# Non-BPS branes as holographic symmetry operators

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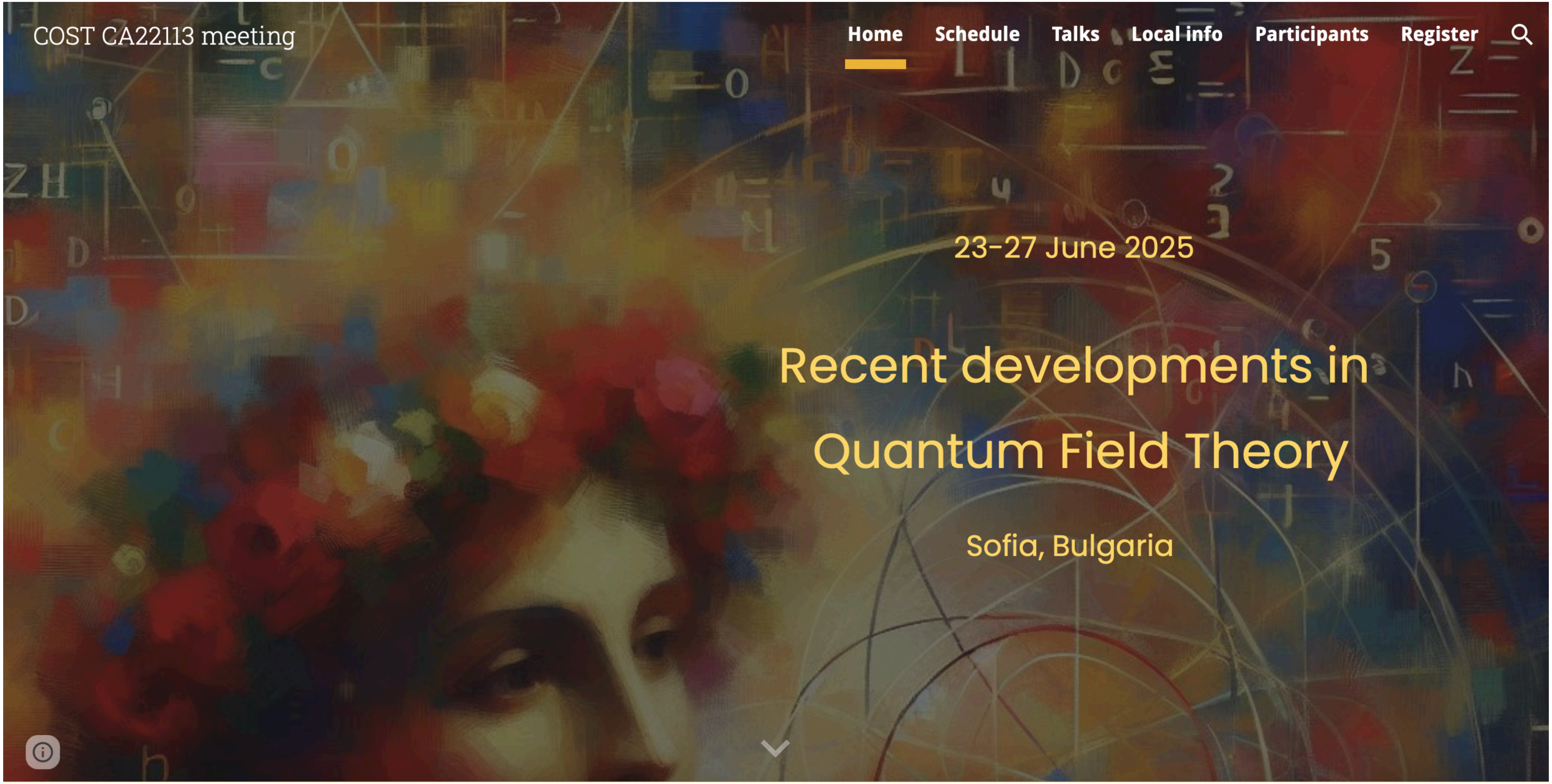
2407.00773, with Oren Bergman, Eduardo Garcia Valdecasas and Francesco Mignosa + ongoing

Warning/apologies: Work in progress!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23-27 June 2025
Recent developments in
Quantum Field Theory
Sofia, Bulgaria

# Invited Speakers

Christopher Beem (Oxford, UK)

Jerome Gauntlett (Imperial, UK)

Alba Grassi* (Geneva and CERN, CH)

Zohar Komargodski* (Stony Brook, US)

Elli Pomoni (DESY, DE)

Silviu Pufu (Princeton, US)

Shlomo Razamat (Technion, IL)

Balt van Rees (Polytechnique, FR)

Alberto Zaffaroni (Milano Bicocca, IT)

*To be confirmed

- Over the last decade there has been a revolution in our understanding of global symmetries in field theory and their fate upon the inclusion of dynamical gravity.

- It was key to realize that symmetries are associated to the existence of defect operators supported on submanifolds of spacetime on which they depend topologically

Gaiotto, Kapustin, Seiberg & Willett'14

  - extension to higher-form symmetries (continuous or discrete)

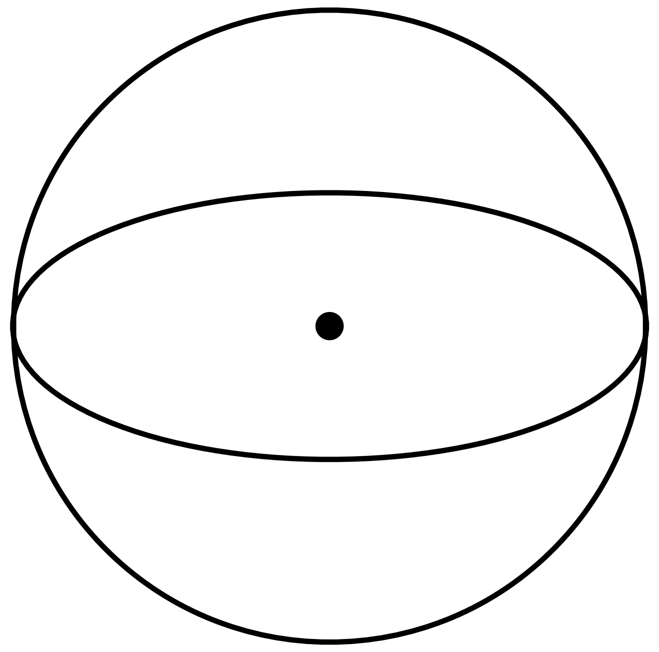
  - possible "exotic OPEs" for defects (non-invertible symmetries)

- Let us consider the simplest case of abelian 0-form global symmetries in QFT's in d-dimensions. Noether's theorem ensures (barring anomalies) the existence of a conserved 1-form current

- The modern perspective is that associated to the symmetry there are symmetry operators defined on d-1 dimensional manifolds whose dependence on the manifold is only topological

- On a charge q operator, the symmetry operator acts as

$$\mathcal{O}_q(x)\,U_\alpha(M) = e^{iq\alpha}\,\mathcal{O}_q \qquad \leftrightarrow$$



$$= e^{iq\alpha} \ .$$

- It is natural to ask for the holographic realization of the defect operators.
- The continuous case is particularly puzzling. In particular, they depend on a continuous parameter whose String Theory origin is obscure.

**Today**: propose a holographic realization of continuous symmetry defects and study some of its consequences

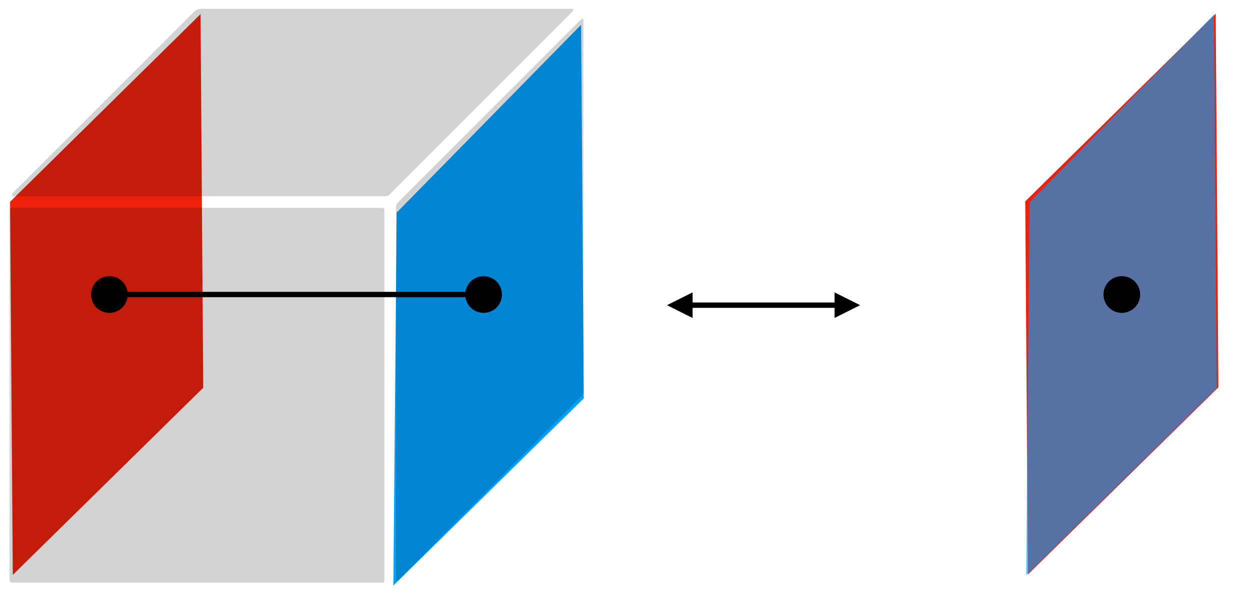
(also Cvetic, Heckman, Hubner & Torres '23)
Waddleton'24)

# Contents

# Continuous symmetries in holography and non-BPS D-branes

- It is very interesting to study holographic CFT's. A natural question is what is the bulk avatar of the global symmetry

- For discrete symmetries this is closely connected to the SymmTFT: one couples the d-dimensional QFT to a d+1 dimensional TQFT on an interval, so that the TQFT and its bc. encode the possible forms of the global symmetries of the QFT



holography is a natural realization of this

- Continuous symmetries are, from this point of view, more complicated. Yet it is natural to ask about the bulk realization of these

There have been proposals for SymmTFT. See:

Brennan & Sun' 24
Antinucci & Benini '24
Apurzzi, Bedogna & Dondi '24
Bonetti, del Zotto & Minasian' 24
Gagliano & Garcia Etxebarria'24

- First of all, let us consider 10d string theory: it has RR potentials. Associated to those there would higher form U(1) global symmetries…but the existence of dynamical branes explicitly breaks these symmetries.

- Consider a Dp brane: it sources a p+2-form RR field strength satisfying

$$\frac{1}{2\pi}\int_{M_{8-p}} \star F_{p+2} = 1\,.$$

- Alternatively, the Dp brane links with a 7-p dimensional object: it would be a “wrong dimension” brane. String theory contains something like this: non-BPS branes!!!

$$S_{\widetilde{Dp}} = -\int_{\Sigma_{p+1}} d^{p+1}x\, e^{-\Phi}\, V(T)\sqrt{-\det(G_{\mu\nu} + \partial_\mu T \partial_\nu T)} + \int_{\Sigma_{p+1}} W(T)\, dT \wedge C_p\,.$$

Sen ’04

**Warning**: not a low-energy effective theory, but rather a theory whose eom. reproduce the BCFT describing the unstable D-brane

- The relevant facts here are

  - $V(T), W(T) > 0 \qquad \& \qquad \lim_{T\to\infty} V(T) \sim \lim_{T\to\infty} W(T) \sim e^{-\frac{T}{\sqrt{2}}}.$
  - $V(0) = \tilde{T}_p = \sqrt{2}\, T_p\,.$
  - The Dp-1 brane is a tachyon kink in the non-BPS Dp, which requires

$$\int_{-\infty}^{\infty} dT\, V(T) = \int_{-\infty}^{\infty} dT\, W(T) = T_{p-1}\,.$$

- In the presence of RR-field strengths we can integrate by parts the WZ action finding

$$S_{WZ} = \int_{\Sigma_{p+1}} Z(T)\, F_{p+1}\,.$$

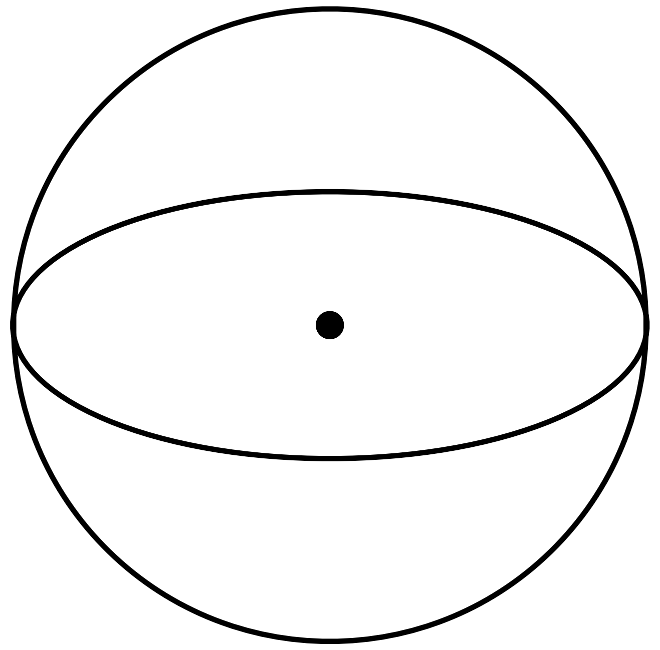
This is the result of the integration by parts. Crucially, note that the above requirements do not fix the integration constant (for a D-brane it would be a gauge transformation, but not here)

$$Z(T) = \int_{T_0}^{T} dT'\, W(T')\,, \qquad \int_{-\infty}^{\infty} dT'\, W(T') = T_p = -\int_{T_0}^{-\infty} dT'\, W(T') + \int_{T_0}^{\infty} dT'\, W(T') = Z(\infty) - Z(-\infty)$$

Similar to Cornalba, Costa & Penedones '04

- As a consequence, after the non-BPS brane decays to its vacuum it leaves behind a phase $e^{i\frac{\alpha}{2\pi}\int F_{p+1}}$.

- So if we link q Dp's with a non-BPS D7-p we find



$$= e^{iq\alpha} \quad .$$

- Thus the non-BPS brane measures the linked charge.

- However it is non-topological: D-branes are dynamical and the would-be symmetry is explicitly broken.

- Let us now go to AdS

$$ds^2 = \frac{dz^2 + d\vec{x}^2}{z^2} + ds^2_{\mathcal{X}} \, .$$

- Because of the z-factor, branes pushed to the boundary are infinitely heavy and become non-dynamical.
- Thus, a non-BPS D-brane at the boundary of AdS is topological!

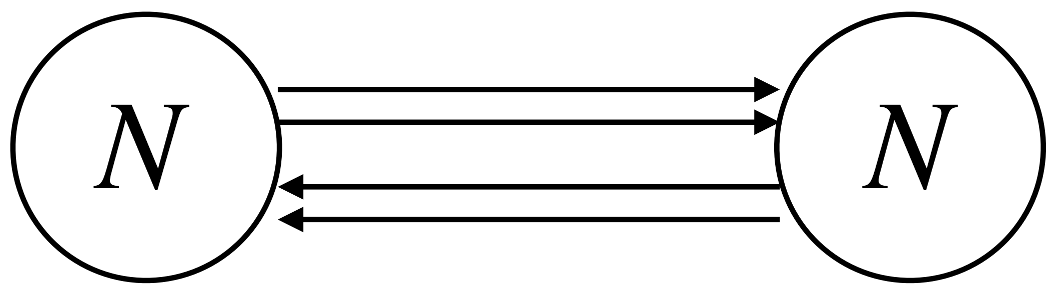
↓

Natural candidates for the holographic realization of U(1) symmetry defects (note the emergence of the parameter labelling the operator!)

(see also Cvetic, Heckman, Hubner & Torres '23, in terms of fuxbranes)

(Waddleton'24)

- Let us see an explicit example: consider the Klebanov-Witten (conformal) field theory (with gauge group $SU(N) \times SU(N)$)



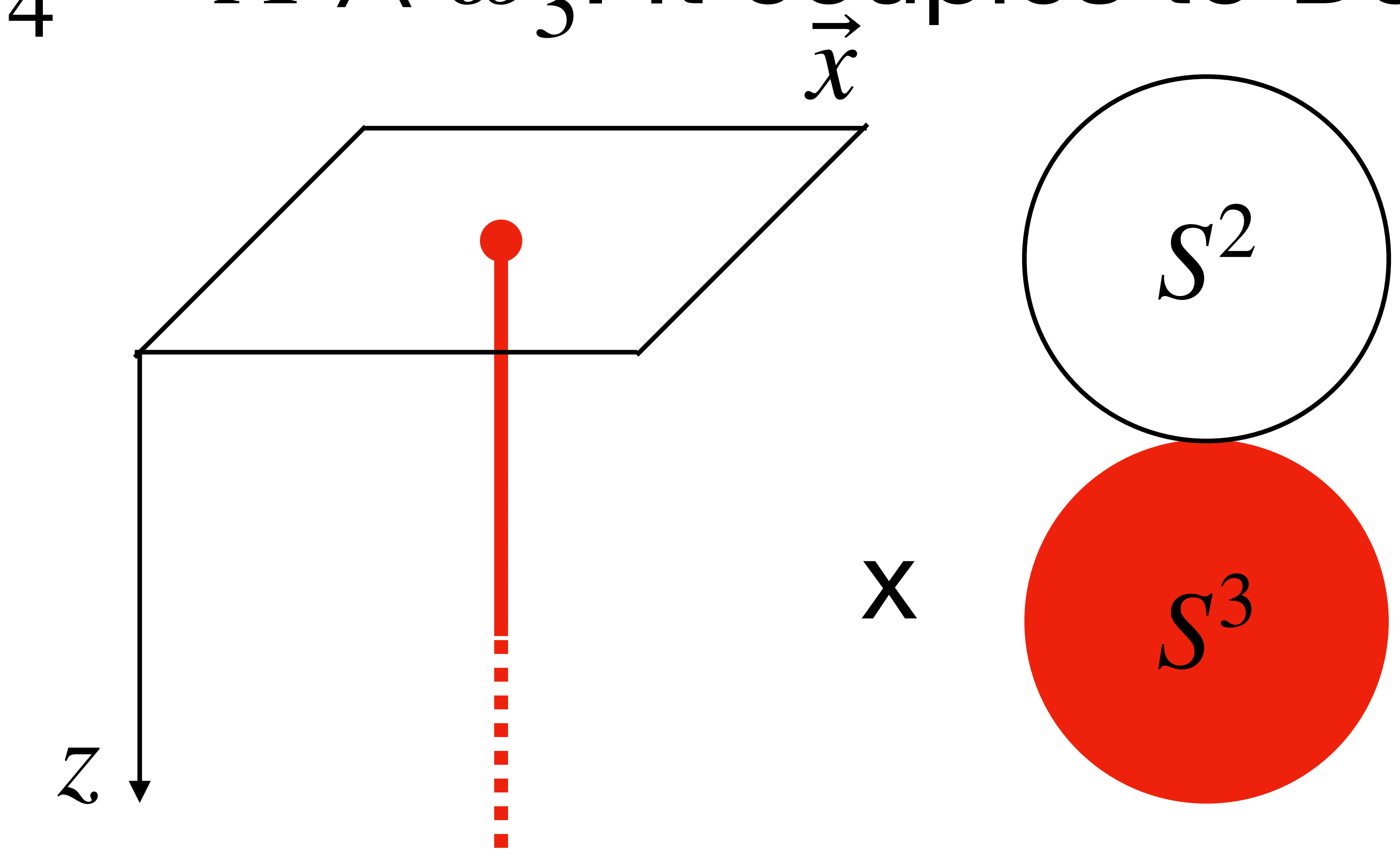

$$W = \epsilon^{ij}\,\epsilon^{mn}\,A_i\,B_m\,A_j\,B_n\,.$$

- There is a global U(1) baryonic symmetry $(A_i, B_m) \to (e^{i\theta}A_i, e^{-i\theta}B_m)$ under which determinant operators are charged (baryons).

- The holographic dual is $AdS_5 \times T^{1,1}$. For our purposes $T^{1,1} \sim S^2 \times S^3$.

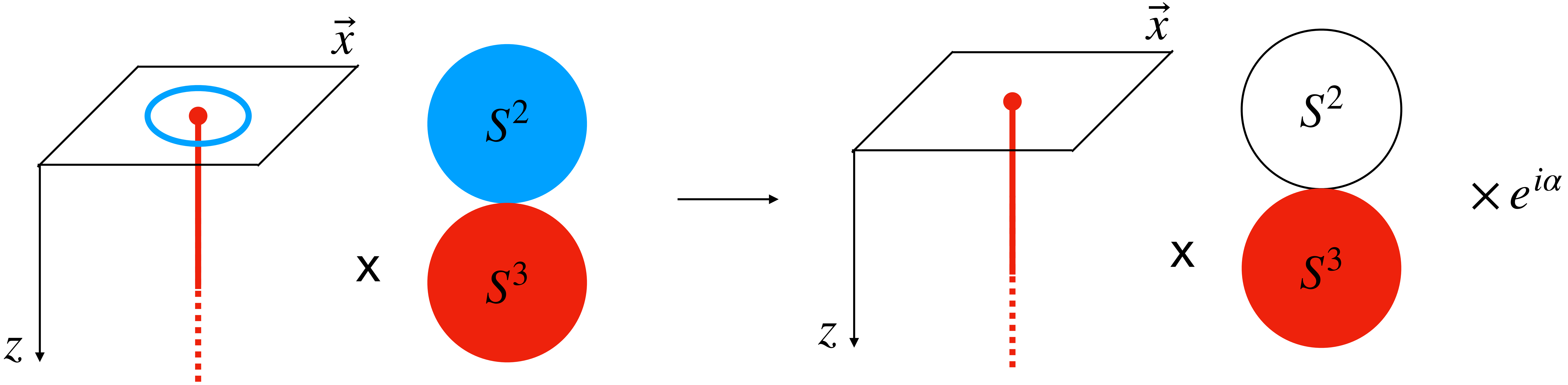
- The baryonic gauge field in $AdS_5$ comes from $C_4 = A \wedge \omega_3$. It couples to D3-branes wrapping the $S^3$: the baryons

Wilson line for baryon gauge field in AdS ⟵



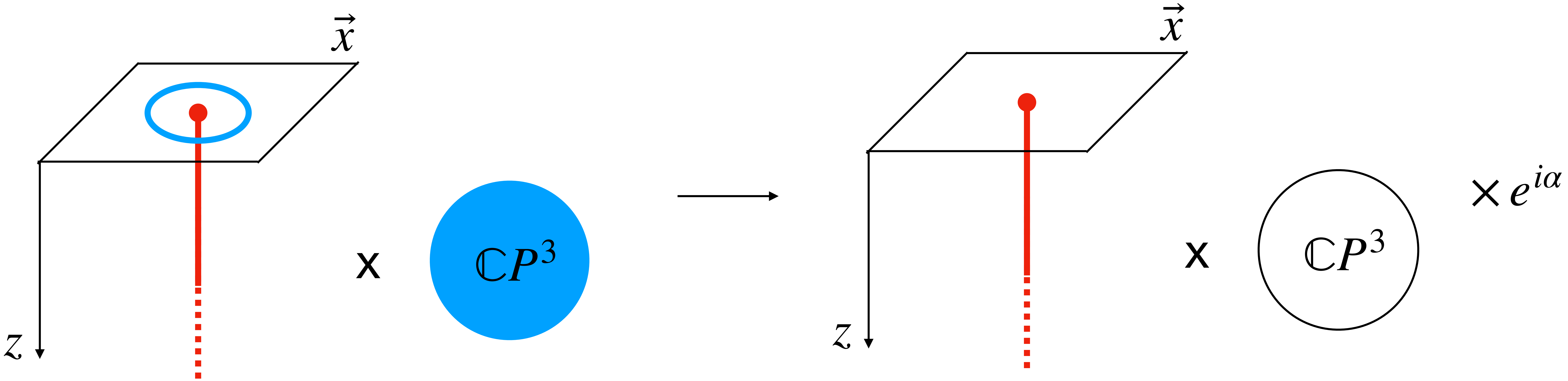

Gubser & Klebanov '98

- Such D3 produces a 5-form field strength on the transverse space: consider a non-BPS D4 linking with the D5…




- …so we recover the expected action of the symmetry operator.

- Another example is the ABJM theory $U(N)_k \times U(N)_{-k}$, which has a $U(1)_{\text{monopole}}$ symmetry generated by $J = \star\, \mathrm{Tr}(F_1 + F_2)$.
- The holographic dual (take k big) is $AdS_4 \times \mathbb{C}P^3$
- The charged states are D0 branes, which link with non-BPS D7



- Consider now $SU(N)\ \mathcal{N}=4 \leftrightarrow AdS_5 \times S^5$. There is really no continuous symmetry.

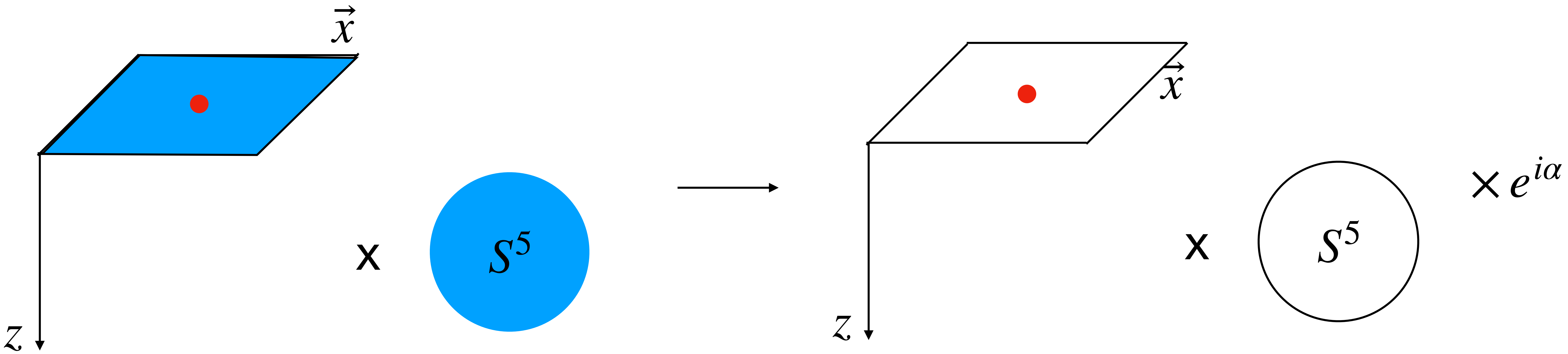
- But there is the $\theta$ parameter which one may imagine changing it: it corresponds to a -1 form “symmetry”.

Aloni, Garcia-Valdecasas, Reece & Suziki ’24

- The word symmetry is a bit of an abuse, as the theory changes. For instance, the instanton action changes.

- Yet holographically they don’t seem too different…

- Consider a D8 wrapping the internal space




It links with a D-1 brane: $e^{iC_0} \to e^{iC_0 + i\alpha}$, which means that the effect of the D8 is $\theta \to \theta + \alpha$. But in this case this is the 0-form symmetry associated to $C_0$… which is broken by D-1 branes (the D-1 can indeed go to the boundary)

# non-BPS D-branes and brane/antibrane pairs?

- A non-BPS Dp brane can be regarded as an intermediate step in the decay of a Dp+1/anti Dp+1 system…can our symmetry operators be regarded in this way? (we are in curved space, not completely obvious).

Sen '03

Bah, Jefferson, Roumpedakis & Waddleton '24

- The action is now

$$S = \int V(|T|, Y)\left(\sqrt{G + F + |DT|^2}\,|_{D5} + \sqrt{G + F + |DT|^2}\,|_{\overline{D5}}\right)$$

- Consider a D5/anti D5 separated a distance Y. Close to T=0

$$V \sim 1 + |T|^2 (Y^2 - 1) + \cdots \ ,$$

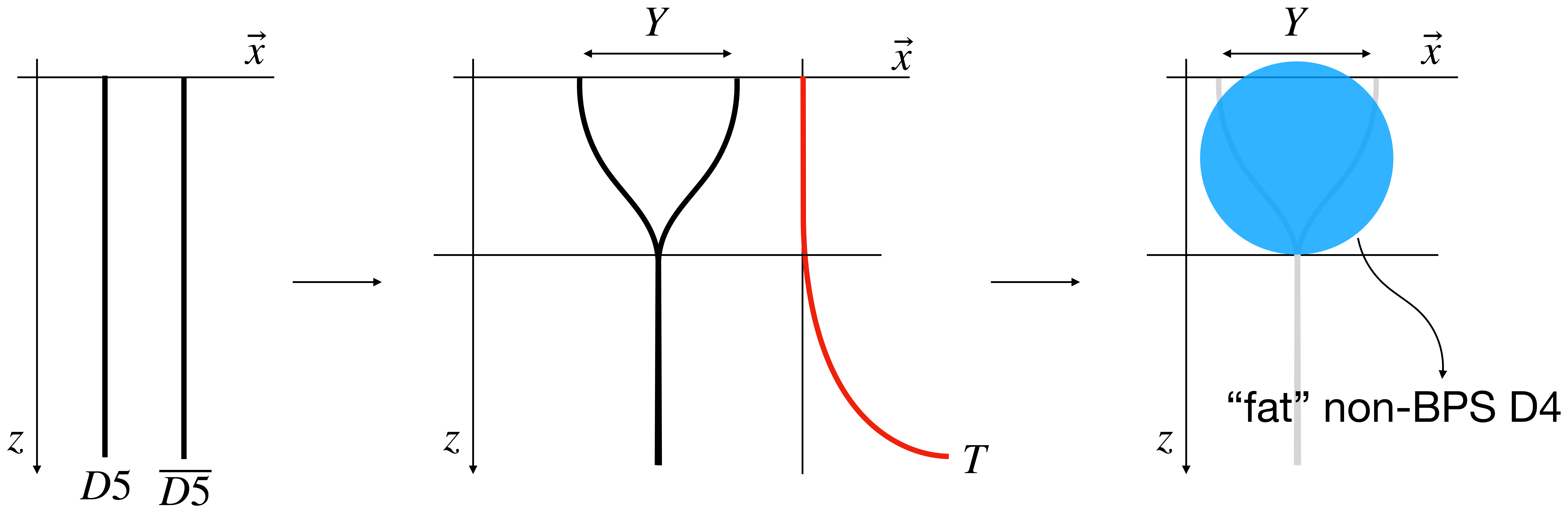
- so as T rolls down its potential Y becomes very massive and freezes to 0. The equation of motion for (the real part of) T is

$$\partial_x \left( \frac{V \partial_x T}{\sqrt{1 + \partial_x T^2}} \right) - 3 \frac{V \partial_x T}{\sqrt{1 + \partial_x T^2}} - \frac{\partial V}{\partial T} \sqrt{1 + \partial_x T^2} = 0 \,, \qquad (x = \log z) \,.$$

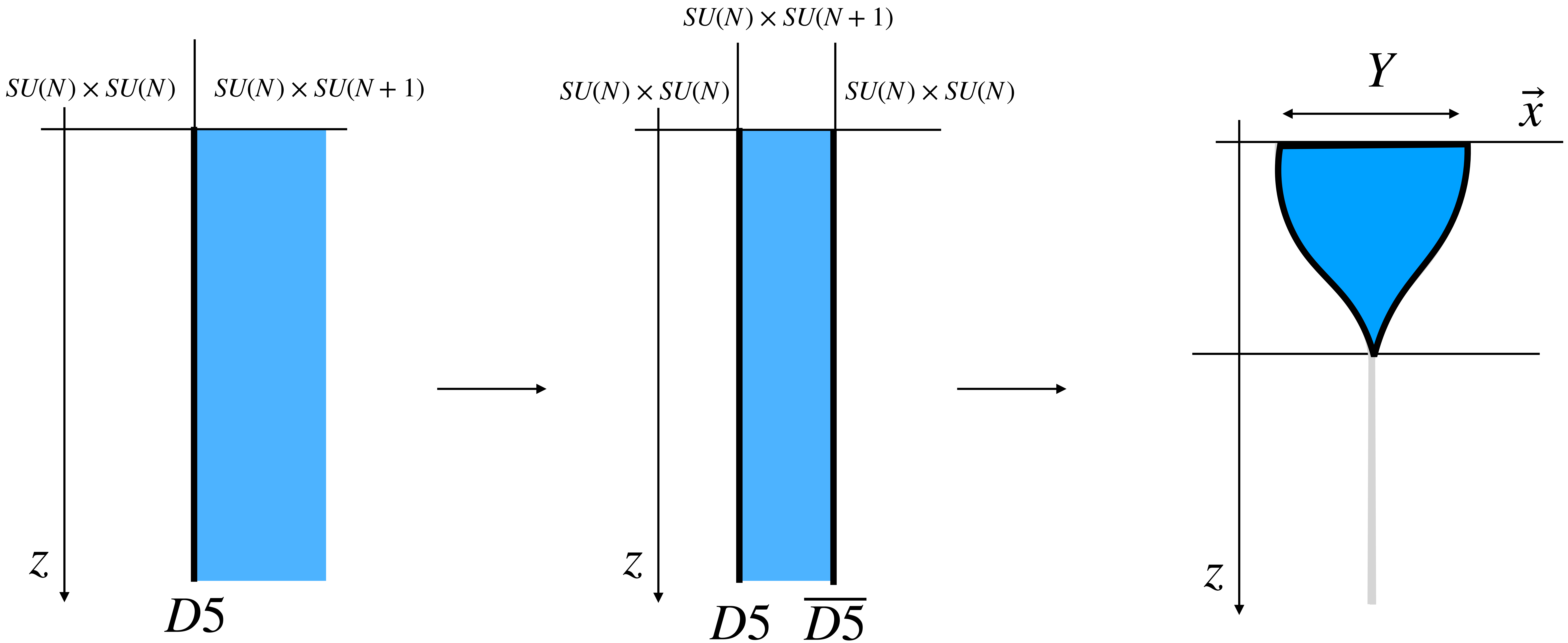
- This cannot be solved exactly, but one can qualitatively argue that T vanishes at the boundary and rolls down its potential as we go deep in the bulk

- As a consequence we must re-examine our assumption that Y=0…but only close to the boundary where T is basically 0. It turns out that $Y \sim \mathrm{ct}$ close to the boundary.

- The upshot is that the D5/anti D5 system looks like



- The D5 alone is well-known to represent a DW increasing the rank, so




Only baryons feel this: as they are dragged across a string is created

# Conclusions (and open directions)

- Non-BPS branes at the AdS boundary look like ideal candidates for symmetry operators of continuous symmetries: they naturally link with charged objects and they hide the continuous parameter labelling the operator

- These non-BPS D-branes can be regarded as Dp+1/anti Dp+1: indeed only baryons feel this and as dragged across a string is created

- A natural extension/check of the proposal is to explore the spectroscopy of non-BPS branes in a given background and match it to the corresponding symmetries

- Interestingly, this seems to include candidates for symmetry operators of discrete symmetries as well

- A natural question is whether there is a relation to the proposed symTFT's for continuous symmetries?

(in the end the charged/charge objects and their linkings are the same)

roughly $S = i \int_{M_{D+1}} dC_p \wedge F_{D-p}$ with $C_p \sim U(1), F_{D-p} \sim \mathbb{R}$

# Many thanks