Lessons from Non-Supersymmetric Strings

Augusto Sagnotti

Scuola Normale Superiore and INFN – Pisa

SCUOLA NORMALE SUPERIORE

INFN Istituto Nazionale di Fisica Nucleare

STRING THEORY AS A BRIDGE BETWEEN GAUGE THEORY AND GRAVITY

ROMA "LA SAPIENZA", FEBRUARY 17-19 2025

The (SUSY) 10D-11D Hexagon

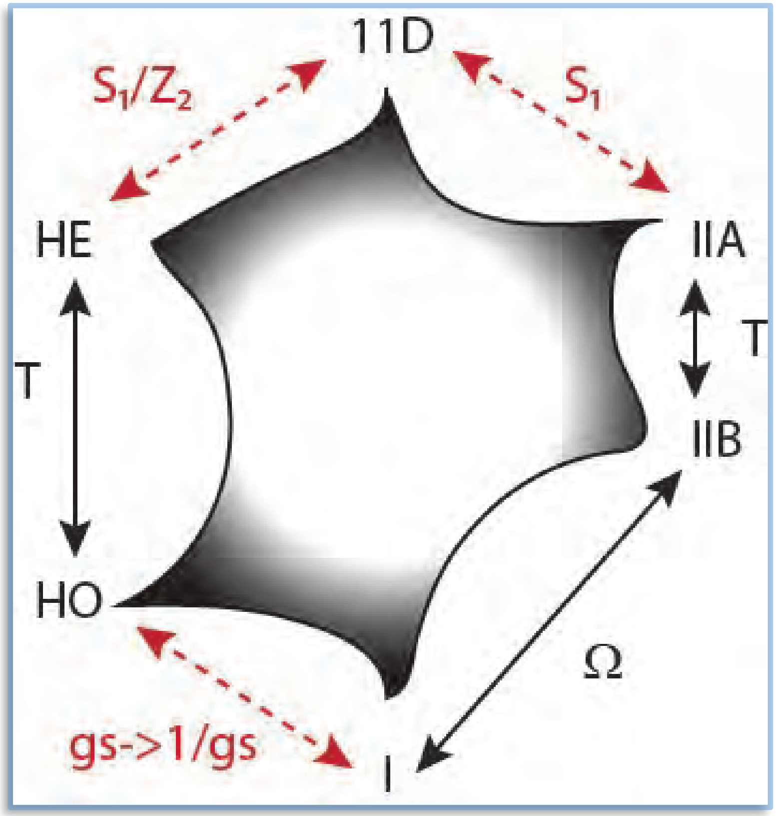
- Perturbative → **Solid arrows**
- **[** 10&11D supergravity → **Dashed arrows]**
- **Highest point** of (SUSY) String Theory

BUT:

- Exhibits **dramatically our limitations**

(Witten, 1995)

- **SUSY**: **stabilizes** the 10D Minkowski vacua



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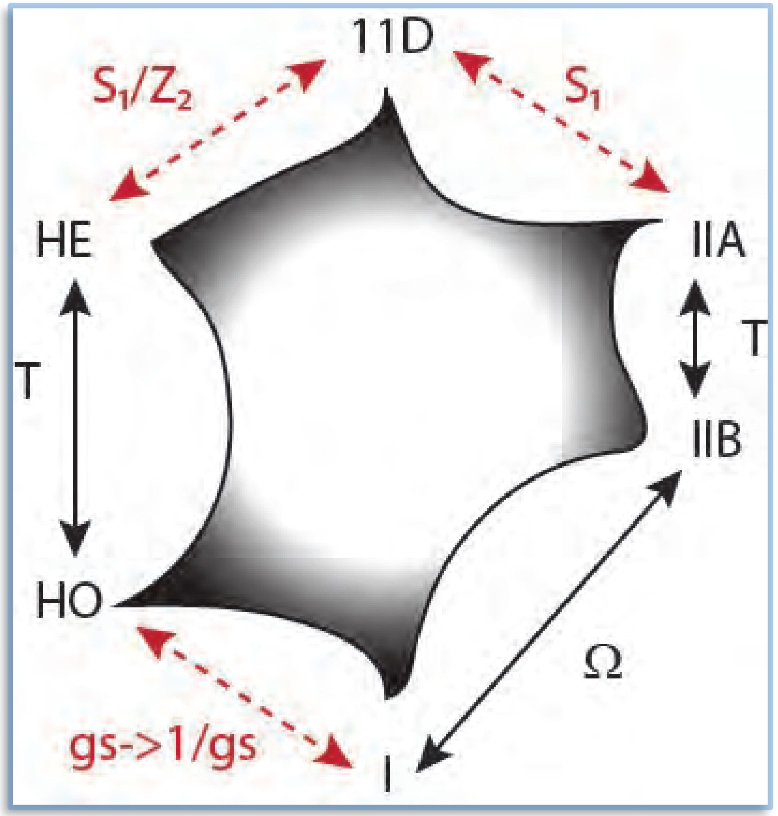
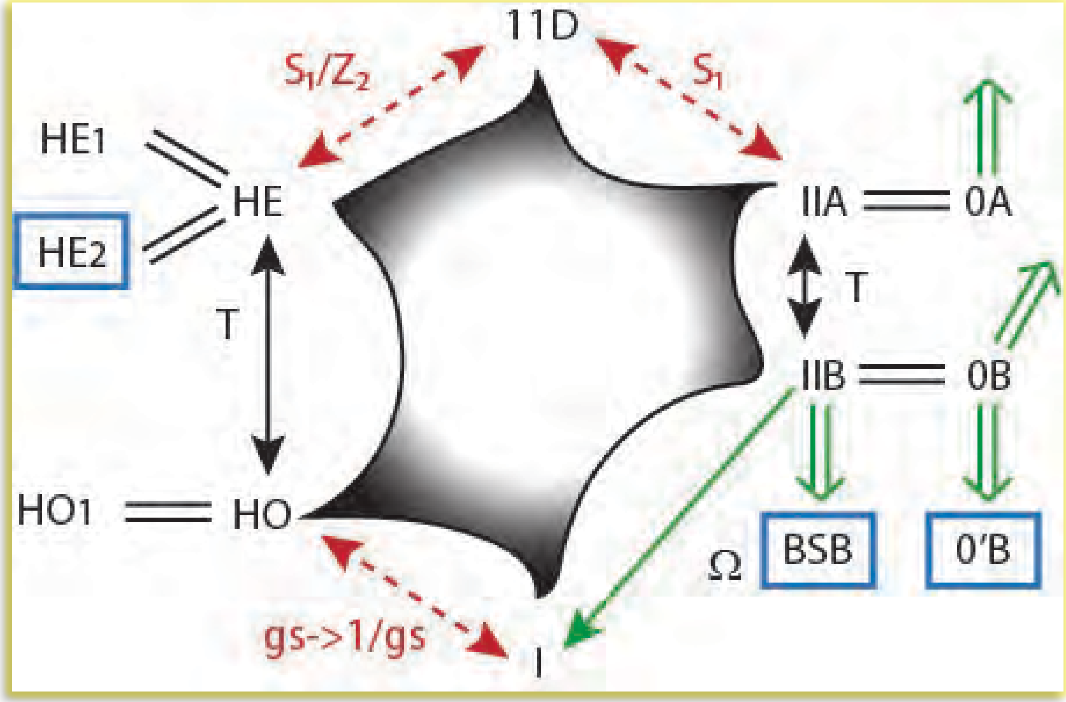


BROKEN SUSY ?

The 10D-11D Zoo



- Non-SUSY closed & orientifolds

(Seiberg, Witten, 1986)
(Dixon, Harvey, 1986)
(Bianchi, AS, 1990)

∃ 3 non-SUSY non-tachyonic strings

- SO(16)xSO(16) — (Dixon, Harvey, 1986) (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)
- 0'B U(32) — (AS, 1995)
- [BSB: Usp(32)] — (Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)

- **NO SUPERSYMMETRY → TACHYONS (typically) : We are still UNABLE to cope with them**
- **∃ three 10D theories WITHOUT SUPERSYMMETRY BUT NO TACHYONS:**

1) Heterotic variant 2) Exotic descendant of "tachyonic 0B" 3) Brane SUSY breaking

I. 10D Tachyon-Free Models

Non-Tachyonic 10D String Models

SO(16)xSO(16) Heterotic

(Dixon, Harvey, 1987)
(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)

$$O_{2n} = \frac{\theta^n\left[{0 \atop 0}\right](0|\tau) + \theta^n\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,, \qquad S_{2n} = \frac{\theta^n\left[{1/2 \atop 0}\right](0|\tau) + i^{-n}\,\theta^n\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}$$

$$V_{2n} = \frac{\theta^n\left[{0 \atop 0}\right](0|\tau) - \theta^n\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,, \qquad C_{2n} = \frac{\theta^n\left[{1/2 \atop 0}\right](0|\tau) - i^{-n}\,\theta^n\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}$$

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1 - q^n)\,, \qquad q = e^{2\pi i\tau}\,,$$

$$\theta\left[{\alpha \atop \beta}\right](z|\tau) = \sum_{n\in Z} q^{\frac{1}{2}(n+\alpha)^2}\, e^{i2\pi(n+\alpha)(z-\beta)}$$

ORBIFOLD of the HE (E_8xE_8) heterotic by $(-1)^{FL+F1+F2}$:

$$\mathcal{T}_{HE} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{1}{(Im\tau)^4\,\eta^8\,\bar\eta^8}\,(V_8 - S_8)\,(\bar O_8 + \bar S_8)\,(\bar O_8 + \bar S_8)$$

$$\mathbf{1}: \quad \mathcal{T}_{HE} \to \frac{1}{2}\left[1 + (-1)^{F_L+F_1+F_2}\right]\mathcal{T}_{HE}$$

$$\mathbf{2}: \quad \frac{1}{2}(-1)^{F_L+F_1+F_2}\mathcal{T}_{HE} \to \left\{1 + \left[\tau \to -\frac{1}{\tau}\right] + \left[\tau \to \tau+1\right]\right\}\frac{1}{2}(-1)^{F_L+F_1+F_2}\mathcal{T}_{HE}$$

$$\mathcal{T} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{1}{(Im\tau)^4\,\eta^8\,\bar\eta^8}\left[O_8(\bar V_{16}\,\bar C_{16} + \bar C_{16}\,\bar V_{16}) + V_8(\bar O_{16}\,\bar O_{16} + \bar S_{16}\,\bar S_{16}) - S_8(\bar O_{16}\,\bar S_{16} + \bar S_{16}\,\bar O_{16}) - C_8(\bar V_{16}\,\bar V_{16} + \bar C_{16}\,\bar C_{16})\right]$$

Non-Tachyonic 10D String Models

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The First Tachyonic 10D Orientifold

(Bianchi, AS, 1990)

$$O_{2n} = \frac{\theta^n\left[{0 \atop 0}\right](0|\tau) + \theta^n\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,, \qquad S_{2n} = \frac{\theta^n\left[{1/2 \atop 0}\right](0|\tau) + i^{-n}\,\theta^n\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}$$

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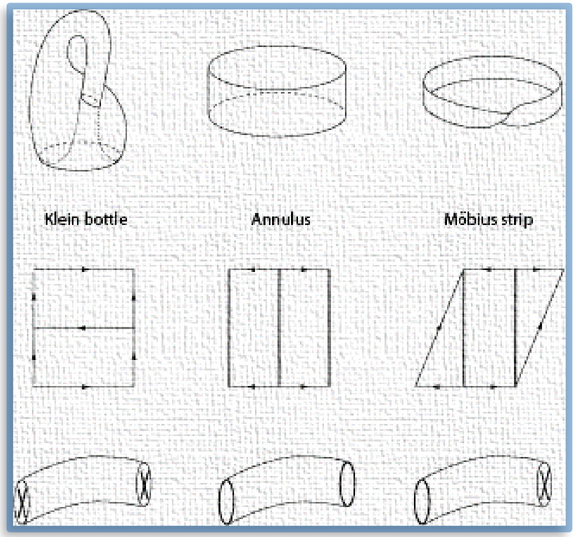

$$g_{\mu\nu},\ B_{\mu\nu},\ \phi,\ A^{1,2}_{\mu\nu\rho},\ A^{1,2}_{\mu}$$

0A

(Dixon, Harvey, 1987)
(Seiberg, Witten, 1987)

From 0A:

$$\frac{1}{2}\mathcal{T} + \mathcal{K} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\, \frac{|O_8|^2 + |V_8|^2 + S_8\overline{C}_8 + C_8\overline{S}_8}{(Im\tau)^4\ \eta^8\,\bar\eta^8} + \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{O_8 + V_8}{(\tau_2)^4\ \eta^8}\,[2i\tau_2]$$

$$\mathcal{A} + \mathcal{M} = \int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{\left[\frac{N_1{}^2}{2} + \frac{N_2{}^2}{2}\right](O_8 + V_8) - N_1\,N_2\ (S_8 + C_8)}{(\tau_2)^4\ \eta^8}\,[i\tau_2/2] - \left[\frac{N_1 + N_2}{2}\right]\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{\left(-\hat O_8 + \hat V_8\right)}{(\tau_2)^4\ \hat\eta^8}\,[i\tau_2/2 + 1/2]$$

$$\widetilde{\mathcal{K}} + \widetilde{\mathcal{A}} + \widetilde{\mathcal{M}} = \int_0^\infty d\ell\, \frac{\frac{2^5}{2} + 2^{-5}\left[\frac{N_1{}^2}{2} + \frac{N_2{}^2}{2}\right](O_8 + V_8) - 2^{-5}\,N_1\,N_2\ (O_8 - V_8)}{\eta^8}\,[i\ell] - 2\left[\frac{N_1 + N_2}{2}\right]\int_0^\infty d\ell\, \frac{\left(\hat O_8 + \hat V_8\right)}{\hat\eta^8}\,[i\ell + 1/2]$$

The First Tachyonic 10D Orientifold

(Bianchi, AS, 1990)

$$O_{2n} = \frac{\theta^n\begin{bmatrix}0\\0\end{bmatrix}(0|\tau) + \theta^n\begin{bmatrix}0\\1/2\end{bmatrix}(0|\tau)}{2\,\eta^n(\tau)}\,, \qquad S_{2n} = \frac{\theta^n\begin{bmatrix}1/2\\0\end{bmatrix}(0|\tau) + i^{-n}\,\theta^n\begin{bmatrix}1/2\\1/2\end{bmatrix}(0|\tau)}{2\,\eta^n(\tau)}$$

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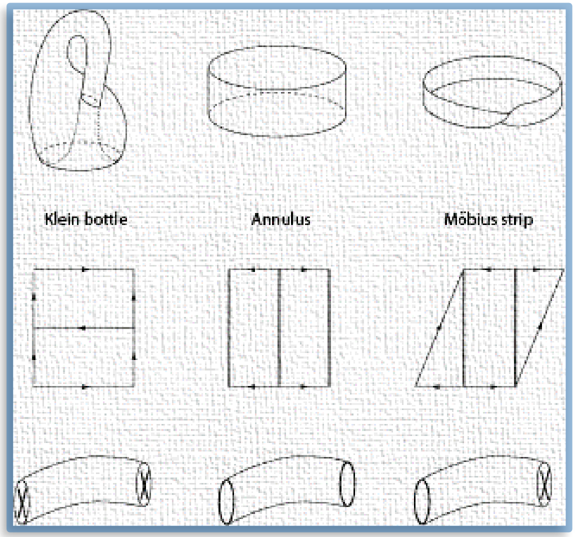

$$g_{\mu\nu},\; B_{\mu\nu},\; \phi,\, A^{1,2}_{\mu\nu\rho},\; A^{1,2}_{\mu}$$

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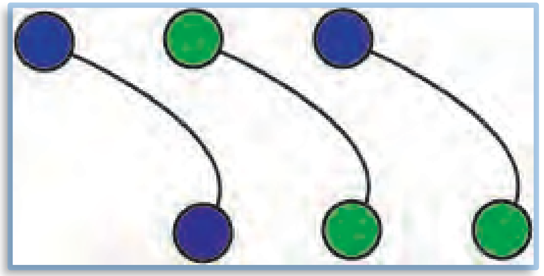
2 different types of Chan-Paton charges:



Non-Tachyonic 10D String Models

0'B Orientifold

(AS, 1995)

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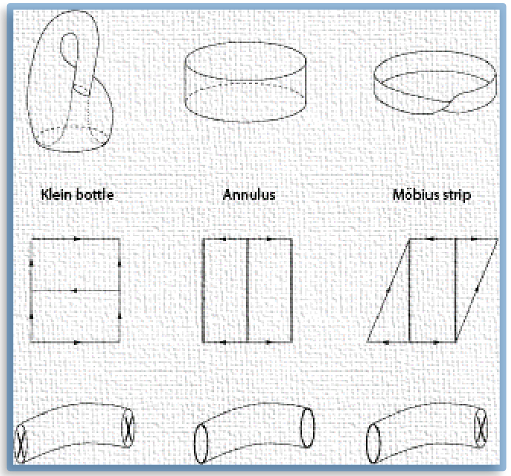

$g_{\mu\nu},\ B^{1,2}_{\mu\nu},\ \phi^{1,2},\ A_{\mu\nu\rho\sigma}$

0B

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$$\mathcal{A} + \mathcal{M} = \int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\,\frac{\mathcal{N}\overline{\mathcal{N}}\,V_8 - \frac{1}{2}\left(\mathcal{N}^2 + \overline{\mathcal{N}}^2\right)C_8}{(\tau_2)^4\,\eta^8}\,[i\tau_2/2] - \frac{\mathcal{N}+\overline{\mathcal{N}}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\,\frac{\hat{C}_8}{(\tau_2)^4\,\hat\eta^8}\,[i\tau_2/2 + 1/2]$$

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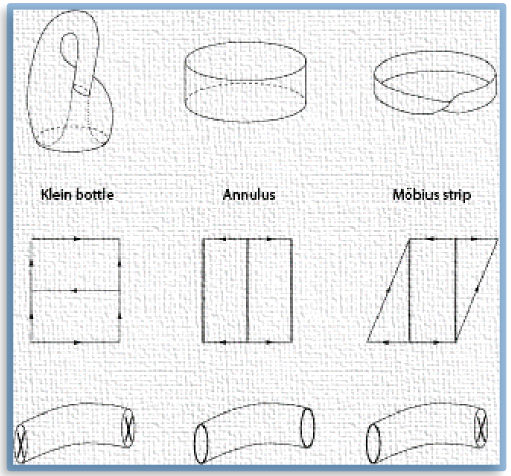

$g_{\mu\nu},\ B^{1,2}_{\mu\nu},\ \phi^{1,2},\ A_{\mu\nu\rho\sigma}$

0B

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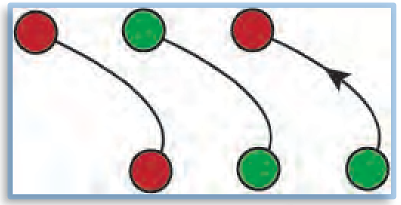
"Complex" charges : U(32)



OTHER PECULIAR FEATURES:

- **Simplest occurrence of generalized Green-Schwarz**
- **U(1) anomalous → SU(32)**

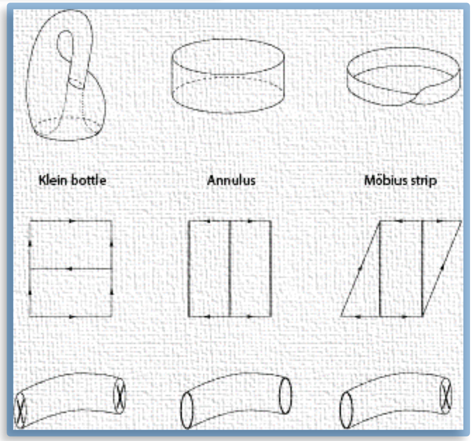
Non-Tachyonic 10D String Models

0'B Orientifold

$$O_8 = \frac{\theta^4\left[{0 \atop 0}\right](0|\tau) + \theta^4\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^4(\tau)}\,, \qquad S_8 = \frac{\theta^4\left[{1/2 \atop 0}\right](0|\tau) + \theta^4\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}$$

$$V_8 = \frac{\theta^4\left[{0 \atop 0}\right](0|\tau) - \theta^4\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^4(\tau)}\,, \qquad C_8 = \frac{\theta^4\left[{1/2 \atop 0}\right](0|\tau) - \theta^4\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^4(\tau)}$$

$$\begin{pmatrix} O_8 \\ V_8 \\ -S_8 \\ -C_8 \end{pmatrix} \xrightarrow{\textbf{s}} \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} O_8 \\ V_8 \\ -S_8 \\ -C_8 \end{pmatrix}$$



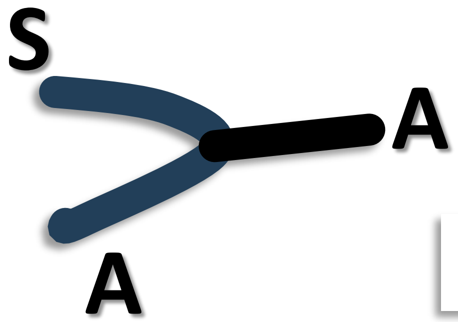
"- signs for Ramond": (Schellekens and Warner, 1987)

$$\mathcal{K} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{\varepsilon_o\,O_8 + \varepsilon_v\,V_8 + \varepsilon_s\,(-S_8) + \varepsilon_c\,(-C_8)}{(\tau_2)^4\,\eta^8}\,[2i\tau_2]$$

Standard choice: $\varepsilon_i = (1,1,1,1)$

$$\varepsilon_i = (1,1,-1,-1)$$

$$\varepsilon_i = (-1,1,1,-1)$$



Can change ε_i compatibly with the FUSION RULES (as in 2D WZW models of ADE series)

(Fioravanti, Pradisi, AS, 1990)
(Pradisi, AS, Stanev, 1994)

Non-Tachyonic 10D String Models

Usp(32) Orientifold

(Sugimoto, 1999)

$$O_{2n} = \frac{\theta^n \begin{bmatrix}0\\0\end{bmatrix}(0|\tau) + \theta^n \begin{bmatrix}0\\1/2\end{bmatrix}(0|\tau)}{2\,\eta^n(\tau)} , \qquad S_{2n} = \frac{\theta^n \begin{bmatrix}1/2\\0\end{bmatrix}(0|\tau) + i^{-n}\,\theta^n \begin{bmatrix}1/2\\1/2\end{bmatrix}(0|\tau)}{2\,\eta^n(\tau)}$$

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$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1 - q^n) , \qquad q = e^{2\pi i \tau} ,$$

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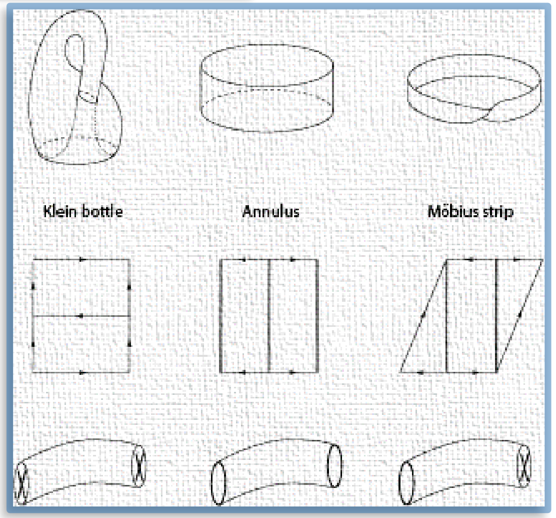


$$\mathcal{T}_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|V_8 - S_8|^2}{(Im\tau)^4\, \eta^8\, \bar\eta^8} \longrightarrow \frac{1}{2}\mathcal{T} + \mathcal{K} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|V_8 - S_8|^2}{(Im\tau)^4\, \eta^8\, \bar\eta^8} + \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2} \frac{V_8 - S_8}{(\tau_2)^4\, \eta^8} [2i\tau_2]$$

$$\mathcal{A} + \mathcal{M} = \frac{1}{2}\mathcal{N}^2 \int_0^\infty \frac{d\tau_2}{(\tau_2)^2} \frac{V_8 - S_8}{(\tau_2)^4\, \eta^8} [i\tau_2/2] - \frac{1}{2}\mathcal{N}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2} \frac{-\hat V_8 - \hat S_8}{(\tau_2)^4\, \hat\eta^8} [i\tau_2/2 + 1/2]$$

Differs from the SUSY SO(32) just in ONE SIGN → NON-LINEAR SUSY

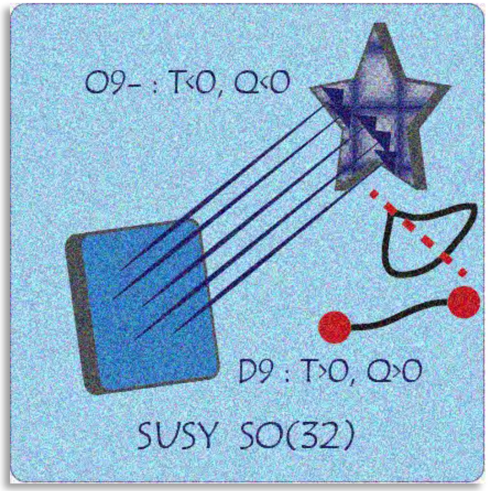
- **Massless BOSE : SO(32) → Usp(32)**
- **Massless FERMI: STILL ANTISYMMETRIC, NOW REDUCIBLE → GOLDSTINO**

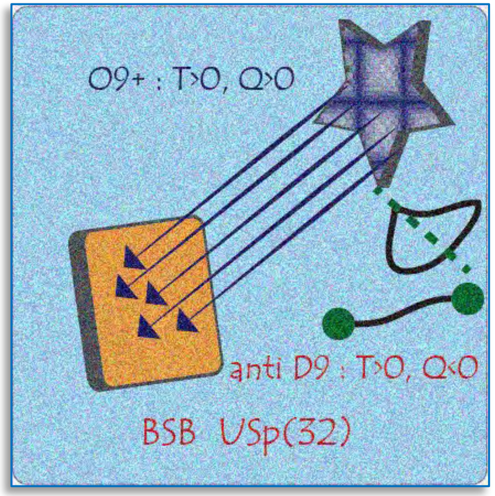
Brane SUSY Breaking (BSB)

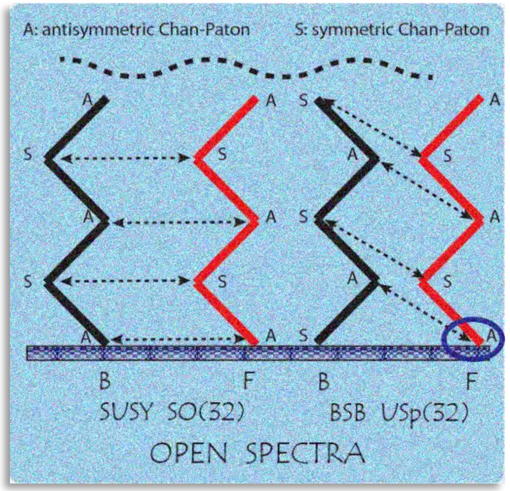
(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

- ❖ **NO TACHYONS**
- ❖ **Non-linear SUSY: ∃ goldstino!**

(Dudas, Mourad, 2000)
(Pradisi, Riccioni, 2001)







NON-LINEAR REALIZATIONS: USUALLY limits of linear ones. WHERE ARE THE "HIGGS" MODES HERE?

In D=10 BSB IS AN OPTION, in lower dimensions **IT CAN BE INEVITABLE** with special Klein-bottle projections

SUSY IN CLOSED SPECTRUM, NOT IN OPEN: puzzle noted in Rome in the early '90's (see hep-th/9302099), with **M. Bianchi and G. Pradisi** [See also 2403.02392 for some recent developments]

Non-SUSY → Back-Reaction on the Vacuum

- **Dual Role** of **Vacuum amplitudes** in **String Theory:**
  - a. **Consistency conditions**
  - b. **Backreaction on vacuum**

- **AT BEST: Double expansion** in powers of $\alpha' R$ and $g_s = e^{\phi}$

- **VERY DIFFICULT: one can at least EXPLORE the dominant terms … AND YET …**

1. SO(16) x SO(16) :

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2 - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,\mathrm{tr}\,\mathcal{F}^2\right] - T + \ldots\right\}$$

2. 0'B, USp(32):

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

Tadpole potential

II. The Climbing Scalar

(Different Cosmologies with $V = e^{\gamma\phi}$ for $\gamma < \gamma_c$ & $\gamma \geq \gamma_c$)

Cosmology: "Critical"Potential & Climbing Scalar

WHAT POTENTIALS LEAD TO SLOW-ROLL, AND WHERE ?

(Dudas, Kitazawa, AS, 2010)

$$ds^2 = -dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x} \quad \Longrightarrow \quad \ddot{\phi} + 3\dot{\phi}\sqrt{\frac{1}{3}\dot{\phi}^2 + \frac{2}{3}V(\phi)} + V' = 0$$

Driving force from V' *vs* friction from V

- **IF V does not vanish : a convenient gauge "makes the damping term neater"** *(Dudas and Mourad, 2000)*

$$ds^2 = e^{2\mathcal{B}(t)}\, dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\, d\mathbf{x}\cdot d\mathbf{x}$$

$$V e^{2\mathcal{B}} = V_0$$

$$\tau = t\sqrt{\frac{d-1}{d-2}}, \qquad \varphi = \phi\sqrt{\frac{d-1}{d-2}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

- **NOW: driving from logV** *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ **Quadratic potential?** **Far away from origin** *(Linde, 1983)*

❖ **Exponential potential?** **YES or NO**

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

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$$m\ddot{x} + b\dot{x} = f$$

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$V = e^{2\gamma\varphi}$: Climbing & Descending Scalars

(HERE we work with $\gamma_c = 1$)

(Halliwell, 1987;..., Dudas and Mourad, 2000; Russo, 2004)
(Dudas, Kitazawa, AS, 2010)

Follow solutions back to the initial singularity:

- $\gamma < 1$? Both signs of speed allowed

a. **"Climbing" solution** (φ climbs, then descends):

b. **"Descending" solution** (φ only descends):

Limiting τ- speed (LM attractor): $$\mathbf{v}_{\mathbf{lim}} = - \frac{\gamma}{\sqrt{1-\gamma^2}}$$

(Lucchin and Matarrese, 1985)

$\gamma = 1$ is "critical": LM attractor & descending solution disappear for $\gamma \geq 1$

$$V_S \sim e^{-\phi}$$
$$V_E = e^{\frac{3}{2}\phi}$$
$$\Downarrow$$
$$\left(V_E = e^{2\gamma\varphi}\right)$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^2} + \gamma\left(1 + \dot{\varphi}^2\right) = 0$$

$$\ddot{\varphi} + \dot{\varphi}\,|\dot{\varphi}| + \gamma\,\dot{\varphi}^2 \simeq 0 \longrightarrow \dot{\varphi} = \frac{C}{t}$$

$$|\mathbf{C}| = \frac{\mathbf{1}}{\mathbf{1} + \epsilon\gamma}\,, \quad \epsilon = \pm 1$$

$$V = Te^{2\gamma\varphi}$$

$V = e^{2\gamma\varphi}$: *Climbing & Descending Scalars*

(HERE we work with $\gamma_c = 1$)

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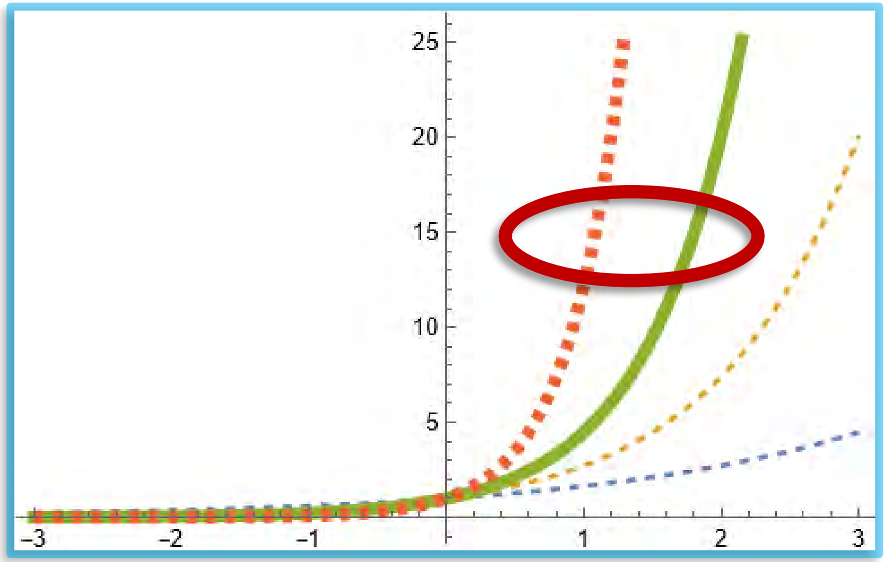

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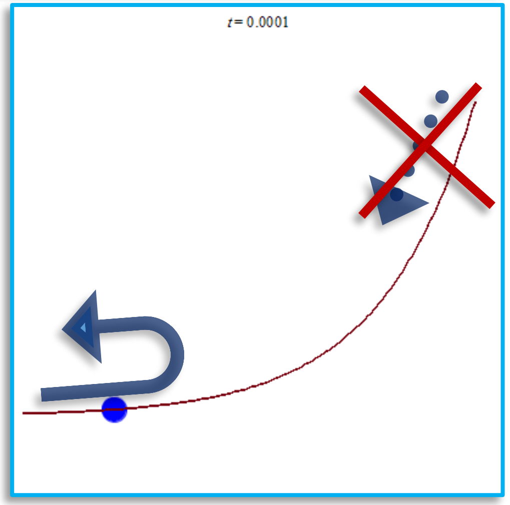


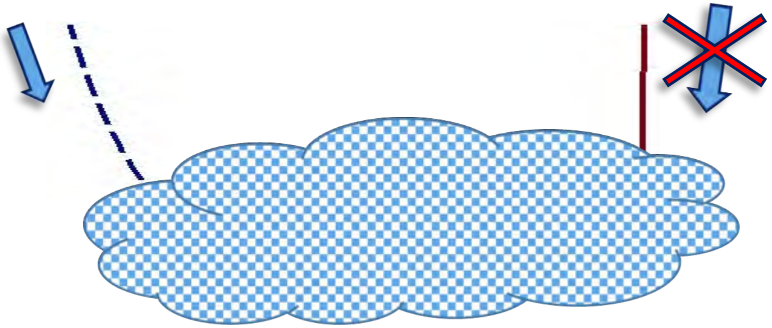
Cosmology: a Climbing Scalar as Trigger of Inflation?

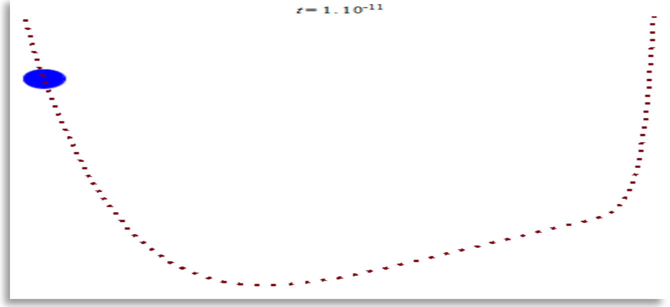
CLIMBING & SLOW-ROLL ? With (super)critical Exponential (e.g. + Starobinsky) → **FIXED INITIAL CONDITIONS**

$$V(\phi) = T\, e^{2\varphi} + v(\phi)$$

$$e.g.\ \ v(\phi) = v_0 \left(1 - e^{-\frac{2}{3}\varphi}\right)^2$$

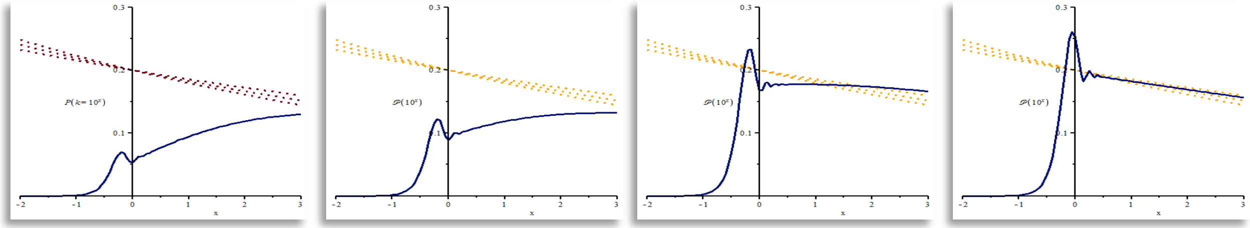
(Dudas, Kitazawa, Patil, AS, 2013)
(Kitazawa, AS, 2014)





DAMPED LOW END of primordial power spectrum → POSSIBLY: damping of first CMB multipoles (cfr. lack-of-power)

[+ enhanced tensor-to-scalar ratio at the transition]



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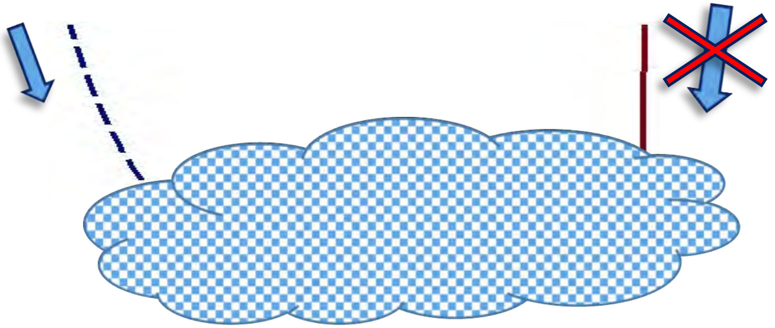

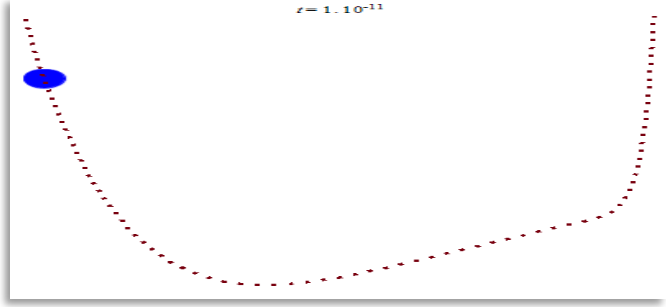

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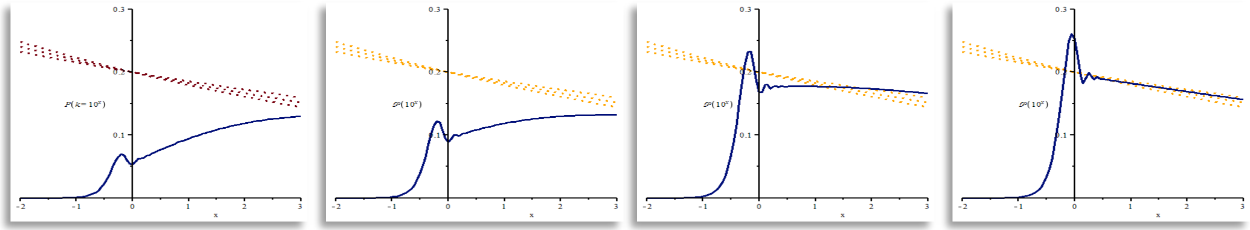
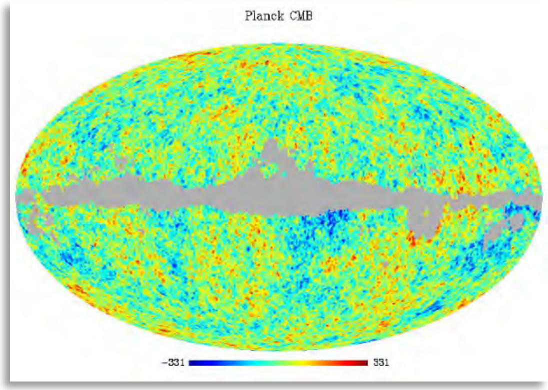
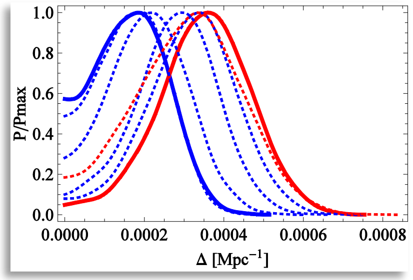


$$P(k) \sim k^{3-3\nu} \longrightarrow P(k) \sim \frac{k^3}{\left[k^2 + \Delta^2\right]^\nu}$$

[Extends Chibisov-Mukhanov tilt by Δ]





$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\mathrm{Mpc}^{-1}$$

$$\Delta_{infl} \sim 10^{12} - -10^{14}\ GeV \ \text{for}\ N \sim 60$$

[**RED** : + 30−degree extended mask]

(Gruppuso, Mandolesi, Natoli, Kitazawa, AS, 2015)
(+ Lattanzi, 2017)

Climbing Scalar : Instability of Isotropy

(Basile, Mourad, AS, 2018)

- **COSMOLOGY : the issue is the time evolution of perturbations**
- **INITIALLY (large η) V is negligible: tensor perturbations evolve as**

$$h''_{ij} + \frac{1}{\eta}\, h'_{ij} + \mathbf{k}^2\, h_{ij} = 0$$

$$h_{ij} \sim A_{ij}\, J_0(k\eta) + B_{ij}\, Y_0(k\eta) \qquad (\mathbf{k} \neq 0)$$

$$h_{ij} \sim A_{ij} + B_{ij}\, \log\left(\frac{\eta}{\eta_0}\right) \qquad (\mathbf{k} = 0)$$

- **NOTE: logarithmic growth for k=0 (instability of isotropy) !!**
- **RESONATES with**

(Kim, Nishimura, Tsuchiya, 2018)
(Anagnostopoulos, Auma, Ito, Nishimura, Papadoulis, 2018)

(HINT of) Dynamical origin of compactification ?

III. Dudas-Mourad Vacua

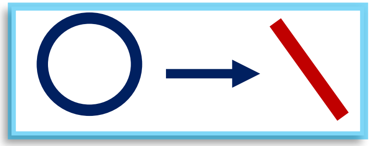
(Stable tadpole-driven compactifications on intervals)

9D Dudas-Mourad Vacua

(Dudas, and Mourad, 2000, 2001)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] - \frac{1}{12}\, \mathcal{H}_3^2 - \frac{1}{4}\, e^{-\phi}\, \mathrm{tr}\, \mathcal{F}^2 - T\, e^{-\phi} + \ldots \right\}$$

9D solutions → T : DRIVES compactification & KK CIRCLE → INTERVAL
[HERE for Usp(32) and U(32), & similar for SO(16) x SO(16)]



❖ **SPONTANEOUS COMPACTIFICATIONS:** INTERVALS of FINITE length ~ $\frac{1}{\sqrt{T}}$

❖ FINITE 9D Planck mass & gauge coupling

- At ends: g_s → (∞,0) & curvature diverges
- ASYMPTOTICS: Kasner-like (FREE!)

$$e^{\phi} = e^{u+\phi_0}\, u^{\frac{1}{3}}$$

$$ds^2 = e^{-\frac{u}{6}}\, u^{\frac{1}{18}}\, dx^2 + \frac{2}{3\,T\,u^{\frac{3}{2}}}\, e^{-\frac{3}{2}(u+\phi_0)}\, du^2$$

$$u \to 0 \;:\; ds^2 \sim (\mu_0\,\xi)^{\frac{2}{9}}\, dx^2 + d\xi^2\,, \quad e^{\phi} \sim (\mu_0\,\xi)^{\frac{4}{3}}$$

$$u \to \infty \;:\; ds^2 \sim [\mu_0\,(\xi_m - \xi)]^{\frac{2}{9}}\, dx^2 + d\xi^2\,, \quad e^{\phi} \sim [\mu_0\,(\xi_m - \xi)]^{-\frac{4}{3}}$$

- EXTENSIONS: $V_E = T\, e^{\frac{3}{2}\phi} \longrightarrow V_E = T\, e^{\gamma\,\phi}$ Orientifold γ : "CRITICAL" !

- ARE large values of Curvature & g_s INEVITABLE in these non-SUSY compactifications?
- STABILITY ?

Dudas-Mourad Vacua : Stability , I

(Basile Mourad, AS, 2018)

❖ **Dudas-Mourad: ∃STRONG COUPLING END but STABLE VACUUM !**

- **SETUP : Scalar perturbations:**

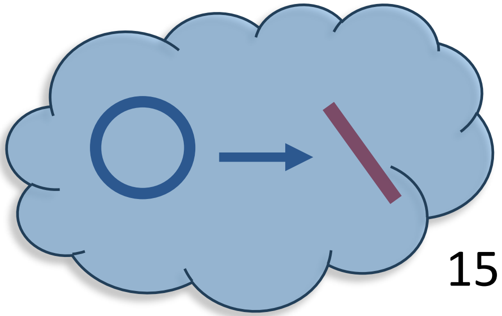
$$ds^2 \;=\; e^{2\Omega(z)}\left[(1+A)\,dx^\mu\,dx_\mu \;+\; (1-7A)\,dz^2\right] \;,$$

$$A'' + A'\left(24\,\Omega' \;-\; \frac{2}{\phi'}\,e^{2\Omega}\,V_\phi\right) \;+\; A\left(m^2 \;-\; \frac{7}{4}\,e^{2\Omega}\,V \;-\; 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) \;=\; 0$$

❖ **Schrödinger-like form:**

$$\begin{aligned} m^2\,\Psi \;&=\; \left(b \;+\; \mathcal{A}^\dagger\,\mathcal{A}\right)\Psi \\ \mathcal{A} \;&=\; \frac{d}{dr} \;-\; \alpha(r)\;, \qquad \mathcal{A}^\dagger \;=\; -\,\frac{d}{dr} \;-\; \alpha(r)\;, \qquad b \;=\; \frac{7}{2}\,e^{2\Omega}\,V\,\frac{1}{1 \;+\; \frac{9}{4}\,\alpha_O\,y^2} \;>\; 0 \end{aligned}$$

BUT: Boundary Conditions !



Singular Potentials & Self-Adjoint Extensions

(Mourad, AS, 2023)

❖ **SELF-ADOINT EXTENSIONS (boundary conditions) → COMPLETE SETS of NORMALIZABLE modes**

$$V(z) \sim \frac{\mu^2 - \frac{1}{4}}{z^2} \,, \qquad V(z) \sim \frac{\tilde{\mu}^2 - \frac{1}{4}}{(z_m - z)^2} \longrightarrow \psi \sim z^{\frac{1}{2} \pm \mu} \,, \qquad \psi \sim (z_m - z)^{\frac{1}{2} \pm \tilde{\mu}}$$

- **Two choices at z=0 ONLY IF $\mu < 1$ (and similarly at right end)**
- **Gravity and dilaton for $\gamma \leq \gamma_c$:** $\mu = \tilde{\mu}$

❖ **The possible self—adjoint extensions depend on μ**

a) **$\mu \geq 1$: UNIQUE b.c. → SCALAR MODES (MASSIVE)**

b) **$\mu < 1$: b.c. $\in$ SL(2,R) x U(1) → [indep.: AdS_3 boundary (θ_1,θ_2)] → TENSOR & VECTOR MODES**

STABILITY ANALYSIS ($m^2 > 0$) → EXACT LEGENDRE EIGENVALUE EQUATION

$$H = \mathcal{A}^\dagger \mathcal{A} \qquad V_\pm = \left(\frac{\pi}{z_m}\right)^2 \left[\frac{\left(\mu^2 - \frac{1}{4}\right)}{\sin^2\left(\frac{\pi\, z}{z_m}\right)} - \left(\frac{1}{2} \pm \mu\right)^2\right]$$

Legendre functions (& Exact zero modes)

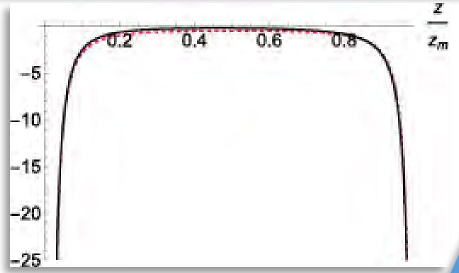
Dudas-Mourad Vacua : Stability , II

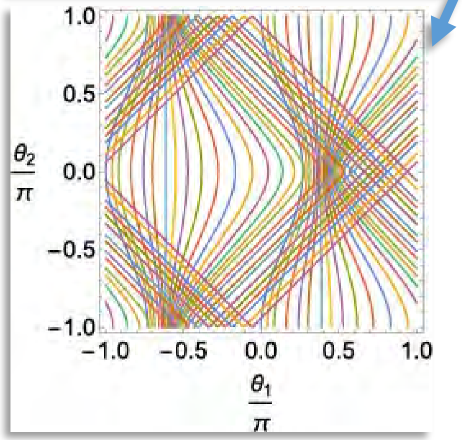
(Mourad, AS, 2023)

- (Singular) potentials closely approximated by Legendre ones
- Exact eigenvalue equations
- Vertical adjustments: compare with the exact zero modes

$$V_{\pm} \;=\; \left(\frac{\pi}{z_m}\right)^2 \left[\frac{\left(\mu^2 \;-\; \frac{1}{4}\right)}{\sin^2\left(\frac{\pi\, z}{z_m}\right)} \;-\; \left(\frac{1}{2} \pm \mu\right)^2\right]$$

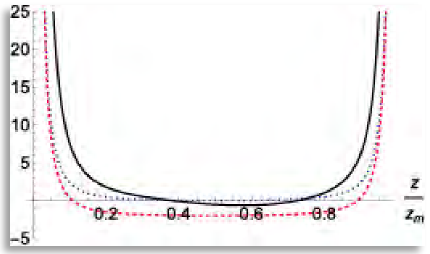
Tensor Modes (μ=0) :





Contour lines of fixed tachyon mass

Scalar Modes (μ=1)



UNIQUE b.c.
[up to vertical adjustment]
(massive scalar)

$$m^2 \;\simeq\; \left(\frac{\pi}{z_m}\right)^2 \left[n(n+1) - \frac{7}{8}\right] \;,\; n = 1, 2, \ldots$$

$$m^2 \;\simeq\; \left(\frac{\pi}{z_m}\right)^2 \; n\,(n\,+\,1) \;,\; n = 0, 1, 2, \ldots$$

UNIQUE stable b.c. (π,0)
(massless 9D graviton !)

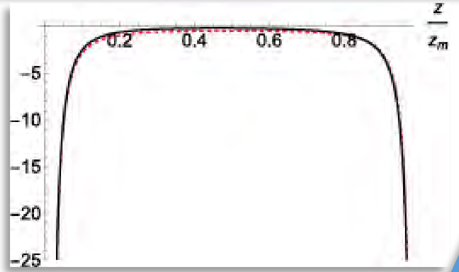
Dudas-Mourad Vacua : Stability , II

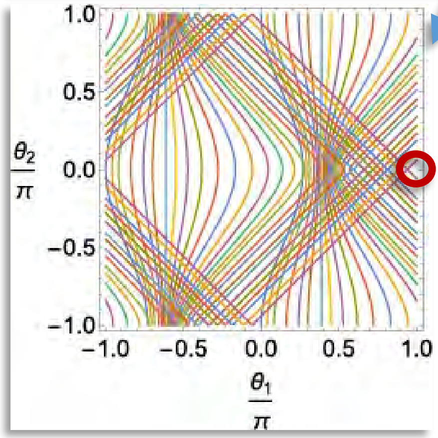
(Mourad, AS, 2023)

- (Singular) potentials closely approximated by Legendre ones
- Exact eigenvalue equations
- Vertical adjustments: compare with the exact zero modes

$$V_{\pm} \ = \ \left(\frac{\pi}{z_m}\right)^2 \left[\frac{\left(\mu^2 \ - \ \frac{1}{4}\right)}{\sin^2\left(\frac{\pi\, z}{z_m}\right)} \ - \ \left(\frac{1}{2} \pm \mu\right)^2\right]$$

Tensor Modes (μ=0) :





Contour lines of fixed tachyon mass

$$m^2 \ \simeq \ \left(\frac{\pi}{z_m}\right)^2 \ n\,(n\,+\,1) \ \ , \ n = 0,1,2,\ldots$$

UNIQUE stable b.c. (π,0)
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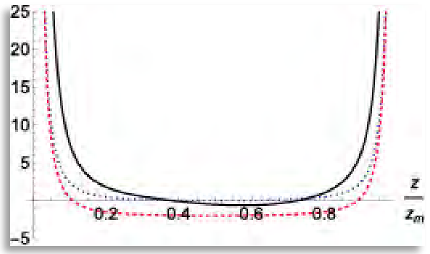

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$$m^2 \ \simeq \ \left(\frac{\pi}{z_m}\right)^2 \left[n(n+1) - \frac{7}{8}\right] \ , \ n = 1,2,\ldots$$

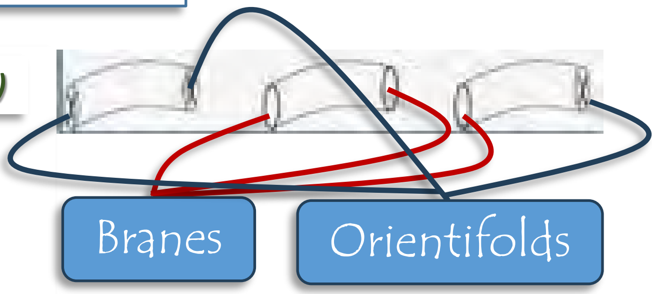
IV. Recent Developments

- ***Branes***
- ***Generalized Dudas-Mourad-Like Vacua***
- ***Bounded g_s (& Insights on the nature of endpoints)***

Branes, I (no Tadpole potential)

(Dudas, Mourad, AS, 2001)

- ∃ BPS CHARGED (anti) D-branes in SUSY strings: REVEALED by FORM POTENTIALS *(Polchinski, 1995)*
- ∃ additional UNCHARGED branes: brane-antibrane BOUND STATES *(Sen, 1998)*
- One can classify BRANES in (non-) SUSY strings by CFT TECHNIQUES (IGNORING the TADPOLE)



Ex: charged Dp in IIB (consistent for odd p)

$$\tilde{A}_{9p} \sim (n\bar{d}+d\bar{n})\,(V_{p-1}O_{9-p}-O_{p-1}V_{9-p}) + \left(e^{-\frac{i\pi}{4}(p-1)}\,n\bar{d}+e^{\frac{i\pi}{4}(p-1)}\,d\bar{n}\right)(S_{p-1}S_{9-p}-C_{p-1}C_{9-p})$$

$$A_{9p} \sim (n\bar{d}+d\bar{n})\,[(O_{p-1}+V_{p-1})\,(S_{9-p}+C_{9-p})-(S_{p-1}+C_{p-1})\,(O_{9-p}+V_{9-p})]$$
$$+ \left(n\bar{d}+e^{\frac{i\pi(n-5)}{2}}\,d\bar{n}\right)[(O_{p-1}-V_{p-1})\,(S_{9-p}-C_{9-p})] + \left(e^{\frac{-i\pi(n-5)}{2}}\,n\bar{d}+d\bar{n}\right)[(S_{p-1}-C_{p-1})\,(O_{9-p}-V_{9-p})]$$

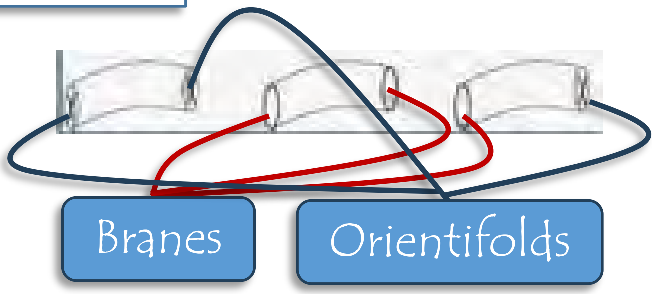
Chiral Spectra

$$p=1,5\ :\ A_{9p} \sim (n\bar{d}+d\bar{n})\,(O_{p-1}S_{9-p}+V_{p-1}C_{9-p}-C_{p-1}O_{9-p}-S_{p-1}V_{9-p})$$

$$p=-1,3,7\ :\ A_{9p} \sim n\bar{d}\,(O_{p-1}S_{9-p}+V_{p-1}C_{9-p}-C_{p-1}O_{9-p}-S_{p-1}V_{9-p})$$
$$+\ d\bar{n}\,(O_{p-1}C_{9-p}+V_{p-1}S_{9-p}-S_{p-1}O_{9-p}-C_{p-1}V_{9-p})$$

Branes, II (no Tadpole potential)

(Dudas, Mourad, AS, 2001)



In general:

	IIA	IIB	SO(32) SUSY	USp(32) BSB	O'B
CHARGED	D_0, D_2, D_4, D6, D8	$D_{-1}, D_1, D_3, D_5, D_7, D_9$	D_1(SO), D_5(Usp), D_9(SO)	D_1(USp), D_5(SO), D_9(USp)	D_{-1}(U), D_1(U), D_3(U), D_5, D_7(U), D_9(U)
UNCHARGED	$D_{-1}, D_1, D_3, D_5, D_7, D_9$	D_0, D_2, D_4, D_6, D_8	$D_0, D_2, D_4, D_6, D_8, D_{-1}, D_3, D_5$	D_0, D_2, D_4, D6, D8, D_{-1}, D_3, D_5	D_0, D_2, D_4, D_6, D_8

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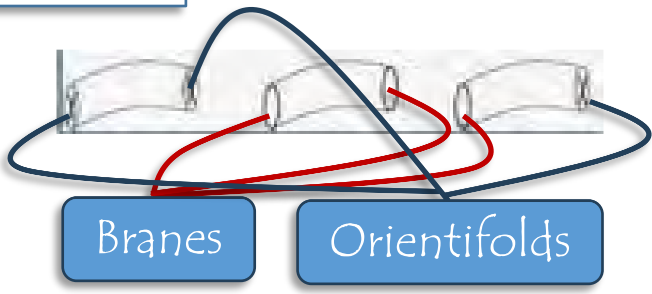

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	IIA	IIB	SO(32) SUSY	USp(32) BSB	O'B
CHARGED	D_0, D_2, D_4, D6, D8	$D_{-1}, D_1, D_3, D_5, D_7, D_9$	D_1(SO), D_5(Usp), D_9(SO)	D_1(USp), D_5(SO), D_9(USp)	D_{-1}(U), D_1(U), D_3(U), D_5, D_7(U), D_9(U)
UNCHARGED	$D_{-1}, D_1, D_3, D_5, D_7, D_9$	D_0, D_2, D_4, D_6, D_8	$D_0, D_2, D_4, D_6, D_8, D_{-1}, D_3, D_5$	D_0, D_2, D_4, D6, D8, D_{-1}, D_3, D_5	D_0, D_2, D_4, D_6, D_8

WHAT HAPPENS WITH THE TADPOLE POTENTIAL ?

Non-SUSY Branes & Tadpole Potential

(Mourad, Raucci, AS, 2024)

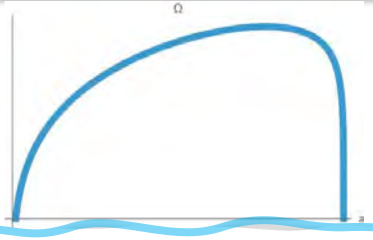
- **NAIVELY: the whole spacetime should collapse around branes**

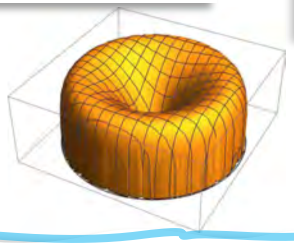
(Antonelli, Basile, 2019)

$$ds^2 = e^{2\,\Omega(z)}\left(dx_9^2 + dz^2\right)$$

$$ds^2 \sim z^{\frac{1}{4}}\left(dx^2 + dz^2\right), \qquad ds^2 \sim (z_m - z)^{\frac{1}{4}}\left|\log\left(z_m - z\right)\right|^{\frac{1}{4}}\left(dx^2 + dz^2\right)$$

Intuition (drawn from Dudas-Mourad):





$$ds^2 = e^{2A(r)}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + e^{2B(r)}\,dr^2 + e^{2C(r)}\,\ell^2\,\gamma_{mn}(\xi)\,d\xi^m\,d\xi^n$$

$$X = (p+1)A + (7-p)C\,, \qquad W = (p+1)A + (8-p)C + \frac{\gamma}{2}\,\phi\,, \qquad K = \phi + 8\,\gamma\,A$$

$$X'' = \frac{(7-p)^2}{\ell^2}\,e^{2X} - T\,e^{2W}\,, \qquad W'' = \frac{(8-p)(7-p)}{\ell^2}\,e^{2X} + \frac{1}{2}\left(\gamma^2 - \frac{9}{4}\right)T\,e^{2W}\,, \qquad K'' = 0\,,$$

$$\begin{aligned} 0 =\ & \frac{(7-p)(8-p)}{\ell^2} - T\,e^{2W} \\ & + \frac{8(7-p)(W')^2 - 16(8-p)W'X' + 4(8-p)\left(\gamma^2 - \frac{9}{4}\right)(X')^2 + \frac{p+1}{2}(K')^2}{(p+1) + 4(7-p)\gamma^2} \end{aligned}$$

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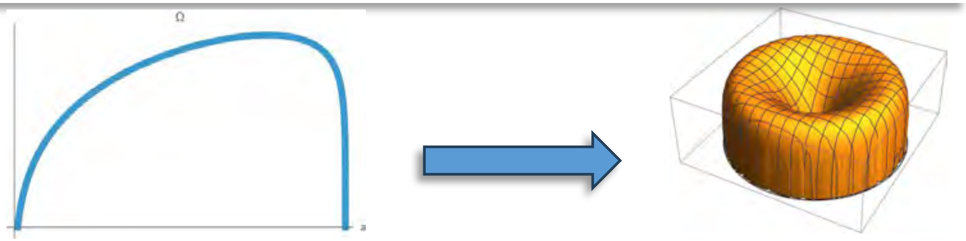
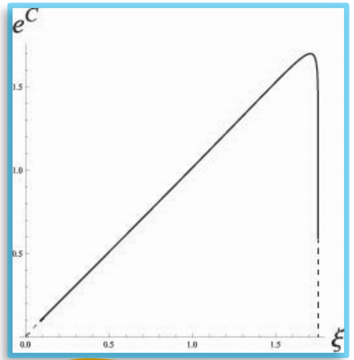
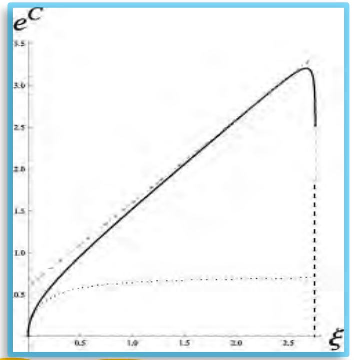
Intuition (drawn from 9D Dudas-Mourad(DM)):



NO Brane:



Brane:



CORRECT, BUT NOT the right VIEW!

SPHERICALLY SYMMETRIC spacetimes DO ClOSE (around branes) within distances O($\frac{1}{\sqrt{T}}$) !

→ LOWER-DIMENSIONAL DM-LIKE VACUA!

In the 9D Dudas-Mourad vacuum
(expect it also for lower-dimensional DM-like ones)

- **(Deformed) UNCHARGED branes are EXACT SOLUTIONS**
- **CONSISTENT ASYMPTOTICS of CHARGED branes away from cores**

$$ds^2 = e^{2A(z,r)}dx_{p+1}^2 + e^{2B(z,r)}\,dy^i\,dy^i + e^{2D(z,r)}dz^2$$

$$A = \Omega(z) \mp \sqrt{\frac{7-p}{7(p+1)}}\log\left[\frac{1+v(r)}{1-v(r)}\right]\ ,\quad v(r) = \left(\frac{v_0}{r}\right)^{6-p}$$

$$B = \Omega(z) \pm \sqrt{\frac{(7-p)(p+1)}{7(6-p)^2}}\log\left[\frac{1+v(r)}{1-v(r)}\right] + \frac{\log\left[1-v^2(r)\right]}{6-p}$$

$$D = \Omega(z)$$

D=4 with Fluxes on T^5 x I

(Mourad, AS, 2023)

❖ Five-form flux in IIB → ϕ CONSTANT, SPATIAL INTERVAL of length l

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu}{\left[h\, \sinh(\tilde{r})\right]^{\frac{1}{2}}} + \left[\sinh(\tilde{r})\right]^{\frac{1}{2}} \left[\ell^2\, e^{-\frac{\sqrt{10}}{2}\,\tilde{r}}\, d\tilde{r}^2 + (2\,\Phi\,\ell)^{\frac{2}{5}}\; e^{-\frac{\sqrt{10}}{10}\,\tilde{r}}\, \left(d\,\tilde{y}^i\right)^2\right]$$

$$\mathcal{H}_5^{(0)} = \frac{1}{2\,h}\,\frac{dx^0 \wedge \ldots \wedge dx^3 \wedge d\tilde{r}}{\left[\sinh(\tilde{r})\right]^2} + \Phi\, d\tilde{y}^1 \wedge \ldots \wedge d\tilde{y}^5$$

❖ FINITE gs , BUT STILL CURVATURE SINGULARITY]

USED EXTENSIVELY: (Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen, 2001)

- SUSY BREAKING scale ~ $1/\ell$
- SUSY recovered asymptotically at the r=0 end
- Less familiar tensor eqs: (+ Einstein eqs.)
- Interval of FINITE length : $\ell \sim H^{\frac{1}{4}}\, \rho^{\frac{5}{4}}$
- PERTURBATIONS: → Schrödinger-like systems ∃ STABLE BOUNDARY CONDITONS!
  (Hypergeometric setup)

A Closer Look at the Interval, I

(Mourad, AS, 2022, 2023)

One can "explore" the interval with a probe brane :

$$\frac{\mathcal{S}}{V_3} = -T_3 \int dt\, e^{4A(r(t))} \sqrt{1 - e^{2(B-A)(r(t))}\, \dot{r}(t)^2} + [2\times] q_3 \int b[r(t)]\, dt$$

$$b(r) = -\frac{1}{4\,\rho\, H}\left[\coth\left(\frac{r}{\rho}\right) - 1\right]$$

$$E = \frac{T_3\ e^{4A(r(t))}}{\sqrt{1 - e^{2(3A+5C)(r(t))}\, \dot{r}(t)^2}} - q_3\, b$$

The probe brane feels the potential below:

$$V(r) = T_3 e^{4A} - q_3 b = \frac{1}{2\,|H|\,\rho}\left[\frac{T_3}{\sinh\left(\frac{r}{\rho}\right)} + \frac{q_3\, \mathrm{sign}(H)}{2}\left(\coth\left(\frac{r}{\rho}\right) - 1\right)\right]$$

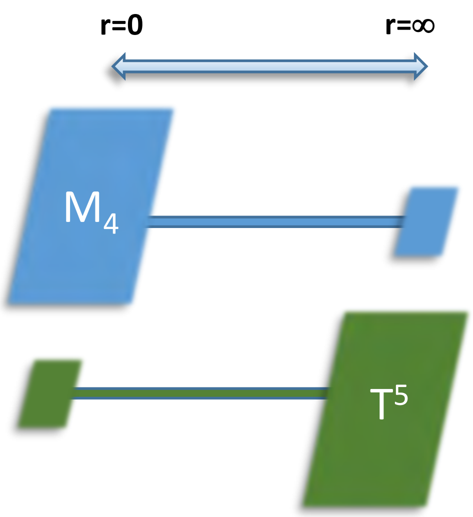
$$\mathbf{V} \sim \boxed{\frac{\mathbf{1}}{\mathbf{r}}\left[\mathbf{T_3} + [\mathbf{2}\times]\frac{\mathbf{q_3}}{\mathbf{2}}\,\mathrm{sign}\,(\mathbf{H})\right]}$$

BPS r=0 endpoint ! (consistently w. Killing spinor emerging as ρ→∞)

NO FORCE: if T3 and q3 are TUNED (the factor "2 x" is a matter of "INTERNAL DISPUTE")

A Closer Look at the Interval, II

(Mourad, AS, 2022, 2023)



Einstein action with York-Gibbons-Hawking term & its variation :

$$\mathcal{S}_{grav} = \frac{1}{2\,k_{10}^2}\int_{\mathcal{M}} d^9x\,dr\,\sqrt{-\tilde{\mathbf{g}}}\,\mathcal{N}\left[\tilde{\mathbf{R}} + \mathcal{K}_{\mathbf{mn}}\mathcal{K}_{\mathbf{pq}}\left(\tilde{\mathbf{g}}^{\mathbf{mn}}\tilde{\mathbf{g}}^{\mathbf{pq}} - \tilde{\mathbf{g}}^{\mathbf{mp}}\tilde{\mathbf{g}}^{\mathbf{nq}}\right)\right]$$

$$\tilde{g}_{mn} = g_{mn}\,, \qquad \mathcal{N}_m = g_{mr}\,, \qquad \mathcal{N}^2 + \mathcal{N}^m\mathcal{N}_m = g_{rr}\,, \qquad \mathcal{K}_{mn} = \frac{1}{2\,\mathcal{N}}\left(\partial_r\,\tilde{g}_{mn} - \widetilde{D}_{(m}\mathcal{N}_{n)}\right)$$

$$\mathbf{G}_{\mathbf{mn}} - \mathbf{T}_{\mathbf{mn}} + \frac{\delta(\mathbf{r}-\mathbf{R}^\star)}{\mathcal{N}}\left[\mathcal{K}_{\mathbf{mn}} - \tilde{\mathbf{g}}_{\mathbf{mn}}\,\mathcal{K}\right] - \frac{\delta(\mathbf{r}-\mathbf{r}^\star)}{\mathcal{N}}\left[\mathcal{K}_{\mathbf{mn}} - \tilde{\mathbf{g}}_{\mathbf{mn}}\,\mathcal{K}\right] = \mathbf{0}\,,$$

This reveals TENSION (and CHARGE) of an EFFECTIVE BPS O_3 orientifold at r=0

Neat realization of "dynamical cobordism" (HERE protected by SUSY)

(McNamara, Vafa, 2019)
(Uranga et al, 2021)
(Blumehagen et al, 2021)
(Raucci, 2022)

$$G_{\mu\nu} - T_{\mu\nu} + H\,\tilde{g}_{\mu\nu}\sqrt{-\det\,\tilde{g}_{\mu\nu}}\;\delta(z-z^\star) = 0$$

$$\mathcal{S}_T = \frac{H}{k_{10}^2}\int d^9x\,\sqrt{-\det\,\tilde{g}_{\mu\nu}}\,\Big|_{z^\star} \longrightarrow \boxed{\mathbf{T} = -\frac{\mathbf{\Phi}}{\mathbf{k_{10}^2}}}$$

- **LESS CLEAR at the other NON-SUSY end (BUT opposite charge)**

Summarizing

- ❖ **Tadpoles → Dudas-Mourad vacua: BOUNDARIES play a key role !**

✓ **STABILITY:** NO tachyon modes emerge [cfr UNSTABLE AdS x S !]

(Basile, Mourad, AS, 2018)
(Raucci, 2023)
(Mourad, AS, 2023)

- (Proportional) Tension & charge of EFFECTIVE (SUSY) ORIENTIFOLD in a SPECIAL SETTING
- ∃ (explicit) correspondence with work on "Dynamical Cobordism"

  [See: (Bergshoeff, Riccioni et al, 2006 –) for a wide zoo of lower-dimensional branes built via SUGRA U-dualities]

(McNamara, Vafa, 2019)
(Uranga et al, 2021)
(Blumehagen et al, 2021)
(Raucci, 2022)

✓ **COSMOLOGY:** climbing & inflation → (lack-of-power [enhanced tens.-to-scal. ratio] [& non-Gaussianities?]

- INTRIGUING **INSTABILITY OF ISOTROPY** (k=0) in "climbing scalar" Cosmology : **4D by accident?**

➢ **BRANES & TADPOLES → ∃ DEFORMED (un)charged branes in Dudas-Mourad vacua**

(See: 2406.14296, 2406.16327)

Thank You