A compendium of logarithmic corrections in AdS/CFT

Nikolay Bobev

Instituut voor Theoretische Fysica, KU Leuven

String Theory as a Bridge between Gauge Theory and Quantum Gravity

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The Log Team

Marina David (Leuven)

Junho Hong (Leuven $\to$ Seoul)

Valentin Reys (Saclay $\to$ Paris)

Xuao Zhang (Leuven $\to$ Beijing)

Motivation

Supersymmetric localization allows for the exact calculation of physical observables in supersymmetric QFTs.

Apply this tool to SCFTs with holographic duals in string and M-theory.

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Goal: Describe recent progress on these topics for 3d SCFTs with AdS_4 duals in M-theory focusing on the $\log N$ terms in the large N expansion.

Plan

- Motivation ✓
- Supersymmetric partition functions
- Log corrections from supergravity
- The unbearable lightness of the KK scale
- Breaking supersymmetry: black holes and thermal observables
- Outlook

Supersymmetric partition functions

[Aharony,Bergman,Jafferis,Maldacena]; [Kapustin,Willett,Yaakov]; [Drukker,Mariño,Putrov]; [Mariño,Putrov]; [Fuji,Hirano,Moriyama]; [Herzog,Klebanov,Pufu,Tesileanu]; [Benini,Zaffaroni]; [Closset,Kim]; [Benini,Hristov,Zaffaroni]; [Liu,Pando Zayas,Rathee,Zhao]; [NPB,Hong,Reys]; [Nosaka]; [Hatsuda]; [Hristov]; [Chester,Kalloor,Sharon]; [Bhattacharya^2,Minwalla,Raju]; [Kim]; [Choi,Hwang,Kim]; [Choi,Hwang]; [Nian,Pando Zayas]; [NPB,Choi,Hong,Reys]; [NPB,De Smet,Hong,Reys,Zhang]

ABJM and holography

The ABJM theory: $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ 3d CS-matter theory with two pairs of bi-fundamental chirals $(A_{1,2}, B_{1,2})$ and superpotential

$$\mathcal{W} = \mathrm{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)\,.$$

For $k > 2$ it has $\mathcal{N} = 6$ supersymmetry and $\mathrm{SU}(4)_R \times \mathrm{U}(1)_b$ global symmetry. Describes N M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$.

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- In the limit of fixed k and large N, the ABJM theory is dual to the M-theory background $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

$$(L/\ell_{\mathrm{P}})^6 \sim k\,N\,.$$

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- At large k and fixed 't Hooft coupling $\lambda = N/k$ the theory is dual to type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$

$$k\, g_{\mathrm{st}} = L/\ell_{\mathrm{s}} \sim \lambda^{1/4} \,.$$

  Perturbative type IIA string theory at large k and small g_{st}, i.e. fixed λ and large N.

ABJM on S^3

The path integral on S^3 can be computed by supersymmetric localization and reduces to a matrix model

$$Z(N,k) = \frac{1}{N!^2} \int \frac{d^N\mu}{(2\pi)^N} \frac{d^N\nu}{(2\pi)^N} \, \exp\left[\frac{ik}{4\pi} \sum_{i=1}^{N} (\mu_i^2 - \nu_i^2) \right] \frac{\Pi_{i<j} \left[2\sinh(\frac{\mu_i - \mu_j}{2}) \right]^2 \left[2\sinh(\frac{\nu_i - \nu_j}{2}) \right]^2}{\Pi_{i,j} \left[2\cosh(\frac{\mu_i - \nu_j}{2}) \right]^2}$$

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Different methods have been used to study $Z(N,k)$ at large N

- Map to CS theory on $S^3/\mathbb{Z}_2$ (or topological strings on $\mathbb{P}^1 \times \mathbb{P}^1$) and solve with large N techniques. Applies at large N, fixed N/k.
- Study the large N limit at fixed k numerically.
- Map the problem to a free Fermi gas on the real line with non-standard kinetic term. Valid at large N and finite k.

ABJM on S^3 - An Airy Tale

The path integral on a squashed S^3 with real mass deformation can be computed by supersymmetric localization and reduces to a matrix model.

$$Z_{S^3}(N,k;m_a,b) \overset{!}{=} e^{\mathcal{A}(k,\Delta,b)} C^{-\frac{1}{3}} \mathrm{Ai}[C^{-\frac{1}{3}}(N-B)] + \mathcal{O}(\mathrm{e}^{-\sqrt{N}}) \,,$$

with fixed k, large N and

$$C = \frac{2}{\pi^2 k} \frac{(b+b^{-1})^{-4}}{\prod_{a=1}^4 \Delta_a} \,, \quad B = \frac{k}{24} - \frac{1}{12k} \sum_{a=1}^{4} \frac{1}{\Delta_a} + \frac{1 - \frac{1}{4}\sum_a \Delta_a^2}{3k(b+b^{-1})^2 \prod_{a=1}^4 \Delta_a} \,,$$

and

$$\Delta_1 = \frac{1}{2} - \mathrm{i}\,\frac{m_1+m_2+m_3}{b+b^{-1}} \,, \qquad \Delta_2 = \frac{1}{2} - \mathrm{i}\,\frac{m_1-m_2-m_3}{b+b^{-1}} \,,$$
$$\Delta_3 = \frac{1}{2} + \mathrm{i}\,\frac{m_1+m_2-m_3}{b+b^{-1}} \,, \qquad \Delta_4 = \frac{1}{2} + \mathrm{i}\,\frac{m_1-m_2+m_3}{b+b^{-1}} \,.$$

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The large N expansion takes the explicit form

$$-\log Z_{S^3} = \frac{2}{3\sqrt{C}} N^{\frac{3}{2}} - \frac{B}{\sqrt{C}} N^{\frac{1}{2}} + \frac{1}{4}\log N - \mathcal{A} + \frac{1}{4}\log\frac{32}{k} + \mathcal{O}(N^{-\frac{1}{2}})\,.$$

The topologically twisted index

The topologically twisted index (TTI) is the partition function of 3d $\mathcal{N} = 2$ SCFTs on $S^1 \times \Sigma_{\mathfrak{g}}$. Supersymmetry is preserved by a topological twist on $\Sigma_{\mathfrak{g}}$.

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Using supersymmetric localization the path integral can be reduced to a matrix integral and computed at large N and fixed k. The free energy $F_{S^1 \times \Sigma_{\mathfrak{g}}} = -\log Z_{S^1 \times \Sigma_{\mathfrak{g}}}$, takes the simple form:

$$F_{S^1 \times \Sigma_{\mathfrak{g}}} = \frac{\pi \sqrt{2k \Delta_1 \Delta_2 \Delta_3 \Delta_4}}{3} \sum_{a=1}^{4} \frac{\mathfrak{n}_a}{\Delta_a} \left(\hat{N}_{\Delta}^{\frac{3}{2}} - \frac{\mathfrak{c}_a}{k} \hat{N}_{\Delta}^{\frac{1}{2}} \right) + \frac{1 - \mathfrak{g}}{2} \log \hat{N}_{\Delta} - \hat{f}_0(k, \Delta, \mathfrak{n}) + \mathcal{O}(e^{-\sqrt{N}}) \,,$$

where $\sum_{a=1}^{4} \Delta_a = 2$, $\sum_{a=1}^{4} \mathfrak{n}_a = 2(1 - \mathfrak{g})$, and

$$\hat{N}_{\Delta} \equiv N - \frac{k}{24} + \frac{1}{12k} \sum_{a=1}^{4} \frac{1}{\Delta_a} \,, \qquad \mathfrak{c}_a = \frac{\prod_{b \neq a} (\Delta_a + \Delta_b)}{8 \Delta_1 \Delta_2 \Delta_3 \Delta_4} \sum_{b \neq a} \Delta_b \,.$$

The holographic dual is given by (Euclidean) supersymmetric static Reissner-Nordström BHs in AdS_4. The TTI computes the entropy of these BHs.

The superconformal index

The superconformal index (SCI), or $S^1 \times_\omega S^2$ partition function, counts $\frac{1}{16}$-BPS operators in 3d $\mathcal{N} = 2$ SCFTs.

It is useful to consider the Cardy-like limit $\omega \to 0$. The SCI can then be analyzed with similar tools as the TTI.

For the ABJM theory at fixed k and large N we find the following ω^{-1} and ω^0 results (for $\Delta_a = 1/2$)

$$
\begin{aligned}
&\log Z_{S^1 \times_\omega S^2}(N, k, \omega) \\
&\quad = -\frac{\pi\sqrt{2k}}{3}\left[\left(\frac{1}{2\omega} + 1\right)\left(N - \frac{k}{24} + \frac{2}{3k}\right)^{\frac{3}{2}} - \frac{3}{k}\left(N - \frac{k}{24} + \frac{2}{3k}\right)^{\frac{1}{2}}\right] \\
&\qquad - \frac{2}{\omega}\hat{g}_0(k) - \frac{1}{2}\log\left(N - \frac{k}{24} + \frac{2}{3k}\right) + \hat{f}_0(k) + \mathcal{O}(\mathrm{e}^{-\sqrt{N}}) + \mathcal{O}(\omega)\,.
\end{aligned}
$$

This index captures the entropy of supersymmetric AdS_4 Kerr-Newman black holes.

$\log N$ results in 3d $\mathcal{N} = 2$ susy QFT

Theory	$\mathcal{M}_3$	log coefficient $\mathcal{C}$	Ref.	10/11d bulk
M2-brane theories (class I)				
$(S^7/\mathbb{Z}_k)_{\text{free}}$ (†)	S^3_b	$-\frac{1}{4}$	[25, 26, 44, 53]	✓ [16] (s.c.)
	$S^1 \times \Sigma_\mathfrak{g}$	$-\frac{1}{2}(1-\mathfrak{g})$	[18, 44]	✓ [17] (s.c.)
	$S^1 \times_\omega S^2$	$-\frac{1}{2}$	[45]	✗
$(S^7/\mathbb{Z}_{N_f})_{\text{f.p.}}$ (†)	S^3_b	$-\frac{1}{4}$	[40, 42, 46, 54]	✗
	$S^1 \times \Sigma_\mathfrak{g}$	$-\frac{1}{2}(1-\mathfrak{g})$	[46]	
	$S^1 \times_\omega S^2$	$-\frac{1}{2}$	[45]	
$N^{010}/\mathbb{Z}_k$	$S^3_{b=1}$	$-\frac{1}{4}$	[26, 46]	✗
	$S^1 \times \Sigma_\mathfrak{g}$	$-\frac{1}{2}(1-\mathfrak{g})$	[46]	✓ [20]
	$S^1 \times_\omega S^2$	$-\frac{1}{2}$	[55]	✗
$V^{52}/\mathbb{Z}_k$	$S^1 \times \Sigma_\mathfrak{g}$	$-\frac{1}{2}(1-\mathfrak{g})$	[46]	✓ [20]
	$S^1 \times_\omega S^2$	$-\frac{1}{2}$	[55]	✗
$Q^{111}/\mathbb{Z}_k$	$S^1 \times \Sigma_\mathfrak{g}$	$-\frac{1}{2}(1-\mathfrak{g})$	[46]	✓ [20]
	$S^1 \times_\omega S^2$	$-\frac{1}{2}$	[55]	
M5-brane theories (class II)				
A_{N-1}	S^3_b	$-\frac{1}{2}$	[56, 57]	✗
	$S^1 \times \Sigma_{\mathfrak{g}>1}$	$(b_1(\mathcal{H}_3)-1)(1-\mathfrak{g})$	[19, 58]	✓ [58]
	$S^1 \times_\omega S^2$	$b_1(\mathcal{H}_3)-1$	[58]	
D_N	S^3_b	0	[57]	✗
	$S^1 \times \Sigma_{\mathfrak{g}>1}$			
	$S^1 \times_\omega S^2$			
IIA theories (class III)				
$\mathbb{CP}^3$ (†)	S^3_b	$-\frac{1}{6}$	[44, 59]	✗
	$S^1 \times \Sigma_\mathfrak{g}$	$\frac{2}{3}(1-\mathfrak{g})$	[44, 60]	
$\mathbb{CP}^3_{\text{def}}$	$S^3_{b=1}$	$-\frac{1}{6}$	[61]	✗
S^6_{def}	$S^3_{b=1}$	$-\frac{2}{9}$ (fixed k)	[62]	✗
		$-\frac{1}{6}$ ('t Hooft)		
	$S^1 \times \Sigma_\mathfrak{g}$	$-\frac{7}{18}(1-\mathfrak{g})$	[63]	

Log corrections from supergravity

[Camporesi,Higuchi]; [Vassilevich]; [Sen]; [Bhattacharyya,Grassi,Mariño,Sen]; [Liu,Pando Zayas,Rathee,Zhao]; [Pando Zayas,Xin]; [Hristov,Reys]; [David,Godet,Liu,Pando Zayas]; [NPB-David-Hong-Reys-Zhang]

Log corrections

There are log corrections to the (semi-classical) BH entropy $(\mathrm{Area} \gg G_{\mathrm{N}})$

$$S_{\mathrm{BH}} = \frac{\mathrm{Area}}{4G_{\mathrm{N}}} + s_0 \log \frac{\mathrm{Area}}{G_{\mathrm{N}}} + \dots$$

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Ashoke Sen: s_0 can be computed by 1-loop contributions of all "light" fields in the BH background. Agreement with string theory UV calculations for BPS black holes. "IR window into UV physics!"

Example: 4d Schwarzschild in GR + n_s fields of spin $s = 0, \frac{1}{2}, 1, \frac{3}{2}$.

$$s_0 = \frac{1}{90}\left(2n_0 + 7n_{\frac{1}{2}} - 26n_1 - \frac{233}{2}n_{\frac{3}{2}} + 289\right) .$$

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Here: Log corrections in AdS$_4$, i.e. $\log \frac{L^2}{G_\text{N}}$ with $L^2 \gg G_\text{N}$.

$$F_{S^3}(b,\Delta) = f_{\frac{3}{2}}(b,\Delta)N^{\frac{3}{2}} + f_{\frac{1}{2}}(b,\Delta)N^{\frac{1}{2}} + \frac{1}{4}\log N + \ldots$$

$$F_{S^1\times\Sigma_\mathfrak{g}}(\mathfrak{n},\Delta) = g_{\frac{3}{2}}(\mathfrak{n},\Delta)N^{\frac{3}{2}} + g_{\frac{1}{2}}(\mathfrak{n},\Delta)N^{\frac{1}{2}} + \frac{1-\mathfrak{g}}{2}\log N + \ldots$$

$$F_{S^1\times_\omega S^2}(\omega) = h_{\frac{3}{2}}(\omega)N^{\frac{3}{2}} + h_{\frac{1}{2}}(\omega)N^{\frac{1}{2}} + \frac{1}{2}\log N + \ldots$$

The coefficient of $\log N$ does NOT depend on continuous parameters!

Heat kernel in 4d

Study the log term in the (Euclidean) path integral of GR+EFT in AdS$_4$ with cutoff scale Λ

$$-\log Z_{\mathrm{GR+EFT}} = \frac{1}{16\pi G_{\mathrm{N}}} S_{\mathrm{cl}}(\mathring{\phi}) + \mathcal{C} \log L\Lambda + \ldots .$$

All fields ϕ with $\mathrm{mass}_\phi < \Lambda$ contribute to $\mathcal{C}$. Use the heat kernel method to compute $\mathcal{C}$.

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Input: The kinetic operator $\mathcal{Q}_\phi$ and the number of zero modes

$$\mathcal{C} = \sum_\phi \int d^4x \sqrt{g}\, a_4(x, \mathcal{Q}_\phi) + \mathcal{C}_{\text{ZM}} .$$

The Seeley-de Witt coefficient $a_4(x, \mathcal{Q}_\phi)$ depends on the background fields

$$16\pi^2 a_4(x, \mathcal{Q}_\phi) = a_E E_4 + c W^2 + b_1 R^2 + b_2 R F_{\mu\nu} F^{\mu\nu} .$$

Straightforward to calculate $a_4(x, \mathcal{Q}_\phi)$ for massive fields of spin ≤ 2.

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Subtlety: It is in general hard to compute $\mathcal{C}_{\text{ZM}}$. Rigorous results only for AdS$_4$ and AdS$_2 \times \Sigma_{\mathfrak{g}}$.

Massive scalar

The background: Any solution of 4d Einstein-Maxwell theory

$$S_{\mathrm{EM}} = -\frac{1}{16\pi G_{\mathrm{N}}} \int d^4x \sqrt{q} \left[R + \frac{6}{L^2} - F_{\mu\nu} F^{\mu\nu} \right] .$$

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The field: Scalar fields ϕ of mass m, charge q and action

$$S_\phi = \int d^4x \sqrt{g}\ \phi[-\mathcal{D}^\mu \mathcal{D}_\mu + m^2]\phi \,, \qquad \mathcal{D}_\mu = \nabla_\mu - {\rm i}qA_\mu \,.$$

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The Seeley-de Witt coefficients:

$$a_E = \frac{1}{360} \,, \quad c = \frac{1}{120} \,, \quad b_1 = \frac{1}{288} [(mL)^2 + 2]^2 \,, \quad b_2 = \frac{1}{144} (qL)^2 \,.$$

Seeley-de Witt Coefficients

Bulk contribution to the SdW coefficient for massive fields of spin ≤ 2.

spin	mass	a_E	c	b_1
0	$(mL)^2 = -2$	$\frac{1}{360}$	$\frac{1}{120}$	0
0	m	$\frac{1}{360}$	$\frac{1}{120}$	$\frac{1}{288}\left((mL)^2+2\right)^2$
1/2	0	$-\frac{11}{720}$	$-\frac{1}{40}$	0
1/2	m	$-\frac{11}{720}$	$-\frac{1}{40}$	$\frac{1}{144}(mL)^2\left((mL)^2-2\right)$
1	0	$\frac{31}{180}$	$\frac{1}{10}$	0
1	m	$\frac{31}{180}+\frac{1}{360}$	$\frac{1}{10}+\frac{1}{120}$	$\frac{1}{288}\left(3(mL)^4-12(mL)^2+4\right)$
3/2	$mL = 1$	$\frac{589}{720}$	$\frac{137}{120}$	0
3/2	m	$\frac{589}{720}-\frac{11}{720}$	$\frac{137}{120}-\frac{1}{40}$	$\frac{1}{72}\left((mL)^4-8(mL)^2+11\right)$
2	0	$\frac{571}{180}$	$\frac{87}{20}$	0
2	m	$\frac{571}{180}+\frac{31}{180}+\frac{1}{360}$	$\frac{87}{20}+\frac{1}{10}+\frac{1}{120}$	$\frac{5}{288}\left((mL)^4-8(mL)^2+8\right)$

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3/2	m	$\frac{589}{720}-\frac{11}{720}$	$\frac{137}{120}-\frac{1}{40}$	$\frac{1}{72}\left((mL)^4-8(mL)^2+11\right)$
2	0	$\frac{571}{180}$	$\frac{87}{20}$	0
2	m	$\frac{571}{180}+\frac{31}{180}+\frac{1}{360}$	$\frac{87}{20}+\frac{1}{10}+\frac{1}{120}$	$\frac{5}{288}\left((mL)^4-8(mL)^2+8\right)$

Important: For $s = 1, \frac{3}{2}, 2$ massless fields need ghosts while massive fields need Stückelberg "friends".

Log-Bootstrap

Study 4d sugra backgrounds and impose that $\mathcal{C}$ does not depend on continuous parameters.

- AdS-Taub-NUT ($\mathrm{U}(1) \times \mathrm{U}(1)$ squashed S^3)

$$
\begin{aligned}
ds^2 &= f_1^2 dx^2 + f_2^2 dy^2 + \frac{1}{f_1^2}(d\psi + y^2 d\phi)^2 + \frac{1}{f_2^2}(d\psi + x^2 d\phi)^2 \,, \\
f_1^2 &= \frac{L^2(y^2 - x^2)}{(x^2-1)(b^4-x^2)} \,, \qquad f_2^2 = \frac{L^2(y^2-x^2)}{(y^2-1)(y^2-b^4)} \,, \\
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- Euclidean Romans solution (Euclidean susy RN black hole in AdS_4)

$$ds^2 = \left[\left(\frac{r}{L} + \frac{\kappa L}{2r}\right)^2 - \frac{q^2}{4r^2}\right] d\tau^2 + \left[\left(\frac{r}{L} + \frac{\kappa L}{2r}\right)^2 - \frac{q^2}{4r^2}\right]^{-1} dr^2 + r^2 ds^2_{\Sigma_\mathfrak{g}} \,,$$

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This leads to the strong constraint

$$\boxed{c^{\mathrm{tot}} = b_1^{\mathrm{tot}} = b_2^{\mathrm{tot}} = 0}$$

Top-down KK supergravity

Consider the concrete example of 11d sugra on S^7 dual to the ABJM theory at $k = 1$ and large N.

The resulting 4d $\mathcal{N} = 8$ gauged sugra is not a standard EFT, it has infinitely many fields!

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Organize the KK modes into $\mathcal{N} = 8$ multiplets and compute the SdW coefficients.

Spectrum on $\mathrm{AdS}_4 \times S^7$

spin	Dynkin label	Δ
2	$(n,0,0,0)_{n\geq 0}$	$3+\frac{n}{2}$
3/2	$(n,0,0,1)_{n\geq 0}$ $(n-1,0,1,0)_{n\geq 1}$	$\frac{5}{2}+\frac{n}{2}$ $\frac{7}{2}+\frac{n}{2}$
1	$(n,1,0,0)_{n\geq 0}$ $(n-1,0,1,1)_{n\geq 1}$ $(n-2,1,0,0)_{n\geq 2}$	$2+\frac{n}{2}$ $3+\frac{n}{2}$ $4+\frac{n}{2}$
1/2	$(n+1,0,1,0)_{n\geq 0}$ $(n-1,1,1,0)_{n\geq 1}$ $(n-2,1,0,1)_{n\geq 2}$ $(n-2,0,0,1)_{n\geq 2}$	$\frac{3}{2}+\frac{n}{2}$ $\frac{5}{2}+\frac{n}{2}$ $\frac{7}{2}+\frac{n}{2}$ $\frac{9}{2}+\frac{n}{2}$
0_+	$(n+2,0,0,0)_{n\geq 0}$ $(n-2,2,0,0)_{n\geq 2}$ $(n-2,0,0,0)_{n\geq 2}$	$1+\frac{n}{2}$ $3+\frac{n}{2}$ $5+\frac{n}{2}$
0_-	$(n,0,2,0)_{n\geq 0}$ $(n-2,0,0,2)_{n\geq 2}$	$2+\frac{n}{2}$ $4+\frac{n}{2}$

Massive $\mathcal{N}=8$ supermultiplets at KK level n.

Top-down KK supergravity

At each KK level n one has $c(n) = b_1(n) = b_2(n) = 0$!

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For the total a_E coefficient one finds the divergent sum

$$a_E = -\frac{1}{72}\sum_{n=0}^{\infty}(n+1)(n+2)(n+3)^2(n+4)(n+5)\,.$$

Unclear how to regulate this sum. If we take $a_E = -1/3$ we find

$$\boxed{\mathcal{C}(\partial\mathcal{M}) = -\frac{1}{4}\chi(\mathcal{M})}\,.$$

Perfect agreement with all susy localization results in the ABJM theory!

Similar results for a number of other AdS_4 vacua in M-theory with explicitly known KK spectra.

The unbearable lightness of the KK scale

Assumption: The UV completion of GR+EFT in AdS_4 is holographic, i.e. there is a dual family of 3d CFTs with a suitable large N limit.

Consider such a GR+EFT with **finitely** many fields and a cutoff Λ such that $L\Lambda \sim N^\alpha$ with $\alpha > 0$ (or $L\Lambda \sim \lambda^\beta$ for a marginal coupling λ).

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A new tool to delineate the landscape of scale separated AdS_4 vacua?

Black holes and thermal observables

[Witten]; [Horowitz,Myers]; [NPB,Charles,Hristov,Reys]; [NPB,Hong,Reys];
[Iliesiu,Koloğlu,Mahajan,Perlmutter,Simmons-Duffin]; [Luo,Wang]; [Benjamin,Lee,Ooguri,Simmons-Duffin]

BHs and thermal observables

Using the results above we can also compute the leading corrections to the entropy of any large asymptotically $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ black hole.

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Example: AdS-Schwarzschild black hole

$$S_{\mathrm{Sch}}^{\mathrm{ABJM}} = \frac{2\pi r_+^2}{L^2}\frac{\sqrt{2k}}{3}\left(N^{\frac{3}{2}} + \frac{16-k^2}{16k}N^{\frac{1}{2}}\right) + \frac{2\pi}{\sqrt{2k}}N^{\frac{1}{2}} - \frac{1}{2}\log N$$

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Consider a 3d CFT on $S^1_\beta \times \mathbb{R}^2$. The vev of the stress-energy tensor and the thermal free energy are

$$\langle \mathcal{T}^{00}\rangle = \frac{2}{3}\frac{b_{\mathcal{T}}}{\beta^3}\,, \qquad F_{S^1_\beta\times\mathbb{R}^2} = \frac{f_{\mathcal{T}}}{\beta^3}\,, \qquad 3f_{\mathcal{T}} = b_{\mathcal{T}}\,.$$

To compute $f_{\mathcal{T}}$ in the bulk use the "AdS soliton". For the ABJM theory we find

$$b_{\mathcal{T}} = -\frac{8\pi^2\sqrt{2k}}{27}N^{\frac{3}{2}} + \frac{\pi^2(k^2-16)}{27\sqrt{2k}}N^{\frac{1}{2}} + 0\times\log N\ldots\,.$$

Somewhat surprisingly we find that to this order at large N $b_{\mathcal{T}} = -\frac{\pi^3}{72}C_{\mathcal{T}}$!

Summary

- Presented exact results for the large N limit of the partition function of the ABJM theory on S^3, $S^1 \times \Sigma_{\mathfrak{g}}$, and $S^1 \times_\omega S^2$ focusing on the $\log N$ contribution.
- Discussed how these log terms can be reproduced by supergravity and string/M-theory via AdS/CFT.
- Important for understanding the entropy of supersymmetric AdS_4 Reissner-Nordström and Kerr-Newman black holes.
- New constraints on gravity + EFTs in AdS?
- Application of these results to non-supersymmetric black hole thermodynamics and CFT thermal observables.

Outlook

Results I did not discuss

- Large N supersymmetric partition functions for other 3d $\mathcal{N}=2$ holographic SCFTs via supersymmetric localization.
- Similar logarithmic correction results for the holographically dual AdS_4 backgrounds in string/M-theory.
- Similar large N and holographic results for 3d $\mathcal{N}=2$ SCFTs arising from M5-branes (class $\mathcal{R}$ SCFTs).

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- Similar large N and holographic results for 3d $\mathcal{N} = 2$ SCFTs arising from M5-branes (class $\mathcal{R}$ SCFTs).

Some open questions

- A better understanding of the simplicity and universality of the logarithmic corrections.
- Derivation from (or lessons for) type IIA string theory and M-theory?
- OSV-type conjecture for AdS black holes?
- Application of the "unbearable lightness" constraint to candidate scale separated AdS_4 vacua?

Grazie Mille!