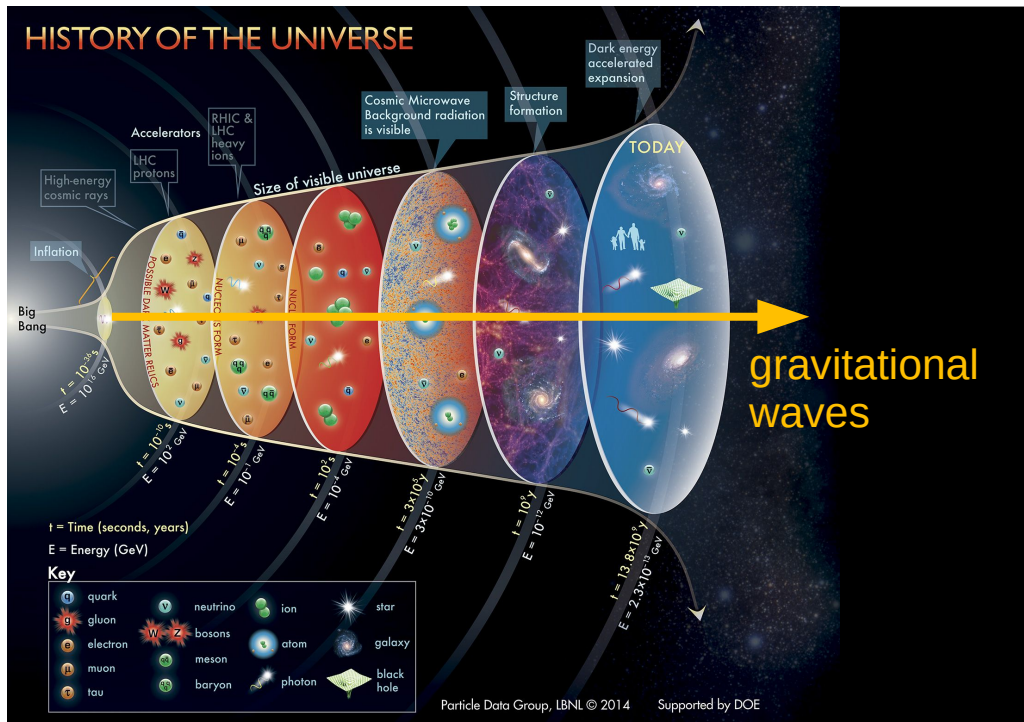


Searching for high-frequency gravitational waves with axion detectors



Valerie Domcke
CERN

Cosmic Whispers Online Seminar
November 26, 2024

Based on work with
Sebastian Ellis, Camilo Garcia-Cely,
Joachim Kopp, Sung Mook Lee
and Nick Rodd

high frequency ($> \text{kHz}$) GW sources

Cosmological

- sourced by violent cosmological event in the early Universe
- stochastic GW background (SGWB): stationary, isotropic, broad spectrum
- GW frequency determined by Hubble horizon at sourcing time
→ high frequency = early Universe
- observationally bounded by BBN and CMB (extra radiation)
- vanilla cosmology: SGWB from cosmic inflation & CGWB very small. But in many BSM models, saturating BBN bound is easy

Astrophysical

high frequency ($> \text{kHz}$) GW sources

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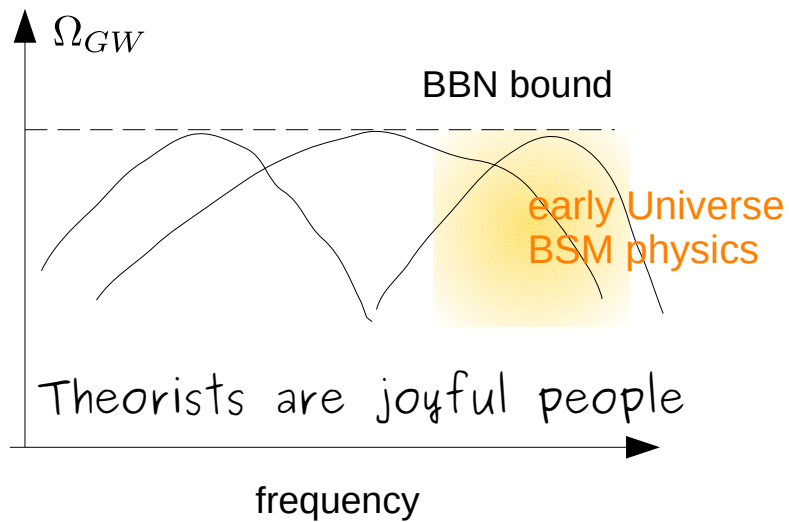
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Astrophysical

- localized GW sources, both coherent and incoherent signals possible
- no known astrophysical objects emit (significantly) in UHF band
- eg mergers of light primordial black holes or exotic compact objects, superradiance
- large signals require near-by events
→ rare events with GW strain far above BBN bound are possible
- SGWB from unresolved sources, typically harder to detect

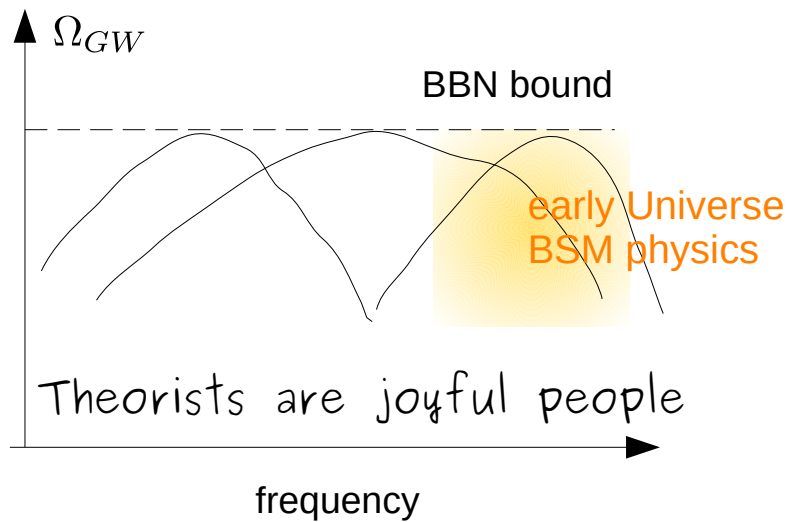
UHF GW searches are searches for new physics

challenges in UHF GW detection



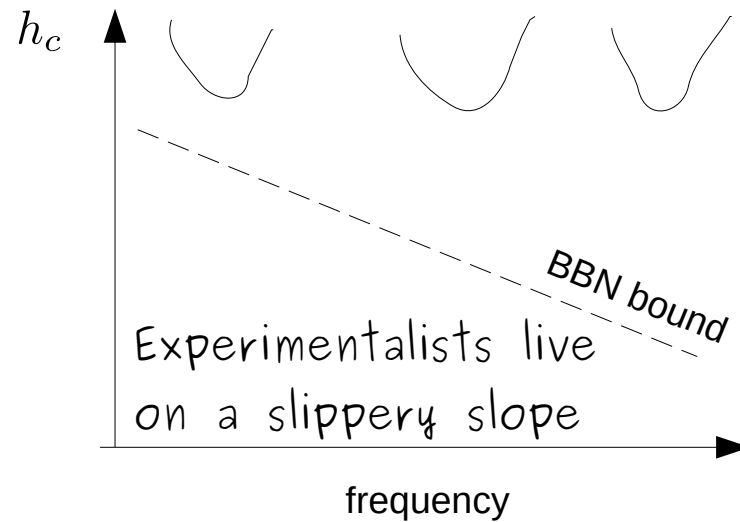
CMB/BBN bound constrains energy

challenges in UHF GW detection



CMB/BBN bound constrains energy

$$\Omega_{GW} \propto f^2 h_c^2$$

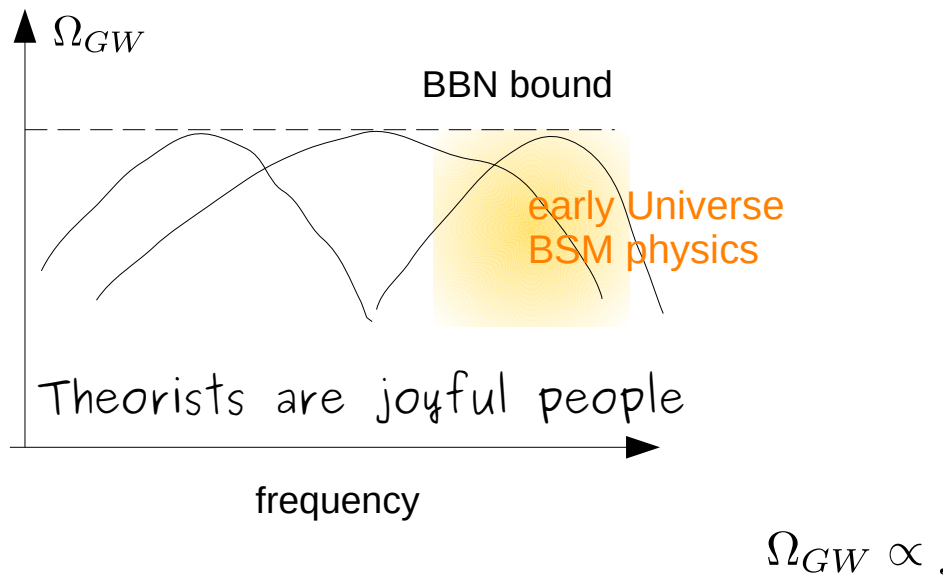


experiments measure displacement

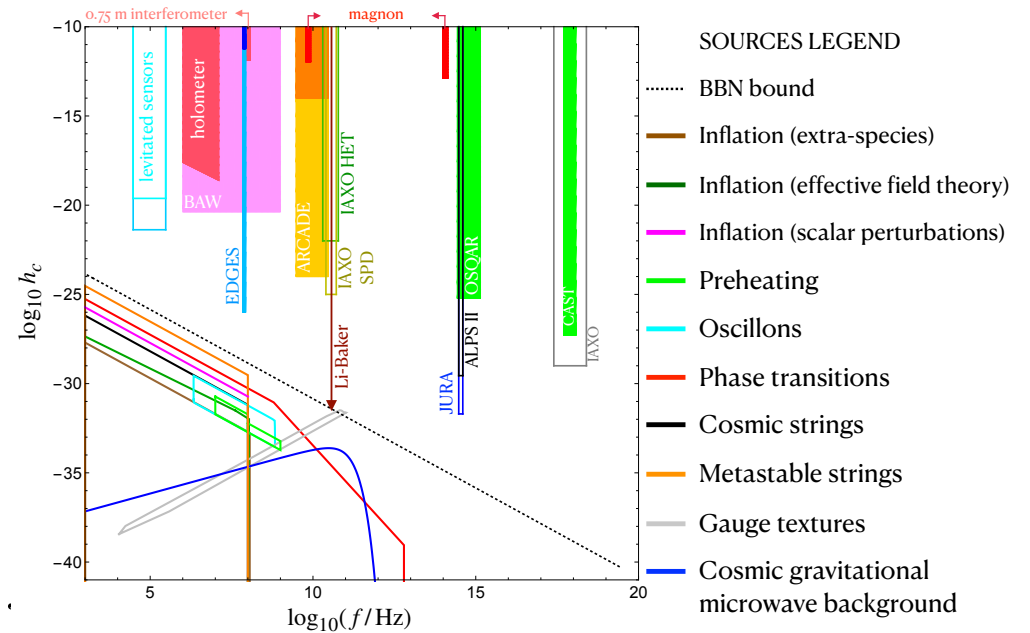
challenges in UHF GW detection

update coming!

UHF GW initiative Living Review:
<https://arxiv.org/abs/2011.12414>



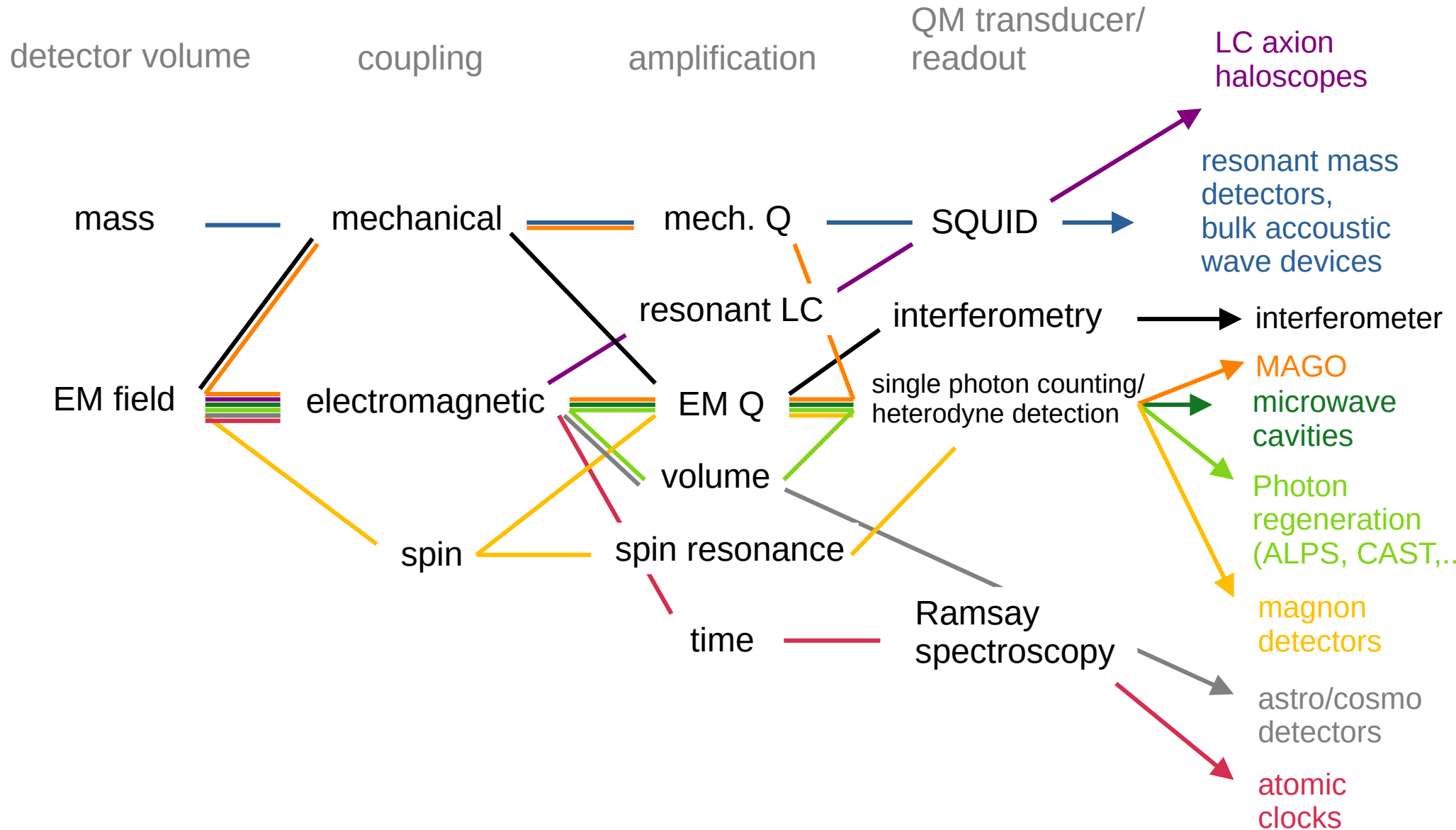
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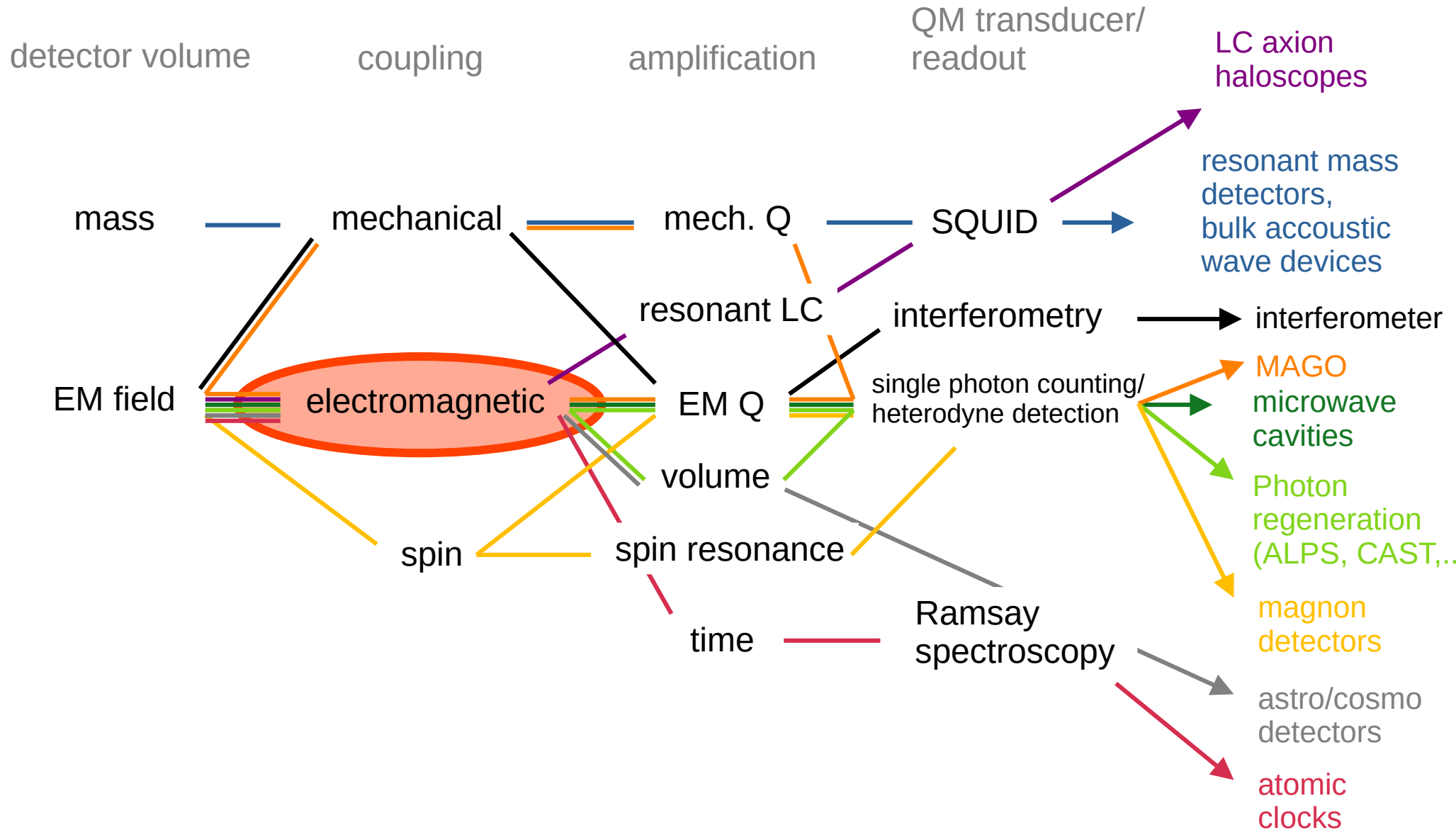
experiments measure displacement

UHF GW initiative:
<https://www.ctc.cam.ac.uk/activities/UHF-GW.php>

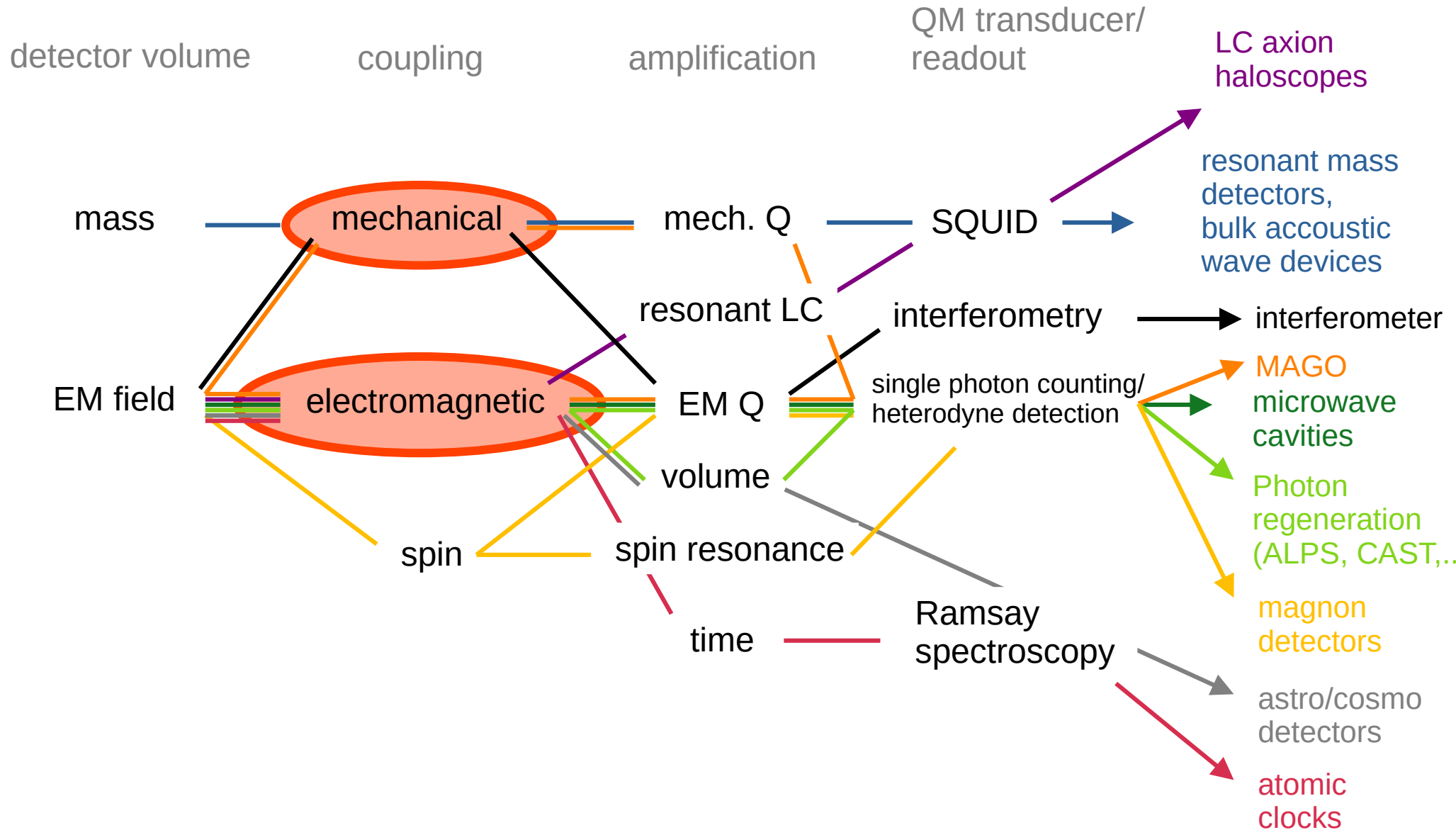
detection strategies



detection strategies



detection strategies



GW electrodynamics

Classical electrodynamics + linearized GR, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $F_{\mu\nu} = \bar{F}_{\mu\nu} + F_{\mu\nu}^h$:

$$\partial_\mu (\sqrt{-g} g^{\mu\alpha} F_{\alpha\beta} g^{\beta\nu}) = 0$$

$$\rightarrow \partial_\nu F_h^{\mu\nu} = j_{\text{eff}}^\mu = (-\nabla \cdot \mathbf{P}, \nabla \times \mathbf{M} + \partial_t \mathbf{P})$$

effective current
effective polarization vector
effective magnetization vector

with

$$P_i = -h_{ij} \bar{E}_j + \frac{1}{2} h \bar{E}_i + h_{00} \bar{E}_i - \epsilon_{ijk} h_{0j} \bar{B}_k,$$

$$M_i = -h_{ij} \bar{B}_j - \frac{1}{2} h \bar{B}_i + h_{jj} \bar{B}_i + \epsilon_{ijk} h_{0j} \bar{E}_k,$$

induced at linear order in h
in presence of external E,B field

VD, Garcia-Cely, Rodd `22

Direct analogy with axion electrodynamics

$$\mathcal{L} \supset g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B} \rightarrow \mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$$

McAllister et al `18
Tobar, McAllister, Goryachev `19
Quellet, Bogorad `19

effective source terms in Maxwell's equation due to GW

Dielectric GW haloscope

Maxwell equations:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 0, & \nabla \times \mathbf{B} - \underline{\epsilon} \dot{\mathbf{E}} &= \underline{j_{\text{eff}}}, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{E} + \dot{\mathbf{B}} &= 0,\end{aligned}$$

effective current induced by Gws
dielectric constant

Dielectric GW haloscope

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effective current induced by Gws
dielectric constant

particular solutions in medium and vacuum

$$\mathbf{E}_m^p = \frac{c_\theta B_0}{\epsilon - 1} (h_\times \hat{\mathbf{p}} + h_+ \hat{\mathbf{s}}) e^{-i\omega(t - \hat{\mathbf{k}} \cdot \mathbf{x})} \sim h B_0$$

$$\mathbf{E}_v^p = -\frac{B_0}{2} \left[i\omega x (h_\times \hat{\mathbf{p}} + h_+ \hat{\mathbf{s}}) + h_\times s_\theta \hat{\mathbf{k}} \right] e^{-i\omega(t - \hat{\mathbf{k}} \cdot \mathbf{x})} \sim h B_0 \omega x$$

Dielectric GW haloscope

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ALPs, IAXO, ...
as GW detectors

[Ejlli et al `19]



resonant conversion
in vacuum

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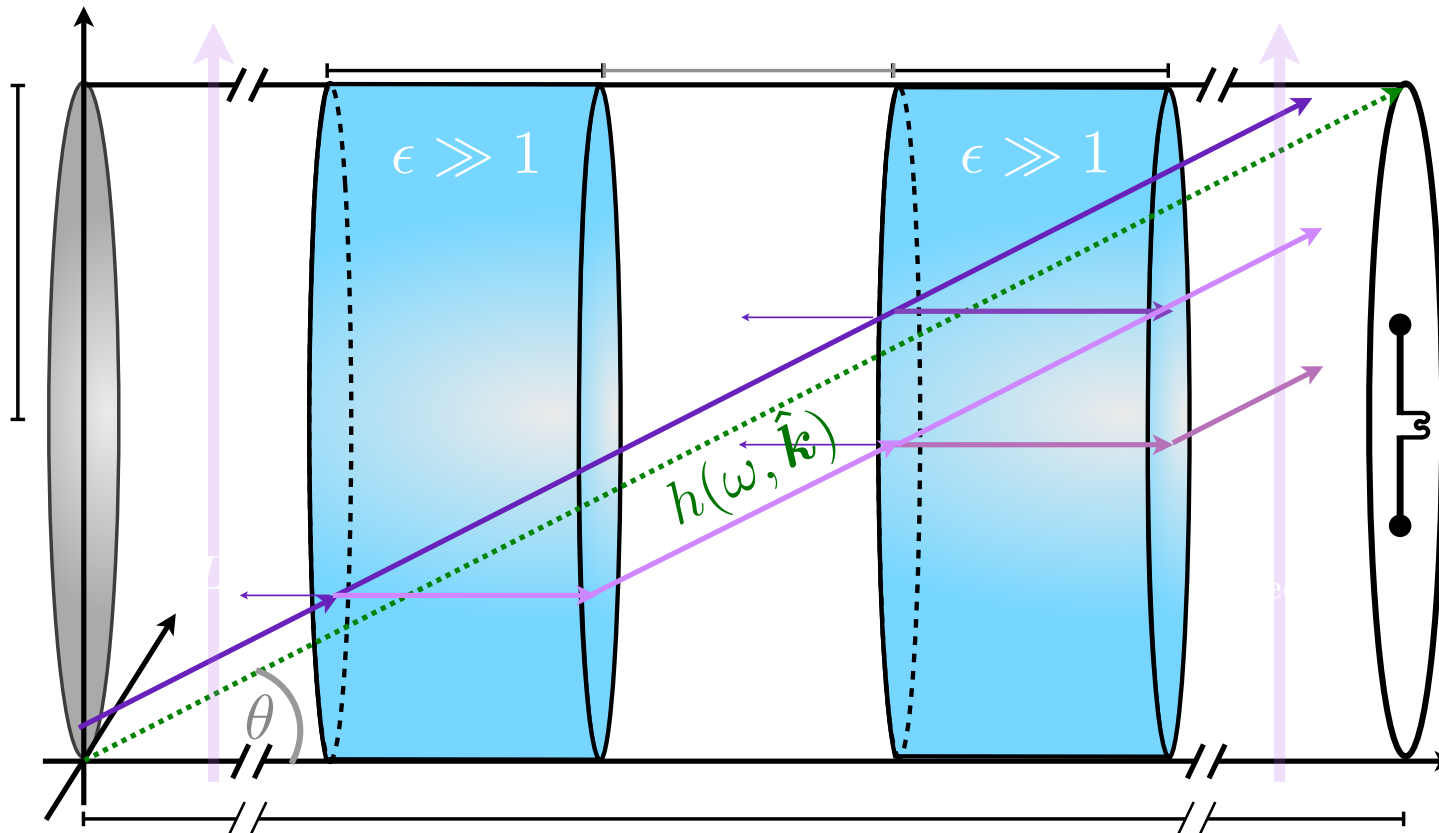


resonant conversion
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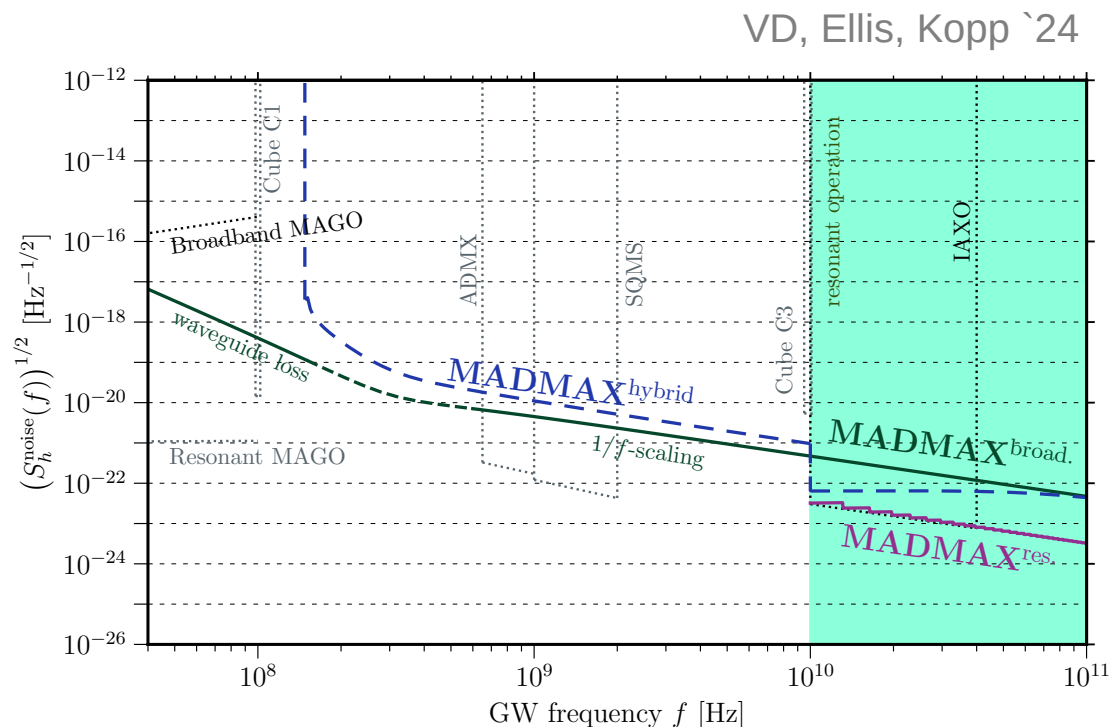
Boundary conditions at surface of dielectric medium

→ EM waves sourced at surfaces of dielectric disks

Dielectric GW haloscope



Dielectric GW Haloscope



GWs vs axions

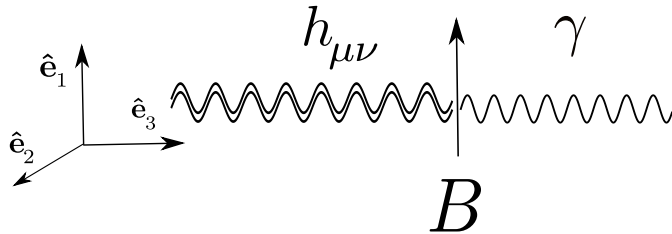
GWs are relativistic:

- + resonant conversion in vacuum
- + relaxed requirement on disk surface
- new requirement on effective disk width

- MADMAX can be operated as GW detector
- Hybrid resonant / broadband mode particularly interesting

photon (re-)generation experiments

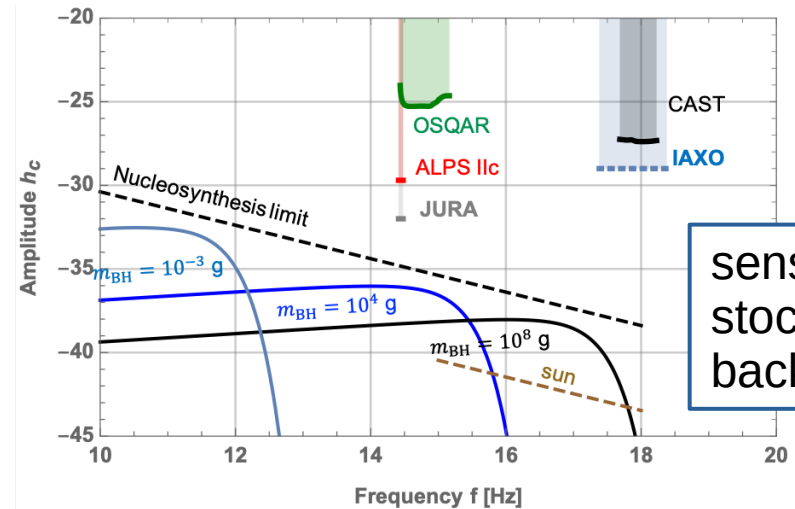
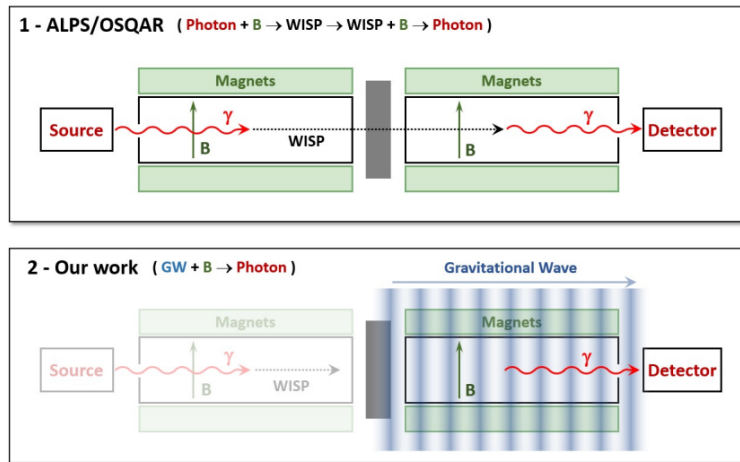
[Gertsenshtein '62, Boccaletti et al '70, Raffelt, Stodolsky '88]



analogous to axion to photon conversion

Light-shining-through-the-wall (LSW) experiments, helioscopes:

Ejilli et al '19



sensitivity to stochastic GW background

Microwave cavities:

Berlin et al '21, '23

astro/cosmo environments:

Fujita et al '20, VD, Garcia-Cely '21, Feng et al '22, Liu et al '23, Ito et al '23, Ramazanov et al '23,...

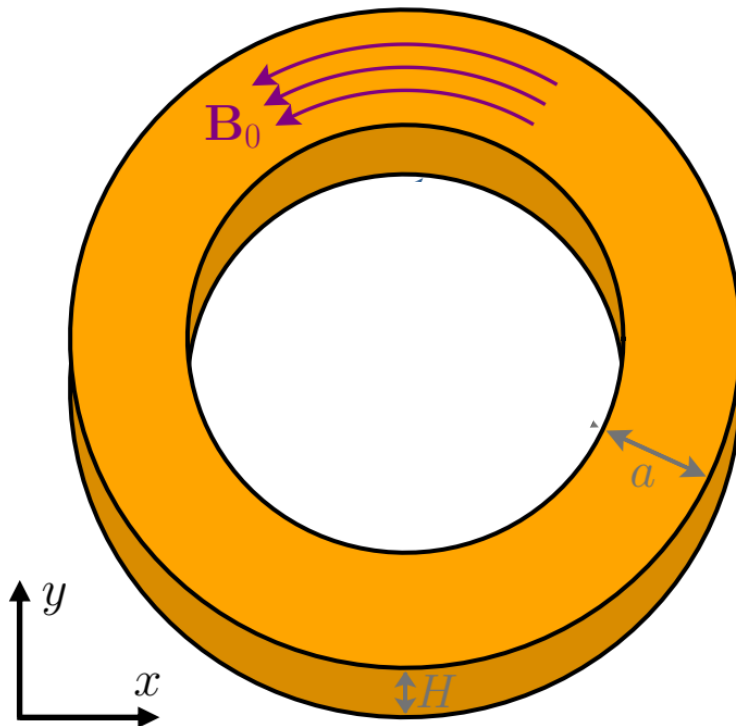
high frequency gravitational waves

GW signal in low-mass axion haloscopes

eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Lee, Rodd '22, '23

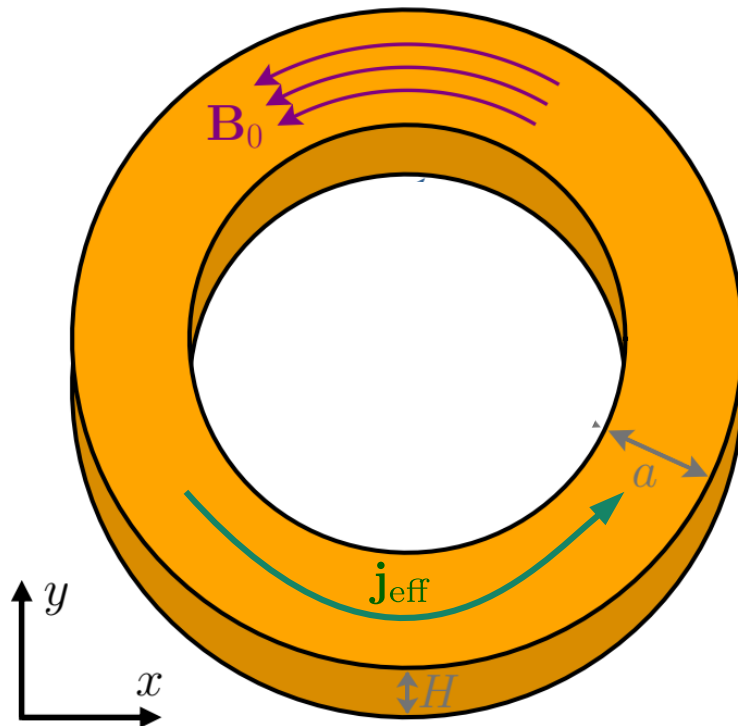
static magnetic field (i.e. rigid detector)



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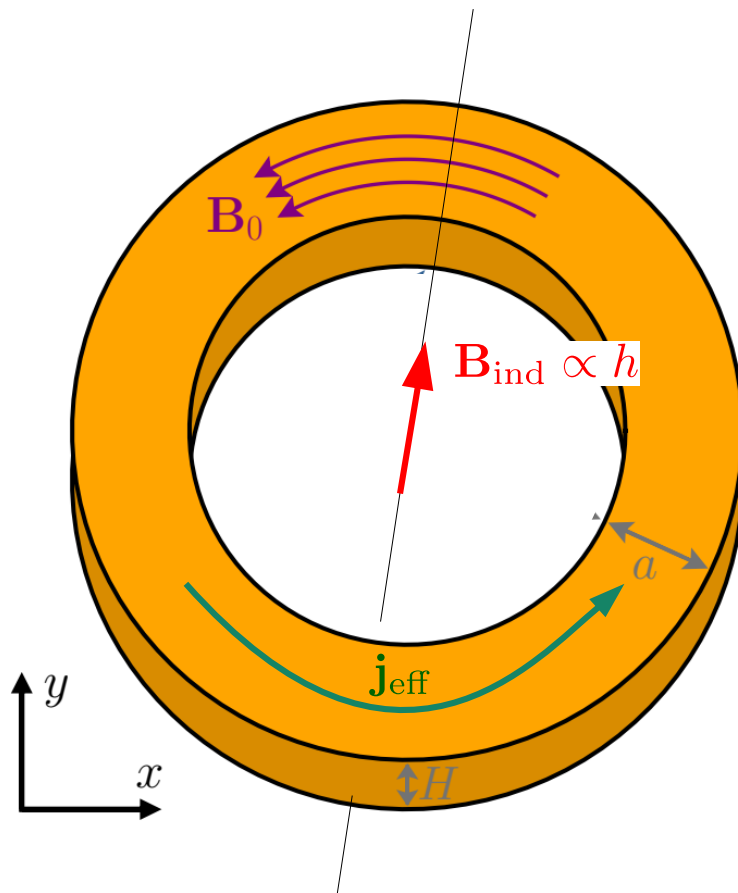
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effective current $\sim h(\omega L)^2 B_0$

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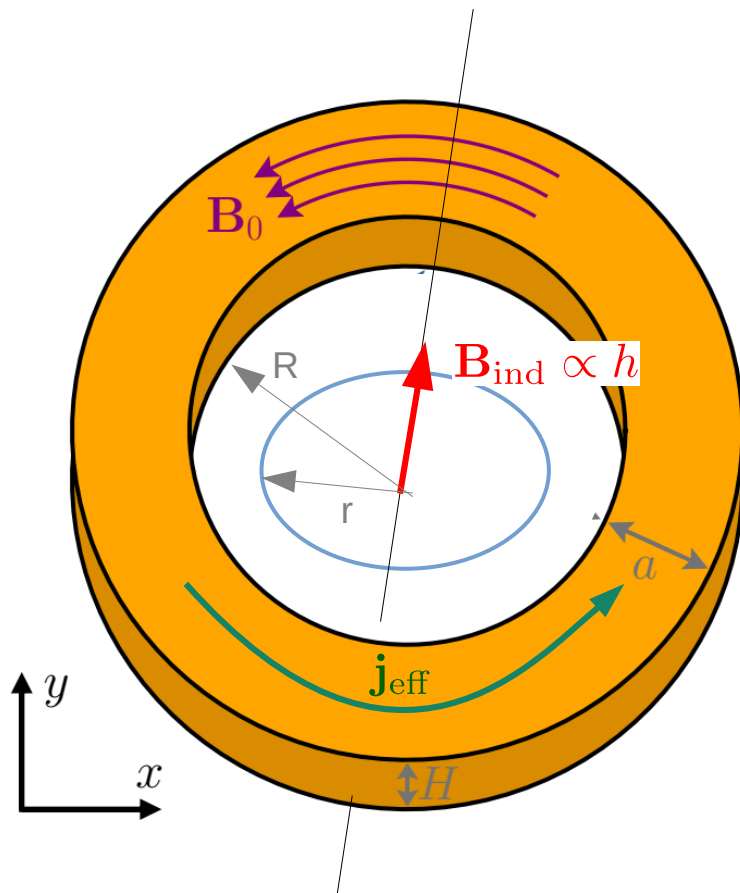
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induced oscillating magnetic field

measure magnetic flux ($\sim h$)
through pickup loop

at leading order in (ωR) :

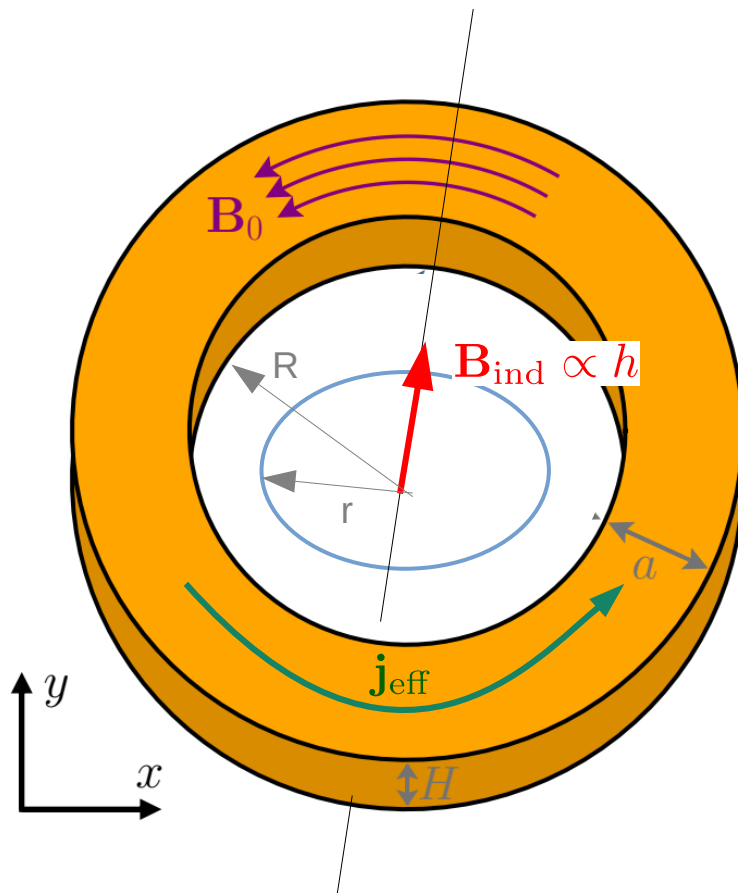
$$\Phi_{\text{gw}} = \frac{i e^{-i\omega t}}{16\sqrt{2}} h \times \omega^3 B_0 \pi r^2 R a (a + 2R) s_{\theta_h}^2$$

$$\sim (\omega L)^3 h B_0 L^2$$

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$$\Phi_a = e^{-i\omega t} g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} B_0 \pi r^2 R \ln(1 + a/R)$$

$$\sim (\omega L) g_{a\gamma\gamma} a B_0 L^2$$

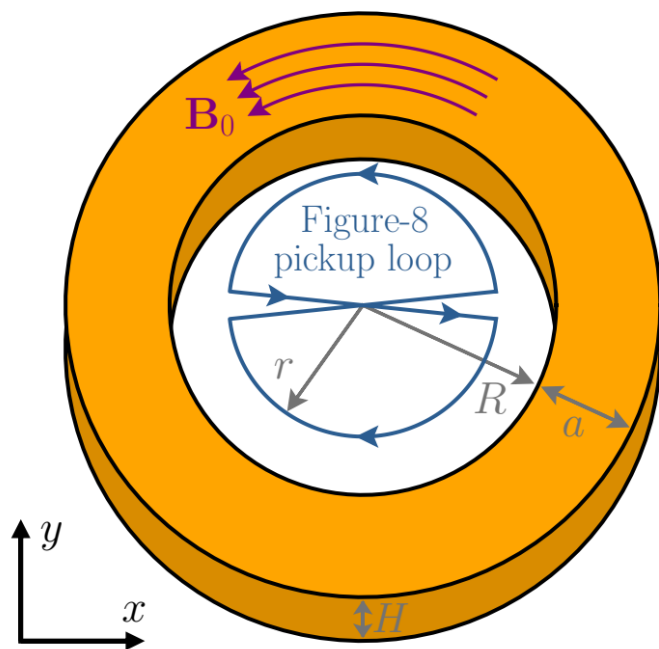
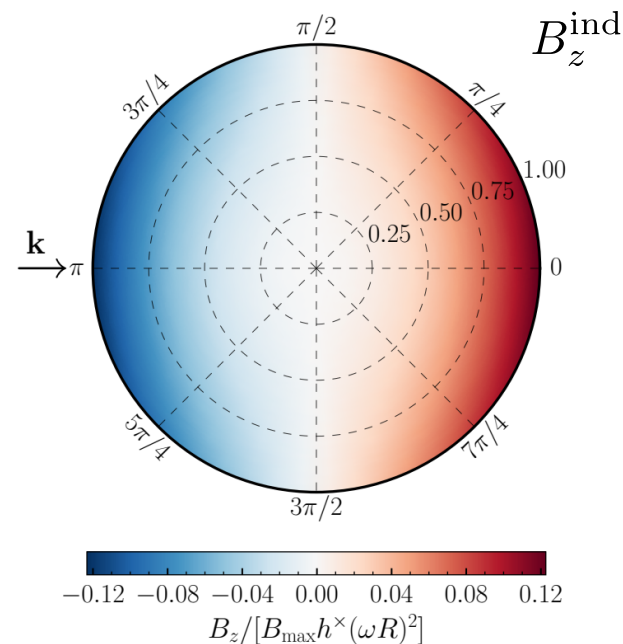
match to axion induced flux to estimate sensitivity to GW signals

optimized pickup loop geometry

spin 2 structure of GW : angular modulation of induced B field

leading order $(\omega R)^2$ contribution captured by breaking cylindrical symmetry, e.g. using a figure-8 pickup loop

[VD, Garcia-Cely, Lee, Rodd `23] Symmetries and selection rules

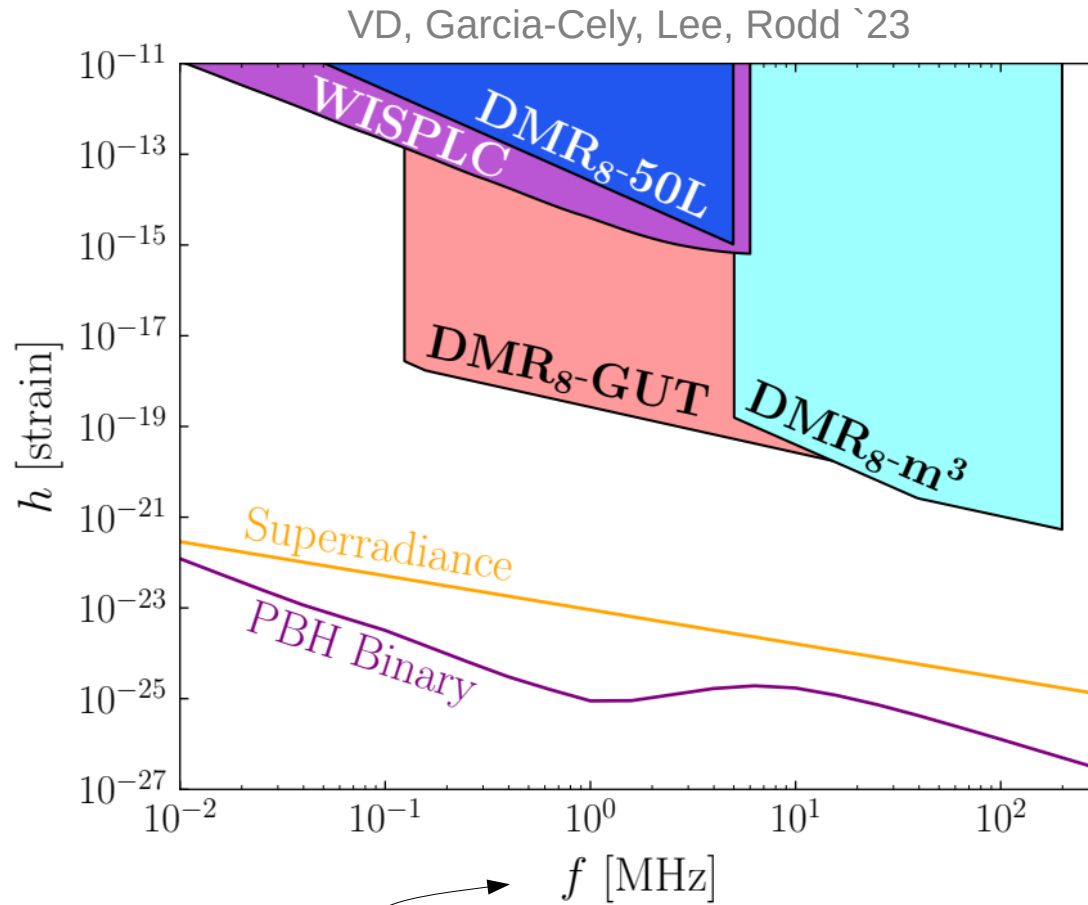


$$\Phi_{\text{gw},8} = \frac{e^{-i\omega t}}{3\sqrt{2}} \omega^2 B_0 r^3 R \ln(1 + a/R) s_{\theta_h} \times (h^\times s_{\phi_h} - h^+ c_{\theta_h} c_{\phi_h})$$

$$\sim (\omega L)^2 h B_0 L^2$$

parametric improvement for modified pickup loop

Low mass haloscopes



$$\Phi_h(h^+, h^\times; \phi_h, \theta_h) = \mathcal{R}_c \Phi_a(g_{a\gamma\gamma}),$$

coherence ratio factor:
coherence & observation time

← sensitivity to signal with
 $\mathcal{R}_c = 1$

← sensitivity to 'axion-like' signal

← \mathcal{R}_c computed assuming
 $Q_r = 10^4, T_m = 10^3 \text{ s} (10^{-3} \text{ s})$

axion haloscopes as **MHz** GW detectors

still far away from BBN bound, but clear synergies of UHF GW and axion searches

mechanical coupling

mechanical response to GWs

$$\omega_n \sim n v_s L^{-1} \sim 10^{-5} L^{-1} \quad \text{mech. eigenmodes}$$

$$\omega_{\text{gw}} \ll \omega_n$$

rigid limit

$$\omega_{\text{gw}} \sim \omega_n$$

on resonance
(Weber Bars)

$$\omega_n \ll \omega_{\text{gw}} \ll L^{-1}$$

free-falling limit

$$L^{-1} \ll \omega_{\text{gw}}$$

response function
may be suppressed
by oscillation pattern

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on resonance
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free-falling limit

$$L^{-1} \ll \omega_{\text{gw}}$$

mechanical deformations less stiff than
Maxwell's equations by factor $v_s/c \sim 10^{-5}$

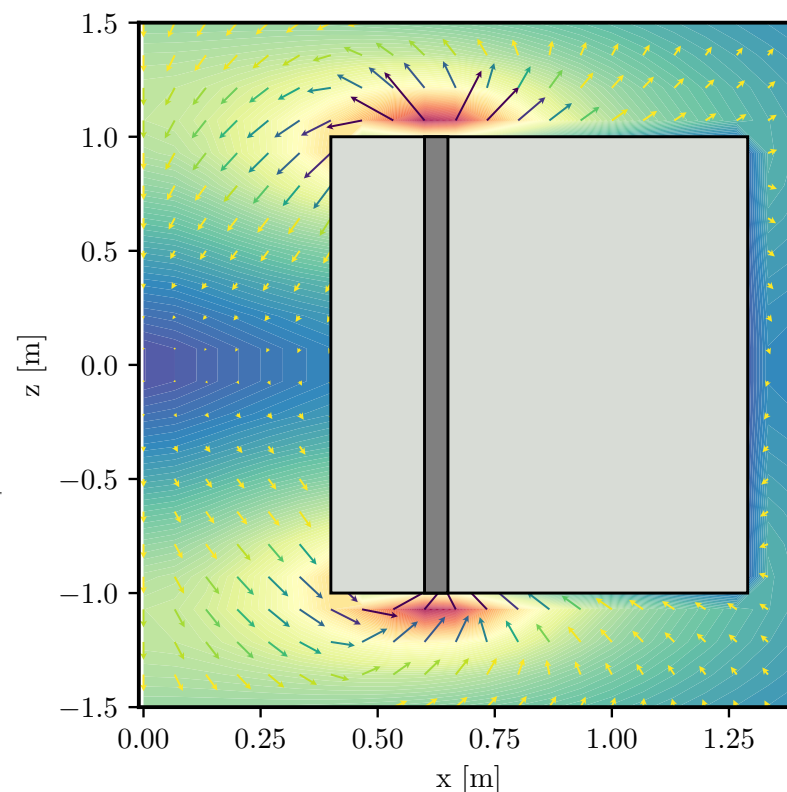
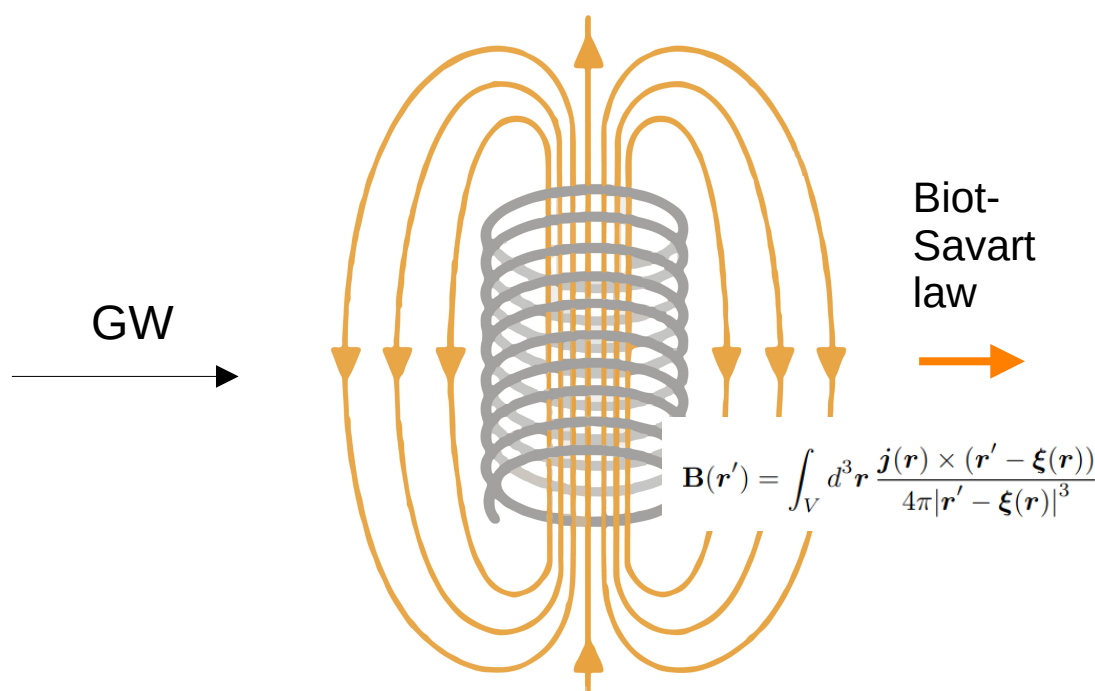
response function
may be suppressed
by oscillation pattern

- mechanical coupling can be significantly more efficient
- challenge of transducing mechanical deformation to EM signal for quantum readout

Magnetic Weber Bar

VD, Ellis, Rodd '24

GW acts as a mechanical force on (current-carrying) wires:

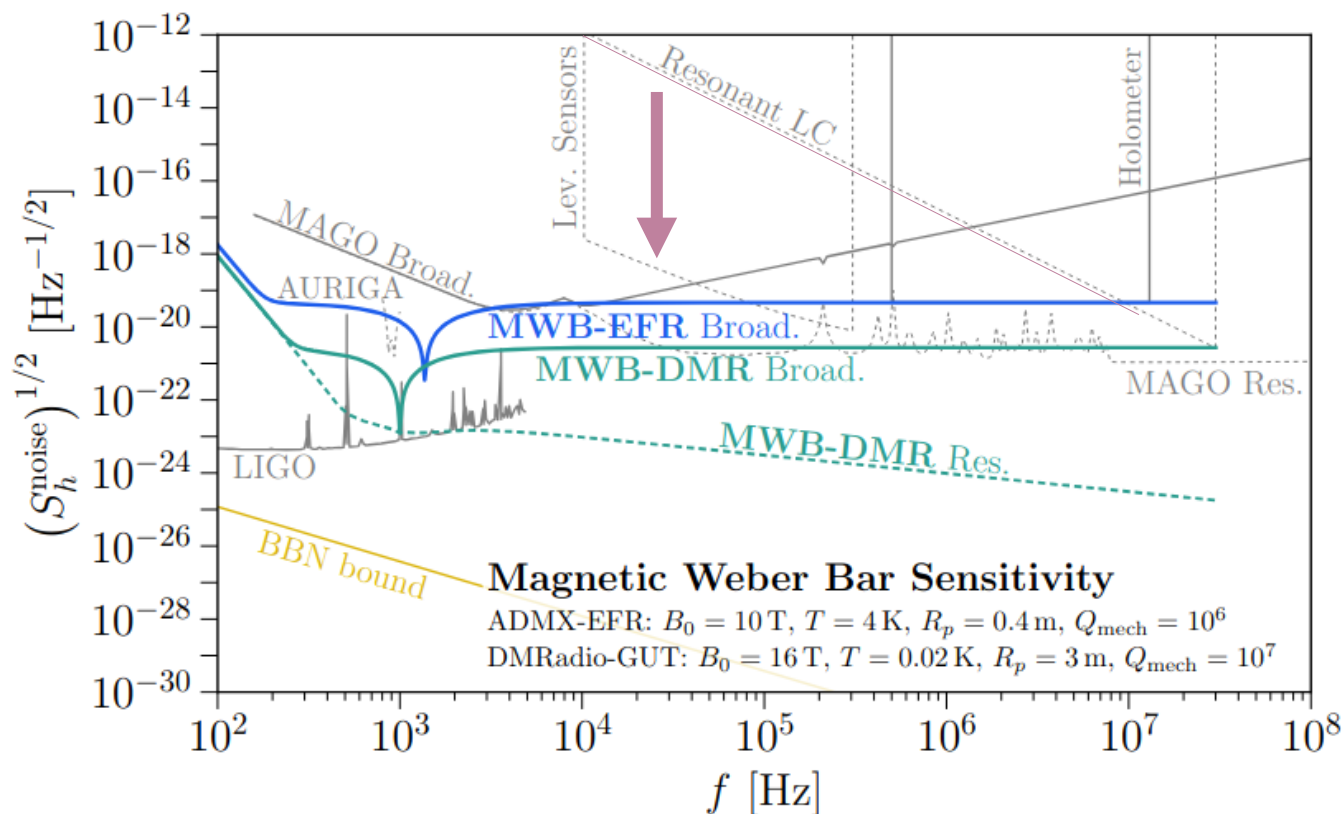


➔ Induced AC magnetic field $\sim h B$, read out with pickup loop + SQUID

Magnetic Weber Bar

VD, Ellis, Rodd '24

mechanical vs
EM coupling



3 effects at $O(h)$:

- deformation of magnet coil
- motion of pickup loop
- modulation of supercurrent

Noise contributions:

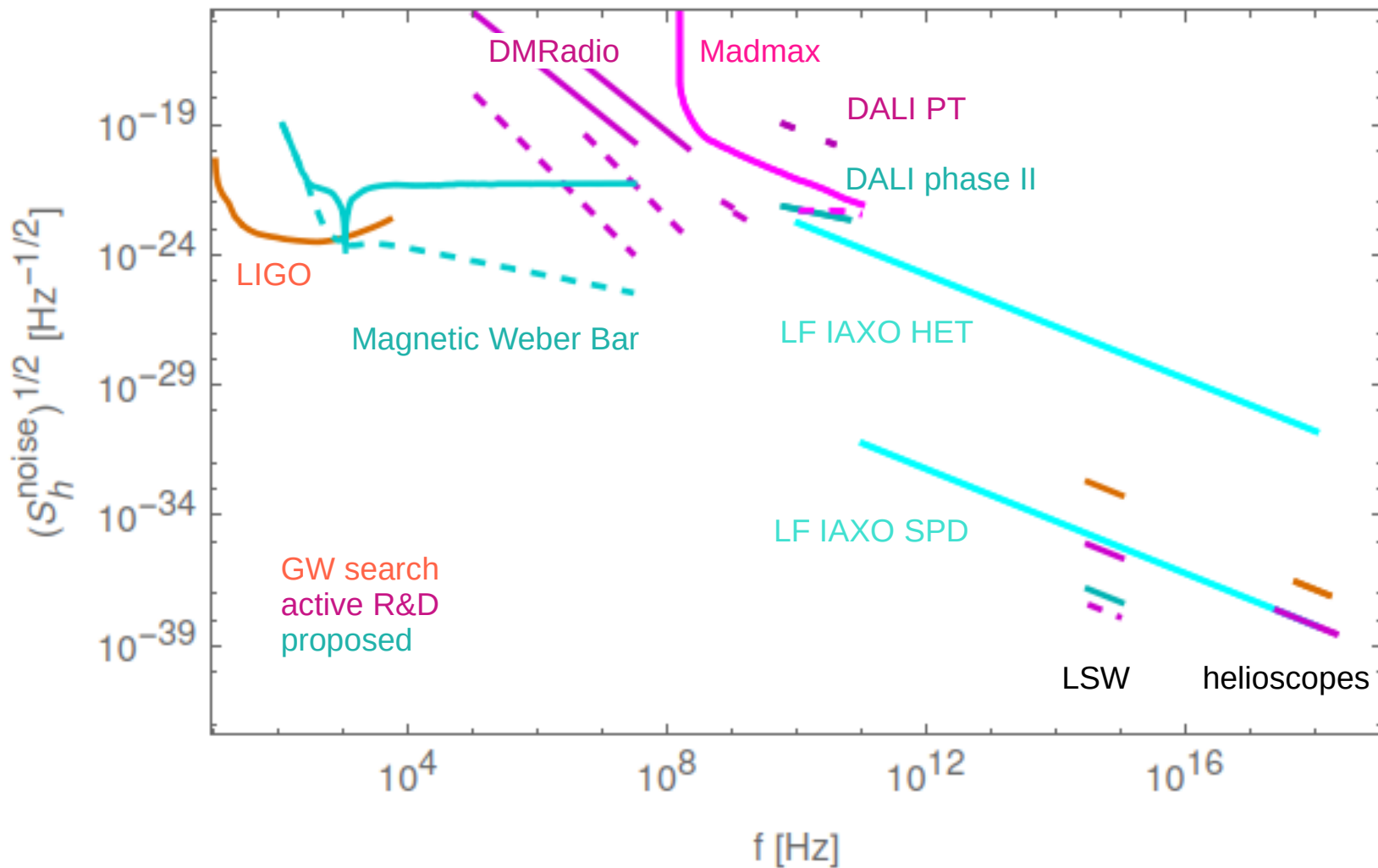
SQUID, thermal mechanical, seismic, thermal noise of resonant readout

Very competitive broadband sensitivity !

Charting the GW landscape

UHG GW initiative Living Review:
<https://arxiv.org/abs/2011.12414>

update coming!



Conclusions and Outlook

Synergies between axion and high-frequency GW searches

$$(g_{a\gamma\gamma}a)F_{\mu\nu}\tilde{F}^{\mu\nu} \leftrightarrow hF_{\mu\nu}F^{\mu\nu}$$

Key differences to leverage:

pseudoscalar versus spin 2 : optimize geometry

massive axion versus relativistic GWs : different resonance conditions

mechanical coupling of GWs : powers of $\omega L, v_s/c$, can be missed by NDA

...

A lot of room for new ideas!

... and an advertisement:

CERN TH visitor program

<https://theory.cern/visitor-info>

short-term visits typically O(week)

long term visits (> 3 months, usually sabbaticals)

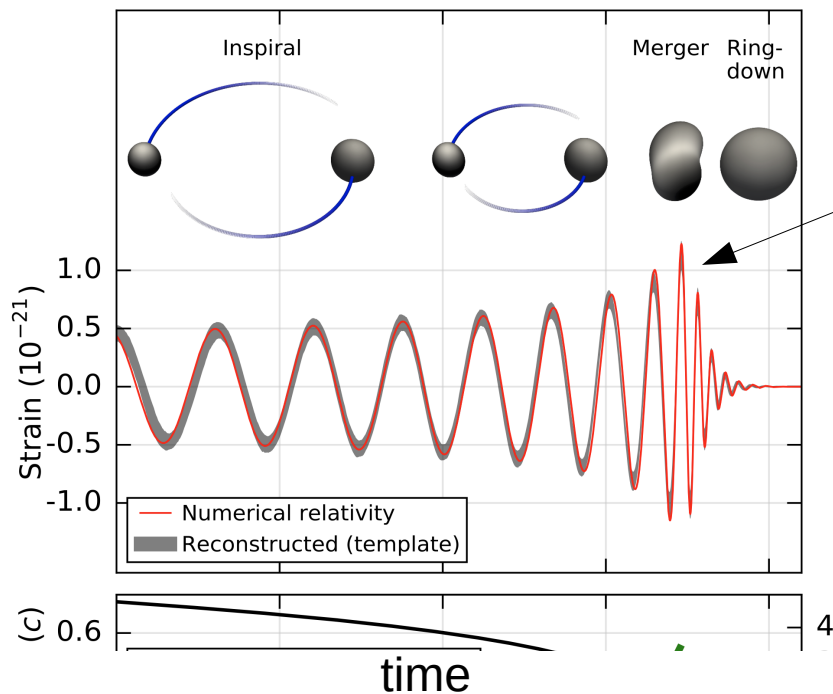
consider applying!

backup slides

astrophysical sources

Example:
mergers of light primordial black holes

$$h_{+, \times}^{\text{PBH}} \simeq 10^{-23} \left(\frac{10 \text{ kpc}}{D} \right) \left(\frac{m_{\text{PBH}}}{10^{-5} M_{\odot}} \right)^{5/3} \left(\frac{f}{100 \text{ MHz}} \right)^{2/3}$$



$$f_{\text{ISCO}} = 220 \text{ MHz} \left(\frac{10^{-5} M_{\odot}}{m_{\text{PBH}}} \right)$$

astrophysical sources

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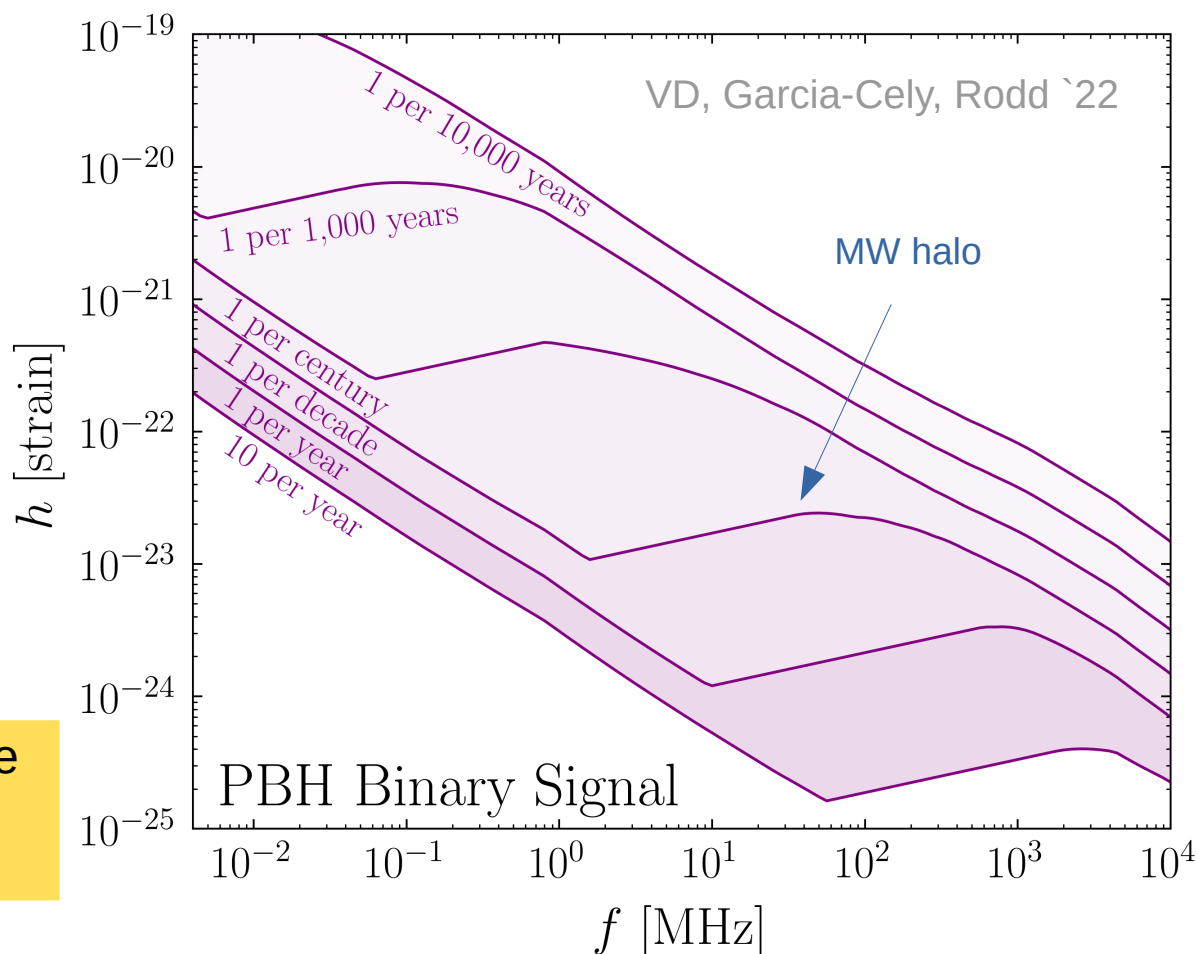
event rate:

$$\langle \Gamma \rangle = \int_0^{\infty} dr 4\pi r^2 \delta(r) R_0(m_{\text{PBH}}, f_{\text{PBH}})$$

MW halo merger rate

$$\times \Theta \left[Q^{1/4} h_{+, \times}^{\text{PBH}}(f, m_{\text{PBH}}, r) - h_{\text{th}} \right]$$

large GW amplitudes possible for rare events, but signal duration very short!



see also Franciolini, Maharana, Muia `22

BBN bound

radiation energy after electron decoupling:

$$\rho_{rad} = \frac{\pi^2}{30} \left(2 + \frac{7}{4} \left(\frac{4}{11} \right)^{4/3} (3.046 + \Delta N_{eff}) \right) T^4$$

photons neutrinos BSM

at BBN or CMB decoupling:

$$\rho_{GW}(T) < \Delta \rho_{rad}(T) \quad \Rightarrow \quad \left(\frac{\rho_{GW}}{\rho_\gamma} \right)_{T_{BBN,CMB}} \leq \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \Delta N_{eff} \simeq 0.05$$

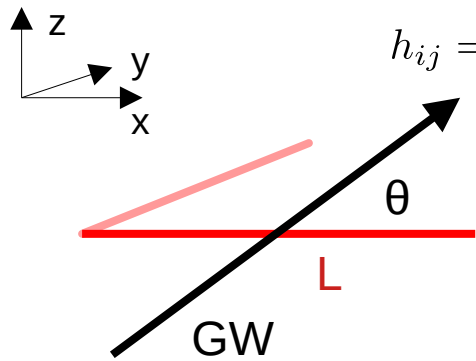
→ at BBN, CMB decoupling ~ 5 % GW energy density allowed

today: $\frac{\rho_{GW}^0}{\rho_c^0} = \Omega_\gamma^0 \left(\frac{g_s^0}{g_s(T)} \right)^{4/3} \frac{\rho_{GW}(T)}{\rho_\gamma(T)} \leq 10^{-5} \Delta N_{eff} \simeq 10^{-6}$

note: constraint
on *total* GW energy

→ today, energy fraction < 10⁻⁶ (for GWs present at BBN / CMB decoupling)

warm-up: LIGO



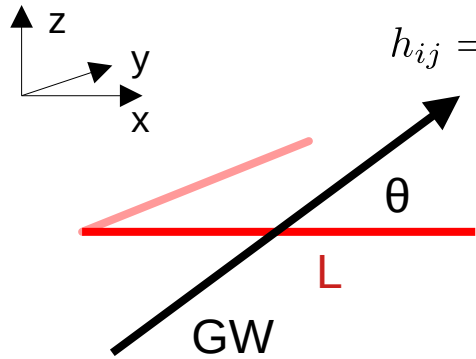
$$h_{ij} = h_+ e_{ij}^+ \cos[\omega(t - c_\theta x - s_\theta z)]$$

VD, Kopp '24

$$\partial_\nu F_h^{\mu\nu} = j_{\text{eff}}^\mu$$

$$\vec{A} = \vec{A}_0 + \Delta\vec{A} = A_0 \cos[\omega_\gamma(t - x)]\hat{e}_z + \Delta\vec{A}(t, \vec{x})$$

warm-up: LIGO



$$h_{ij} = h_+ e_{ij}^+ \cos[\omega(t - c_\theta x - s_\theta z)]$$

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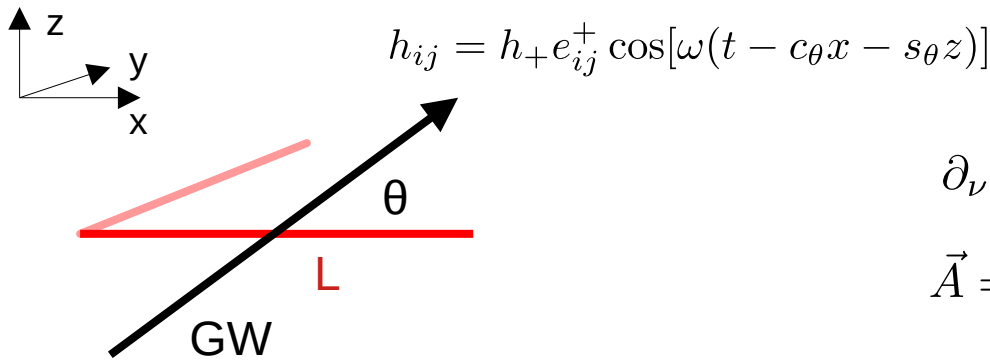
$$\vec{A} = \vec{A}_0 + \Delta\vec{A} = A_0 \cos[\omega_\gamma(t - x)]\hat{e}_z + \Delta\vec{A}(t, \vec{x})$$

particular solution along x-axis:

$$\Delta A_z^p(t, x) = - \sum_{\lambda=\pm} \lambda \frac{A_0 h_+}{4} \frac{\omega_\gamma}{\omega} (1 + c_\theta) \cos[(\omega_\gamma + \lambda\omega)t - (\omega_\gamma + \lambda\omega c_\theta)x] + \mathcal{O}(\omega_\gamma/\omega)^0$$

warm-up: LIGO

VD, Kopp '24



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Add plane waves with frequencies $\omega_\gamma \pm \omega$ to match boundary condition $\vec{A}(t, 0) = \vec{A}_0(t, 0)$

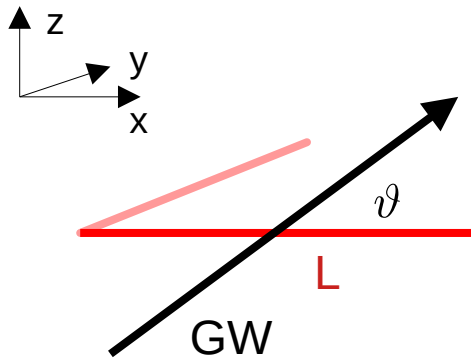
→ full solution for EM wave propagating in GW background

→ compute phase shift: $\Delta\phi = -\frac{1}{2} h_+ \omega_\gamma L \sin^2 \theta$, $\Delta\phi_{\text{LIGO}} = h_+ \omega_\gamma L \cos(2\theta)$

→ at next order in ω/ω_γ : amplitude modulation, rotation of polarization



Interferometers revisited



$$h_{ij} = h_+ e_{ij}^+ \cos[\omega(t - c_\theta x - s_\theta z)]$$

$$\mathbf{A} = \bar{\mathbf{A}} + \mathcal{A} = A_0 \cos[\omega_\gamma(t - x)] \hat{e}_z + \mathcal{A},$$

$$\mathcal{A}_i = a_i^- \cos_- + a_i^+ \cos^+$$

$$\cos_+ \equiv \cos[(\omega_\gamma + \omega_g)t - \omega_\gamma x - \hat{\mathbf{k}}_g \cdot \mathbf{x}]$$

$$\cos_- \equiv \cos[(\omega_\gamma - \omega_g)t - \omega_\gamma x + \hat{\mathbf{k}}_g \cdot \mathbf{x}]$$

$$\longrightarrow a_x^\pm = A_0 h_+ s_\theta \frac{(\omega_g \pm \omega_\gamma(1 - c_\theta))}{4(\omega_g \pm \omega_\gamma)} = \frac{1}{4} A_0 h^+ s_\theta (1 - c_\theta) + \mathcal{O}(\omega_g/\omega_\gamma),$$

$$a_y^\pm = \frac{1}{4} A_0 h_\times,$$

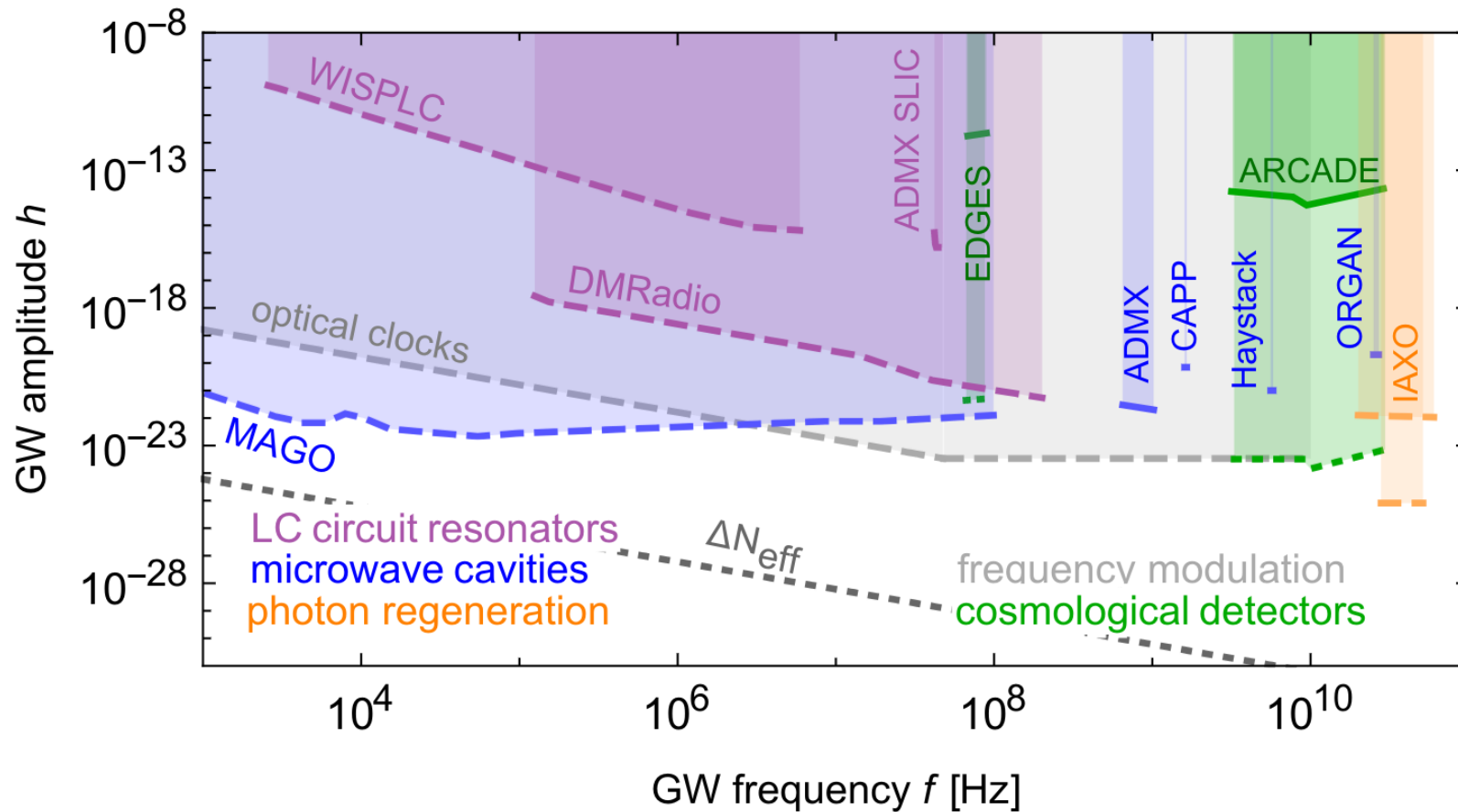
$$a_z^\pm = -A_0 h_+ \frac{\omega_\gamma}{\omega_g} (8\omega_\gamma(\omega_g \pm \omega_\gamma))^{-1} \cdot \left(2c_\theta(\omega_g^2 \pm \omega_g \omega_\gamma + \omega_\gamma^2) \pm \omega_\gamma(\omega_g \pm 2\omega_\gamma - \omega_g c_{2\theta}) \right)$$

$$= \mp \frac{1}{4} A_0 h^+ \frac{\omega_\gamma}{\omega_g} (1 + c_\theta) + \mathcal{O}[(\omega_g/\omega_\gamma)^0].$$

change in polarization

electromagnetic HF GW detectors

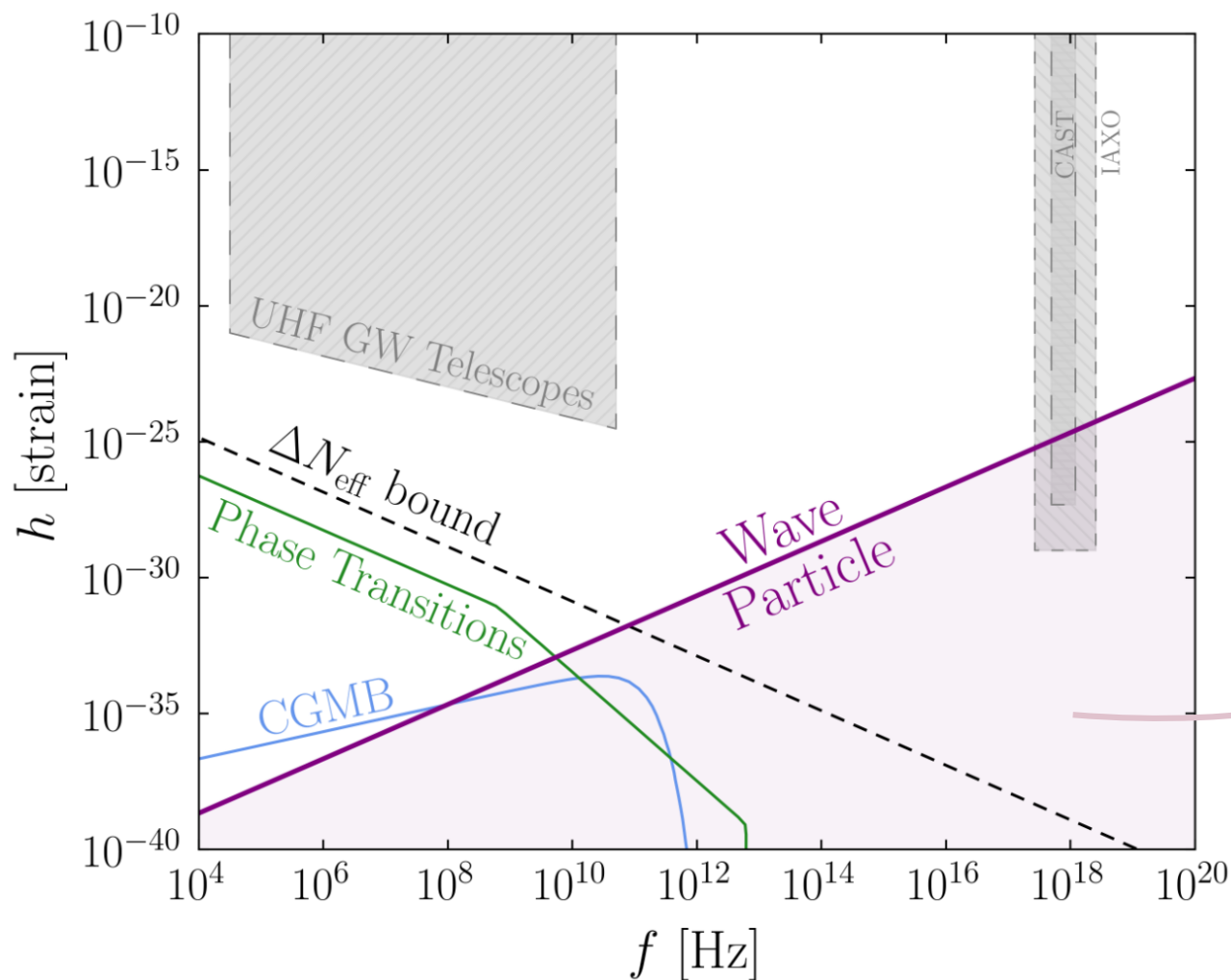
VD, Moriond proceedings '23



exploit synergies with axion searches

tests of quantum gravity ?

Carney, VD, Rodd '23



- dilute graviton gas vs classical GW
- CAST has the sensitivity to detect single gravitons (the source is the issue)
- Rigorous test of quantisation de facto impossible

see also F. Dyson '13

wave versus particle regime

energy density of GW:

$$\rho \sim h^2 \omega^2 M_{\text{pl}}^2$$

number of GW 'quanta' in de-Broglie volume:

$$n = \rho/\omega, \quad \lambda_{\text{dB}} \sim 1/\omega \quad \Rightarrow \quad n \lambda_{\text{dB}}^3 \sim h^2 M_{\text{pl}}^2 / \omega^2$$

single graviton limit:

$$N = n \lambda_{\text{dB}}^3 < 1 \quad \Rightarrow \quad h \lesssim \omega / M_{\text{pl}}$$

(at LIGO, $N \sim 10^{37} (h/10^{-22})^2$)