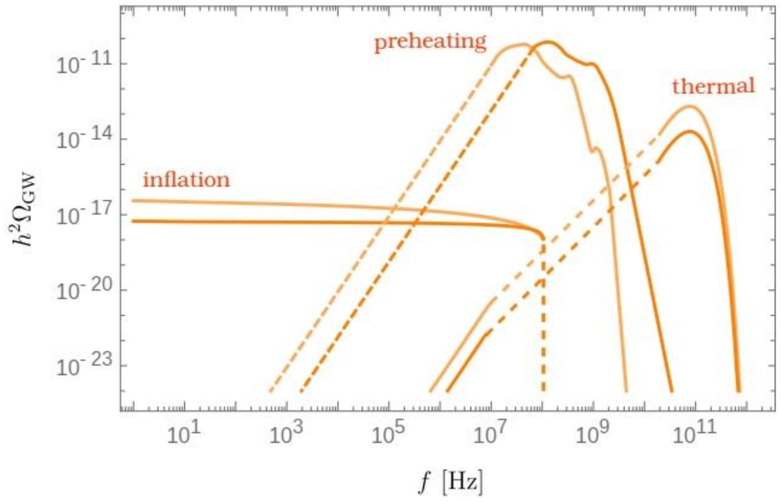


From Wikipedia

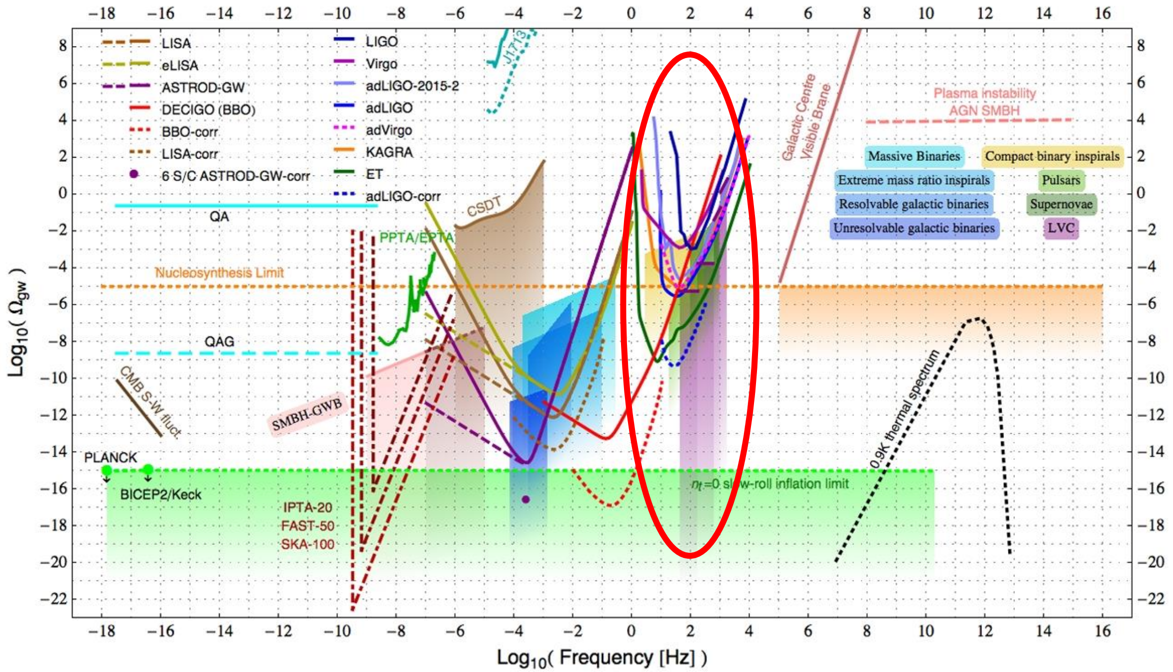
GW in High Frequency

- Cosmic source:

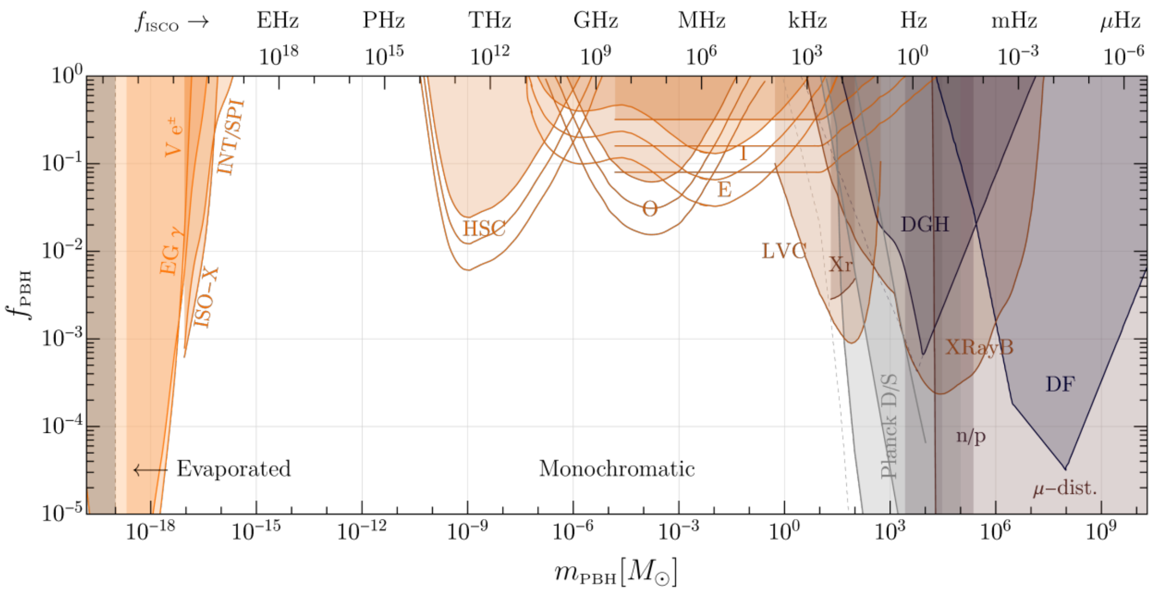
  Inflations, preheating, thermal fluctuation
- Astro source: BH mergers
- PBH merger (cosmic origin)



Standard Model +Axion +Seesaw +Higgs inflation model, from arXiv:2203.00621

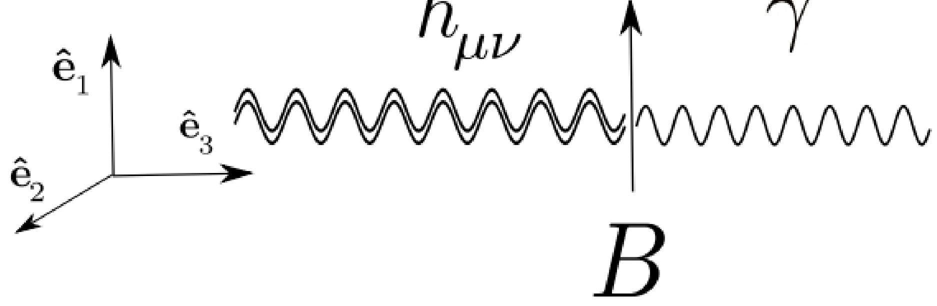


From arXiv: 1511.00231



From arXiv:2205.02153

Gertsenshtein Effect



- the action including plasma effect

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\mathcal{R} + \int d^4x\sqrt{-g}\left(-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + j^\mu \mathcal{A}_\mu\right) \qquad j^\mu = -\frac{1}{c}\omega_{\rm pl}^2\mathcal{A}^\mu$$

- For (ultra)relativistic particles, Graviton-photon mixing reduce to linear system

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$\mathcal{A}_\mu = \overline{A}_\mu + A_\mu$$
$$n_{\rm pl} = -e^2 n_e/(2\omega^2 m_e),$$

$$\partial_l \begin{pmatrix} h_\times(\omega,l) \\ h_+(\omega,l) \\ A_x(\omega,l) \\ A_y(\omega,l) \end{pmatrix} = iK(l)\begin{pmatrix} h_\times(\omega,l) \\ h_+(\omega,l) \\ A_x(\omega,l) \\ A_y(\omega,l) \end{pmatrix},\ K(l) = \begin{pmatrix} \omega & 0 & -i\frac{1}{2}\kappa\mathbf{B}_x(l) & i\frac{1}{2}\kappa\mathbf{B}_y(l) \\ 0 & \omega & i\frac{1}{2}\kappa\mathbf{B}_y(l) & i\frac{1}{2}\kappa\mathbf{B}_x(l) \\ i\frac{1}{2}\kappa\mathbf{B}_x(l) & -i\frac{1}{2}\kappa\mathbf{B}_y(l) & \omega(1+n_{\rm pl}) & 0 \\ -i\frac{1}{2}\kappa\mathbf{B}_y(l) & -i\frac{1}{2}\kappa\mathbf{B}_x(l) & 0 & \omega(1+n_{\rm pl}) \end{pmatrix}$$

- neglect the Cotton-Mouton effect and the Faraday Rotation, as they are quadratic in B

Constant magnetic field

- Analytically solution

$$\mathcal{P}(\triangle l) = \frac{1}{4}\kappa^2 B^2 l_{osc}^2 \sin^2\left(\frac{\triangle l}{l_{osc}}\right) \qquad l_{osc} = \frac{2}{\sqrt{\kappa^2 B^2 + n_{\mathrm{pl}}^2 \omega^2}}$$

- Typical values:

e.g., $B \sim nGs$, $X_e(z \sim 20) \sim 10^{-4}$,

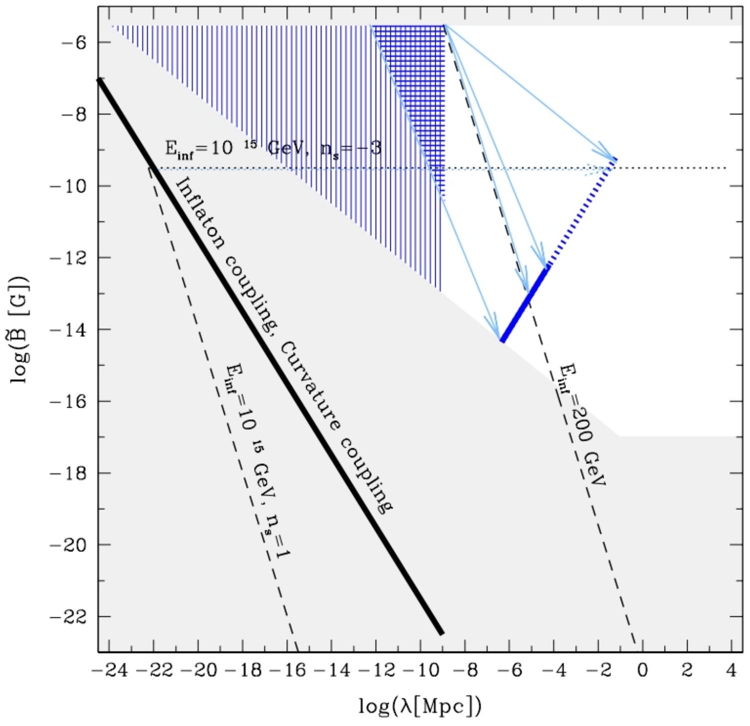
$\omega \sim MHz$, $l_{osc} \sim 10^{15} m$, $\mathcal{P}(\Delta l > l_{osc}) \sim 10^{-32}$

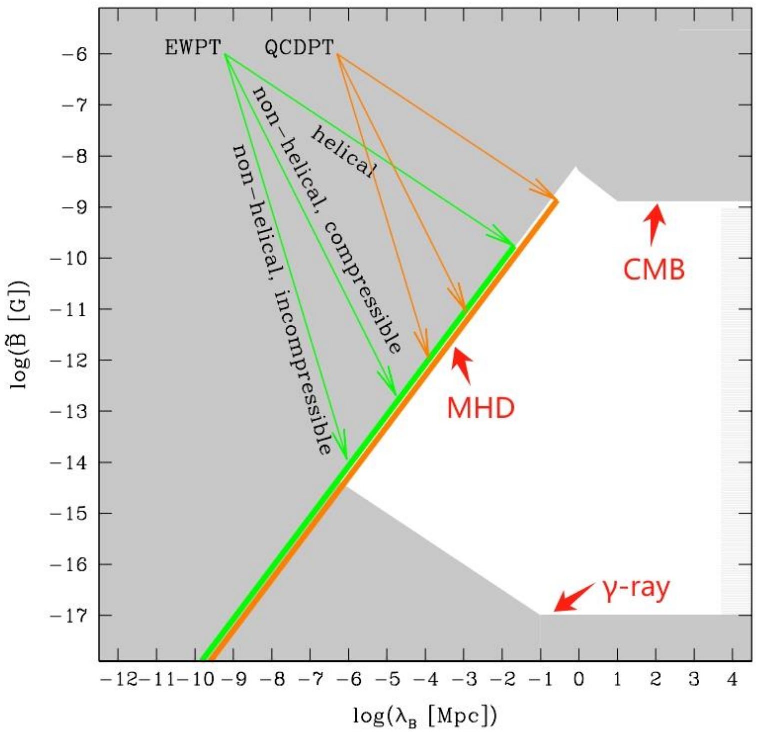
$\omega \sim GHz$, $l_{osc} \sim 10^{18} m \sim 100 pc$, $\mathcal{P} \sim 10^{-26}$

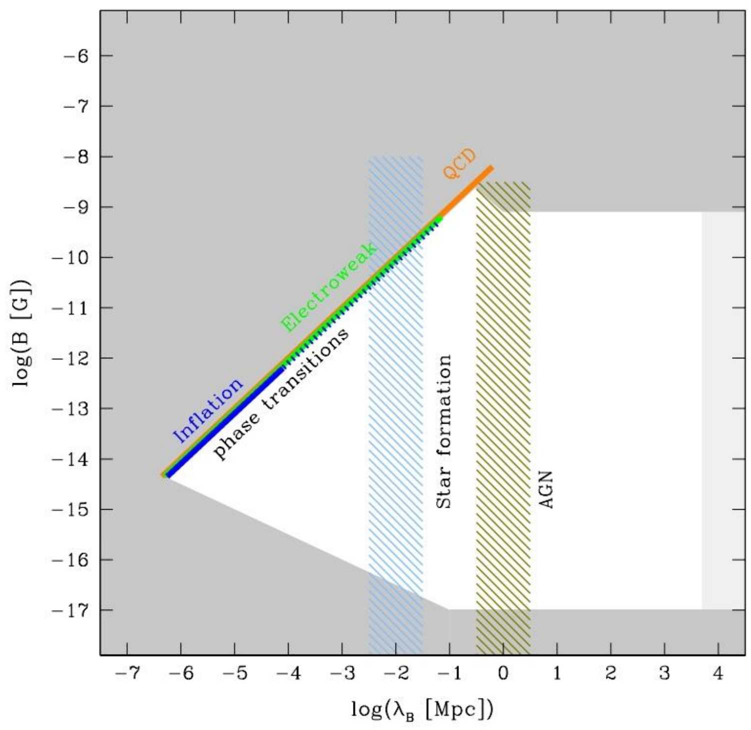
$\omega \sim THz$, $l_{osc} \sim 10^{21} m \sim 0.1 Mpc$, $\mathcal{P} \sim 10^{-20}$

Stochastic primordial magnetic field

- Its origin remans a mystery
- Cosmological source: Inflation (blue or red spectrum) + Phase transition (blue spectrum)
- CMB+Magnetohydrodynamics MHD+Gamma-ray Emission
- Stochastic nature. Magnetic waves produced in huge amount of uncorrelated region at primordial universe expand and superimpose at present day. (Just like SGWB)
- Astrophysical source: Star formation, Galactic wind etc.







From arxiv:1303.7121

Statistics of magnetic field

$$\langle \mathbf{B}_i(\mathbf{x})\mathbf{B}_j(\mathbf{x}')\rangle = \frac{1}{(2\pi)^3}\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}'-\mathbf{x})}\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)P_B(k),$$

- Energy density $\rho_B = \langle \mathbf{B}^2(\mathbf{x})\rangle/2 = \int dk k^2 P_B(k)/(2\pi^2),$
- Root mean square (RMS) $B \equiv \sqrt{\langle \mathbf{B}^2(\mathbf{x})\rangle}$
- Correlation/Integral length $\lambda_B = 2\pi\rho_B^{-1}\int \rho_B(k)k^{-1}dk$
- Average strength at scale λ $B_\lambda = \frac{8\pi}{\lambda^3}P_B(2\pi/\lambda)$
- Average strength at λ_B $B_{\lambda_B} = \frac{8\pi}{\lambda_B^3}P_B(2\pi/\lambda_B)$
- Use either RMS or B_{λ_B} to normalize P_B
- Transverse Part

$$\langle \mathbf{B}_x(l)\mathbf{B}_x(l') + \mathbf{B}_y(l)\mathbf{B}_y(l')\rangle = \frac{1}{(2\pi)^3}\int d^3k e^{ik\cos\theta(l'-l)}\left(1+\cos^2\theta\right)P_B(k),$$

Stochastic magnetic field

- Suggest that B coherently oscillates at scale larger than 1pc, that is much larger than the wavelength of incident graviton of interests (typically, MHz)
- Replace B by $B(l)$ in the Mixing EoM and perturbatively solve it

$$\begin{pmatrix} h_\times(l) \\ h_+(l) \\ A_x(l) \\ A_y(l) \end{pmatrix} = \begin{pmatrix} \mathcal{U}_{11} & \mathcal{U}_{12} & \mathcal{U}_{13} & \mathcal{U}_{14} \\ \mathcal{U}_{21} & \mathcal{U}_{22} & \mathcal{U}_{23} & \mathcal{U}_{24} \\ \mathcal{U}_{31} & \mathcal{U}_{32} & \mathcal{U}_{33} & \mathcal{U}_{34} \\ \mathcal{U}_{41} & \mathcal{U}_{42} & \mathcal{U}_{43} & \mathcal{U}_{44} \end{pmatrix} \begin{pmatrix} h_\times(l_0) \\ h_+(l_0) \\ A_x(l_0) \\ A_y(l_0) \end{pmatrix} \begin{matrix} 1 \\ \\ \\ 0 \end{matrix}$$

$$K_0 = \begin{pmatrix} \omega & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \\ 0 & 0 & \omega\left(n_{\mathrm{pl}}+1\right) & 0 \\ 0 & 0 & 0 & \omega\left(n_{\mathrm{pl}}+1\right) \end{pmatrix}$$

$$\mathcal{U}\left(l, l_0\right) = e^{i(l-l_0)K_0} + i e^{i(l-l_0)K_0} \int_{l_0}^{l} dl' e^{-i(l'-l_0)K_0} \delta K\left(l'\right) e^{i(l'-l_0)K_0} + O\left(\delta K^2\right).$$

$$\delta K(l) = \begin{pmatrix} 0 & 0 & -i\frac{1}{2}\kappa\mathbf{B}_x & i\frac{1}{2}\kappa\mathbf{B}_y \\ 0 & 0 & i\frac{1}{2}\kappa\mathbf{B}_y & i\frac{1}{2}\kappa\mathbf{B}_x \\ i\frac{1}{2}\kappa\mathbf{B}_x & -i\frac{1}{2}\kappa\mathbf{B}_y & 0 & 0 \\ -i\frac{1}{2}\kappa\mathbf{B}_y & -i\frac{1}{2}\kappa\mathbf{B}_x & 0 & 0 \end{pmatrix}$$

$$\left|\mathcal{U}_{31}\left(l, l_0\right)\right|^2 = \left|\mathcal{U}_{42}\left(l, l_0\right)\right|^2 = \frac{1}{4}\kappa^2 \int_{l_0}^{l} dl_1 \int_{l_0}^{l} dl_2 e^{-2i(l_1-l_2)l_{osc0}^{-1}} \mathbf{B}_x\left(l_1\right) \mathbf{B}_x\left(l_2\right),$$

$$\left|\mathcal{U}_{32}\left(l, l_0\right)\right|^2 = \left|\mathcal{U}_{41}\left(l, l_0\right)\right|^2 = \frac{1}{4}\kappa^2 \int_{l_0}^{l} dl_1 \int_{l_0}^{l} dl_2 e^{-2i(l_1-l_2)l_{osc0}^{-1}} \mathbf{B}_y\left(l_1\right) \mathbf{B}_y\left(l_2\right),$$

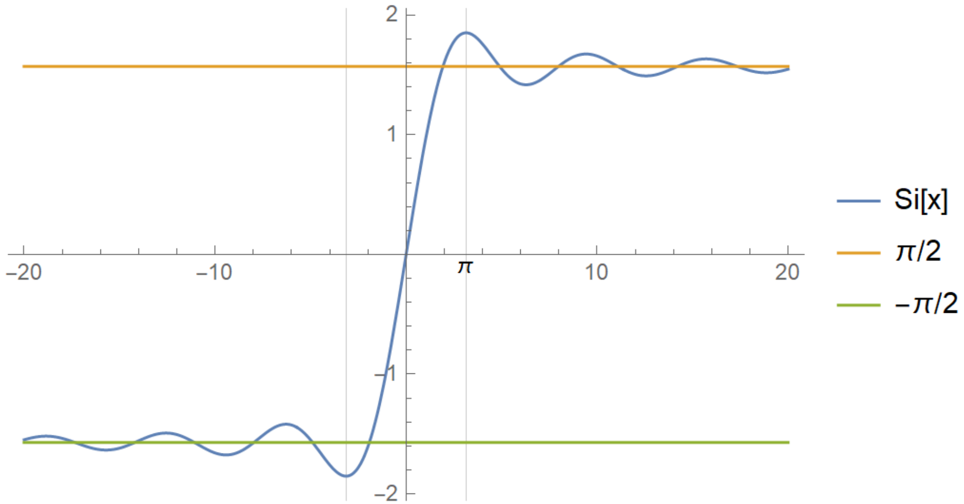
$$\langle\mathcal{P}(l)\rangle = \frac{\left\langle\left|A_x(l)\right|^2\right\rangle + \left\langle\left|A_y(l)\right|^2\right\rangle}{\left\langle\left|h_\times\left(l_0\right)\right|^2\right\rangle + \left\langle\left|h_+\left(l_0\right)\right|^2\right\rangle} = \frac{1}{2}\left(\left|\mathcal{U}_{31}\right|^2 + \left|\mathcal{U}_{32}\right|^2 + \left|\mathcal{U}_{41}\right|^2 + \left|\mathcal{U}_{42}\right|^2\right),$$

Conversion Probability

$$
\begin{aligned}
\mathcal{P}(\Delta l) = \frac{\kappa^2}{8\pi^2} \int \frac{1}{k} P_B(k) dk \Bigg\{ & 2k + \frac{1}{\Delta l}\left(\sin\left(\left(2l_{\rm osc}^{-1} - k\right)\Delta l\right) - \sin\left(\left(2l_{\rm osc}^{-1} + k\right)\Delta l\right)\right) \\
& + 2\frac{4l_{\rm osc}^{-2} + k^2}{2l_{\rm osc}^{-1} - k} \sin^2\left(\frac{1}{2}\left(2l_{\rm osc}^{-1} - k\right)\Delta l\right) - 2\frac{4l_{\rm osc}^{-2} + k^2}{2l_{\rm osc}^{-1} + k} \sin^2\left(\frac{1}{2}\left(2l_{\rm osc}^{-1} + k\right)\Delta l\right) \\
& + 4l_{\rm osc}^{-1}\left(\mathrm{Ci}\left(\left|\left(2l_{\rm osc}^{-1} + k\right)\Delta l\right|\right) - \mathrm{Ci}\left(\left|\left(2l_{\rm osc}^{-1} - k\right)\Delta l\right|\right) + \ln\left|\frac{2l_{\rm osc}^{-1} - k}{2l_{\rm osc}^{-1} + k}\right| \right) \\
& + \Delta l\left(4l_{\rm osc}^{-2} + k^2\right)\left(\mathrm{Si}\left(\left(2l_{\rm osc}^{-1} + k\right)\Delta l\right) - \mathrm{Si}\left(\left(2l_{\rm osc}^{-1} - k\right)\Delta l\right)\right)\Bigg\}, \qquad \text{(II.10)}
\end{aligned}
$$

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos(x')}{x'} dx' \ \text{ and } \ \mathrm{Si}(x) = \int_0^x \frac{\sin(x')}{x'} dx'.$$

- the circle matters in large distance limit
- For monochromatic spectrum $P_B \sim B^2 \delta(k - k_B)/k_B^2$, when $2l_{osc}^{-1} - k_B < 0$, the probability converges to $\mathcal{P} \sim \kappa^2 B^2 \lambda_B \Delta l$ in large Δl

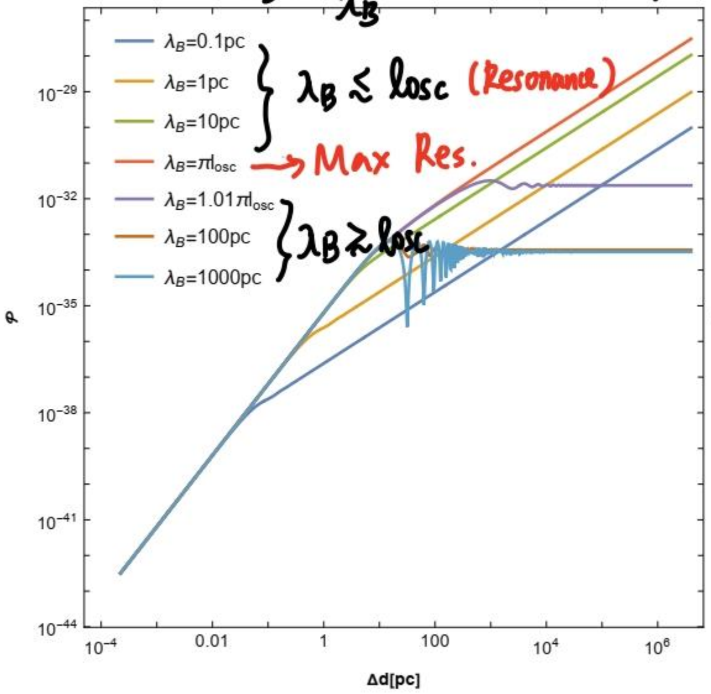


Stochastic Resonance

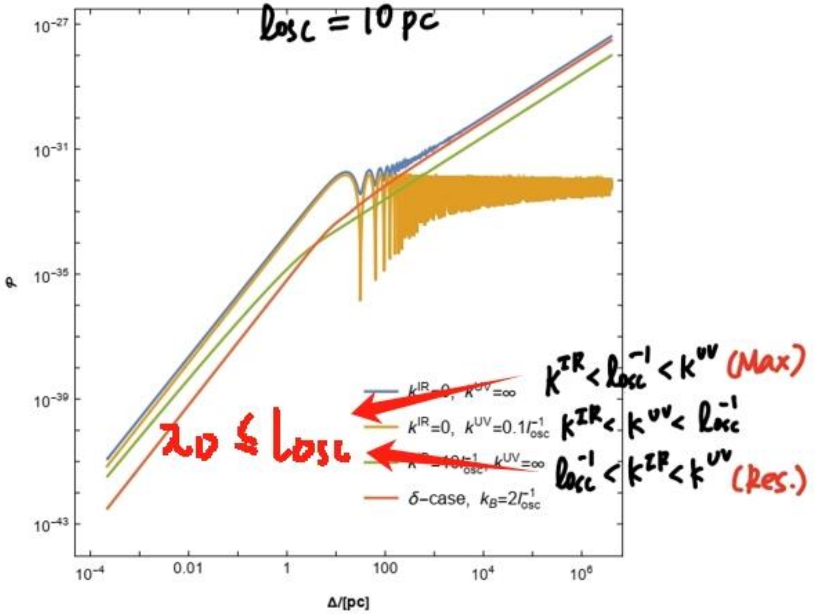
- Two extreme cases: monochromatic and truncated scale invariant spectrum
- Motivation: magnetogenesis from the phase transition and inflation
- Infrared k^{IR} and ultraviolet cutoffs k^{UV} (physical meaning will be given later)

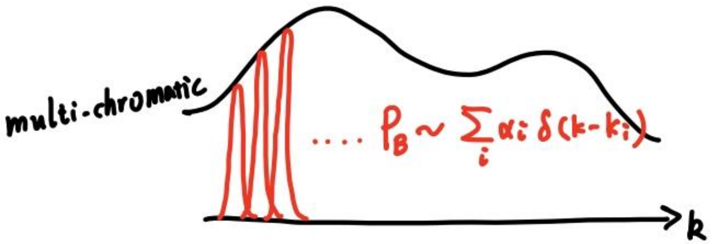
$$P_B(k) = \frac{B^2}{k_B^2}\delta(k-k_B)$$

$k_B \sim \frac{1}{\lambda_B}$ $\quad$ $l_{osc} = 10\,pc$



$$P_B(k) = \frac{B^2}{k^3}, \quad k^{IR} < k < k^{UV}$$





Generalize to any multichromactic case

Envelope of probability curves

monochromatic power spectrum	$\lambda_B > l_{osc}$	$\triangle l \gtrsim l_{osc}$	$\triangle l < l_{osc}$
		$\mathcal{P} \simeq \kappa^2 B^2 l_{osc}^2$	$\mathcal{P} \simeq \kappa^2 B^2 \triangle l^2$
	$\lambda_B \lesssim l_{osc}$	$\triangle l \gtrsim \lambda_B$	$\triangle l < \lambda_B$
		$\mathcal{P} \simeq \kappa^2 B^2 \lambda_B \triangle l$	$\mathcal{P} \simeq \kappa^2 B^2 \triangle l^2$
scale invariant power spectrum	$k^{IR} \lesssim l_{osc}^{-1} \lesssim k^{UV}$	$\triangle l \gtrsim l_{osc}$	$\triangle l < l_{osc}$
		$\mathcal{P} \simeq \kappa^2 B^2 l_{osc} \triangle l$	$\mathcal{P} \simeq \kappa^2 B^2 \triangle l^2$
	$l_{osc}^{-1} < k^{IR} < k^{UV}$	$\triangle l \gtrsim 1/k^{IR}$	$\triangle l < 1/k^{IR}$
		$\mathcal{P} \simeq \kappa^2 B^2 \triangle l / k^{IR}$	$\mathcal{P} \simeq \kappa^2 B^2 \triangle l^2$
	$k^{IR} < k^{UV} < l_{osc}^{-1}$	$\triangle l \gtrsim l_{osc}$	$\triangle l < l_{osc}$
		$\mathcal{P} \simeq \kappa^2 B^2 l_{osc}^2$	$\mathcal{P} \simeq \kappa^2 B^2 \triangle l^2$

Oscillatory magnetic field

- Oscillatory B: $B(l) = B\cos(k_B(l - l_0))$

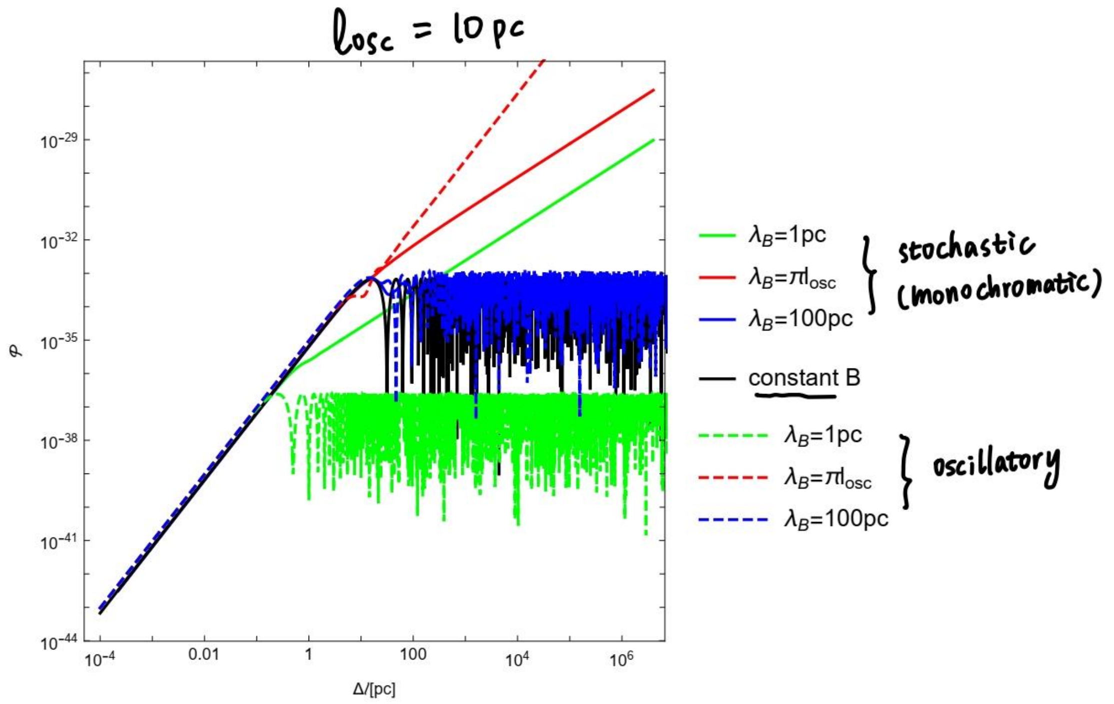
  narrow resonant band: $\lambda_B = \pi l_{osc}$

  quadratic growth $\mathcal{P} \sim \Delta l^2$

- Stochastic case:

  broad resonant band $\lambda_B \leq \pi l_{osc}$

  linear growth $\mathcal{P} \sim \Delta l$



$$\mathcal{P}(\Delta l) = \frac{1}{4}\kappa^2 B^2 \frac{1}{(-4l_{\rm osc}^{-2}+k_B^2)^2}\left\{4l_{\rm osc}^{-2}\cos^2(k_B\Delta l)+4l_{\rm osc}^{-2}+k_B^2\sin^2(k_B\Delta l)\right.$$
$$\left. -8l_{\rm osc}^{-2}\cos(k_B\Delta l)\cos(2l_{\rm osc}^{-1}\Delta l) - 4l_{\rm osc}^{-1}k_B\sin(2l_{\rm osc}^{-1}\Delta l)\sin(k_B\Delta l)\right\}.$$

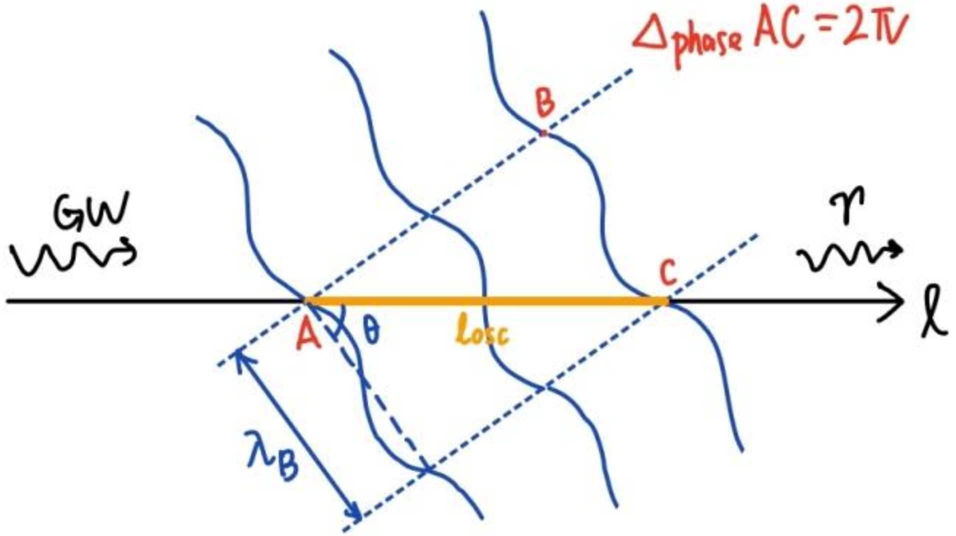
Oscillatory case

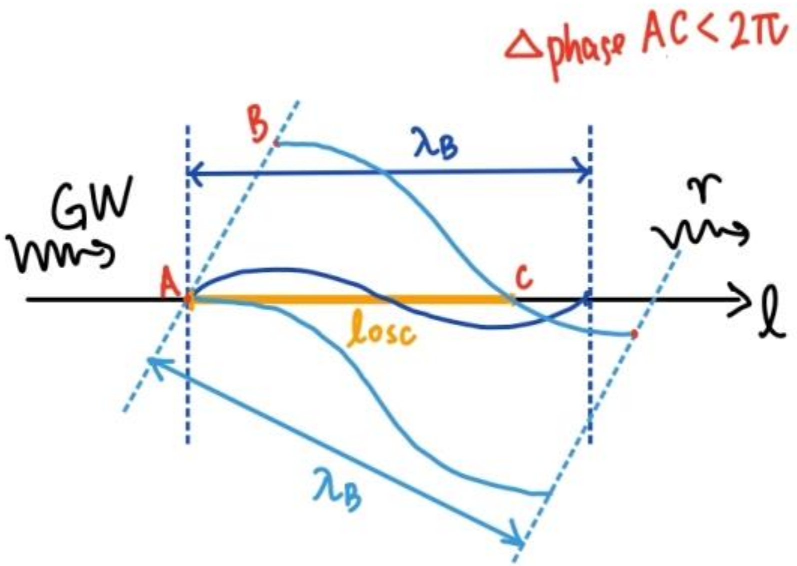
$$\begin{aligned}\mathcal{P}(\Delta l) = {} & \frac{\kappa^2}{8\pi^2}\int \frac{1}{k}P_B(k)dk\left\{2k+\frac{1}{\Delta l}\left(\sin\left(\left(2l_{\rm osc}^{-1}-k\right)\Delta l\right)-\sin\left(\left(2l_{\rm osc}^{-1}+k\right)\Delta l\right)\right)\right.\\ & +2\frac{4l_{\rm osc}^{-2}+k^2}{2l_{\rm osc}^{-1}-k}\sin^2\left(\frac{1}{2}\left(2l_{\rm osc}^{-1}-k\right)\Delta l\right)-2\frac{4l_{\rm osc}^{-2}+k^2}{2l_{\rm osc}^{-1}+k}\sin^2\left(\frac{1}{2}\left(2l_{\rm osc}^{-1}+k\right)\Delta l\right)\\ & +4l_{\rm osc}^{-1}\left(\mathrm{Ci}(\left|\left(2l_{\rm osc}^{-1}+k\right)\Delta l\right|)-\mathrm{Ci}(\left|\left(2l_{\rm osc}^{-1}-k\right)\Delta l\right|)+\ln\left|\frac{2l_{\rm osc}^{-1}-k}{2l_{\rm osc}^{-1}+k}\right|\right)\\ & \left.+\Delta l\left(4l_{\rm osc}^{-2}+k^2\right)\left(\mathrm{Si}\left(\left(2l_{\rm osc}^{-1}+k\right)\Delta l\right)-\mathrm{Si}\left(\left(2l_{\rm osc}^{-1}-k\right)\Delta l\right)\right)\right\},\end{aligned} \quad \text{(II.10)}$$

Stochastic case

An intuitive interpretation

- For monochromatic case, it is a superposition of plane waves propagating in random directions but all with the same wavelength
- The resonance in single oscillatory B field case happens at $\lambda_B \simeq l_{osc}$, equivalent to phase shift 2π of the B field
- In stochastic context, when $\lambda_B \lesssim l_{osc}$, always exists an angle θ satisfying $\lambda_B = cos\theta l_{osc}$, such that the phase difference of the magnetic field between points A and C is 2π





Universe Expansion

- Mixing EoM (work in comoving framework)

$$\partial_l \begin{pmatrix} A \\ h \end{pmatrix} = iK(a,l) \begin{pmatrix} A \\ h \end{pmatrix}, K(a,l) = a \begin{pmatrix} (n_{\rm pl}(a)+1)\,\omega(a) & -\frac{1}{2}\kappa B(a,l) \\ -\frac{1}{2}\kappa B(a,l) & \omega(a) \end{pmatrix}$$

$$\omega(a) = \omega_0/a \qquad B(a,l) = B_0(l)/a^2 \qquad n_{\rm pl}(a) = -e^2 n_{b0} X_e(a)/(2a\omega_0^2 m_e),$$

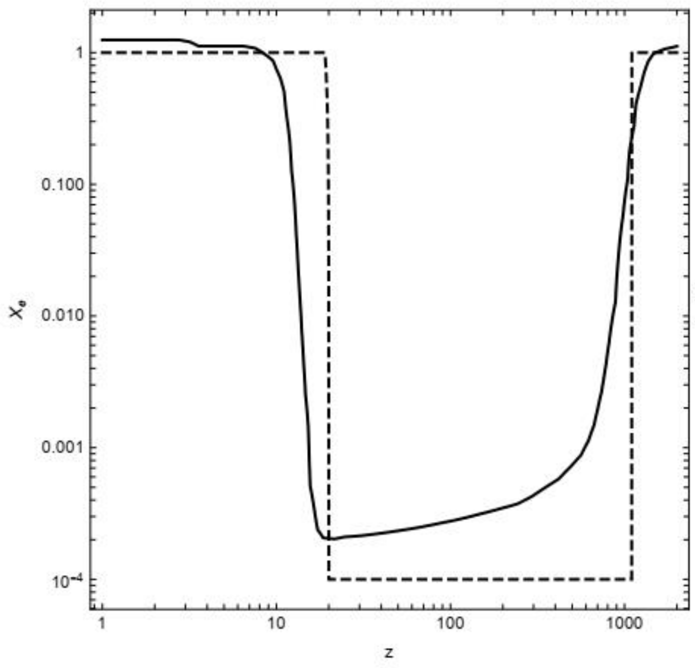
- $X_e(a)$ is the ionization fraction
- Steady approximation
- Redshift discretization (from Recombination to present day)

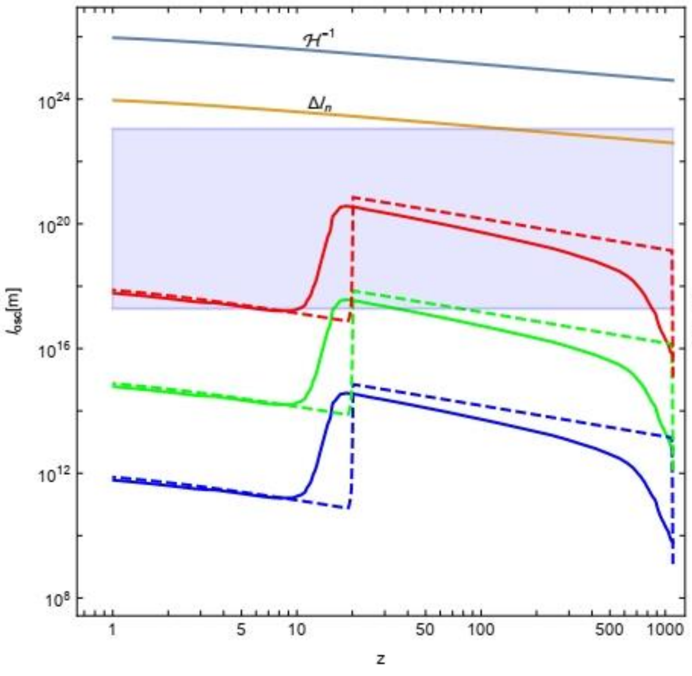
$$[z_1, z_2, \cdots, z_N] \qquad z_{n+1} = (1-\epsilon) z_n \qquad \triangle l = \triangle\tau = \epsilon\mathcal{H}^{-1}.$$

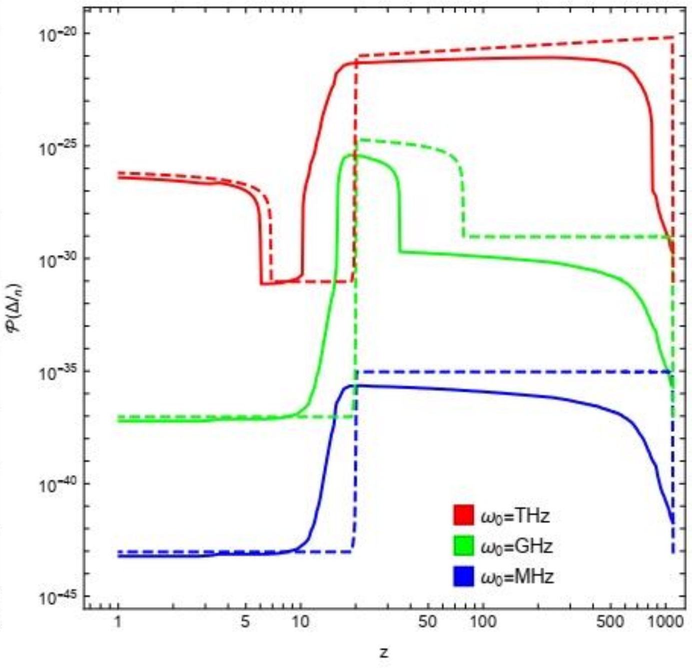
- Sum over each redshift slice $\mathcal{P}_{\rm total} = \sum_{n=1}^{N-1} \mathcal{P}^{\rm exp}(\triangle l_n).$
- In resonant region, the linear growth pattern of conversion probability indicate independence of ϵ on results.

Conversion Probability

- Simplified $X_e(a)$ captures main features.
- Conversion maximize at z~20 just before the reionization
- Higher frequency, larger oscillation length l_{osc}, higher conversion probability







Magnetic field in cosmology

- Modelled in two power laws $P_B(z,k) = (1+z)^4 P_{B_0}(k) = (1+z)^4 \begin{cases} A\,(k/k_0)^{n_L} & k^{IR} < k \leq k_0, \\ A\,(k/k_0)^{n_S} & k_0 < k < k_D, \end{cases}$
- Two representative cases:

1, phase transition magnetogenesis: $n^L = 2$

k^{IR} is normally set to the Hubble radius at the phase transition time.

2, inflation magnetogenesis: $n^L = -3$

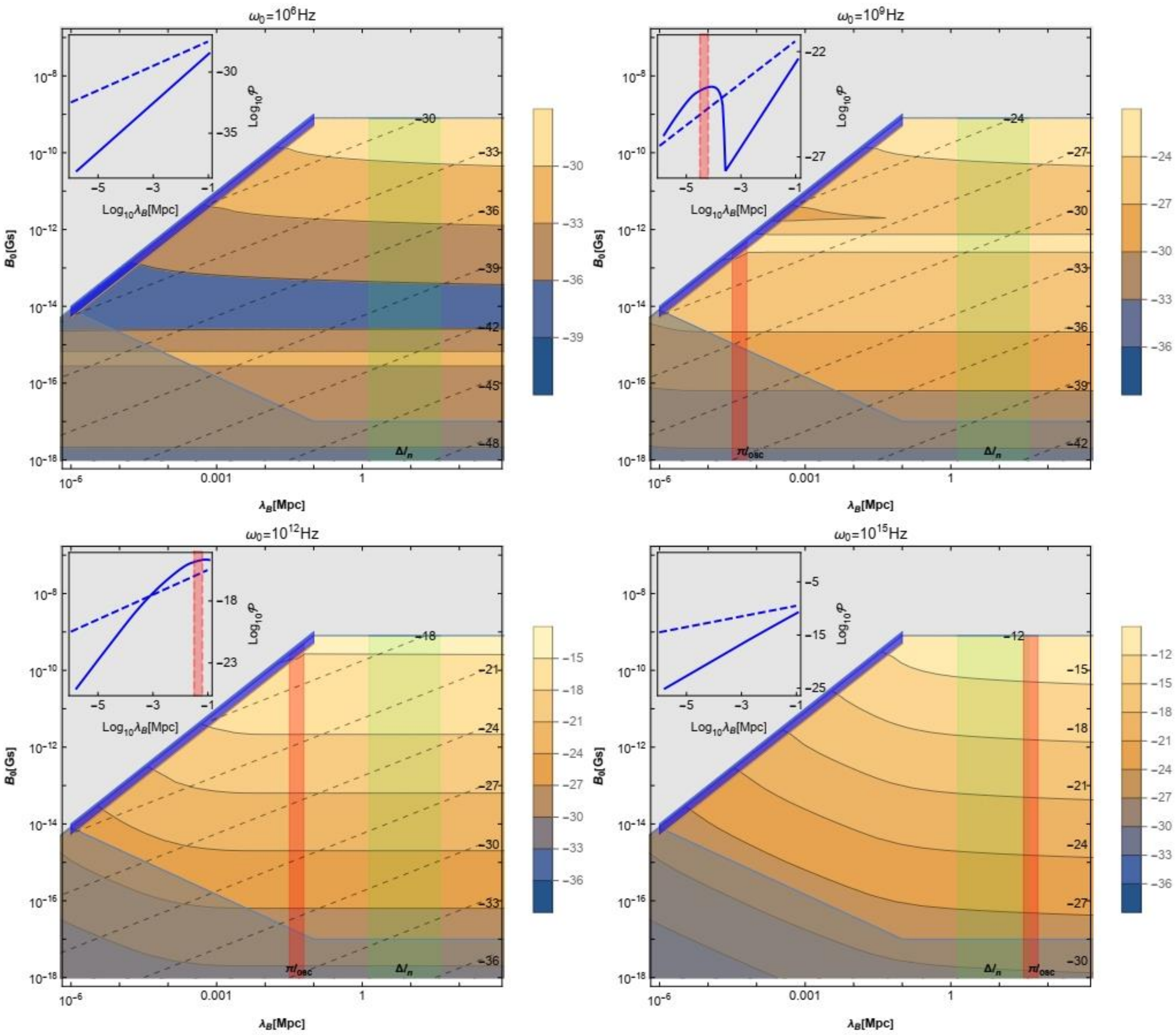
k^{IR} is the scale when magnetogenesis starts during inflation. Its value can either be larger or smaller than the current Hubble radius.

- At small scale, MHD leads to a universal Kolmogorov slope $n^S = -11/3$
- At smaller scale, energy is damped away via the viscosity of charged plasma during recombination, leads to an ultraviolet cutoff

$$k_D \simeq \mathcal{O}(100)(10^{-9}\mathrm{Gauss}/B_0)\mathrm{Mpc}^{-1}$$

Numerical results

- Focus on the High Frequency window
- Phase transition && inflation magnetogenesis
- Compared to the Ref. *V. Domcke and C. Garcia-Cely. Phys. Rev. Lett., 126(2):021104, 2021.* (dashed line), which is based on a domain-like model.
- The difference can be very large, e.g., at $B_0 < 10^{-12}$Gauss or $\lambda_B > Mpc$, the conversion probability is $5 \sim 10$ orders larger
- Distinct features related to the magnetic spectrum can be used to distinguish the primordial magnetogenesis models



Compared to Domain-like model

- Most literature adopt the domain-like model, the size of each domain equals to the correlation length of B field.
- Easier to handle, stochastics are manifested by the random direction in different domains.
- Our model: perturbation method is robust, magnetic field is stochastic, no "unnatural" configuration
- Both models has linear relation in distance:

  domain-like model: the classical accumulative rule $\mathcal{P}\sim\mathcal{P}_{cell}N\sim\mathcal{P}_{cell}\Delta l/\lambda_B$

  our model: $\mathcal{P}\sim\Delta l$ under resonant condition
- Stochastic Resonance effect + evolution of PMF, especially in broad B-λ_B region which could only possibly fulfilled by the truncated (nearly) scale invariant spectrum, makes our results dramatically different to others.

Domain Model vs. Our Model

phase transition	$\omega_0 = 10^6\mathrm{Hz}(\triangle l_n > l_{\rm osc}, \triangle l_n > \lambda_B)$	$\lambda_B > l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 l_{\rm osc}^2 D/\triangle l_n$
	$\omega_0 = 10^9\mathrm{Hz}(\triangle l_n > l_{\rm osc}, \triangle l_n > \lambda_B)$	$\lambda_B > l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 l_{\rm osc}^2 D/\triangle l_n$
		$\lambda_B \lesssim l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \lambda_B D$
	$\omega_0 = 10^{12,15}\mathrm{Hz}(\triangle l_n > l_{\rm osc}, \triangle l_n > \lambda_B)$	$\lambda_B \lesssim l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \lambda_B D$
scale invariant (inflation)	$\omega_0 = 10^6\mathrm{Hz}(\triangle l_n > l_{\rm osc}, \lambda_B > l_{\rm osc})$	$,\lambda_D \lesssim l_{\rm osc} \lesssim \lambda_B$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 l_{\rm osc} D$
		$l_{\rm osc} < \lambda_D < \lambda_B$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 l_{\rm osc}^2 D/\triangle l_n$
	$\omega_0 = 10^9\mathrm{Hz}(\triangle l_n > l_{\rm osc})$	$\lambda_D \lesssim l_{\rm osc} \lesssim \lambda_B$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 l_{\rm osc} D$
		$\lambda_D < \lambda_B < l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \lambda_B D(\triangle l_n \gtrsim \lambda_B)$
			$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \triangle l_n D(\triangle l_n < \lambda_B)$
		$l_{\rm osc} < \lambda_D < \lambda_B$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 l_{\rm osc}^2 D/\triangle l_n$
	$\omega_0 = 10^{12}\mathrm{Hz}(\triangle l_n > l_{\rm osc}, \lambda_D < l_{\rm osc})$	$\lambda_D \lesssim l_{\rm osc} \lesssim \lambda_B$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 l_{\rm osc} D$
		$\lambda_D < \lambda_B < l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \lambda_B D$
	$\omega_0 = 10^{15}\mathrm{Hz}(\triangle l_n < l_{\rm osc}, \lambda_D < l_{\rm osc})$	$\lambda_D \lesssim l_{\rm osc} \lesssim \lambda_B$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \triangle l_n D$
		$\lambda_D < \lambda_B < l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \lambda_B D(\triangle l_n \gtrsim \lambda_B)$
			$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \triangle l_n D(\triangle l_n < \lambda_B)$
domain-like model	all $\omega_0(\lambda_B = \triangle l_n)$	$\lambda_B > l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 l_{\rm osc}^2 D/\lambda_B$
		$\lambda_B \lesssim l_{\rm osc}$	$\mathcal{P}_{\rm total} \simeq \kappa^2 B^2 \lambda_B D$

Axion-photon conversion

- Axion-like-particle, $L_{a\gamma} = -\frac{1}{4}\int d^4x\sqrt{-g}g_{a\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}a\,.$
- Similar dynamics for the ultra-relativistic ALP,

$$\left(\omega - i(1+z)\frac{\mathrm{d}}{\mathrm{d}l} + M\right)\begin{pmatrix} A_x \\ A_y \\ a \end{pmatrix} = 0, \qquad M = \begin{pmatrix} \Delta_{xx} & \Delta_{xy} & \frac{1}{2}g_{a\gamma}B_x \\ \Delta_{xy} & \Delta_{yy} & \frac{1}{2}g_{a\gamma}B_y \\ \frac{1}{2}g_{a\gamma}B_x & \frac{1}{2}g_{a\gamma}B_y & \Delta_a \end{pmatrix}$$

- Potential to impact on radio wave:

1,The stronger coupling than graviton-photon mixing

2, ALP is massive. Under the mass-equal condition $m_\gamma \simeq m_a$, probability can be significantly enhanced, because oscillation length becomes large $l_{osc} \simeq \frac{1}{\Delta_{pl}-\Delta_a}$, where $\Delta_{pl}= m_\gamma^2/\omega$ and $\Delta_a= m_a^2/\omega$

Parameter space of interest

- Rough estimation:

Graviton-photon case: $\kappa \approx 10^{-19} GeV^{-1}$

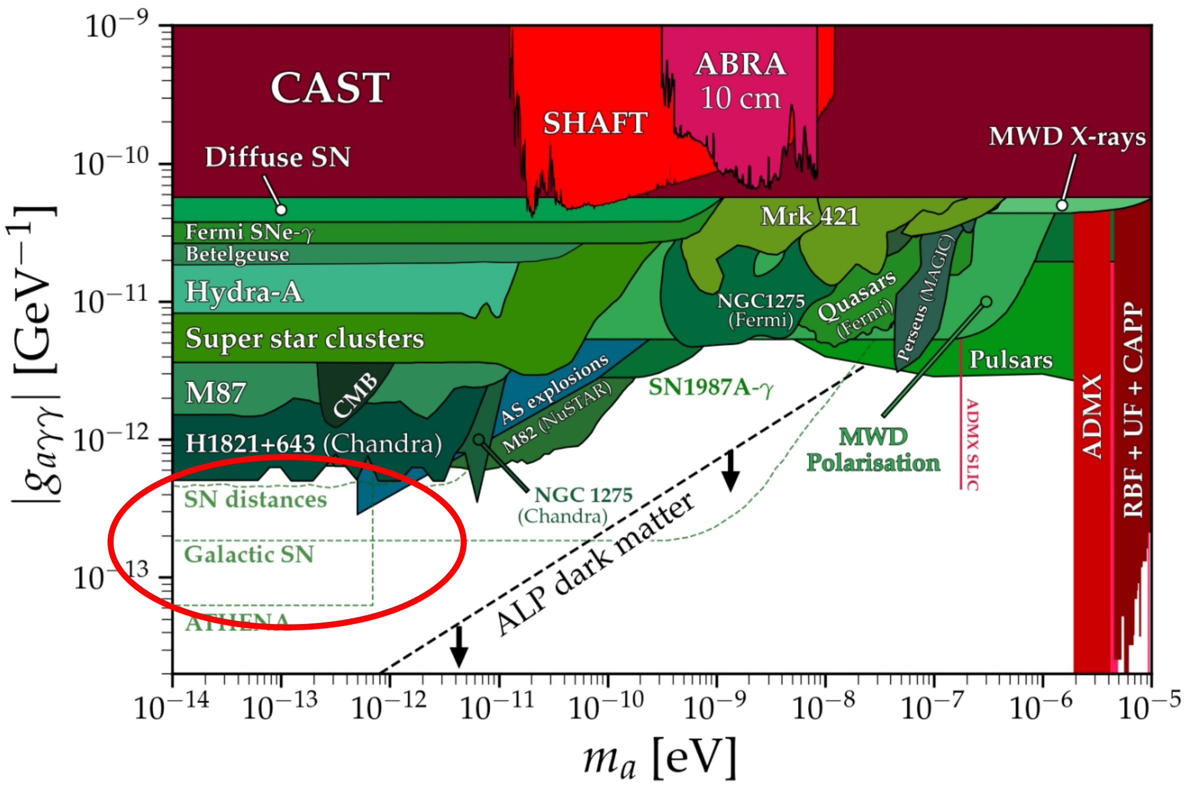
$\omega \sim GHz,\ l_{osc} \sim 10^{18} m,\ \ \mathcal{P} \sim 10^{-26}$

ALP-photon case: $g_{a\gamma} \approx 10^{-12} GeV^{-1}$,

$\omega \sim GHz,\ l_{osc} \sim 10^{18} m,\ \ \mathcal{P} \sim 10^{-12}$

- The effective mass of extragalactic plasma: $m_a \approx 10^{-14} - 10^{-12}$eV.

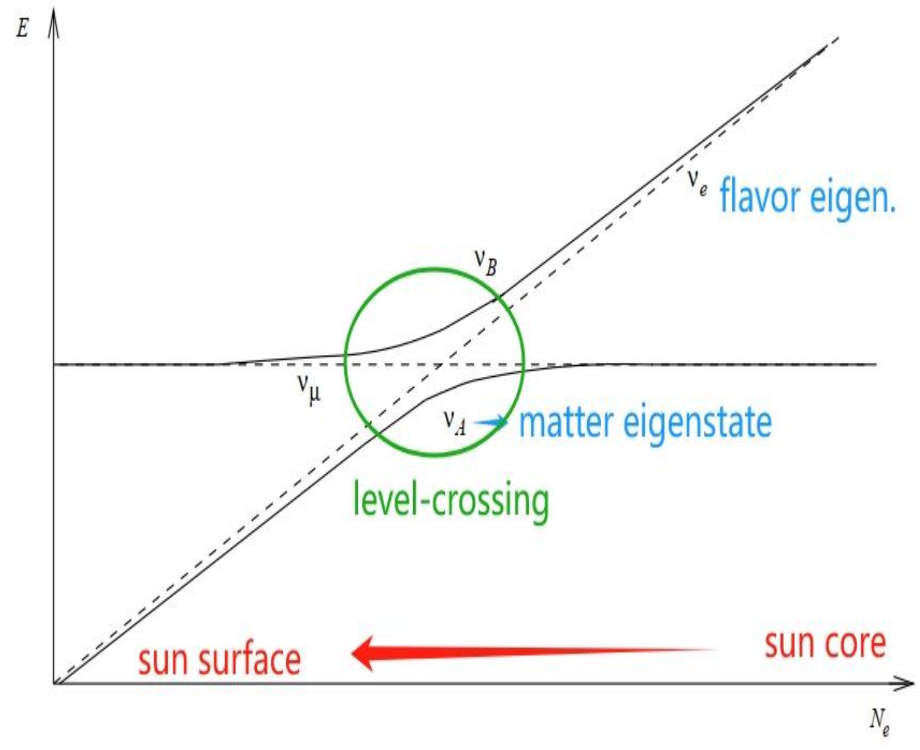
The mass-equal resonance enlarge l_{osc}



https://cajohare.github.io/AxionLimits/

Mass-equal resonance

- Two-level system with a time dependent Hamiltonian.
- As the time-dependent parameter varies, the energy levels approach each other and "cross“, enhancing the transition between two energy/flavor states.
- MSW resonance: neutrino oscillation in varying matter density (solar sphere, earth atmosphere).
- Landau-Zener approach widely used in phenomena of atomic and molecular transition.



A refined perturbative approach

- To handle the Mass-equal resonance, we refine the perturbative approach as the graviton-photon case, by replacing the constant $\Delta = \Delta_{pl} - \Delta_a$ (i.e. constant l_{osc}) to a linear expansion $\Delta(z) = \Delta(z_c) + (z - z_c)\Delta'(z_c)$ at interval $z \in [z_i, z_f]$
- The conversion probability is

$$\mathcal{P}(z_i, z_{i+1}) = \frac{3\pi}{4(2\pi)^2} \frac{(1+z_c)^3 g_{a\gamma}^2}{H_c |\Delta'(z_c)|} \int dk k^2 P_B(k) W(t_1, t_2)$$

$$t_1 = \frac{(z_c - z_f)\,\Delta'(z_c) - \triangle(z_c)}{(1+z_c)\,k}, \quad t_2 = \frac{(z_c - z_i)\,\Delta'(z_c) - \triangle(z_c)}{(1+z_c)\,k},$$

$$W(t_1, t_2) = \begin{cases} |t_1 - t_2|, & |t_1| < 1 \quad |t_2| < 1 \\ |t_1 + 1|, & |t_1| < 1 \quad t_2 < -1 \\ |t_1 - 1|, & |t_1| < 1 \quad t_2 > 1 \\ |t_2 + 1|, & t_1 < -1 \quad |t_2| < 1 \\ |t_2 - 1|, & t_1 > 1 \quad |t_2| < 1 \\ 2, & t_1 < -1 \quad t_2 > 1 \quad or \quad t_1 > 1 \quad t_2 < -1 \\ \simeq 0, & \text{others} \end{cases}.$$

Two limit of the probability

1, In mass-equal resonance when $\Delta(z_{res}) = 0$, the conversion probability for a monochromatic case during the redshift interval ϵz_{res} is

$$\mathcal{P} \simeq 8.6 * 10^{-4} \left(\frac{g_{a\gamma}}{10^{-12}\mathrm{GeV}^{-1}}\right)^2 \left(\frac{B_0}{\mathrm{nGs}}\right)^2 \left(\frac{\omega_0}{\mathrm{GHz}}\right) \left(\frac{m_a}{10^{-12}\mathrm{eV}}\right)^{1/3} \mathrm{Min}\left(\epsilon z_{res}\left(\frac{\mathrm{GHz}}{\omega_0}\right)\left(\frac{\lambda_B}{\mathrm{kpc}}\right), 1\right).$$

- Landau-Zener approximation gives almost the same result without Min- function.
- Landau-Zener approximation is based on the domain like model, thus only valid when $\lambda_B > W_r$ with W_r the resonance width.
- Our approach can extend to the case with arbitrary small λ_B, where a suppression is revealed as a consequence of stochastic nature.

2, slowly varying oscillation length $\Delta' \ll 1$

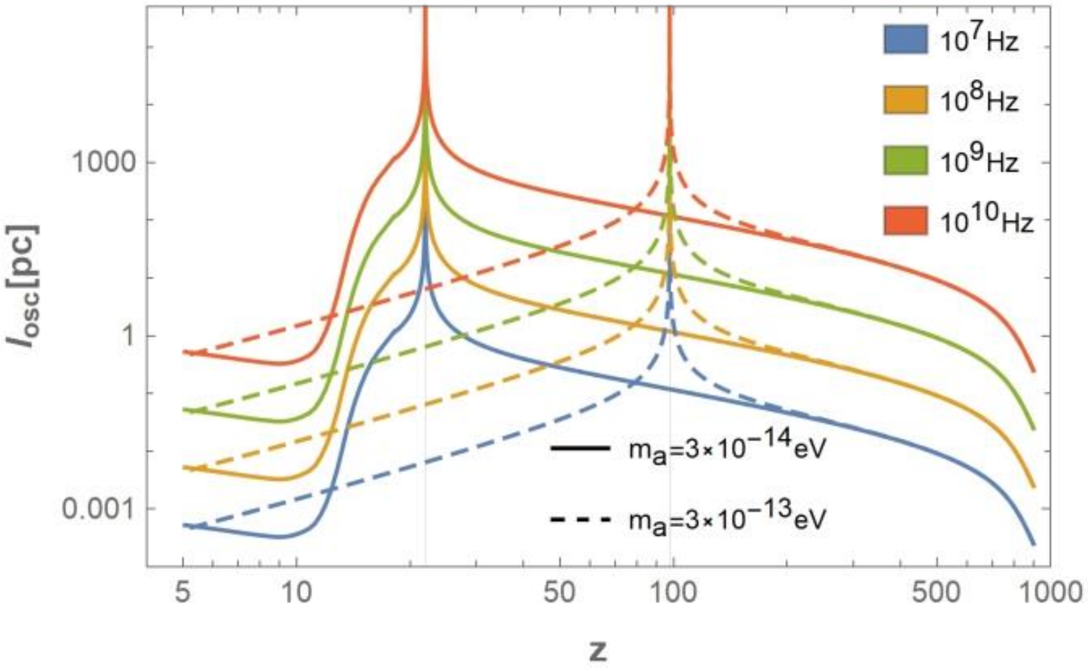
- When $-1 < t_1 \simeq t_2 < 1$, equivalent to $\lambda_B \lesssim l_{osc}$, the probability is

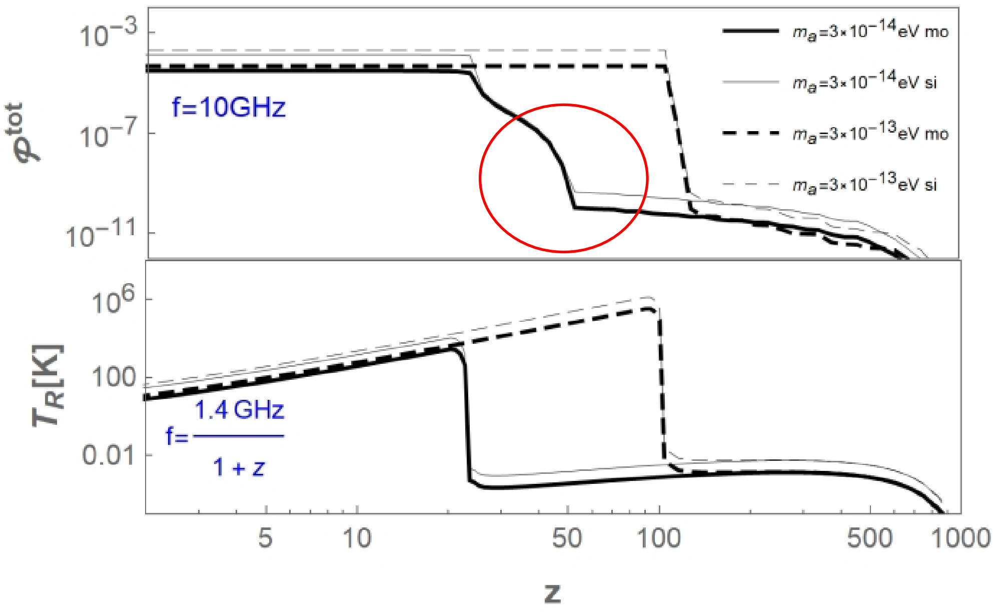
$$\mathcal{P} \simeq \frac{3}{32} z_c^2 g_{a\gamma}^2 B_0^2 \lambda_B \triangle l,$$

- reduce to previous result of linear growth pattern

Two typical cases of ALP mass

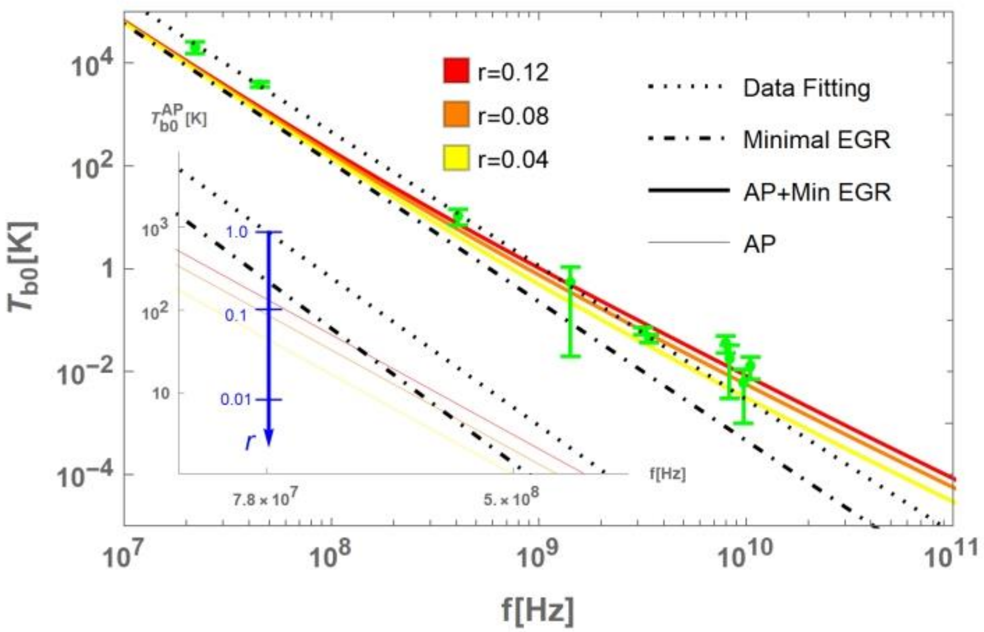
- Peak in small and large redshift $z{\sim}20$ and $z{\sim}100$ (aim to impact on 21cm)
- Sharp feature at mass-equal redshift, MSW-type resonance
- Soft feature when $l_{osc} \gtrsim \lambda_B$(red circle), Stochastic resonance

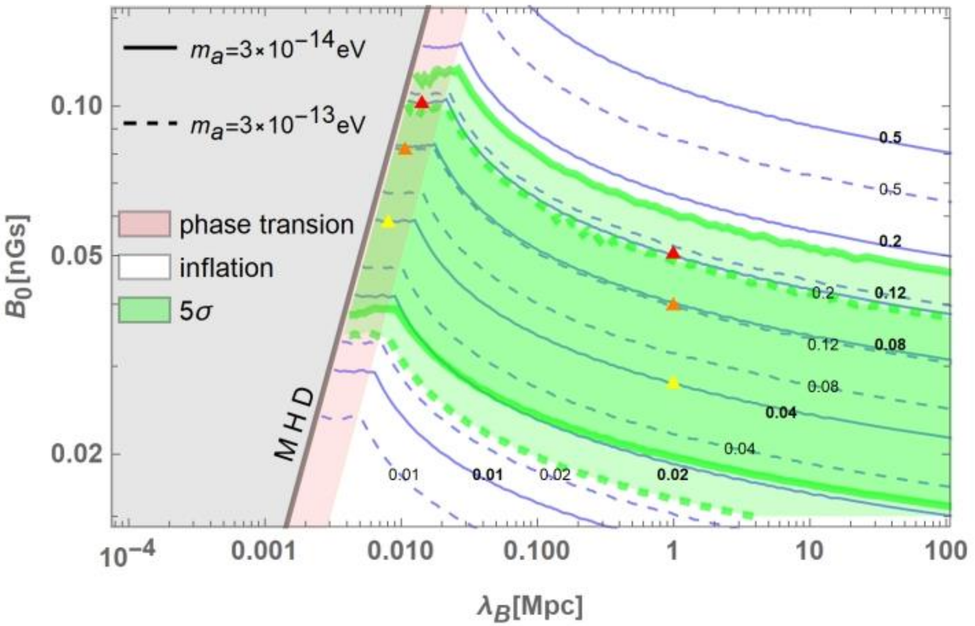




Radio Excess

- ARCADE2 and other lower low-frequency surveys have detected an excess in radio emission at frequency 22MHz-10GHz, fitted as $T - T_0 = 869.3\text{K}\left(\frac{f}{78MHz}\right)^{-2.6}$
- Minimal extra-galactic radiation contributes ~20% of the observed excess.
- Parameter $r = T^{AP}/T^{obs}|_{78MHz}$
- Axion induced radiation has power-law scaling as f^{-2}
- Our model superimposing on Minimal Extra-galactic radiation can be best fitted with $\chi^2_{min} = 49$ (12 $d.o.f$), which is 5σ away from the Minimal Extra-galactic radiation model ($\chi^2 = 117$).
- Green region denotes the 5σ improvement to the null hypothesis. Overlaps with the region bounded by r≃0.03 and r≃0.15



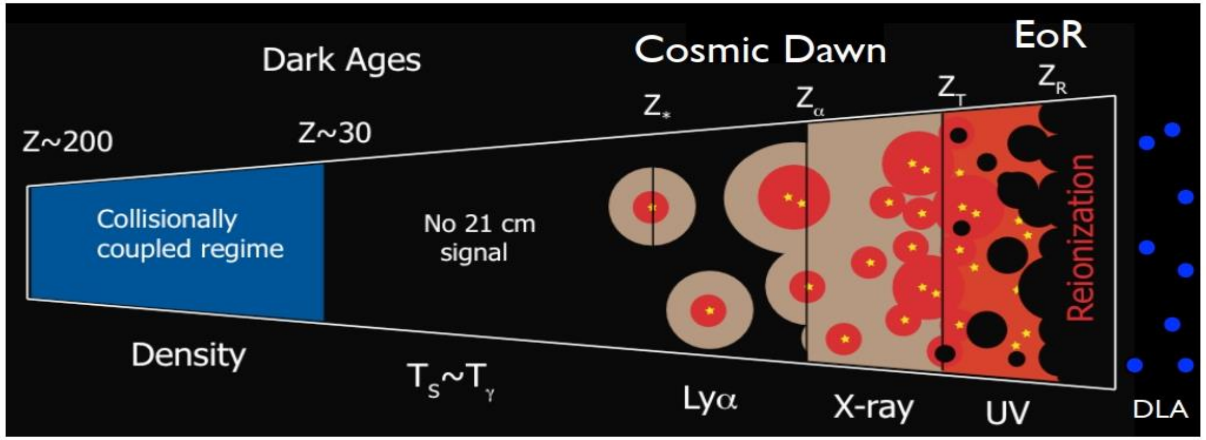


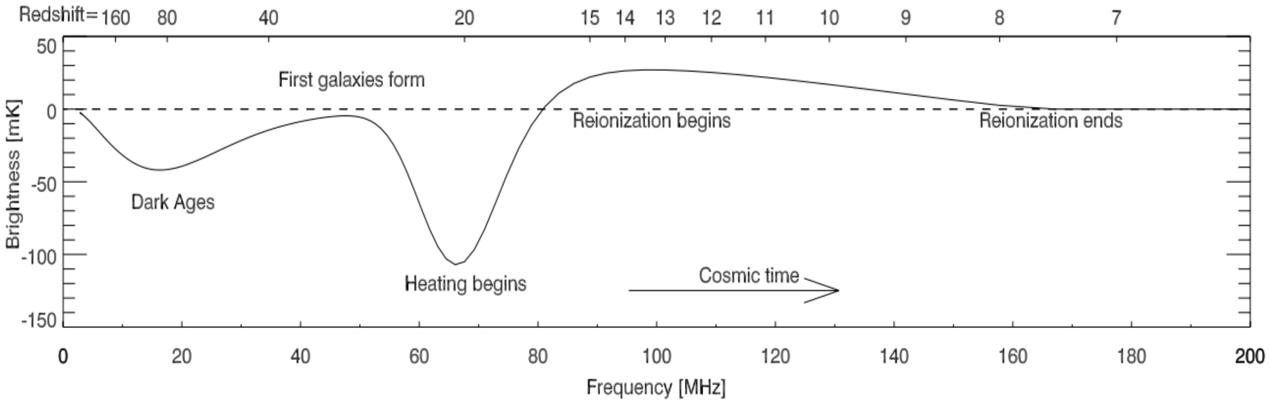
21cm absorption

- Another anomalous radio signal is the 21cm hydrogen line absorption, detected by EDGES.
- The absorption trough exhibits depth of $-0.5^{+0.2}_{-0.5}$K at around 78MHz, which is twice deeper than the value predicted in SM at 99% C.L.
- Two solutions: $T_{21cm} \sim 1 - \frac{T_R}{T_{gas}}$

1, cooling of the gas temperature T_{gas} through interaction with CDM

2, heating the background T_R by additional photon injection from Dark Radiation

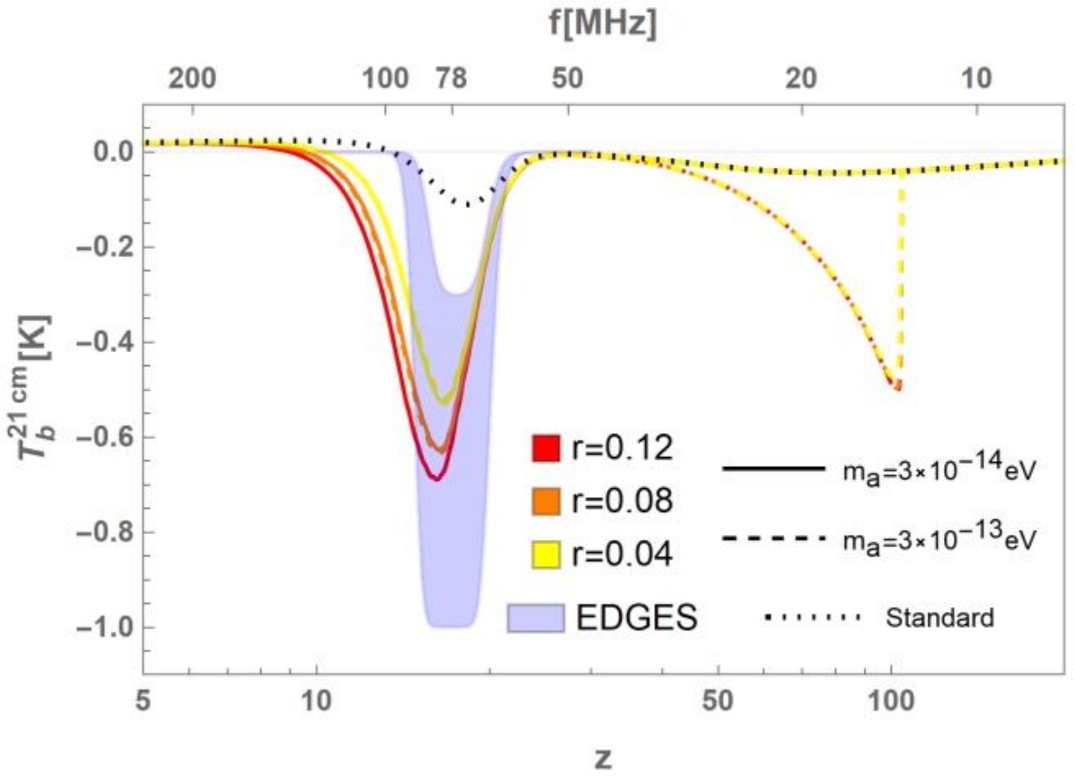


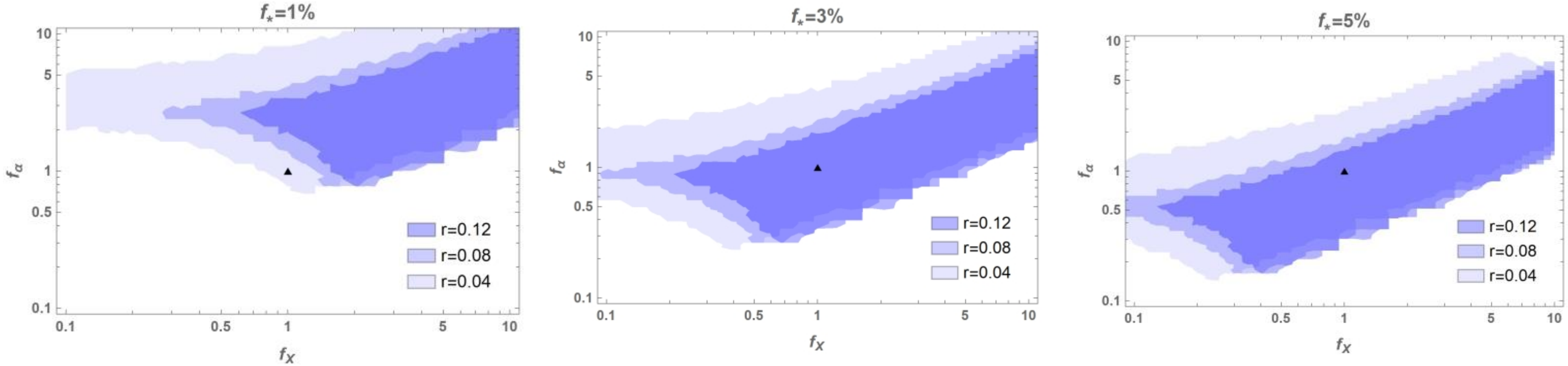


From arXiv:1109.6012

21cm absorption

- Taking into account the energy transfer between total radiation background and intergalactic medium.
- A deep trough at high redshift is predicted for $m_a \sim 10^{-13}$eV, as the mass-equal resonance is excited at higher redshift.
- Heating process driven by Lyman-α and X-ray flux are determined by normalized emissivity factors f_α and f_X, which is highly model-dependent regarding the star formation and dynamics.
- degeneracy between f_α , f_X and the star formation efficiency f_*





Conclusion

- We observed the broad resonance with a linear growth pattern for the particle-mixing in a stochastic background.
- The resonant condition is different for peaked and scale invariant spectra.
- Taking into account the evolution of PMF, the conversion probability distribution in B-λ_B parameter space is different to results obtained from domain-like model.
- Apply our model in the ALP-photon mixing, we can give a simultaneous explanation on Radio excess and 21cm absorption signal.

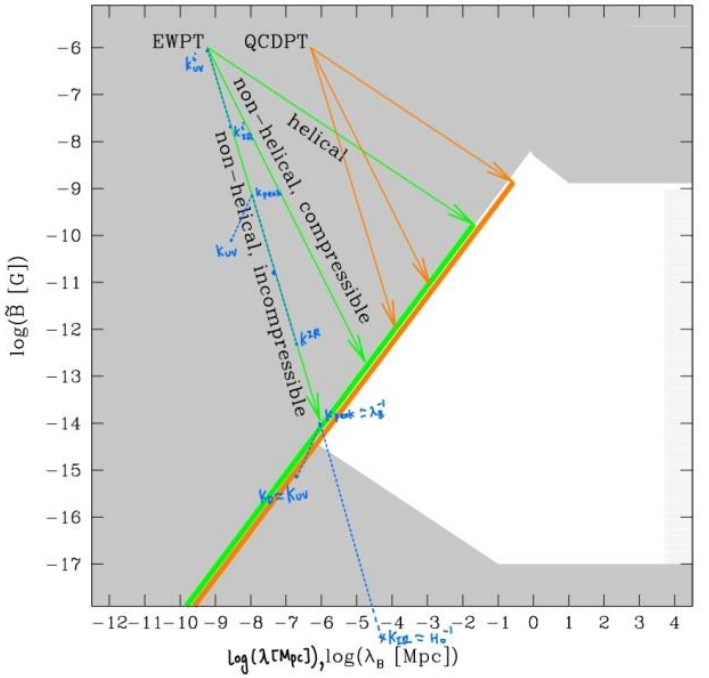
Future work

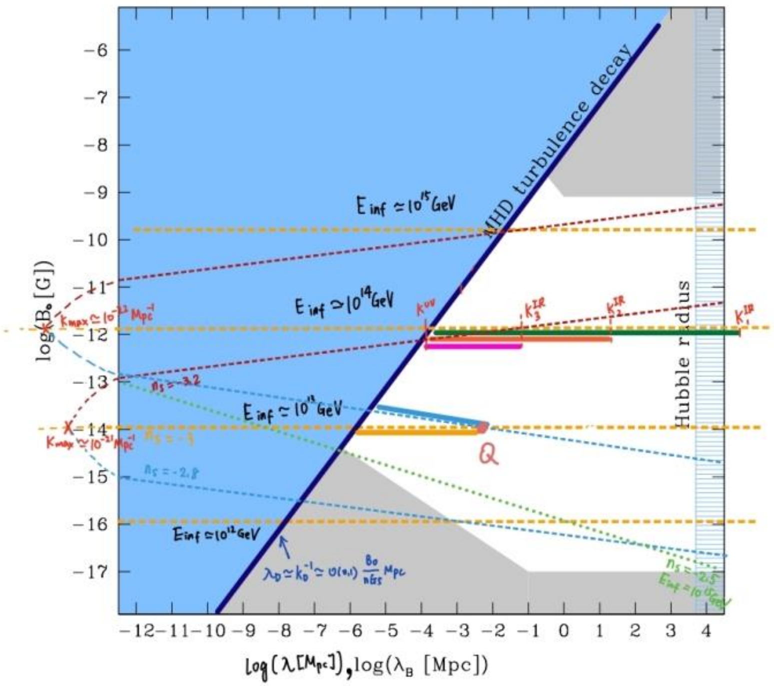
- Consider the Gamma-ray physics in high energy regime (LHASSO experiment).
- Consider the cosmic string/ PBH with a correlated analysis on EM and GW phenomena
- Generalize to nonlinear electrodynamics and f(R) gravity scenario.
- It is much likely that conversion probability in the stochastic background has a universal scaling property given by a simple formula $\mathcal{P} \approx \lambda_B W_R$, where W_R is the width of the resonant region determined by $\lambda_B < l_{osc}$. (work in progress)

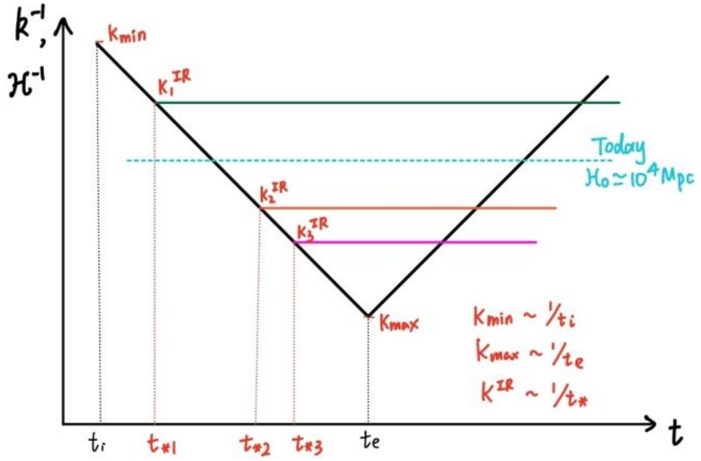
Thank you

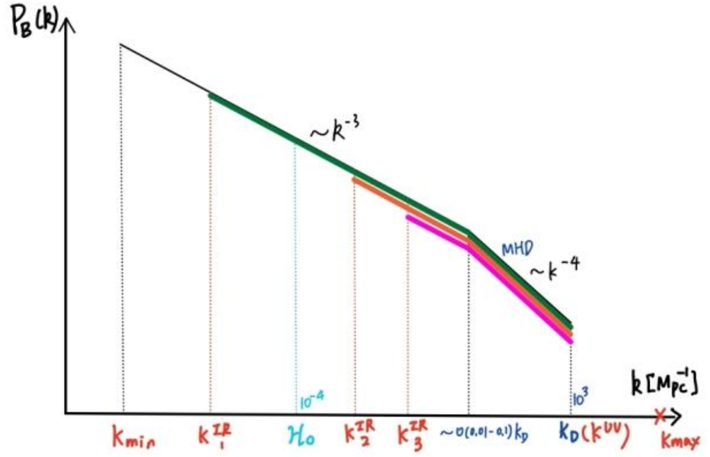
Evolution of PMF

- See Review *Ruth Durrer, Andrii Neronov , "Cosmological Magnetic Fields: Their Generation, Evolution and Observation"*
- Phase transition (Electroweak and QCD) survive in narrow stripe region today
- The broad region can be fulfilled by (nearly) scale invariant spectrum with different k_{IR}: the larger λ_{IR}, the smaller k_{IR}, the earlier start time of megnetogenesis during inflation.









Brightness temperature

- Transition from ALPs to photons proportional to the conversion probability

$$\Omega_\gamma(\omega, T(z)) \;=\; \Omega_{ALP}(\omega, T(z))\mathcal{P}^{tot}(\omega, z).$$

- Brightness temperature from ALPs at frequency ω and temperature T (redshift dependent) is

$$T_b(\omega, T) \;=\; \frac{\pi^4}{15}\left(\frac{T}{\omega}\right)^3 \frac{\Omega_\gamma(\omega, T)}{\mathbf{\Omega}_\gamma(T)} T = \frac{\pi^4}{15}\frac{T^4}{\omega^3}\frac{\Omega_{ALP}(\omega, T)}{\mathbf{\Omega}_\gamma(T)}\mathcal{P}^{tot}(\omega, z),$$

- Brightness temperature at present time is ,

$$T_b(\omega_0) \;=\; \frac{\pi^4}{15}\frac{T_0^4}{\omega_0^3}\frac{\Omega_{ALP0}(\omega_0)}{\mathbf{\Omega}_{\gamma 0}}\mathcal{P}^{tot}(\omega_0).$$

which is used in simulation and compared to ARCADE2. Here $\mathcal{P}^{tot}(\omega_0)$ is the total conversion probability accumulated from recombination until now.

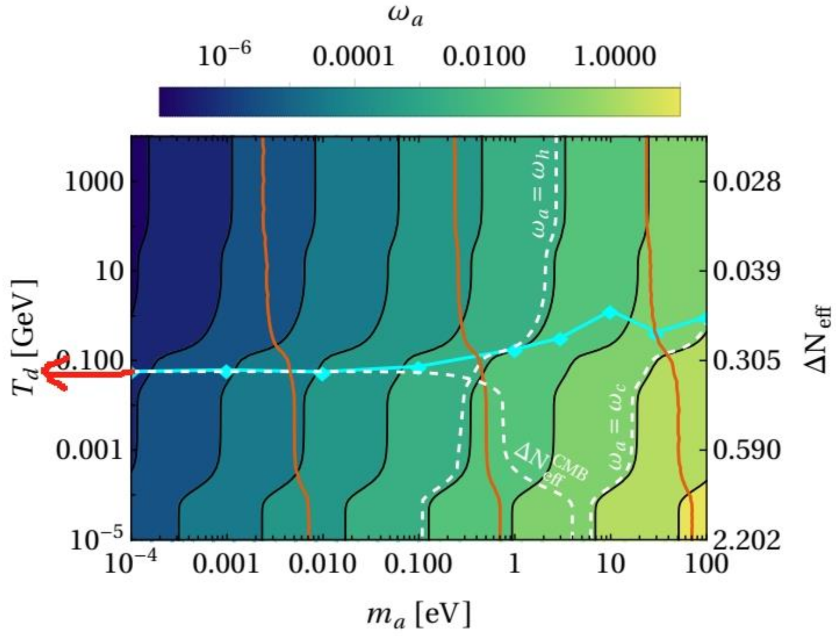
- Brightness temperature at any redshift with comoving 21cm frequency $\widetilde{\omega} = 2\pi * 1.4GHz/(1+z)$ is

$$T_b(z) \;=\; \frac{\pi^4}{15}\left(\frac{T_0}{\widetilde{\omega}}\right)^3 \frac{\Omega_{ALP}(\widetilde{\omega})}{\mathbf{\Omega}_\gamma}\mathcal{P}^{tot}(\widetilde{\omega}, z)T_0(1+z),$$

- Including the CMB radiation, the total temperature relevant to 21cm physics at given redshift is $T_b(z) + T_0(1+z)$, which is used in simulation and compared to EDGES.

Parameter space of ALP

- Assuming a frequency-independent spectrum for the abundance of ALPs in this work, thus ALP energy density is $\boldsymbol{\Omega}_{ALP} \simeq \Omega_{ALP}(\omega)$
- Ratio of ALP to photon energy density is constrained by $\boldsymbol{\Omega}_{ALP}/\boldsymbol{\Omega}_0 = 0.23\Delta N_{eff} \lesssim 0.06$ with the extra effective number of neutrino species $\Delta N_{eff} \lesssim 0.3$ (*Planck)*
- For ALP abundance to satisfy ΔN_{eff} constraint, they must decouple from the primordial plasma at an energy scale ≳0.1 GeV (≃0.1 GeV to saturate the ΔN_{eff} bound)
- Within this same parameter range, ALPs can also exist as non-relativistic particles. With a significantly suppressed misalignment angle, $\theta \sim 0.01$, these non-relativistic ALPs could be CMD



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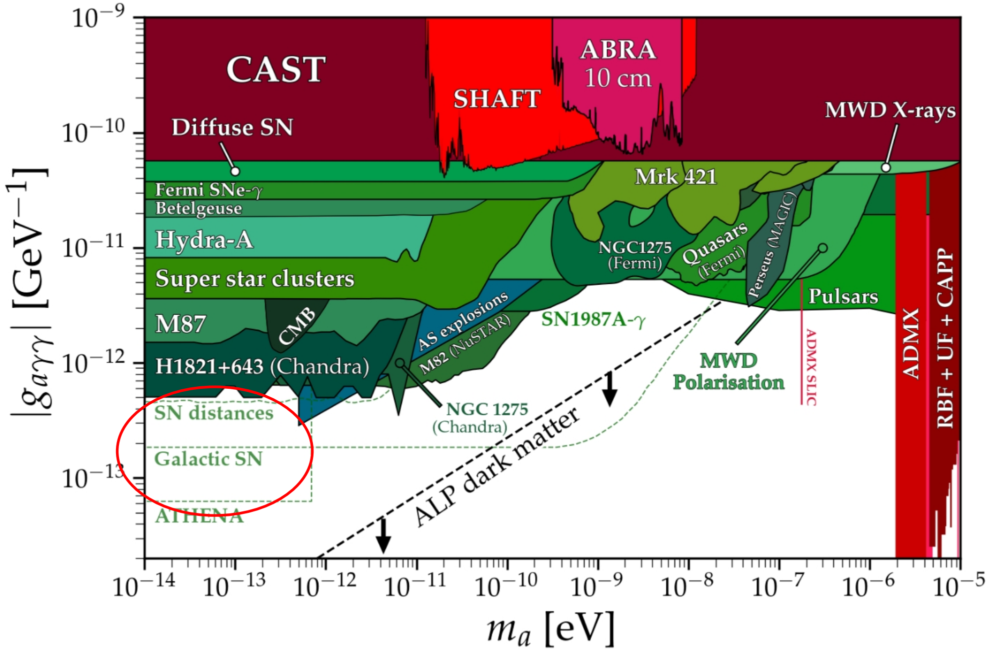

https://cajohare.github.io/AxionLimits/