Axion Potentials and Moduli Stabilization
in
Realistic Heterotic M-Theory

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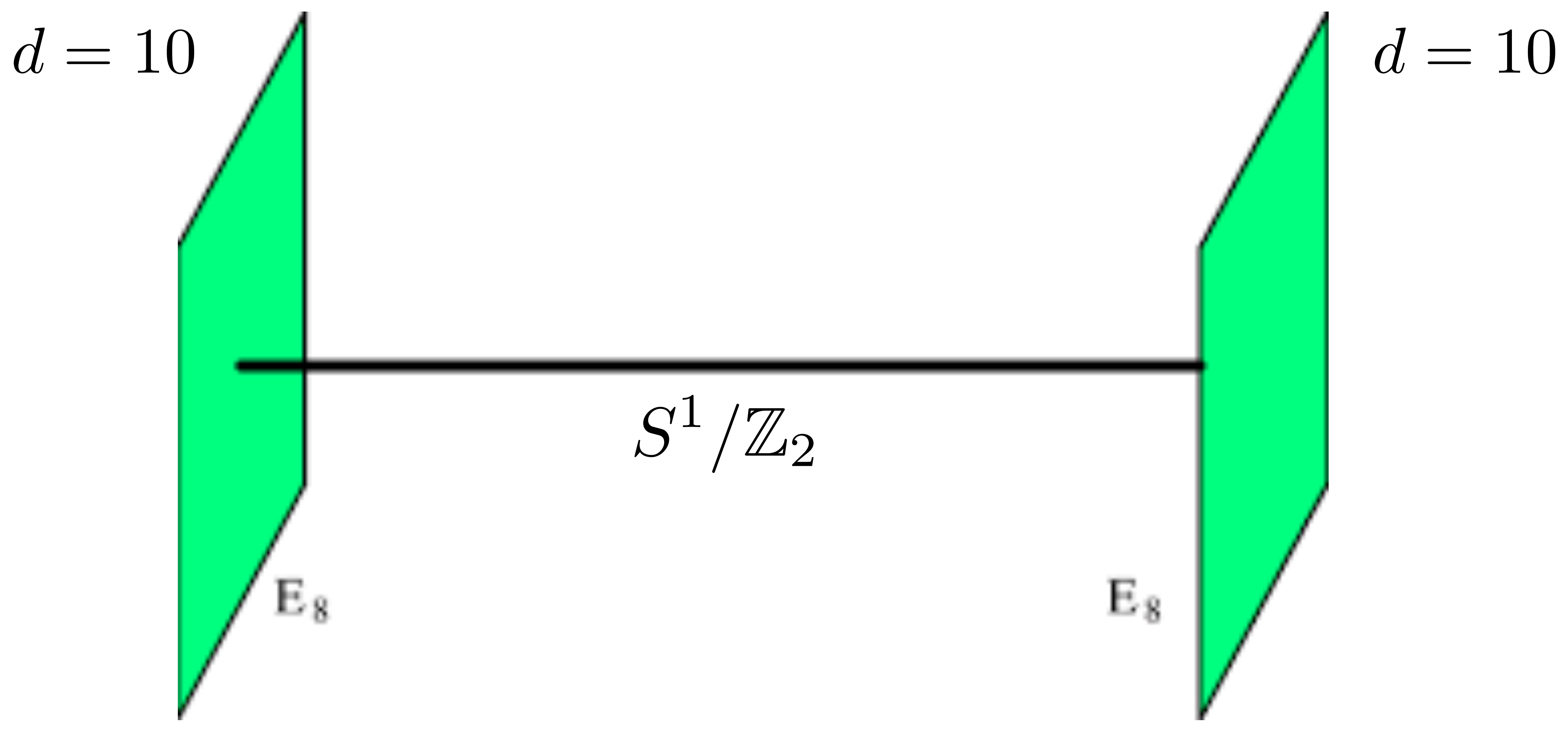
Brief Review of a Realistic Heterotic M-Theory Vacuum

<u>Horava-Witten Theory</u> :

d=11 M-Theory compactified on an $S^1/\mathbb{Z}_2$ orbifold $\Longrightarrow$ an N=1 E_8 supermultiplet on each of the two d=10 orbifold fixed planes

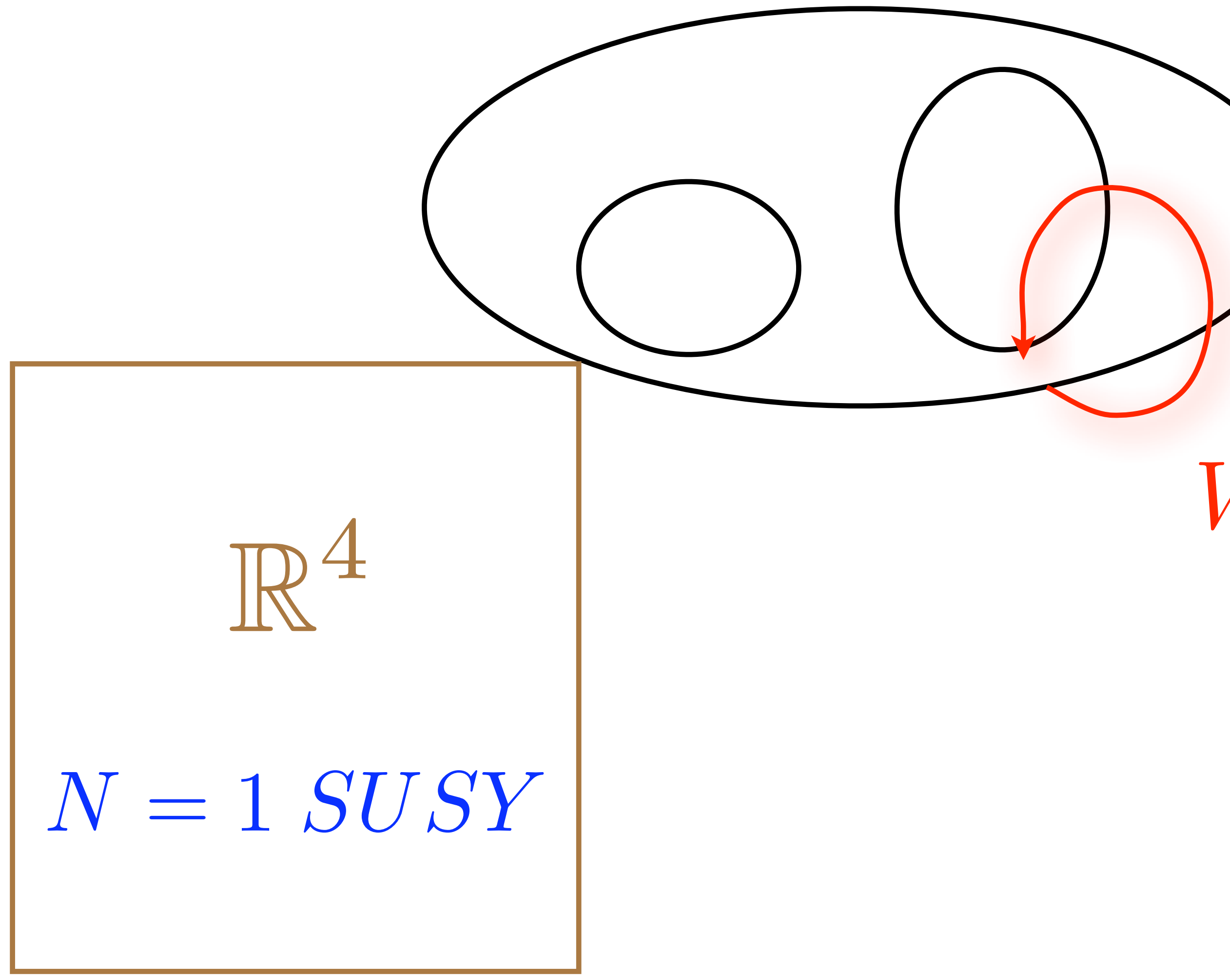


observable sector

hidden sector

Observable Sector: SU(4) Heterotic Compactification:

$X, d = 6$ "Schoen" CY $\quad$ $h^{1,1} = 3 \Rightarrow$ three Kähler moduli T^i



$V\ , G = SU(4)$ "slope" stable holomorphic vector bundle

$W\ , F = \mathbb{Z}_3 \times \mathbb{Z}_3$ homotopy

$\mathbb{R}^4$ Theory Gauge Group:

$$G = SU(4) \Rightarrow \quad E_8 \longrightarrow \ Spin(10)$$

Choose the $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines to be

$$\chi_{T_{3R}} = e^{iY_{T_{3R}} \frac{2\pi}{3}} \ , \quad \chi_{B-L} = e^{iY_{B-L} \frac{2\pi}{3}}$$

where the generators Y_{B-L} and $Y_{T_{3R}}$ arise "naturally" and is called the "canonical basis". $\Rightarrow$

$$Spin(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$$

$\mathbb{R}^4$ Theory Spectrum:

$n_r = (h^1(X, U_R(V)) \otimes \mathbf{R})^{\mathbb{Z}_3 \times \mathbb{Z}_3} \Longrightarrow$ 3 families of quarks/leptons

$$Q = (U, D)^T = (\mathbf{3}, \mathbf{2}, 0, \frac{1}{3}), \quad u = (\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{2}, -\frac{1}{3}), \quad d = (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{2}, -\frac{1}{3})$$
$$L = (N, E)^T = (\mathbf{1}, \mathbf{2}, 0, -1), \quad \nu = (\mathbf{1}, \mathbf{1}, -\frac{1}{2}, 1), \quad e = (\mathbf{1}, \mathbf{1}, \frac{1}{2}, 1)$$

and 1 pair of Higgs-Higgs conjugate fields

$$H = (\mathbf{1}, \mathbf{2}, \frac{1}{2}, 0), \quad \bar{H} = (\mathbf{1}, \mathbf{2}, -\frac{1}{2}, 0)$$

under $SU(3)_C \times SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$.

We refer to this theory as the B-L MSSM.

Wilson Line Breaking:

$\pi_1\big(X/(\mathbb{Z}_3 \times \mathbb{Z}_3)\big) = \mathbb{Z}_3 \times \mathbb{Z}_3 \Longrightarrow$ 2 independent classes of non-contractible curves. $\Longrightarrow$ each Wilson line has a mass scale $M_{\chi_{T_{3R}}}$, $M_{\chi_{B-L}}$

At a generic region of moduli space $M_{\chi_{T_{3R}}} \simeq M_{\chi_{B-L}} (\simeq M_U)$

which we henceforth assume. We find that

$$M_U = 3.15 \times 10^{16} \text{ GeV}$$

Soft Supersymmetry Breaking:

At this scale, we statistically scatter the 24 soft supersymmetry parameters in the range $(\frac{M}{f}, Mf)$ where, to make all sparticle masses CERN accessible, we choose $M = 2.7\ TeV$, $f = 3.3$.

The RG scaling results are subjected to all present phenomenological constraints-- namely

A) B-L symmetry is radiatively broken at $M_{B-L} > 2.5$ TeV

B) EW symmetry is radiatively broken at $M_Z = 91.2$ GeV

C) The Higgs mass is given by $M_{H^0} = 125.36 \pm 0.82$ GeV

In addition, we will enforce that all sparticle masses exceed their present experimental bounds. These are given by

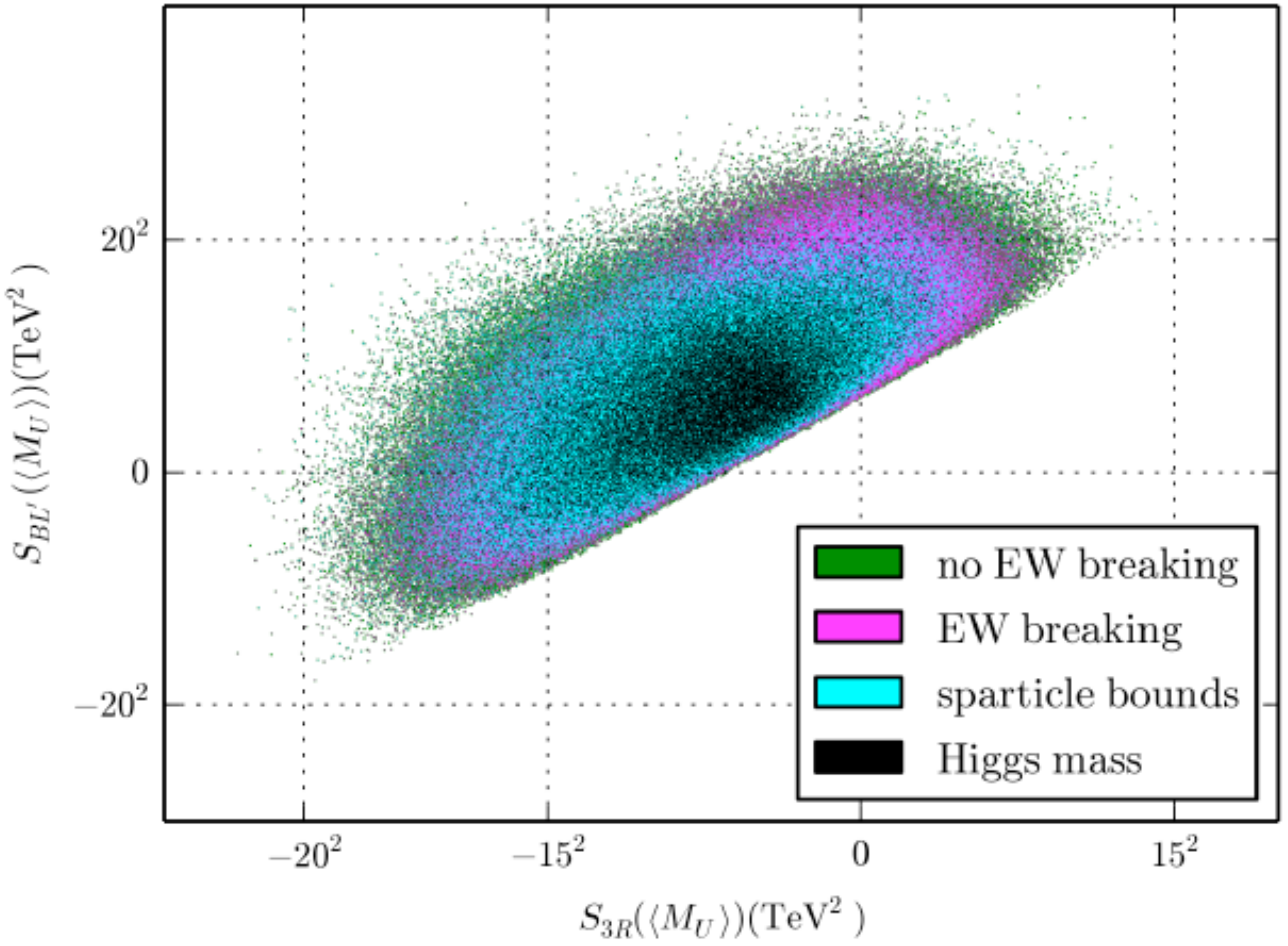
Particle(s)	Lower Bound
Left-handed sneutrinos	45.6 GeV
Charginos, sleptons	100 GeV
Squarks, except for stop or sbottom LSP's	1000 GeV
Stop LSP (admixture)	450 GeV
Stop LSP (right-handed)	400 GeV
Sbottom LSP	500 GeV
Gluino	1300 GeV
Z_R	2500 GeV

We find that most of the RGE scaling behaviour is dominated by the two parameters

$$S_{BL'} = \text{Tr}\left(2m_{\tilde{Q}}^2 - m_{\tilde{u}^c}^2 - m_{\tilde{d}^c}^2 - 2m_{\tilde{L}}^2 + m_{\tilde{\nu}^c}^2 + m_{\tilde{e}^c}^2\right),$$

$$S_{3R} = m_{H_u}^2 - m_{H_d}^2 + \text{Tr}\left(-\frac{3}{2}m_{\tilde{u}^c}^2 + \frac{3}{2}m_{\tilde{d}^c}^2 - \frac{1}{2}m_{\tilde{\nu}^c}^2 + \frac{1}{2}m_{\tilde{e}^c}^2\right)$$

$\Longrightarrow$ we can plot our results in a two-dimensional space. We find that out of 10 million random initial points in SUSY breaking parameter space, all points that break B-L symmetry with $M_{B-L} > 2.5\ TeV$ are



Of these, there are 44,884 "valid" black points that satisfy all phenomenological requirements.

How does the B-L MSSM create those 24 soft SUSY breaking parameters?

Hidden Sector: For simplicity, we will assume the hidden sector vector bundle consists of a single holomorphic line bundle given by

$$\mathcal{L} = \mathcal{O}_X(2,1,3)$$

extended to $\mathcal{V} = \mathcal{L} \oplus \mathcal{L}^{-1}$ so as to embed its U(1) structure group as (1,-1) into the SU(2) of

$$SU(2) \times E_7 \subset E_8$$

It follows that the low energy gauge group is

$$G = U(1) \times E_7$$

and with respect to this group

$$\underline{\mathbf{248}} \rightarrow (0, \underline{\mathbf{133}}) \oplus \big((1, \underline{\mathbf{56}}) \oplus (-1, \underline{\mathbf{56}})\big) \oplus \big((2, \underline{\mathbf{1}}) \oplus (0, \underline{\mathbf{1}}) \oplus (-2, \underline{\mathbf{1}})\big)$$

One can explicitly compute the associated low energy spectrum using the Euler characteristic.

The result is

	$U(1)\times E_7$	Cohomology	Index χ
	$(0,\underline{\mathbf{133}})$	$H^\bullet(X,\mathcal{O}_X)$	0
	$(0,\underline{\mathbf{1}})$	$H^\bullet(X,\mathcal{O}_X)$	0
	~~$(-1,\underline{\mathbf{56}})$~~	~~$H^\bullet(X,L)$~~	~~8~~
Left chiral supermultiplets	$(1,\underline{\mathbf{56}})$	$H^\bullet(X,L^{-1})$	-8
	~~$(-2,\underline{\mathbf{1}})$~~	~~$H^\bullet(X,L^2)$~~	~~58~~
Left chiral supermultiplets	$(2,\underline{\mathbf{1}})$	$H^\bullet(X,L^{-2})$	-58

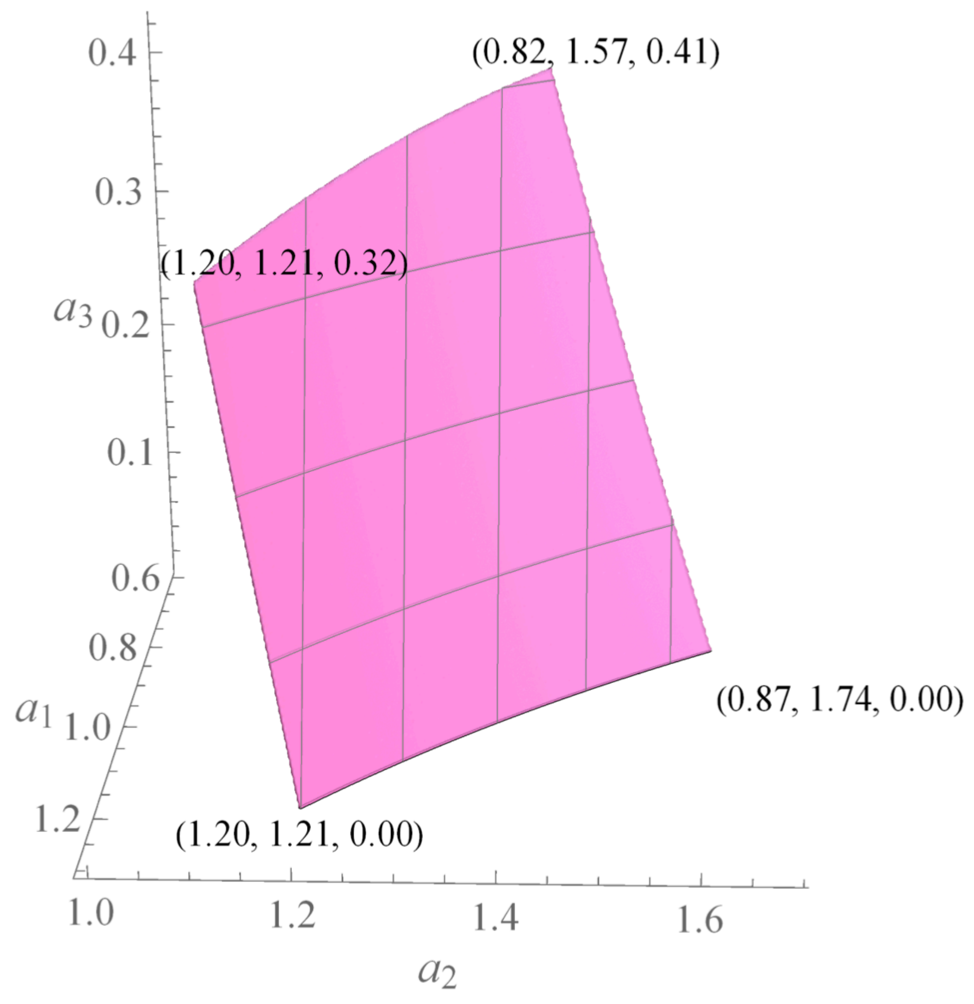
Since the left chiral fields all have positive U(1) charges, it follows that the Abelian U(1) gauge group is anomalous. This is cancelled by the Green-Schwartz mechanism which generates an inhomogeneous shift in the imaginary part of the dilaton and each Kahler modulus given by

$$\delta_\theta S = -2i\pi a \epsilon_S^2 \epsilon_R^2 \tfrac{1}{2}\beta_i^{(2)} l^i \theta \equiv k_S \theta,$$

$$\delta_\theta T^i = -2ia\epsilon_S \epsilon_R^2 l^i \theta \equiv k_T^i \theta \text{ , i=1,2,3}$$

$\Rightarrow$ the imaginary part of the dilaton and of each Kahler modulus transforms like an "axion".

It can be shown that, for the hidden sector bundle $\mathcal{L} = \mathcal{O}_X(2,1,3)$, all physical and mathematical constraints for the above explicit observable sector and the hidden sector (such as slope stable SU(4) bundle, polystable $\mathcal{L} \oplus \mathcal{L}^{-1}$ bundle, ...) can be satisfied for Kahler moduli in the "magenta" region given by



Gaugino Condensation: Condensation of the hidden sector E_7 gauginos produces a non-perturbative superpotential

$$W_{gc} = M_U^3 exp\left(- \frac{6\pi}{b_L \hat{\alpha}_{GUT}} \mathrm{S} \right)$$

where for the above matter spectrum

$$b_L = 6$$

This superpotential spontaneously breaks N=1 supersymmetry and produces the 24 soft SUSY breaking terms in the observable sector. For the Kahler moduli in the "magenta" region, randomly scattering remaining parameters, we reproduce most of the "valid" black points shown above. That is, this B-L MSSM theory is physically realistic.

Stabilizing Moduli and Axion Potentials:

Henceforth, for simplicity, we will consider only the "universal" breathing modulus T and ignore the other two Kahler moduli. That is, we consider the complex fields S and T only. The associated Kahler potentials are

$$K_S = -\kappa_4^{-2} \ln(S + \bar{S}) \; ,$$
$$K_T = -3\kappa_4^{-2} \ln(T + \bar{T})$$

The above inhomogeneous transformations become

$$\delta_\theta S = 2i\pi a \epsilon_S^2 \epsilon_R^2 \beta l \theta \; \equiv k_S \theta$$
$$\delta_\theta T = -2ia\epsilon_S \epsilon_R^2 l\theta \; \equiv k_T \theta$$

The Green-Schwartz mechanism also generates a mass for the U(1) gauge boson given by

$$m_{\rm anom} = \sqrt{2\langle g_2^2 \Sigma^2 \rangle}$$

where

$$g_2^2 = \frac{\pi \hat{\alpha}_{\rm GUT}}{\text{ReS}} \quad \text{and} \quad \Sigma^2 = g_{S\bar{S}} k_S \bar{k}_S + g_{T\bar{T}} k_T \bar{k}_T$$

D-Term Potential: Ignoring the homogenous transformations on hidden matter, the inhomogeneous U(1) transformations on S and T lead to a D-term potential energy given by

$$V_D = \frac{1}{2ReS} \mathcal{P}^2$$

where

$$\mathcal{P} = i k_S \partial_S K + i k_T \partial_T K = -\frac{a \epsilon_S \epsilon_R^2}{\kappa_4^2} \left(-\frac{1}{s} \pi \beta \epsilon_S l + \frac{3l}{t} \right)$$

with $ReS = s$ and $ReT = t$. Expanding s and t around their vevs leads to the FI term

$$\text{FI} = -\frac{a \epsilon_S \epsilon_R^2}{\kappa_4^2} \left(-\frac{1}{\langle s \rangle} \pi \beta \epsilon_S l + \frac{3l}{\langle t \rangle} \right)$$

To preserve N=1 supersymmetry, one must set FI=0 $\Longrightarrow$

Expanding

$$\langle s \rangle = \frac{\pi \epsilon_S \beta}{3} \langle t \rangle = .230 F^{4/3} \beta < t >$$

$$S =< s > + \delta S, \; T =< t > + \delta T$$

one finds that the Lagrangian for $\delta S, \delta T$ has off-diagonal kinetic energy and mass terms.

However, defining two complex scalar fields ξ^1, ξ^2 by

$$\begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} = \mathbf{U} \begin{pmatrix} \delta S \\ \delta T \end{pmatrix} , \quad \mathbf{U} = \frac{1}{\langle \Sigma \rangle} \begin{pmatrix} \langle g_{S\bar{S}} \bar{k}_S \rangle & \langle g_{T\bar{T}} \bar{k}_T \rangle \\ \sqrt{\langle g_{S\bar{S}} g_{T\bar{T}} \rangle} \langle \bar{k}_T \rangle & -\sqrt{\langle g_{S\bar{S}} g_{T\bar{T}} \rangle} \langle \bar{k}_S \rangle \end{pmatrix}$$

diagonalizes both the kinetic energy and mass terms. Specifically, we find

$$m_{\xi^1} = \sqrt{2 \langle g_2^2 \Sigma^2 \rangle} = m_{\text{anom}} \quad , \; m_{\xi^2} = 0$$

That is

$$\mathcal{L} \supset -\partial^\mu \bar{\xi}^1 \partial_\mu \xi^1 - \partial^\mu \bar{\xi}^2 \partial_\mu \xi^2 - m_{\text{anom}}^2 \xi^1 \bar{\xi}^1$$

Plugging in the B-L MSSM expressions values for ϵ_S, ϵ_R , we find that

$$m_{\rm anom} = (\frac{3.39l}{F\beta^{1/2}})\frac{M_U}{<t>^{3/2}}$$

Therefore, for

$$<t> \lesssim (\frac{3.39l}{F\beta^{1/2}})^{2/3} \Rightarrow m_{\rm anom} \gtrsim M_U$$

both the U(1) vector boson and ξ^1 can be integrated out of the low energy effective theory. We do this henceforth.

Inverting the previous field redefinition

$$\begin{pmatrix} \delta S \\ \delta T \end{pmatrix} = \mathbf{U}^{-1} \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix}$$

we find that U^{-1} in the B-L MSSM is given by

$$U^{-1} = \frac{i<t>}{M_P} \begin{pmatrix} \mathbf{2.00F^{4/3}}\beta & -\mathbf{1.15F^{4/3}}\beta \\ -2.880 & -5.007 \end{pmatrix}$$

Writing

$$\delta S = \delta s + i\sigma \ , \ \delta T = \delta t + i\chi$$

defining

$$\frac{\xi^2}{M_P} = \tilde{\eta} + i\tilde{\phi}$$

it follows that

$$s = < s > + < t > 1.12 F^{4/3} \beta \tilde{\phi} \quad , \quad t = < t > + < t > 5.00 \tilde{\phi}$$

$$\sigma = - < t > 1.15 F^{4/3} \beta \tilde{\eta} \quad , \quad \chi = - < t > 5.00 \tilde{\eta}$$

F-Term Potential: The same sign charges of the matter chiral superfields disallow a perturbative superpotential in the hidden sector. However, non-perturbative superpotentials can occur for the complex structure moduli, the dilaton and the Kahler moduli.

(1) Complex Structure: There is a flux induced superpotential, $W_{\rm flux}$, for the complex structure moduli. Suffice it here to say that one can solve this for local minima that do not break supersymmetry. These depend on three real parameters A,B and $< c >$.

(2) Dilaton: As already mentioned, condensation of the gauginos associated with E_7 induces a non-perturbative superpotential W_{gc} in the hidden sector given by

$$W_{gc} = M_U^3 exp\left(-\frac{6\pi}{b_L \hat{\alpha}_{GUT}}\mathrm{S}\right)$$

(3) Kahler Modulus: String worldsheets wrapping isolated holomorphic curves can produce a non-perturbative "instanton" superpotential, W_I. Summing over the isolated curves in the universal cohomology class $\mathcal{C}$ associated with T, this has the form

$$W_I = \mathcal{P}e^{-\tau T} \quad \text{where} \quad \mathcal{P} = \sum_{i=1}^{n[\mathcal{C}]} \mathcal{P}_i \quad \longleftarrow \text{Gromov} - \text{Witten invariant}$$

For the B-L MSSM

$$\tau = 61.618\frac{v_{\mathcal{C}}}{v^{1/3}} \quad , \quad \mathcal{P} = pe^{i\theta_p}$$

"Reasonable" values for n and $\mathcal{P}$ are

$$10^{-2} \lesssim \frac{v_C}{v^{1/3}} \lesssim 1 \quad , \quad 3.368 \lesssim p \lesssim 3.368 \times 10^2$$

Note: the Beasley-Witten theorem does not apply because of the $\mathbb{Z}_3 \times \mathbb{Z}_3$ isometry. Using these non-perturbative superpotentials, one can compute the F-term potential energy V_F. Assuming , $FI = 0$ we find

$$
\begin{aligned}
V_F(\langle t\rangle,\tilde{\eta},\tilde{\phi}) =& \frac{M_U^4}{F^{4/3}\beta\langle t\rangle^4\langle c\rangle^3(0.230+1.15\tilde{\phi})(1+5.00\tilde{\phi})^3} \\
&\times \Big[1.138\, F^{-4/3}\tilde{d}(A^2+B^2) \\
&+1.32\times 10^{-6}\tilde{d}^{-1}\big((1+19.0F^{2/3}\beta\langle t\rangle(1+5.01\tilde{\phi})^2+3\big) \\
&\quad \times \exp[-19.0\, F^{2/3}\beta\langle t\rangle(1+5.01\,\tilde{\phi})] \\
&-(2.43\times 10^{-3}F^{-2/3})\,\big(1+19.0\, F^{2/3}\beta\langle t\rangle(1+5.01\,\tilde{\phi})\big) \\
&\quad \times \exp[-9.48\, F^{2/3}\beta\langle t\rangle(1+5.00\,\tilde{\phi})] \\
&\quad \times sgn(A)\sqrt{A^2+B^2}\cos[47.5\, F^{2/3}\beta\langle t\rangle\tilde{\eta} - \arctan(\frac{B}{A})] \\
&+2.62\times 10^{-6}\tilde{d}^{-1}\,p\,\big(5.50+\langle t\rangle(19.0\, F^{2/3}\beta(1+5.01\,\tilde{\phi})+3\tau(1+5.00\,\tilde{\phi}))\big) \\
&\quad \times \exp[-(9.49\, F^{2/3}\beta(1+5.01\,\tilde{\phi})+\tau(1+5.00\,\tilde{\phi}))\langle t\rangle] \\
&\quad \times \cos[(-47.5\, F^{2/3}\beta+5.00\,\tau)\langle t\rangle\tilde{\eta}+\theta_p] \\
&+4.36\times 10^{-7}\tilde{d}^{-1}p^2\big(3+(3+2\,\tau\langle t\rangle(1+5.005\,\tilde{\phi}))^2\big) \\
&\quad \times \exp[-2\tau\langle t\rangle(1+5.00\,\tilde{\phi})] \\
&-2.43\times 10^{-3}\, F^{-2/3}\, p\,(1+2\tau\langle t\rangle(1+5.00\,\tilde{\phi})) \\
&\quad \times \exp[-\tau\langle t\rangle(1+5.00\,\tilde{\phi})] \\
&\quad \times sgn(A)\sqrt{A^2+B^2}\cos[5.00\,\tau\langle t\rangle\tilde{\eta}+\theta_p-\arctan(\frac{B}{A})]\Big]
\end{aligned}
$$

This complicated expression can be simplified by recognizing the following.

1) It follows from

$$s = <s> + <t> 1.12 F^{4/3} \beta \tilde{\phi} \quad , \quad t = <t> + <t> 5.00 \tilde{\phi}$$

that a non-zero value of $\tilde{\phi}$ simply raises the value of V_D above zero. Hence, we can take

$$<\tilde{\phi}> = 0$$

2) For the physical values of F, β, τ in the B-L MSSM, the exponential prefactors in the second, third and fourth terms in V_F are strongly exponentially supressed relative to the first, fifth and sixth terms. Hence, we can

ignore these three suppressed terms

V_F then simplifies to

$$V_F(< t >, \tilde{\eta}) = \frac{M_U^4}{0.230 F^{4/3} \beta\langle t\rangle^4 \langle c\rangle^3} \{1.14\, F^{-4/3}(A^2 + B^2)$$

$$+ 4.36 \times 10^{-7} p^2 \left(3 + (3 + 2\,\tau\langle t\rangle)^2\right) \times \exp[-2\tau\langle t\rangle]$$

$$-2.43 \times 10^{-3}\, p\, F^{-2/3} \sqrt{A^2 + B^2}\, (1 + 2\tau\langle t\rangle) \times \exp[-\tau\langle t\rangle]\; sgn(A)\; \cos[5.00\,\tau\langle t\rangle \tilde{\eta} + \theta_p - \arctan(\frac{B}{A})]\} \leftarrow$$

3) Note that if we choose

$$sgn(A) > 0$$

the minus sign in the last term ensures V_F will be minimized by choosing the

$$cos = +1$$

It follows that

$$< \tilde{\eta} >= \frac{2\pi n + arctan(B/A) - \theta_p}{5.00\tau < t >} \quad , \quad n \in \mathbb{Z}$$

Using this, V_F then simplifies to

$$V_F(< t >) = \frac{M_U^4}{0.230 F^{4/3} \beta \langle t \rangle^4 \langle c \rangle^3} \{1.14\, F^{-4/3}(A^2 + B^2)$$

$$+\, 4.36 \times 10^{-7} p^2 \left(3 + (3 + 2\,\tau\langle t \rangle)^2\right) \times \exp[-2\tau\langle t \rangle]$$

$$-2.43 \times 10^{-3}\, p\, F^{-2/3} \sqrt{A^2 + B^2}\, (1 + 2\tau\langle t \rangle) \times \exp[-\tau\langle t \rangle] \}$$

Although they do not affect the existence or locations of extrema, the parameters A, B, β, $\langle c \rangle$, F do affect details like the magnitude of V_F at an extremum and the masses of $\tilde{\tilde{\phi}}$ and $\tilde{\eta}$. We take, without loss of generality

$$A = 1/3, B = A/\sqrt{3}, < c >= 1/\sqrt{3}, F = 1.5$$

which can be shown to give a complex structure vacuum state that preserves N=1 supersymmetry. Additionally, the B-L MSSM vacuum requires one to choose

$$l = 1 \quad \text{and} \quad \beta = 6.42$$

Recall that

$$< t > \lesssim \left(\frac{3.39 l}{F \beta^{1/2}}\right)^{2/3} \Rightarrow m_{\rm anom} \gtrsim M_U$$

$\Longrightarrow$ in this example

$$< t > \lesssim < t >_{bound} = 0.925$$

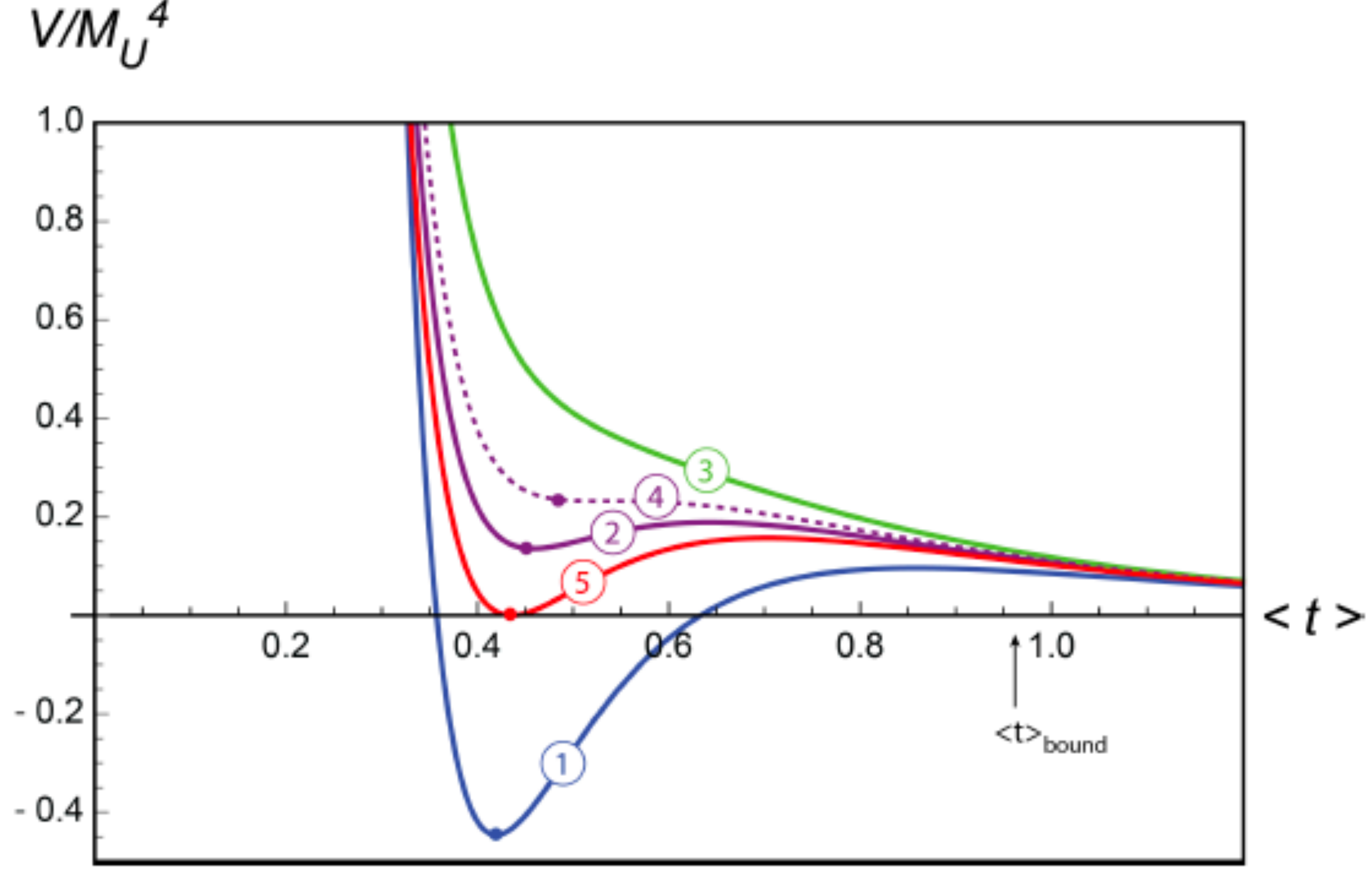
Having chosen these parameters, $V_F(< t >)$ still is a function of τ and p.
Choosing, as an example,

$$\tau = 3$$

and five different values for p given by

$$p = (\; 250 \; , \; 200 \; , \; 175 \; , \; 192 \; , \; 210 \;)$$

we find that



The values of $m_{\rm anom}$ and $m_{\tilde{\phi}}$, $m_{\tilde{\eta}}$ at the minima of the blue red and purple curves are found to be

	$m_{\rm anom}$	m_ϕ	m_η
blue ($V_{\rm min} < 0$)	1.0×10^{17} GeV	1.7×10^{15} GeV	1.7×10^{15} GeV
red ($V_{\rm min} = 0$)	9.7×10^{16} GeV	1.2×10^{15} GeV	1.5×10^{15} GeV
purple ($V_{\rm min} > 0$)	9.0×10^{16} GeV	9.2×10^{14} GeV	1.4×10^{15} GeV

It is important to note that the blue curve (and most curves for a wide range of p) have a negative cosmological constant. However, as exemplified by the red and purple curves, a small number have a vanishing or positive cosmological constant and, hence, de Sitter vacua ! $\Rightarrow$

potential violation of one of the Swampland conjectures !

However, our results are consistent with another of the Swampland conjectures, namely the well-established Transplanckian Censorship Conjecture (TCC). This postulates, that

For large values of the moduli fields, with canonically normalized kinetic energy, there is a positive lower bound on the gradient of the potential when V>0, namely

$$\frac{|\nabla V|}{V} \geq \sqrt{2}$$

in four-dimensional spacetime.

Our previous expression for V_F is only valid for $<t> \lesssim (\frac{3.39l}{F\beta^{1/2}})^{2/3} \Rightarrow m_{\rm anom} \gtrsim M_U$.

Therefore, to consider large values of t we have to consider the generic form of V_F given for $S = s + i\sigma$, $T = t + i\chi$ by

$$
\begin{aligned}
V_F =& \frac{M_U^4}{st^3\langle c\rangle^3}[(\frac{1.14}{F^{4/3}})\ \tilde{d}(A^2+B^2) \\
&+1.32\times 10^{-6}\tilde{d}^{-1}((1+2bs)^2+3)e^{-2bs} \\
&-(\frac{2.43\times 10^{-3}}{F^{2/3}})(1+2bs)e^{-bs}sgn(A)\sqrt{A^2+B^2}\cos(b\sigma+\arctan(\frac{B}{A})) \\
&+2.62\times 10^{-6}p\tilde{d}^{-1}(1+2bs+3(\tau t+\frac{3}{2}))e^{-bs-\tau t}\cos(b\sigma-\tau\chi+\theta_p) \\
&+4.36\times 10^{-7}p^2\tilde{d}^{-1}(3+(2\tau t+3)^2)e^{-2\tau t} \\
&-(\frac{2.43\times 10^{-3}}{F^{2/3}})p(1+2\tau t)e^{-\tau t}sgn(A)\sqrt{A^2+B^2}\cos(\tau\chi-\theta_p+\arctan(\frac{B}{A}))]
\end{aligned}
$$

Taking $s = .230F^{4/3}\beta t$ sets $V_D = 0$. Then, except for the first term, all other terms in V_F are strongly suppressed by a factor of $e^{-\mathbf{c}t}$ for differing positive coefficients $\mathbf{c}$.

Therefore, in the large t limit

$$V_F \propto \frac{1}{t^4}$$

with a positive constant of proportionality. However, the kinetic energy for t is given by

$$\kappa_4^2 \frac{\partial^2 K}{\partial T \partial \bar{T}} (\partial T \partial \bar{T})|_{\mathrm{Im}T=0} = \frac{3}{4} \frac{(\partial t)^2}{t^2}$$

To rewrite the kinetic energy in terms of a canonically normalized field, define

$$\Phi = \sqrt{3/2} \ln t$$

Then the kinetic energy for Φ is

$$\frac{1}{2}(\partial \Phi)^2$$

and V_F becomes

$$V_F \propto e^{-4\sqrt{2/3}\Phi}$$

It follows that

$$\frac{|\nabla V|}{V} = \frac{|dV_F/d\Phi|}{V_F} = 4\sqrt{2/3} > \sqrt{2}$$

Hence our theory exceeds the lower bound of $\sqrt{2}$ in the large field limit. Hence,

Our theory is consistent with the Swampland TCC conjecture !