On the large charge/ matrix models duality

Alba Grassi

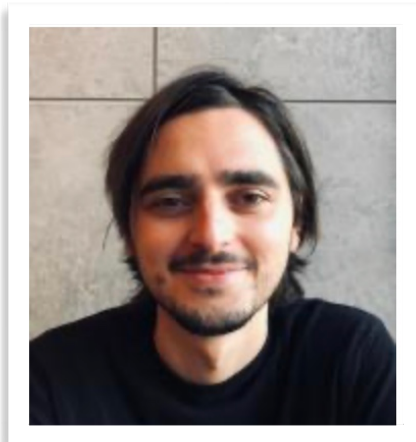
CERN

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

FNSNF
Swiss National Science Foundation

UNIVERSITÉ DE GENÈVE

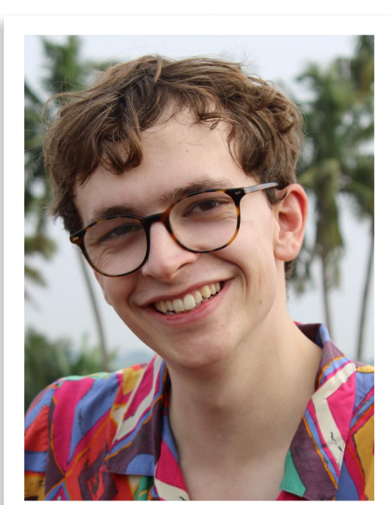
Based on

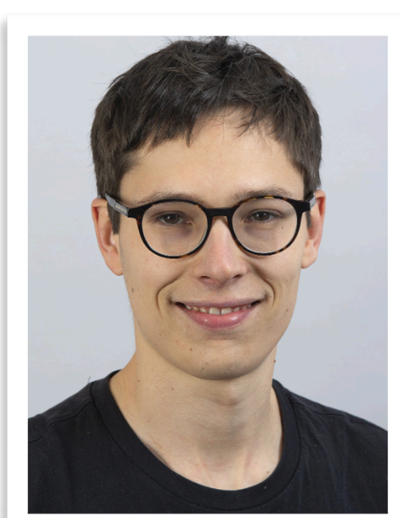


ArXiv 1908.10306 with Zohar Komargodski

and Luigi Tizzano

ArXiv 2408.07391 with Cristoforo Iossa





ArXiv 2502.xxxx with A Brown, F Galvagno, C Iossa, C Wen

Introduction

Strongly coupled QFT are mostly inaccessible to analytic methods

→ look for simpler limits

Examples:

- large rank of the gauge group ['t Hooft, …]
- large spin [Alday - Maldacena, Komargodski-Zhiboedov, …]
- large charge [Alday - Maldacena, Alvarez-Gaume, Maeda, Hellerman, Orlando, Reffert, Watanabe - Monin, Pirtskhalava, Rattazzi, Seibold …]

Introduction

In this talk we will study a class of correlators in the limit of a "large number of insertions" $\rightarrow$ large R charge

➡ Emergence of a "dual" description in terms of matrix models where the rank of matrices is related to the "number of insertions"

Introduction

! the matrix model we discuss in this talk are different from the ones appearing usually in susy localization

→ In susy localization (eg Pestun or AdS/CFT) we typically have one matrix model where rank of matrices = rank of gauge group.

→ In our framework we multiple matrix models where the rank of matrices = nb of operator insertions

K “types” of operators → K matrix models (or possible multi-cuts) coupled in a non trivial way

Our examples

Our two examples:

- $\mathcal{N} = 2$, SU(N) SQCD: a vector multiplet and $N_f = 2N$ fundamental hypermultiplets
- $\mathcal{N} = 4$, SU(N) SYM: a vector multiplet and an adjoint hypermultiplet

Within these theories, we can consider extremal or integrated correlators. Today I will mostly focus on extremal correlators of Coulomb branch operators:

$$\mathcal{O}_{\vec{n}} = \prod_{k=2}^{N} \left(\mathrm{Tr}(\varphi^k)\right)^{n_k}$$

where φ is the complex scalar in the vector multiplet. Hence we have N-1 independent generators

$$\phi_k = \mathrm{Tr}(\varphi^k) \qquad k = 2, \ldots N$$

Extremal correlators

These theories have a global $u(1)$ R symmetry and $\mathcal{O}_{\vec{n}} = \prod_{k=2}^{N} \left(\mathrm{Tr}(\varphi^k)\right)^{n_k}$ satisfy:

$$\rightarrow \quad \Delta(\mathcal{O}_{\vec{n}}) = \frac{R(\mathcal{O}_{\vec{n}})}{2} = \sum_{k=2}^{N} n_k k$$

scaling dimension — R-charge

We study extremal correlators of coulomb branch operators

$$\langle \mathcal{O}_{\vec{i}_1}(x_1)\mathcal{O}_{\vec{i}_2}(x_2)\cdots\mathcal{O}_{\vec{i}_m}(x_m)\overline{\mathcal{O}}_{\vec{n}}(y)\rangle_{\mathbb{R}^4}$$

where $\displaystyle\sum_{k=1}^{m} \Delta\left(\mathcal{O}_{\vec{i}_k}\right) = \Delta\left(\overline{\mathcal{O}}_{\vec{n}}\right)$

Extremal correlators

It is possible to show that the x_i and y dependence is completely fixed

[Papadodimas, Baggio et al, …]

$$\langle \mathcal{O}_{\vec{i}_1}(x_1)\mathcal{O}_{\vec{i}_2}(x_2)\cdots\mathcal{O}_{\vec{i}_m}(x_m)\overline{\mathcal{O}}_{\vec{n}}(y)\rangle_{\mathbb{R}^4} = \boxed{G_{\vec{i}_1,\ldots,\vec{i}_m,\vec{n}}(\tau,\bar{\tau})}\prod_{k=1}^{m}\frac{1}{|y-x_k|^{2\Delta\left(\mathcal{O}_{\vec{i}_k}\right)}}$$

τ is the coupling of the theory: $\tau = \frac{4\pi}{g_{\rm YM}^2}\mathrm{i} + \frac{\theta}{2\pi}$

we are interested in these objects

Extremal correlators

Define: $\mathcal{O}_k(\infty) := \lim_{y\to\infty} |y|^{2\Delta_k}\mathcal{O}_k(y)$

$$\underbrace{\langle \mathcal{O}_{\vec{i}_1}(0)\mathcal{O}_{\vec{i}_2}(0)\cdots\mathcal{O}_{\vec{i}_m}(0)}\overline{\mathcal{O}}_{\vec{n}}(\infty)\rangle_{\mathbb{R}^4} = G_{\vec{i}_1,\ldots,\vec{i}_m,\vec{n}}(\tau,\bar{\tau})$$

$$\sim \mathcal{O}_{\sum_{k=1}^m \vec{i}_k}(0)$$

[OPE+ suitable normalisation]

We reduce the computation of extremal correlators to the computation of a minimal set of two point functions

How do these correlators behave when $\sum_{k=2}^{N} n_k k$ is large?

Extremal correlators

Next:

- Review of rank 1 case [AG-Komargodski-Tizzano]
- Generalization to higher rank case [AG-Iossa]
- Semiclassical interpretation [Brown-Galvagno-AG–Iossa-Wen]

Rank 1 example - SU(2)

For rank 1 theories the Coulomb branch operators are: $\mathcal{O}_n = \left(\mathrm{Tr}\varphi^2\right)^n$

We have [Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu]

$$G_n(\tau,\bar{\tau}) = \langle \mathcal{O}_n(0)\overline{\mathcal{O}}_n(\infty)\rangle_{\mathbb{R}^4} = \frac{\det_{k,\ell=0,\ldots n}\langle \mathcal{O}_k(N)\overline{\mathcal{O}}_l(S)\rangle_{S^4}}{\det_{k,\ell=0,\ldots n-1}\langle \mathcal{O}_k(N)\overline{\mathcal{O}}_l(S)\rangle_{S^4}}$$

where $\langle \mathcal{O}_k(N)\overline{\mathcal{O}}_l(S)\rangle_{S^4}$ are the correlation functions on the sphere which, for the theories of interest to us, are known explicitly thanks to supersymmetric localization

Q: where do the determinants in (1) come from?

Rank 1 example - SU(2)

Although $\mathbb{R}^4$ and S^4 are conformally equivalent, the dictionary between the correlation functions on S^4 and the correlation functions on $\mathbb{R}^4$ is highly nontrivial because of operator mixing

[Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu]

Indeed, on the sphere, a Coulomb branch operator of dimension Δ, can mix with any other operator of dimension Δ − 2k, k = 1, 2, 3, .. because of the presence of a scale, i.e. the radius of the sphere.

$$\mathcal{O}_\Delta \;\to\; \mathcal{O}_\Delta + R\mathcal{O}_{\Delta-2} + R^2\mathcal{O}_{\Delta-4} + \cdots + R^{\Delta/2}\mathbb{1}$$

R: Ricci scalar

For example $\mathcal{O}_2 = \mathrm{Tr}\varphi^2$ mixes with the identity i.e.

$$\langle(\mathrm{Tr}\varphi^2)\rangle_{S^4} \neq 0$$

Rank 1 example - SU(2)

To compute correlators on $\mathbb{R}^4$ starting from S^4 we need to disentangle this mixing. This can be achieved by identifying a suitable basis of operators, where those with different scaling dimensions are orthogonal. This is done by performing a Gram-Schmidt (GS) orthogonalization with operators of lower dimension. The scalar product in GS is the two point function on S^4.

[Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu]

Rank 1 example - SU(2)

The result of this GS procedure is

$$G_n(\tau,\bar\tau) = \langle \mathcal{O}_n(0)\overline{\mathcal{O}}_n(\infty)\rangle_{\mathbb{R}^4} = \frac{\det_{k,\ell=0,\ldots n}\langle \mathcal{O}_k(N)\overline{\mathcal{O}}_l(S)\rangle_{S^4}}{\det_{k,\ell=0,\ldots n-1}\langle \mathcal{O}_k(N)\overline{\mathcal{O}}_l(S)\rangle_{S^4}}$$

[Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu]

where $\langle \mathcal{O}_k(N)\overline{\mathcal{O}}_l(S)\rangle_{S^4}$ are the correlation functions on the sphere which are known explicitly thanks to supersymmetric localization

Example $\mathcal{N}=2$ SU(2) SQCD

$$G_1(\tau,\bar\tau) = \frac{6}{(\operatorname{Im}\tau)^2} - \frac{135\zeta(3)}{2\pi^2}\frac{1}{(\operatorname{Im}\tau)^4} + \frac{1575\zeta(5)}{4\pi^3}\frac{1}{(\operatorname{Im}\tau)^5} + O\left(\frac{1}{(\operatorname{Im}\tau)^6}\right)$$

$$+\cos\theta\, e^{-\operatorname{Im}\tau}\left(\frac{6}{(\operatorname{Im}\tau)^2} + \frac{3}{\pi}\frac{1}{(\operatorname{Im}\tau)^3} - \frac{135\zeta(3)}{2\pi^2}\frac{1}{(\operatorname{Im}\tau)^4} + O\left(\frac{1}{(\operatorname{Im}\tau)^5}\right)\right) + O\left(e^{-2\operatorname{Im}\tau}\right)$$

Rank 1 example - SU(2)

The result of this GS procedure is

$$G_n(\tau,\bar\tau) = \langle \mathcal{O}_n(0)\overline{\mathcal{O}}_n(\infty)\rangle_{\mathbb{R}^4} = \frac{\det_{k,\ell=0,\ldots n}\langle \mathcal{O}_k(N)\overline{\mathcal{O}}_l(S)\rangle_{S^4}}{\det_{k,\ell=0,\ldots n-1}\langle \mathcal{O}_k(N)\overline{\mathcal{O}}_l(S)\rangle_{S^4}}$$

[Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu]

→ Next: we need to find a way of studying the large n regime.

→ Our strategy: rewrite these determinant as matrix models

[AG,Tizzano,Komargodski, ..]

From determinants to matrix models

If

$$\mu_n = \int_{\mathbb{R}_+} x^n w(x) \mathrm{d}x, \qquad D_n = \det \left(\mu_{i+j}\right)_{i,j=0}^{n-1} \qquad \text{(Hankel determinant)}$$

Then D_n has the following multidimensional representation (Andrèief–Gram–Hein identity)

$$D_n = \frac{1}{n!} \int_{\mathbb{R}_+^n} \mathrm{d}^n x \prod_{1 \le i,j \le n} \left(x_i - x_j\right)^2 \prod_{i=1}^{n} w(x_i)$$

From determinants to matrix models

In the rank 1 example the relevant matrix models are of the form

$$Z(n) = \int [DW] e^{-V(W)}$$

where W are Wishart matrices: $W = A^{\dagger}A$, with A an $n \times n$ complex matrix.

This can be written as a multidimensional integral over the eigenvalues x_i of W :

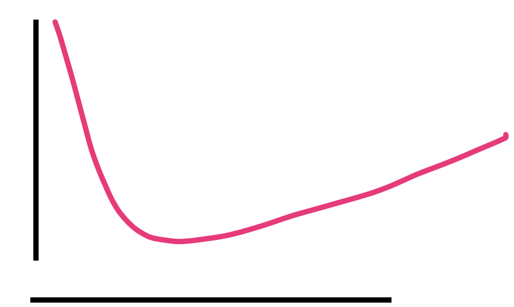
$$Z(n) = \frac{1}{n!} \int_0^{\infty} \mathrm{d^n x} \prod_{i<j} (x_i - x_j)^2 \prod_{i=1}^{n} e^{-V(x_i)}$$

From determinants to matrix models

$$Z(n) = \frac{1}{n!}\int_0^\infty \mathrm{d^n x} \prod_{i<j}(x_i - x_j)^2 \prod_{i=1}^{n} e^{-V(x_i)}$$

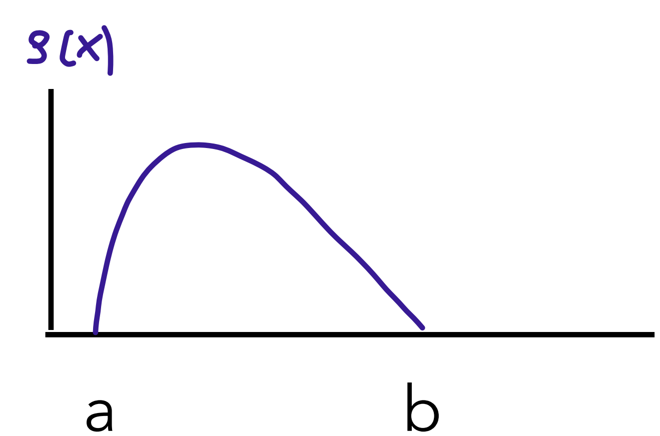
confining potential

repulsive interaction



When n is large, the two interactions reach an equilibrium and the eigenvalues distribute according to [Marchenko-Pastur]

$$\rho(x) = \frac{1}{2\pi x}\sqrt{(b-x)(x-a)}$$



where a and b depend on the potential V(x)

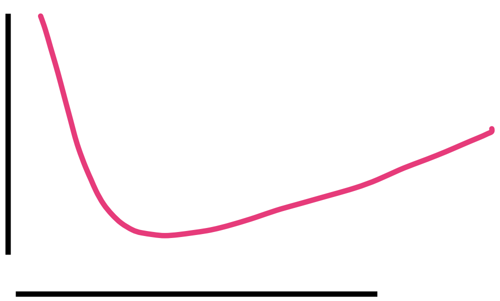
[AG,Tizzano,Komargodski]

Rank 1 example - SU(2)

When we apply the Andrèief-Gram-Hein identity to the determinants appearing in $G_n(\tau,\bar{\tau}) = \langle \mathcal{O}_n(0)\overline{\mathcal{O}}_n(\infty)\rangle_{\mathbb{R}^4}$, we find that these are Wishart matrix models where n is the rank of the Wishart matrices and the potential is

SU(2), $\mathcal{N}=4$ SYM $\qquad V(x) = nx - \frac{1}{2}\log x$

confining potentials



SU(2), $\mathcal{N}=2$ SQCD $\qquad V(x) = nx - \frac{1}{2}\log x - \log Y(\sqrt{\lambda x})$

where $\lambda = n/\mathrm{Im}\tau$ and $Y(x) = \left|\frac{G(1+2ix)G(1-2ix)}{G(1+ix)^4G(1-ix)^4}\right|^2$

G are Barnes functions.

[AG,Tizzano,Komargodski]

Rank 1 example - SU(2)

SU(2), $\mathcal{N} = 4$ SYM

$$G_n^{\mathcal{N}=4} = \frac{Z^{\mathcal{N}=4}(n+1)}{Z^{\mathcal{N}=4}(n)} = \sqrt{2\pi}\left(\frac{1}{\mathrm{Im}\tau}\right)^{2n+\frac{3}{2}} \Gamma(2n+2)$$

exact

SU(2), $\mathcal{N} = 2$ SQCD

$$G_n^{\mathcal{N}=2} = \frac{Z^{\mathcal{N}=2}(n+1)}{Z^{\mathcal{N}=2}(n)} = G_n^{\mathcal{N}=4} \times \frac{\left\langle Y(\sqrt{\lambda}x)\right\rangle_{n+1}^{\mathcal{N}=4}}{\left\langle Y(\sqrt{\lambda}x)\right\rangle_{n}^{\mathcal{N}=4}}$$

emergent

large n analysis using matrix model tools

[AG,Tizzano,Komargodski]

Rank 1 example - SU(2)

Let us focus on SU(2), $\mathcal{N} = 2$ SQCD. We can study the matrix model in 2 limits

1) n large, $\mathrm{Im}\tau$ fixed:

$$\log G_n = 2\log \mathrm{Im}\tau + 2n\left(1 - 2\log \mathrm{Im}\tau\right) + \log \Gamma\left[2n + \frac{5}{2}\right] + \mathcal{O}\left(\mathrm{e}^{-\sqrt{n}}\right)$$

→ We can prove EFT prediction [Maeda, Hellerman, Orlando, Reffert, Watanabe]

This simplicity is a consequence of non trivial identities $Y(x)$. For example

$$\int_0^\infty \log\left(e^{8x^2\log 2}x^{-1}Y(x)\right)dx = 0$$

[AG,Tizzano,Komargodski]

Rank 1 example - SU(2)

Let us focus on SU(2), $\mathcal{N} = 2$ SQCD. We can study the matrix model in 2 limits

2) n large, $\lambda = n/\mathrm{Im}\tau$ fixed: $\log G_n = \sum_{k\geq 0} n^{1-k} C_k(\lambda)$

→ In this limit we get new non-perturbative predictions beyond EFT

The fact that this double scaling limit even exists in the SCFT is non trivial [Bourget, Gomez, Russo]

Example: $C_0(\lambda) = 2\log\lambda - 2$

$$C_1(\lambda) = \frac{3}{2}\log\lambda + \int \mathrm{d}x\rho(x)\big(G\ \mathit{Barnes\ functions}\big)$$

$\sim$ Bessels K functions

Rank 1 example - SU(2)

What can we do with this matrix models (MM)?

- perform a systematic large n expansion and prove the prediction of large charge EFT + double scaling limit

- go beyond and compute analytically some non-perturbative effects

- The MM techniques can be extended to higher theories, where we do not have EFT type predictions yet

↖

This is what I want to discuss now

Next: matrix models for higher rank

An example: SU(3)

ArXiv 2408.07391 with Cristoforo Iossa

Rank 2 example - SU(3)

In SU(3) theories Coulomb branch operators are built from these two generators

$$\phi_3 = \mathrm{Tr}\varphi^3, \quad \phi_2 = \mathrm{Tr}\varphi^2$$

More precisely we have operators of the form: $\Phi_n^m = \left(\mathrm{Tr}\varphi^2\right)^n \left(\mathrm{Tr}\varphi^3\right)^m$

$$\Delta(\Phi_m^n) = \frac{R(\Phi_m^n)}{2} = 3m + 2n$$

scaling dimension R-charge

Any other CB operator can be written as a combinations of Φ_n^m

Rank 2 example - SU(3)

Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu:

→ to compute correlators on $\mathbb{R}^4$ we go on S^4 and disentangle the operator mixing by applying Gram-Schmidt with operator of lower and equal dimensions

To analyze the large m and/or n regime it, we need to make this systematic.

For this is convenient to work in another basis of operators.

$$\Phi_n^m = (\phi_2)^n \, (\phi_3)^m \longrightarrow \mathcal{O}_n^m$$

Example: SU(3) $\mathcal{N} = 4$ SYM

Let me first focus on SU(3) $\mathcal{N} = 4$ SYM. It is convenient to work with

$$\mathcal{O}_n^m = \phi_2^n \mathcal{O}_0^m$$

where $\mathcal{O}_0^m$ is constructed starting with $(\phi_3)^m$ and then do a GS orthogonalization procedure wrt operator with the same dimension. This gives (eg m even)

$$\mathcal{O}_0^m = \phi_3^m + \sum_{\frac{m}{2} > j \geq 0} c_j \, \phi_3^{2j} \phi_2^{3(m/2-j)}$$

determined by GS on operators with the same dimension

eg: $\mathcal{O}_0^4 = \phi_3^4 - \frac{1}{8}\phi_2^3\phi_3^2 + \frac{1}{576}\phi_2^6$

Recall: $\phi_3 = \mathrm{Tr}\varphi^3, \quad \phi_2 = \mathrm{Tr}\varphi^2$

Example: SU(3) $\mathcal{N} = 4$ SYM

… after some algebra ….

$$\langle \mathcal{O}_n^m \overline{\mathcal{O}_k^\ell} \rangle_{\mathbb{R}^4}^{\mathcal{N}=4} = \delta_{nk}\delta_{m\ell} G_n^m(\tau, \bar{\tau})$$

$Z^{(m)}(n)$ is a Wishart matrix model

with

$$G_n^m(\tau, \bar{\tau}) = \frac{Z_J\left(1 + \left\lfloor \frac{m}{2} \right\rfloor\right)}{Z_J\left(\left\lfloor \frac{m}{2} \right\rfloor\right)} \frac{Z^{(m)}(n+1)}{Z^{(m)}(n)}$$

[AG-Iossa]

$Z_J(n)$ is a Jacobi matrix model

Example: SU(3) $\mathcal{N} = 4$ SYM

More precisely the two matrix models are

n: size of matrices

Recall : $O^m_n = \phi_2^n O^m_0$

$O^m_0 = \phi_3^m + GS$

(1) $$Z^{(m)}(n) = \frac{1}{n!}\int_{\mathbb{R}^n_+} \mathrm{d}^n y \prod_{i<j}(y_i - y_j)^2 \prod_{i=1}^{n} \mathrm{e}^{-2\pi \mathrm{Im}\tau y_i} y_i^{3m+3}$$

m: size of matrices

$O^m_n = \phi_2^n O^m_0$

$= \phi_3^m + GS$

(2) $$Z_{\mathrm{J}}\left(\left\lfloor \frac{m}{2} \right\rfloor\right) = \frac{1}{\left(\left\lfloor \frac{m}{2} \right\rfloor\right)!}\int_{[0,1]^{\lfloor \frac{m}{2} \rfloor}} \mathrm{d}^{\lfloor \frac{m}{2} \rfloor} x \prod_{i<j}\left(x_i - x_j\right)^2 \prod_{i=1}^{\lfloor \frac{m}{2} \rfloor} \sqrt{\frac{1}{x_i} - 1}$$

Example: SU(3) $\mathcal{N} = 4$ SYM

These two matrix models are exactly solvable and we obtain

$$G_n^m(\tau, \bar{\tau}) = \frac{Z_{\mathrm{J}}\left(1 + \left\lfloor \frac{m}{2} \right\rfloor\right)}{Z_{\mathrm{J}}\left(\left\lfloor \frac{m}{2} \right\rfloor\right)} \frac{Z^{(m)}(n+1)}{Z^{(m)}(n)}$$

$$= \frac{\Gamma(n+1)\Gamma(3m+n+4)}{3^{m+1} 2^{6m+2n+1} \pi^{3m+2n} \mathrm{Im}\tau^{3m+2n}}$$

Therefore the large m, n asymptotic follows easily

[AG-Iossa]

Example: SU(3) $\mathcal{N}=4$ SYM

One key insight from this simple example is the existence of a class of operators $\mathcal{O}_0^m$ whose correlation functions exhibit behavior analogous to those in SU(2) theories at all orders in the 1/m expansion.

[AG-Iossa]

$$G_0^m = \langle \mathcal{O}_0^m \overline{\mathcal{O}_0^m} \rangle_{\mathbb{R}^4}^{\mathcal{N}=4} = 2^{-6m-1} 3^{-m-1} \pi^{-3m} \mathrm{Im}(\tau)^{-3\mathrm{m}} \Gamma(3\mathrm{m}+4)$$

$$3m = R/2$$

$$4 = \frac{N(N-1)}{2} + 1$$

Recall: $\mathcal{O}_0^m = \phi_3^m + \mathrm{GS}$

Example: SU(N) $\mathcal{N} = 4$ SYM

This can be generalize to any SU(N)   [Brown-Galvagno-AG-Iossa-Wen]

$$\mathcal{O}_0^m = \phi_N^m + \mathrm{GS}$$

$$G_0^m = \langle \mathcal{O}_0^m \overline{\mathcal{O}_0^m} \rangle_{\mathbb{R}^4}^{\mathcal{N}=4} = \frac{N^{2m}}{(N \mathrm{Im}\tau)^{Nm}} \frac{\Gamma\left(\frac{R}{2} + \alpha_N + 1\right)}{\Gamma\left(\alpha_N + 1\right)} (1 + \mathcal{O}(\mathrm{e}^{-m}))$$

where $\phi_N = \mathrm{Tr}(\varphi^N)$ and $R = 2mN,\ \alpha_N = \frac{1}{2} N(N-1)$

These operators provide a promising starting point to extend the EFT techniques of Hellermann et al. at higher rank

Next: SU(3) $\mathcal{N} = 2$ SQCD

[AG-Iossa]

Example: SU(3) $\mathcal{N} = 2$ SQCD

Punchline: correlators of $\mathcal{N} = 2$ are expectation values of $Z_{\rm G}$ within the $\mathcal{N} = 4$ matrix models

m: controlled by Jacobi matrix model $\sim \left(\mathrm{Tr}(\varphi^3)\right)^m$

n: controlled by Wishart matrix model $\sim \left(\mathrm{Tr}(\varphi^2)\right)^n$

$Z_{\rm G} \sim$ couple the two models

where $Z_{\rm G}(a_1, a_2) = \prod_{i \neq j} H(\mathrm{i}(a_i - a_j)) \prod_{i=1}^{3} H(\mathrm{i}a_i)^{-6}$, $H(a) = G(1+a)G(1-a)$ and $a_3 = -(a_1 + a_2)$

→ systematic large m, n expansions with different phases

(happy to discuss after the talk if interested)

Example: SU(3) $\mathcal{N}=2$ SQCD

Case I $n=0,\ m\to\infty\,,\ \mathrm{Im}\tau\to\infty\,,\ \lambda=\dfrac{m}{2\pi\mathrm{Im}\tau}$ fixed. [AG-Iossa]

$$\log G_0^m=\langle \mathcal{O}_0^m\overline{\mathcal{O}_0^m}\rangle_{\mathbb{R}^4}^{\mathcal{N}=2}=\int_0^1 \mathrm{d}x\sigma_{\mathrm{J}}(x)\log\left(Z_{\mathrm{G}}(x,3\lambda)\right)+\mathcal{O}(1/m)$$

eigenvalue density of Jacobi model

$$\sigma_{\mathrm{J}}(x)=\frac{1}{\pi\sqrt{x(1-x)}}$$

$\sim$ bunch of Barnes functions

weak coupling : $$\log G_0^m=\sum_{k=1}^{\infty}\frac{9(-1)^k2^{k+2}\left(3^k-1\right)\lambda^{k+1}\zeta(2k+1)\Gamma\left(k+\frac{3}{2}\right)}{\sqrt{\pi}(k+1)^2k!}$$

Example: SU(3) $\mathcal{N} = 2$ SQCD

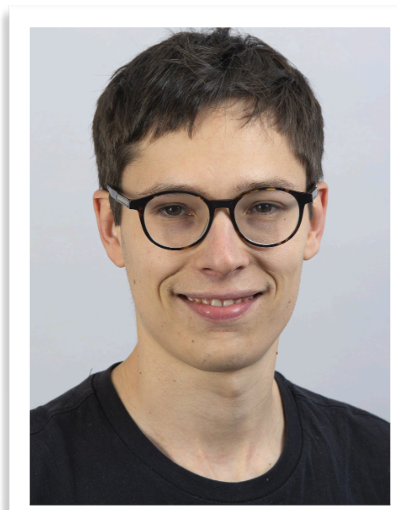
Case I $n = 0,\ m \to \infty\,,\ \mathrm{Im}\tau \to \infty\,,\ \lambda = \dfrac{m}{2\pi \mathrm{Im}\tau}$ fixed. [AG-Iossa]

$$\log G_0^m = \int_0^1 \mathrm{d}x \sigma_{\mathrm{J}}(x) \log\left(Z_G(x, 3\lambda)\right) + \mathcal{O}(1/m)$$

strong coupling: $\log G_0^m = -\,9\lambda \log 3 + \log \lambda - \mathscr{C}_{np}(\sqrt{6\lambda}) + 3\,\mathscr{C}_{np}(\sqrt{2\lambda})$

$$\mathscr{C}_{np}(\sqrt{\lambda}) = \sum_{n\geq 1} \frac{6}{n^2\pi^2}\left(K_0(2n\pi\sqrt{\lambda}) + 2n\pi\sqrt{\lambda}K_1(2n\pi\sqrt{\lambda})\right) = \mathrm{e}^{-2\pi\sqrt{\lambda}}\left(\frac{6\sqrt[4]{\lambda}}{\pi} + \frac{33\sqrt[4]{\frac{1}{\lambda}}}{8\pi^2}\mathcal{O}\left(\left(\frac{1}{\lambda}\right)^{3/4}\right)\right)$$

From matrix models to semiclassics

ArXiv 2502.xxxx with A Brown, F Galvagno, C Iossa, C Wen

Semiclassics

Thanks to the matrix model structure, we are able to derive the large m and n behavior of such correlators and see some nice and simple structure emerging.

Can we derive this by semiclassical analysis of the path integral?

$$\int \mathscr{D}[\mathrm{fields}]\, e^{-S_{\mathrm{SYM}}}\ \mathscr{O}_{\mathrm{n}}^{\mathrm{m}}\ \mathscr{O}_{\mathrm{n}}^{\mathrm{m}}$$

In the following we study this question for $\mathscr{N} = 4$ SYM

Semiclassics

One difficulty for higher ranks was that it was not clear what the "nice operators" were.

Brown-Galvagno-
AG-Iossa-Wen

The nice operators are

$$\mathcal{O}_0^m = \phi_N^m + GS$$

determined by GS on operators with the same dimension

Recall: $\phi_N = \mathrm{Tr}(\varphi^N)$

Semiclassics

When considering these operators, our results are compatible with the following classical configuration of the scalar field: [Brown-Galvagno-AG-Iossa-Wen]

$$\varphi^{\mathrm{cl}} = \sqrt{\frac{m}{\mathrm{Im}\tau}}\left(\mathrm{e}^{\mathrm{i}\theta}\Omega^N + \mathrm{e}^{-\mathrm{i}\theta}\overline{\Omega}^N\right), \quad \theta \in [0,2\pi]$$

where $\Omega^N_{k\ell} = \delta_{k\ell}\mathrm{e}^{\frac{\mathrm{i}\pi\ell}{N}}$. This configuration breaks $SU(N) \ \rightarrow \ U(1)^{N-1}$

This implies that

$$\langle \mathcal{O}_0^m \mathcal{O}_0^m \rangle_{\mathbb{R}^4} = \frac{1}{\mathrm{Im}\tau^{3m}}\Gamma\left(Nm+1\right)\left(Nm\right)^{\alpha_N}\left(1 + \mathcal{O}\left(\frac{1}{m}\right)\right)$$

$$\alpha_N = \frac{1}{2}N(N-1)$$

→ compatible with our results

More general correlators

We can use this insight and go beyond extremal correlators

$$\int \mathscr{D}[\text{fields}]\, e^{-S_{SYM}}\ \mathcal{O}_0^{m}\ \mathcal{O}_0^{m}(\ldots)$$

not necessarily coulomb branch operators

→ ArXiv 2502.xxxx with A Brown, F Galvagno, C Iossa, C Wen

Summary and Outlook

Using the guideline coming from matrix models, we start the exploration of the large charge regime of 1/2BPS correlators for higher rank theories:

- emergence of a multiple matrix model description where the size of the matrices in each model corresponds to the maximal number of insertions of each generator.

- this allows to study make new prediction for the behavior in the large charge sector of higher rank theories.

- we identified a class of operator whose behavior is similar to the rank 1 case.

- partial interpretation from semiclassics in $\mathcal{N} = 4$

Summary and Outlook

Many open questions

- EFT-type description at higher rank?
- connection to integrability
- Matrix model for SU(N) with N>3
- gravity meaning of our special operators
- study more general observables
- how general is the statement that regimes of large quantum numbers behave as matrix models?
- matrix models beyond SCFT

• • •

Thank you for your attention

More general correlators

For example, we consider the following 4 point function of 1/2 BPS operators in $\mathcal{N} = 4$ (not extremal)

$$G_m(x, Y) = \frac{\langle \mathcal{O}_0^m(x_1, Y_1)\mathcal{O}_0^m(x_2, Y_2)\phi_2(x_3, Y_3)\phi_2(x_4, Y_4)\rangle_{\mathbb{R}^4}}{\langle \mathcal{O}_0^m(x_1, Y_1)\mathcal{O}_0^m(x_2, Y_2)\rangle_{\mathbb{R}^4}}$$

Y_i: polarization vectors, if we align them in a certain way we get Coulomb branch operators.

fixed by superconformal symmetry [Eden, Petkou, Schubert, Sokatchev]

$$G_m(x, Y) = G_m^{\text{free}}(x, Y) + I_4(x, Y)\mathcal{T}_m(u, v, \tau, \bar{\tau})$$

$$u = z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad v = (1-z)(1-\bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Interesting part, coupling dependent

More general correlators

Our semiclassical approach implies that [Brown-Galvagno-AG-Iossa-Wen]

$$\mathcal{T}_m(u, v, \tau, \bar{\tau}) = \frac{K}{2} \sum_{s=1}^{K-1} L(u, v; 4\lambda \sin^2 \frac{\pi s}{N})$$

where

$$L(u, v; a) = \frac{1}{u} \left[\left(\sum_{\ell=0}^{\infty} (-a)^{\ell} P^{(\ell)}(u, v) \right)^2 - 1 \right]$$

ℓ−loop ladder Feynman integral, known in closed form [Usyukina-Davydychev]

masses associated to the massive fluctuations around our classical configuration

Extra technicalities

Standard Normalization:

$$\langle O_I(0)\overline{O}_J(\infty)\rangle = \delta_{I,J}$$

$$\langle O_I(0)O_K(0)\overline{O}_{K+I}(\infty)\rangle = C_{K,I}$$

(structure constants)

Our Normalization:

$$\langle O_I(0)\overline{O}_J(\infty)\rangle = G_{2I}\delta_{I,J}$$

$$\langle O_I(0)O_K(0)\overline{O}_{K+I}(\infty)\rangle = G_{2(K+I)}$$

$$O_K \to O_K\sqrt{G_{2K}}$$

$$C_{I,K} = \sqrt{\frac{G_{2(I+K)}}{G_{2I}G_{2K}}}$$

Example: SU(3) $\mathcal{N} = 2$ SQCD

[AG-Iossa]

Case II $m, n \to \infty$, $\text{Im}\tau \to \infty$, $\lambda = \dfrac{m}{2\pi \text{Im}\tau}$, $\kappa = \dfrac{n}{2\pi \text{Im}\tau}$ fixed, $\beta = \dfrac{n}{m}$

$$\log G^{m}_{\beta m} = \left(1 + \kappa \partial_\kappa + \beta \partial_\beta\right) \left(\int_a^b \mathrm{d}y \rho_{\text{MP}}(y) \int_0^1 \mathrm{d}x \sigma_{\text{J}}(x) \log\left(Z_{\text{G}}(x, \kappa y)\right) \right) + \mathcal{O}(1/m)$$

density of Wishart model

$$\rho_{\text{MP}}(x) = \frac{1}{2\pi x} \sqrt{(b-x)(x-a)}$$

$$a = 2 + 3\beta^{-1} - 2\sqrt{1 + 3\beta^{-1}}$$

$$b = 2 + 3\beta^{-1} + 2\sqrt{1 + 3\beta^{-1}}$$

density of Jacobi model

$$\sigma_{\text{J}}(x) = \frac{1}{\pi \sqrt{x(1-x)}}$$

$\sim$ bunch of Barnes functions

[AG-Iossa]

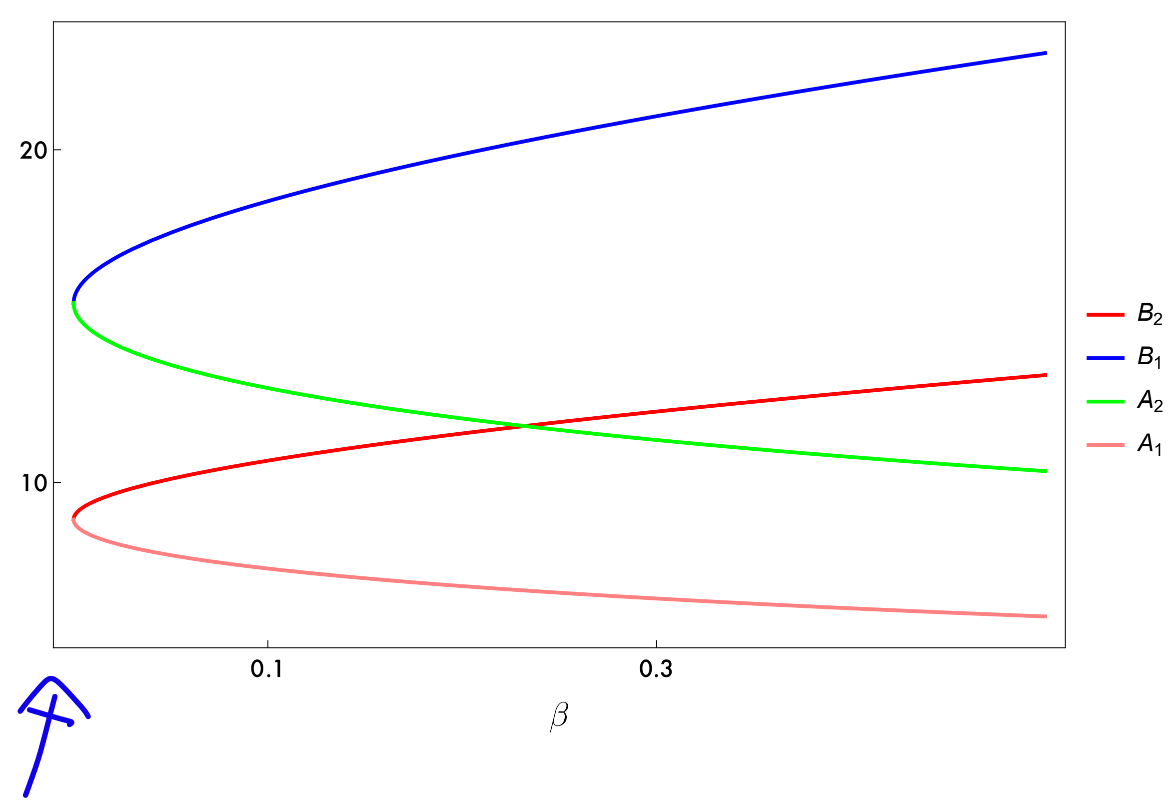
Example: SU(3) $\mathcal{N}=2$ SQCD

Case II $m, n \to \infty$, $\mathrm{Im}\tau \to \infty$, $\lambda = \dfrac{m}{2\pi \mathrm{Im}\tau}$, $\kappa = \dfrac{n}{2\pi \mathrm{Im}\tau}$ fixed, $\beta = \dfrac{n}{m}$

At strong 't Hooft coupling (λ, κ large) we find 4 instantons actions

$$A_i(\beta)\sqrt{\lambda} = 3^{\frac{i-1}{2}} \frac{2\sqrt{6}\pi}{\sqrt{2\beta + 2\sqrt{\beta(\beta+3)} + 3}} \sqrt{\lambda}$$

$$B_i(\beta)\sqrt{\lambda} = 3^{\frac{i-1}{2}} \frac{2\pi\sqrt{6\lambda}}{\sqrt{\left(2\beta - 2\sqrt{\beta(\beta+3)} + 3\right)}}$$



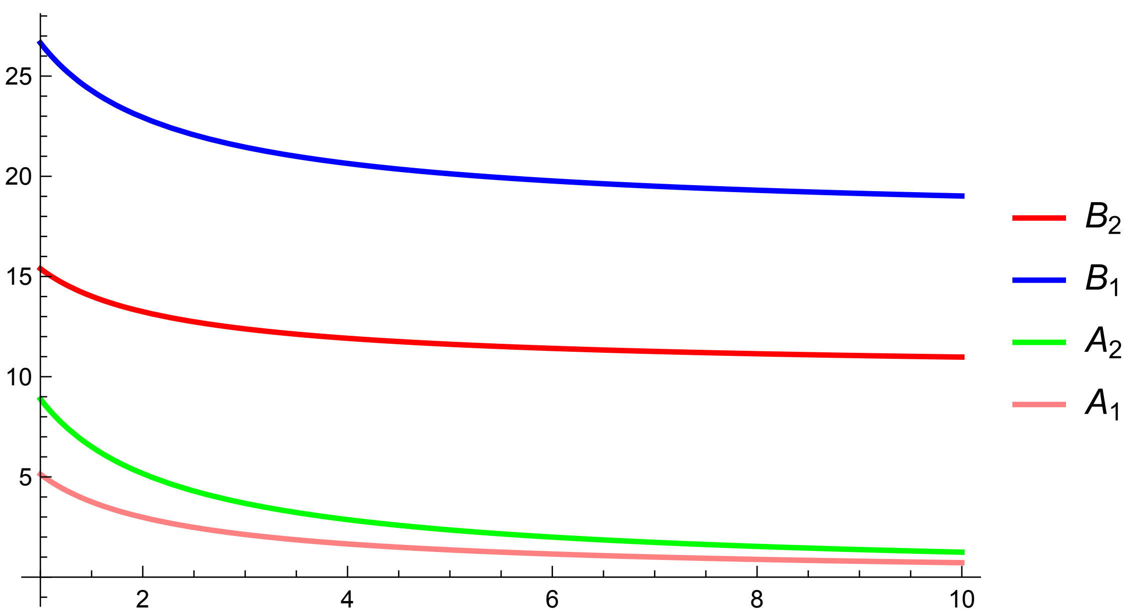
if $\beta \to 0$: we recover **Case I** and $A_i(0) = B_i(0)$

Example: SU(3) $\mathcal{N} = 2$ SQCD

[AG-Iossa]

Case II $m, n \to \infty$, $\text{Im}\tau \to \infty$, $\lambda = \dfrac{m}{2\pi \text{Im}\tau}$, $\kappa = \dfrac{n}{2\pi \text{Im}\tau}$ fixed, $\beta = \dfrac{n}{m}$

$$\tilde{A}_i(\beta)\sqrt{\kappa} = 3^{\frac{i-1}{2}} \frac{2\sqrt{6}\pi}{\sqrt{\beta}\sqrt{2\beta + 2\sqrt{\beta(\beta+3)} + 3}}\sqrt{\kappa} \qquad \tilde{B}_i(\beta)\sqrt{\kappa} = 3^{\frac{i-1}{2}} \frac{2\pi\sqrt{6}}{\sqrt{\beta}\sqrt{\left(2\beta - 2\sqrt{\beta(\beta+3)} + 3\right)}}\sqrt{\kappa}$$



if $\beta \to \infty$: two instanton actions vanishes $A_i(\beta)\sqrt{\lambda} \to 0$

new perturbative series emerging

[AG-Iossa]

Example: SU(3) $\mathcal{N} = 2$ SQCD

Indeed in the **limit** $\boldsymbol{\beta = \frac{n}{m} \to \infty}$ a new perturbative series at large $\kappa = \dfrac{n}{2\pi \mathrm{Im}\tau}$ emerges

$$\lim_{\beta\to\infty} \log G^m_{\beta m} = \log\left(\frac{\kappa}{6\sqrt{3}}\right) - 6\kappa\log(3) - 2 + F^{\mathrm{p}}(\kappa) + \mathcal{O}(\mathrm{e}^{-B_i \kappa^{1/2}})$$

$$F^{\mathrm{p}}(\kappa) = \sum_{n\geq 0} \frac{3\ 2^{-3n-\frac{1}{2}}\left(3^{n+\frac{3}{2}}-1\right)(n+1)\pi^{-2n-\frac{9}{2}}\kappa^{-n-\frac{1}{2}}\zeta(2n+3)\Gamma\left(n+\frac{1}{2}\right)^3}{\Gamma(n+1)}$$

This is in fact a divergent sum. Its Borel transform has poles on the real axis but its median Borel summation agree with the following exact answer.

$$\lim_{\beta\to\infty} \log G^m_{\beta m} = -12\int_0^\infty \frac{e^x}{x(e^x-1)^2}\left(2 + J_0(x\sqrt{2\kappa})^2 - 3J_0\left(x\sqrt{\frac{2\kappa}{3}}\right)^2\right)$$