From Holographic Correlators in the Sky to Euclidean AdS

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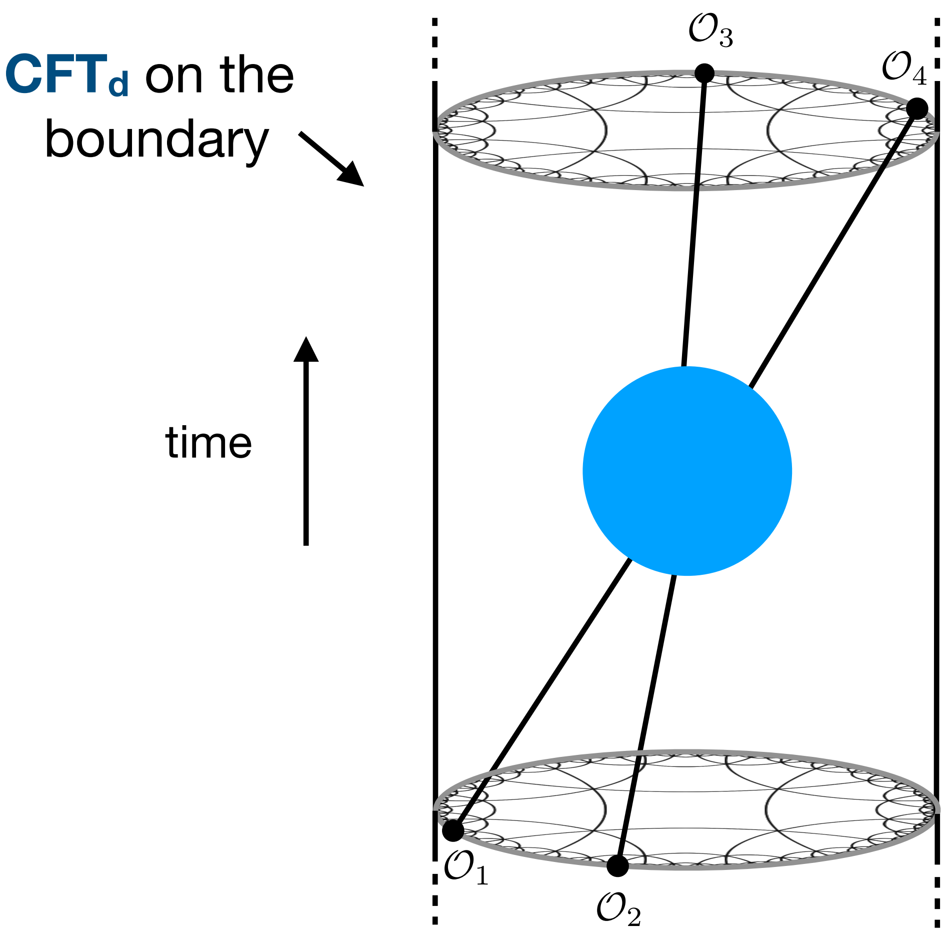
AdS-CFT

Quantum Gravity in AdS_{d+1} = [non-gravitational] CFT in $\mathbb{M}^d$

Observables ?! ⟷ **Correlation functions**



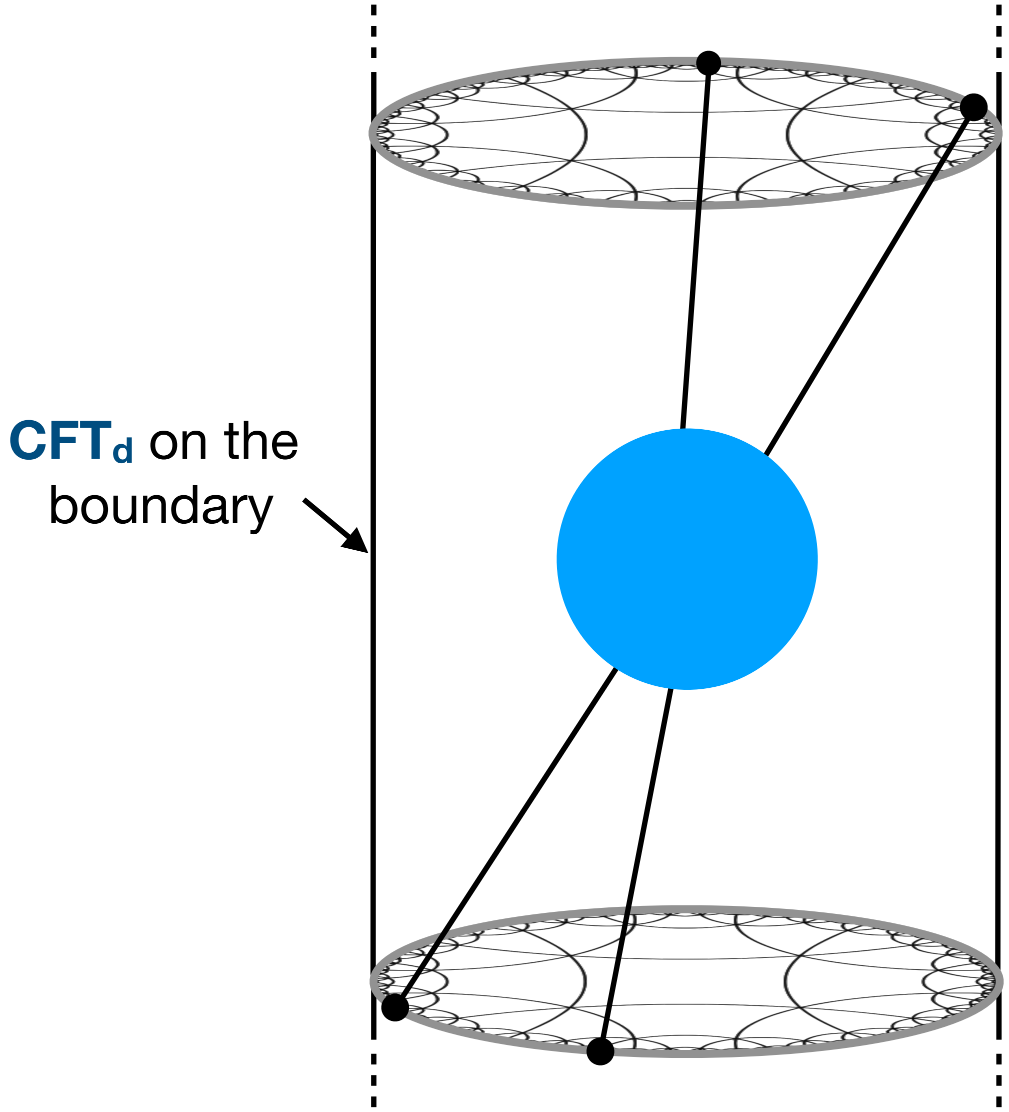
Constrained non-perturbatively by the Conformal Bootstrap:

- Conformal symmetry
- Unitarity
- Associative OPE

$$(\mathcal{O}_1\mathcal{O}_2)\,\mathcal{O}_3 = \mathcal{O}_1\,(\mathcal{O}_2\mathcal{O}_3)$$

[Belavin, Polyakov, Zamolodchikov 1984; Rattazzi, Rychkov, Tonni, Vichi 2008]

AdS-CFT

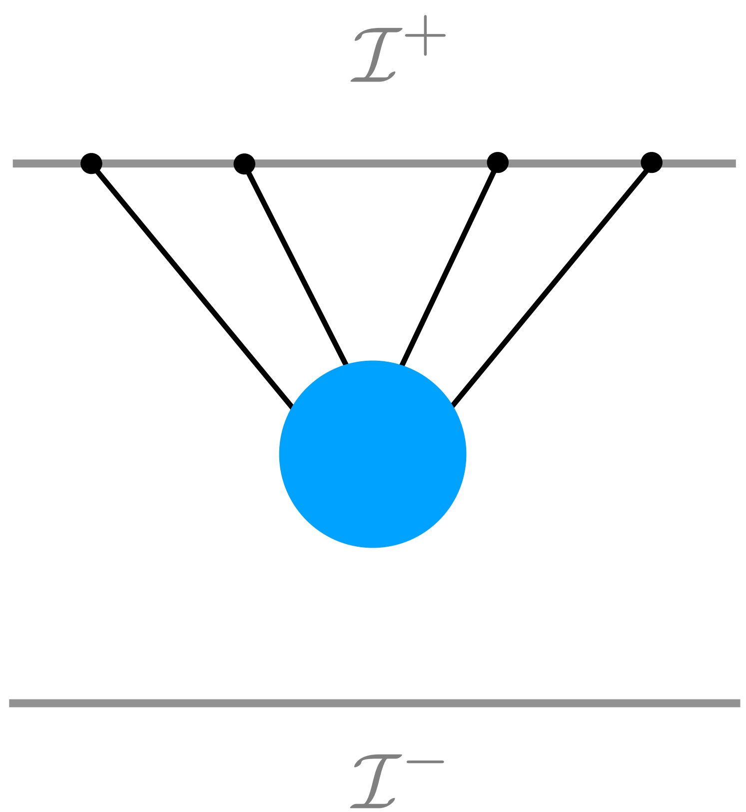


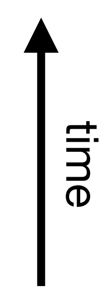
Can we extend this understanding to our own universe?

Holography for all Λs?

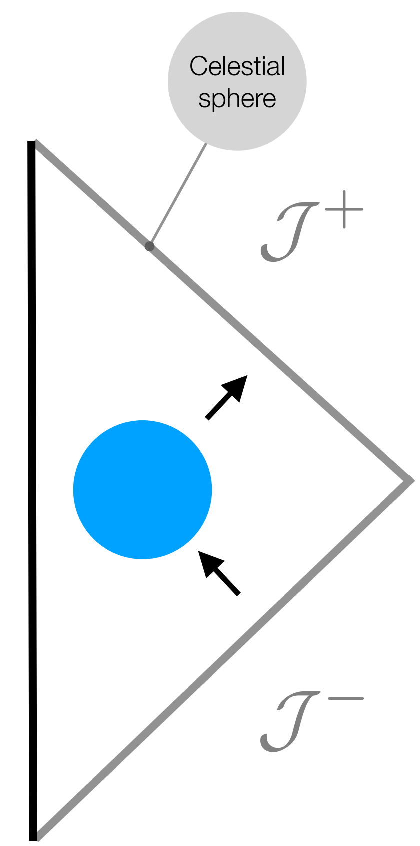
The maximally symmetric cousins of AdS

$\Lambda > 0$ de Sitter





$\Lambda = 0$ Minkowski



- Cosmological scales
- Primordial inflation

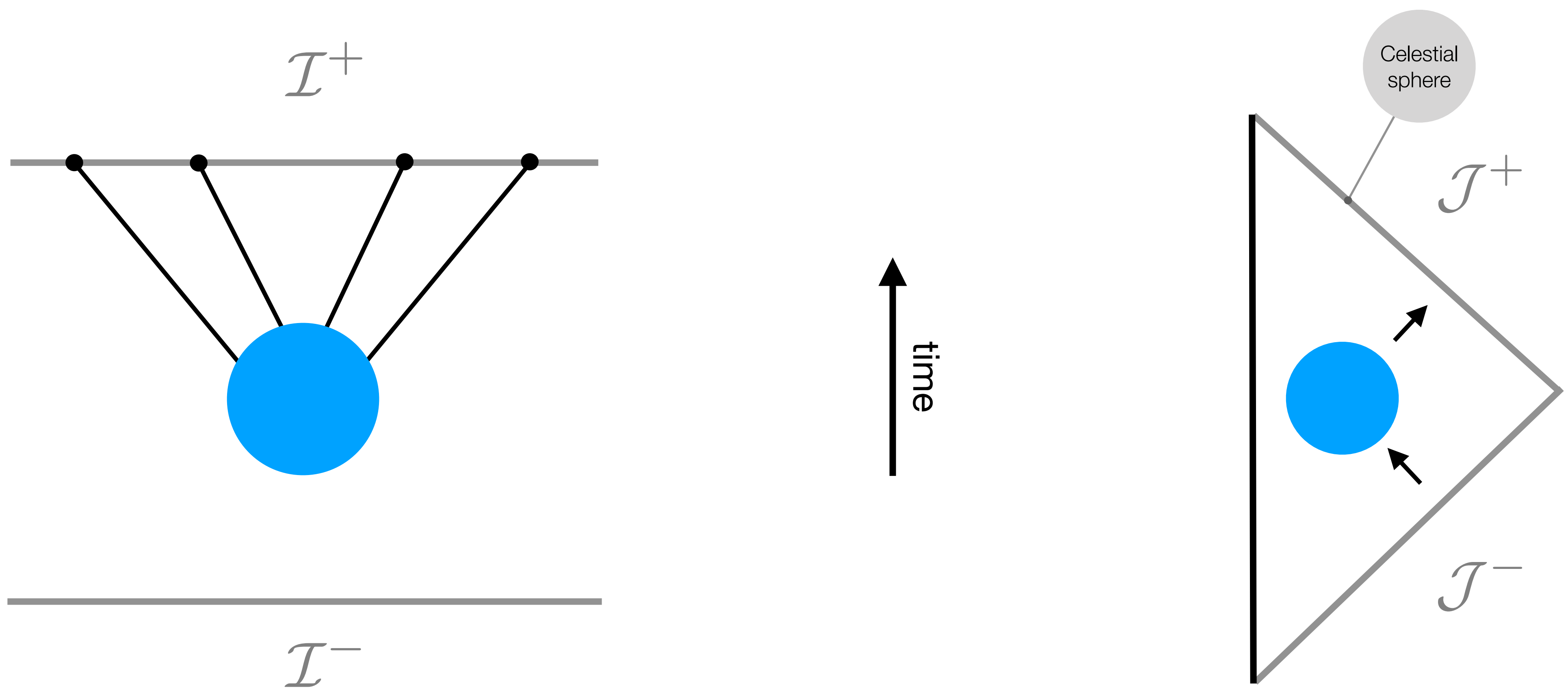
- intermediate scales

Holography for all Λs?

The maximally symmetric cousins of AdS

$\Lambda > 0$ de Sitter

$\Lambda = 0$ Minkowski



Cosmological Bootstrap

[Arkani-Hamed and Maldacena ’15]

[Arkani-Hamed and Benincasa ’17]

[Arkani-Hamed, Baumann, Lee and Pimentel ’18]

[Sleight and Taronna ’19] [Pajer et al ’20] […]

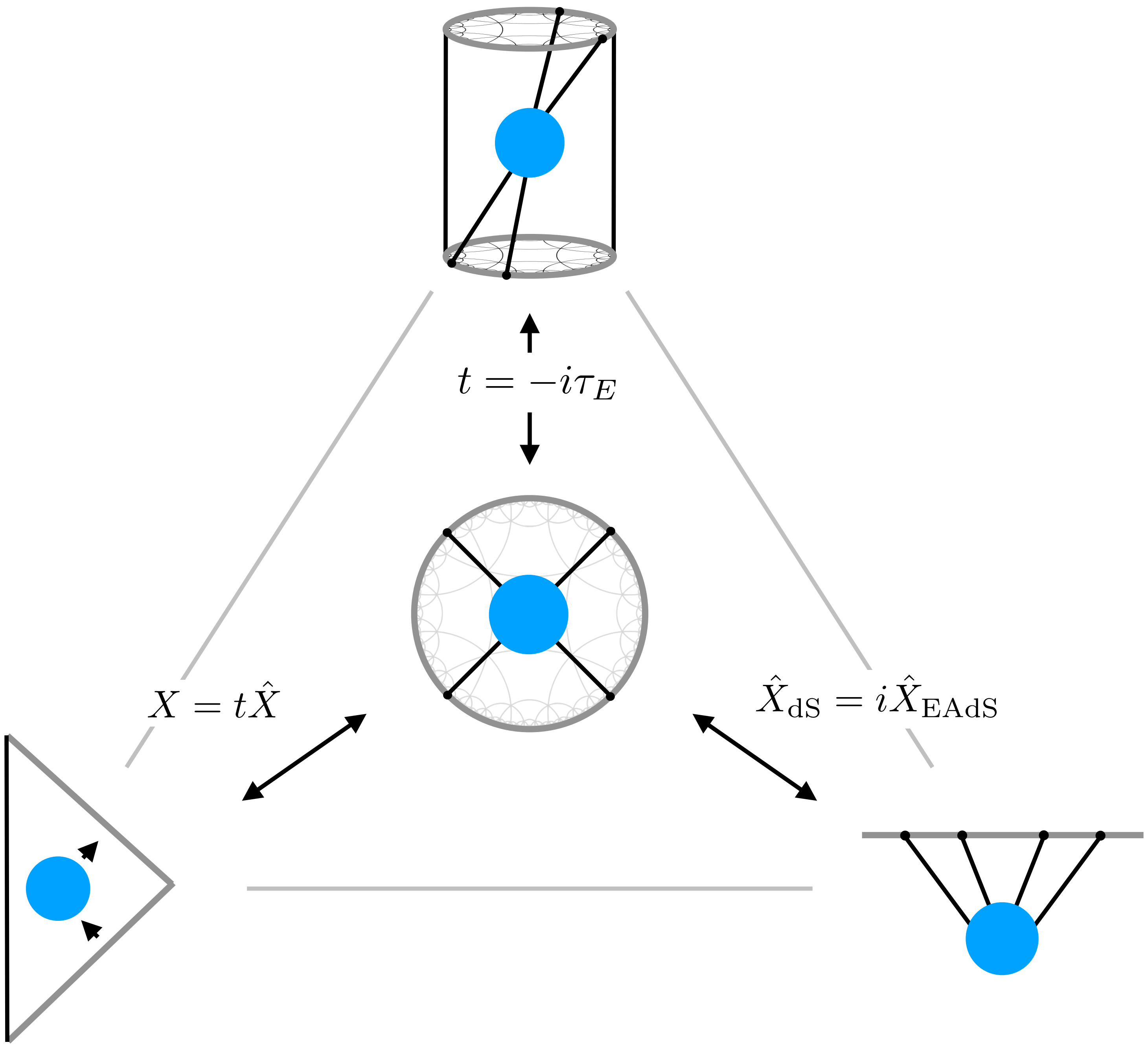
Celestial holography

[de Boer and Solodukhin '03]

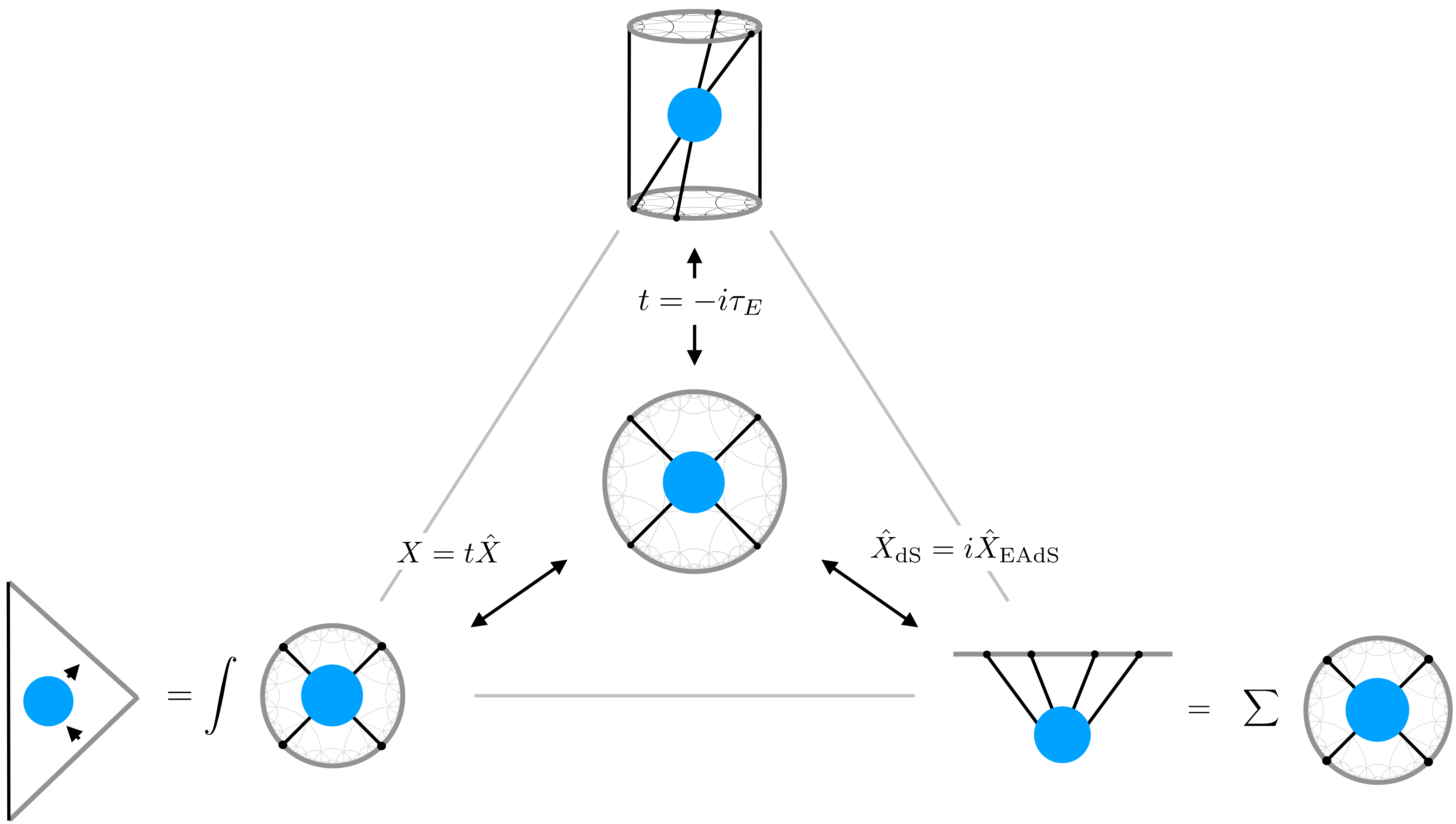
[Strominger ’17] [Pasterski, Shao, Strominger ’17]

[Pasterski, Shao ’17] […]

Holography for all Λs in Euclidean AdS?



Holography for all Λs in Euclidean AdS?



[2301.01810 CS MT, 2401.16591 LI CS MT]

[2007.09993 CS MT, 2109.02725 CS MT, 2407.16652 AC CS MT]

Based on work with: A. Chopping, L. Iacobacci, M. Taronna

Outline

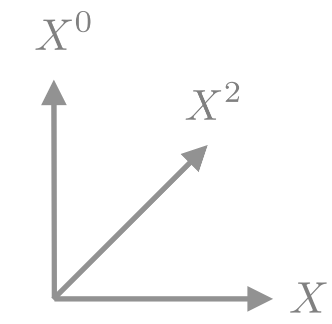
I. $\Lambda < 0$

II. $\Lambda > 0$

III. $\Lambda = 0$

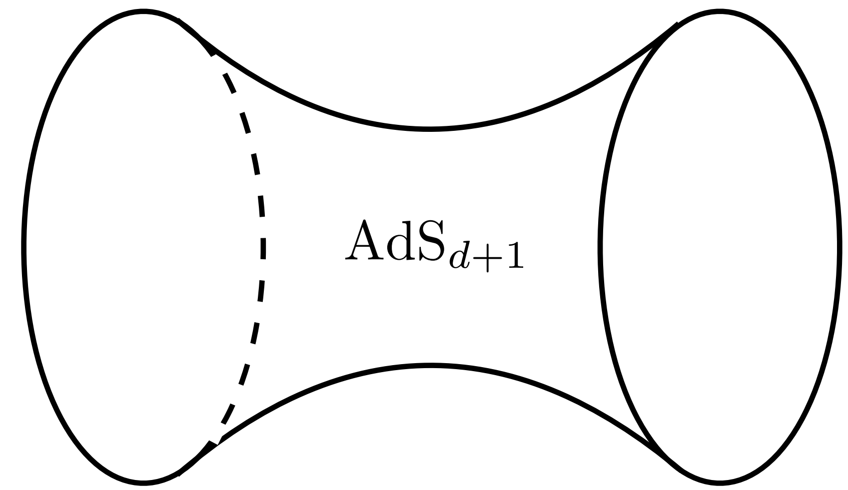
$$\Lambda < 0$$

Anti-de Sitter space-time



$\mathrm{AdS}_{d+1} \subset \mathbb{R}^{d,2}$:

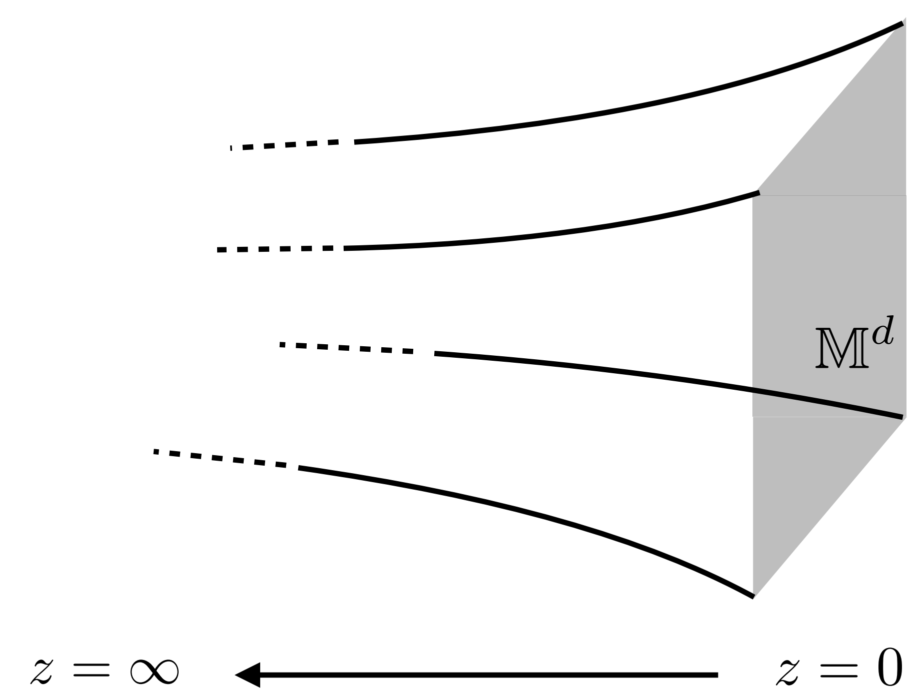
$$-\left(X^0\right)^2 - \left(X^{d+1}\right)^2 + \sum_{i=1}^{d} \left(X^i\right)^2 = -R_{\mathrm{AdS}}^2$$



Isometry group: $SO\left(d,2\right)$ = conformal group in $\mathbb{M}^d$

Poincaré coordinates:

$$\mathrm{d}s^2 = R_{\mathrm{AdS}}^2 \frac{\mathrm{d}z^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$

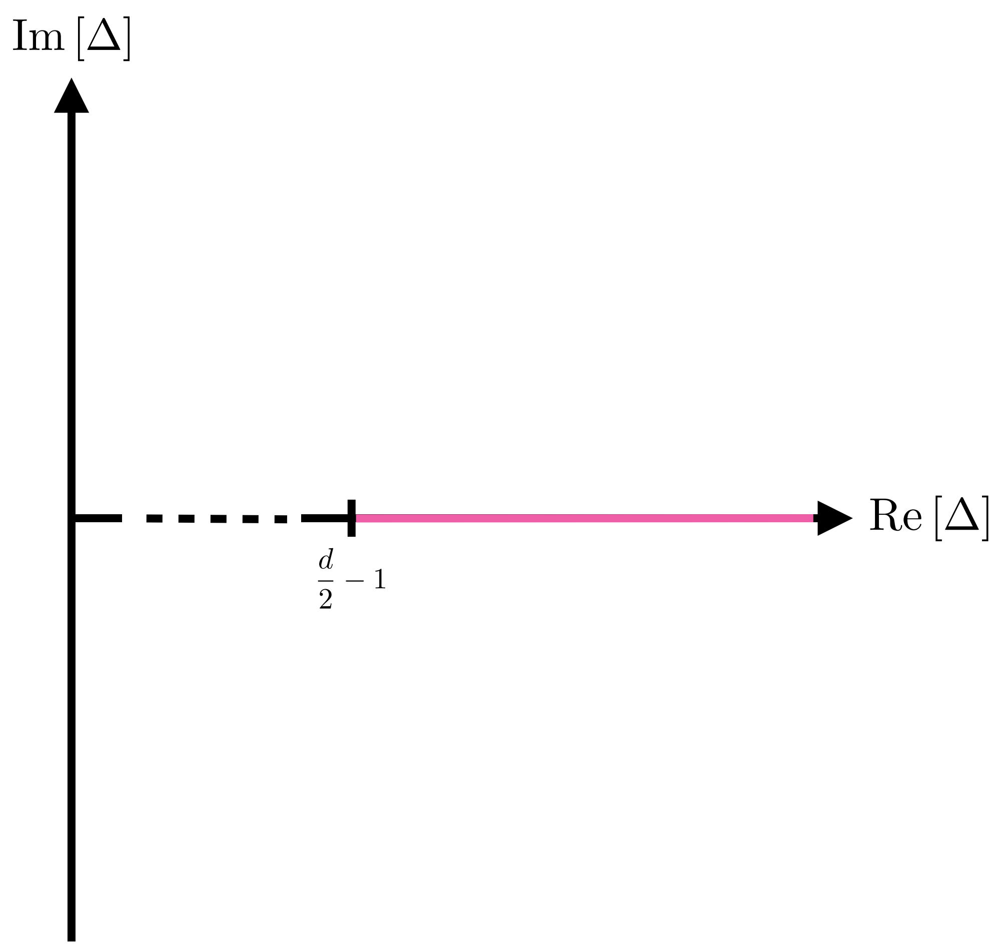


Particles in AdS

Particles in AdS_{d+1} $\longleftrightarrow$ unitary irreducible representations of $SO(d,2)$

Labelled by a scaling dimension Δ and spin J. Unitarity constrains Δ :

E.g. Spin J=0 representations



Notes:

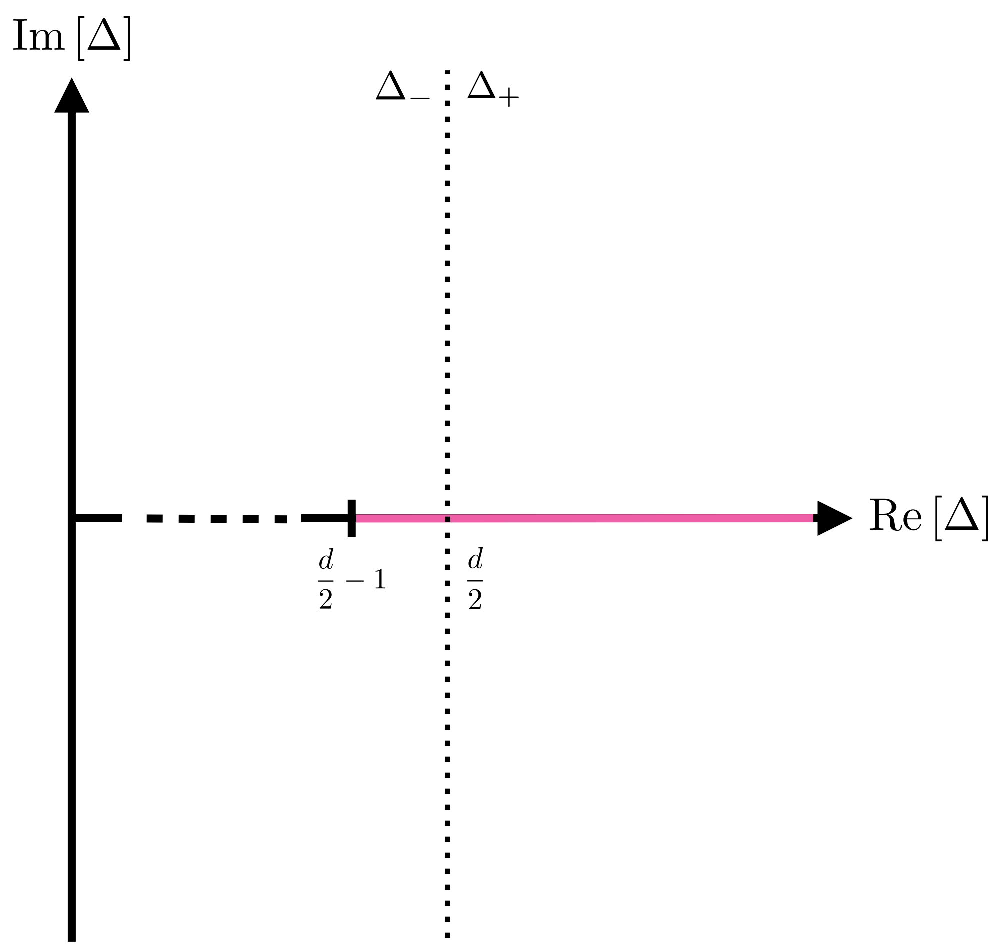
- $\Delta \in \mathbb{R}$
- Bounded from below $\Delta \geq \frac{d}{2} - 1$

Particles in AdS

Particles in AdS$_{d+1}$ $\longleftrightarrow$ unitary irreducible representations of $SO(d,2)$

Labelled by a scaling dimension Δ and spin J. Can be realised by fields in AdS$_{d+1}$:

E.g. Spin J=0 representations



Quadric Casimir equation

$\langle \mathcal{C}_2 \rangle = \Delta(\Delta - d)$

$$\left(\nabla^2 - m^2\right)\varphi = 0 \quad \leftrightarrow \quad \left(\mathcal{C}_2 - \langle \mathcal{C}_2 \rangle\right)\varphi = 0$$

$$m^2 R^2_{\mathrm{AdS}} = \Delta(\Delta - d)$$

Boundary behaviour ($\Delta_- = d - \Delta_+$):

$$\lim_{z\to 0} \varphi(z,x) = O_{\Delta_+}(x)\, z^{\Delta_+} + O_{\Delta_-}(x)\, z^{\Delta_-}$$

Dirichlet boundary condition

Neuman boundary condition

N.B. Δ_- may be ruled out by unitarity

$O_{\Delta_\pm}(x)$ transform as primary fields with scaling dimension $\Delta_\pm$ in Minkowski CFT$_d$

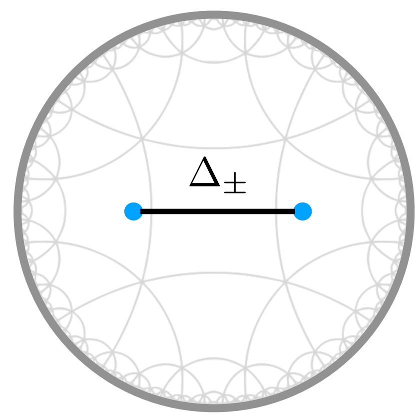
AdS boundary correlators

$$\lim_{z\to 0} z^{-(\Delta_1+\ldots+\Delta_n)}\langle\varphi_1(x_1,z)\ldots\varphi_n(x_n,z)\rangle \overset{!}{=} \langle\mathcal{O}_{\Delta_1}(x_1)\ldots\mathcal{O}_{\Delta_n}(x_n)\rangle$$

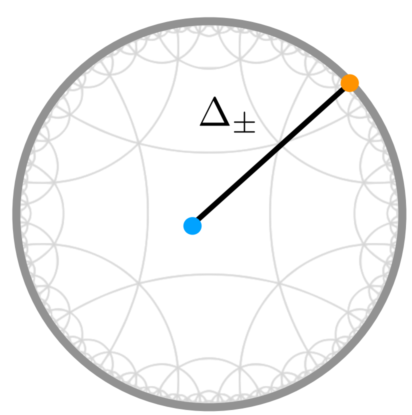
Feynman rules:

Bulk-to-bulk propagator, $\Delta_\pm$ boundary condition:

$\Delta_{+/-}$ Dirichlet / Neumann b.c.



Bulk-to-boundary propagator, $\Delta_\pm$ boundary condition:

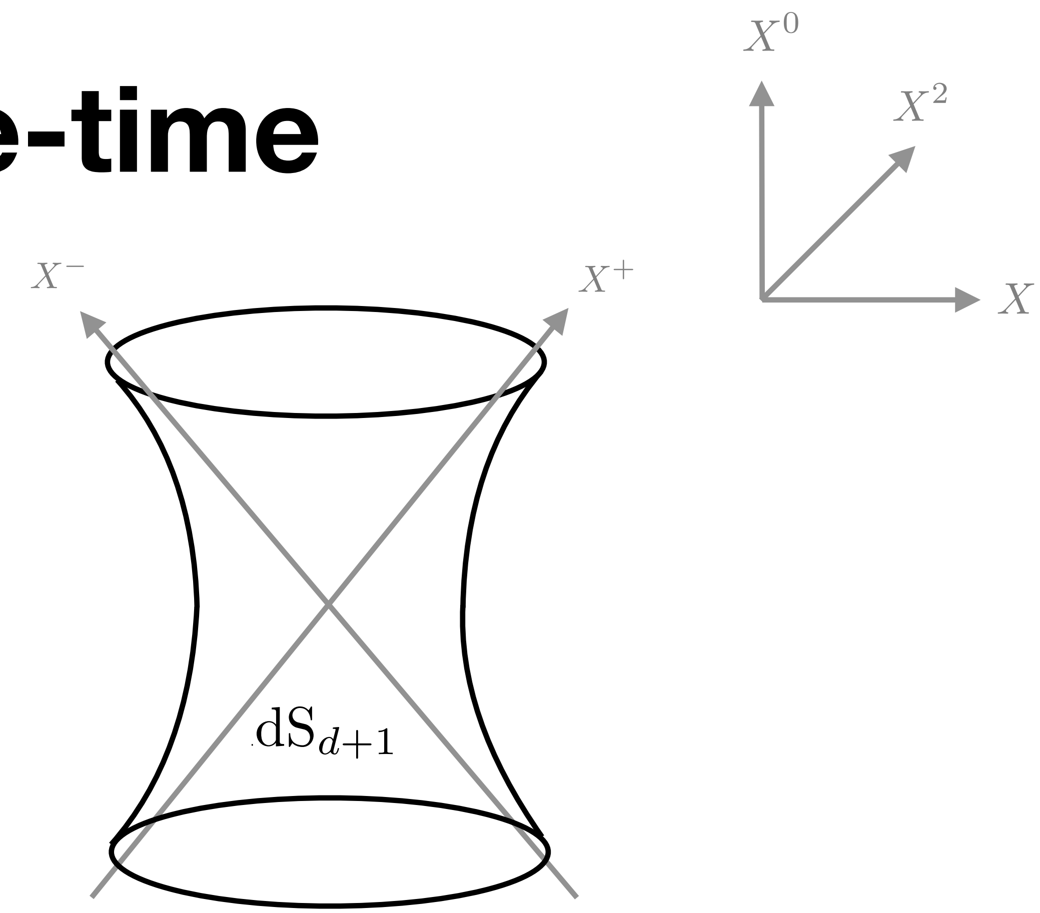


$$\Lambda > 0$$

de Sitter space-time

$\mathrm{dS}_{d+1} \subset \mathbb{M}^{d+2}$:

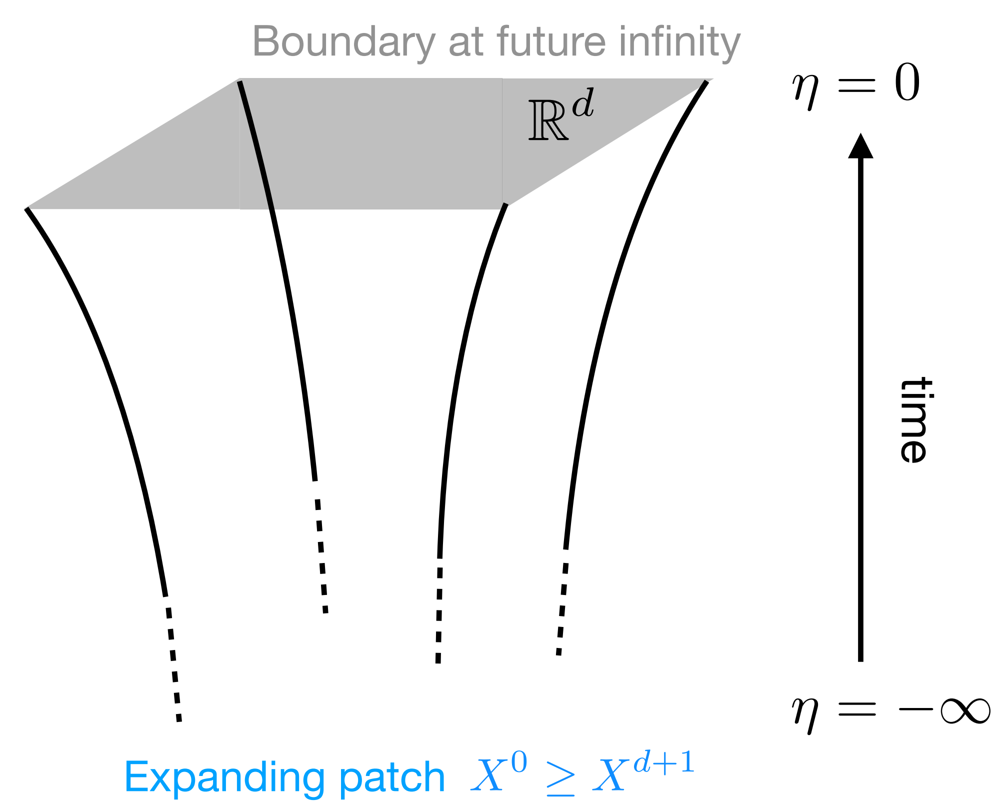
$$-\left(X^0\right)^2 + \sum_{i=1}^{d+1} \left(X^i\right)^2 = R_{\mathrm{dS}}^2$$



Isometry group: $SO\,(d+1,1)$ = conformal group in $\mathbb{R}^d$

Poincaré coordinates:

$$\mathrm{d}s^2 = R_{\mathrm{dS}}^2 \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{\eta^2}$$



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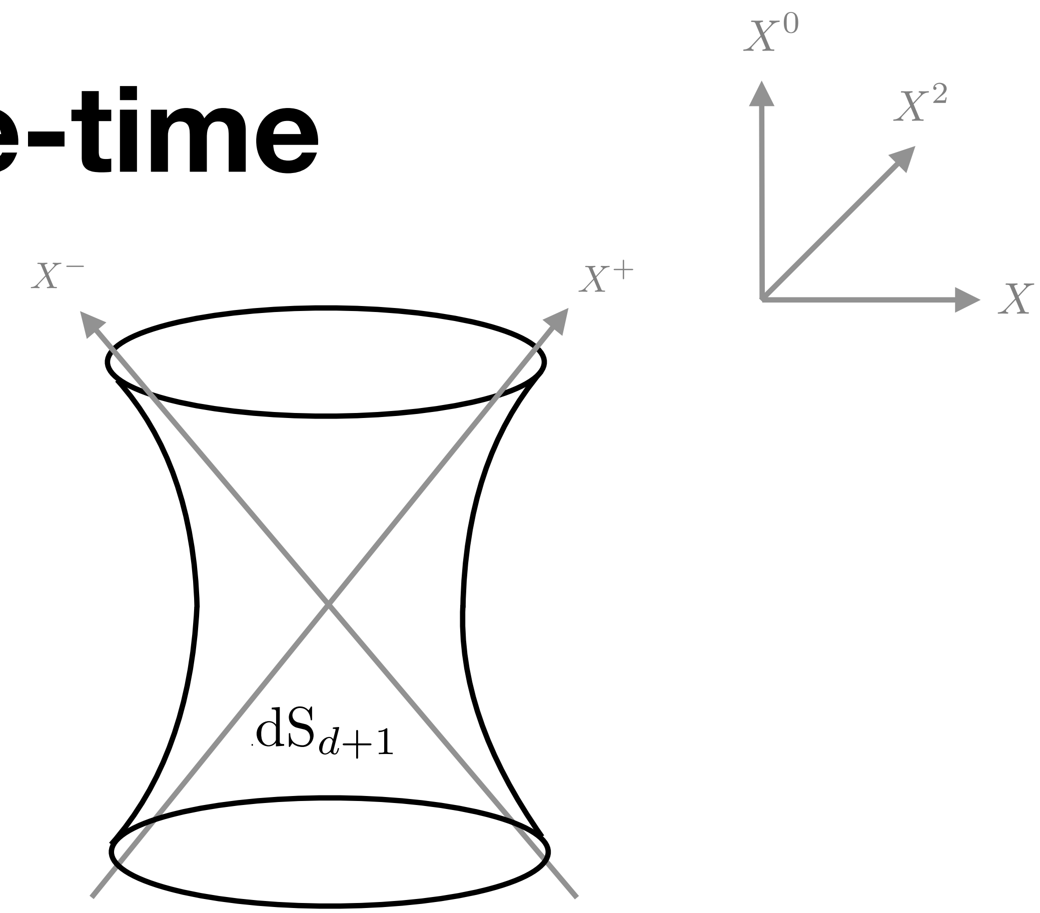

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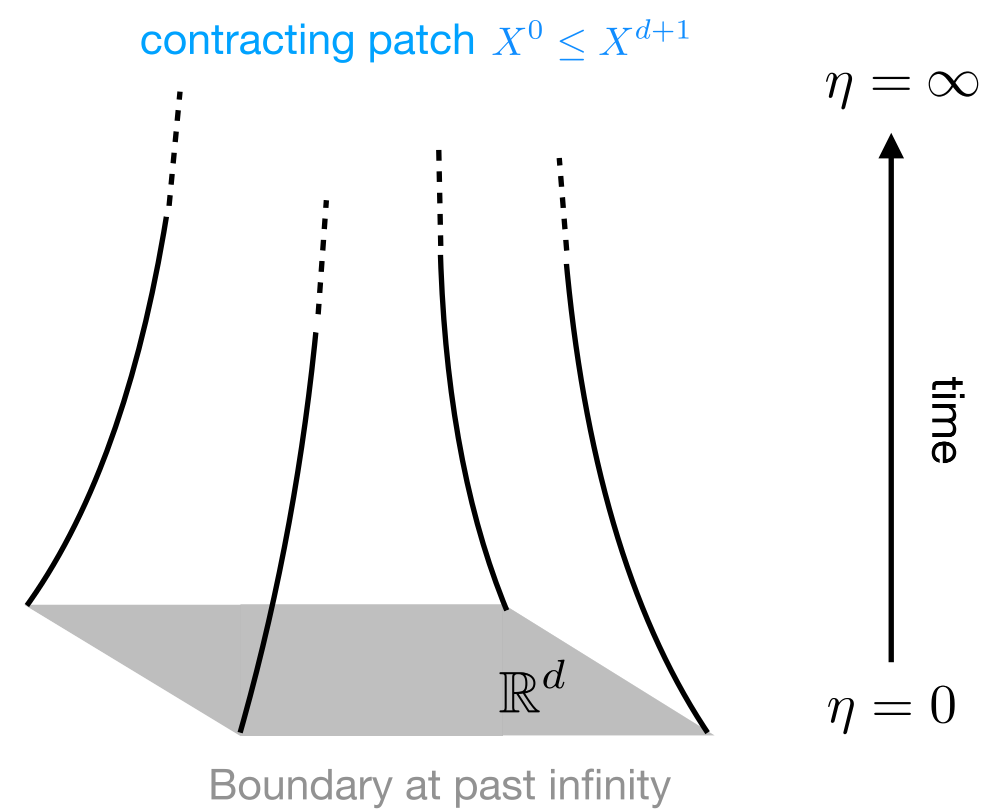
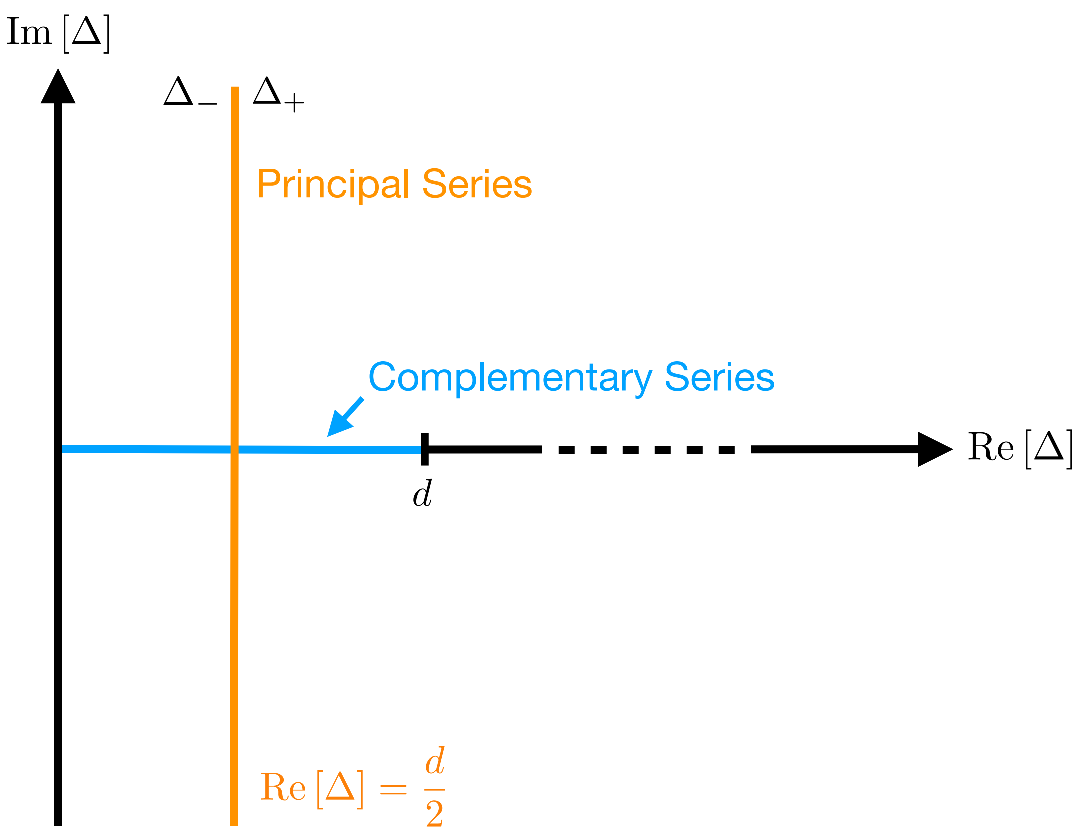


Particles in dS

Particles in dS_{d+1} $\longleftrightarrow$ unitary irreducible representations of $SO(d+1,1)$

Labelled by a scaling dimension Δ and spin J. Unitarity constrains Δ :

E.g. Spin J=0 representations



Notes:

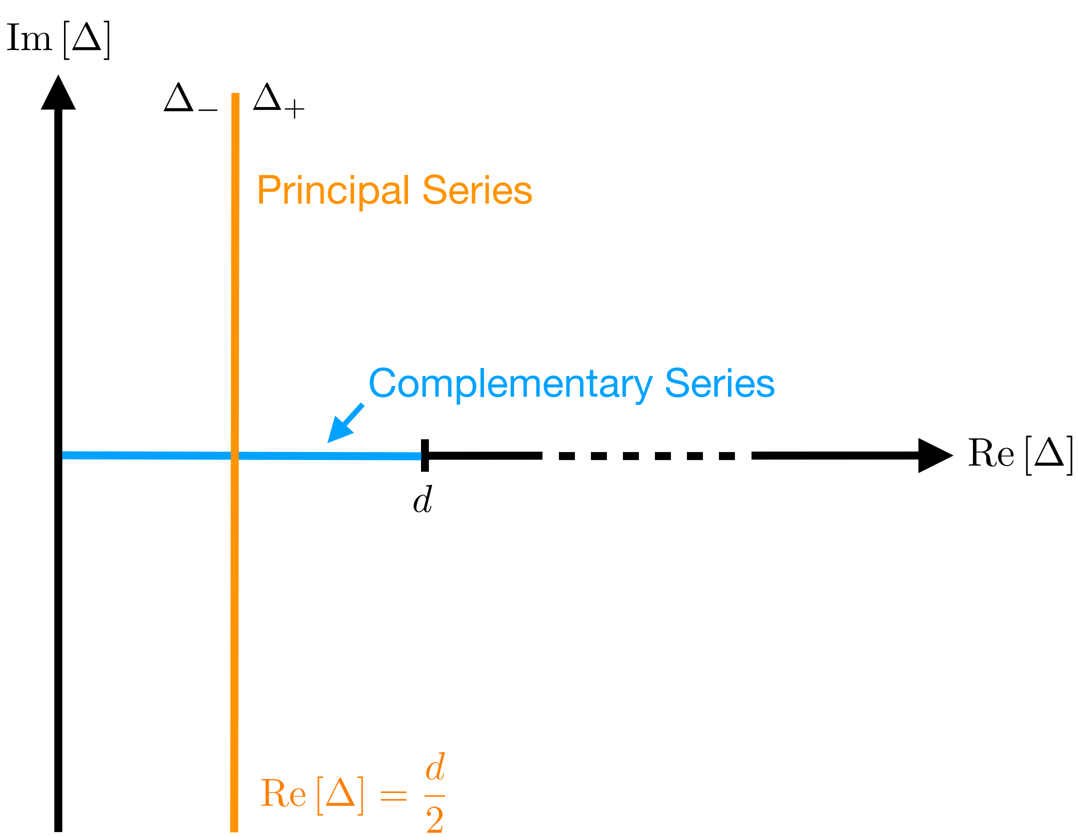
- Both Δ_+ and Δ_- are unitary
- Δ can be complex - Principal Series

Particles in dS

Particles in dS_{d+1} $\longleftrightarrow$ unitary irreducible representations of $SO(d+1,1)$

Labelled by a scaling dimension Δ and spin J. Can be realised by fields in dS_{d+1}.

E.g. Spin J=0 representations



Quadric Casimir equation

$$\langle \mathcal{C}_2 \rangle = \Delta\,(d-\Delta)$$

$$\left(\nabla^2 - m^2\right)\varphi = 0 \quad \leftrightarrow \quad \left(\mathcal{C}_2 - \langle \mathcal{C}_2 \rangle\right)\varphi = 0$$

$$m^2 R_{\mathrm{dS}}^2 = \Delta\,(d-\Delta)$$

Boundary behaviour:

$$\lim_{\eta\to 0}\varphi\,(\eta, x) = O_{\Delta_+}(\mathbf{x})\,\eta^{\Delta_+} + O_{\Delta_-}(\mathbf{x})\,\eta^{\Delta_-}$$

Determined by the initial state

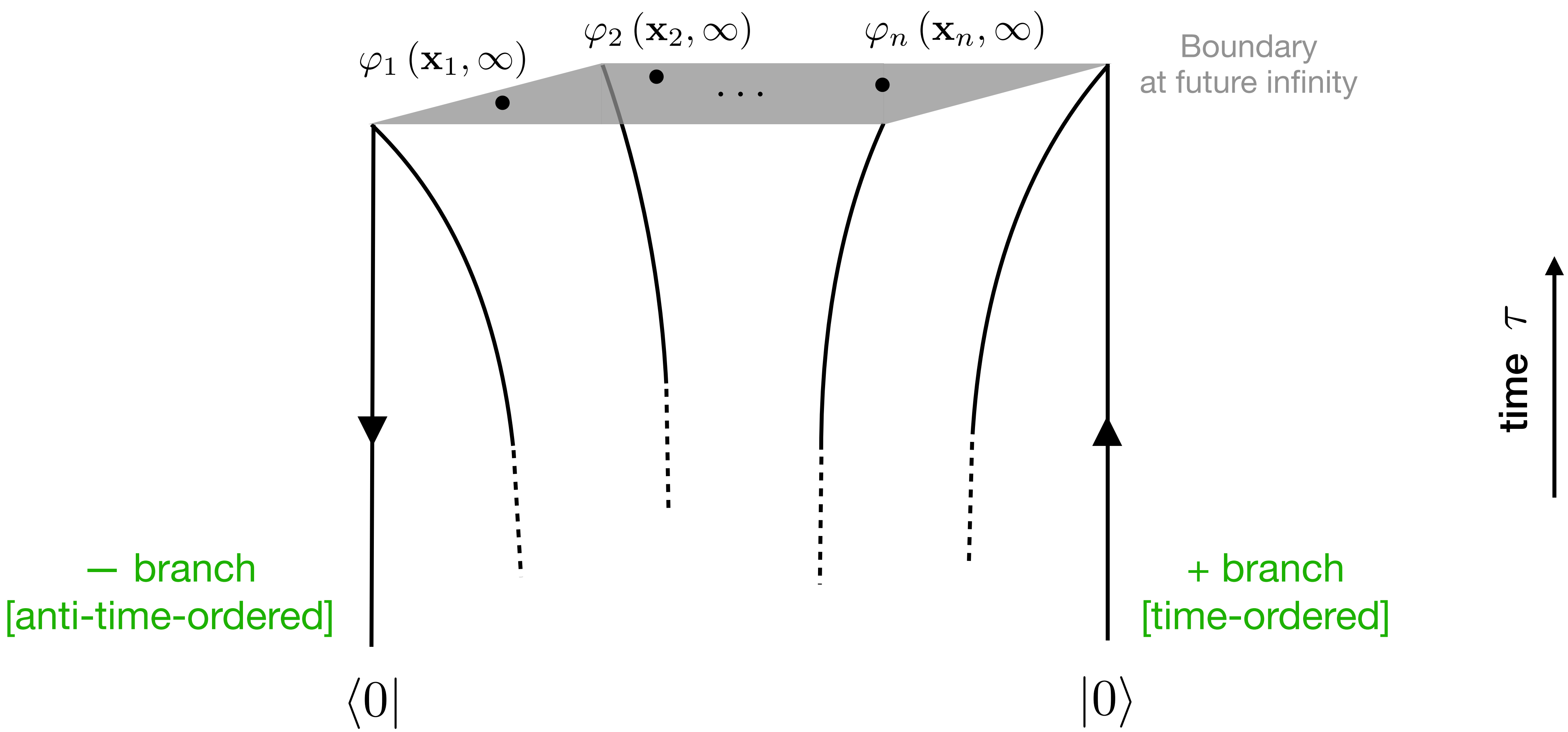
$O_{\Delta_\pm}(\mathbf{x})$ transform as primary fields with scaling dimension $\Delta_\pm$ in Euclidean CFT_d

dS Boundary Correlators

[in-in formalism for late-time correlators]

[Maldacena ’02, Weinberg '05]

$$\lim_{\tau\to\infty} \langle\Omega|\varphi_1(\mathbf{x}_1,\tau)\ldots\varphi_n(\mathbf{x}_n,\tau)|\Omega\rangle$$



Take $|0\rangle$ to be the free theory vacuum

dS Boundary Correlators

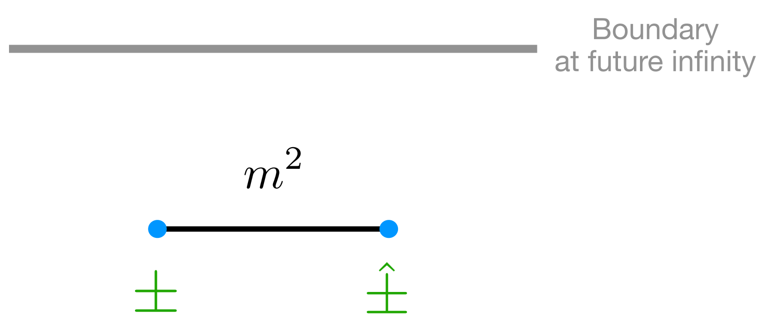
[in-in formalism for late-time correlators]

[Maldacena '02, Weinberg '05]

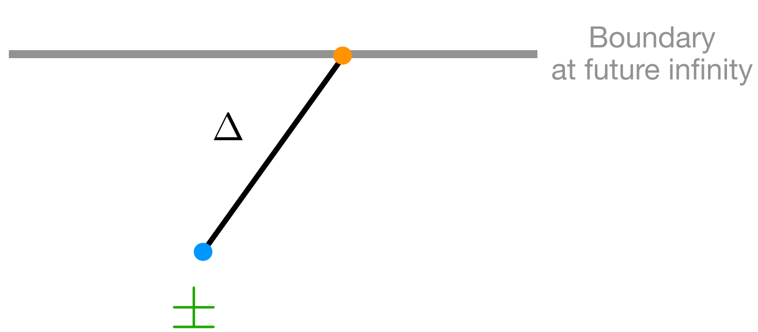
$$\lim_{\tau\to\infty} \langle\Omega|\varphi_1(\mathbf{x}_1,\tau)\ldots\varphi_n(\mathbf{x}_n,\tau)|\Omega\rangle$$

Feynman rules:

$\pm$ bulk-to- $\hat{\pm}$ bulk propagator:



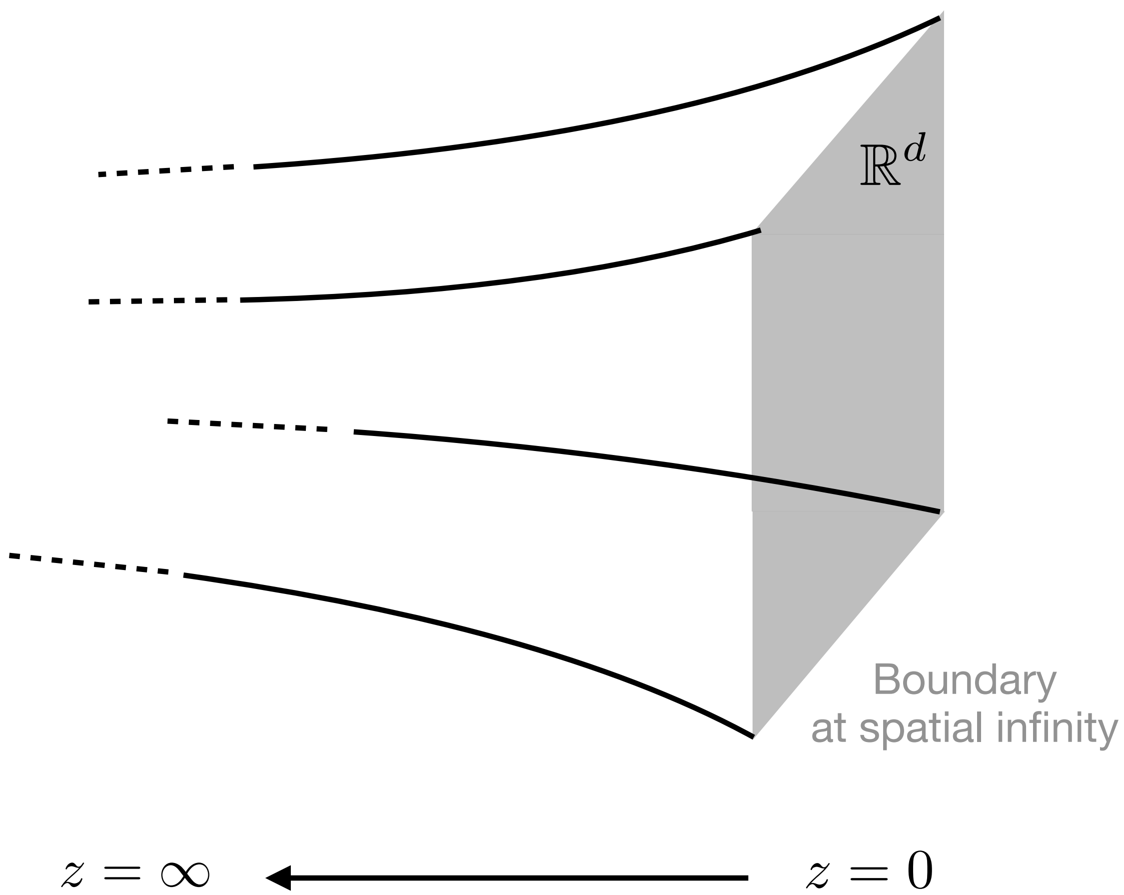
$\pm$ bulk-to-boundary propagator:



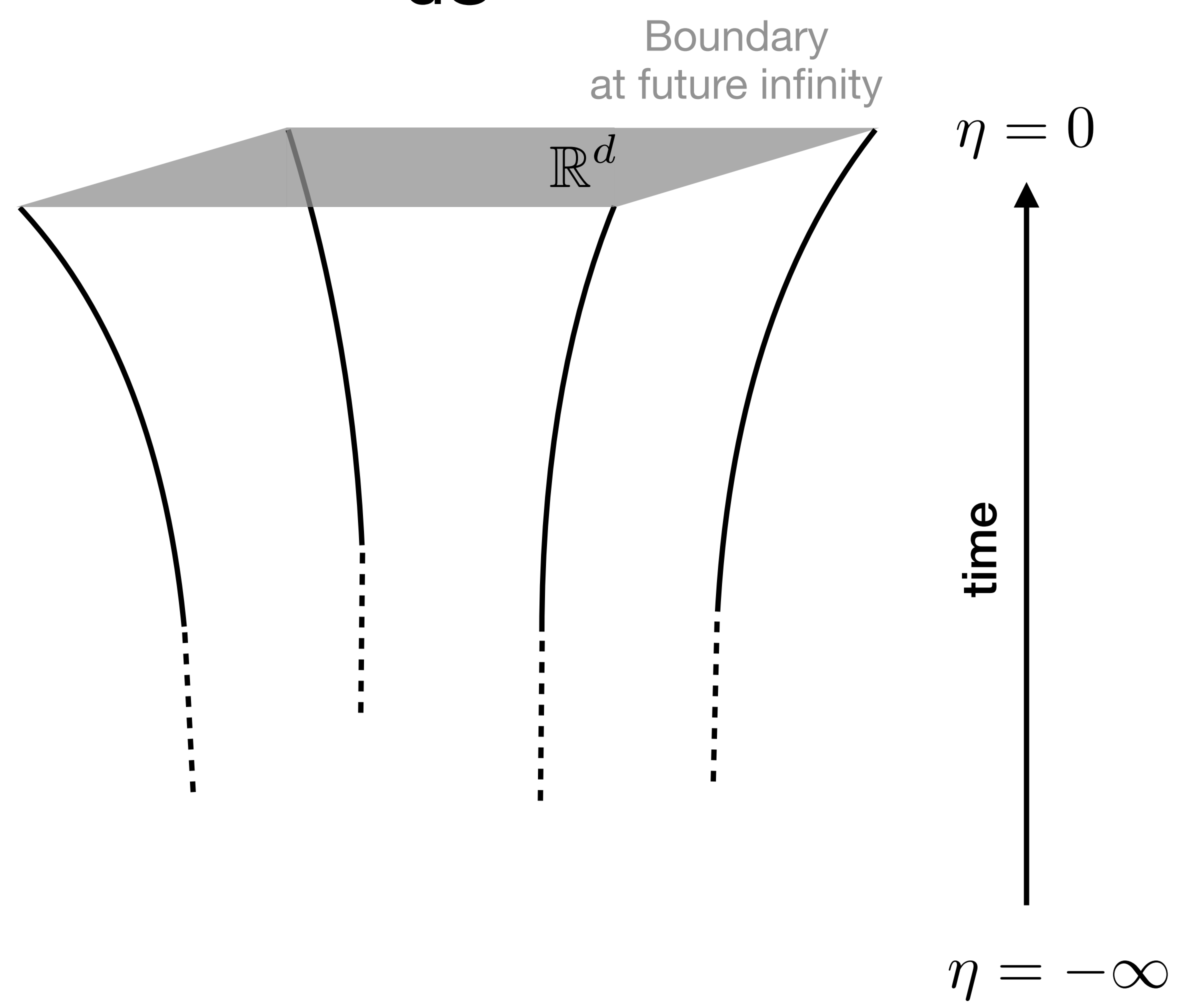
Sum contributions from each branch (±) of the time (in-in) contour!

From dS to Euclidean AdS

Euclidean AdS



dS



$$\mathrm{d}s^2 = R_{\mathrm{AdS}}^2 \frac{\mathrm{d}z^2 + \mathrm{d}\mathbf{x}^2}{z^2} \qquad \longleftrightarrow \qquad \mathrm{d}s^2 = R_{\mathrm{dS}}^2 \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{\eta^2}$$

EAdS and dS are identified under:

$$R_{\mathrm{AdS}} = \pm i R_{\mathrm{dS}} \qquad z = \pm i\,(-\eta)$$

From dS to Euclidean AdS

[Bunch-Davies vacuum]

Wightman function is defined as having the same light-cone singularities as in Minkowski space:

at short distances

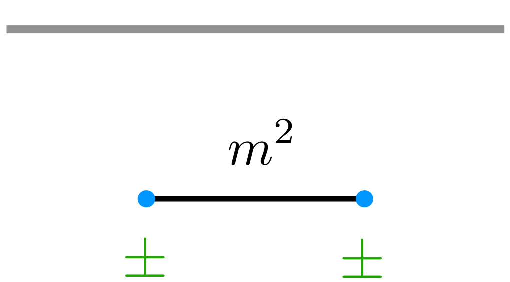
$$G(\sigma) = A\, {}_2F_1\left(\begin{matrix}\Delta_+, \Delta_- \\ \frac{d+1}{2}\end{matrix}; \sigma\right) \approx \frac{1}{\left[(x-y)^2\right]^{\frac{d-2}{2}}} \frac{\Gamma\left(\frac{d+1}{2}\right)}{2\,(d-1)\,\pi^{(d+1)/2}}$$

flat space expression

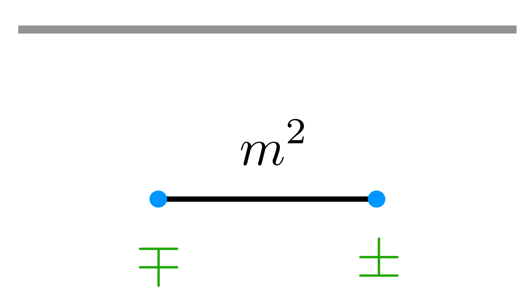
[B. Allen '86]

invariant distance: $\sigma(x,y) = \dfrac{R^2 + X(x)\cdot Y(y)}{2R^2}, \quad X^2 = R^2, \quad Y^2 = R^2.$

Propagators in the in-in formalism correspond to different $i\epsilon$ prescriptions:



$= \quad G(\sigma \mp i\epsilon)\,,$



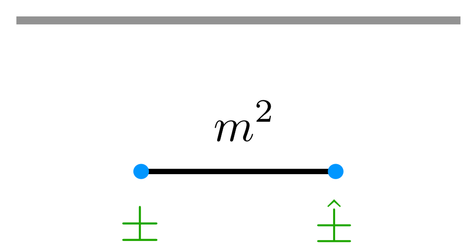
$= \quad G\left(\sigma \mp i\epsilon\, \mathrm{sgn}\left(\eta_{\mp} - \eta_{\pm}\right)\right)$

These are obtained from $G(\sigma)$ by replacing: $\quad \eta_- = \eta_-\,(1 + i\epsilon)\,, \qquad \eta_+ = \eta_+\,(1 - i\epsilon)$

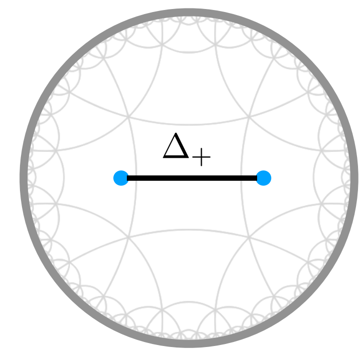
From dS to Euclidean AdS

[Bunch-Davies vacuum] [C.S. and M. Taronna ’19 ’20 ’21]

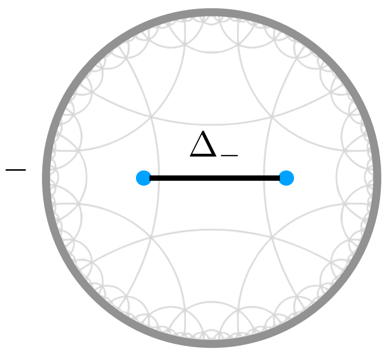
$\pm$ bulk-to- $\hat{\pm}$ bulk propagator:



$= e^{\mp \frac{i\pi}{2}\Delta_+} e^{\hat{\mp}\frac{i\pi}{2}\Delta_+}$

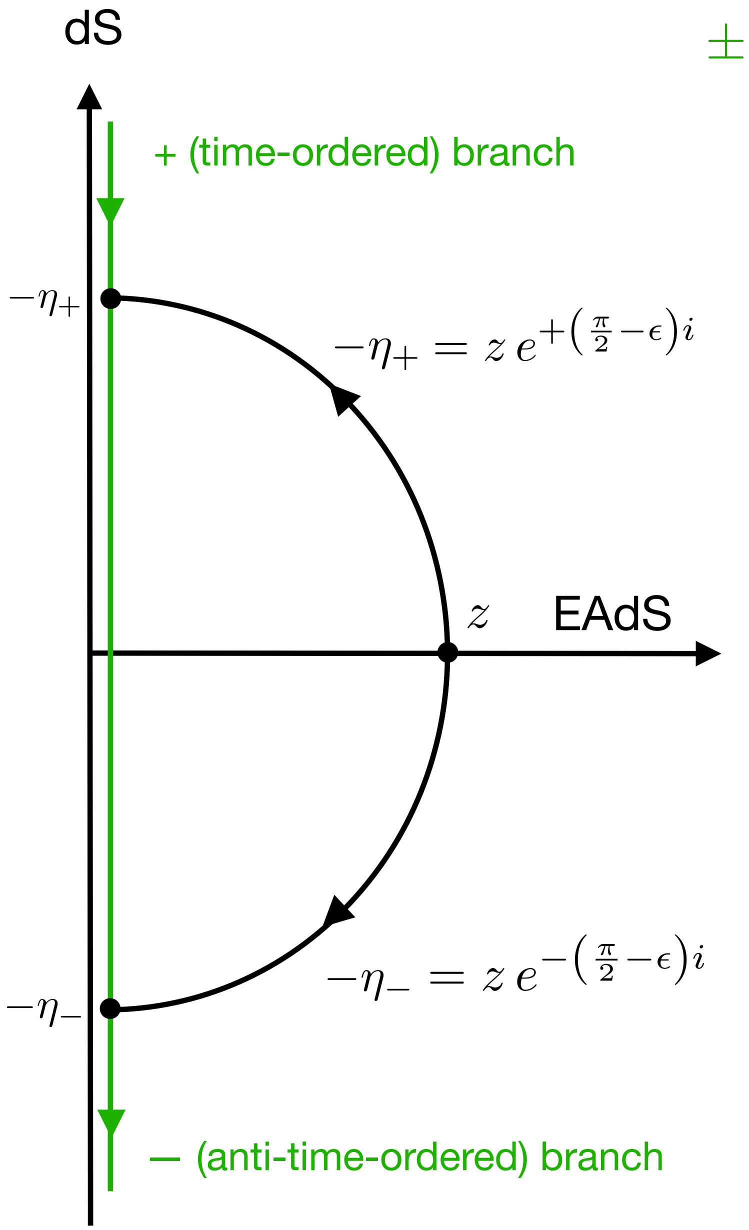


$+ e^{\mp \frac{i\pi}{2}\Delta_-} e^{\hat{\mp}\frac{i\pi}{2}\Delta_-}$

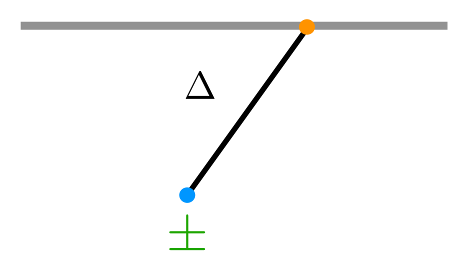


Dirichlet boundary condition

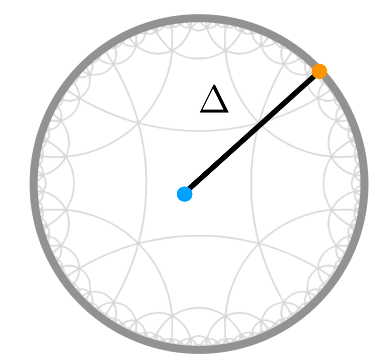
Neumann boundary condition



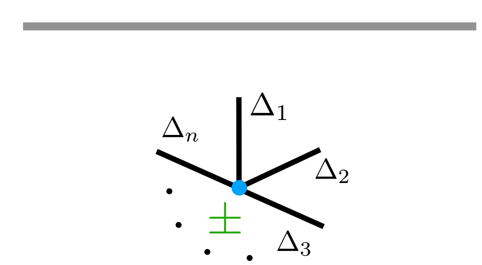
$\pm$ bulk-to-boundary propagator:



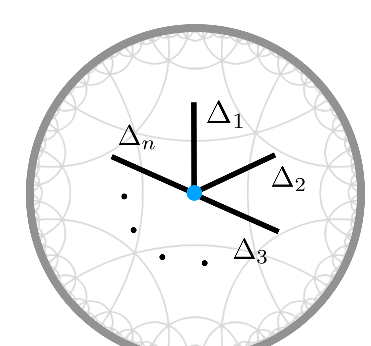
$= e^{\mp \frac{i\pi}{2}\Delta}$



$\pm$ bulk integrals:



$= e^{\pm(d-1)\frac{\pi i}{2}}$

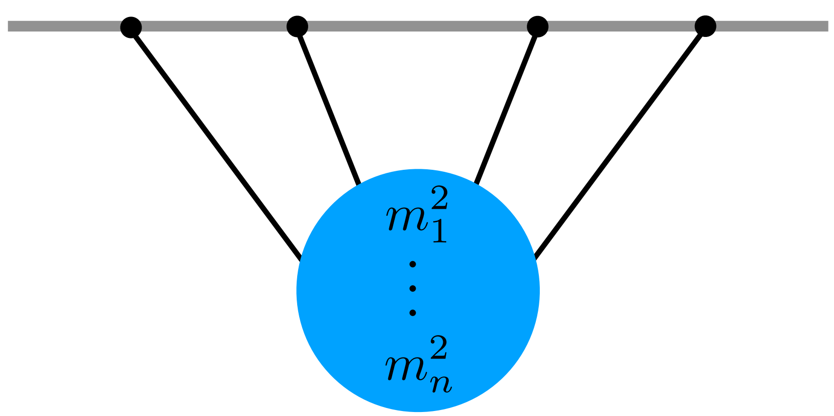


From dS to Euclidean AdS

[Bunch-Davies vacuum] [C.S. and M. Taronna ’20 ’21]

dS boundary correlators are perturbatively recast as Witten diagrams in EAdS:

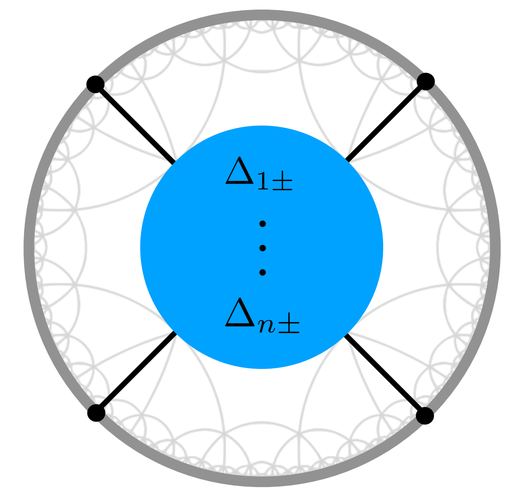
e.g. four-points



Combines contributions from each branch of the in-in contour

$$= \sum_{\Delta_{1\pm}\ldots\Delta_{n\pm}} C_{\Delta_{1\pm}\ldots\Delta_{n\pm}}$$

Sum over boundary conditions for exchanged particles



Notzs:

- Contributions from both $\Delta_\pm$ modes, which is not always possible in AdS
- $\Delta_{i\,\pm} \in$ Unitary Irreducible Representation of **dS** isometry group

From dS to Euclidean AdS

[Generic dS invariant vacuum] [A. Chopping, C.S., M. Taronna '24]

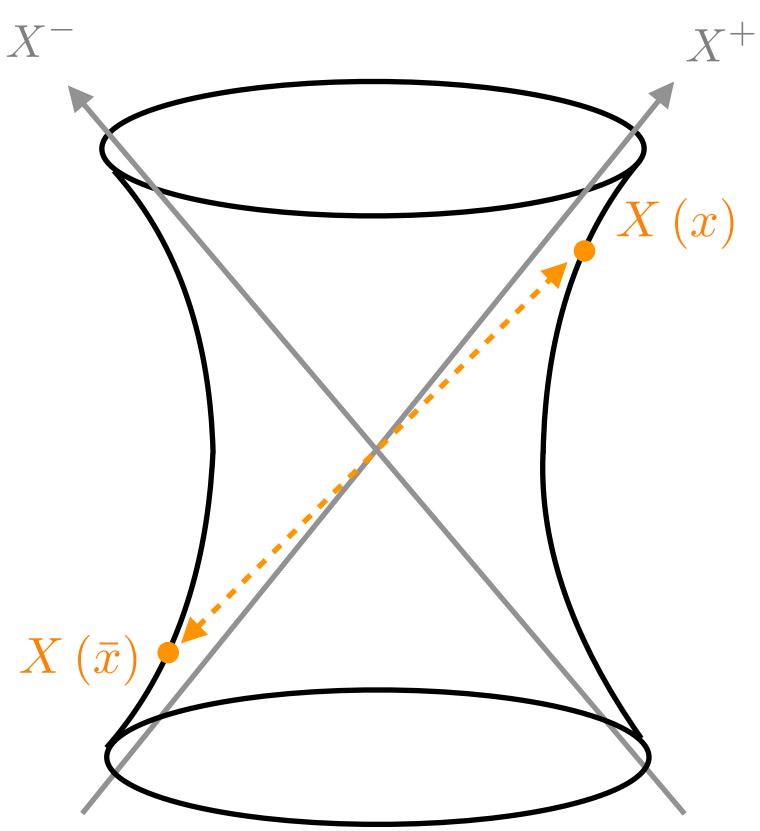
The Wightman function now has a singularity for antipodal points:

$$G(\sigma) = A\,{}_2F_1\left(\begin{matrix}\Delta_+,\Delta_-\\ \frac{d+1}{2}\end{matrix};\sigma\right) + B\,{}_2F_1\left(\begin{matrix}\Delta_+,\Delta_-\\ \frac{d+1}{2}\end{matrix};\bar{\sigma}\right)$$

Bunch-Davies solution — Antipodal transform

[B. Allen '86]

where $\bar{\sigma}(x,y) = \sigma(\bar{x},y)$ with antipodal transformation $X(\bar{x}) = -X(x)$



In Poincaré coordinates: $x = (\eta, \mathbf{x})\,, \quad \bar{x} = (-\eta, \mathbf{x})$

Upshot:

- 2pt functions are a combination of Bunch-Davies (BD) ones with points antipodally transformed to the **contracting patch**.
- In turn, perturbative late time correlators are a combination of BD ones with boundary points antipodally transformed. I.e. to the **past boundary**.

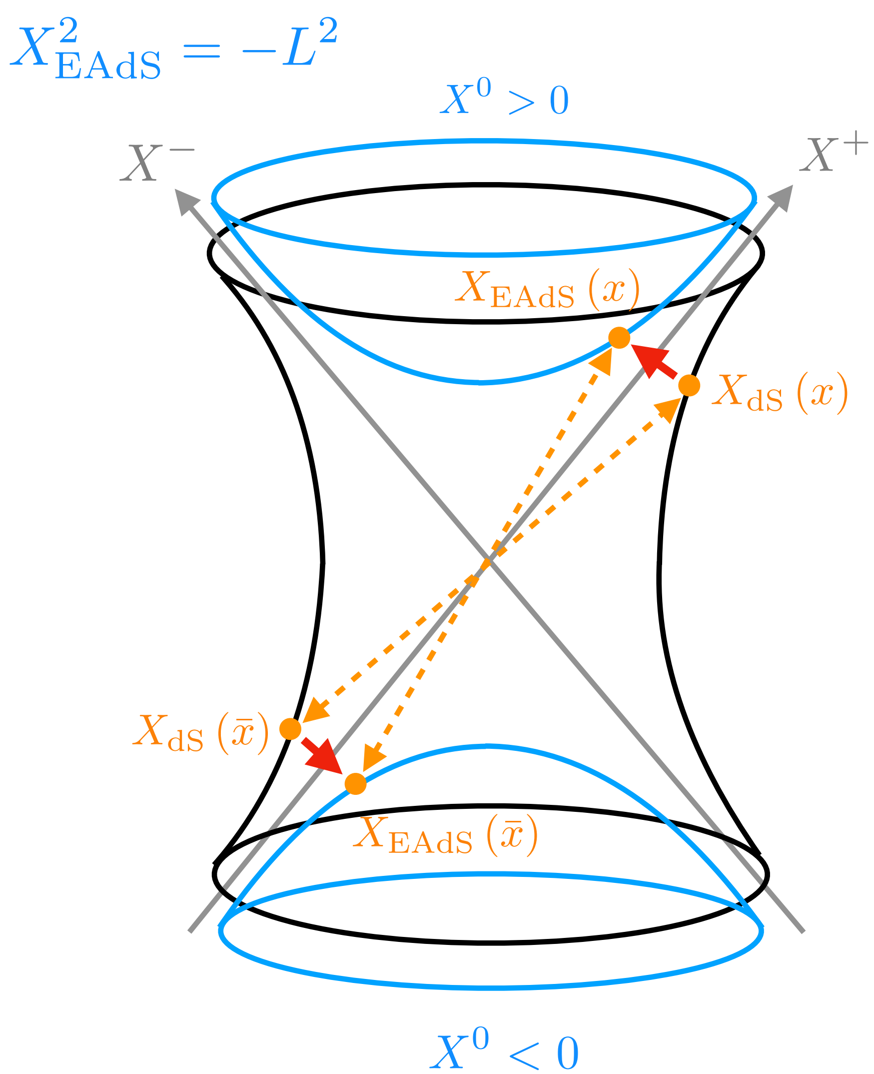
From dS to Euclidean AdS

[Generic dS invariant vacuum] [A. Chopping, C.S., M. Taronna '24]

Under analytic continuation $\eta \to \pm iz$ to EAdS:

Points in the **expanding patch** of dS continue to the **upper sheet** $(X^0 > 0)$ of EAdS.

Their **antipodes** in the contracting patch continue to the **lower sheet** $(X^0 < 0)$ of EAdS.



Perturbative late-time correlators are a combination of EAdS Witten diagrams, but with some points antipodally transformed to the boundary of the **lower sheet of EAdS**!

In momentum space the antipodal transformation corresponds to a sign change in the modulus:

$$k \to e^{\pm \pi i} k, \quad k = |\mathbf{k}|$$

see e.g. mode function:

$$f_{\mathbf{k}}(\eta) = (-\eta)^{\frac{d}{2}} \frac{\sqrt{\pi}}{2} e^{\frac{\pi \nu}{2}} H^{(2)}_{i\nu}(-k\eta)$$

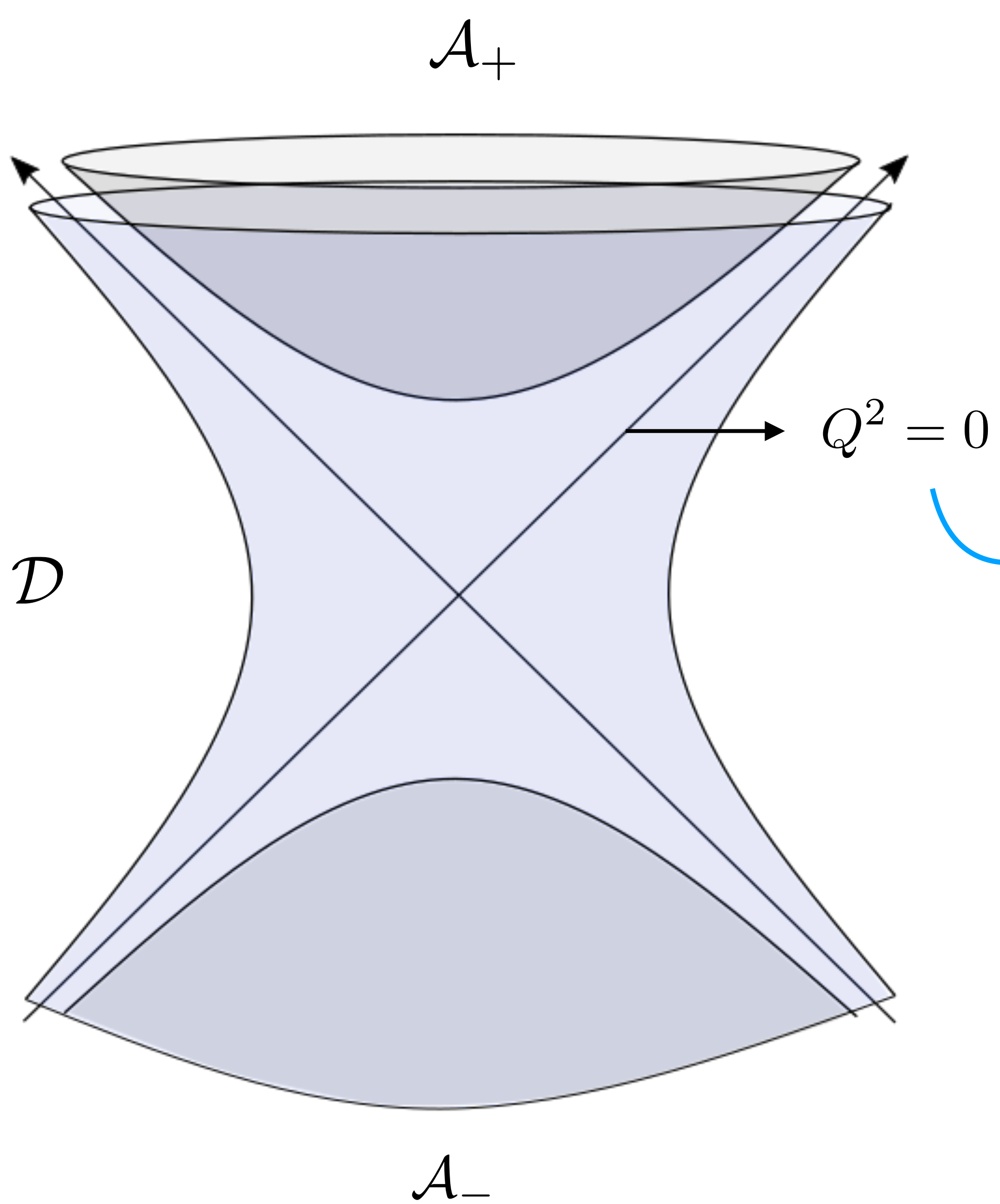
$k\eta \to k(-\eta) = (-k)\eta$

$$\Lambda = 0$$

Hyperbolic slicing of Minkowski space

[de Boer and Solodukhin '03]

(d+2)-dimensional Minkowski space $\mathbb{M}^{d+2}$, coordinates $X^A, \quad A = 0, \ldots d+1$



$\mathcal{A}_\pm : \quad X^2 = -t^2$ (i.e. $EAdS_{d+1}$, radius t)

$\mathcal{D} : \quad X^2 = R^2$ (i.e. dS_{d+1}, radius R)

Conformal boundary:

$$Q^2 = 0, \quad Q \equiv \lambda Q, \quad \lambda \in \mathbb{R}^+$$

Introduce projective coordinates:

$$\xi_i = Q^i / Q^0, \quad i = 1, \ldots, d+1$$

$$\xi_1^2 + \ldots + \xi_{d+1}^2 = 1$$

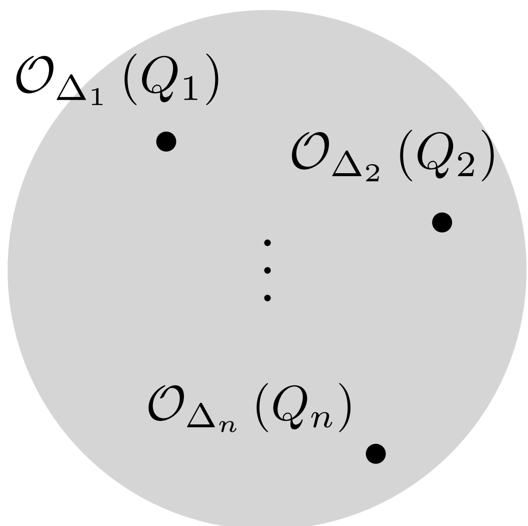
d-dimensional Celestial sphere

$SO\,(d+1,1)$ acts on the celestial sphere as the Euclidean conformal group!

Minkowski boundary correlators

[C.S. and M. Taronna '23]

Radial Mellin transform of Minkowski correlators implements a radial reduction onto the hyperbolic slicing:



Hyperbolic coordinate

$$= \prod_i \lim_{\hat{X}_i \to Q_i} \int_0^\infty \frac{dt_i}{t_i} t_i^{\Delta_i} \left\langle \phi_1(t_1 \hat{X}_1) \ldots \phi_n(t_n \hat{X}_n) \right\rangle$$

Mellin transform radial coordinate

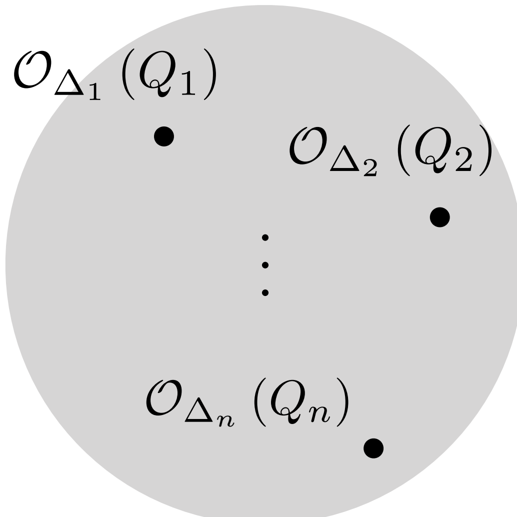
Celestial correlators then arise in the boundary limit $\hat{X}_i \to Q_i$!

Note: Celestial correlators are not celestial *amplitudes* [Pasterski, Shao Strominger '17] which are scattering amplitudes in a conformal basis. In particular:

celestial amplitudes ~ LSZ (celestial correlators)

Minkowski boundary correlators

[C.S. and M. Taronna ’23; L. Iacobacci, C.S. and M. Taronna ’24]



$$= \prod_i \lim_{\hat{X}_i \to Q_i} \int_0^\infty \frac{dt_i}{t_i}\, t_i^{\Delta_i} \left\langle \phi_1(t_1 \hat{X}_1) \dots \phi_n(t_n \hat{X}_n) \right\rangle$$

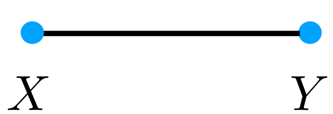
Hyperbolic coordinate

Mellin transform

radial coordinate

Feynman rules:

Bulk-to-bulk propagator:

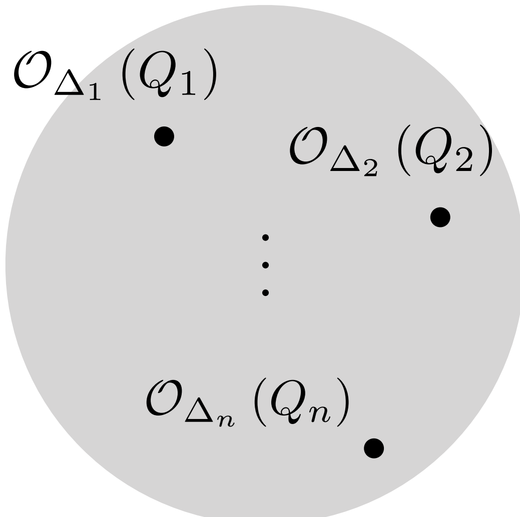
$G_F(X,Y) =$ 

Bulk-to-boundary propagator:

$$G_\Delta^{\text{flat}}(X,Q) = \lim_{\hat{Y} \to Q} \int_0^\infty \frac{dt}{t} t^\Delta G_F\left(X, t\hat{Y}\right)$$

Minkowski boundary correlators

[C.S. and M. Taronna ’23; L. Iacobacci, C.S. and M. Taronna ’24]



$$= \prod_i \lim_{\hat{X}_i \to Q_i} \int_0^\infty \frac{dt_i}{t_i} t_i^{\Delta_i} \left\langle \phi_1(t_1 \hat{X}_1) \dots \phi_n(t_n \hat{X}_n) \right\rangle$$

Hyperbolic coordinate

Mellin transform

radial coordinate

Feynman rules in the hyperbolic slicing**:**

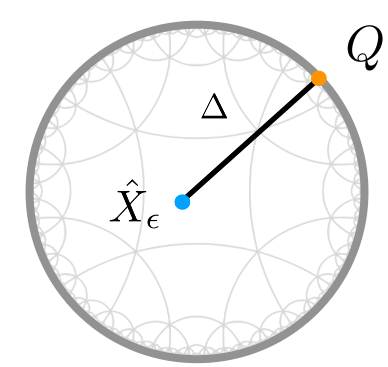
Bulk-to-bulk propagator:

$$G_F(X,Y) =$$

Bulk-to-boundary propagator:

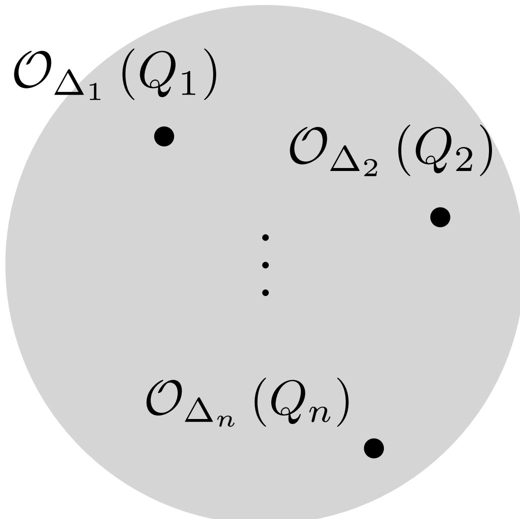
radial reduction onto extended unit hyperboloid

$$G_\Delta^{\text{flat}}(X,Q) = \lim_{\hat{Y}\to Q} \int_0^\infty \frac{dt}{t} t^\Delta G_F\left(X, t\hat{Y}\right) = \mathcal{K}_\Delta^{(m)}\left(\sqrt{X^2+i\epsilon}\right) \times$$



Minkowski boundary correlators

[C.S. and M. Taronna ’23; L. Iacobacci, C.S. and M. Taronna ’24]



$$= \prod_i \lim_{\hat{X}_i \to Q_i} \int_0^\infty \frac{dt_i}{t_i} t_i^{\Delta_i} \left\langle \phi_1(t_1 \hat{X}_1) \dots \phi_n(t_n \hat{X}_n) \right\rangle$$

Hyperbolic coordinate

Mellin transform

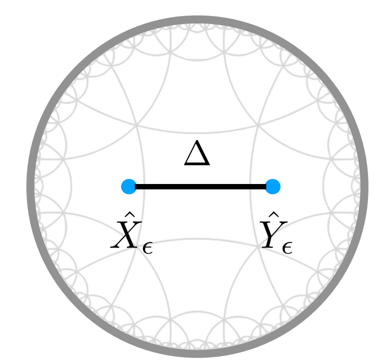
radial coordinate

Feynman rules in the hyperbolic slicing:

Bulk-to-bulk propagator:

$$G_F(X,Y) =$$

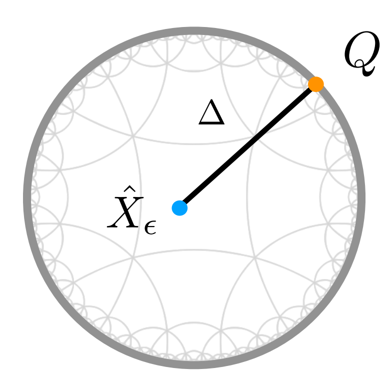
$$= \frac{1}{2} \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i} \mathcal{K}_\Delta^{(m)}\left(\sqrt{X^2+i\epsilon}\right) \mathcal{K}_{d-\Delta}^{(m)}\left(\sqrt{Y^2+i\epsilon}\right)$$



radial reduction onto extended unit hyperboloid

Bulk-to-boundary propagator:

$$G_\Delta^{\text{flat}}(X,Q) = \lim_{\hat{Y} \to Q} \int_0^\infty \frac{dt}{t} t^\Delta G_F\left(X, t\hat{Y}\right) = \mathcal{K}_\Delta^{(m)}\left(\sqrt{X^2+i\epsilon}\right) \times$$

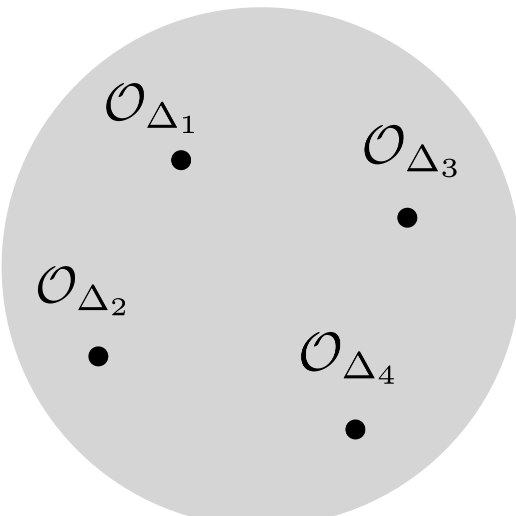


From the Celestial Sphere to EAdS

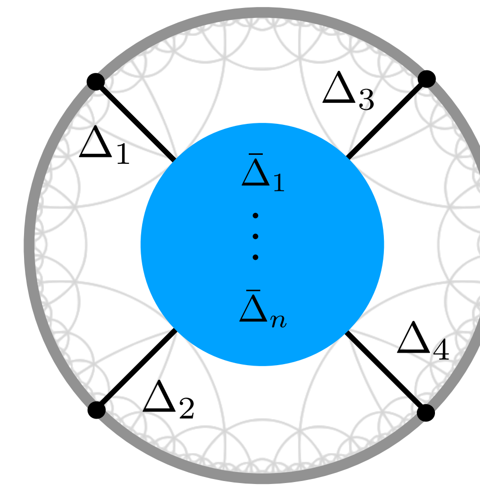
[C.S. and M. Taronna '23; L. Iacobacci, C.S. and M. Taronna '24]

In general, for exchanges of particles of mass $m_i, \quad i = 1, \ldots, n$

Unitary Principal Series representations of SO(d+1,1)



$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\bar{\Delta}_1}{2\pi i} \cdots \frac{d\bar{\Delta}_n}{2\pi i} \, C_{\bar{\Delta}_1 \ldots \bar{\Delta}_n}(m_1, \ldots, m_n)$$



Minkowski exchanges are a *continuum* of EAdS exchanges

Makes manifest conformal symmetry

From the Celestial Sphere to EAdS

[C.S. and M. Taronna '23; L. Iacobacci, C.S. and M. Taronna '24]

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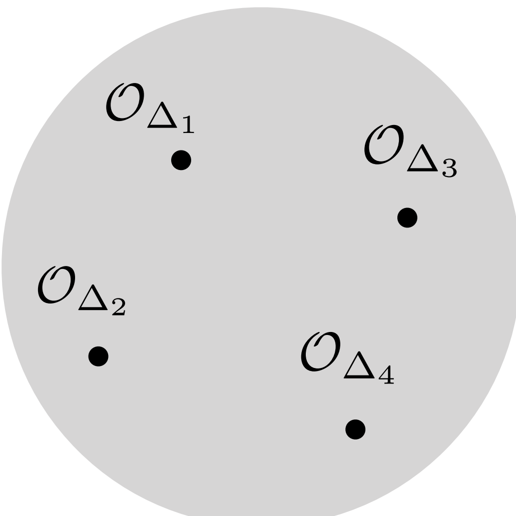
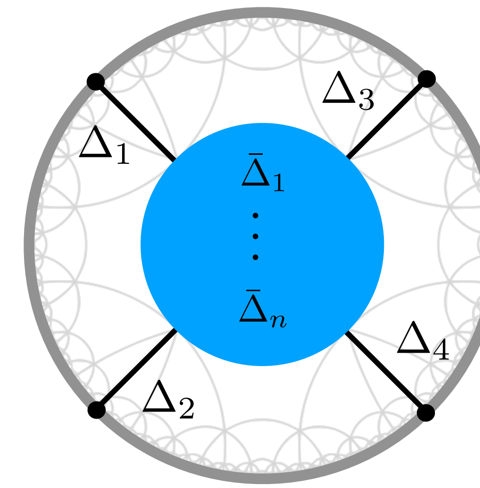


Unitary Principal Series representations of SO(d+1,1)

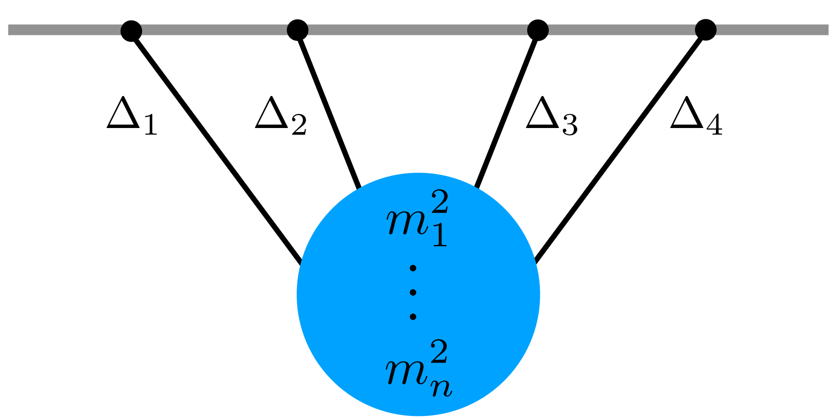
$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\bar{\Delta}_1}{2\pi i} \ldots \frac{d\bar{\Delta}_n}{2\pi i} \, C_{\bar{\Delta}_1 \ldots \bar{\Delta}_n}(m_1, \ldots, m_n)$$



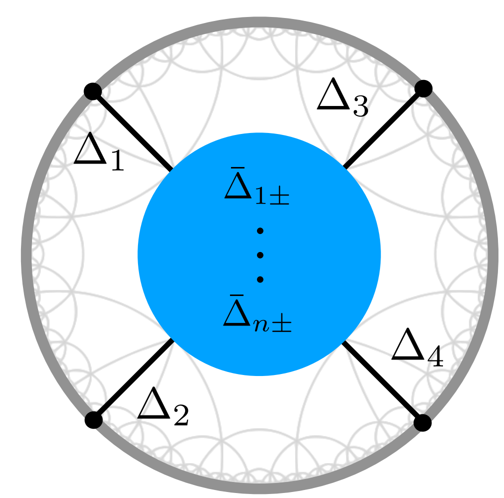
Minkowski exchanges are a *continuum* of EAdS exchanges

Makes manifest conformal symmetry

Compare with de Sitter:



$$= \sum_{\bar{\Delta}_{1\pm} \ldots \bar{\Delta}_{n\pm}} C_{\bar{\Delta}_{1\pm} \ldots \bar{\Delta}_{n\pm}}$$



dS exchanges are a *discrete sum* of EAdS exchanges

Outlook

- Perturbative dS and celestial correlators have a similar analytic structure to AdS.

→ Conformal partial wave expansion:

Spectral density

$$\langle \mathcal{O}(\mathbf{x}_1)\,\mathcal{O}(\mathbf{x}_2)\,\mathcal{O}(\mathbf{x}_3)\,\mathcal{O}(\mathbf{x}_4)\rangle = \sum_{J=0}^{\infty} \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i}\,\rho_J(\Delta)\;\mathcal{F}_{\Delta,J}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)$$

Conformal Partial Wave

[dS: Sleight, Taronna ’20; Celestial: Pacifico, Sleight, Taronna ’25]

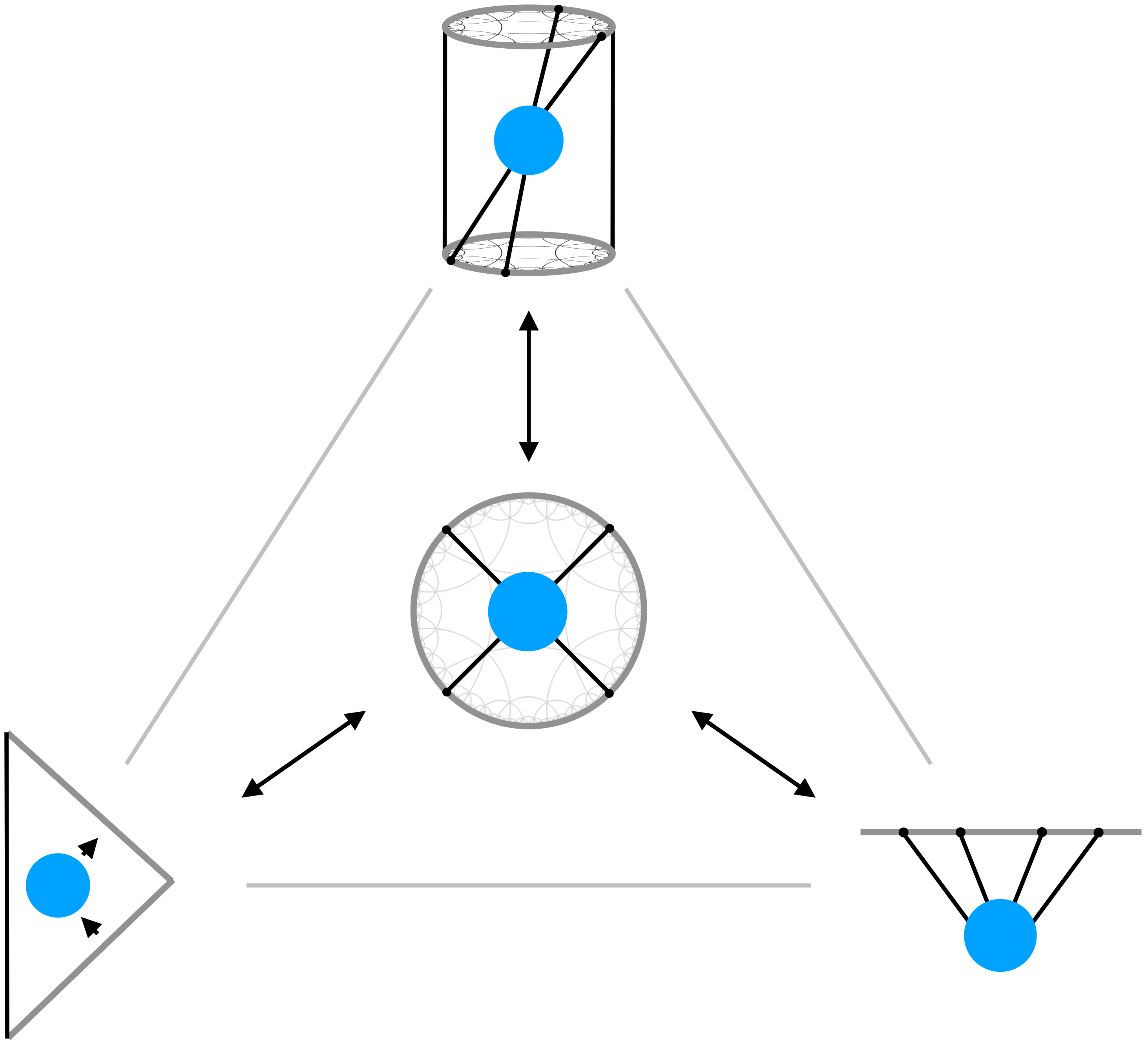
Non-perturbative Bootstrap of Euclidean CFTs dual to physics in dS/Minkowski space?

$SO(d+1,1)$ Unitarity: $\rho_J(\Delta) \geq 0$

[dS: Hogervorst, Penedones, Vaziri ’21, di Pietro, Komatsu, Gorbenko ’21]

[Celestial: Iacobacci, Sleight, Taronna ’22, Pacifico, Sleight, Taronna ’25]

- Probe non-perturbative structure with integrable models?



Thank you.