S-matrix tools for bound waveform modelling

Riccardo Gonzo

THE UNIVERSITY *of* EDINBURGH

String Theory as a Bridge between Gauge Theory and Quantum Gravity (Sapienza, Rome)

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Motivation and introduction (I)

- The recent discovery of gravitational waves calls for new analytical techniques to study the two-body problem.

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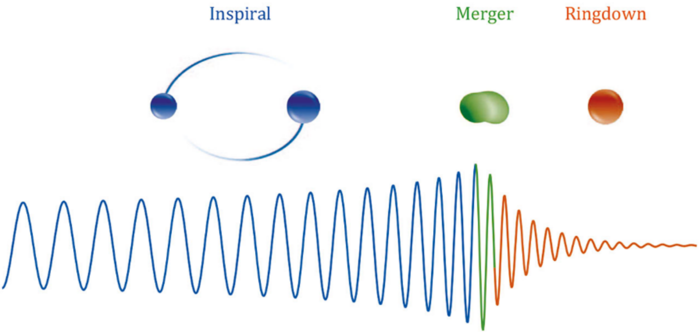

Image credit: 1610.03567 (Antelis, Moreno)

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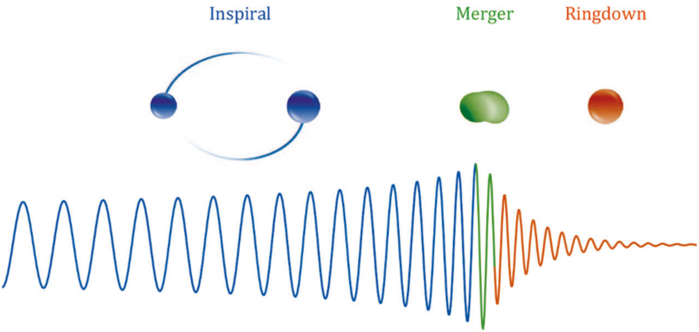

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- Today: focus on the inspiral phase, where we can model compact objects as point particles in the spirit of effective field theory [Goldberger,Rothstein]

Motivation and introduction (II)

- Idea: use particle field theory tools

Real world	EFT of point particles
Compact objects of mass M	Point particles of mass M
Spin effects of magnitude a	Spinning particles of classical spin a
Tidal effects, GR curvature corrections	Higher-dimensional operators
Absorption effects	Non-unitary absorption dofs

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- Why scattering amplitudes? Few years ago someone called our attention to it . . .

[Submitted on 29 Oct 2017]

High-energy gravitational scattering and the general relativistic two-body problem

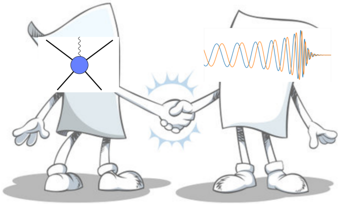
Thibault Damour

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys.\ Rev.\ D {\bf 94}, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.

Why amplitudes? Why Post-Minkowskian expansion? (I)

- Why amplitudes? (adapted to scattering orbits... bound orbits? Stay tuned!)

Amplitudes are gauge-invariant, universal objects which encode in a compact and analytic way the perturbative dynamics for point particles.
New perspective on GR!

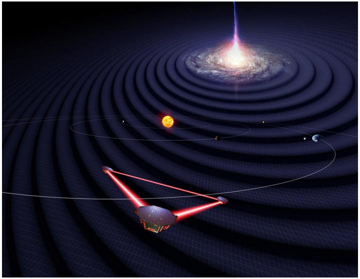


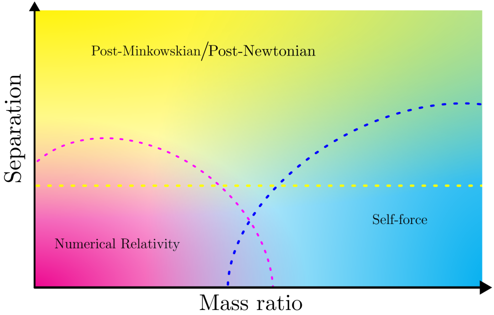
Advantages:
1) analytic compact expressions
2) many physical insights (clean setup)
3) scalable and flexible formalism
(spin, tidal effects, beyond GR)
4) great synergy with PN and GSF

Disadvantages:
1) need scatter-to-bound map
2) need resummation scheme

Why amplitudes? Why Post-Minkowskian expansion? (II)

- To model accurately the entire parameter space of the two-body dynamics, various communities need to work together: Gravitational self-force (GSF), Post-Minkowskian (PM), Post-Newtonian (PN) and numerical relativity (NR)





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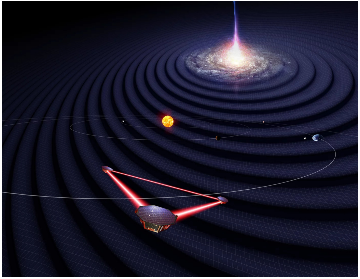

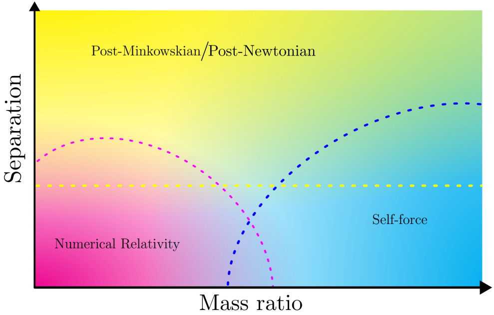

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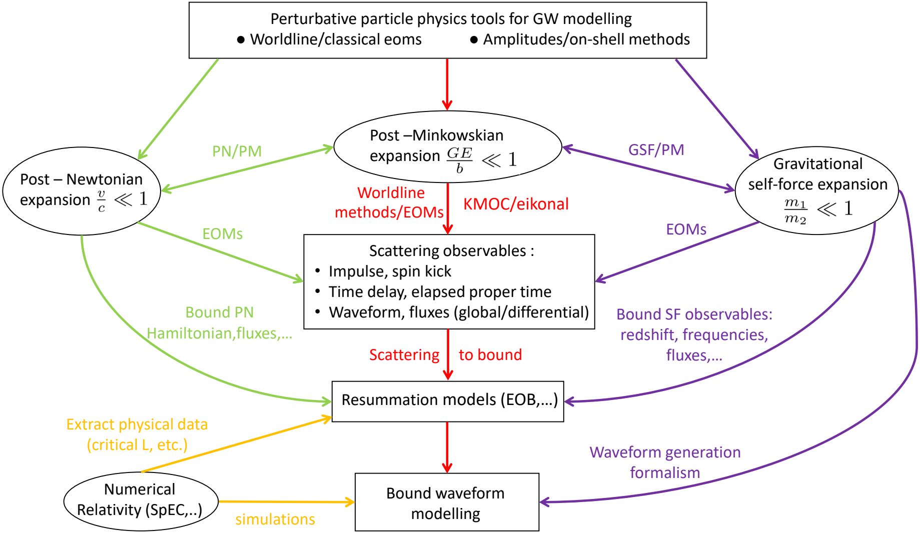
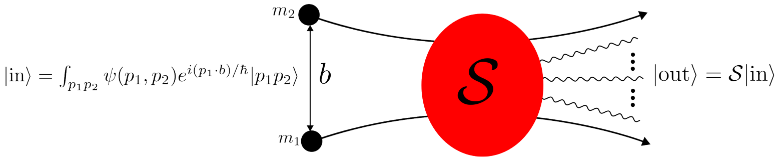
- Important deadline: being ready for LISA mission's planned launch $\sim$ 2035

Particle physics for GWs modelling: workflow



The Post-Minkowskian two-body scattering problem (I)

- Two-body scattering in GR: consider as initial state two massive particles separated by an impact parameter b^μ [Kosower,Maybee,O'Connell=KMOC]



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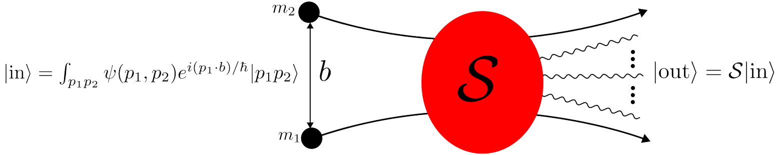
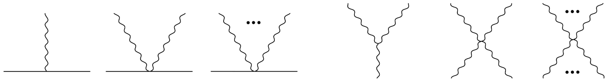


- The dynamics of the evolution is determined by the action

$$S = -\frac{1}{16\pi G_N}\int \mathrm{d}^4 x \sqrt{-g} R + S_{\text{matter}} + S_{\text{GF}}$$

where we perform the perturbative expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,, \quad \kappa = \sqrt{32\pi G_N} \rightarrow \text{Post-Minkowskian expansion in } G_N\,.$$



The Post-Minkowskian two-body scattering problem (II)

- Conservative 4-pt amplitude $\mathcal{M}_4(p_1, p_2; p_1', p_2')$: in the classical limit $\hbar \to 0$

$$p_1^\mu := p_A^\mu + \hbar \frac{\bar{q}^\mu}{2}\,, \qquad (p_1')^\mu := p_A^\mu - \hbar \frac{\bar{q}^\mu}{2}\,, \qquad s = (p_A + p_B)^2\,,$$
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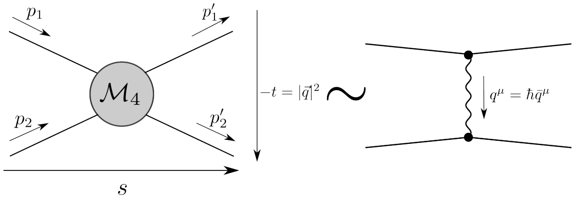
where p_A, p_B are the classical momenta and q is the momentum transfer.



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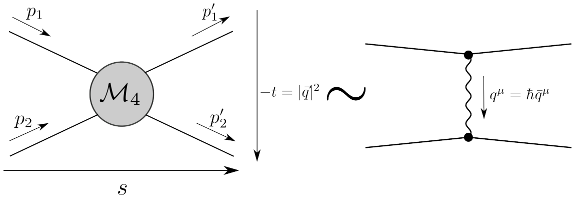

- Generalization for the $4+M$-pt amplitude $\mathcal{M}_{4+M}(p_1, p_2; p_1', p_2', k_1, \ldots, k_M)$

$$q_{1,2}^\mu = p_{1,2}^\mu - (p_{1,2}')^\mu = \hbar \bar{q}_{1,2}^\mu\,, \qquad k_j^\mu = \hbar \bar{k}_j^\mu\,, j = 1, \ldots, M\,.$$

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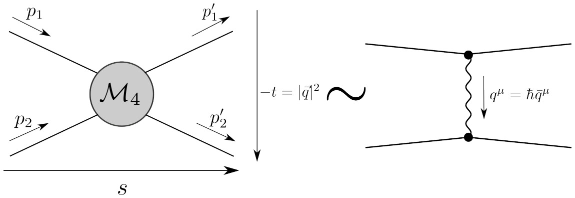

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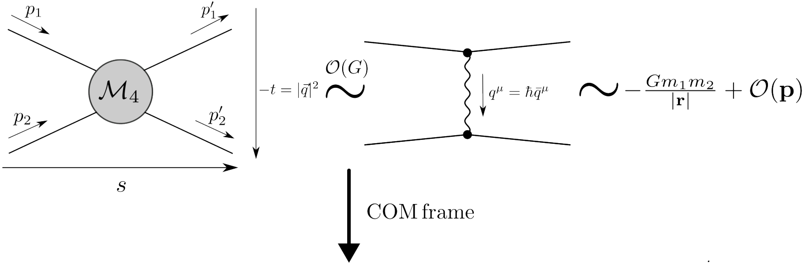
- Main lesson: only wavevectors $\bar{q}_{1,2}^\mu, \bar{k}_j$ are classical, need to restore $\hbar$!

The Post-Minkowskian two-body scattering problem (III)

- A conservative state-of-art PM Hamiltonian can be extracted from 4-pt amplitudes [Cheung,Solon,Rothstein;Bern,Parra-Martinez,Roiban,Ruf,Shen, Solon,Zeng] (or equivalently the scattering angle [Di Vecchia, Heissenberg, Russo, Veneziano; Dlapa, Kälin, Liu, Porto; Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch; Damgaard, Hansen, Planté, Vanhove])



$$H_{\rm PM}(\mathbf{p},\mathbf{r}) = \sum_{i=1,2} \sqrt{|\mathbf{p}|^2 + m_i^2} + \sum_{j=1}^{\infty} c_j\left(|\mathbf{p}|^2\right)\left(\frac{G}{|\mathbf{r}|}\right)^j$$

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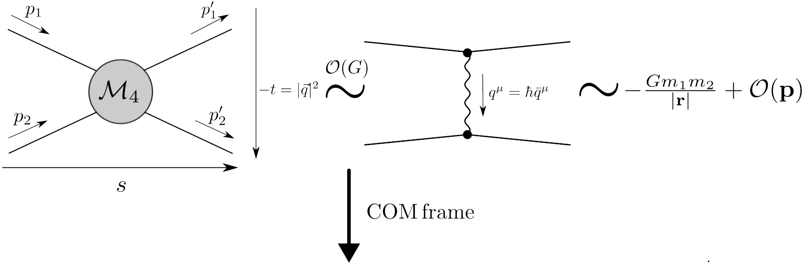

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- Relevant to bound orbits, except for subtle non-local-in-time effects! [Cho, Dlapa,Kälin,Liu,Porto] The EOB implementation is already promising for GW modelling [Buonanno,Mogull,Patil,Pompili;Buonanno,Jakobsen,Mogull]

The Post-Minkowskian scattering waveform (I)

- We can compute classical observables $\mathcal{O}$ with in-in expectation values

$$\langle\psi_{\text{in}}|\mathcal{S}^\dagger\mathcal{O}\mathcal{S}|\psi_{\text{in}}\rangle\Big|_{\hbar\to 0} = 2\Re i\langle\psi_{\text{in}}|\mathcal{O}T|\psi_{\text{in}}\rangle\Big|_{\hbar\to 0} + \langle\psi_{\text{in}}|T^\dagger\mathcal{O}T|\psi_{\text{in}}\rangle\Big|_{\hbar\to 0}$$

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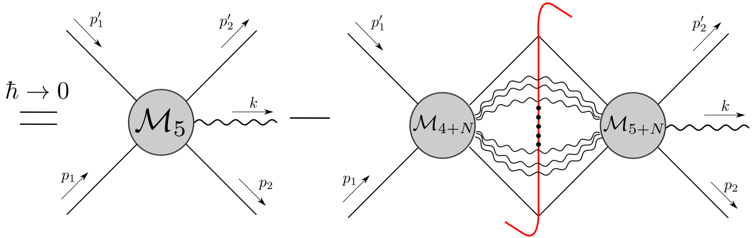
- The on-shell expectation value of the time-domain waveform relevant for the inspiral phase is [Cristofoli,RG,Kosower,O'Connell]

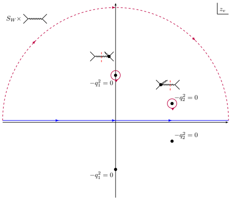
$$h_{\mu\nu}^{\text{hyp}}(x) = \langle \psi_{\text{in}} | \mathcal{S}^\dagger h_{\mu\nu}(x) \mathcal{S} | \psi_{\text{in}} \rangle$$



The Post-Minkowskian scattering waveform (II)

- Use on-shell tools:

  Simplify the phase space integration of the 5-pt amplitude using S-matrix analyticity and unitarity (factorization into 3-pt and 4-pt amplitudes)



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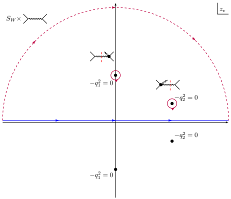


- Result: new compact representation of the tree-level scattering waveform! [Kovacs,Thorne; Jakobsen,Mogull,Plefka,Steinhoff; De Angelis,RG,Novichkov]

$$h^{(0)>}(x) = \frac{G_N^2 m_1 m_2}{|\vec{x}|\sqrt{-b^2}} \frac{1}{\bar{w}_1^2 \bar{w}_2^2 \sqrt{1+T_2^2}\left(\gamma + \sqrt{\left(1+T_1^2\right)\left(1+T_2^2\right)} + T_1 T_2\right)}$$

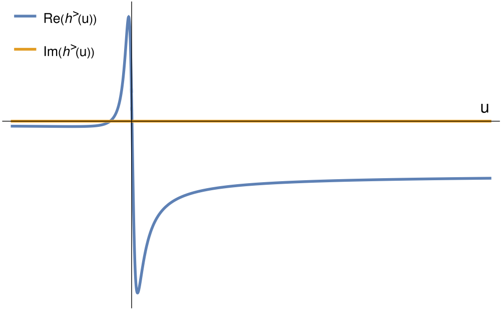
$$\times \left(\frac{3\bar{w}_1 + 2\gamma\left(2T_1T_2\bar{w}_1 - T_2^2\bar{w}_2 + \bar{w}_2\right) - \left(2\gamma^2-1\right)\bar{w}_1}{\gamma^2-1} f_{1,2}^2 \right.$$

$$- \frac{4\gamma T_2 \bar{w}_2 f_1 + 2\left(2\gamma^2-1\right)\left[T_1\left(1+T_2^2\right)\bar{w}_2 f_1 + T_2(T_1T_2\bar{w}_1 + \bar{w}_2) f_2\right]}{\sqrt{\gamma^2-1}} f_{1,2}$$

$$\left. + 4\left(1+T_2^2\right)\bar{w}_2 f_1 f_2 - 4\gamma\left(1+T_2^2\right)\bar{w}_2\left(f_1^2+f_2^2\right) + 2\left(2\gamma^2-1\right)\left(1+2T_2^2\right)\bar{w}_2 f_1 f_2 \right) + (1\leftrightarrow 2)$$

The Post-Minkowskian scattering waveform (III)

- The tree-level scattering waveform in the equatorial plane looks like



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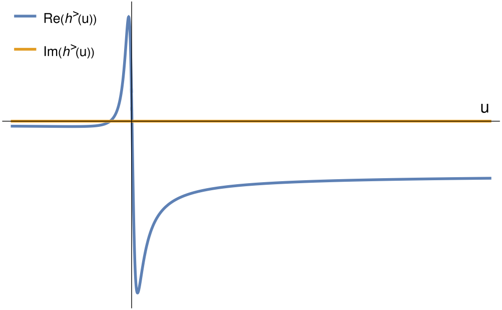

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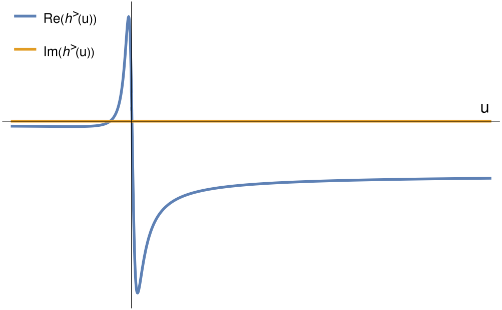


Most of the energy is released during the closest approach ($\sim$ periastron)!

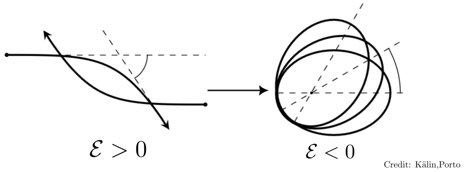
- Relevant for hyperbolic encounters or dynamical capture events (short-duration, burst-like waveform): possible LISA sources?
- Very different compared to (quasi)-periodic bound waveforms for inspiralling compact binaries. . . is it possible to establish a connection?

From scattering to bound dynamics

- Classical scattering amplitudes describe hyperbolic encounters. If we define

$$\mathcal{E} := \frac{E - m_1 - m_2}{\mu}\,, \qquad p_\infty^2 = -\tilde{p}_\infty^2 = \frac{E^2 - (m_1 + m_2)^2}{2 m_1 m_2}\,,$$

we have $\mathcal{E}, p_\infty^2 > 0$ for scattering orbits and $\mathcal{E}, p_\infty^2 < 0$ for bound orbits.



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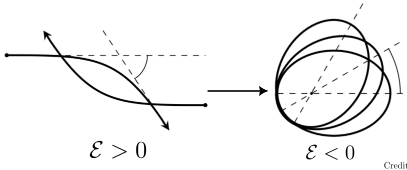

- Powerful analytic method to extract bound state physics from amplitudes: gauge invariant map between scattering and bound observables:

$$\boxed{\mathcal{O}^{>}(\mathcal{E} > 0, J, c_X, a_1, a_2, m_1, m_2) \to \mathcal{O}^{<}(\mathcal{E} < 0, J, c_X, a_1, a_2, m_1, m_2)}.$$

First derived in PM for aligned-spin binaries [Kälin,Porto] (hints in 1985 [Damour, DeRuelle]!), extended to fluxes [Cho,Kälin,Porto;Saketh,Vines, Steinhoff,Buonanno], waveforms [Adamo,RG,Ilderton]; proved recently at geodesic order [RG,Shi;RG,Lewis,Pound]; hints for misaligned spin [RG,Shi]

Warm up: geodesics in Schwarzschild (I)

- Consider the motion of a spinless particle in Schwarzschild with the action

$$S\left[x^{\mu}(\tau), p_{\mu}(\tau)\right] = \int \mathrm{d}\tau\, p_{\mu}\dot{x}^{\mu} - \frac{e}{2}\left(g^{\mu\nu}p_{\mu}p_{\nu} + m^2\right) \,,$$

where (x^{μ}, p_{μ}) are canonically conjugate variables and $g_{\mu\nu} = \bar{g}^{\mathrm{Schw}}_{\mu\nu}$.

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- Using Hamilton-Jacobi theory [Carter], we use the constants of motion $P_i = (m, E, L)$ and we transform to (X^i, P_i) with the generating function

$$W\left(t, r, \varphi; P_i\right) = -Et + L\varphi + I_{r,0}\left(r; P_i\right) \,, \quad I_{r,0}\left(r; P_i\right) = \int_{r_{\mathrm{m}}}^{r} \mathrm{d}r \; p_{r,0}\left(r; P_i\right) \,.$$

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- The new Hamilton's equations are (with the Hamiltonian $H_0 = -m_1^2/2$)

$$X^i = \frac{\partial W}{\partial P_i} , \qquad m_1 \frac{\mathrm{d}X^i}{\mathrm{d}\tau} = \frac{\partial H_0}{\partial P_i} = -m_1 \delta^i_1 .$$

Direct connection with observables $(\Delta\varphi, \Delta t, \Delta\tau)$![RG,Lewis,Pound;Schmidt]

Warm up: geodesics in Schwarzschild (II)

- For scattering orbits (hyperbolic trajectory with a single turning point $r_{\rm m}$) we define the (ϵ-regularized) radial action

$$I_{r,0}^{>,\epsilon}(P_i) = 2\int_{r_{\rm m}}^{+\infty} \mathrm{d}r\, r^{\epsilon}\, p_{r,0}(r; P_i)\,,$$

while for elliptic bound orbits (radial motion constrained between r_- and r_+)

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- HJ theory allows to derive a complete basis of scattering and bound observables and the first law of black hole dynamics [RG,Lewis,Pound;Le Tiec]

$$\delta I_{r,0}^{>,\epsilon} = -(\pi + \chi_0)\delta L + \Delta t_0^{\epsilon}\delta E - \Delta\tau_0^{\epsilon}\delta m_1\,.$$

$$\delta I_{r,0}^{<} = -(2\pi + \Delta\Phi_0)\delta L + \frac{2\pi}{\Omega_{r,0}}\delta E - \frac{2\pi}{\Omega_{r,0}}\langle z\rangle_0 \delta m_1\,.$$

Scatter-to-bound map at geodesic order

- Remarkable analytic continuation between scattering and bound planar orbits [Kälin,Porto; Adamo,RG,Ilderton; Di Vecchia,Heissenberg,Russo,Veneziano]

$$\int_{\mathcal{C}_r^>} = 2\int_{r_m(p_\infty,L)}^{\infty}, \qquad \int_{\mathcal{C}_r^<} = 2\int_{r_-(\tilde{p}_\infty,L)}^{r_+(\tilde{p}_\infty,L)}, \quad r_\pm(\tilde{p}_\infty,L) \overset{\mathcal{E}\leq 0}{=} r_m(\pm i\tilde{p}_\infty,L)\,,$$

with $p_\infty = p_r(r\to\infty)$ so that at 0SF order $(p_r(\tilde{p}_\infty) = \mp p_r(\pm i\tilde{p}_\infty))$ [RG,Shi]

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- Scatter-to-bound maps for gauge-invariant observables [RG,Lewis,Pound]

$$\begin{aligned}
\Delta\Phi_0 &= \chi_0(i\tilde{p}_\infty,L,m_1) + \chi_0(-i\tilde{p}_\infty,L,m_1)\,,\\
\frac{2\pi}{\Omega_{r,0}} &= \lim_{\epsilon\to 0}[\Delta t_0^\epsilon(i\tilde{p}_\infty,L,m_1) + \Delta t_0^\epsilon(-i\tilde{p}_\infty,L,m_1)]\,,\\
\frac{2\pi\langle z\rangle_0}{\Omega_{r,0}} &= \lim_{\epsilon\to 0}[\Delta\tau_0^\epsilon(i\tilde{p}_\infty,L,m_1) + \Delta\tau_0^\epsilon(-i\tilde{p}_\infty,L,m_1)]\,.
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- Connection with the S-matrix? Spin (non-planar motion)? Radiative effects?

Scattering and bound observables from 4-pt amplitudes (I)

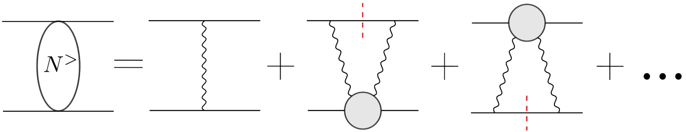
- Natural connection between the radial action and the conservative S-matrix

$$\mathcal{S} = \exp\left(\frac{i}{\hbar}\hat{N}\right) , \quad N\left(E, q, m_1, m_2\right) := \langle p_1' p_2' | \hat{N} | p_1 p_2 \rangle \Big|_{\hbar \to 0} ,$$

$$N^{>,\epsilon}\left(E, L, \{m_a\}\right) = \frac{4 E p_\infty}{\hbar} \int \frac{\mathrm{d}^{2+2\epsilon} q}{(2\pi)^{2+2\epsilon}} e^{-i(b(L)\cdot q)/\hbar} N\left(E, q, \{m_a\}\right) ,$$

$$N^{>,\epsilon}(p_\infty, L) = \frac{i}{\hbar}\left(\oint_{\mathcal{C}_r^>} dr\, r^\epsilon\, p_{r,\mathrm{COM}}(r, p_\infty^2, L) + \pi L\right) = \frac{i}{\hbar}\left(I_r^{>,\epsilon} + \pi L\right) ,$$

where $p_{r,\mathrm{COM}}$ is the center-of-mass radial momentum. This is the "amplitude-action" relation! [Bern et al.;Kol,O'Connell,Telem] A full proof was given recently [Damgaard,Hansen,Plante,Vanhove]



Direct connection of $\hat{N}$ with the classical Bethe-Salpeter kernel [Adamo,RG]

Scattering and bound observables from 4-pt amplitudes (II)

- The S-matrix is a generating functional for classical observables: in the spinless conservative case (i.e., no on-shell gravitons) [RG,Lewis,Pound]

$$\delta N^{>,\epsilon}(E,L,\{m_a\}) = -\Delta\chi\,\delta L + \Delta T^{\epsilon}\,\delta E - \sum_{a=1,2}\Delta\tau_a^{\epsilon}\,\delta m_a$$

$$\big\downarrow \quad \delta N^{<}(E,L,\{m_a\}) = \delta N^{>,\epsilon}(E,L,\{m_a\}) - \delta N^{>,\epsilon}(E,-L,\{m_a\})$$

$$\delta N^{<}(E,L,\{m_a\}) = -\Delta\Phi\,\delta L + \frac{2\pi}{\Omega_r}\delta E - \sum_{a=1,2}\frac{2\pi}{\Omega_r}\langle z_a\rangle\delta m_a$$

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- Novel IR finite scattering observables: global and proper time differences

$$\Delta t^{\mathrm{rel}}_{0,L_{\mathrm{ref}}} = \lim_{\epsilon\to 0}\left[\Delta t_0^{\epsilon}(P_i) - \Delta t_0^{\epsilon}(P_{i,\mathrm{ref}})\Big|_{\mathcal{O}\left(\frac{m_1 m_2}{L_{\mathrm{ref}}}\right)}\right],$$

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- In the classical GSF approach, natural extension of the first law to the dissipative case using the pseudo-Hamiltonian formulation!
- Can we extend the scatter-to-bound map to spinning binaries? Waveforms?

Scattering and bound observables for spinning binaries

- The motion of aligned-spin binaries is still planar: trivial extension of the spinless case $I_r^{>}(E, L, a_1, a_2, \{m_a\})$, same scattering/bound observables!

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- Classical scattering observables for generic spinning binaries can be extracted by recursively applying Dirac brackets, [RG,Shi]

$$\Delta\lambda^\mu = \sum_{j=1} \frac{1}{j!} \underbrace{\{I_r^{>}, \{I_r^{>}, \ldots, \{I_r^{>}}_{j \text{ times}}, \lambda^\mu\} \ldots\}\}\,, \qquad \lambda^\mu \in \{v_1^\mu, v_2^\mu, s_1^\mu, s_2^\mu\}\,.$$

Independently confirmed by [Kim,Kim,Lee]; used to derive state-of-art 2-loop observables! [Apkinar,Febres-Cordero,Kraus,Smirnov,Zeng]

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- For a linear in spin probe in Kerr, we can use again action-angle variables and compute the bound frequencies $K^{\phi r} = \Delta\Phi, K^{\theta r}, K^{\phi s r}$ [Witzany;RG,Shi]

Type of observable	Position space	Spin space
Scattering	Δv_1^μ $(\Delta\varphi,\ \Delta\theta)$	Δs_1^μ
Bound	$K^{\phi r},\ K^{\theta r}$	$K^{\phi s r}$

  At this order the scatter-to-bound map holds (at the level of the action), but hard to generalize to all orders in spin for both bodies! More work to do ...

From scattering to bound waveforms (I)

- We propose a scatter-to-bound map for PM waveforms [Adamo,RG,Ilderton]

$$\boxed{h^{<\text{dyn}}(u, \hat{n}; \tilde{p}_\infty, L) = h^{>\text{dyn}}(u, \hat{n}; p_\infty = +i\tilde{p}_\infty, L)\,, \qquad \mathcal{E} < 0\,.}$$

How can this be verified?

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- Use the Post-Newtonian expansion: the waveform in the center-of-mass frame admits a multipole expansion [Bini,Damour,Geralico; Bini,Damour,De Angelis,Geralico,Herderschee,Roiban,Teng;Georgoudis,Heissenberg,Russo]

$$h^{>}\left(u = \frac{b}{p_\infty c}\tilde{u}^{>},\hat{n}\right) = \frac{4G_N}{c^4}\left(W_{\text{N}}^{>} + \frac{1}{c}W_{0.5\text{PN}}^{>} + \frac{1}{c^2}W_{1\text{PN}}^{>} + \dots\right),$$

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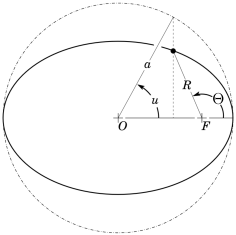
But PN multipoles can be computed independently with the quasi-Keplerian parametrization for hyperbolic and elliptic orbits! [Damour,Deruelle]



From scattering to bound waveforms (II)

- We find a B2B map between radiative multipoles for hyperbolic and elliptic orbits up to 1PN [Adamo, RG, Ilderton; Junker, Schäfer]

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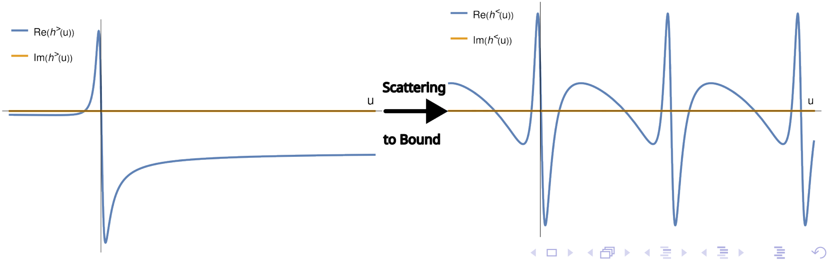
- We need a resummation in the eccentricity to recover the bound waveform periodicity in the time u from PM waveforms

$$n^{>}t = e_t^{>}\sinh(\mathrm{v}) - \mathrm{v} + \mathcal{O}(1/c)\,, \qquad n^{<}t = \mathrm{u} - e_t^{<}\sin(\mathrm{u}) + \mathcal{O}(1/c)\,.$$



Elephant in the room: hereditary effects in GR (I)

- The scattering-to-bound map naively breaks down when (non-local in time) hereditary effects are present! [Cho,Kälin,Porto; Dlapa,Liu,Kälin,Porto]

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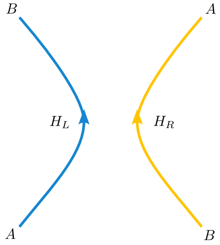
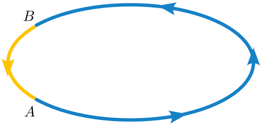
- What is the origin of the map? The 0SF (quasi-Keplerian) trajectory

$$r(\pi,\chi) = \frac{pM}{1+e\cos(\chi)}\,, \qquad \frac{\mathrm{d}t}{\mathrm{d}\chi} = \mathcal{F}_t(\chi,\pi)\,, \quad (p,e) = \mathcal{F}_\pi(E,L)\,,$$

involves a single branch hyperbola in the scattering vs an ellipse in the bound





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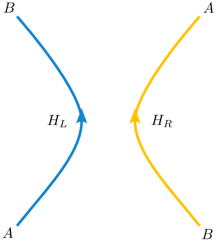

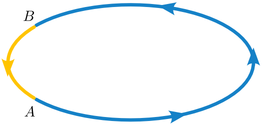

- Use the geometry: all conics are equivalent in the projective plane. Therefore, we need the second branch of the hyperbola (unphysical scattering) to get a full periodic scattering system! [RG,Lewis,Pound]

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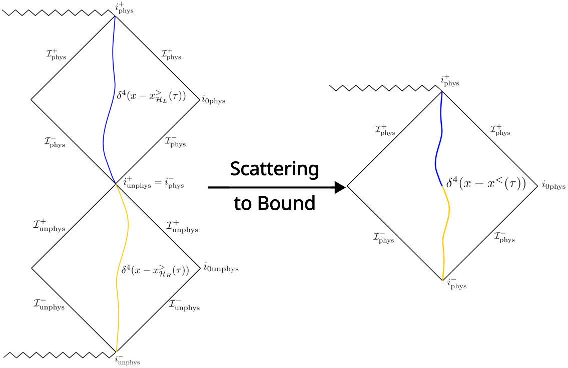
- What changes? Hereditary effects accumulate along both the physical and unphysical trajectory, giving a complete map between the periodic scattering and bound 1SF pseudo-Hamiltonian at the integrand level



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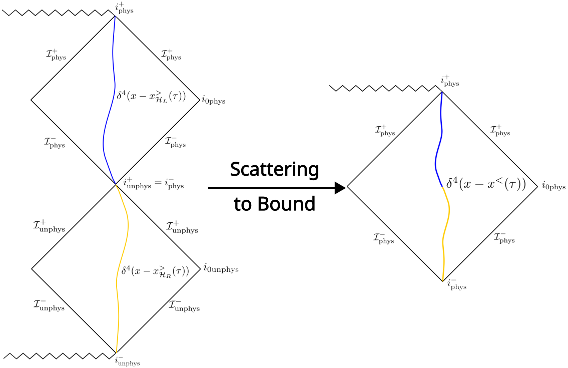

- New definition of a "periodic scattering" system: potential implementation in the amplitude/worldline formalism! [RG,Lewis,Kavanagh,Pound,Usseglio]

Summary table of the boundary to bound dictionary

- For aligned-spin binaries we find a conjectural scatter-to-bound dictionary [Kälin,Porto;Saketh,Vines,Steinhoff,Buonanno;Cho,Kälin,Porto;Adamo,RG; Heissenberg;Adamo,RG,Ilderton;Damour,Deruelle;RG,Shi;RG,Lewis,Pound]

Bound observable	Scattering observable
$\Delta\Phi(\tilde{p}_\infty; L, a, c_X)$	$\chi(-i\tilde{p}_\infty; L, a, c_X) + \chi(+i\tilde{p}_\infty; L, a, c_X)$
$\frac{2\pi}{\Omega_r}(\tilde{p}_\infty; L, a, c_X)$	$\Delta t^\epsilon(-i\tilde{p}_\infty; L, a, c_X) + \Delta t^\epsilon(+i\tilde{p}_\infty; L, a, c_X)$
$\frac{2\pi}{\Omega_r}\langle z\rangle(\tilde{p}_\infty; L, a, c_X)$	$\Delta\tau^\epsilon(-i\tilde{p}_\infty; L, a, c_X) + \Delta\tau^\epsilon(+i\tilde{p}_\infty; L, a, c_X)$
$\Delta E^<_{\text{rad}}(\tilde{p}_\infty; L, a, c_X)$	$\Delta E^>_{\text{rad}}(-i\tilde{p}_\infty; L, a, c_X) + \Delta E^>_{\text{rad}}(+i\tilde{p}_\infty; L, a, c_X)$
$\Delta J^<_{\text{rad}}(\tilde{p}_\infty; L, a, c_X)$	$\Delta J^>_{\text{rad}}(-i\tilde{p}_\infty; L, a, c_X) + \Delta J^>_{\text{rad}}(+i\tilde{p}_\infty; L, a, c_X)$
$h^{<\text{dyn}}(u; \tilde{p}_\infty, L, a, c_X)$	$h^{>\text{dyn}}(u; +i\tilde{p}_\infty, L, a, c_X)$

which is valid at least up to 3PM/0SF/3PN order for integrated observables and tree-level/1PN for waveforms. Need to study tail effects at higher orders!

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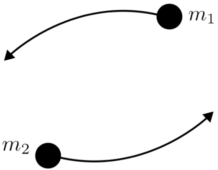
$\Delta E^<_{\text{rad}}(\tilde{p}_\infty; L, a, c_X)$	$\Delta E^>_{\text{rad}}(-i\tilde{p}_\infty; L, a, c_X) + \Delta E^>_{\text{rad}}(+i\tilde{p}_\infty; L, a, c_X)$
$\Delta J^<_{\text{rad}}(\tilde{p}_\infty; L, a, c_X)$	$\Delta J^>_{\text{rad}}(-i\tilde{p}_\infty; L, a, c_X) + \Delta J^>_{\text{rad}}(+i\tilde{p}_\infty; L, a, c_X)$
$h^{<\text{dyn}}(u; \tilde{p}_\infty, L, a, c_X)$	$h^{>\text{dyn}}(u; +i\tilde{p}_\infty, L, a, c_X)$

  which is valid at least up to 3PM/0SF/3PN order for integrated observables and tree-level/1PN for waveforms. Need to study tail effects at higher orders!
- Hints towards a generalization to misaligned spin [RG,Shi]

$$\text{(projected-) spin kick } \hat{l}_\mu \Delta s^\mu \leftrightarrow \text{intrinsic spin precession } K^{\phi_s r} = \Omega_s/\Omega_r$$

Summary and future directions

- Promising application of novel particle physics tools for the binary dynamics in the Post-Minkowskian regime (field relatively new! $\sim$ 7 years)



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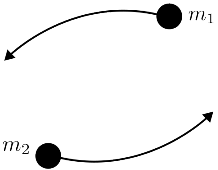

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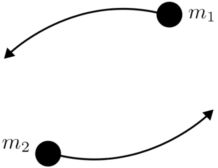

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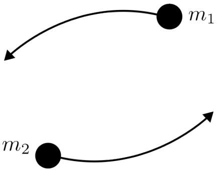

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- Scattering waveforms can be themselves useful to model hyperbolic encounters/dynamical capture events
- Resummation of perturbative methods is needed for direct application to LISA waveform modelling (EOB, GSF, ...)→Exciting direction for the future!

Self-force and amplitudes annual meetings 2025-2026

Excited about Self-force & Amplitudes?

Please join us for the 2nd annual workshop on Self-force & Amplitudes in Southampton on 9-12 September 2025! http://indico.global/event/4539/ (with C.Kavanagh,Z.Nasipak,J.Plefka,A.Pound) and/or for the Nordita program in April 2026 (with L.Cangemi,P.di Vecchia,C.Kavanagh,A.Pound,G.Pratten)

Memories of the 1st Self-force&Amplitudes workshop at the Higgs Centre in 2024!