Novel AdS vacua from dynamical open strings

Giuseppe Sudano

Università di Roma 'Tor Vergata' & INFN Roma 2

Mostly based on [Balaguer, Bevilacqua, Dibitetto, Fernández-Melgarejo, GS '24, Bevilacqua, Dibitetto, GS (to appear)]

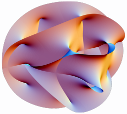
Meeting PRIN 'String Theory as a bridge between Gauge Theories and Quantum Gravity'
17 Feb 2025, Sapienza University

Effective field theory approach to String Theory

String Theory is a highly constrained framework. For instance, the absence of anomalies imposes the number of spacetime dimensions to be D = 10, as also required in order for the gravitons to be massless.

Realistic field theories in lower dimensions can be recovered with an effective field theory approach:

- Top-down: the extra dimensions curl up in a small enough compact space, called internal manifold;



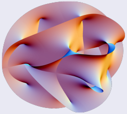
- Bottom-up: to establish some criteria that a low energy theory should meet in order to admit a UV completion in a Quantum Gravity theory (swampland program).

Effective field theory approach to String Theory

String Theory is a highly constrained framework. For instance, the absence of anomalies imposes the number of spacetime dimensions to be $D = 10$, as also required in order for the gravitons to be massless.

Realistic field theories in lower dimensions can be recovered with an effective field theory approach:

- Top-down: the extra dimensions curl up in a small enough compact space, called internal manifold;



- Bottom-up: to establish some criteria that a low energy theory should meet in order to admit a UV completion in a Quantum Gravity theory (swampland program).

The scalar potential and the vacua

Upon compactification, the fields of the theory give rise to:

- scalars known as moduli, e. g. in IIA SUGRA, if $M = (\mu, m)$,

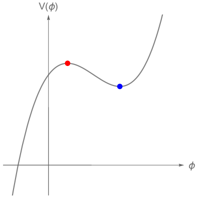
$$C_{MNR} \quad \rightarrow \quad C_{mnr} \; ;$$

- gauge fluxes (p-form fields integrated in the compact manifold), e. g. in IIA SUGRA

$$\oint_{\mathcal{C}_3} H_{(3)} \quad , \quad \oint_{\mathcal{C}_4} F_{(4)}$$

- metric fluxes associated to the geometry of the internal manifold.

Fluxes induce a scalar potential for the moduli, whose extrema correspond to the vacua.



A vacuum is a field configuration such that the metric is maximally symmetric. This condition can be realized if all macroscopic fields vanish, except scalars which can be constant.

The scalar potential and the vacua

Upon compactification, the fields of the theory give rise to:

- scalars known as moduli, e. g. in IIA SUGRA, if $M = (\mu, m)$,

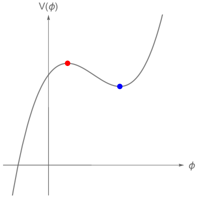
$$C_{MNR} \quad \to \quad C_{mnr} \;;$$

- gauge fluxes (p-form fields integrated in the compact manifold), e. g. in IIA SUGRA

$$\oint_{\mathcal{C}_3} H_{(3)} \quad , \quad \oint_{\mathcal{C}_4} F_{(4)}$$

- metric fluxes associated to the geometry of the internal manifold.

Fluxes induce a scalar potential for the moduli, whose extrema correspond to the vacua.



A vacuum is a field configuration such that the metric is maximally symmetric. This condition can be realized if all macroscopic fields vanish, except scalars which can be constant.

Gauged Supergravity and the embedding tensor formalism

Supergravity theories (SUGRA) provide a low-energy description of the lower-dimensional setting arising from flux compactifications. Supersymmetry (SUSY) constraints the field content, the isometries of the scalar manifold and the global symmetry group G_0.

A gauging of a certain subgroup $G \subseteq G_0$ is needed in order to have a potential and moduli stabilization in a vacuum:

- the symmetry of vector fields is promoted to a non-Abelian gauge symmetry G;
- scalar fields get minimally coupled to the vector fields ${A_\mu}^M$.

The embedding tensor formalism allows to perform the gauging in a G_0-covariant way:

$$\Theta_M{}^\alpha, \quad X_M = \Theta_M{}^\alpha t_\alpha \ .$$

[Nicolai, Samtleben '01]

The embedding tensor transforms in

$$\Theta \in V' \otimes \text{adj} = \theta_1 \oplus \cdots \oplus \theta_n \ .$$

Two types of constraints select the allowed irreps:

- $\mathbb{P}_1(\Theta) = 0$;
- $\mathbb{P}_2(\Theta \otimes \Theta) = 0$ linked to the closure of the gauge algebra

$$[X_M, X_N] = X_{MN}{}^P X_P \ .$$

Gauged Supergravity and the embedding tensor formalism

Supergravity theories (SUGRA) provide a low-energy description of the lower-dimensional setting arising from flux compactifications. Supersymmetry (SUSY) constraints the field content, the isometries of the scalar manifold and the global symmetry group G_0.

A gauging of a certain subgroup $G \subseteq G_0$ is needed in order to have a potential and moduli stabilization in a vacuum:

- the symmetry of vector fields is promoted to a non-Abelian gauge symmetry G;
- scalar fields get minimally coupled to the vector fields $A_\mu{}^M$.

The embedding tensor formalism allows to perform the gauging in a G_0-covariant way:

$$\Theta_M{}^\alpha, \quad X_M = \Theta_M{}^\alpha t_\alpha \ .$$

[Nicolai, Samtleben '01]

The embedding tensor transforms in

$$\Theta \in V' \otimes \text{adj} = \theta_1 \oplus \cdots \oplus \theta_n \ .$$

Two types of constraints select the allowed irreps:

- $\mathbb{P}_1(\Theta) = 0$;
- $\mathbb{P}_2(\Theta \otimes \Theta) = 0$ linked to the closure of the gauge algebra

$$[X_M, X_N] = X_{MN}{}^P X_P \ .$$

Gauged Supergravity and the embedding tensor formalism

Supergravity theories (SUGRA) provide a low-energy description of the lower-dimensional setting arising from flux compactifications. Supersymmetry (SUSY) constraints the field content, the isometries of the scalar manifold and the global symmetry group G_0.

A gauging of a certain subgroup $G \subseteq G_0$ is needed in order to have a potential and moduli stabilization in a vacuum:

- the symmetry of vector fields is promoted to a non-Abelian gauge symmetry G;
- scalar fields get minimally coupled to the vector fields $A_\mu{}^M$.

The embedding tensor formalism allows to perform the gauging in a G_0-covariant way:

$$\Theta_M{}^\alpha, \quad X_M = \Theta_M{}^\alpha t_\alpha \ .$$

[Nicolai, Samtleben '01]

The embedding tensor transforms in

$$\Theta \in V' \otimes \mathrm{adj} = \theta_1 \oplus \cdots \oplus \theta_n \ .$$

Two types of constraints select the allowed irreps:

- $\mathbb{P}_1(\Theta) = 0$;
- $\mathbb{P}_2(\Theta \otimes \Theta) = 0$ linked to the closure of the gauge algebra

$$[X_M, X_N] = X_{MN}{}^P X_P \ .$$

Gauged Supergravity and the embedding tensor formalism

Supergravity theories (SUGRA) provide a low-energy description of the lower-dimensional setting arising from flux compactifications. Supersymmetry (SUSY) constraints the field content, the isometries of the scalar manifold and the global symmetry group G_0.

A gauging of a certain subgroup $G \subseteq G_0$ is needed in order to have a potential and moduli stabilization in a vacuum:

- the symmetry of vector fields is promoted to a non-Abelian gauge symmetry G;
- scalar fields get minimally coupled to the vector fields ${A_\mu}^M$.

The embedding tensor formalism allows to perform the gauging in a G_0-covariant way:

$$ {\Theta_M}^\alpha , \quad X_M = {\Theta_M}^\alpha t_\alpha \ . $$

[Nicolai, Samtleben '01]

The embedding tensor transforms in

$$ \Theta \in V' \otimes \mathrm{adj} = \theta_1 \oplus \cdots \oplus \theta_n \ . $$

Two types of constraints select the allowed irreps:

- $\mathbb{P}_1(\Theta) = 0$;
- $\mathbb{P}_2(\Theta \otimes \Theta) = 0$ linked to the closure of the gauge algebra

$$ [X_M, X_N] = {X_{MN}}^P X_P \ . $$

The potential from the embedding tensor formalism

A gauge-invariant Lagrangian is obtained via:

- the minimal coupling of scalars through a covariant derivative

$$\boxed{\partial_\mu \quad \to \quad D_\mu = \partial_\mu - g A_\mu{}^M \Theta_M{}^\alpha t_\alpha} \; ;$$

- gauge-covariant field strengths.

An invariant Lagrangian under SUSY transformations is

$$\mathcal{L}_{\text{gauged}} = \mathcal{L}_{\text{ungauged}}[\partial \to D] + \mathcal{L}_{\text{top}} + \mathcal{L}_{\text{YUK}} + \mathcal{L}_{\text{pot}} \; ,$$

with the further terms given by:

- gauge-covariant topological term $\mathcal{L}_{\text{top}}$;
- Yukawa-like bilinears for fermions, e. g. in half-maximal SUGRA,

$$e^{-1}\mathcal{L}_{\text{YUK}} = g \left(A_1{}^{\alpha\beta}\, \bar{\psi}_{\mu\alpha}\, \gamma^{\mu\nu}\, \psi_{\nu\beta} + \; A_2{}^{\alpha\beta}\, \bar{\psi}_{\mu\alpha}\, \gamma^{\mu}\, \chi_\beta \; + \; A_{3A\beta}{}^{\alpha}\, \bar{\psi}_{\mu\alpha}\, \gamma^{\mu}\, \lambda^{A\beta} \right) + \text{h.c.} \; ,$$

  with A_1, A_2, A_3 linear in the embedding tensor;
- a potential, quadratic in the embedding tensor components, e. g. in half-maximal SUGRA

$$e^{-1}\mathcal{L}_{\text{pot}} = -g^2 \left(|A_1|^2 - |A_2|^2 - |A_3|^2 \right) \; .$$

Fermionic shifts are also needed to have invariance under SUSY

$$\delta\psi_\mu{}^\alpha \to \delta\psi_\mu{}^\alpha + g A_1{}^{\alpha\beta}\gamma_\mu \varepsilon_\beta \quad , \quad \delta\chi^\alpha \to \delta\chi^\alpha + g A_2{}^{\alpha\beta}\varepsilon_\beta \quad , \quad \delta\lambda_{A\alpha} \to \delta\lambda_{A\alpha} + g A_{3A\alpha}{}^{\beta}\varepsilon_\beta \; .$$

The potential from the embedding tensor formalism

A gauge-invariant Lagrangian is obtained via:

- the minimal coupling of scalars through a covariant derivative

$$\boxed{\partial_\mu \quad \to \quad D_\mu = \partial_\mu - g A_\mu{}^M \Theta_M{}^\alpha t_\alpha} \; ;$$

- gauge-covariant field strengths.

An invariant Lagrangian under SUSY transformations is

$$\mathcal{L}_{\text{gauged}} = \mathcal{L}_{\text{ungauged}}[\partial \to D] + \mathcal{L}_{\text{top}} + \mathcal{L}_{\text{YUK}} + \mathcal{L}_{\text{pot}} \;,$$

with the further terms given by:

- gauge-covariant topological term $\mathcal{L}_{\text{top}}$;
- Yukawa-like bilinears for fermions, e. g. in half-maximal SUGRA,

$$e^{-1}\mathcal{L}_{\text{YUK}} = g\left(A_1{}^{\alpha\beta}\,\bar{\psi}_{\mu\alpha}\,\gamma^{\mu\nu}\,\psi_{\nu\beta} + \; A_2{}^{\alpha\beta}\,\bar{\psi}_{\mu\alpha}\,\gamma^{\mu}\,\chi_\beta \; + \; A_{3A\beta}{}^{\alpha}\,\bar{\psi}_{\mu\alpha}\,\gamma^{\mu}\,\lambda^{A\beta}\right) + \text{h.c.} \;,$$

  with A_1, A_2, A_3 linear in the embedding tensor;
- a potential, quadratic in the embedding tensor components, e. g. in half-maximal SUGRA

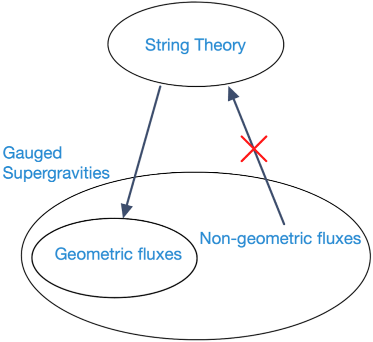
$$e^{-1}\mathcal{L}_{\text{pot}} = -g^2\,\left(|A_1|^2 - |A_2|^2 - |A_3|^2\right) \,.$$

Fermionic shifts are also needed to have invariance under SUSY

$$\delta\psi_\mu{}^\alpha \to \delta\psi_\mu{}^\alpha + gA_1{}^{\alpha\beta}\gamma_\mu\varepsilon_\beta \quad , \quad \delta\chi^\alpha \to \delta\chi^\alpha + gA_2{}^{\alpha\beta}\varepsilon_\beta \quad , \quad \delta\lambda_{A\alpha} \to \delta\lambda_{A\alpha} + gA_{3A\alpha}{}^{\beta}\varepsilon_\beta \;.$$

10-dimensional interpretation of the embedding tensor components

Some of the embedding tensor components in gauged SUGRA have a 10-dimensional interpretation as fluxes.

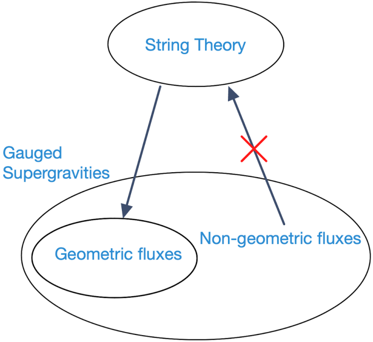


The embedding tensor encompasses all possible gaugings in all possible duality frames: a duality covariant description of string compactifications is allowed by T-folds and S-folds.

The potential provided by the gauging, completely fixed by SUSY, is an alternative writing for the 10-dimensional potential, arising from the compactification procedure and quadratic in the fluxes.

10-dimensional interpretation of the embedding tensor components

Some of the embedding tensor components in gauged SUGRA have a 10-dimensional interpretation as fluxes.



The embedding tensor encompasses all possible gaugings in all possible duality frames: a duality covariant description of string compactifications is allowed by T-folds and S-folds.

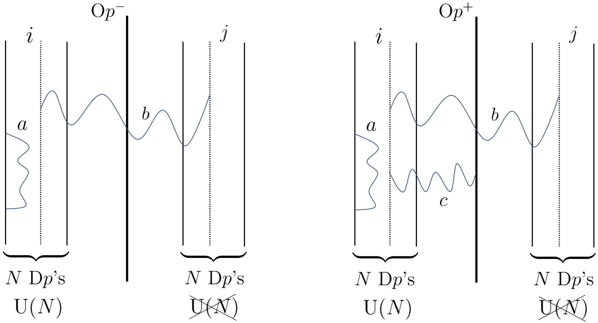
The potential provided by the gauging, completely fixed by SUSY, is an alternative writing for the 10-dimensional potential, arising from the compactification procedure and quadratic in the fluxes.

Dynamical open strings

The embedding tensor formalism has been exploited in order to find and characterize novel vacuum solutions when the dynamics of open strings is taken into account.

A massless vector multiplet is associated to each open string state ending in a Dp-brane and a super Yang-Mills theory is defined along the worldvolume of the branes. An enhancement of the gauge group occurs when parallel branes are in stacks:

$$U(1)^N \quad \rightarrow \quad U(N)\ .$$



Orientifolds (Op-planes) are stuck at the fixed locus of the involution

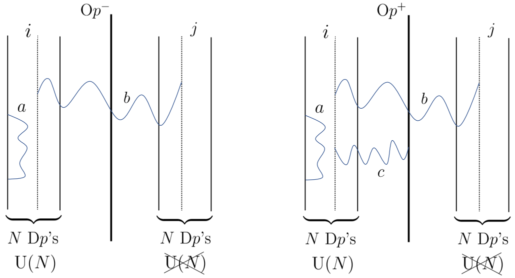
$$\Omega_{O_p} = \Omega \, \sigma_{O_p} \, \sigma_{F_L} \ .$$

Parallel orientifolds to a stack of D-branes change the gauge group. The states surviving the orientifold projection are such that (λ_{ij} Chan-Paton factor)

$$\lambda = -M \lambda^T M^{-1} \ ,$$

with:

- $M = \mathbb{1}_{2N}$: Op$^-$, gauge group is SO(2N);
- $M = \begin{pmatrix} \mathbb{0}_N & \mathbb{1}_N \\ -\mathbb{1}_N & \mathbb{0}_N \end{pmatrix}$: Op$^+$, gauge group USp(2N).



Dynamical open strings

Orientifolds (Op-planes) are stuck at the fixed locus of the involution

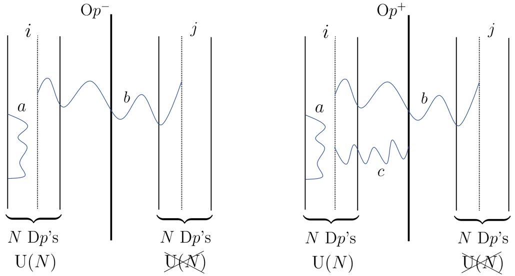
$$\Omega_{O_p} = \Omega\ \sigma_{O_p}\ \sigma_{F_L}\ .$$

Parallel orientifolds to a stack of D-branes change the gauge group. The states surviving the orientifold projection are such that (λ_{ij} Chan-Paton factor)

$$\lambda = -M\lambda^T M^{-1}\ ,$$

with:

- $M = \mathbb{1}_{2N}$: $\mathrm{O}p^-$, gauge group is SO(2N);
- $M = \begin{pmatrix} \mathbb{0}_N & \mathbb{1}_N \\ -\mathbb{1}_N & \mathbb{0}_N \end{pmatrix}$: $\mathrm{O}p^+$, gauge group USp(2N).



Flux compactification down to 4 dimensions

We analyze the flux compactification of massive type IIA SUGRA on twisted tori down to 4 dimensions, in presence of smeared parallel D6-branes and O6-planes:

$$\text{D6/O6}\ :\quad \underbrace{-\ |\ -\ -\ -}_{\text{4D spacetime}}\quad \underbrace{\underbrace{-\ -\ -}_{y^a}\underbrace{\cdot\ \cdot\ \cdot}_{y^i}}_{y^m}\ .$$

$(38+6\mathfrak{N})$ scalars are projected in by the orientifold involution.

Type IIA Field	σ_{O6}	$(-1)^{F_L}\Omega$	physical dof's
Φ	$+$	$+$	1
g_{ab}	$+$	$+$	6
g_{ij}	$+$	$+$	6
B_{ai}	$-$	$-$	9
C_i	$-$	$-$	3
C_{abc}	$+$	$+$	1
C_{aij}	$+$	$+$	9
C_{abijk}	$-$	$-$	3
Y^{Ii}	$-$	$-$	$3\,\mathfrak{N}$
$\mathcal{A}^I{}_a$	$+$	$+$	$3\,\mathfrak{N}$

Type IIA Flux	σ_{O6}	$(-)^{F_L}\Omega$
$\omega_{ab}{}^c$	$+$	$+$
$\omega_{ij}{}^c$	$+$	$+$
$\omega_{ai}{}^j$	$+$	$+$
H_{ijk}	$-$	$-$
H_{abi}	$-$	$-$
$F_{(0)}$	$+$	$+$
F_{ai}	$-$	$-$
F_{abij}	$+$	$+$
F_{abcijk}	$-$	$-$
$\mathcal{F}^I{}_{ab}$	$+$	$+$

Flux compactification down to 4 dimensions

We analyze the flux compactification of massive type IIA SUGRA on twisted tori down to 4 dimensions, in presence of smeared parallel D6-branes and O6-planes:

$$\text{D6/O6} \;:\quad \underbrace{- \mid - \; - \; -}_{\text{4D spacetime}} \;\; \underbrace{\underbrace{- \; - \; -}_{y^a} \underbrace{\cdot \; \cdot \; \cdot}_{y^i}}_{y^m} \; .$$

$(38 + 6\mathfrak{N})$ scalars are projected in by the orientifold involution.

Type IIA Field	σ_{O6}	$(-1)^{F_L}\Omega$	physical dof's
Φ	$+$	$+$	1
g_{ab}	$+$	$+$	6
g_{ij}	$+$	$+$	6
B_{ai}	$-$	$-$	9
C_i	$-$	$-$	3
C_{abc}	$+$	$+$	1
C_{aij}	$+$	$+$	9
C_{abijk}	$-$	$-$	3
Y^{Ii}	$-$	$-$	$3\,\mathfrak{N}$
$\mathcal{A}^I{}_a$	$+$	$+$	$3\,\mathfrak{N}$

Type IIA Flux	σ_{O6}	$(-)^{F_L}\Omega$
$\omega_{ab}{}^c$	$+$	$+$
$\omega_{ij}{}^c$	$+$	$+$
$\omega_{ai}{}^j$	$+$	$+$
H_{ijk}	$-$	$-$
H_{abi}	$-$	$-$
$F_{(0)}$	$+$	$+$
F_{ai}	$-$	$-$
F_{abij}	$+$	$+$
F_{abcijk}	$-$	$-$
$\mathcal{F}^I{}_{ab}$	$+$	$+$

How to get moduli stabilization?

A flux-induced potential for the moduli can be found with different approaches:

- dimensional reduction from $D = 10$ to $d = 4$;
- $\mathcal{N} = 4, d = 4$ gauged SUGRA.

The compactification Ansatz for the metric reads

$$ds^2_{(10)} = \tau^{-2}\, g_{\mu\nu}\, dx^\mu\, dx^\nu + \rho\,(\,\sigma^2\, M_{ab}\, e^a\, e^b + \sigma^{-2}\, M_{ij}\, e^i\, e^j\,)\ ,$$

where e^m are the Maurer-Cartan forms, such that

$$\mathrm{d}e^m + \frac{1}{2}\,\omega_{np}{}^m\, e^n \wedge e^p = 0\ , \qquad \omega_{[mn}{}^r\, \omega_{p]r}{}^q = 0\ , \qquad \omega_{mn}{}^n = 0\ .$$

How to get moduli stabilization?

A flux-induced potential for the moduli can be found with different approaches:

- dimensional reduction from $D = 10$ to $d = 4$;
- $\mathcal{N} = 4, d = 4$ gauged SUGRA.

The compactification Ansatz for the metric reads

$$ds^2_{(10)} = \tau^{-2}\, g_{\mu\nu}\, dx^\mu\, dx^\nu + \rho\,(\,\sigma^2\, M_{ab}\, e^a\, e^b + \sigma^{-2}\, M_{ij}\, e^i\, e^j\,)\ ,$$

where e^m are the Maurer-Cartan forms, such that

$$\mathrm{d}e^m + \frac{1}{2}\,\omega_{np}{}^m\, e^n \wedge e^p = 0\ , \qquad \omega_{[mn}{}^r\, \omega_{p]r}{}^q = 0\ , \qquad \omega_{mn}{}^n = 0\ .$$

The SO(3) truncation

We perform an SO(3) consistent truncation, i. e. we only retain scalar fields and fluxes which are singlets under the diagonal part of

$$\boxed{\mathrm{SO}(3)_a \times \mathrm{SO}(3)_i \times \mathrm{SO}(3)_I}$$

The scalar manifold consists of $6 + 2\frac{\mathfrak{N}}{3}$ scalars (we mainly focus on $\mathfrak{N} = 3$):

- from the closed-string sector: ρ , τ , σ , C_{abc} , C_{aij} , B_{ai} ;
- from the open-string sector: $\left(\mathcal{A}^I_a\,,\, Y^{Ii}\right) = \left(\mathcal{A}\,\delta^I_a\,,\, Y\,\delta^{Ii}\right)$.

Type IIA	fluxes	Parametrization
F_{aibjck}	a_0	$F_{aibjck} = \varepsilon_{abc}\,\varepsilon_{ijk}\,a_0$
F_{aibj}	a_1	$F_{aibj} = \varepsilon_{abc}\,\varepsilon_{ijk}\,\delta^{ck}\,a_1$
F_{ai}	a_2	$F_{ai} = \delta_{ai}\,a_2$
F_0	a_3	$F_0 = a_3$
H_{ijk}	b_0	$H_{ijk} = \varepsilon_{ijk}\,b_0$
${\omega_{ij}}^c$	b_1	${\omega_{ij}}^c = \varepsilon_{ijd}\,\delta^{cd}\,b_1$
H_{abk}	c_0	$H_{abk} = \varepsilon_{abk}\,c_0$
${\omega_{ka}}^j = {\omega_{bk}}^i$	c_1	${\omega_{ka}}^j = \varepsilon_{kal}\,\delta^{lj}\,c_1$
${\omega_{bc}}^a$	$\bar{c}_1$	${\omega_{bc}}^a = \varepsilon_{bcd}\,\delta^{ad}\,\bar{c}_1$
${\mathcal{F}^K}_{ab}$	g_0	$\mathcal{F}^K_{ab} = \varepsilon_{abc}\,\delta^{cK}\,g_0$
${g_{IJ}}^K$	g_1	${g_{IJ}}^K = \epsilon_{IJL}\delta^{LK}\,g_1$

The SO(3) truncation

We perform an SO(3) consistent truncation, i. e. we only retain scalar fields and fluxes which are singlets under the diagonal part of

$$\boxed{\mathrm{SO}(3)_a \times \mathrm{SO}(3)_i \times \mathrm{SO}(3)_I}$$

The scalar manifold consists of $6 + 2\frac{\mathfrak{N}}{3}$ scalars (we mainly focus on $\mathfrak{N} = 3$):

- from the closed-string sector: ρ , τ , σ , C_{abc} , C_{aij} , B_{ai} ;
- from the open-string sector: $\left(\mathcal{A}_a^I \, , \, Y^{Ii}\right) = \left(\mathcal{A}\, \delta_a^I \, , \, Y\, \delta^{Ii}\right)$.

Type IIA	fluxes	Parametrization
F_{aibjck}	a_0	$F_{aibjck} = \varepsilon_{abc}\, \varepsilon_{ijk}\, a_0$
F_{aibj}	a_1	$F_{aibj} = \varepsilon_{abc}\, \varepsilon_{ijk}\, \delta^{ck}\, a_1$
F_{ai}	a_2	$F_{ai} = \delta_{ai}\, a_2$
F_0	a_3	$F_0 = a_3$
H_{ijk}	b_0	$H_{ijk} = \varepsilon_{ijk}\, b_0$
${\omega_{ij}}^c$	b_1	${\omega_{ij}}^c = \varepsilon_{ijd}\, \delta^{cd}\, b_1$
H_{abk}	c_0	$H_{abk} = \varepsilon_{abk}\, c_0$
${\omega_{ka}}^j = {\omega_{bk}}^i$	c_1	${\omega_{ka}}^j = \varepsilon_{kal}\, \delta^{lj}\, c_1$
${\omega_{bc}}^a$	$\bar{c}_1$	${\omega_{bc}}^a = \varepsilon_{bcd}\, \delta^{ad}\, \bar{c}_1$
${\mathcal{F}^K}_{ab}$	g_0	$\mathcal{F}^K_{ab} = \varepsilon_{abc}\, \delta^{cK}\, g_0$
${g_{IJ}}^K$	g_1	${g_{IJ}}^K = \epsilon_{IJL} \delta^{LK}\, g_1$

The potential from dimensional reduction

Exploiting the compactification Ansatz for the metric after SO(3) truncation,

$$ds^2_{(10)} = \tau^{-2}\, g_{\mu\nu}\, dx^\mu\, dx^\nu + \rho\,(\,\sigma^2\,\delta_{ab}\, e^a\, e^b + \sigma^{-2}\,\delta_{ij}\, e^i\, e^j\,)\ ,$$

the scalar potential arises from the terms containing only moduli, in the actions both for the bulk and the sources.

$$\mathcal{S}_{\text{bulk}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\ \sqrt{-g^{(10)}}\Big(e^{-2\Phi}\big(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\big) - \frac{1}{4}\sum_{p=0}^{5}\frac{|F_{(2p)}|^2}{(2p)!}\Big)$$

The presence of a stack of coincident D6-branes and O6-planes is taken into account by two contributions: Dirac-Born-Infeld (DBI) and Wess-Zumino (WZ) actions.

[Myers '99]

$$S^{\text{DBI}}_{\text{D6}} = -T_{\text{D6}}\int_{\text{WV(D6)}} \mathrm{d}^7x\ \text{Tr}\left(e^{-\hat\Phi}\sqrt{-\det(\mathbb{M}_{MN})\det(\mathbb{Q}^i_{\ j})}\right)$$

$$S^{\text{WZ}}_{\text{D6}} = \mu_{\text{D6}}\int_{\text{WV(D6)}} \text{Tr}\left\{\text{P}\left[e^{i\lambda\iota_Y\iota_Y}\left(\hat{C}\wedge e^{\hat{B}_{(2)}}\right)\wedge e^{\lambda\mathcal{F}}\right]\right\}$$

$$S^{\text{DBI}}_{\text{O6}} = -T_{\text{O6}}\int \mathrm{d}^7x\ e^{-\Phi}\sqrt{-\det(G_{MN})}\ , \qquad S^{\text{WZ}}_{\text{O6}} = \mu_{\text{O6}}\int_{WV(\text{O6})} C_{(7)}$$

The potential from dimensional reduction

Exploiting the compactification Ansatz for the metric after SO(3) truncation,

$$ds^2_{(10)} = \tau^{-2}\, g_{\mu\nu}\, dx^\mu\, dx^\nu + \rho\,(\,\sigma^2\,\delta_{ab}\, e^a\, e^b + \sigma^{-2}\,\delta_{ij}\, e^i\, e^j\,)\ ,$$

the scalar potential arises from the terms containing only moduli, in the actions both for the bulk and the sources.

$$\mathcal{S}_{\text{bulk}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\,\sqrt{-g^{(10)}}\Big(e^{-2\Phi}\big(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\big) - \frac{1}{4}\sum_{p=0}^{5}\frac{|F_{(2p)}|^2}{(2p)!}\Big)$$

The presence of a stack of coincident D6-branes and O6-planes is taken into account by two contributions: Dirac-Born-Infeld (DBI) and Wess-Zumino (WZ) actions.

[Myers '99]

$$S^{\text{DBI}}_{\text{D6}} = -T_{\text{D6}}\int_{\text{WV(D6)}} \mathrm{d}^7x\ \text{Tr}\left(e^{-\hat{\Phi}}\sqrt{-\det(\mathbb{M}_{MN})\det(\mathbb{Q}^i_{\ j})}\right)$$

$$S^{\text{WZ}}_{\text{D6}} = \mu_{\text{D6}}\int_{\text{WV(D6)}} \text{Tr}\left\{\text{P}\left[e^{i\lambda\iota_Y\iota_Y}\left(\hat{\boldsymbol{C}}\wedge e^{\hat{B}_{(2)}}\right)\wedge e^{\lambda\mathcal{F}}\right]\right\}$$

$$S^{\text{DBI}}_{\text{O6}} = -T_{\text{O6}}\int \mathrm{d}^7x\ e^{-\Phi}\sqrt{-\det(G_{MN})}\ , \qquad S^{\text{WZ}}_{\text{O6}} = \mu_{\text{O6}}\int_{WV(\text{O6})} C_{(7)}$$

Gauged SUGRA description

Reduction of massive type IIA down to 4 dimensions on twisted tori admits a description in terms of 4d $\mathcal{N} = 4$ gauged SUGRA (half-maximal SUSY due to the presence of sources). The multiplet content consists of one gravity and $6 + \mathfrak{N}$ vector multiplets, which amounts to the boson fields: 1 graviton, $12 + \mathfrak{N}$ vector fields and $38 + 6\mathfrak{N}$ scalars.

The global symmetry of the theory is given by

$$\boxed{G_{\text{global}} = \text{SL}(2, \mathbb{R}) \times \text{SO}(6, 6 + \mathfrak{N})}$$

and the scalar manifold is its non-compact part

$$\mathcal{M}_{\text{scalar}} = \underbrace{\frac{\text{SL}(2, \mathbb{R})}{\text{SO}(2)}}_{M_{\alpha\beta}} \times \underbrace{\frac{\text{SO}(6, 6 + \mathfrak{N})}{\text{SO}(6) \times \text{SO}(6 + \mathfrak{N})}}_{M_{MN}}$$

We can also write $M = \mathcal{V}\mathcal{V}^T$, where $\mathcal{V}$ is a vielbein.

The embedding tensor $f_{\alpha MNP}$ needs to satisfy the quadratic constraints

$$f_{\alpha R[MN} f^{R}_{\beta PQ]} = 0 \ , \qquad \varepsilon^{\alpha\beta} f_{\alpha MNR} f^{R}_{\beta PQ} = 0 \ .$$

Gauged SUGRA description

Reduction of massive type IIA down to 4 dimensions on twisted tori admits a description in terms of 4d $\mathcal{N}=4$ gauged SUGRA (half-maximal SUSY due to the presence of sources). The multiplet content consists of one gravity and $6+\mathfrak{N}$ vector multiplets, which amounts to the boson fields: 1 graviton, $12+\mathfrak{N}$ vector fields and $38+6\mathfrak{N}$ scalars.

The global symmetry of the theory is given by

$$\boxed{G_{\text{global}} = \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SO}(6,6+\mathfrak{N})}$$

and the scalar manifold is its non-compact part

$$\mathcal{M}_{\text{scalar}} = \underbrace{\frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)}}_{M_{\alpha\beta}} \times \underbrace{\frac{\mathrm{SO}(6,6+\mathfrak{N})}{\mathrm{SO}(6)\times\mathrm{SO}(6+\mathfrak{N})}}_{M_{MN}}$$

We can also write $M = \mathcal{V}\mathcal{V}^T$, where $\mathcal{V}$ is a vielbein.

The embedding tensor $f_{\alpha MNP}$ needs to satisfy the quadratic constraints

$$f_{\alpha R[MN} f^{R}_{\beta PQ]} = 0 \,, \qquad \varepsilon^{\alpha\beta} f_{\alpha MNR} f^{R}_{\beta PQ} = 0 \,.$$

Gauged SUGRA description

Reduction of massive type IIA down to 4 dimensions on twisted tori admits a description in terms of 4d $\mathcal{N}=4$ gauged SUGRA (half-maximal SUSY due to the presence of sources). The multiplet content consists of one gravity and $6+\mathfrak{N}$ vector multiplets, which amounts to the boson fields: 1 graviton, $12+\mathfrak{N}$ vector fields and $38+6\mathfrak{N}$ scalars.

The global symmetry of the theory is given by

$$\boxed{G_{\text{global}} = \text{SL}(2,\mathbb{R}) \times \text{SO}(6,6+\mathfrak{N})}$$

and the scalar manifold is its non-compact part

$$\mathcal{M}_{\text{scalar}} = \underbrace{\frac{\text{SL}(2,\mathbb{R})}{\text{SO}(2)}}_{M_{\alpha\beta}} \times \underbrace{\frac{\text{SO}(6,6+\mathfrak{N})}{\text{SO}(6)\times\text{SO}(6+\mathfrak{N})}}_{M_{MN}}$$

We can also write $M = \mathcal{V}\mathcal{V}^T$, where $\mathcal{V}$ is a vielbein.

The embedding tensor $f_{\alpha MNP}$ needs to satisfy the quadratic constraints

$$f_{\alpha R[MN} f_{\beta PQ]}^{R} = 0 \ , \qquad \varepsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}^{R} = 0 \ .$$

The scalar potential

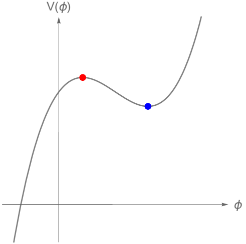
The gauging induces the scalar potential

$$V = \frac{1}{64} f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta}\Big[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \Big(\frac{2}{3}\eta^{MQ} - M^{MQ}\Big)\eta^{NR}\eta^{PS}\Big] + \\ - \frac{1}{144} f_{\alpha MNP} f_{\alpha QRS} \varepsilon^{\alpha\beta} M^{MNPQRS} \, ,$$

with

$$\eta_{MN} = \eta^{MN} = \begin{pmatrix} \mathbb{O}_6 & \mathbb{I}_6 & \mathbb{O}_{6,\mathfrak{N}} \\ \mathbb{I}_6 & \mathbb{O}_6 & \mathbb{O}_{6,\mathfrak{N}} \\ \mathbb{O}_{\mathfrak{N},6} & \mathbb{O}_{\mathfrak{N},6} & \mathbb{I}_{\mathfrak{N}} \end{pmatrix} \, , \quad M_{MNPQRS} \equiv \varepsilon_{\underline{mnpqrs}} \mathring{\mathcal{V}}_{\underline{M}}{}^{\underline{m}} \mathring{\mathcal{V}}_{\underline{N}}{}^{\underline{n}} \mathring{\mathcal{V}}_{\underline{P}}{}^{\underline{p}} \mathring{\mathcal{V}}_{\underline{Q}}{}^{\underline{q}} \mathring{\mathcal{V}}_{\underline{R}}{}^{\underline{r}} \mathring{\mathcal{V}}_{\underline{S}}{}^{\underline{s}} \, .$$

[Schon, Weidner '06]



The dictionary between the approaches

A dictionary allows to compare the outcomes of the different approaches:

$$C_{abc} = \epsilon_{abc}\chi \ , \qquad C_{aij} = \epsilon_{aij}\chi_1 \ , \qquad B_{ai} = \delta_{ai}\chi_2 \ ,$$
$$e^{\phi} = \frac{1}{\tau\sigma^3} \ , \qquad e^{\varphi_1} = \frac{\sigma}{\tau} \ , \qquad e^{\varphi_2} = \frac{1}{\rho} \ .$$

Type IIA	Fluxes	$d=4\,\mathcal{N}=4$
F_{aibjck}	a_0	$-f_{+\bar{a}\bar{b}\bar{c}}$
F_{aibj}	a_1	$f_{+\bar{a}\bar{b}\bar{k}}$
F_{ai}	a_2	$-f_{+\bar{a}\bar{j}\bar{k}}$
$F_{(0)}$	a_3	$f_{+\bar{i}\bar{j}\bar{k}}$
H_{ijk}	b_0	$-f_{-\bar{a}\bar{b}\bar{c}}$
H_{abk}	c_0	$f_{+\bar{a}\bar{b}k}$
ω_{ij}^{c}	b_1	$f_{-\bar{a}\bar{b}\bar{k}}$
$\omega_{ka}^{j} = \omega_{bk}^{i}$	c_1	$f_{+\bar{a}\bar{j}k} = f_{+\bar{i}\bar{b}k}$
ω_{bc}^{a}	$\bar{c}_1$	$f_{+a\bar{b}\bar{c}}$
${\mathcal{F}^K}_{ab}$	g_0	$f_{+\bar{a}\bar{b}K}$
${g_{IJ}}^K$	g_1	f_{+IJK}

Through this dictionary, the terms in the gauged SUGRA potential can be identified with those arising from the dimensional reduction:

- the terms with the same scaling in the dilatons as the square of the embedding tensor components can be identified with the reduction of the R-R sector and modifications of the field-strengths due to the WZ action

$$\boxed{\boldsymbol{F} = \mathrm{d}\boldsymbol{C} + H_{(3)} \wedge \boldsymbol{C} + \boldsymbol{F}^{\mathrm{flux}} \wedge e^{-B_{(2)}} + \Delta \boldsymbol{F}_{\mathrm{WZ}}}$$

- the other terms arise from the bulk action for the NS-NS sector or the DBI actions both for the D6-branes and the orientifold, as long as we take the tadpole cancellation condition into account

$$\boxed{\oint_{\mathcal{M}_3{}^{(i)}} (\mathrm{d}F_2 - H_3 \wedge F_0) = N_{D6}\, T_{D6} + T_{O6}}$$

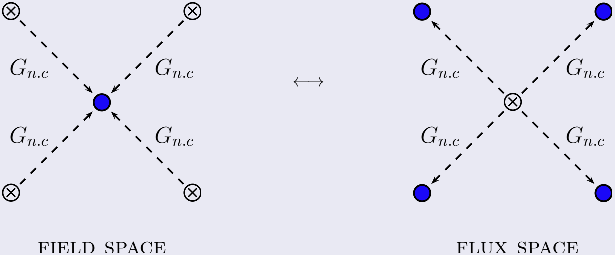
The vacua of the theory

The vacua of the theory are the extrema of the potential

$$\partial_\Phi V|_{\Phi_0} = 0 \ .$$

Given the homogeneity of the scalar manifold, we can choose to work in the flux picture.

[Dibitetto, Guarino, Roest '11]



The vacua also have to satisfy the quadratic constraints:

$$c_1(c_1 - \bar{c}_1) = 0 \ , \quad b_1(c_1 - \bar{c}_1) = 0 \ , \quad a_3\, c_0 + 2\, a_2\, c_1 - a_2\, \bar{c}_1 = 0 \ , \quad g_0 g_1 = 0$$

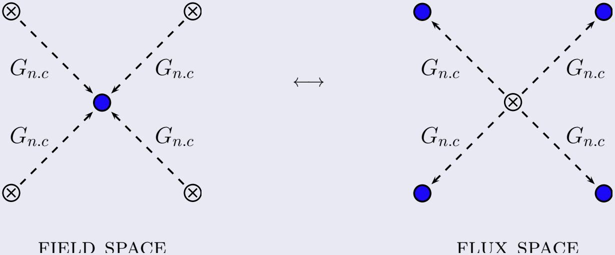
The vacua of the theory

The vacua of the theory are the extrema of the potential

$$\partial_\Phi V|_{\Phi_0} = 0 \ .$$

Given the homogeneity of the scalar manifold, we can choose to work in the flux picture.

[Dibitetto, Guarino, Roest '11]



The vacua also have to satisfy the quadratic constraints:

$$c_1(c_1 - \bar{c}_1) = 0 \ , \quad b_1(c_1 - \bar{c}_1) = 0 \ , \quad a_3\, c_0 + 2\, a_2\, c_1 - a_2\, \bar{c}_1 = 0 \ , \quad g_0 g_1 = 0$$

Solutions in the origin

Novel AdS$_4$ vacua have been found in this setting ($\Lambda < 0$).

- They are perturbatively stable if all the eigenvalues of the canonically normalized mass matrix

$$\mathcal{L}_{\text{KIN}} = -\frac{1}{2} K_{AB}\, \partial\phi^A \partial\phi^B \quad \longrightarrow \quad \boxed{(m^2)^A{}_B = \frac{1}{|\Lambda|} K^{AC} \frac{\partial^2 V}{\partial\phi^C \partial\phi^B}}$$

  are above the Breitenlohner-Freedman bound

$$m^2 \geq -\frac{3}{4} \, .$$

- The vacua with residual SUSY are such that

$$A_1^{ij} q_j = \sqrt{-3V} q^i \, , \qquad A_2^{ij} q_j = 0 \, .$$

Even though not completely general, we have started by looking for vacua in the origin of the scalar manifold.

Solution	a_0	a_1	a_2	a_3	b_0	b_1	c_0	$c_1 = \bar{c}_1$	g_0	g_1
A	λ	0	0	$s_1 \frac{\sqrt{5} + 3\sqrt{13}}{14} \lambda$	0	$-\frac{11 + \sqrt{65}}{14} \lambda$	0	$-\lambda$	$s_2 \frac{5\sqrt{10} + \sqrt{26}}{14} \lambda$	0
B	λ	0	0	$s_1 \frac{\sqrt{5} - 3\sqrt{13}}{14} \lambda$	0	$\frac{-11 + \sqrt{65}}{14} \lambda$	0	$-\lambda$	$s_2 \frac{5\sqrt{10} - \sqrt{26}}{14} \lambda$	0

Solutions in the origin

Novel AdS$_4$ vacua have been found in this setting ($\Lambda < 0$).

- They are perturbatively stable if all the eigenvalues of the canonically normalized mass matrix

$$\mathcal{L}_{\text{KIN}} = -\frac{1}{2} K_{AB}\, \partial\phi^A \partial\phi^B \quad \longrightarrow \quad \boxed{(m^2)^A{}_B = \frac{1}{|\Lambda|} K^{AC} \frac{\partial^2 V}{\partial\phi^C \partial\phi^B}}$$

  are above the Breitenlohner-Freedman bound

$$m^2 \geq -\frac{3}{4} \,.$$

- The vacua with residual SUSY are such that

$$A_1^{ij} q_j = \sqrt{-3V} q^i \,, \qquad A_2^{ij} q_j = 0 \,.$$

Even though not completely general, we have started by looking for vacua in the origin of the scalar manifold.

Solution	a_0	a_1	a_2	a_3	b_0	b_1	c_0	$c_1 = \bar{c}_1$	g_0	g_1
A	λ	0	0	$s_1 \frac{\sqrt{5}+3\sqrt{13}}{14}\lambda$	0	$-\frac{11+\sqrt{65}}{14}\lambda$	0	$-\lambda$	$s_2 \frac{5\sqrt{10}+\sqrt{26}}{14}\lambda$	0
B	λ	0	0	$s_1 \frac{\sqrt{5}-3\sqrt{13}}{14}\lambda$	0	$\frac{-11+\sqrt{65}}{14}\lambda$	0	$-\lambda$	$s_2 \frac{5\sqrt{10}-\sqrt{26}}{14}\lambda$	0

Solutions out of the origin

Other vacua can be found when some scalars are out of the origin.

Solution	a_0	a_1	a_2	a_3	b_0	b_1	c_0	c_1 , $\bar{c}_1$	g_0	g_1
1	$\left(s_2\frac{3}{2}-\frac{1}{2}\mathcal{A}^2\right)\lambda$	$s_1\frac{1}{2}\sqrt{\frac{3}{5}}\lambda$	$-s_2\frac{\lambda}{6}$	$s_1\frac{1}{2}\sqrt{\frac{5}{3}}\lambda$	$-s_1s_2\frac{\lambda}{\sqrt{15}}$	$\frac{\lambda}{3}$	$s_1s_2\frac{\lambda}{\sqrt{15}}$	λ	0	$-\frac{\lambda}{\mathcal{A}}$
2	$\left(s_2\frac{5}{3}-\frac{1}{2}\mathcal{A}^2\right)\lambda$	0	0	$s_1\frac{\sqrt{5}}{3}\lambda$	0	$\frac{\lambda}{3}$	0	λ	0	$-\frac{\lambda}{\mathcal{A}}$
3	$\left(s_2-\frac{1}{2}\mathcal{A}^2\right)\lambda$	$-s_1\frac{\lambda}{\sqrt{3}}$	$s_2\frac{\lambda}{3}$	$s_1\frac{\lambda}{\sqrt{3}}$	$s_1s_2\frac{\lambda}{\sqrt{3}}$	$\frac{\lambda}{3}$	$-s_1s_2\frac{\lambda}{\sqrt{3}}$	λ	0	$-\frac{\lambda}{\mathcal{A}}$
4	$\left(s_2\sqrt{5}-\frac{1}{2}\mathcal{A}^2\right)\lambda$	0	0	$s_1\lambda$	0	λ	0	λ	0	$-\frac{\lambda}{\mathcal{A}}$

- Solutions 1 are SUSY, Solutions 2, 3, 4 are not.
- Solutions 1, 3 and 4 $(s_2 = +)$ are perturbatively stable.

Stable non-SUSY vacua would deserve greater attention, since on grounds of the AdS swampland conjecture, destabilizing mechanisms should exist.

[Ooguri, Vafa '17]

Some simple generalizations of these solutions have been also found in the instance $\mathfrak{N} = 6$.

Other vacua can be found when some scalars are out of the origin.

Solution	a_0	a_1	a_2	a_3	b_0	b_1	c_0	c_1 , $\bar{c}_1$	g_0	g_1
1	$\left(s_2\frac{3}{2}-\frac{1}{2}\mathcal{A}^2\right)\lambda$	$s_1\frac{1}{2}\sqrt{\frac{3}{5}}\lambda$	$-s_2\frac{\lambda}{6}$	$s_1\frac{1}{2}\sqrt{\frac{5}{3}}\lambda$	$-s_1 s_2\frac{\lambda}{\sqrt{15}}$	$\frac{\lambda}{3}$	$s_1 s_2\frac{\lambda}{\sqrt{15}}$	λ	0	$-\frac{\lambda}{\mathcal{A}}$
2	$\left(s_2\frac{5}{3}-\frac{1}{2}\mathcal{A}^2\right)\lambda$	0	0	$s_1\frac{\sqrt{5}}{3}\lambda$	0	$\frac{\lambda}{3}$	0	λ	0	$-\frac{\lambda}{\mathcal{A}}$
3	$\left(s_2-\frac{1}{2}\mathcal{A}^2\right)\lambda$	$-s_1\frac{\lambda}{\sqrt{3}}$	$s_2\frac{\lambda}{3}$	$s_1\frac{\lambda}{\sqrt{3}}$	$s_1 s_2\frac{\lambda}{\sqrt{3}}$	$\frac{\lambda}{3}$	$-s_1 s_2\frac{\lambda}{\sqrt{3}}$	λ	0	$-\frac{\lambda}{\mathcal{A}}$
4	$\left(s_2\sqrt{5}-\frac{1}{2}\mathcal{A}^2\right)\lambda$	0	0	$s_1\lambda$	0	λ	0	λ	0	$-\frac{\lambda}{\mathcal{A}}$

- Solutions 1 are SUSY, Solutions 2, 3, 4 are not.
- Solutions 1, 3 and 4 ($s_2 = +$) are perturbatively stable.

Stable non-SUSY vacua would deserve greater attention, since on grounds of the AdS swampland conjecture, destabilizing mechanisms should exist.

[Ooguri, Vafa ‘17]

Some simple generalizations of these solutions have been also found in the instance $\mathfrak{N} = 6$.

The reliability of our setup

The SUGRA approximation is reliable as long as the higher-loop expansion in g_s and the higher-derivative expansion in α' are under control.

We look for the scaling of fields and fluxes with respect to

$$\mathbb{R}^+_\Omega \equiv \mathbb{R}^+_{\text{trombone}} \times \mathbb{R}^+_\rho \times \mathbb{R}^+_\tau$$

Scalars	ρ	τ	σ
Ω weights	Ω^2	Ω^6	Ω^0

Fluxes	$H_{(3)}$	$F_{(p)}$	ω	$\mathcal{F}^I$
Ω weights	Ω^2	Ω^{2+p}	Ω^0	Ω^4

Scales	g_s	$\frac{Vol_6}{(2\pi\ell_s)^6}$	$\frac{\lvert\Lambda\rvert}{M_{\text{Pl}}^4}$	$\frac{\ell_{\text{KK}}}{\ell_{\text{AdS}}}$
Ω weights	Ω^{-3}	Ω^6	Ω^{-14}	Ω^0

An explicit perturbative corner for the fluxes is:

$$\lambda = 6\,m\,n \quad , \qquad \Omega^2 = \sqrt{5}\,p \quad , \qquad \mathcal{A} = -\frac{6\,n}{5\,p^2} \quad ,$$

$$a_0 = 250\,m\,n\,p^4 - 108\,m\,n^3\,, \;\; a_3 = 10\,m\,n\,p\,, \;\; b_1 = 2\,m\,n\,, \;\; c_1 = 6\,m\,n\,, \;\; g_1 = m\,.$$

Flux compactification down to 7 dimensions

We examine the flux compactification of massive type IIA SUGRA on 3-spheres down to 7 dimensions, in presence of localized parallel D6-branes and O6-planes:

$$\text{D6/O6} \ : \quad \underbrace{- \,|\, - \ - \ - \ - \ - \ -}_{\text{7D spacetime}} \ \underbrace{\cdot \ \cdot \ \cdot}_{y^i}$$

S^3 is the topology of the internal manifold of all supersymmetric AdS$_7$ vacua.

[Apruzzi, Fazzi, Rosa, Tomasiello '13]

The orientifold involution projects in 1 graviton, a 3-form, $6 + \mathfrak{N}$ vector fields and $(10 + 3\mathfrak{N})$ scalars.

IIA fields	$\mathbb{Z}_2$-even components	7D fields
g_{MN}	$g_{\mu\nu}$	graviton ($\times 1$)
	g_{ij}	scalars ($\times 6$)
B_{MN}	$B_{\mu i}$	vectors ($\times 3$)
Φ	Φ	scalar ($\times 1$)
C_M	C_i	scalars ($\times 3$)
C_{MNP}	$C_{\mu\nu\rho}$	3-form ($\times 1$)
	$C_{\mu jk}$	vectors ($\times 3$)
${\mathcal{A}^I}_\mu$	${\mathcal{A}^I}_\mu$	vectors ($\times \mathfrak{N}$)
Y^{iI}	Y^{iI}	scalars ($\times 3\mathfrak{N}$)

Flux compactification down to 7 dimensions

We examine the flux compactification of massive type IIA SUGRA on 3-spheres down to 7 dimensions, in presence of localized parallel D6-branes and O6-planes:

$$\text{D6/O6} \;:\; \underbrace{- \mid - \; - \; - \; - \; - \; -}_{\text{7D spacetime}} \; \underbrace{\cdot \;\cdot\; \cdot}_{y^i}$$

S^3 is the topology of the internal manifold of all supersymmetric AdS_7 vacua.

[Apruzzi, Fazzi, Rosa, Tomasiello '13]

The orientifold involution projects in 1 graviton, a 3-form, $6+\mathfrak{N}$ vector fields and $(10+3\mathfrak{N})$ scalars.

IIA fields	$\mathbb{Z}_2$-even components	7D fields
g_{MN}	$g_{\mu\nu}$	graviton $(\times 1)$
	g_{ij}	scalars $(\times 6)$
B_{MN}	$B_{\mu i}$	vectors $(\times 3)$
Φ	Φ	scalar $(\times 1)$
C_M	C_i	scalars $(\times 3)$
C_{MNP}	$C_{\mu\nu\rho}$	3-form $(\times 1)$
	$C_{\mu jk}$	vectors $(\times 3)$
${\mathcal{A}^I}_\mu$	${\mathcal{A}^I}_\mu$	vectors $(\times \mathfrak{N})$
Y^{iI}	Y^{iI}	scalars $(\times 3\mathfrak{N})$

These warped compactifications admit a low-energy description in terms of 7d $\mathcal{N}=1$ gauged SUGRA. One gravity multiplet and $(3+\mathfrak{N})$ vector multiplets give rise to the boson field content of the theory: 1 graviton, 1 2-form, $6+\mathfrak{N}$ vectors, $10+3\mathfrak{N}$ scalars.

$$\boxed{G_0 \;=\; \mathbb{R}^+_X \,\times\, \mathrm{SO}(3,3+\mathfrak{N})}$$

The scalar manifold is its non-compact part

$$\underbrace{\mathbb{R}^+_X}_{X} \times \underbrace{\frac{\mathrm{SO}(3,3+\mathfrak{N})}{\mathrm{SO}(3)\,\times\,\mathrm{SO}(3+\mathfrak{N})}}_{M_{AB}} \;.$$

We focus on the instance $\mathfrak{N}=3$. According to the SO(3) truncation, the brane-position moduli are

$$Y^i = Y\delta^{iI}t_I \;.$$

The embedding tensor

As long as we focus on the closed-string sector, the global symmetry group is $\mathrm{SO}(3,3) \approx \mathrm{SL}(4)$. The gauging deformations surviving the linear constraints are:

$$\Theta \quad \in \quad \underbrace{\mathbf{10}'_{(+1)}}_{Q_{(mn)}} \quad \oplus \quad \underbrace{\mathbf{10}_{(+1)}}_{\tilde{Q}^{(mn)}} \quad \oplus \quad \underbrace{\mathbf{6}_{(+1)}}_{\xi_{[mn]}} \, .$$

A Stueckelberg-like mass θ for the 2-form is another deformation.

$$\xi_{mn} = 0 \, , \quad \tilde{Q}^{mn} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{q} \end{pmatrix} , \quad Q_{mn} = \begin{pmatrix} q & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & q & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

satisfy the quadratic constraints

$$\theta \, \xi_{mn} = 0 \, , \qquad \left(\tilde{Q}^{mp} + \xi^{mp} \right) Q_{pn} \; - \; \frac{1}{4} \left(\tilde{Q}^{pq} \, Q_{pq} \right) \delta^m_n = 0 \, .$$

IIA fluxes	Deformations
$F_{(0)}$	$\sqrt{2} \, \tilde{q}$
H_{ijk}	$\frac{1}{\sqrt{2}} \, \theta \, \epsilon_{ijk}$
Θ_{ij}	$q \, \delta_{ij}$
${g_{IJ}}^K$	$g_{YM} {\varepsilon_{IJ}}^K$

The embedding tensor

As long as we focus on the closed-string sector, the global symmetry group is $\mathrm{SO}(3,3) \approx \mathrm{SL}(4)$. The gauging deformations surviving the linear constraints are:

$$\Theta \quad \in \quad \underbrace{\mathbf{10}'_{(+1)}}_{Q_{(mn)}} \quad \oplus \quad \underbrace{\mathbf{10}_{(+1)}}_{\tilde{Q}^{(mn)}} \quad \oplus \quad \underbrace{\mathbf{6}_{(+1)}}_{\xi_{[mn]}} \ .$$

A Stueckelberg-like mass θ for the 2-form is another deformation.

$$\xi_{mn} = 0 \ , \quad \tilde{Q}^{mn} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{q} \end{pmatrix} , \quad Q_{mn} = \begin{pmatrix} q & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & q & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

satisfy the quadratic constraints

$$\theta \, \xi_{mn} = 0 \ , \qquad \left(\tilde{Q}^{mp} + \xi^{mp}\right) Q_{pn} \ - \ \frac{1}{4} \left(\tilde{Q}^{pq} \, Q_{pq}\right) \delta^m_n = 0 \, .$$

IIA fluxes	Deformations
$F_{(0)}$	$\sqrt{2}\,\tilde{q}$
H_{ijk}	$\frac{1}{\sqrt{2}}\,\theta\,\epsilon_{ijk}$
Θ_{ij}	$q\,\delta_{ij}$
${g_{IJ}}^K$	$g_{YM}\,{\varepsilon_{IJ}}^K$

The potential and the vacua

The gauging induces the scalar potential

$$V = \frac{1}{64}\left[\frac{X^2}{2}\, f_{ABC} f_{DEF}\left(\tfrac{1}{3}\, M^{AD} M^{BE} M^{CF} + \left(\tfrac{2}{3}\,\eta^{AD} - M^{AD}\right)\eta^{BE}\eta^{CF}\right) + \right.$$
$$\left. + \,\theta^2 X^{-8} - \tfrac{2}{3}\sqrt{2}\, X^{-3}\,\theta\, f_{ABC}\, M^{ABC}\right],$$

with $M^{ABC} = \epsilon^{abc}\,\mathring{\mathcal{V}}_a{}^A\,\mathring{\mathcal{V}}_b{}^B\,\mathring{\mathcal{V}}_c{}^C$.

We found different families of AdS_7 vacua.

ID	θ	q	$\tilde{q}$	g_{YM}	mass spectrum	SUSY
1	$\frac{\lambda}{4}$	λ	$\frac{\lambda}{2}(2+Y^2)$	$-\frac{\lambda}{2Y}$	$-\frac{8}{15}$ $(\times 1)$ 0 $(\times 6)$ $\frac{16}{15}$ $(\times 5)$ $\frac{8}{3}$ $(\times 1)$	✔
2	$\frac{\lambda}{2}$	λ	$\frac{\lambda}{2}(2+Y^2)$	$-\frac{\lambda}{2Y}$	0 $(\times 8)$ $\frac{4}{5}$ $(\times 1)$ $\frac{12}{5}$ $(\times 1)$	✘
3	$\frac{\lambda}{14}$	λ	$\frac{\lambda}{14}(-16+7Y^2)$	$-\frac{\lambda}{2Y}$	0 $(\times 3)$ $\frac{12}{5}$ $(\times 5)$ $\frac{2}{35}\left(22 \pm \sqrt{1954}\right)$ $(\times 1)$	✘

The potential and the vacua

The gauging induces the scalar potential

$$V = \frac{1}{64}\left[\frac{X^2}{2}\, f_{ABC} f_{DEF}\, \left(\frac{1}{3}\, M^{AD} M^{BE} M^{CF} + \left(\frac{2}{3}\,\eta^{AD} - M^{AD}\right)\eta^{BE}\eta^{CF}\right) + \right.$$
$$\left. + \,\theta^2\, X^{-8} - \frac{2}{3}\sqrt{2}\, X^{-3}\,\theta\, f_{ABC}\, M^{ABC}\right],$$

with $M^{ABC} = \epsilon^{abc}\,\mathring{\mathcal{V}}_a{}^A\,\mathring{\mathcal{V}}_b{}^B\,\mathring{\mathcal{V}}_c{}^C$.

We found different families of AdS_7 vacua.

ID	θ	q	$\tilde{q}$	g_{YM}	mass spectrum	SUSY
1	$\frac{\lambda}{4}$	λ	$\frac{\lambda}{2}(2+Y^2)$	$-\frac{\lambda}{2Y}$	$-\frac{8}{15}$ $(\times 1)$ 0 $(\times 6)$ $\frac{16}{15}$ $(\times 5)$ $\frac{8}{3}$ $(\times 1)$	✔
2	$\frac{\lambda}{2}$	λ	$\frac{\lambda}{2}(2+Y^2)$	$-\frac{\lambda}{2Y}$	0 $(\times 8)$ $\frac{4}{5}$ $(\times 1)$ $\frac{12}{5}$ $(\times 1)$	✘
3	$\frac{\lambda}{14}$	λ	$\frac{\lambda}{14}(-16+7Y^2)$	$-\frac{\lambda}{2Y}$	0 $(\times 3)$ $\frac{12}{5}$ $(\times 5)$ $\frac{2}{35}\left(22 \pm \sqrt{1954}\right)$ $(\times 1)$	✘

The uplift to 10D

All SUSY ($\mathcal{N} = 1$) AdS_7 vacua have been classified. All of them appear in massive type IIA and can be sourced by D6/D8-branes and/or O6/O8 orientifolds. Consistently with their R-symmetry SU(2), their topology is

$$\boxed{\mathrm{AdS}_7 \times S^3}$$

with $S^3 = [0, N] \times S^2$.

Exceptional Field Theory techniques seem to suggest that consistent truncations from 10D down to 7D SUGRA with an arbitrary number of vector multiplets do not exist.

[Malek, Samtleben, Vall Camell '19]

We expect the vacua to solve the Bianchi identities and the equations of motion of

$$\boxed{\mathcal{S}_{\mathrm{NSNS}} + \mathcal{S}_{\mathrm{RR}} + \mathcal{S}_{\mathrm{DBI}} + \mathcal{S}_{\mathrm{WZ}}}$$

A consistent truncation has been found from 10D down to 7D $\mathcal{N} = 1$ SUGRA with no vector multiplets (minimal SUGRA).

[Passias, Rota, Tomasiello '15]

The uplift to 10D

All SUSY ($\mathcal{N} = 1$) AdS_7 vacua have been classified. All of them appear in massive type IIA and can be sourced by D6/D8-branes and/or O6/O8 orientifolds. Consistently with their R-symmetry SU(2), their topology is

$$\boxed{\mathrm{AdS}_7 \times S^3}$$

with $S^3 = [0, N] \times S^2$.

Exceptional Field Theory techniques seem to suggest that consistent truncations from 10D down to 7D SUGRA with an arbitrary number of vector multiplets do not exist.

[Malek, Samtleben, Vall Camell '19]

We expect the vacua to solve the Bianchi identities and the equations of motion of

$$\boxed{\mathcal{S}_{\mathrm{NSNS}} + \mathcal{S}_{\mathrm{RR}} + \mathcal{S}_{\mathrm{DBI}} + \mathcal{S}_{\mathrm{WZ}}}$$

A consistent truncation has been found from 10D down to 7D $\mathcal{N} = 1$ SUGRA with no vector multiplets (minimal SUGRA).

[Passias, Rota, Tomasiello '15]

Minimal SUGRA solution

The uplift Ansatz reads

$$\frac{1}{\sqrt{2}\pi\ell}\mathrm{d}s^2 = g^2 X^{-1/2}\sqrt{-\frac{\alpha}{\ddot{\alpha}}}\mathrm{d}{s_7}^2 + X^{5/2}\sqrt{-\frac{\ddot{\alpha}}{\alpha}}\left(\mathrm{d}z^2 + \frac{\alpha^2}{\dot{\alpha}^2 - 2X^5\alpha\ddot{\alpha}}ds^2_{S^2}\right) ,$$

$$e^{\Phi} = 162 \cdot 2^{1/4}\pi^{5/2}X^{5/4}\frac{(-\alpha/\ddot{\alpha})^{3/4}}{(\dot{\alpha}^2 - 2X^5\alpha\ddot{\alpha})^{1/2}} ,$$

$$B = \pi\ell\left(-z+\frac{\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2X^5\alpha\ddot{\alpha}}\right)\mathrm{vol}_{S^2} , \qquad F_2 = \ell\left(\frac{\ddot{\alpha}}{162\pi^2} + \frac{\pi F_0\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2X^5\alpha\ddot{\alpha}}\right)\mathrm{vol}_{S^2} .$$

$\alpha = \alpha(z)$ is a piecewise cubic function vanishing at the extrema of the interval

$$\boxed{\alpha(0) = \alpha(N) = 0}$$

This Ansatz is solution for both the Bianchi identities and the equations of motion for the bulk if

$$\dddot{\alpha} = -162\pi^3 F_0 , \qquad X = 1, 2^{-1/5} .$$

Minimal SUGRA solution

The uplift Ansatz reads

$$\frac{1}{\sqrt{2}\pi\ell}\mathrm{d}s^2 = g^2 X^{-1/2}\sqrt{-\frac{\alpha}{\ddot{\alpha}}}\mathrm{d}s_7{}^2 + X^{5/2}\sqrt{-\frac{\ddot{\alpha}}{\alpha}}\left(\mathrm{d}z^2 + \frac{\alpha^2}{\dot{\alpha}^2 - 2X^5\alpha\ddot{\alpha}}ds^2_{S^2}\right) ,$$

$$e^{\Phi} = 162 \cdot 2^{1/4}\pi^{5/2}X^{5/4}\frac{(-\alpha/\ddot{\alpha})^{3/4}}{(\dot{\alpha}^2 - 2X^5\alpha\ddot{\alpha})^{1/2}} ,$$

$$B = \pi\ell\left(-z+\frac{\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2X^5\alpha\ddot{\alpha}}\right)\mathrm{vol}_{S^2} , \qquad F_2 = \ell\left(\frac{\ddot{\alpha}}{162\pi^2} + \frac{\pi F_0\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2X^5\alpha\ddot{\alpha}}\right)\mathrm{vol}_{S^2} .$$

$\alpha = \alpha(z)$ is a piecewise cubic function vanishing at the extrema of the interval

$$\boxed{\alpha(0) = \alpha(N) = 0}$$

This Ansatz is solution for both the Bianchi identities and the equations of motion for the bulk if

$$\dddot{\alpha} = -162\pi^3 F_0 , \qquad X = 1, 2^{-1/5} .$$

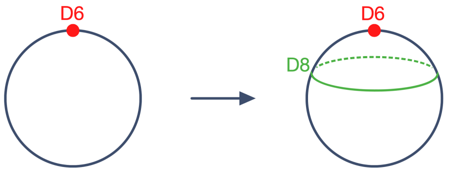
The effect of the sources

The uplift Ansatz also encodes the presence of sources:

- D6-branes at the extrema of the interval where $\ddot{\alpha} \neq 0$.
- D8-branes wrapping a cycle S^2 and sitting where Romans'mass changes value

$$\dddot{\alpha} = -162\pi^3 F_0 \qquad \longrightarrow \qquad \partial_z^4 \alpha \sim f_a \delta(z-a)$$

The open-string dynamics giving rise to our vacua naturally contains the responsible perturbations for the brane polarization via Myers effect.



The tadpole cancellation condition reads

$$\mathrm{d}F_2 - F_0 H_3 = \mathcal{Q}\ \mathrm{Vol}_{S^3}$$

Our vacua are expected to uplift to a solution of the equations of motion for the bulk and non-Abelian brane actions (DBI and WZ).

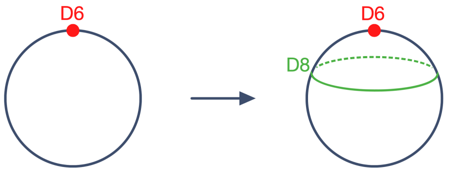
The effect of the sources

The uplift Ansatz also encodes the presence of sources:

- D6-branes at the extrema of the interval where $\ddot{\alpha} \neq 0$.
- D8-branes wrapping a cycle S^2 and sitting where Romans'mass changes value

$$\dddot{\alpha} = -162\pi^3 F_0 \qquad \longrightarrow \qquad \partial_z^4 \alpha \sim f_a \delta(z-a)$$

The open-string dynamics giving rise to our vacua naturally contains the responsible perturbations for the brane polarization via Myers effect.



The tadpole cancellation condition reads

$$\mathrm{d}F_2 - F_0 H_3 = \mathcal{Q}\ \mathrm{Vol}_{S^3}$$

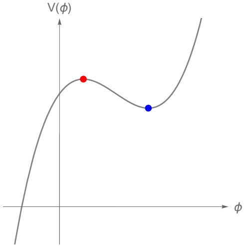
Our vacua are expected to uplift to a solution of the equations of motion for the bulk and non-Abelian brane actions (DBI and WZ).

Conclusions

Gauged SUGRA and the embedding tensor formalism have been powerful tools to discover and analyze novel vacuum solutions in Supergravity when open string dynamics has been taken into account. This has been possible even when a consistent truncation from String Theory is missing.

A few possibilities appear as natural ways for these works to progress:

- to search for both perturbative and non-perturbative mechanisms destabilising the stable non-SUSY vacua; this kind of processes are expected on grounds of the AdS swampland conjecture;
- the new vacua emerge as results of the coupling of closed strings in the bulk and open strings on the branes; this occurrence would suggest non-trivial modifications in the usual holographic setting, which instead exhibits an AdS vacuum in the bulk and a conformal dual on the branes.

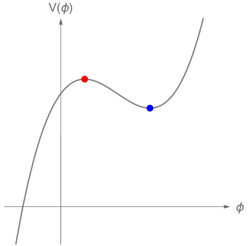


Conclusions

Gauged SUGRA and the embedding tensor formalism have been powerful tools to discover and analyze novel vacuum solutions in Supergravity when open string dynamics has been taken into account. This has been possible even when a consistent truncation from String Theory is missing.

A few possibilities appear as natural ways for these works to progress:

- to search for both perturbative and non-perturbative mechanisms destabilising the stable non-SUSY vacua; this kind of processes are expected on grounds of the AdS swampland conjecture;
- the new vacua emerge as results of the coupling of closed strings in the bulk and open strings on the branes; this occurrence would suggest non-trivial modifications in the usual holographic setting, which instead exhibits an AdS vacuum in the bulk and a conformal dual on the branes.



Many thanks for your attention!

The potential from dimensional reduction

The presence of a stack of coincident D6-branes is taken into account by two contributions: Wess-Zumino (WZ) and Dirac-Born-Infeld (DBI) actions.

[Myers '99]

$$S^{\mathrm{DBI}}_{\mathrm{D6}} = -T_{\mathrm{D6}} \int_{\mathrm{WV(D6)}} \mathrm{d}^7x \ \mathrm{Tr}\left(e^{-\hat{\Phi}} \sqrt{-\det(\mathbb{M}_{MN})\det(\mathbb{Q}^i_j)} \right)$$

$$S^{\mathrm{WZ}}_{\mathrm{D6}} = \mu_{\mathrm{D6}} \int_{\mathrm{WV(D6)}} \mathrm{Tr}\left\{ \mathrm{P}\left[e^{i\lambda\iota_Y\iota_Y} \left(\hat{\boldsymbol{C}} \wedge e^{\hat{B}_{(2)}} \right) \wedge e^{\lambda\mathcal{F}} \right] \right\}$$

$$\mathbb{M}_{MN} = \mathrm{P}\Big[\hat{E}_{MN} + \hat{E}_{Mi}(\mathbb{Q}^{-1}-\delta)^{ij}\hat{E}_{jN}\Big] + \lambda\mathcal{F}_{MN} \ , \qquad \mathbb{Q}^i_j = \delta^i_j + i\lambda[Y^i,Y^k]\hat{E}_{kj} \ ,$$

$$\hat{E}_{\mathcal{M}\mathcal{N}} = \hat{g}_{\mathcal{M}\mathcal{N}} + \hat{B}_{\mathcal{M}\mathcal{N}} \ , \qquad x^{\mathcal{M}} = (x^M, y^i) \ ,$$

$$\lambda = 2\pi\alpha' \ , \quad y^i = \lambda Y^i = \lambda Y^{iI} t_I \ , \quad \iota_Y\iota_Y\Big(\frac{1}{2}C_{ij}\mathrm{d}y^i\wedge\mathrm{d}y^j\Big) = -\frac{1}{2}C_{ij}[Y^i,Y^j] \ ,$$

$$\mathrm{P}[\hat{E}_{MN}] = \hat{E}_{MN} + \lambda D_M Y^i \hat{E}_{iN} + \lambda D_N Y^i \hat{E}_{Mi} + \lambda^2 D_M Y^i D_N Y^j \hat{E}_{ij} \ ,$$

$$D_M Y^i \equiv \partial_M Y^i - i[\mathcal{A}_M, Y^i] \ , \qquad \mathcal{F} = \mathrm{d}\mathcal{A} + i\mathcal{A}\wedge\mathcal{A} \ ,$$

$$[t_I, t_J] = -i g_{IJ}{}^K t_K \ , \quad \mathrm{Tr}[t_I] = 0 \ , \quad \mathrm{Tr}[t_I t_J] = \delta_{IJ} \ .$$

The presence of a stack of coincident D6-branes is taken into account by two contributions: Wess-Zumino (WZ) and Dirac-Born-Infeld (DBI) actions.

[Myers '99]

$$S_{\rm D6}^{\rm DBI} = -T_{\rm D6} \int_{\rm WV(D6)} {\rm d}^7x \; {\rm Tr}\left(e^{-\hat{\Phi}} \sqrt{-\det(\mathbb{M}_{MN}) \det(\mathbb{Q}^i_{\ j})} \right)$$

$$S_{\rm D6}^{\rm WZ} = \mu_{\rm D6} \int_{\rm WV(D6)} {\rm Tr}\left\{ {\rm P}\left[e^{i\lambda \iota_Y \iota_Y} \left(\hat{\boldsymbol{C}} \wedge e^{\hat{B}_{(2)}} \right) \wedge e^{\lambda \mathcal{F}} \right] \right\}$$

$$\mathbb{M}_{MN} = {\rm P}\Big[\hat{E}_{MN} + \hat{E}_{Mi} (\mathbb{Q}^{-1} - \delta)^{ij} \hat{E}_{jN} \Big] + \lambda \mathcal{F}_{MN} \;, \qquad \mathbb{Q}^i_{\ j} = \delta^i_j + i\lambda [Y^i, Y^k] \hat{E}_{kj} \;,$$

$$\hat{E}_{\mathcal{M}\mathcal{N}} = \hat{g}_{\mathcal{M}\mathcal{N}} + \hat{B}_{\mathcal{M}\mathcal{N}} \;, \qquad x^{\mathcal{M}} = (x^M, y^i) \;,$$

$$\lambda = 2\pi\alpha' \;, \quad y^i = \lambda Y^i = \lambda Y^{iI} t_I \;, \quad \iota_Y \iota_Y \left(\frac{1}{2} C_{ij} {\rm d}y^i \wedge {\rm d}y^j \right) = -\frac{1}{2} C_{ij} [Y^i, Y^j] \;,$$

$$ {\rm P}[\hat{E}_{MN}] = \hat{E}_{MN} + \lambda D_M Y^i \hat{E}_{iN} + \lambda D_N Y^i \hat{E}_{Mi} + \lambda^2 D_M Y^i D_N Y^j \hat{E}_{ij} \;,$$

$$D_M Y^i \equiv \partial_M Y^i - i[\mathcal{A}_M, Y^i] \;, \qquad \mathcal{F} = {\rm d}\mathcal{A} + i \mathcal{A} \wedge \mathcal{A} \;,$$

$$[t_I, t_J] = -i g_{IJ}{}^K t_K \;, \quad {\rm Tr}[t_I] = 0 \;, \quad {\rm Tr}[t_I t_J] = \delta_{IJ} \;.$$

The potential from dimensional reduction

The presence of a stack of coincident D6-branes is taken into account by two contributions: Wess-Zumino (WZ) and Dirac-Born-Infeld (DBI) actions.

[Myers '99]

$$S^{\rm DBI}_{\rm D6} = -T_{\rm D6}\int_{\rm WV(D6)} {\rm d}^7x\ {\rm Tr}\left(e^{-\hat\Phi}\sqrt{-\det(\mathbb{M}_{MN})\det(\mathbb{Q}^i_j)}\right)$$

$$S^{\rm WZ}_{\rm D6} = \mu_{\rm D6}\int_{\rm WV(D6)} {\rm Tr}\left\{{\rm P}\left[e^{i\lambda\iota_Y\iota_Y}\left(\hat{\boldsymbol{C}}\wedge e^{\hat B_{(2)}}\right)\wedge e^{\lambda\mathcal{F}}\right]\right\}$$

$$\mathbb{M}_{MN} = {\rm P}\left[\hat E_{MN} + \hat E_{Mi}(\mathbb{Q}^{-1}-\delta)^{ij}\hat E_{jN}\right] + \lambda\mathcal{F}_{MN}\ , \qquad \mathbb{Q}^i_j = \delta^i_j + i\lambda[Y^i,Y^k]\hat E_{kj}\ ,$$

$$\hat E_{\mathcal{M}\mathcal{N}} = \hat g_{\mathcal{M}\mathcal{N}} + \hat B_{\mathcal{M}\mathcal{N}}\ , \qquad x^{\mathcal{M}} = (x^M, y^i)\ ,$$

$$\lambda = 2\pi\alpha'\ , \quad y^i = \lambda Y^i = \lambda Y^{iI}t_I\ , \quad \iota_Y\iota_Y\left(\frac{1}{2}C_{ij}{\rm d}y^i\wedge{\rm d}y^j\right) = -\frac{1}{2}C_{ij}[Y^i,Y^j]\ ,$$

$${\rm P}[\hat E_{MN}] = \hat E_{MN} + \lambda D_MY^i\hat E_{iN} + \lambda D_NY^i\hat E_{Mi} + \lambda^2 D_MY^iD_NY^j\hat E_{ij}\ ,$$

$$D_MY^i \equiv \partial_MY^i - i[\mathcal{A}_M, Y^i]\ , \qquad \mathcal{F} = {\rm d}\mathcal{A} + i\mathcal{A}\wedge\mathcal{A}\ ,$$

$$[t_I, t_J] = -ig_{IJ}{}^Kt_K\ , \quad {\rm Tr}[t_I] = 0\ , \quad {\rm Tr}[t_It_J] = \delta_{IJ}\ .$$

The potential from dimensional reduction

The presence of a stack of coincident D6-branes is taken into account by two contributions: Wess-Zumino (WZ) and Dirac-Born-Infeld (DBI) actions.

[Myers '99]

$$S_{\rm D6}^{\rm DBI} = -T_{\rm D6} \int_{\rm WV(D6)} {\rm d}^7x \ {\rm Tr} \left(e^{-\hat{\Phi}} \sqrt{-\det(\mathbb{M}_{MN}) \det(\mathbb{Q}^i_j)} \right)$$

$$S_{\rm D6}^{\rm WZ} = \mu_{\rm D6} \int_{\rm WV(D6)} {\rm Tr} \left\{ {\rm P} \left[e^{i\lambda \iota_Y \iota_Y} \left(\hat{\boldsymbol{C}} \wedge e^{\hat{B}_{(2)}} \right) \wedge e^{\lambda \mathcal{F}} \right] \right\}$$

$$\mathbb{M}_{MN} = {\rm P}\left[\hat{E}_{MN} + \hat{E}_{Mi} (\mathbb{Q}^{-1} - \delta)^{ij} \hat{E}_{jN} \right] + \lambda \mathcal{F}_{MN} \ , \qquad \mathbb{Q}^i_j = \delta^i_j + i\lambda [Y^i, Y^k] \hat{E}_{kj} \ ,$$

$$\hat{E}_{\mathcal{M}\mathcal{N}} = \hat{g}_{\mathcal{M}\mathcal{N}} + \hat{B}_{\mathcal{M}\mathcal{N}} \ , \qquad x^{\mathcal{M}} = (x^M, y^i) \ ,$$

$$\lambda = 2\pi\alpha' \ , \quad y^i = \lambda Y^i = \lambda Y^{iI} t_I \ , \quad \iota_Y \iota_Y \left(\frac{1}{2} C_{ij} {\rm d}y^i \wedge {\rm d}y^j \right) = -\frac{1}{2} C_{ij} [Y^i, Y^j] \ ,$$

$$ {\rm P}[\hat{E}_{MN}] = \hat{E}_{MN} + \lambda D_M Y^i \hat{E}_{iN} + \lambda D_N Y^i \hat{E}_{Mi} + \lambda^2 D_M Y^i D_N Y^j \hat{E}_{ij} \ ,$$

$$D_M Y^i \equiv \partial_M Y^i - i[\mathcal{A}_M, Y^i] \ , \qquad \mathcal{F} = {\rm d}\mathcal{A} + i\mathcal{A} \wedge \mathcal{A} \ ,$$

$$[t_I, t_J] = -i g_{IJ}{}^K t_K \ , \quad {\rm Tr}[t_I] = 0 \ , \quad {\rm Tr}[t_I t_J] = \delta_{IJ} \ .$$

The modified Bianchi identities

The R-R field strengths in presence of fluxes read

$$\boxed{\boldsymbol{F} = \mathrm{d}\boldsymbol{C} + H_{(3)} \wedge \boldsymbol{C} + \boldsymbol{F}^{\text{flux}} \wedge e^{-B_{(2)}}}$$

[Dall'Agata, Villadoro, Zwirner '09]

These - or equivalently their Bianchi identities - undergo further modifications due to the WZ action. For instance,

$$\int_{10} C_{(7)} \wedge J_{(3)}^{(\mathrm{D6})} \to \mathrm{d}\bar{F}_{(2)} = J_{(3)}^{(\mathrm{D6})} \; .$$

These modifications need to be taken into account in the contributions to the potential, coming from the bulk and the DBI action.

The SO(3) truncation reduces the scalar manifold to be

$$\boxed{\mathcal{M}_{\mathrm{SO}(3)} = \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SO}\left(2, 2 + \frac{\mathfrak{N}}{3}\right)}{\mathrm{SO}(2) \times \mathrm{SO}\left(2 + \frac{\mathfrak{N}}{3}\right)}}$$

which corresponds to a content of $(6 + 2\frac{\mathfrak{N}}{3})$ scalars.

$\mathcal{N} = 1$ description

The truncated theory also admits a $\mathcal{N} = 1$ description. Indeed, under the branching SO(3) $\hookrightarrow$ SU(4), the fundamental representation of the R-symmetry SU(4) splits as

$$\mathbf{4} \rightarrow \mathbf{1} \oplus \mathbf{3} \ .$$

The closed-string sector scalars can have a place in an STU model:

$$S = \ \chi + ie^{-\phi} \ , \quad T = \ \chi_1 + ie^{-\varphi_1} \ , \quad U = \ \chi_2 + ie^{-\varphi_2} \ .$$

The scalar potential, up to the quadratic constraints, reads

$$\boxed{V = e^K \left(\sum_{\Phi} K^{\Phi\bar{\Phi}} |D_\Phi W|^2 - 3|W|^2 \right)}$$

$$\Phi \equiv (S, T, U) \ , \quad K(\Phi, \bar{\Phi}) = -\log(-i(S - \bar{S})) - 3\log(-i(T - \bar{T})) - 3\log(-i(U - \bar{U})) \ .$$

The holomorphic superpotential is given by

$$W(\Phi) = P_F - P_H \, S + 3 P_Q \, T \ ,$$

$$P_F = a_0 - 3\, a_1 \, U + 3\, a_2 \, U^2 - a_3 \, U^3 \ , \quad P_H = b_0 - 3\, b_1 \, U \ , \quad P_Q = c_0 + (2c_1 - \tilde{c}_1)\, U \ .$$

$\mathcal{N} = 1$ description

The truncated theory also admits a $\mathcal{N} = 1$ description. Indeed, under the branching $\mathrm{SO}(3) \hookrightarrow \mathrm{SU}(4)$, the fundamental representation of the R-symmetry SU(4) splits as

$$\mathbf{4} \to \mathbf{1} \oplus \mathbf{3} \ .$$

The closed-string sector scalars can have a place in an STU model:

$$S = \ \chi + i e^{-\phi} \ , \quad T = \ \chi_1 + i e^{-\varphi_1} \ , \quad U = \ \chi_2 + i e^{-\varphi_2} \ .$$

The scalar potential, up to the quadratic constraints, reads

$$\boxed{V = e^{K} \left(\sum_{\Phi} K^{\Phi \bar{\Phi}} |D_\Phi W|^2 - 3|W|^2 \right)}$$

$$\Phi \equiv (S, T, U) \ , \quad K(\Phi, \bar{\Phi}) = -\log(-i(S - \bar{S})) - 3\log(-i(T - \bar{T})) - 3\log(-i(U - \bar{U})) \ .$$

The holomorphic superpotential is given by

$$W(\Phi) = P_F - P_H\, S + 3 P_Q\, T \ ,$$

$$P_F = a_0 - 3\, a_1\, U + 3\, a_2\, U^2 - a_3\, U^3 \ , \quad P_H = b_0 - 3\, b_1\, U \ , \quad P_Q = c_0 + (2 c_1 - \bar{c}_1)\, U \ .$$

$\mathcal{N} = 1$ description

The truncated theory also admits a $\mathcal{N} = 1$ description. Indeed, under the branching $\mathrm{SO}(3) \hookrightarrow \mathrm{SU}(4)$, the fundamental representation of the R-symmetry SU(4) splits as

$$\mathbf{4} \to \mathbf{1} \oplus \mathbf{3} \; .$$

The closed-string sector scalars can have a place in an STU model:

$$S = \; \chi + ie^{-\phi} \; , \quad T = \; \chi_1 + ie^{-\varphi_1} \; , \quad U = \; \chi_2 + ie^{-\varphi_2} \; .$$

The scalar potential, up to the quadratic constraints, reads

$$\boxed{V = e^K \left(\sum_{\Phi} K^{\Phi\bar{\Phi}} |D_\Phi W|^2 - 3|W|^2 \right)}$$

$$\Phi \equiv (S, T, U) \; , \quad K(\Phi, \bar{\Phi}) = -\log(-i(S - \bar{S})) - 3\log(-i(T - \bar{T})) - 3\log(-i(U - \bar{U})) \; .$$

The holomorphic superpotential is given by

$$W(\Phi) = P_F - P_H \, S + 3P_Q \, T \; ,$$

$$P_F = a_0 - 3\, a_1\, U + 3\, a_2\, U^2 - a_3\, U^3 \; , \quad P_H = b_0 - 3\, b_1\, U \; , \quad P_Q = c_0 + (2c_1 - \tilde{c}_1)\, U \; .$$