Spinning up the BLACK HOLE - *string* correspondence

2307.03573 & 2411.18690 WITH

NEJC ČEPLAK, ROBERTO EMPARAN, MARIJA TOMAŠEVIĆ

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UNIVERSITY OF AMSTERDAM

STRING THEORY AS A BRIDGE @ ROMA, 17 FEBRUARY 2025

European Research Council

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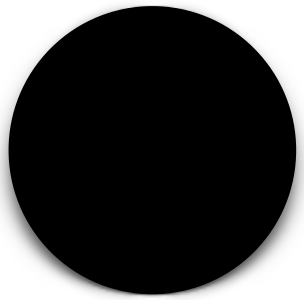
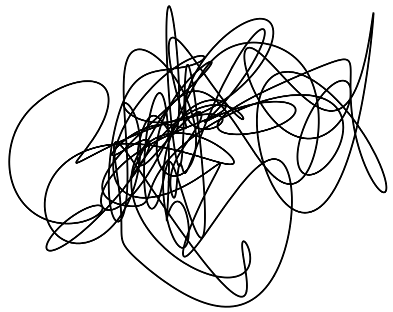


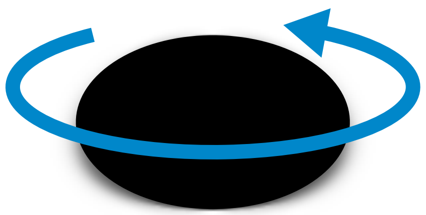
Black Hole - string correspondence

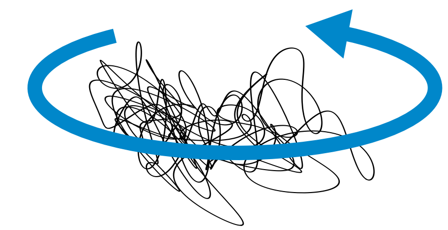


General framework for matching
black holes and massive states of fundamental strings
at a point where their physical properties smoothly agree.

(mass, entropy, temperature, decay rates,…)

Spinning up the BLACK HOLE - *string* correspondence





Plan:

I. Introduction

II. BH-string correspondence (static)

III. BH-string correspondence with rotation

IV. Testing the correspondence with rotation: shapes & sizes

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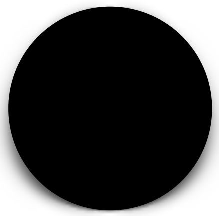
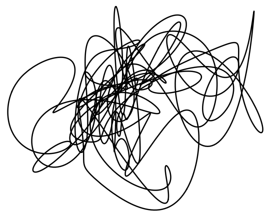
I. Introduction

Why black hole = string?



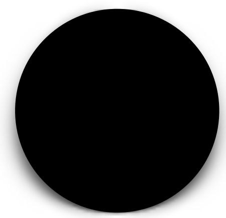
$$S = \frac{A}{4G}$$



Microscopic (statistical) picture for **Schwarzschild** or **Kerr** black holes from strings?

in asymptotically **flat** space in $D \geq 4$

Why black hole = string?



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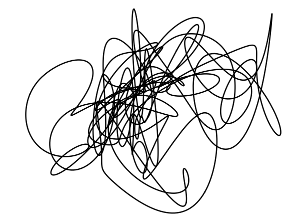

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$\longrightarrow$ **NO: susy, AdS or low dim toy model!**

So cannot expect exact match → henceforth ignore O(1) factors.

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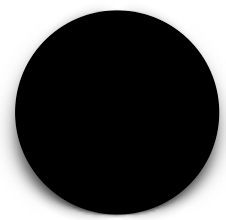

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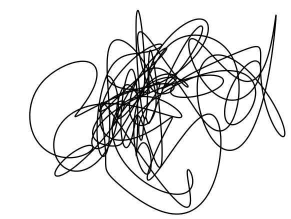

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in asymptotically **flat** space in $D \geq 4$

$\longrightarrow$ **NO: susy, AdS or low dim toy model!**

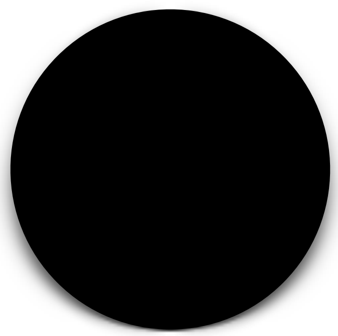
So cannot expect exact match → henceforth ignore O(1) factors.

$\longrightarrow$ **``Broad brush'' picture**

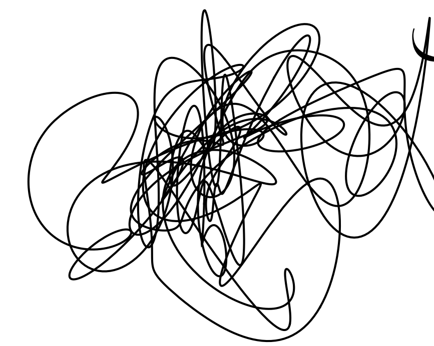
Identify the relevant physics and ignore unnecessary details.

Are black holes = strings ?

BLACK HOLES



fundamental strings



massive

highly degenerate

We are in D=4 for now.

Are black holes = strings ?

BLACK HOLES

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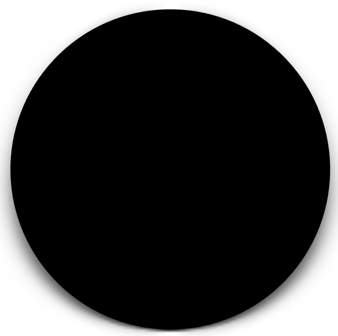

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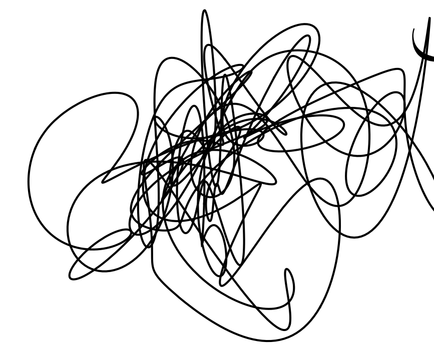

Kerr bound

$$J \leq J_{Kerr} = M^2$$

✓

Regge bound

$$J \leq J_{Regge} = M^2$$

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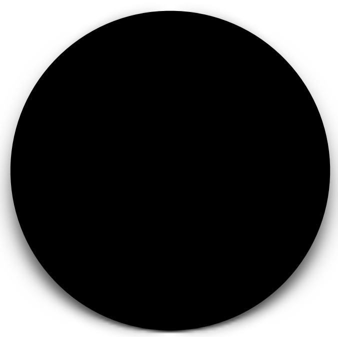

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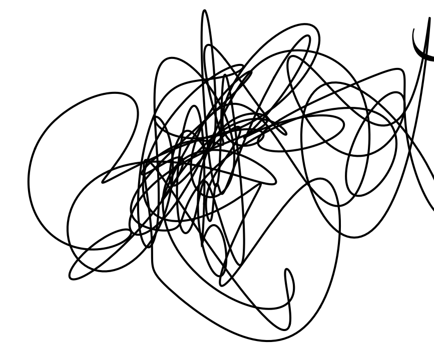

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entropy (Bekenstein)

$$S \sim M^2$$

✗

degeneracy (Hagedorn)

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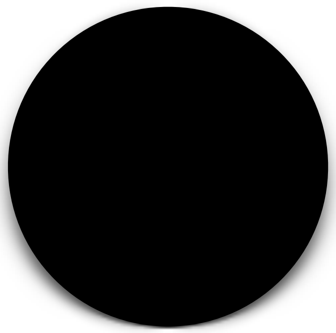

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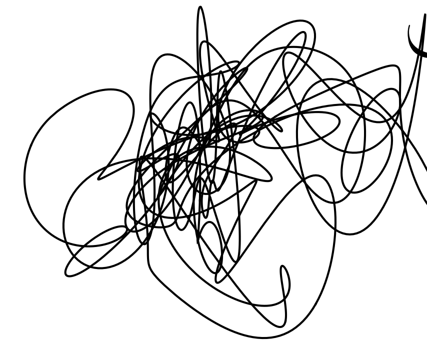

massive

highly degenerate ✓

Kerr bound

$$J \leq J_{Kerr} = M^2/M_P^2$$

Regge bound

$$J \leq J_{Regge} = M^2/M_s^2$$

Mind the units!

entropy (Bekenstein)

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Are black holes = strings ?

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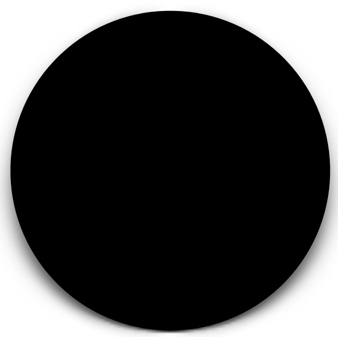

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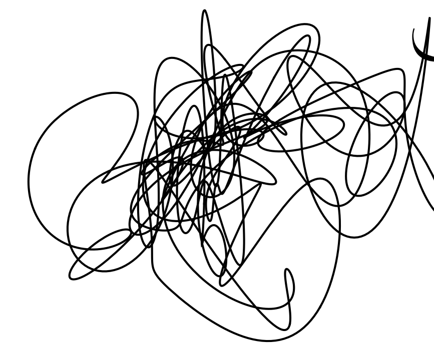

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$M_P^2 = G^{-1}$

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$M_s^2 = l_s^{-2} = \alpha'^{-1}$

$M_s = gM_P$

entropy (Bekenstein)

$$S \sim M^2/M_P^2 \quad = \frac{1}{g^2}\left(g^2 M/M_s\right)^2 \quad =$$

degeneracy (Hagedorn)

$$S \sim M/M_s$$

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``correspondence point''

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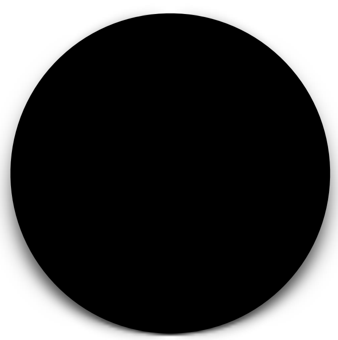

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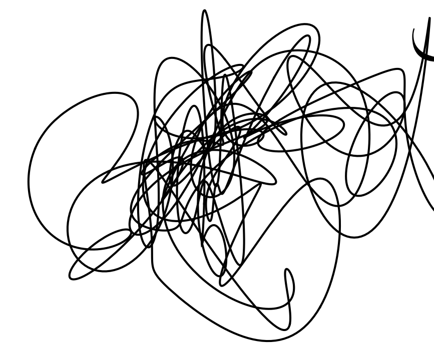

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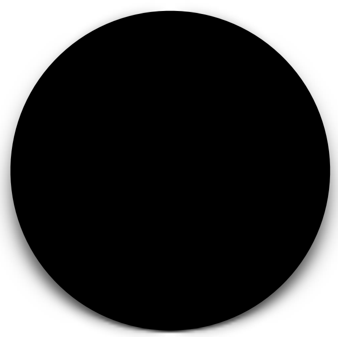

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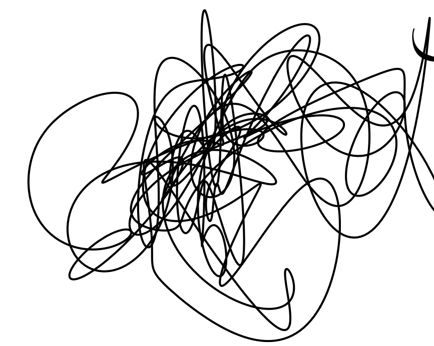

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We are in D=4 for now.

Puzzles with rotation

" $J \leq M^2$ " conflates two different bounds:

$$J_{Kerr} = g^2 \left(\frac{M}{M_s} \right)^2 \ll \left(\frac{M}{M_s} \right)^2 = J_{Regge}$$

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I. No black hole counterpart for strings with $J_{Kerr} < J < J_{Regge}$!

↑ roundish black holes with large degeneracy (J_{Kerr})

↑ thin, long, rigidly rotating rods with small degeneracy (J_{Regge})

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Does the black hole - string correspondence fail?

Resolving the puzzles

Both **puzzles hide an assumption**:

one-to-one matching of *stationary* solutions.

At finite coupling: all objects time-evolve !

susy: no worry
static: no worry
rotating: crucial!

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one-to-one matching of *stationary* solutions.

At finite coupling: all objects time-evolve !

susy: no worry
static: no worry
rotating: crucial!

Instability timescale $\lesseqgtr$ transition timescale.

The puzzles are resolved if we

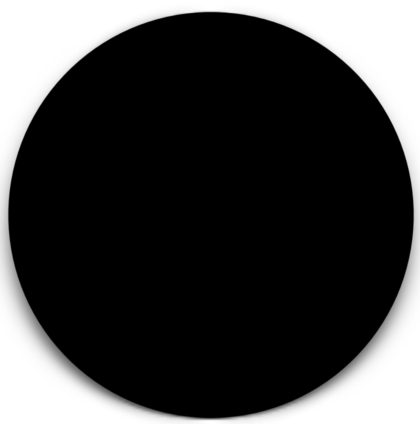
account for dynamics !

II. Black hole - string correspondence

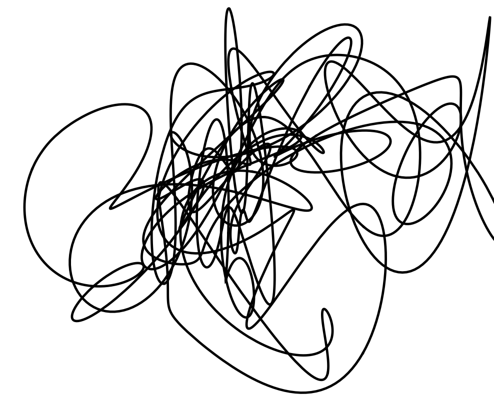
Correspondence principle

[Susskind'93], [Horowitz,Polchinski'96], [Damour,Veneziano'98]

BLACK HOLES



fundamental strings



curvature $\sim \dfrac{1}{g^2 S} \quad \dfrac{1}{\ell_s^2}$

$\sim \dfrac{1}{r_H^2} \sim \dfrac{1}{(GM)^2}$

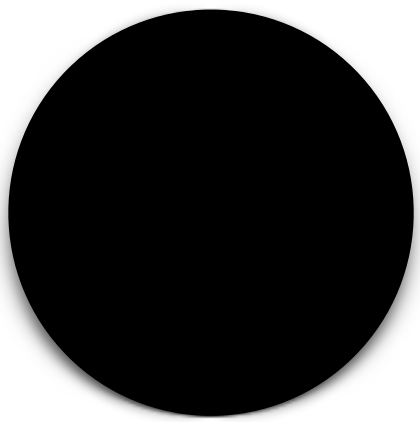
We are in D=4 for now, will generalize to arbitrary D momentarily.

Correspondence principle

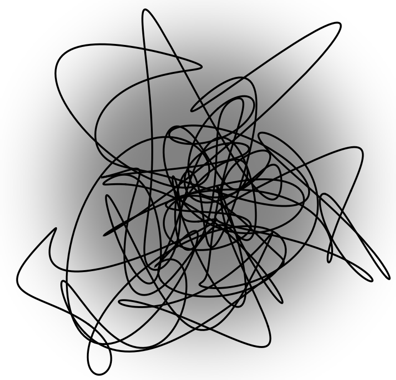
[Susskind'93], [Horowitz,Polchinski'96], [Damour,Veneziano'98]

BLACK HOLES

fundamental strings



$g \searrow$ $\longrightarrow$



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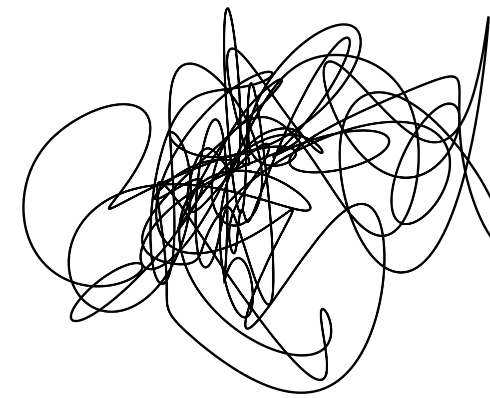

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$\uparrow$

$g^2 = \dfrac{1}{S} \ll 1$

Stringy corrections to geometry important!

We are in D=4 for now, will generalize to arbitrary D momentarily.

Correspondence principle

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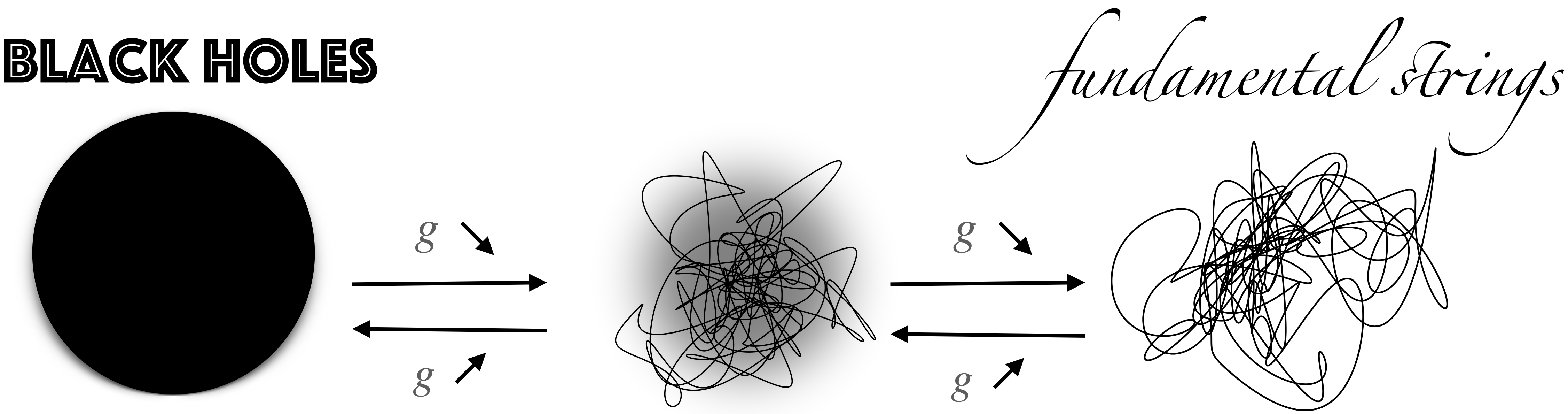

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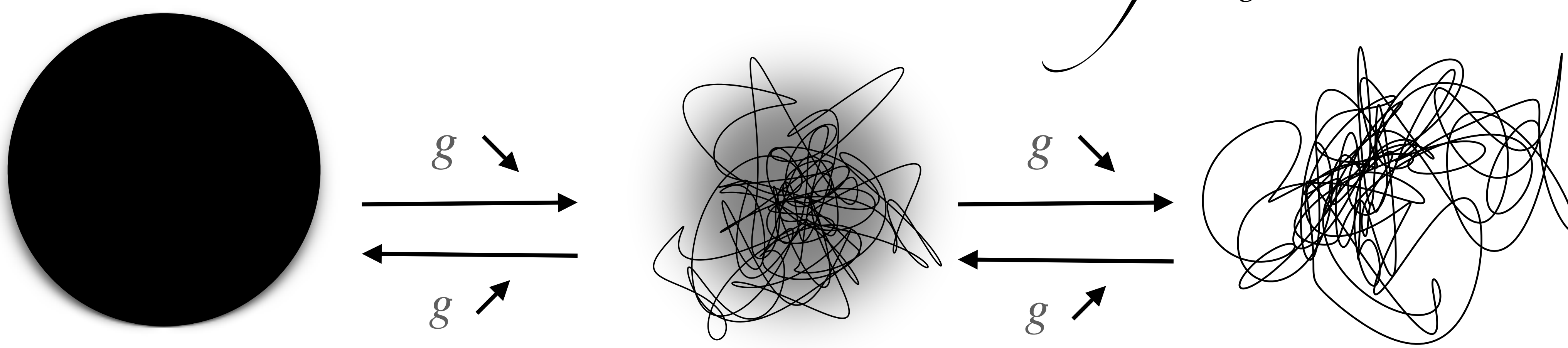
BLACK HOLES *fundamental strings*



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↑

$g^2 = \dfrac{1}{S} \ll 1$

↓

$=$

Stringy corrections to geometry important!

$$S \sim \left(\frac{M}{M_P}\right)^2 \qquad\qquad S \sim \frac{M}{M_s}$$

Statistical interpretation of Bekenstein-Hawking entropy via degeneracy of strings!

We are in D=4 for now, will generalize to arbitrary D momentarily.

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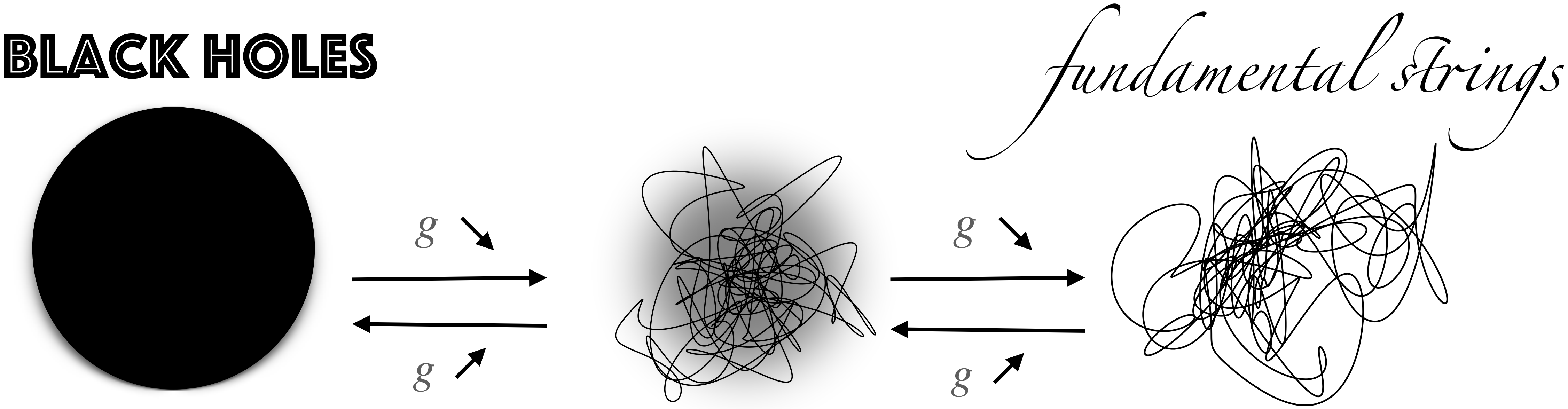

curvature $\sim \frac{1}{(g^2 S)^{\frac{2}{D-2}}} \frac{1}{\ell_s^2}$

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$\uparrow$

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$=$

Stringy corrections to geometry important!

$$S \sim \left(\frac{M}{M_P}\right)^{\frac{D-2}{D-3}}$$

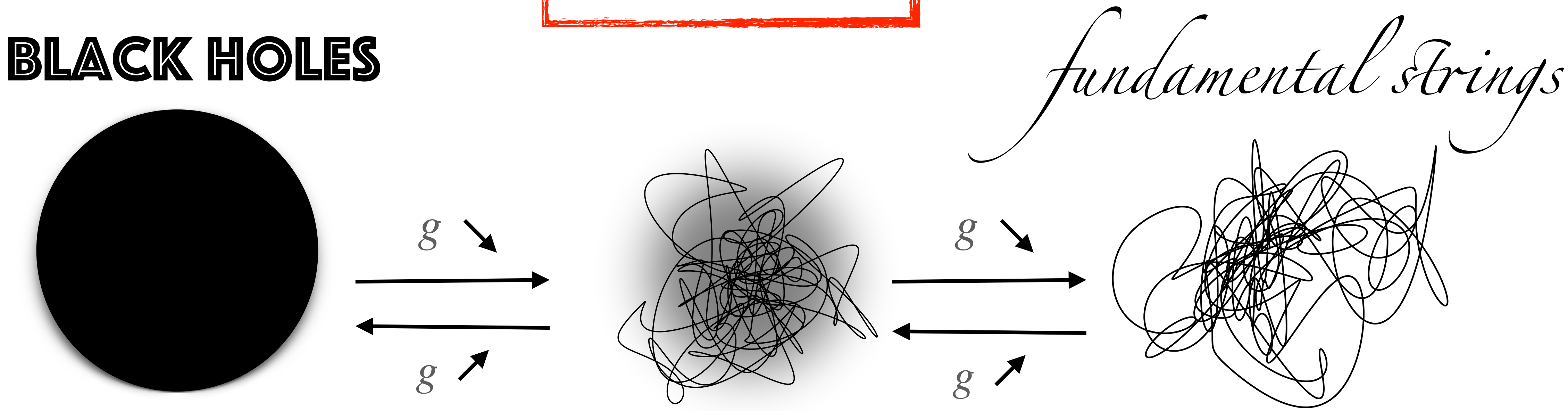
$$S \sim \frac{M}{M_s}$$

Statistical interpretation of Bekenstein-Hawking entropy via degeneracy of strings!

Now we are in D dimensions.

A `` 't Hooft coupling ''

fix S & change g



$g^2 S$: `` 't Hooft coupling ''

$g^2 S \gg 1$ $\quad$ $g^2 S \sim 1$ $\quad$ $g^2 S \ll 1$

A `` 't Hooft coupling ''

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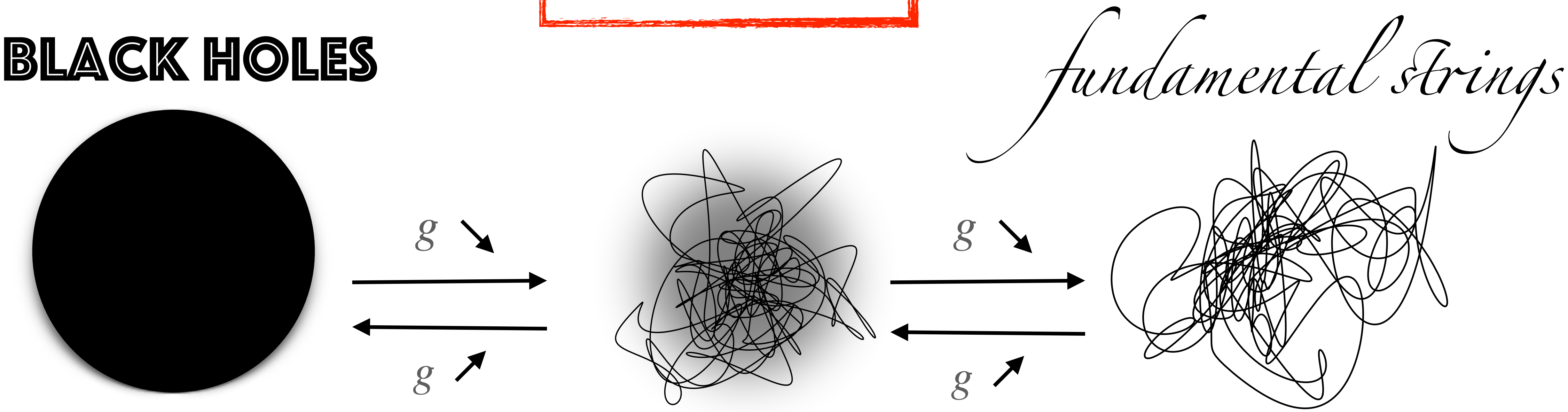


g^2S : `` 't Hooft coupling ''

$g^2S \gg 1$ $\quad$ $g^2S \sim 1$ $\quad$ $g^2S \ll 1$

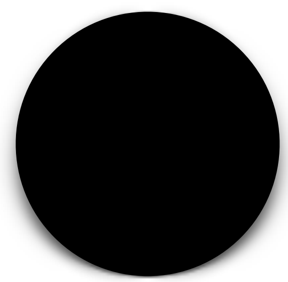
So g^2S plays the same role as $\lambda = g_{YM}^2 N$ in AdS/CFT or gN in D-brane systems.

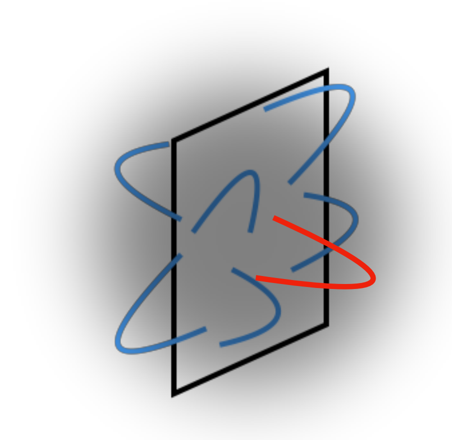
Near vs far from BPS correspondences

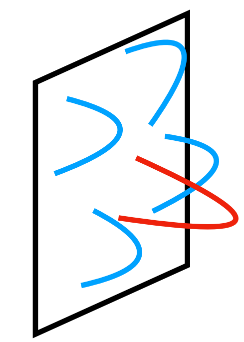
[Strominger,Vafa'96]:

Near

BPS







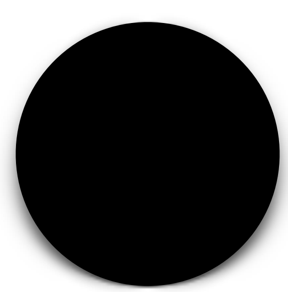
fix N & change g

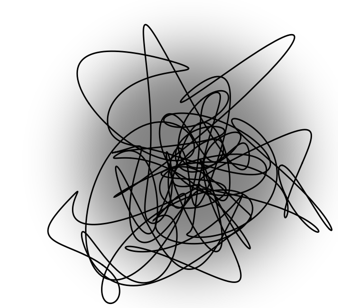
$gN \gg 1$ | $gN \sim 1$ | $gN \ll 1$

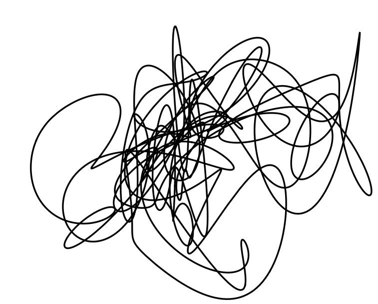
[Horowitz,Polchinski'96]:

Far from

BPS







fix S & change g

$g^2S \gg 1$ | $g^2S \sim 1$ | $g^2S \ll 1$

Black hole - string correspondence

= overarching framework for microscopic understanding of black holes in string theory,

e.g. BH entropy $S = \dfrac{A}{4G}$

General idea:

Interpolate by changing the coupling ≈ adiabatically while holding S, J, Q fixed.

Mass renormalization in general hard to control (prevents precise matching).

Details of the correspondence

Properties of black hole and fundamental string
have to match at correspondence point:

- Mass
- Size
- Decay rates
- …

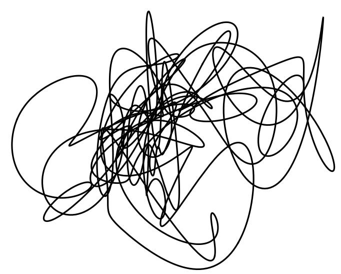
`Correspondence' only if ∃ ≈ adiabaticity.

Also need a `physical realization’ or a `knob’.

Mass

[Susskind'93]

Fixing S as we vary g → mass of black hole and string get renormalized:



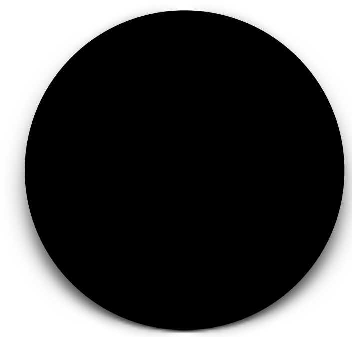
$$M = SM_s - O(g^2)$$

$$=$$

↑

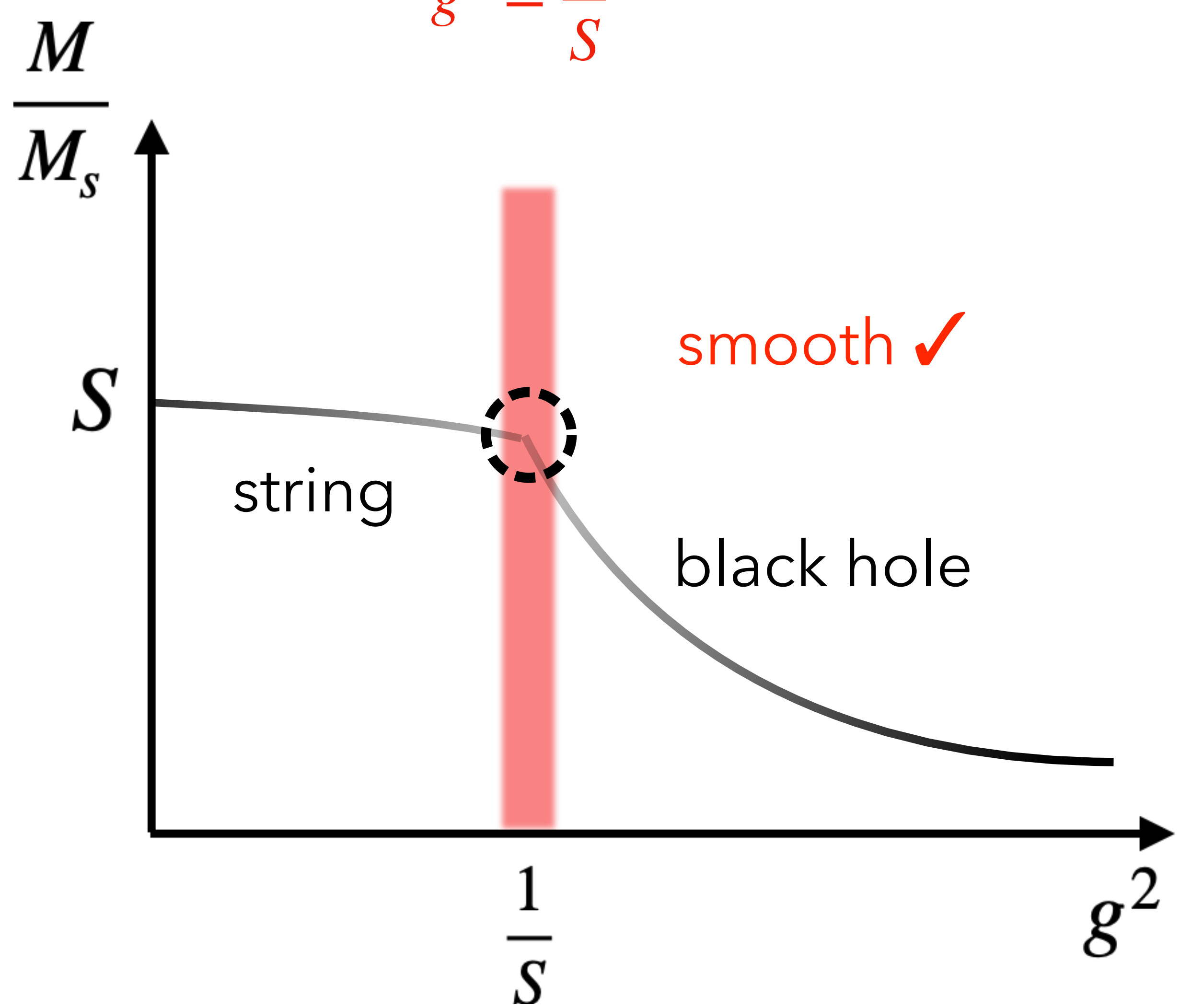
$$g^2 = \frac{1}{S}$$

$$M = \frac{M_s}{g^2}(g^2 S)^{\frac{D-3}{D-2}}$$



adiabats →

curves along which entropy constant



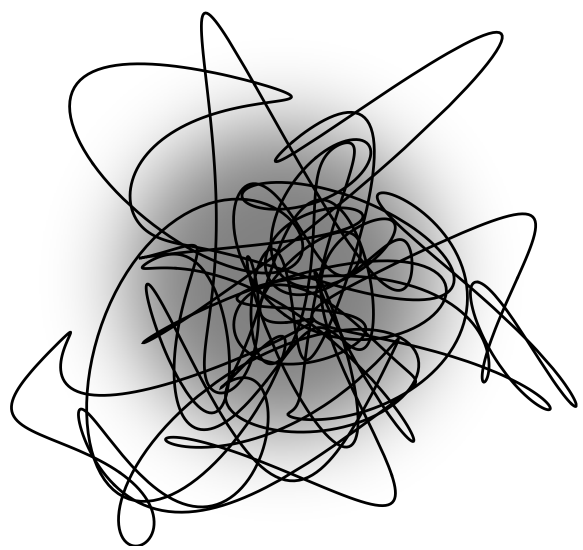
Size

Discrepancy: string much larger than black hole @ correspondence point.

$$\sqrt{L\ell_s} = \sqrt{S}\ell_s \gg \ell_s$$

random-walk string

$$r_H = \ell_s$$

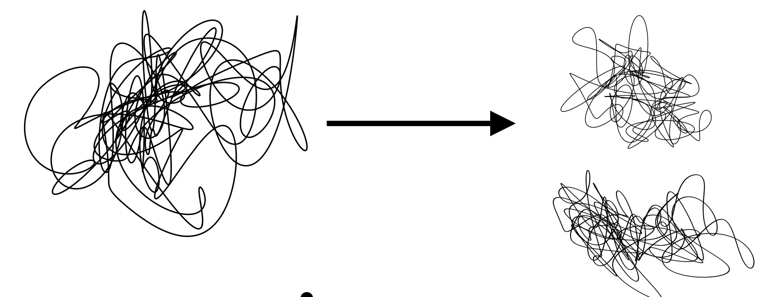


We have neglected the effect of self-gravitation of the string!

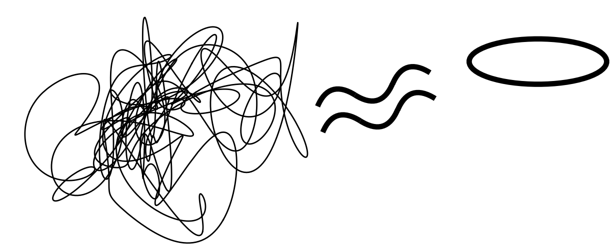
This will shrink the string. [Horowitz,Polchinski'97]

Decay channels

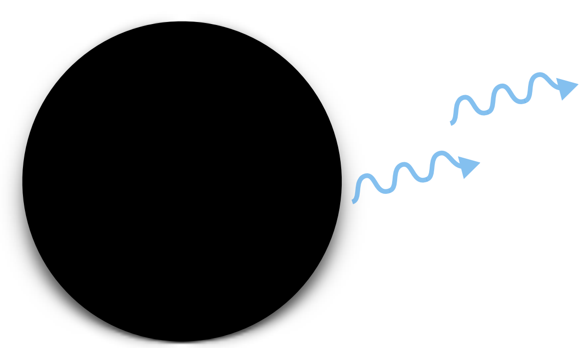
When $0 < g < \infty$: neither strings nor black holes stationary anymore!



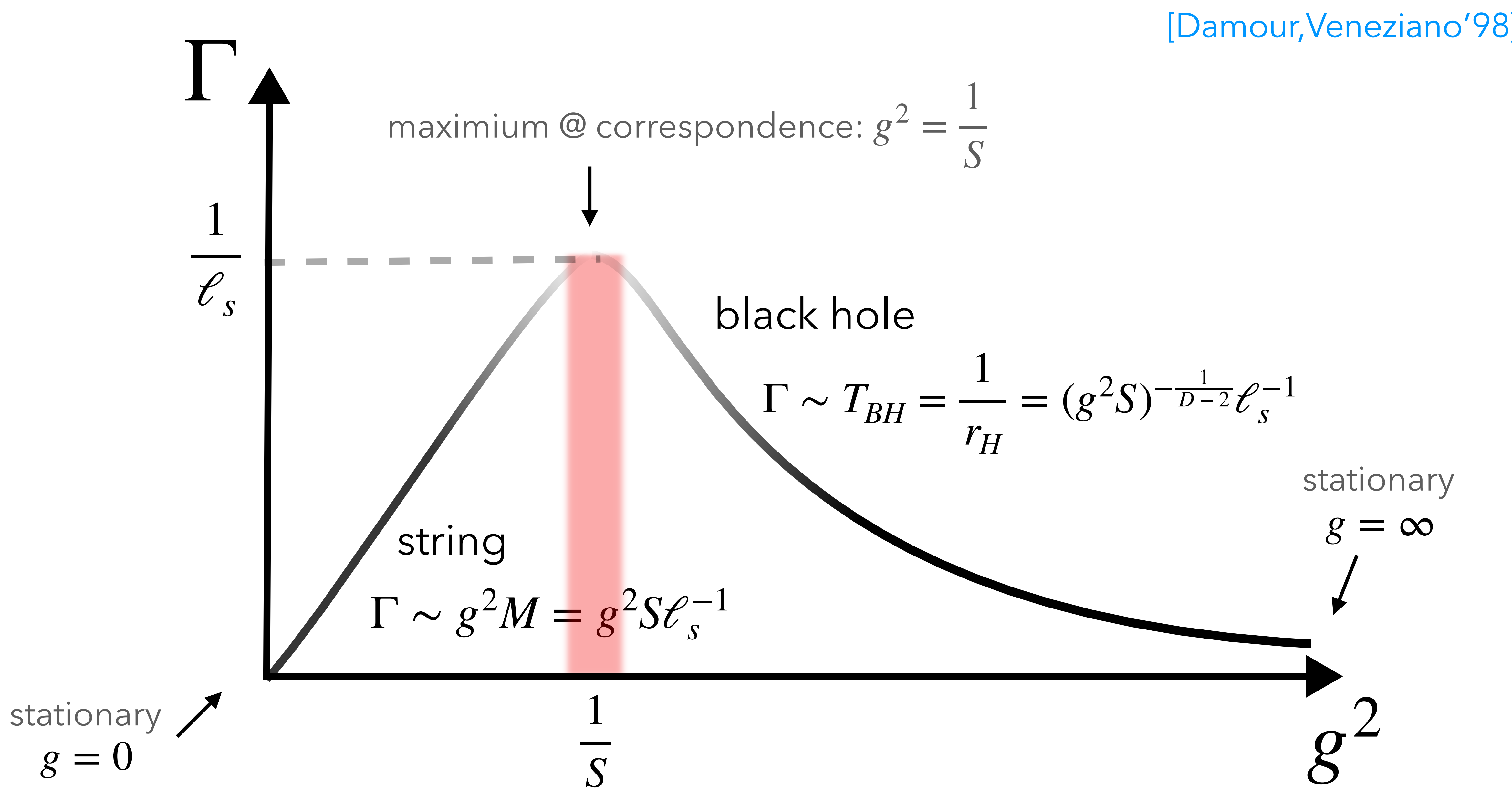
Self-interaction of massive string → decay <
- **fragmentation**
- **emission of light strings**



Quantum effects in black holes → decay via **emission of Hawking radiation**



Decay rates


[Damour,Veneziano'98]
Γ
maximium @ correspondence: $g^2 = \frac{1}{S}$
$\frac{1}{\ell_s}$
black hole
$\Gamma \sim T_{BH} = \frac{1}{r_H} = (g^2 S)^{-\frac{1}{D-2}} \ell_s^{-1}$
stationary
$g = \infty$
string
$\Gamma \sim g^2 M = g^2 S \ell_s^{-1}$
stationary
$g = 0$
$\frac{1}{S}$
g^2

Adiabaticity ?

$g = e^{\phi}$

rate of change $\Delta t_g^{-1} = \frac{\dot{g}}{g} = \dot{\phi}$ of system

fast enough to stay @ const entropy

any finite g: radiation!

slow enough to not excite the state

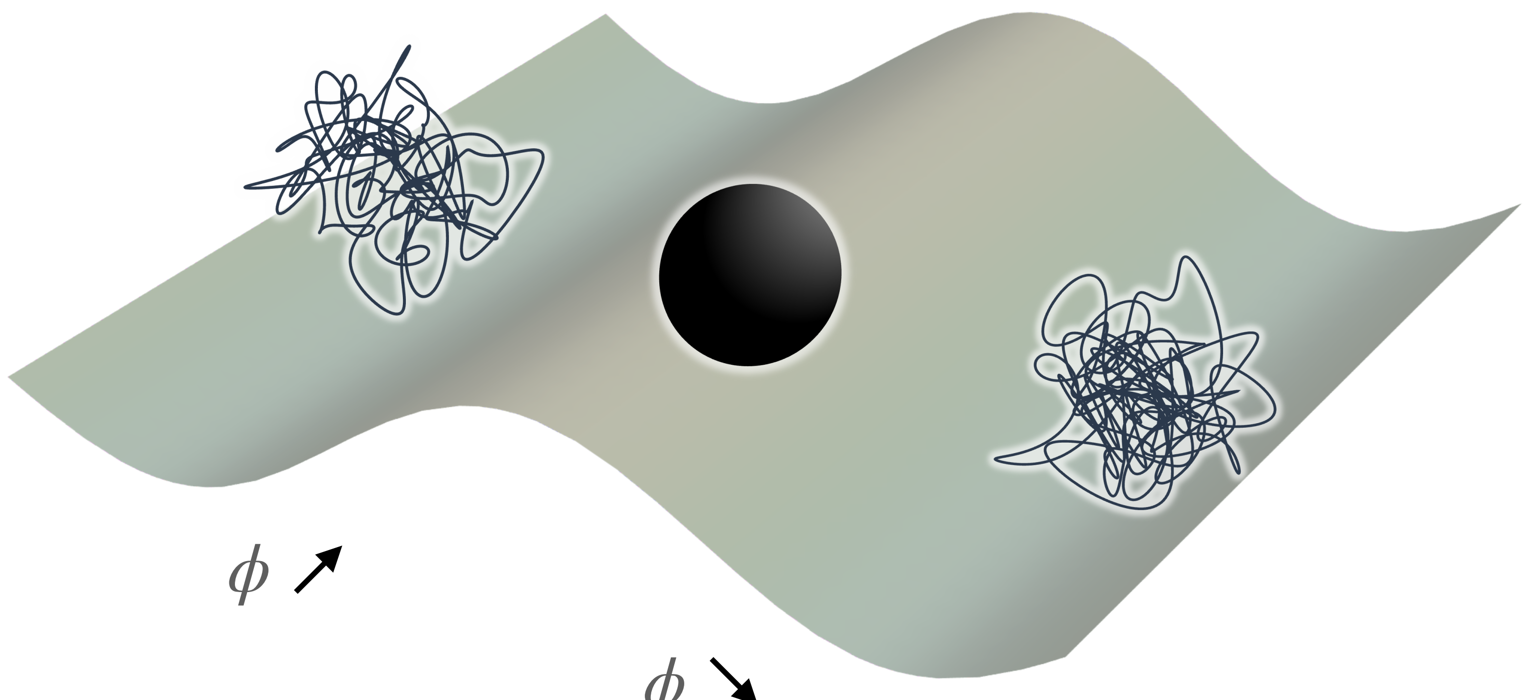
dilaton: long wavelength!

Goldilocks range for the rate of change of g – not too slow and not too fast :

$$\frac{1}{S\ell_s} < \Delta t_g^{-1} < \frac{1}{\ell_s}$$

Correspondence in the `dilaton lab'

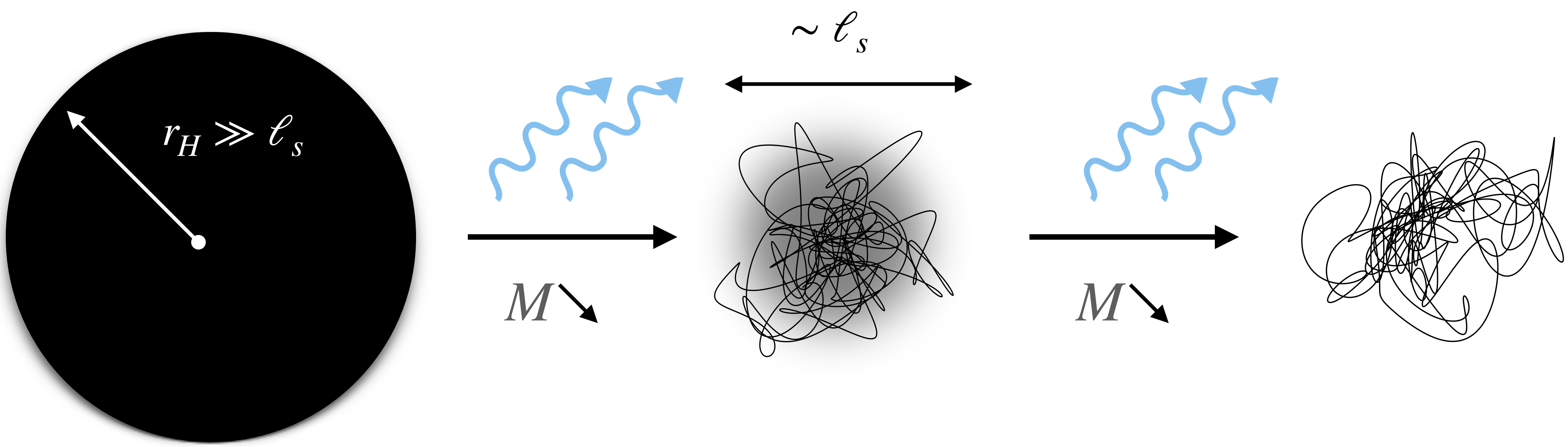
Tuning coupling $g = e^{\phi}$ while entropy S is fixed,
yields within Goldilocks adiabaticity range
transitions between black hole ↔ string ball phases:



[Susskind'93], [Horowitz,Polchinski'96]

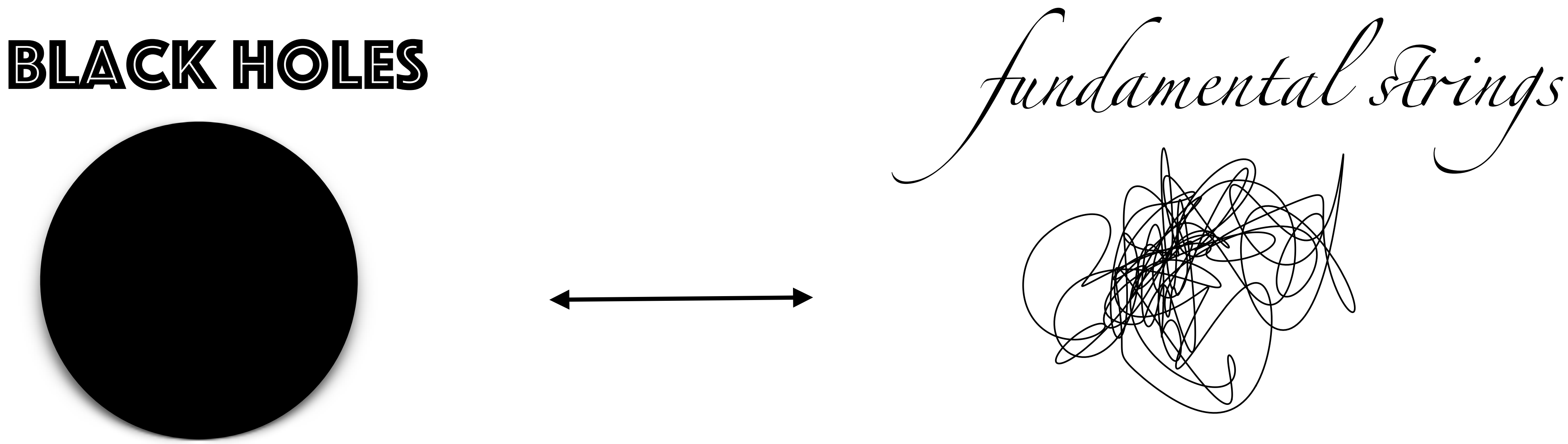
Correspondence in evaporation

Now coupling g stays fixed, but mass decreases as the black hole emits Hawking radiation. Natural expectation: black hole → string ball.



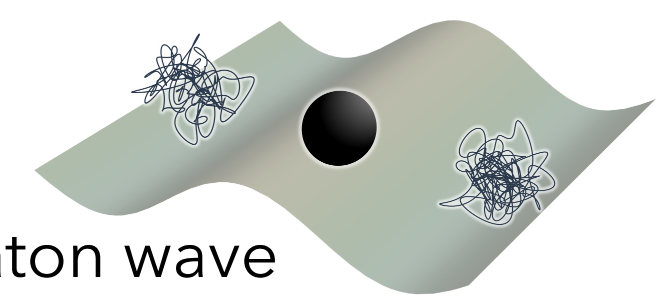
[Bowick,Smolin,Wijewardhana'86]

Summary: **static** correspondence

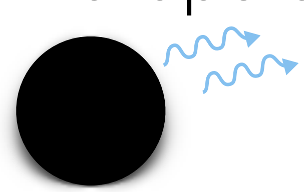


→ relates black holes to weakly coupled strings and thus gives a statistical interpretation of black hole entropy! *

* is **not exact** counting of the entropy



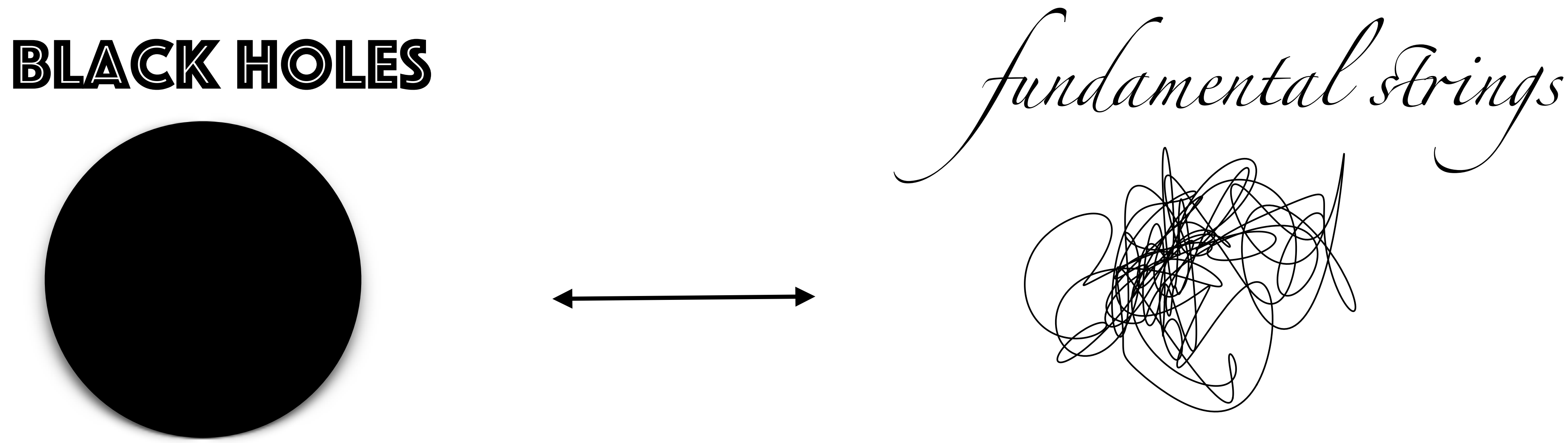
→ is physically realized in evaporation or induced by dilaton wave



→ applies to generic black holes in general dimensions **

** so far only **static** black holes!

Summary: **static** correspondence



Limitations:

Only parametric matching, and only at $g^2 S \sim 1$ (one value!)

Adiabaticity only approximate (but ∃ Goldilocks regime).

Self-gravity necessary (smoothness of transition not guaranteed).

Rotation adds qualitatively new features!

III. Black hole - string correspondence

with rotation

Rotating black holes & fundamental strings

A black hole – string zoo

characteristic length scales:

mass length

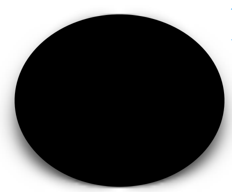
$$\ell_M = (GM)^{\frac{1}{D-3}}$$

spin length

$$\ell_J = \frac{J}{M}$$

[Emparan,Harmark, Niarchos,Obers'09]

$\ell_J < \ell_M$:



[1]

`Kerr regime'

unique, round-ish, dynamically stable

[1] [Myers,Perry'86]

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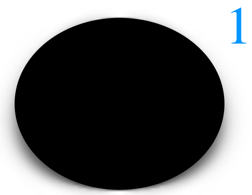

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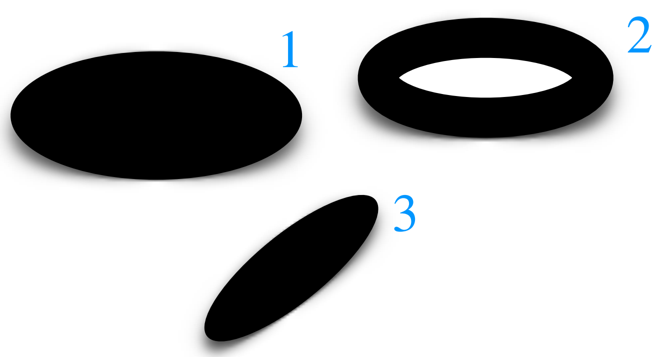

`ultraspinning regime'

different shapes and topologies, dynamically unstable

[1] [Myers,Perry'86] [2] [Emparan,Reall'01] [3] [Andrade,Emparan,Licht,Luna'19]

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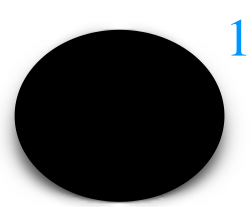

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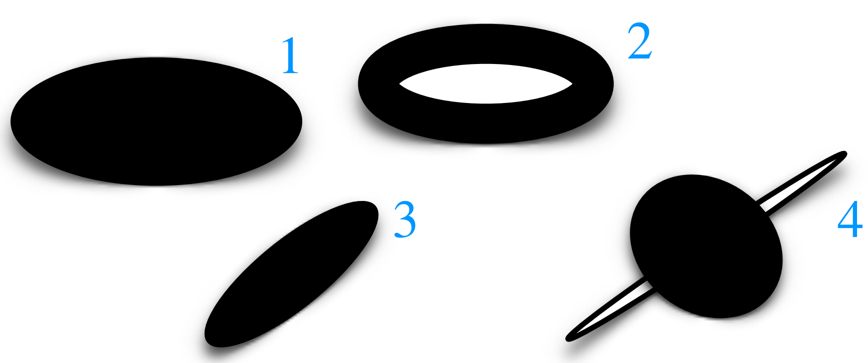

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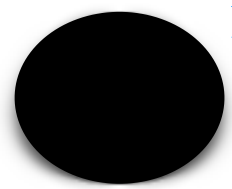
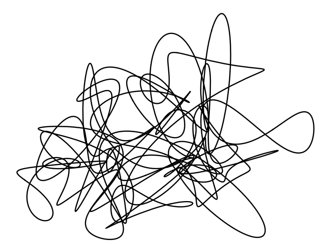


1



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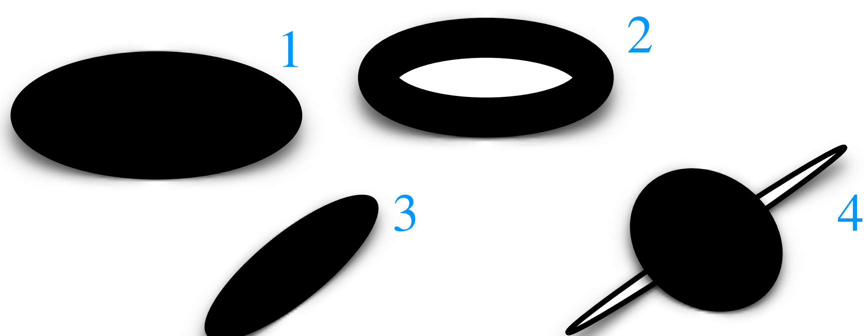

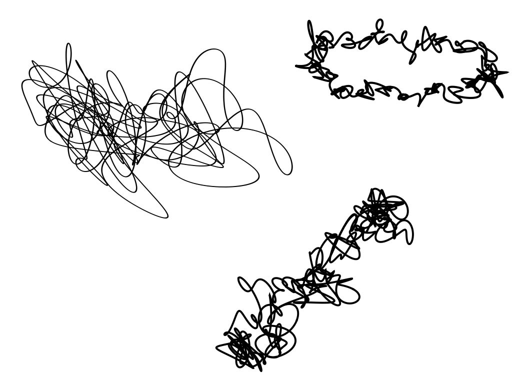

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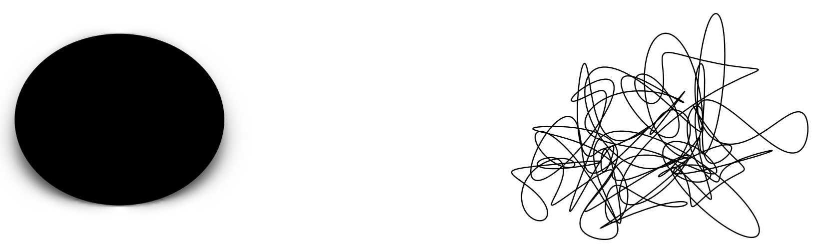

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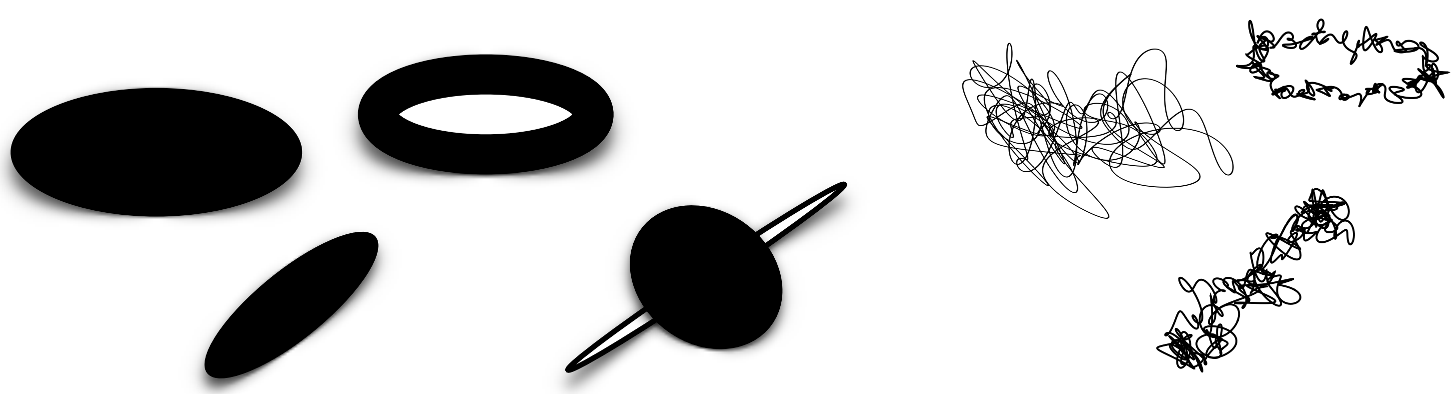


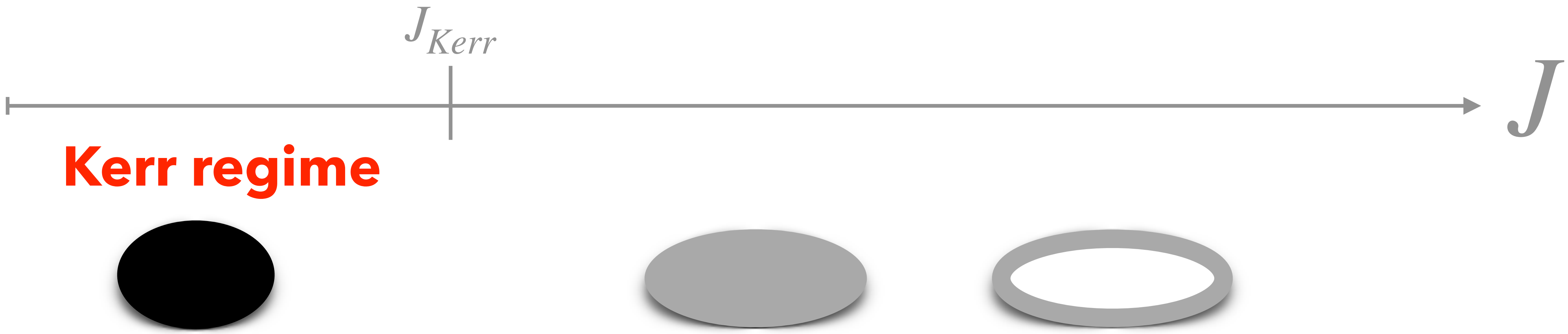
`ultraspinning regime'

different shapes and topologies, dynamically unstable

Focus on **rotation in a single plane**, express results in adiabatic invariant S and J.

Black hole instabilities

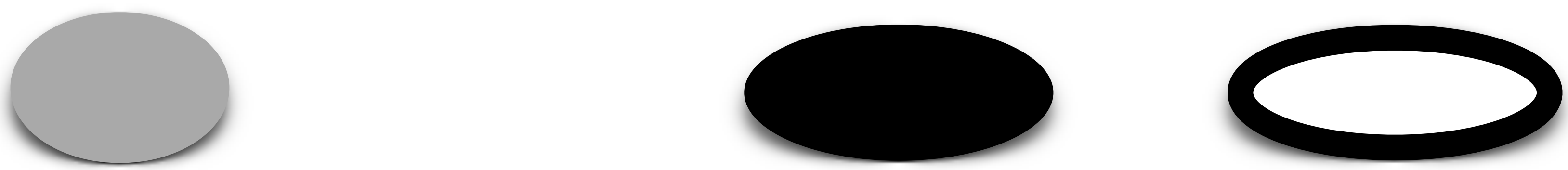
Black hole instabilities



classically stable

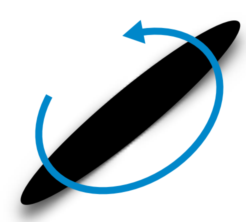
Black hole instabilities

Ultraspinning regime

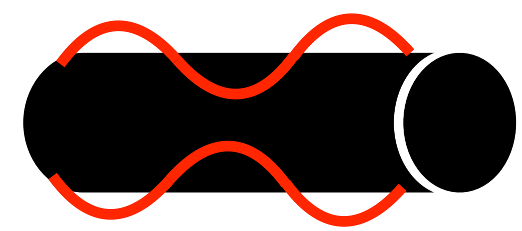


unstable: black holes in $D \geq 6$ & black rings in $D \geq 5$

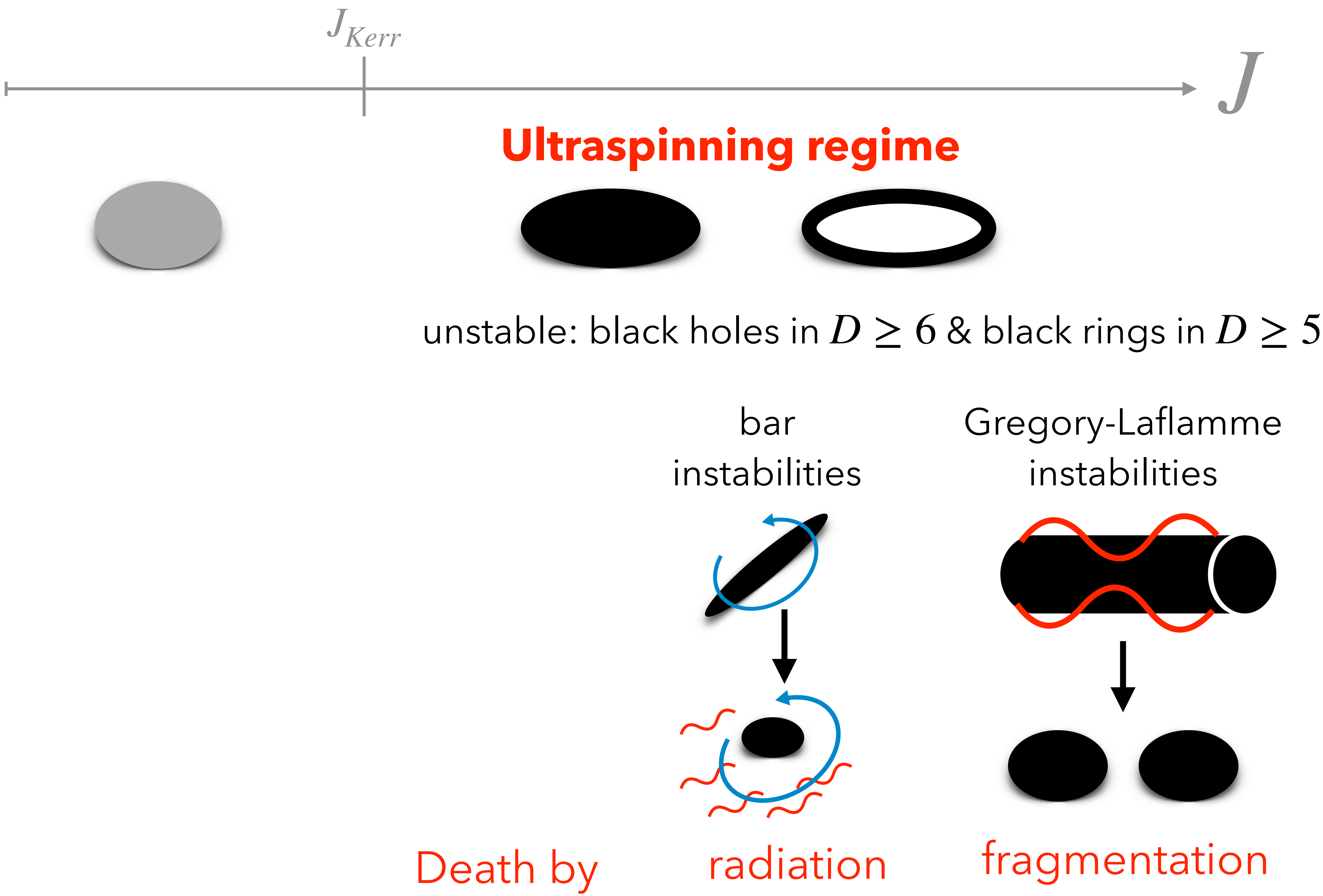
bar
instabilities



Gregory-Laflamme
instabilities

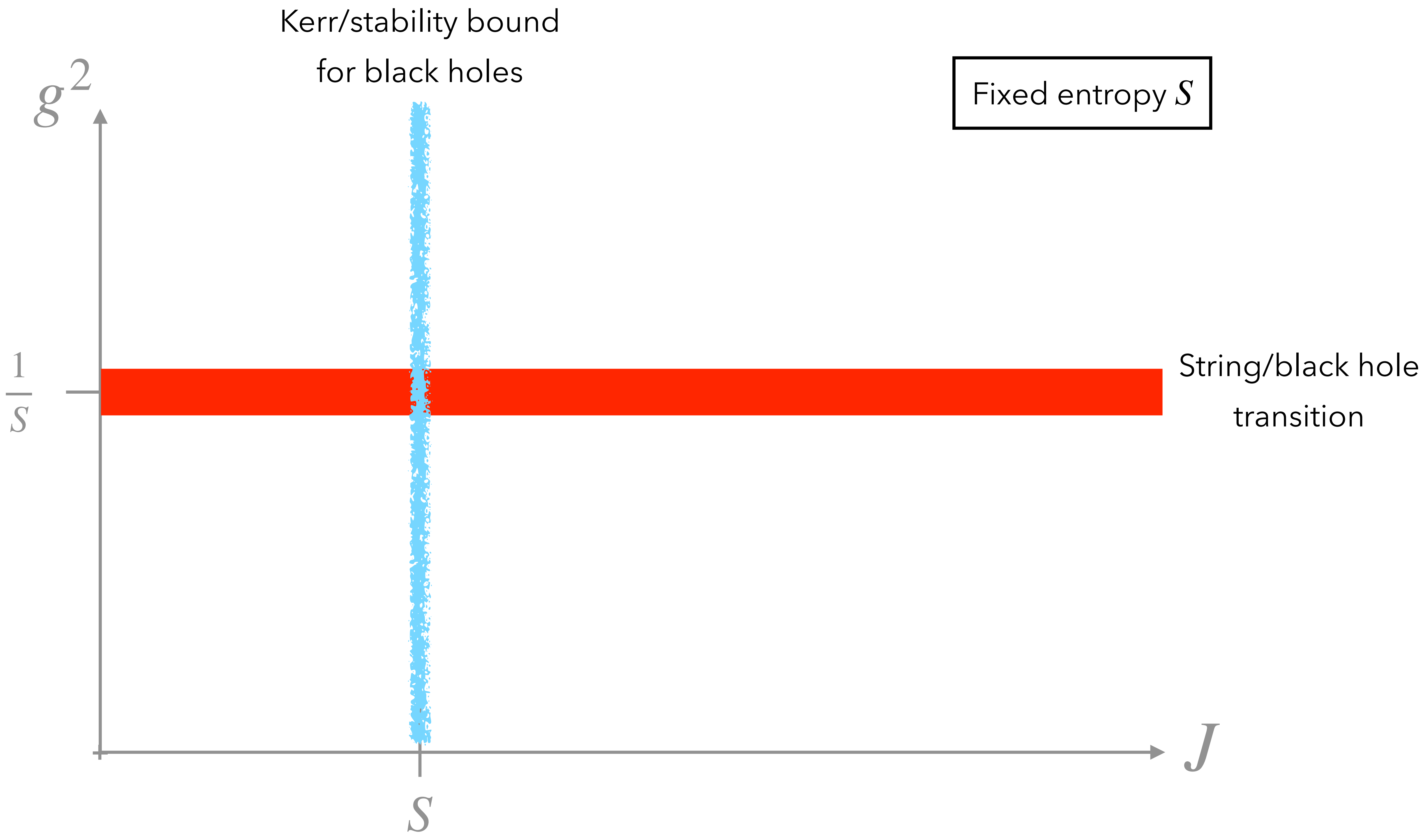


Black hole instabilities

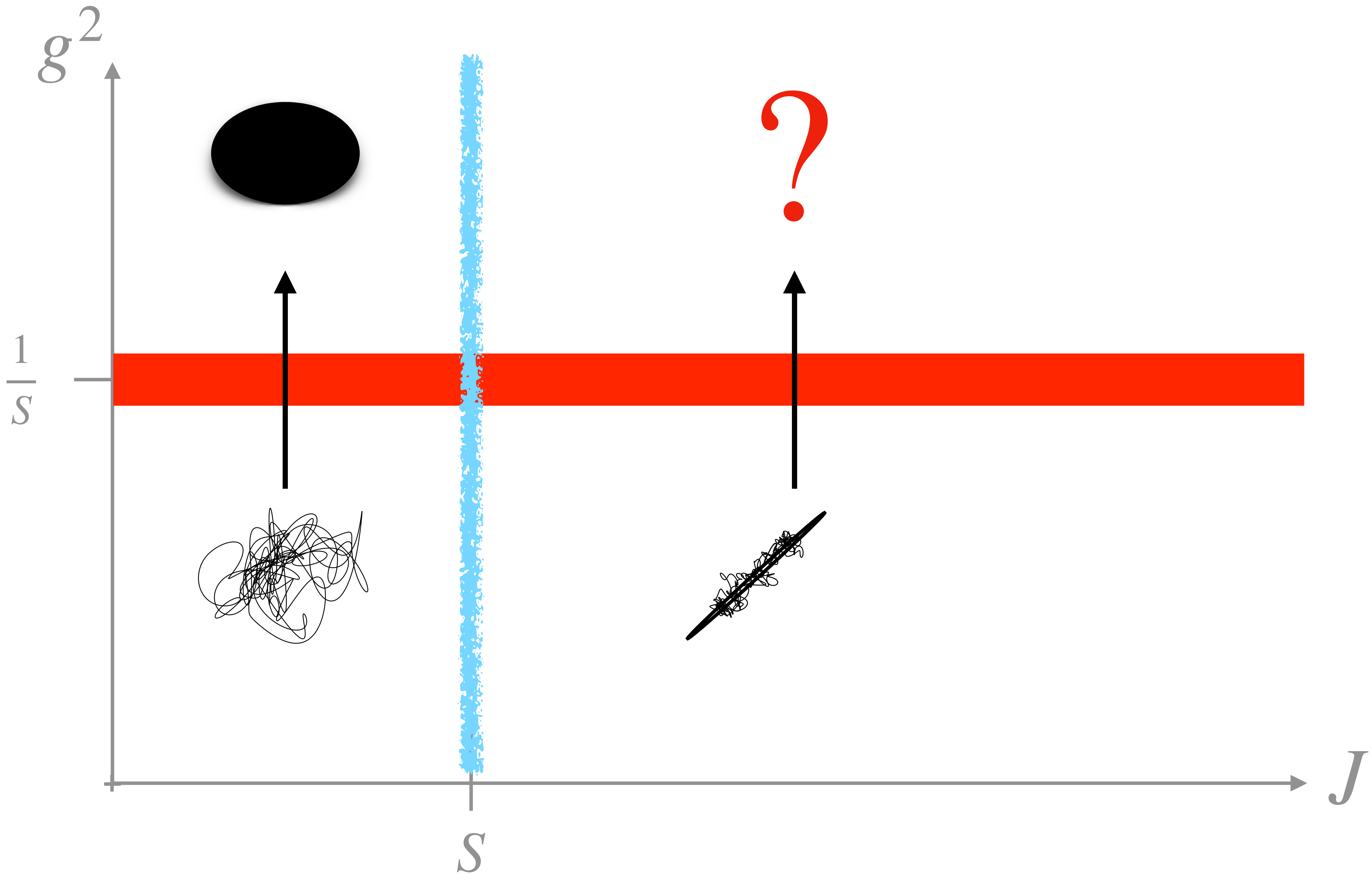


The correspondence with rotation

Correspondence diagram



Strings to black holes



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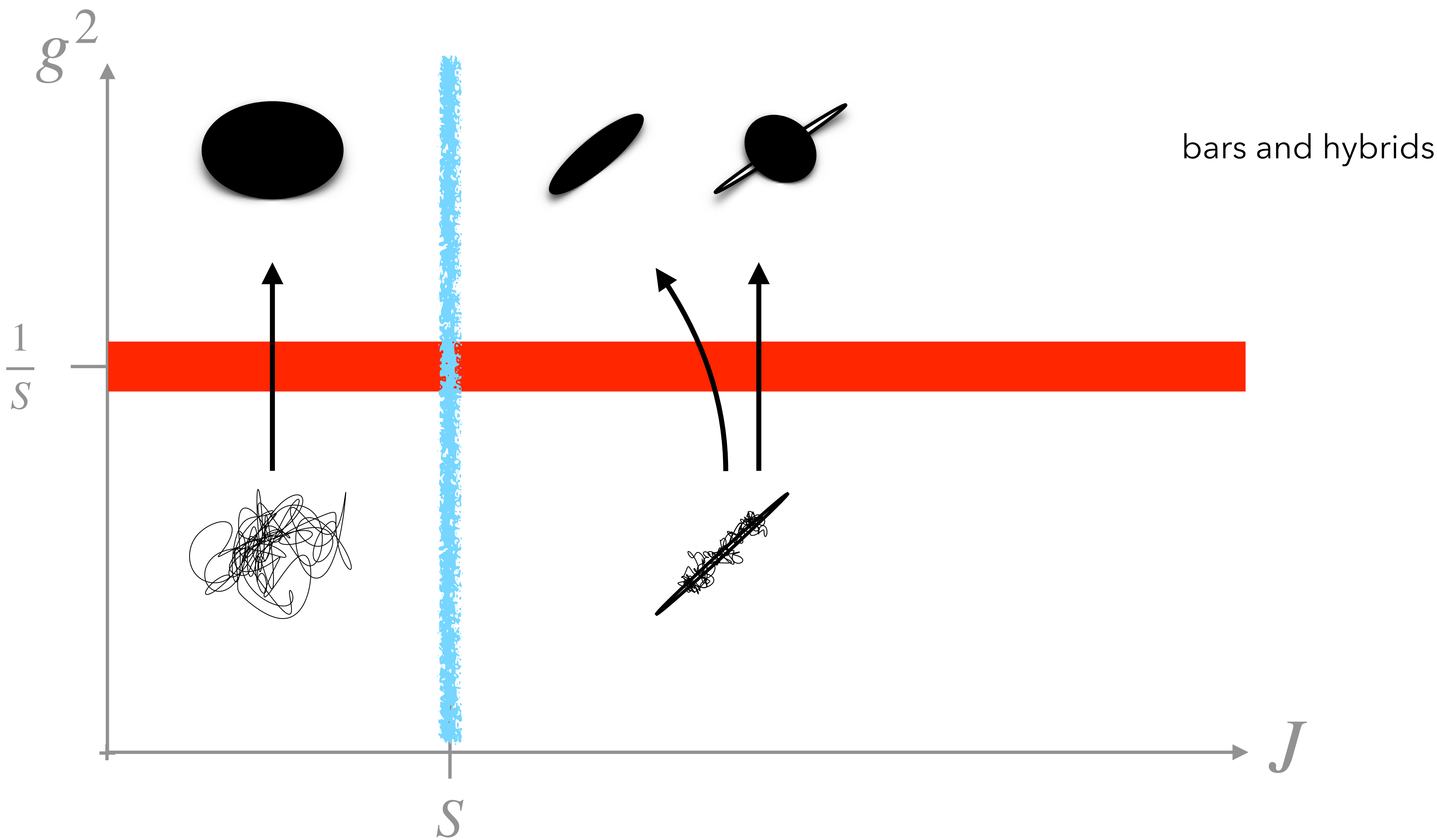
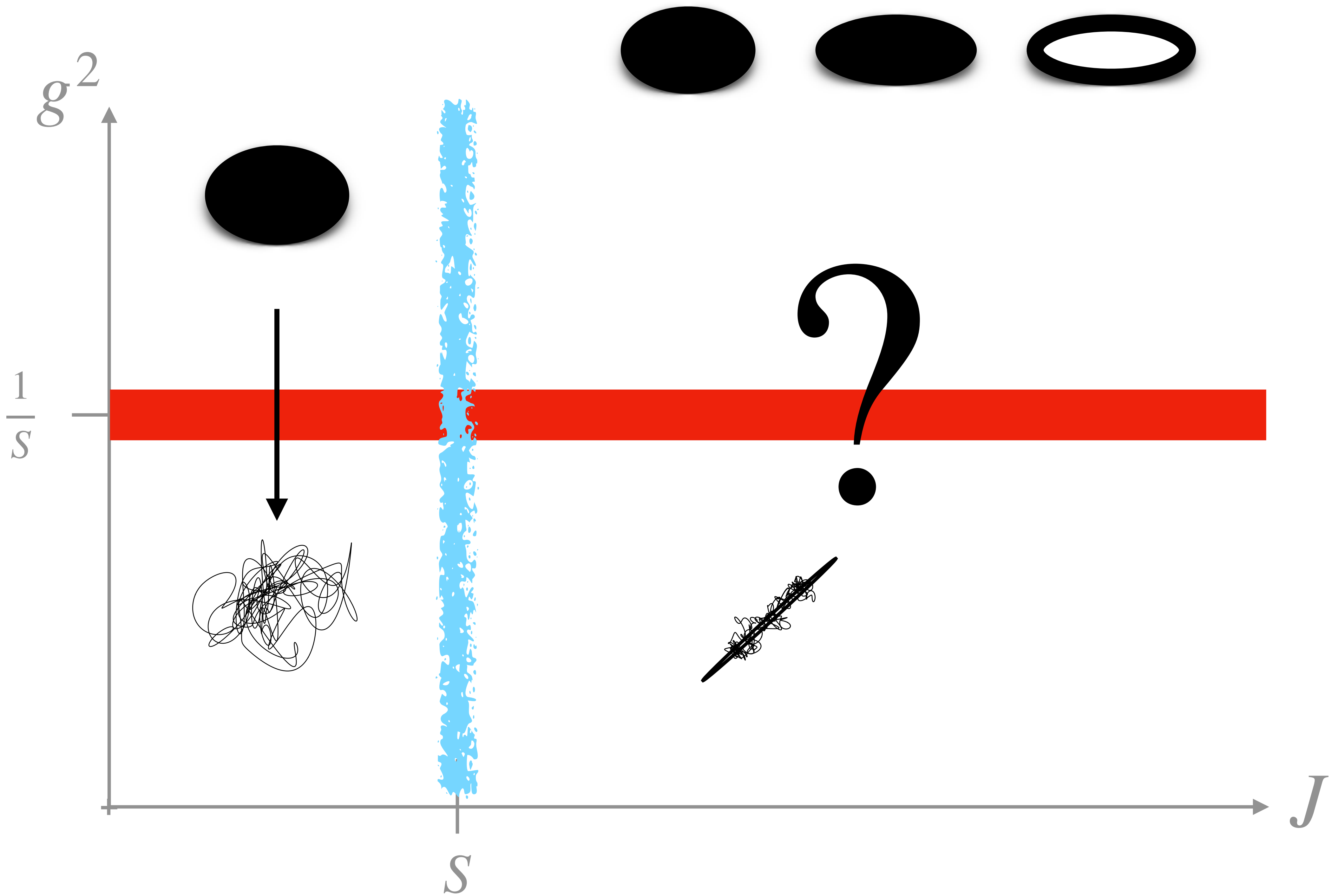


Solves puzzle 1: Highly rotating strings do have black hole counter parts in the form of non-stationary configurations!

Black holes to strings



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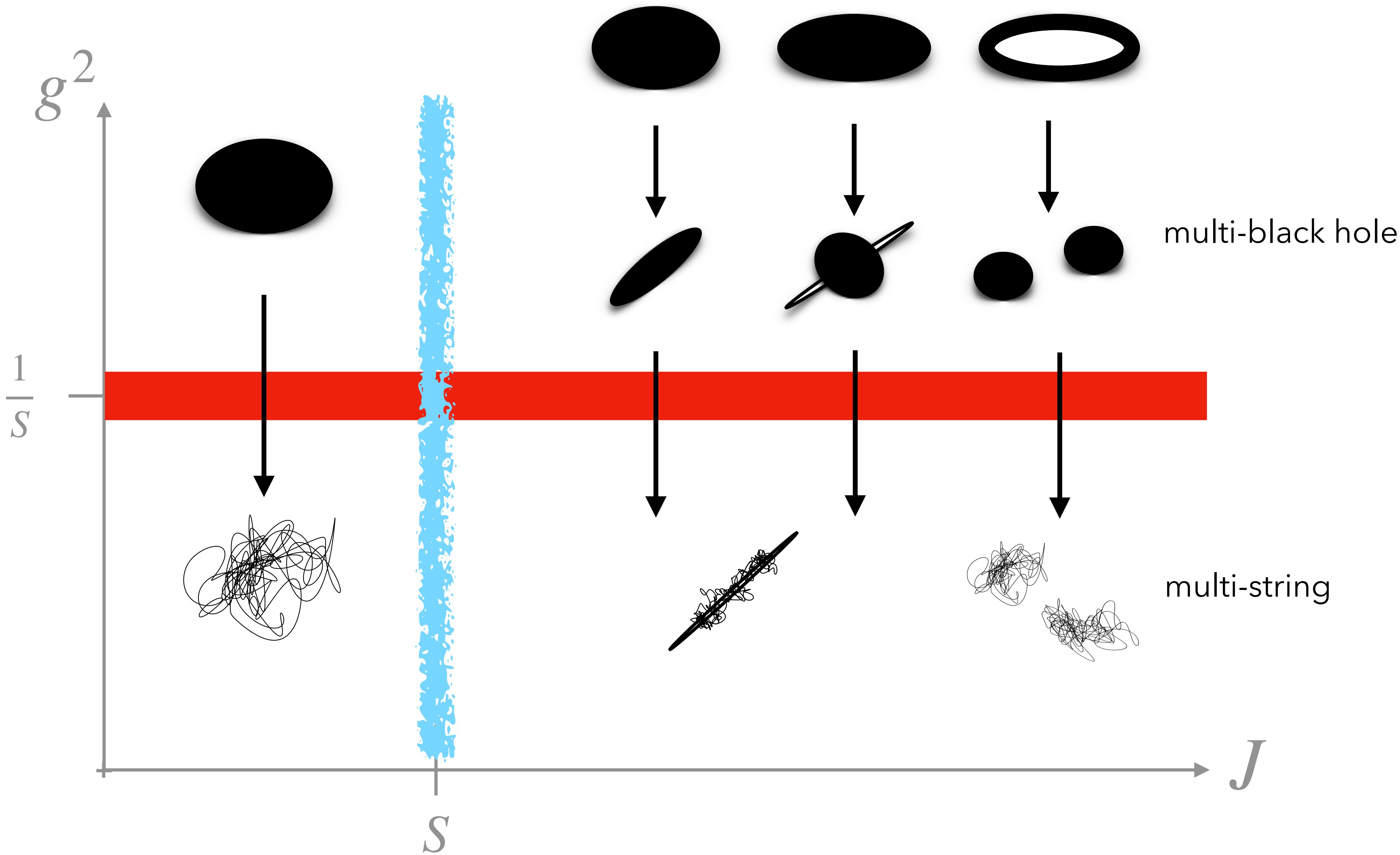
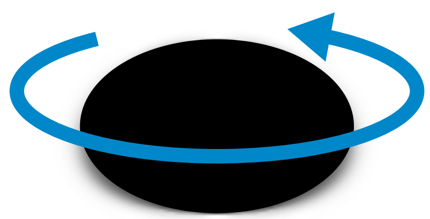
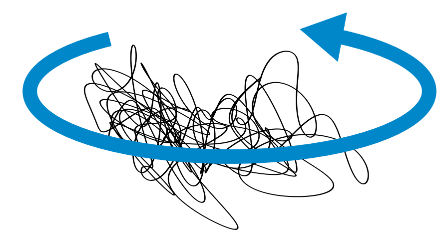


Solves puzzle 2: Ultraspinning black holes are unstable, their decay products have stringy counterparts!

Summary: **rotating** correspondence

BLACK HOLES ⟷ *fundamental strings*



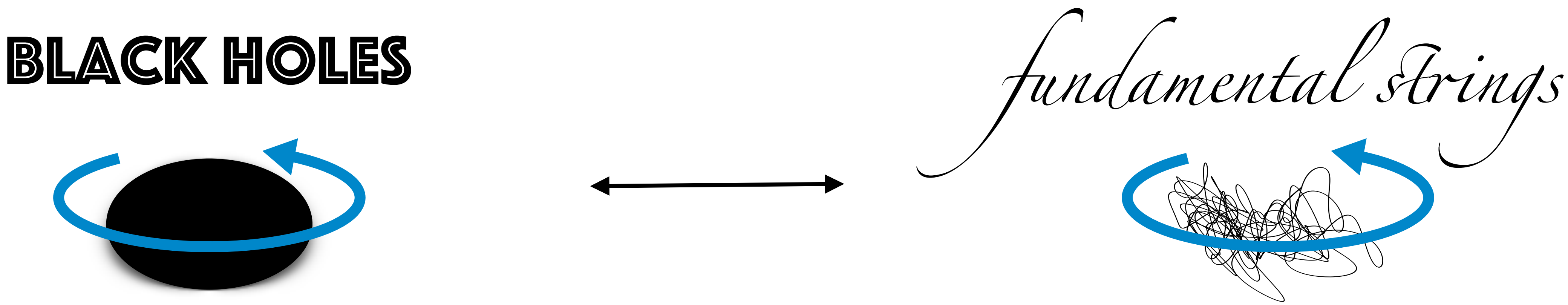


As for static: statistical interpretation of black hole entropy, physically realized in `dilaton lab' or evaporation, applies to <u>generic</u> black holes in any dimension.

New elements:

- Dynamical factors: black hole instabilities and emission of radiation
- Non-stationary phases:
  - black bars
  - black hole – string hybrids
  - multi–string states

Summary: **rotating** correspondence



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  - black hole – string hybrids
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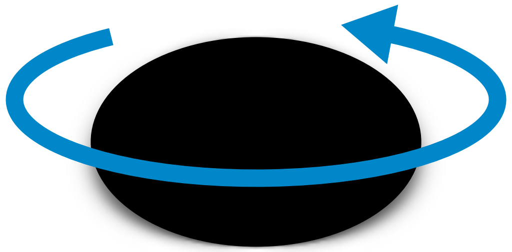
→ Test the correspondence with rotation: shapes and sizes

IV. Testing the correspondence

with rotation

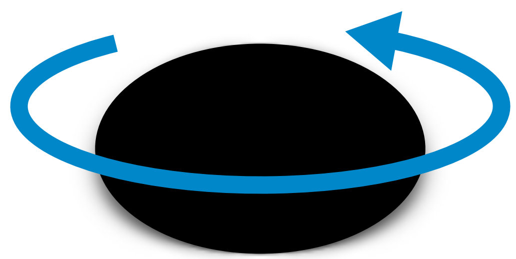
Spinning up a black hole



Mass M

heat TS mechanical energy ΩJ

Spinning up a black hole



Mass M

heat TS mechanical energy ΩJ

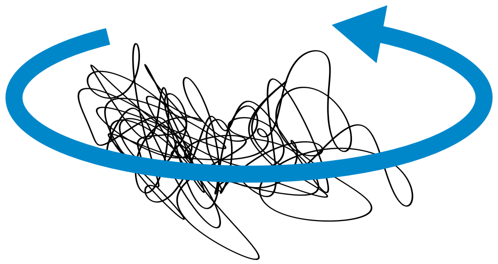
Decreasing TS increases ΩJ for a given total energy.

Geometrical counterpart of heat (entropy) is horizon area.

Overall size of a black hole of mass M must be smaller for larger J.

$$\left(\frac{M}{M_P}\right)^{D-2} \propto S^{D-5}\left(\frac{S^2}{4\pi^2}+J^2\right) \qquad |J| \leq S$$

Spinning up a string

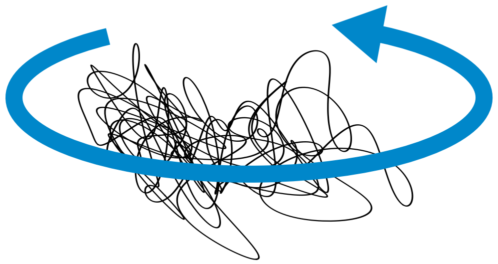


Oscillator excitations for mass M

coherently rotate

jitter in a random-walk

Spinning up a string



Oscillator excitations for mass M

coherently rotate   jitter in a random-walk

Decreasing random-walk increases rotation for a given total energy.

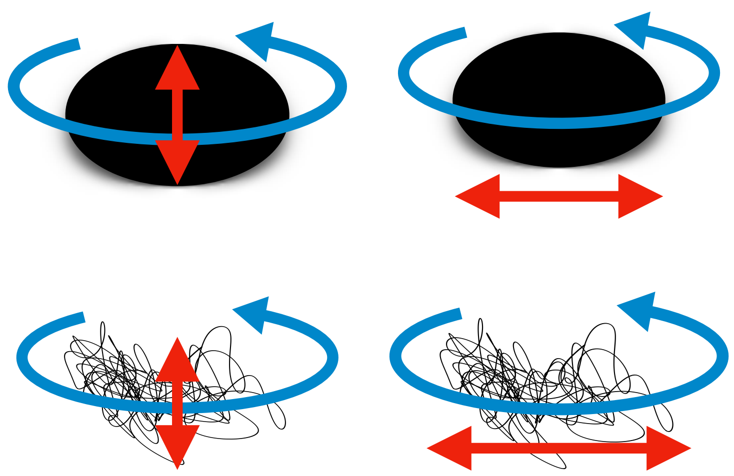
↓

Overall size of the string ball must be smaller for larger J.

$$S^2 \propto \left(\frac{M}{M_s}\right)^2 - 2|J| \qquad |J| = O\left(\frac{M}{M_s}\right)^2$$

What do we expect?

Size ⊥ and || to plane of rotation:



small $J \ll S$:

$$\frac{r_\perp^2}{r_\parallel^2} - 1 \propto -\frac{J^2}{S^2}$$

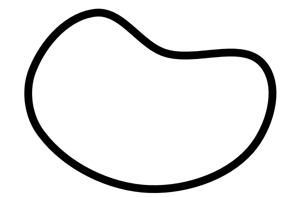
large $J >> S$: pancake effect for string and black hole but no matching

Size and shape of strings

String

$X^\mu(\tau, \sigma)$

$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \pi)$



Open or closed bosonic string in D spacetime dimensions.

Oscillators α_n^μ ($\tilde{\alpha}_n^\mu$) with excitation level n

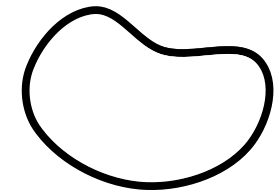
transverse directions

Light-cone gauge: $\alpha_n^\mu \to \alpha_n^i$ with $i = 1, \ldots, D - 2$

String

$X^\mu(\tau,\sigma)$

$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\pi)$



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Light-cone gauge: $\alpha_n^\mu \to \alpha_n^i$ with $i = 1,\ldots,D-2$

Mass: $M^2 \sim N M_s^2$

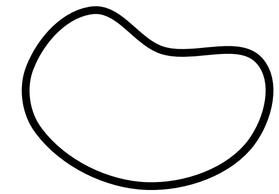
$$N = \sum_{i=1}^{D-2} \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$$

level-counting operator

String

$X^\mu(\tau,\sigma)$

$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\pi)$



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transverse directions

Light-cone gauge: $\alpha_n^\mu \to \alpha_n^i$ with $i = 1,\ldots,D-2$

Mass: $M^2 \sim N M_s^2$ $\qquad N = \sum_{i=1}^{D-2} \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$ level-counting operator

Spin: $J = -i \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^1 \alpha_n^2 - \alpha_{-n}^2 \alpha_n^1)$

coherent rotation in a *single* plane

Static string

Partition function:

$$Z \equiv \mathrm{Tr}(e^{-\beta N}) = \prod_{n=1}^{\infty} \left(\frac{1}{1 - e^{-\beta n}} \right)^{D-2}$$

$$\beta = \frac{1}{T}$$

inverse temperature

Static string

Partition function:

$$Z \equiv \mathrm{Tr}(e^{-\beta N}) = \prod_{n=1}^{\infty} \left(\frac{1}{1 - e^{-\beta n}} \right)^{D-2}$$

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inverse temperature

High-temperature limit $\beta \to 0$:

$$Z(\beta) \sim \beta^{\frac{c}{2}} e^{\frac{c\pi^2}{6\beta}}$$

$$c \equiv D - 2$$

transverse directions

Static string

Partition function: $$Z \equiv \mathrm{Tr}(e^{-\beta N}) = \prod_{n=1}^{\infty} \left(\frac{1}{1 - e^{-\beta n}} \right)^{D-2}$$

$\beta = \frac{1}{T}$

inverse temperature

High-temperature limit $\beta \to 0$: $$Z(\beta) \sim \beta^{\frac{c}{2}} e^{\frac{c\pi^2}{6\beta}}$$

$c \equiv D - 2$

transverse directions

String entropy: $$Z(\beta) = \sum_{n=1}^{\infty} d_n \, e^{-\beta n}$$

large n

saddle point approximation

$$d_n \sim n^{-\frac{c+3}{4}} e^{2\pi\sqrt{\frac{c}{6}n}}$$

[Hardy,Ramanujan'18]

Static string

Partition function:

$$Z \equiv \mathrm{Tr}(e^{-\beta N}) = \prod_{n=1}^{\infty} \left(\frac{1}{1 - e^{-\beta n}} \right)^{D-2}$$

$\beta = \frac{1}{T}$

inverse temperature

High-temperature limit $\beta \to 0$:

$$Z(\beta) \sim \beta^{\frac{c}{2}} e^{\frac{c\pi^2}{6\beta}}$$

$c \equiv D - 2$

transverse directions

String entropy:

$$Z(\beta) = \sum_{n=1}^{\infty} d_n \, e^{-\beta n}$$

$\longrightarrow$ large n

$$d_n \sim n^{-\frac{c+3}{4}} e^{2\pi\sqrt{\frac{c}{6}n}}$$

[Hardy,Ramanujan'18]

saddle point approximation

Strings size:

$$\langle R^2 \rangle = \sum_{n=1}^{\infty} R_n^2 \, e^{-\beta n}$$

$\longrightarrow$

$$R_n^2 \sim \frac{1}{c} \pi \sqrt{\frac{c}{6} n} \, d_n$$

[Mitchell,Turok'87]

$\uparrow$

$$R^2 = (X^i)^2 = \sum_{n=1}^{\infty} \frac{1}{n^2} \alpha_{-n}^i \alpha_n^i \qquad i = 1,2,\ldots,D-2$$

Static string

Partition function:
$$Z \equiv \mathrm{Tr}(e^{-\beta N}) = \prod_{n=1}^{\infty} \left(\frac{1}{1-e^{-\beta n}}\right)^{D-2}$$

$\beta = \frac{1}{T}$

inverse temperature

High-temperature limit $\beta \to 0$:
$$Z(\beta) \sim \beta^{\frac{c}{2}} e^{\frac{c\pi^2}{6\beta}}$$

$c \equiv D-2$

transverse directions

String entropy:
$$Z(\beta) = \sum_{n=1}^{\infty} d_n \, e^{-\beta n} \longrightarrow d_n \sim n^{-\frac{c+3}{4}} e^{2\pi\sqrt{\frac{c}{6}n}}$$

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[Hardy,Ramanujan'18]

saddle point approximation

Strings size:
$$\langle R^2 \rangle = \sum_{n=1}^{\infty} R_n^2 \, e^{-\beta n} \longrightarrow R_n^2 \sim \frac{1}{c}\pi\sqrt{\frac{c}{6}n}\, d_n$$

[Mitchell,Turok'87]

Typical string size $\langle r^2 \rangle_n \sim \frac{R_n^2}{d_n} \sim \sqrt{n} \propto M$ as expected for random walks.

Rotating string

Partition function: $Z \equiv \mathrm{Tr}(e^{-\beta(N-\Omega J)}) = \prod_{n=1}^{\infty} \left(\frac{1}{1-e^{-\beta n}} \right)^{D-4} \frac{1}{(1-e^{-\beta(n+\Omega)})(1-e^{-\beta(n-\Omega)})}$

High-temperature limit $\beta \to 0$: $Z(\beta, \Omega) \sim \beta^{\frac{c}{2}} e^{\frac{c\pi^2}{6\beta}} \frac{\Omega}{\sinh(\pi\Omega)}$

$c \equiv D - 2$

transverse directions

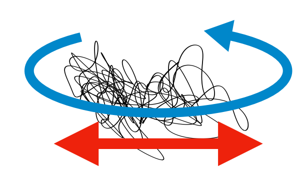
String entropy: $Z(\beta) = \sum_{n=1}^{\infty} d_{n,J}\, e^{-\beta n} e^{\beta \Omega J} \xrightarrow{\text{large } n} d_{n,J}$

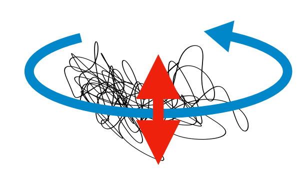
for large $J = O(n)$

[Russo,Susskind'94]

saddle point approximation

Strings size: $\langle R^2 \rangle = \sum_{n=1}^{\infty} R^2_{n,J} e^{-\beta(n-\Omega J)} \longrightarrow R^2_{n,J} \longrightarrow$ $R^2_{n,\parallel}$, $R^2_{n,\perp}$





$R^2 = (X^i)^2 = \sum_{n=1}^{\infty} \frac{1}{n^2} \alpha^i_{-n} \alpha^i_n$

$i = 1,2$

$i = 3, \ldots, D-2$

Rotating string

Partition function:

$$Z \equiv \mathrm{Tr}(e^{-\beta(N-\Omega J)}) = \prod_{n=1}^{\infty}\left(\frac{1}{1-e^{-\beta n}}\right)^{D-4} \frac{1}{(1-e^{-\beta(n+\Omega)})(1-e^{-\beta(n-\Omega)})}$$

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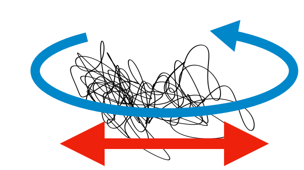
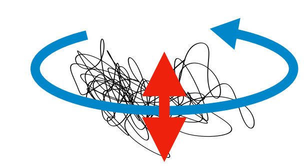
[Russo,Susskind'94]

saddle point approximation

Strings size:

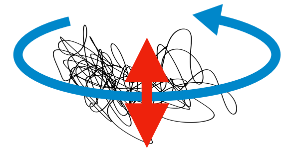
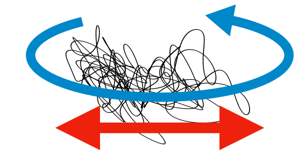
$$\langle R^2 \rangle = \sum_{n=1}^{\infty} R^2_{n,J} e^{-\beta(n-\Omega J)} \longrightarrow R^2_{n,J} < \begin{matrix} R^2_{n,\parallel} \\ R^2_{n,\perp} \end{matrix}$$





What are the typical string sizes $\langle r_J^2 \rangle_n \sim \frac{R^2_{n,J}}{d_{n,J}} < \begin{matrix} \frac{R^2_{n,\parallel}}{d_{n,J}} \\ \frac{R^2_{n,\perp}}{d_{n,J}} \end{matrix}$?

Average string sizes

Transverse to rotation plane:  Along rotation plane: 

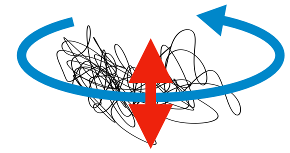
$$\frac{\langle \bar{r}_{\perp}^2 \rangle_n}{\ell_s^2} \propto \begin{cases} \sqrt{n} & J = O(1) \\ \sqrt{n - \mu |J|} & J = O(\sqrt{n}) \\ \sqrt{n - |J|} & J = O(n) \end{cases} \qquad \frac{\langle \bar{r}_{\parallel}^2 \rangle_n}{\ell_s^2} \propto \begin{cases} \sqrt{n} & J = O(1) \\ \sqrt{n - \mu |J|} & J = O(\sqrt{n}) \\ |J| & J = O(n) \end{cases}$$

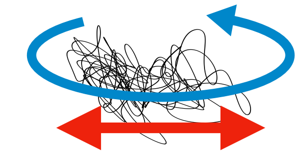
Individually both vary significantly with the strength of interaction.

Note: All numerical factors are dropped!

Pancake effect larger for larger J .

Average string sizes

Transverse to rotation plane: 

Along rotation plane: 

$$\frac{\langle \bar{r}_\perp^2 \rangle_n}{\ell_s^2} \propto \begin{cases} \sqrt{n} & J = O(1) \\ \sqrt{n - \mu |J|} & J = O(\sqrt{n}) \\ \sqrt{n - |J|} & J = O(n) \end{cases}$$

$$\frac{\langle \bar{r}_\parallel^2 \rangle_n}{\ell_s^2} \propto \begin{cases} \sqrt{n} & J = O(1) \\ \sqrt{n - \mu |J|} & J = O(\sqrt{n}) \\ |J| & J = O(n) \end{cases}$$

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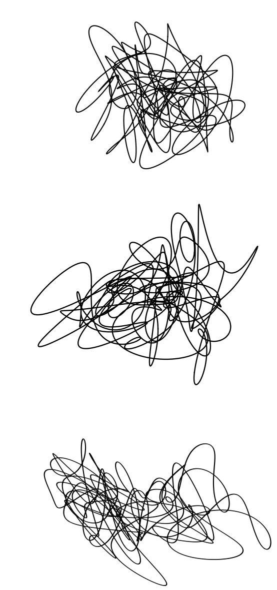
The ratio varies less with g and behaves as approximate adiabatic invariant.

$$\frac{\langle \bar{r}_\perp^2 \rangle_n}{\langle \bar{r}_\parallel^2 \rangle_n} \propto \begin{cases} 1 & J = O(1) \\ \frac{1}{C_\parallel} < 1 & J = O(\sqrt{n}) \\ \frac{S}{|J|} & J = O(n) \end{cases}$$



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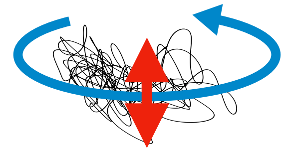
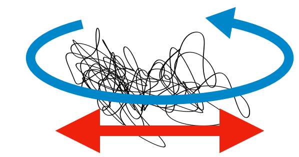
Average string sizes

Transverse to rotation plane: 

subleading orders

$$\frac{\langle \bar{r}_\perp^2 \rangle_n}{\ell_s^2} \propto \begin{cases} \sqrt{n} \; + \gamma_1 + \frac{\gamma_2}{\sqrt{n}} & J = O(1) \\ \sqrt{n - \mu |J|} & J = O(\sqrt{n}) \\ \sqrt{n - |J|} & J = O(n) \end{cases}$$

Along rotation plane: 

subleading orders

$$\frac{\langle \bar{r}_\parallel^2 \rangle_n}{\ell_s^2} \propto \begin{cases} \sqrt{n} \; + \gamma_1 + \frac{\gamma_2}{\sqrt{n}} + \frac{\gamma_\parallel}{\sqrt{n}} J^2 & J = O(1) \\ \sqrt{n - \mu |J|} & J = O(\sqrt{n}) \\ |J| & J = O(n) \end{cases}$$

Individually both vary significantly with the strength of interaction.

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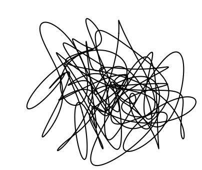
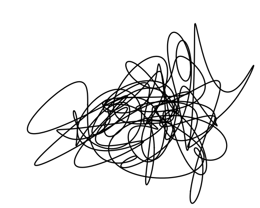
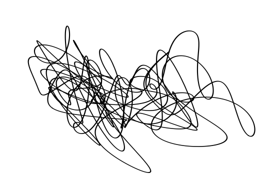
subleading orders

$$\frac{\langle \bar{r}_\perp^2 \rangle_n}{\langle \bar{r}_\parallel^2 \rangle_n} \propto \begin{cases} 1 - \gamma_\parallel \frac{J^2}{n} & J = O(1) & \gamma_\parallel > 1 \\ \frac{1}{C_\parallel} < 1 & J = O(\sqrt{n}) & C_\parallel > 1 \\ \frac{S}{|J|} & J = O(n) & \end{cases}$$







Note: All numerical factors are dropped!

Pancake effect larger for larger J .

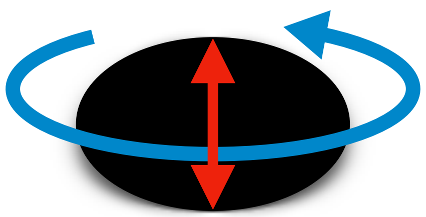
Size and shape of black holes

Black hole sizes

Myers-Perry

Natural to define the transverse characteristic radius at the pole $\theta = 0$ on the horizon:

$$\mathscr{A}_{\perp}^{(D-4)} = \Omega_{D-4}(r_0 \cos\theta)^{D-4}$$



$$r_{\perp} \equiv \left(\frac{\mathscr{A}_{\perp}^{(D-4)}(\theta = 0)}{\Omega_{D-4}} \right)^{\frac{1}{D-4}} = r_0$$

horizon radius

Black hole sizes

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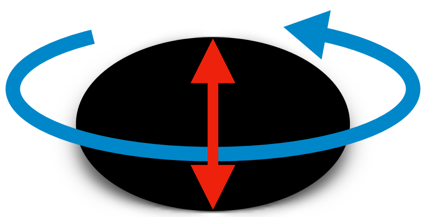
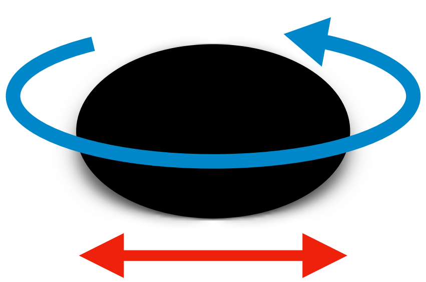


$$r_\perp \equiv \left(\frac{\mathscr{A}_\perp^{(D-4)}(\theta = 0)}{\Omega_{D-4}} \right)^{\frac{1}{D-4}} = r_0$$

horizon radius

Define characteristic radius in rotation plane via horizon area in the (θ, ϕ) directions: *

$$\mathscr{A}_\parallel^{(2)} = \int d\theta d\phi \sqrt{g_{\theta\theta} g_{\phi\phi}} \,|_{r=r_0}$$



$$r_\parallel^{(a_J)} \equiv \sqrt{\frac{\mathscr{A}_\parallel^{(2)}}{4\pi}} = \sqrt{r_0^2 + a_J^2}$$

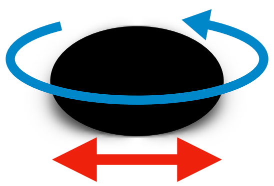
horizon radius   spin length

$$\frac{r_0}{a_J} = \frac{S}{2\pi J}$$

* More physical definition uses critical impact parameter for the capture of a null geodesic in the equatorial plan but form similar to $r_\parallel^{(a_J)}$.

Black hole sizes

$$r_\perp = \frac{D-2}{4\pi}\frac{S}{M} \qquad\qquad r_\parallel = \frac{D-2}{4\pi}\frac{\sqrt{S^2+4\pi^2 J^2}}{M}$$



Individually both get renormalized (since M does).

Black hole sizes

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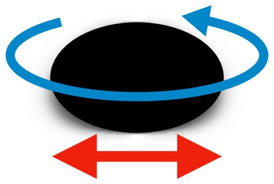

Individually both get renormalized (since M does). The ratio

$$\frac{r_\perp}{r_\parallel} = \frac{S}{\sqrt{S^2+4\pi^2J^2}}$$

is again an approximate adiabatic invariant (since S and J are).

Black hole sizes

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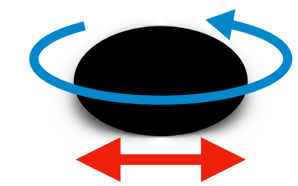

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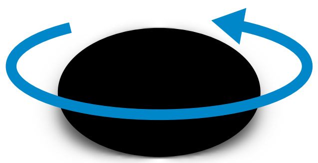
$$\frac{r_\perp^2}{r_\parallel^2} = \begin{cases} 1 - 4\pi^2\frac{J^2}{S^2}, & |J| \ll S \\ C < 1, & |J| \sim S \\ \frac{1}{4\pi^2}\frac{S^2}{J^2}, & |J| \gg S \end{cases}$$

Pancake effect larger for larger J .

Do rotating strings
size up/down like
rotating black holes?

Size: black hole vs string



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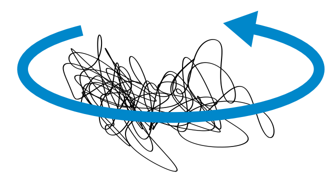
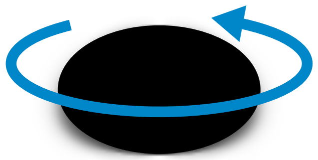


$$\frac{\langle \bar{r}_\perp^2 \rangle_n}{\langle \bar{r}_\parallel^2 \rangle_n} \propto \begin{cases} 1 - \gamma_\parallel \frac{J^2}{S^2}, & J = O(1) \\ \frac{1}{C_\parallel} < 1, & J = O(\sqrt{n}) \\ \frac{S}{|J|}, & J = O(n) \end{cases}$$

Note: Overall numerical factors are dropped!

Size: black hole vs string



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For small J, string and black hole agree: $\propto \frac{J^2}{S^2}$!

Note: Overall numerical factors are dropped!

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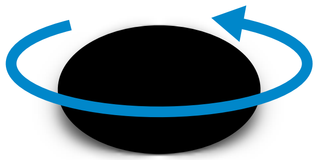

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For small J, string and black hole agree: $\propto \frac{J^2}{S^2}$!

For typical J, ratio (string) $\approx$ ratio (black) at leading order.

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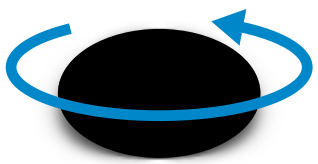
Size: black hole vs string



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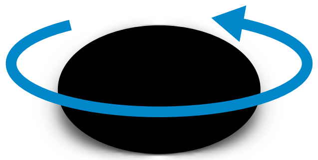
For large J, ratio (string) $\gg$ ratio (black hole) $\Rightarrow$
Black hole gets pancaked more than string. (Note: still no self-gravity)

Note: Overall numerical factors are dropped!

Size: black hole vs string



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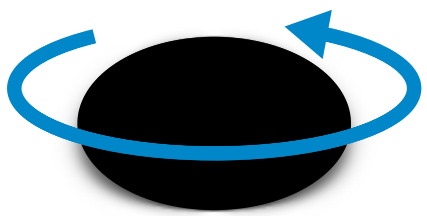
For large J, ratio (string) $\gg$ ratio (black hole) $\Rightarrow$
Black hole gets pancaked more than string. (Note: still no self-gravity)

Mismatch beyond small J expected: no adiabatic correspondence
but dynamics & non-stationary phases.

Note: Overall numerical factors are dropped!

Restoring $\hbar \neq 1$

BLACK HOLES



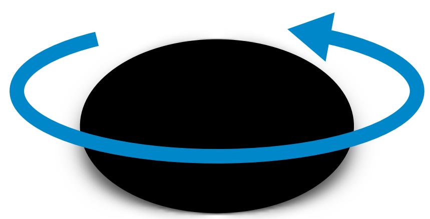
$S \propto \hbar^{-1}$ large (for finite horizon)

fundamental strings

$S \propto n$ n large

Restoring $\hbar \neq 1$

BLACK HOLES



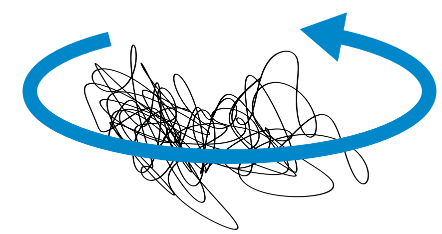
$S \propto \hbar^{-1}$ large (for finite horizon)

$\dfrac{J}{\hbar S}$ large (for semi-classical angular momentum)

$J \to -J$

$\dfrac{J^2}{S^2}$ semi-classical result

fundamental strings

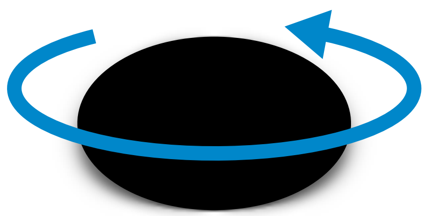


$S \propto n$ $\quad n$ large

$\dfrac{J^2}{n} \propto \dfrac{J^2}{S^2}$ our result ✓

Restoring $\hbar \neq 1$

BLACK HOLES



fundamental strings

$S \propto \hbar^{-1}$ large (for finite horizon)

$\frac{J}{\hbar S}$ large (for semi-classical angular momentum)

$J \to -J$

$\frac{J^2}{S^2}$ semi-classical result

$\frac{\hbar J^2}{(\hbar S)^3}$ quantum correction

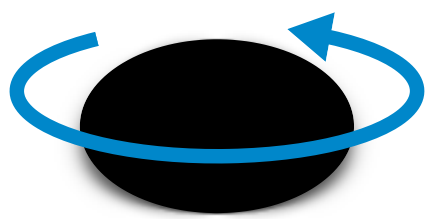
$S \propto n$ n large

$\frac{J^2}{n} \propto \frac{J^2}{S^2}$ our result ✓

$\frac{J^2}{n^{3/2}} \propto \frac{J^2}{S^3}$ our result ✓

Restoring $\hbar \neq 1$

BLACK HOLES



$S \propto \hbar^{-1}$ large (for finite horizon)

$\frac{J}{\hbar S}$ large (for semi-classical angular momentum)

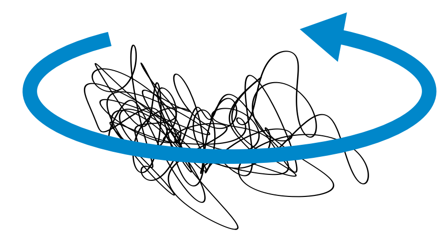
$J \to -J$

$\frac{J^2}{S^2}$ semi-classical result

$\frac{\hbar J^2}{(\hbar S)^3}$ quantum correction

$\frac{\hbar^{-1} J^2}{\hbar S}$ cannot appear

fundamental strings



$S \propto n$ n large

$\frac{J^2}{n} \propto \frac{J^2}{S^2}$ our result ✓

$\frac{J^2}{n^{3/2}} \propto \frac{J^2}{S^3}$ our result ✓

$\vdots$

$\frac{J^2}{n^{1/2}} \propto \frac{J^2}{S}$ does not appear ✓

Thank you!