Test di Fisica Fondamentale con G-GranSasso

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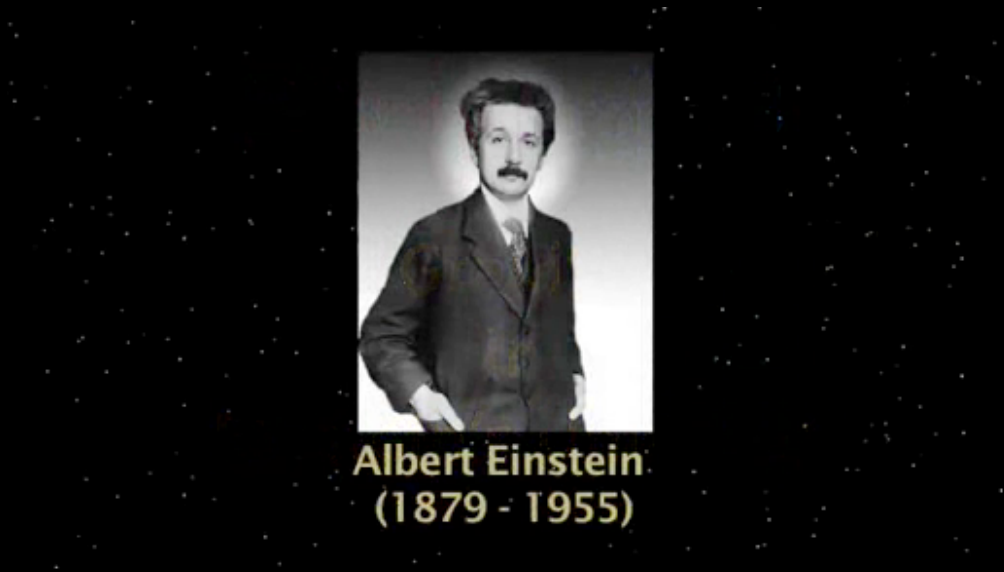
PLAN OF THE TALK

1) **GENERAL RELATIVITY AND MEASUREMENTS**
2) **LOCAL REFERENCE FRAMES**
3) **GENERAL RELATIVITY AND BEYOND**
4) **ALTERNATIVES TO GR**
5) **CONCLUSIONS**

GENERAL RELATIVITY AND MEASUREMENTS

General Relativity in a Nutshell



inertia "here" → $G_{\mu\nu} = 8\pi T_{\mu\nu}$ ← energy "everywhere"

From Covariance to Local Frames

General Covariance requires that physics laws are expressed by means of tensorial equations in a pseudo-Riemannian manifold, which is the (mathematical model of the) four-dimensional space-time.

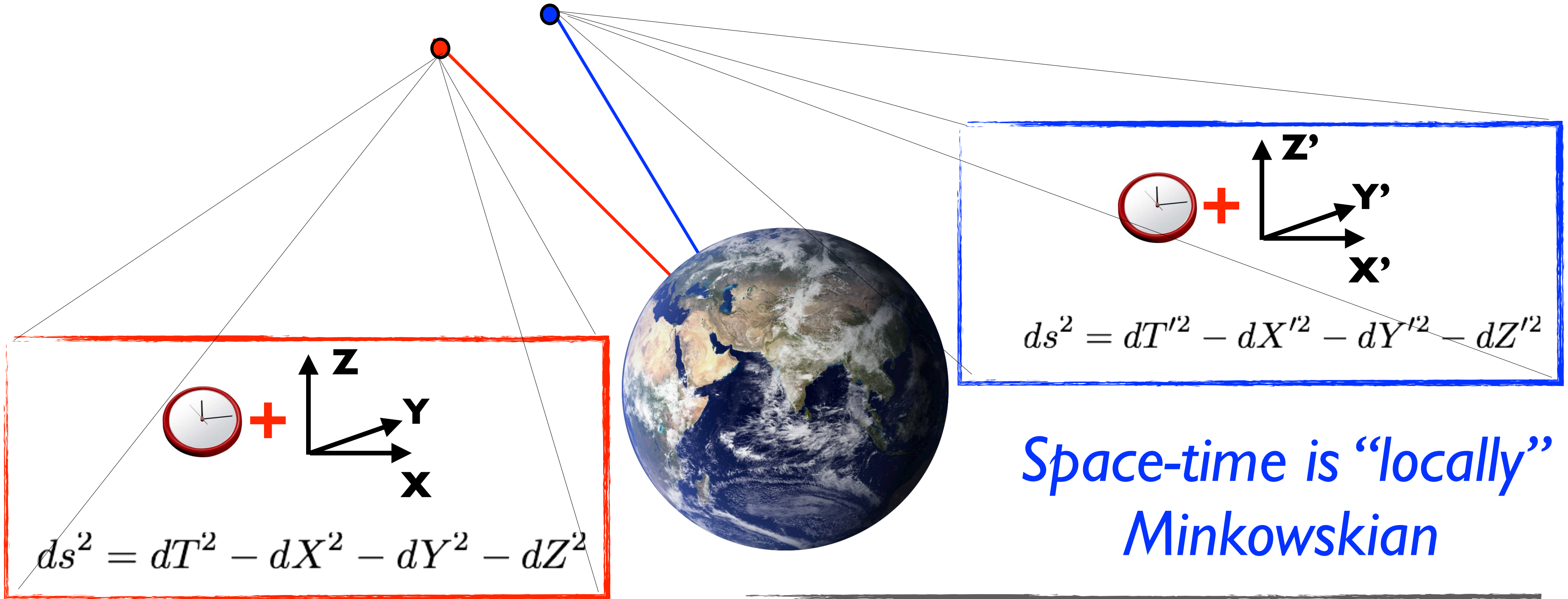
- there are no privileged frames
- within a frame, there are no privileged coordinate sets

Operational definition of measurements

According to the mathematical model underlying GR (but also "metric theories of gravity") in order to operationally define measurements performed in a laboratory we need to

- define the reference frame of the laboratory
- define the, space-time metric in this reference frame

Example: Laboratories in Free Fall



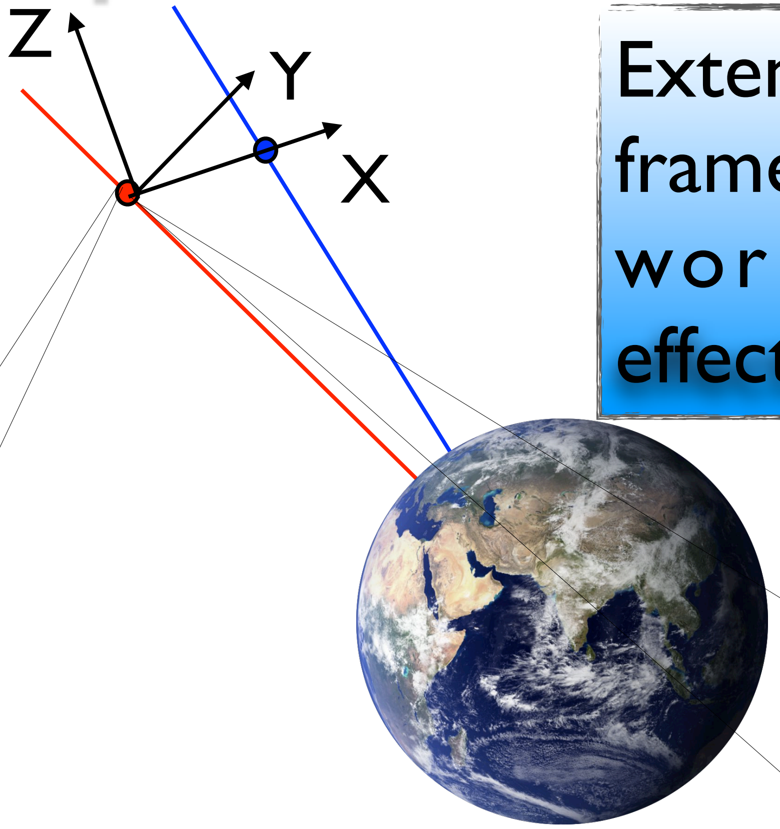
Space-time is "locally" Minkowskian

Space-time is "locally" Minkowskian

Physics is simple when analyzed locally!

GR AND MEASUREMENTS

Example: Laboratories in Free Fall



Extending the reference frame (along the geodesic worldline) curvature effects occur!

$$ds^2 = (1 + R_{0l0n} X^l X^n) dT^2 + (\frac{4}{3} R_{0ljm} X^l X^n) dT dX^J +$$
$$-(\delta_{ik} - \frac{1}{3} R_{ilkm} X^l X^m) dX^i dX^k + O(|X|^3)$$

Laboratories in arbitrary motion

To define the results of measurements in four-dimensional space-time it is necessary to focus on laboratories where measurements are carried out, i.e. on the observers that perform measurements:

- observers possess their own space-time, in the vicinity of their world-lines
- covariant physics laws are then projected onto local space and time
- predictions for the outcome of measurements in the locally Minkowskian spacetime of the observers are obtained

LOCAL REFERENCE FRAMES

Space-time metric in the laboratory

Up to linear displacements from the observer worldline the space-time metric is:

$$ds^2 = (1 + 2\mathcal{A} \cdot \boldsymbol{x})\, dt^2 - d\boldsymbol{x} \cdot d\boldsymbol{x} - 2\,(\Omega \wedge \boldsymbol{x}) \cdot d\boldsymbol{x} dt + O(|\boldsymbol{x}|^2)$$

- $\mathcal{A}$ is the spatial projection of the observer's four-acceleration $\rightarrow$ failure of free fall
- Ω is the precession rate of the local tetrad with respect to a Fermi-Walker transported tetrad $\rightarrow$ rotation of the gyroscopes wit respect to the observer's tetrad
- the observer's frame is non rotating when its axes are Fermi-Walker transported, so Ω measures the rotation rate of the frame

Minkowski spacetime iff *A=0 and Ω=0*

Gravitomagnetic field in a terrestrial laboratory

The space-time metric in a terrestrial laboratory is

$$ds^2 = (1 + 2\mathcal{A} \cdot \boldsymbol{x})\, dt^2 - d\boldsymbol{x} \cdot d\boldsymbol{x} - 2(\boldsymbol{\Omega} \wedge \boldsymbol{x}) \cdot d\boldsymbol{x} dt + O(|\boldsymbol{x}|^2)$$

$\boldsymbol{\Omega}$ is the rotation rate of the laboratory and can be measured by very accurate rotation sensors: ring lasers!

The output of the ring laser is

$$\delta f = \frac{4A}{\lambda P} \boldsymbol{\Omega} \cdot \boldsymbol{u}$$

Definition of the local rotation rate

In order to define $\boldsymbol{\Omega}$, we have to consider that

- the laboratory is fixed on the Earth surface
- the space-time of the rotating Earth can be described by the post-Newtonian metric

$$
\begin{aligned}
ds^2 &= (1 - 2U(R))dT^2 - (1 + 2\gamma U(R))\,\delta_{ij}dX^i dX^j + \\
&2\left[\frac{(1+\gamma+\alpha_1/4)}{R^3}(\boldsymbol{J}_\oplus \times \boldsymbol{R})_i - \alpha_1 U(R) W_i\right] dX^i dT,
\end{aligned}
$$

where $\gamma = 1, \alpha_1 = 0$ in GR; $U(R)$ is the gravitational potential of the Earth, $\boldsymbol{J}_\oplus$ is its angular momentum, W_i measures preferred frames effect.

LOCAL REFERENCE FRAMES

Rotation of a terrestrial laboratory

The rotation rate measured in a terrestrial laboratory is

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_{REL}$$

where $\boldsymbol{\Omega}_0$ is the terrestrial rotation rate and

$$\boldsymbol{\Omega}_{REL} = \boldsymbol{\Omega}_G + \boldsymbol{\Omega}_B + \boldsymbol{\Omega}_W + \boldsymbol{\Omega}_T$$

where

$$\begin{aligned}
\boldsymbol{\Omega}_G &= -(1+\gamma)\frac{GM}{c^2R}\sin\vartheta\Omega_0\boldsymbol{u}_\vartheta \rightarrow \text{Geodetic Precession} \\
\boldsymbol{\Omega}_B &= -\frac{1+\gamma+\alpha_1/4}{2}\frac{G}{c^2R^3}\left[\boldsymbol{J}_\oplus - 3\left(\boldsymbol{J}_\oplus\cdot\boldsymbol{u}_r\right)\boldsymbol{u}_r\right] \rightarrow \text{Lense} - \text{Thirring} \\
\boldsymbol{\Omega}_W &= -\frac{\alpha_1}{4}\frac{GM}{c^2R^2}\boldsymbol{u}_r\wedge\boldsymbol{W} \rightarrow \text{Preferred Frame Effect} \\
\boldsymbol{\Omega}_T &= -\frac{1}{2c^2}\Omega_0^2R^2\sin^2\vartheta\boldsymbol{\Omega}_0 \rightarrow \text{Thomas Precession}
\end{aligned}$$

$$\Omega_G \simeq \frac{M_\oplus}{R_\oplus}\Omega_0 \simeq 6\cdot10^{-10}\Omega_0,\;\; \Omega_B \simeq \zeta\frac{M_\oplus}{R_\oplus}\Omega_0 \simeq 6\cdot10^{-10}\,\zeta\,\Omega_0$$

Gravitomagnetic field in a terrestrial laboratory

The space-time metric in a terrestrial laboratory is

$$ds^2 = (1 + 2\mathcal{A} \cdot \boldsymbol{x})\, dt^2 - d\boldsymbol{x} \cdot d\boldsymbol{x} - 2\,(\boldsymbol{\Omega} \wedge \boldsymbol{x}) \cdot d\boldsymbol{x}dt + O(|\boldsymbol{x}|^2)$$

$$\frac{d\mathbf{p}}{dT} = -m\mathbf{A} + 2m\mathbf{v} \times \boldsymbol{\Omega} \qquad \frac{\hat{D}\widetilde{p}_i}{dT} = m\widetilde{E}_i^G + m\gamma_0 \left(\frac{\tilde{\boldsymbol{v}}}{c} \times \tilde{\boldsymbol{B}}_G\right)_i$$

Analogy

Motion in inertial fields

Motion in GEM fields

GENERAL RELATIVITY AND BEYOND

Was Einstein Right?

- General Relativity (GR) has passed with flying colors many tests in the solar system and in binary pulsar systems

- GR weak field and Newtonian gravity are accurately tested

- Post-Newtonian Parameters (metric theories of gravity) are constrained and in good agreement with GR predictions

GENARAL RELATIVITY AND BEYOND

PPN status report

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	4×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	2.3×10^{-4}	$\eta_N = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		2×10^{-4}	PSR J2317+1439
α_2	spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	4×10^{-20}	pulsar $\dot{P}$ statistics
η_N	Nordtvedt effect	9×10^{-4}	lunar laser ranging
ζ_1	–	2×10^{-2}	combined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	$\ddot{P}_p$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	Lunar acceleration
ζ_4	–	–	not independent

Constrained by ring-lasers

But... is still Einstein Right?

- Data coming from the observation of galactic rotation curves cannot be explained with Newtonian gravity or GR: dark matter is needed

- Light curves of the IaSN and CMB state that the Universe is now undergoing a phase of accelerated expansion, which cannot be accounted for in GR, unless requiring the existence dark energy (cosmic fluid with exotic properties)

- A quantum theory of gravity?

The Fall of GR...?

- The query for dark matter and dark energy perhaps suggests the failure of GR to deal with gravitational interactions on galactic, intergalactic and cosmological scales

- This led to the introduction of several modified theories of gravity which are extension of or alternative to GR

ALTERNATIVES TO GENERAL RELATIVITY

ALTERNATIVES TO GR

Scalar-Tensor Theories

$$S_{ST} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}[F(\varphi)R - Z(\varphi)g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi - 2V(\varphi)] + S_m$$

- Tensor ($g_{\mu\nu}$) + Scalar field (φ)
- Hints: inflation, strings, higher order...

Negligible mass of the scalar field $m_\varphi << m_S$

$$\gamma(\omega) = \frac{1+\omega}{2+\omega} \quad \gamma_{obs} - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

$$\omega > 10^4$$

Very massive scalar field $m_\varphi >> m_S$

dynamics of the scalar field is frozen on the given scale

ALTERNATIVES TO GR

Scalar-Tensor Theories

$$S_{ST} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} [F(\varphi) R - Z(\varphi) g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi - 2V(\varphi)] + S_m$$

- Tensor ($g_{\mu\nu}$) + Scalar field (φ)
- Hints: inflation, strings, higher order...

Intermediate case

$$\gamma = \frac{3 + 2\omega - e^{-m_\varphi / m_S}}{3 + 2\omega + e^{-m_\varphi / m_S}}$$

Measuring γ allows to set contraints on ω at the given scale

Olmo 2005
Perivolaraoupolos 2009
Capone, MLR 2010

ALTERNATIVES TO GR

f(R) Theories

$$S_{HO} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,f(R) + S_m$$

- Higher order theories (richer geometry!)
- Hints: speculation but also...inflation, cosmology

Analogy with scalar-tensor gravity → $$m^2_{HO} = \left[\frac{f' - Rf''}{3f''}\right]$$

Solutions of the *tricky* field equations → $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$U = U_N + \delta U$$

perturbation of the Newtonian potential

f(R) Theories

Some perturbations of the Newtonian potential

$$U = -\frac{GM}{r}\left[1 + \left(\frac{r}{r_c}\right)^{\beta}\right]$$

$$U = -\frac{GM}{r} + \frac{kr^2}{6}$$

$$U = -\frac{GM}{r} + \frac{\alpha}{2} + \frac{\alpha}{2}\ln\frac{r}{2GM}$$

$$U = -\frac{GM}{r}\left[1 + \alpha e^{-\frac{r}{\lambda}}\right]$$

ALTERNATIVES TO GR

Arbitrary Spherically Symmetric Perturbations

$$ds^2 = (1+\phi(r))\,dt^2 - (1+\psi(r))\left(dr^2 + r^2 d\Omega^2\right)$$

where

$$\begin{aligned}\phi(r) &= 2U_N + 2\delta U \\ \psi(r) &= -2U_N + \delta\psi_A(r)\end{aligned}$$

Power Law Perturbations

$$\delta U = \frac{\alpha}{r^{|n|}}$$

$$\delta U = \beta r^{|m|}$$

MLR 2010

Arbitrary Spherically Symmetric Perturbations

Starting from the background metric

$$ds^2 = (1 - 2U_N)dT^2 - (1 + 2\gamma U_N)\,\delta_{ij}dX^i dX^j + \quad 2\left[\frac{(1+\gamma+\alpha_1/4)}{R^3}(J_\oplus \wedge R)_i - \alpha_1 U(R) W_i\right] dX^i dT$$

f.i. we obtain the "geodetic precession"

$$\Omega_G \quad = \quad -(1+\gamma)\,\nabla U_N \wedge V$$

$$ds^2 = (1 - 2U)dT^2 - (1 + 2\gamma U)\,\delta_{ij}dX^i dX^j + \quad 2\left[\frac{(1+\gamma+\alpha_1/4)}{R^3}(J_\oplus \wedge R)_i - \alpha_1 U W_i\right] dX^i dT$$

$$U = U_N + \delta U$$

perturbations of GM terms

$$\Omega_G \rightarrow \Omega_G + \delta\Omega_G$$

Mansouri - Sexl Test Theory

- Test theory of special relativity
- Role of simultaneity and syncrhonziation between frames
- Existence of a privleged frame ("ether frame", e.g. SSB, CMB...) where light propagate isotropically
- Test of anisotropy of light propagation

Generalization of Lorentz transformations

$$t = aT + \vec{\epsilon}\vec{\mathbf{x}}$$

$$\mathbf{x} = d\mathbf{X} + \frac{b - d}{v^2}\mathbf{v}(\mathbf{v}\mathbf{x}) - b\mathbf{v}T$$

ALTERNATIVES TO GR

Mansouri - Sexl Test Theory

Anisotropy of light propagation

$$c(\theta) = \frac{b(1 - v^2)}{a\,[\cos^2\theta + b^2 d^2 (1 - v^2)\sin^2\theta]^{1/2}}$$

Use an accurate sensor (ring laser) to test anisotropy of light propagation w.r.t. some "inertial frame" and set constraints on the theory parameters

ALTERNATIVES TO GR

Standard Model Extension (SME)

- More fundamental theories required to include a quantum description of the gravitational field

- Investigation of the ultimate structure of space and time, and interplay with matter

- Breaking of CPT and Lorentz symmetries?

$$\mathcal{L}_{\rm SME} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm gr} + \delta\mathcal{L}$$

$$S_{\rm EH} = \frac{1}{2\kappa}\int d^4x e(R - 2\Lambda)$$

$$S_{\rm LV} = \frac{1}{2\kappa}\int d^4x e(-uR + s^{\mu\nu}R^T_{\mu\nu} + t^{\kappa\lambda\mu\nu}C_{\kappa\lambda\mu\nu})$$

ALTERNATIVES TO GR

Standard Model Extension (SME)

- Laboratory Experiments
- Space and Solar System Experiments

$$\frac{d\mathbf{S}}{dt} = \Omega \times \mathbf{S}$$

gyro precession rate

$$\delta f = \frac{4A}{\Lambda P}\mathbf{\Omega} \cdot \mathbf{u}$$

ring laser output

$$\Omega^J = \Omega^J_E + \Omega^J_s$$

GR (Ω^J_E)

SME (Ω^J_s)

Bailey and Kostelecky 2006

ROTATING OBSERVERS

Standard Model Extension (SME)

$$\Omega^J = \Omega_E^J + \Omega_{\bar{s}}^J$$

GR ← Ω_E^J $\Omega_{\bar{s}}^J$ → **SME**

where

$$\Omega_{\bar{s}}^J = \tfrac{9}{8}\left(\tilde{i}_{(-1/3)}\bar{s}^{TT} - \tilde{i}_{(-5/3)}\bar{s}^{KL}\hat{\sigma}^K\hat{\sigma}^L\right)\hat{\sigma}^J + \tfrac{5}{4}\tilde{i}_{(-3/5)}\bar{s}^{JK}\hat{\sigma}^K .$$

- corrections parallels to terrestrial rotation rate
- corrections to the geodetic term

Bailey and Kostelecky 2006

CONCLUSIONS

FUNDAMENTAL PHYSICS AND G-GRANSASSO

What we aim at doing with G-GranSasso?

☑ Testing GR and metric theories of gravity in the laboratory

☑ Constraining PPN parameters

What we *could aim* at doing with G-GranSasso?

☐ *Use the ring laser as an accurate sensor to set constraints on theories/models that go beyond the PPN scheme in a terrestrial environment*