What is the Bridge between Gauge Theories and Quantum Gravity at low Energies?

Jan de Boer, Amsterdam

Based on work with Alex Belin, Diego Liska, Pranjal Nayak, Tarek Anous, Julian Sonner, Daniel Jafferis, Boris Post, Martin Sasieta, Jildou Hollander, Andrew Rolph, Ramesh Chandra, Igal Arav, Shira Chapman ‘20-’25

String Theory as a Bridge between GT and QG
Rome, 17 February 2025

CFT

$$Z(\beta) = \sum_i e^{-\beta E_i}$$

=

Exact quantum gravity in AdS

???

=

Semi-classical approximation (GPI)

$$Z(\beta) \sim \exp(c/\beta^{d-1})$$

Saddles + pert corrections

$$Z(\beta) = \sum_i e^{-\beta E_i}$$

$$\rho(E) = \sum \delta(E - E_i)$$

=

???

CFT

GRAVITY

$$Z(\beta) = \int dE \, \rho(E) e^{-\beta E}$$

$$\log \rho(E) \sim E^{\frac{d-1}{d}}$$

=

$$Z(\beta) \sim \exp(c/\beta^{d-1})$$

Saddles + pert corrections

In the CFT, the exact spectrum gets replaced by a continuous “coarse grained” spectral density.

Claim: the right way to do this coarse graining is by replacing the CFT by a statistical average over all sets of CFT data which are semi-classically indistinguishable. Those sets need not obey the axioms of a CFT as long as those violations are not semi-classically detectable.

To make this more precise need to (i) specify which data and (ii) provide a probability distribution (measure) on the space of data.

Statistical physics gives us a preferred method to deal with situations like this (Wigner '55 Balian '68).

Maximize ignorance (=entropy) subject to the constraints imposed by the semi-classical approximation:

$$\int dH \; -\mu[H]\log\mu[H] + \mu[H]\int d\beta\lambda(\beta)\left(\mathrm{Tr}(e^{-\beta H}) - Z(\beta)\right)$$

$$V'(E) = 2\int d\lambda\,\frac{\rho_0(\lambda)}{E-\lambda}$$

One finds

$$\mu[H] \sim \exp\left(\int d\beta\lambda(\beta)\mathrm{Tr}(e^{-\beta H})\right) \sim \exp(-\mathrm{Tr}V(H))$$

where V is arbitrary but needs to be fixed to yield the right partition function (or spectral density).

JdB, Liska, Post, Sasieta '23

This shows that *in the absence of other information* the best description of the Hamiltonian of a theory with a continuous spectral density is in terms of a matrix model.

For a chaotic theory, it may be difficult to obtain more detailed information about the spectrum and this may be the best one can do.

(Black holes are very chaotic Maldacena, Stanford, Shenker '15)

This would resonate with the Bohigas–Giannoni–Schmit (BGS) conjecture(1984) which asserts that the spectral statistics of quantum systems whose classical counterparts exhibit chaotic behavior are described by random matrix theory.

One can play a similar game for much more general choices of data. Suppose for example that we know some correlators of an operator A and we want to extract a probability distribution $\mu[A]$ on the space of operators (viewed as matrices in Hilbert space).

The general picture is one where if one e.g. inputs connected ≤k-point correlators, one gets a “matrix model” with up to k-th order interactions in the exponent.

Lagrange multipliers

$$\int dA d\lambda_i \left(-\mu[A] \log \mu[A] + \sum_i \lambda_i \, \mu[A] \left(f_i[A] - c_i \right) \right)$$

Shannon entropy

Input observations

$$\Rightarrow \mu[A] \sim e^{-\sum_i \lambda_i f_i[A]}$$

Consider for example the finite temperature one and two-point functions of some operator A.

$$\langle A(0)A(t)\rangle_\beta = \sum_{i,j} e^{-\beta(E_i+E_j)/2} e^{i(E_i-E_j)t} |\langle i|A|j\rangle|^2$$

These correlation functions (which can be semi-classically computed by a propagator in a black hole background) can be used to produce a statistical model for $\langle i|A|j\rangle$ and the result is a quadratic matrix model.

This quadratic matrix model is a familiar result. It is usually stated as the so-called Eigenstate Thermalization Hypothesis:

$$\langle E_i|O_a|E_j\rangle = \delta_{ij} f_a(\bar{E}) + e^{-S(\bar{E})/2} g_a(\bar{E}, \Delta E) R^a_{ij}$$

Deutsch ’91
Srednicki ’94
Foini, Kurchan ‘19

$f_a(\bar{E})$: one point functions of simple operators

$g_a(\bar{E}, \Delta E)$: two point functions of simple operators

R^a_{ij} : Gaussian random variables

$$\langle R^a_{ij}\rangle = 0, \qquad \langle R^a_{ij} R^b_{kl}\rangle = \delta^{ab}\delta_{il}\delta_{jk}$$

JdB, Liska, Post, Sasieta, ‘23

As before, this shows that *in the absence of other information* the best description of the matrix elements of a simple operator is in term of ETH.

As before, for a chaotic theory, it may be difficult to obtain more detailed information about these matrix elements and this may be the best one can do.

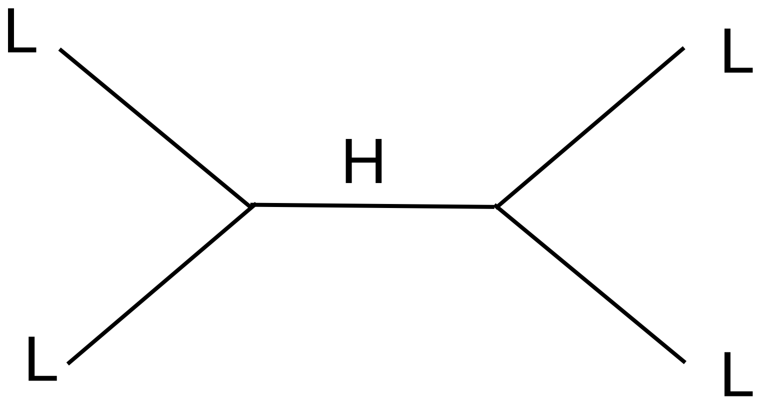
One can also include thermal higher-point functions and these will give rise to higher order moments for the matrices R^a_{ij} (non-gaussianities).

We now apply this logic to semi-classical gravity in AdS.

Conformal field theories are specified by a list of conformal dimensions Δ_i and OPE coefficients C_{ijk}.

The conformal dimensions are described by a matrix model using as input the black hole partition function $Z(\beta)$.

For the OPE coefficients we construct a matrix/tensor model with as input gravitational n-point functions of light operators (L) on various manifolds. Statistics is only relevant for the heavy operators (H) which correspond to black hole microstates and which cannot be distinguished semi-classically from each other.

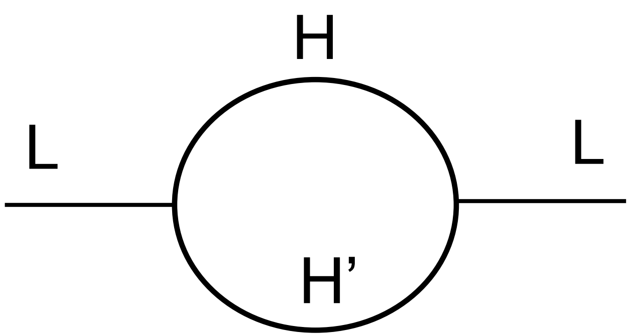


$$\sum_H C^2_{LLH}$$

4 point correlator on S^d

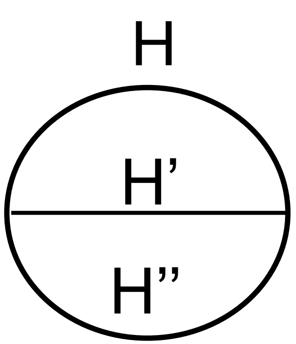
$$\overline{|C_{LLH}|^2} \sim \frac{\Delta_H^{2\Delta_L - 1}}{\Gamma(2\Delta_L)\rho(\Delta_H)}$$

Pappadopulo, Rychkov, Espin, Ratazzi ‘12



$$\sum_{H,H'} C^2_{LHH'}$$

2 point correlator on $S^{d-1}xS^1$

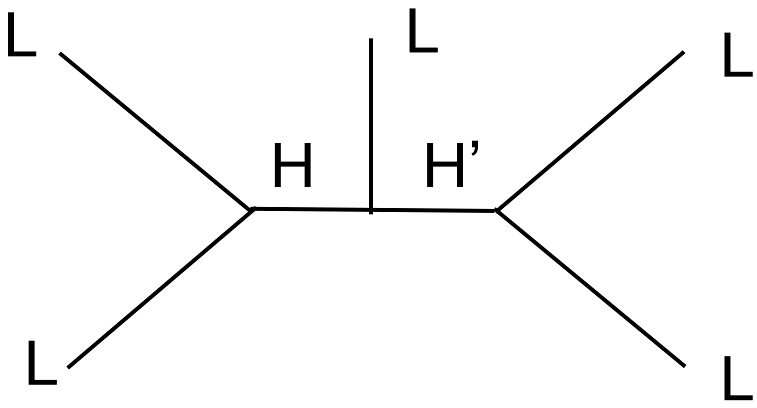


$$\sum_{H,H',H''} C^2_{HH'H''}$$

Connected sum of two times $S^{d-1}xS^1$

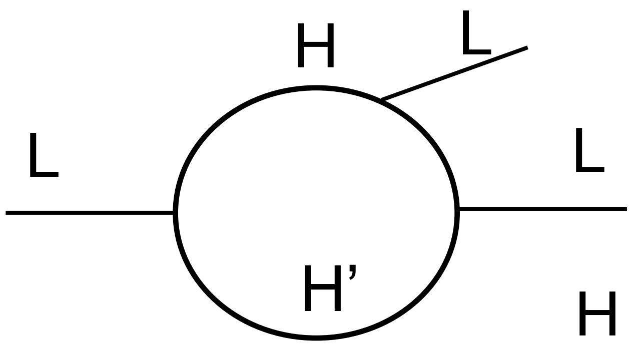
Benjamin, Lee, Ooguri, Simmons-Duffin ‘23

Input gives rise to quadratic matrix model for the C’s



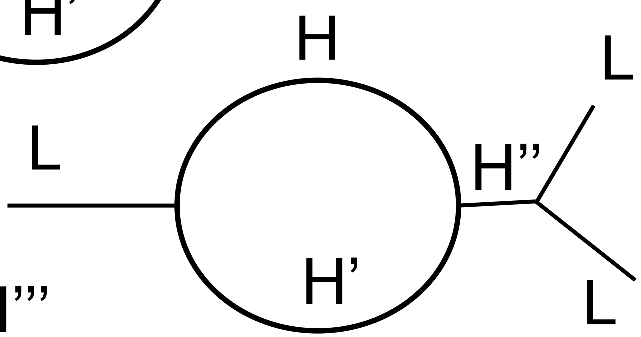
$$\sum_{H,H'} C_{LLH}C_{HLH'}C_{LH'H'}$$

5 point correlator on S^d



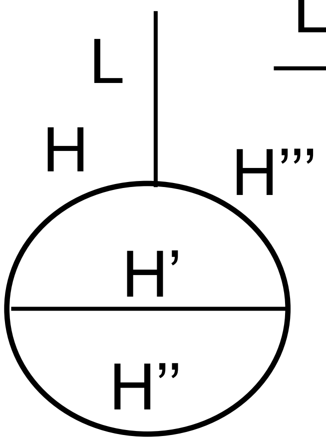
$$\sum_{H,H',H''} C_{LHH'}C_{LH'H''}C_{LH''H}$$

3 point correlator on S^{d-1}xS^1



$$\sum_{H,H',H''} C_{LHH'}C_{HH'H''}C_{H''LL}$$

3 point correlator on S^{d-1}xS^1



$$\sum_{H,H',H'',H'''} C_{HH'H''}C_{H'H''H'''}C_{HH'''L}$$

1 point function on connected sum of two times S^{d-1}xS^1

Input gives rise to cubic tems in matrix model for the C’s

Of course, general computations involve both the OPE coefficients and the spectrum of the theory so there will also be cross-correlations.

Post, Tsiares ‘24

In 2d, the result of all of this is a mixed matrix/tensor model which encodes statistics in the spectrum and statistics of OPE coefficients.

cf Jafferis, Kolchmeyer, Mukhametzhanov, Sonner ’22

In d>2, we do not know what the minimal set of data is to fully describe a CFT, but whatever those are, we get a corresponding statistical model. (Casimir energy on T^{d-1} is not obviously expressible in terms of Δ_i and C_{ijk})

Belin, JdB, Kruthoff, Michel, Shaghoulian, Shyani ’16
Belin, JdB, Kruthoff”18

Semi-classical gravity =

$$\overline{CC} = \int dC\, e^{-V[C]}\, CC$$

+

$$\overline{CCC} = \int dC\, e^{-V[C]}\, CCC$$

+

$$Z(\beta) = e^{\frac{\pi^2 c}{3\beta}} = \int dM e^{-\mathrm{Tr} V[M]}\, \mathrm{Tr}\left(e^{-\beta M}\right)$$

+ interactions

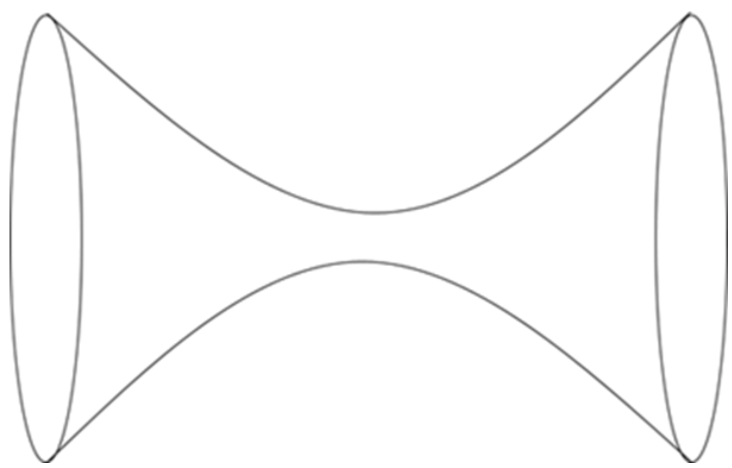
What is this good for???

- It sheds light on the chaotic nature of CFTs
- It explains the factorization problem
- It explains replica wormholes and state overlaps
- It sheds light on how the semi-classical approximation is still compatible with the Page curve
- It is a useful perspective on situations with multiple boundaries or replicas

The factorization problem

Harlow '15
Guica, Harlow '15
Harlow, Jafferis '18
Saad, Shenker, Stanford '19
Marolf, Maxfield, '20
……..

The gravitational path integral includes connected geometries with multiple boundaries



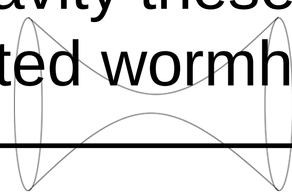
These seem to violate factorization of the CFT on disconnected manifold: $\langle Z(\beta_1)Z(\beta_2)\rangle \neq \langle Z(\beta_1)\rangle\langle Z(\beta_2)\rangle$

This *apparent* lack of factorization arises as follows: above we built a statistical model using gravitational computations with a single boundary.
This produced a “single trace” matrix/tensor model.

Statistical models predict correlations between multiple copies of the theory.

$$\int dH \mu[H] \mathrm{Tr}(e^{-\beta_1 H}) \mathrm{Tr}(e^{-\beta_2 H}) - \int dH \mu[H] \mathrm{Tr}(e^{-\beta_1 H}) \int dH \mu[H] \mathrm{Tr}(e^{-\beta_2 H}) \neq 0$$

We propose that in gravity these correlations precisely correspond to connected wormhole geometries.



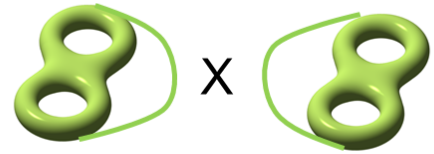
Conjecture: wormholes compute the correlations of the one-sided statistical model. They contain no new information.

Intuition for the conjecture: one-sided computations allow one to reconstruct the bulk Lagrangian. Crossing symmetry is closely related to bulk locality. So all information which is needed to compute wormholes semi-classically is *in principle* available

This conjecture has been tested quite extensively (e.g. Alex Belin, JdB ’20; Chandra, Collier, Hartman, Maloney ’22, JdB, Liska, Post, Sasieta ’23; JdB, Liska, Post ’24; Post Tsiares ‘24;) for computations involving OPE coefficients in AdS3. More general understanding for pure 3d gravity follows from the Virasoro TQFT (Collier, Eberhardt, Zhang ’23 ‘24).

A simple example

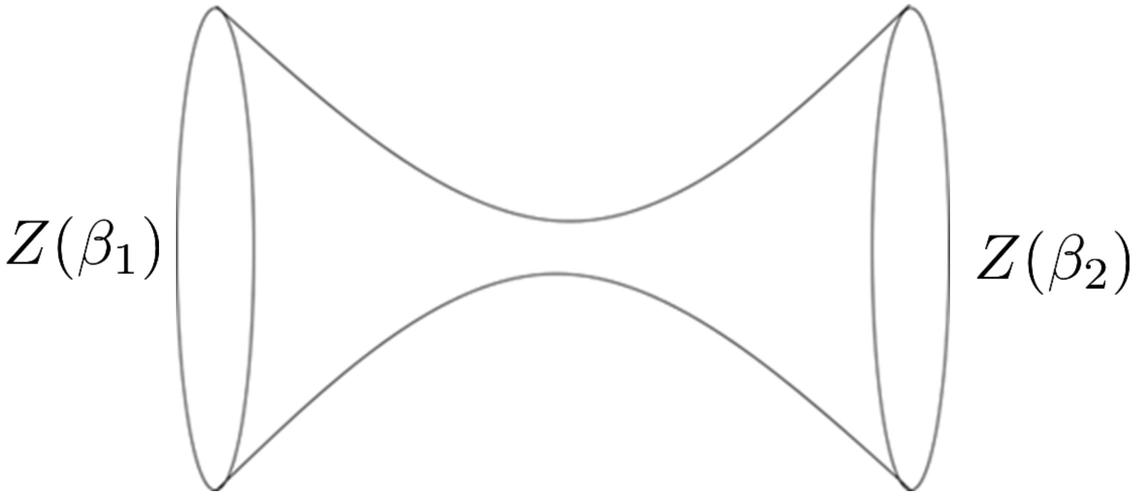
$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij} C^*_{jkk} e^{-3\beta\Delta} \right) \left(\sum_{lmn} C_{llm} C^*_{mmn} e^{-3\beta\Delta} \right) \right\rangle$$



$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij} C^*_{jkk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{llm} C^*_{mnn} e^{-3\beta\Delta} \right) \right\rangle$$

Belin, JdB, ‘20

Test of the conjecture for the spectral part of the theory



Cotler, Jensen ’21 – see also Di Ubaldo, Perlmutter ’23 and Haehl, Reeves, Rozali ‘23

The off-shell gravity computation agrees to leading order with the universal random matrix theory result

$$\langle Z(\beta_1)Z(\beta_2)\rangle = Z(\beta_1)Z(\beta_2) + \frac{1}{2\pi}\frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2} + \dots$$

Ambjørn, Jurkiewicz, Makeenko ’90
Saad, Shenker, Sanford ’19

What about all those other wormholes?

Chandra, JdB, WIP

- Axionic wormholes (Giddings, Strominger ‘88 + many more) – boundary $S^d, T^d, S^1 \times S^{d-1}, \ldots$
- Wormholes supported by complex scalar (multiplets) (Marolf, Santos ‘21 +..) – boundary $S^3, T^3, \ldots$
- Meron wormholes, e.g with SU(2) gauge fields and boundaries S^3 (Maldacena, Maoz ’04 + ..)
- AdS3 wormholes with hyperbolic boundaries (Maldacena, Maoz ’04 + ..)
- Thin shell wormholes
- Double cone wormholes
- Bra-ket wormholes
- Off-shell wormholes

Information recovery in the semiclassical approximation

JdB, Hollander, Rolph '23

Time evolution of an initial state

$$\rho_0 \Rightarrow \overline{\rho(t)} = \int dH\, \mu[H]\, e^{-iHt} \rho e^{iHt}$$

produces a *classical statistical* mixture of states.

In general $S(\overline{\rho(t)})$ will increase: information loss. But since

$$\mathrm{Tr}(\overline{\rho(t)^n}) = \mathrm{Tr}(\rho_0^n) \Rightarrow \overline{S(\rho(t))} = S(\rho_0)$$

a suitable semi-classical replica computation knows that information is actually not lost.

Penington '19
Almheiri, Engelhardt, Marolf, Maxfield '19
Penington, Shenker, Stanford, Yang '19
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '19

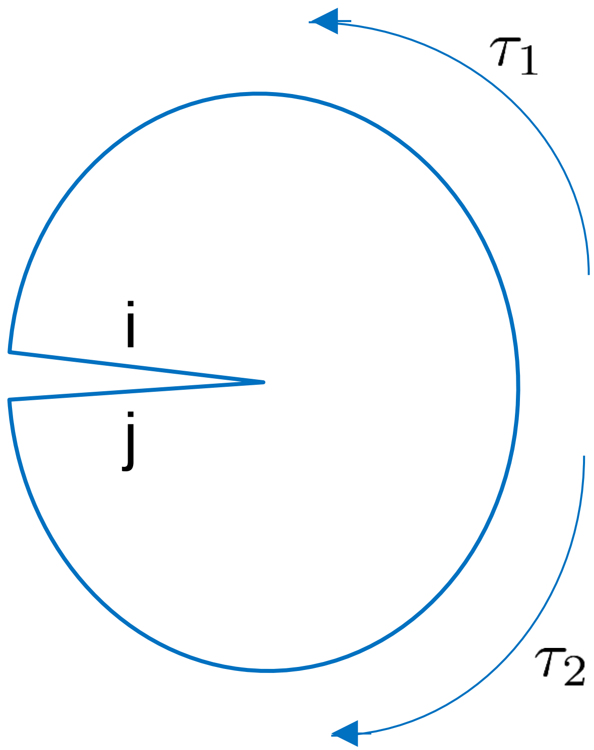
An alternative: state averages

An alternative approach: state averages

So far we looked at semi-classical gravitational computations with a closed boundary.

However, we can also use gravitational path integrals with boundaries to semi-classically produce states.

Freivogel, Nikolapoulou, Rotundo ‘21
Chadra, Hartman ‘22
Penington, Shenker, Stanford, Yang ’19
Bah, Chen, Maldacena ’22
Goel, Lam, Turiaci, Verlinde ‘18
Balasubramanian, Lawrence, Magan, Sasieta ’22
JdB, Liska, Post, Sasieta ‘23



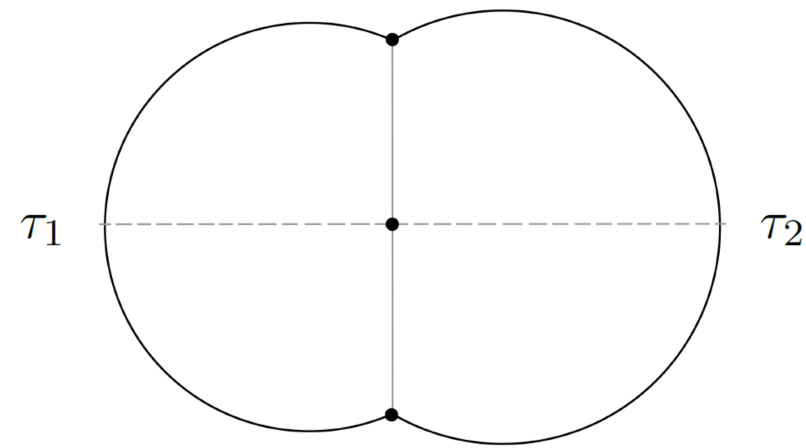
One can derive a suitable state-averaging ansatz for an open path integral.

More precisely, we consider purification of density matrices

$$|\Psi_{\rm sc}\rangle = \sum_{i,\alpha} A_{i\alpha}|E_i\rangle|E_\alpha\rangle \longrightarrow \rho = \sum_{i,j,\alpha} A_{i\alpha}A^\dagger_\alpha|E_i\rangle\langle E_j|$$

and assume such states can be prepared semiclassically (e.g. TFD state or PETS states). We then compute semiclassical overlaps of the form

$$\mathbf{Z}(\tau_1,\tau_2) := \langle\Psi_{\rm sc}|\,\mathrm{e}^{-\tau_1 H_L}\mathrm{e}^{-\tau_2 H_R}\,|\Psi_{\rm sc}\rangle =$$



and apply the maximal ignorance philosophy to obtain a quadratic matrix model for A.

Result:

$$\langle E_i|\rho|E_j\rangle = \delta_{ij}\,\bar{\rho}(E_i) + \frac{e^{-\beta \bar{E}_{ij}}}{Z(\beta)} e^{-S(\bar{E}_{ij})/2} j(\bar{E}_{ij},\omega_{ij})^{1/2} R_{ij}$$

Provides an alternative picture to OPE/spectral statistics. It more directly describes a coarse graining at the level of states. It is in particular useful for cutting/gluing constructions of correlation functions.

It reproduces many results of the OPE/spectral statistics picture.

Interesting feature: $\overline{S(\rho|\rho_\beta)} \sim \mathcal{O}(1)$

JdB, Liska, Post, Sasieta, ‘23

A few general observations for state averages

Arav, JdB, Chapman, WIP

Suppose we consider an ensemble of states which is invariant under unitaries (typicality in microcanonical window)

$$\mu[\rho] = \mu[U^\dagger \rho U]$$

Then

$$\overline{\rho \otimes \ldots \otimes \rho} = \sum_{\sigma \in S_n} a_\sigma S_\sigma$$

where a_σ only depends on the conjugacy class of σ.

This is a useful picture! (cf Liu, Vardman '20)

For example, for a random pure state on a bipartite AB system, the answer for the measure on the space of reduced density matrices reads

$$\overline{\rho_A \otimes \ldots \otimes \rho_A} = \frac{\Gamma(d_A d_B)}{\Gamma(d_A d_B + n)} \sum_{\sigma \in S_n} d_B^{|\sigma|} S_\sigma$$

where $|\sigma|$ is the number of cycles in the permutation σ .
With group theoretic identities one can now derive

$$\mathrm{Tr}(\overline{\rho_A^n}) = \frac{1}{\binom{d_A d_B + n - 1}{n}} \sum_{r=0}^{\min(d_A, d_B, n)-1} (-1)^r \binom{n-1}{r} \binom{d_A + n - r - 1}{n} \binom{d_B + n - r - 1}{n}$$

Arav, Chapman, JdB, WIP

In the presence of a Hamiltonian H

Stationary ensemble: $\frac{d}{dt}\mu[\rho] = 0$

$$\Longrightarrow \overline{\rho \otimes \rho} = f(H_1, H_2) + Sg(H_1, H_2)$$

Static ensemble: $\mu[\rho] \neq 0$ only if $[\rho, H] = 0$

Ergodic ensemble: conserved charges are constant over the ensemble.

An ensemble of pure states which is stationary and ergodic implies subsystem ETH.

Dymarsky, Lashkari, Liu ‘16

A simpler example of state averaging

In the same spirit, suppose we prepare semiclassically a set of states $|\psi_i\rangle$ with some high energy E. If these states form black holes (possibly after some time evolution) they become semi-classically indistinguishable.

Model these states as $|\psi_i\rangle = C_{ia}|\psi_a\rangle$ with $(C_i)_a$ some random unit norm vector acting in a microcanonical energy window and $|\psi_a\rangle$ some fixed orthonormal basis for that window.

Semiclassical computation yields $\langle\psi_i|\psi_j\rangle = \delta_{ij}$

Maximal ignorance principle yields the flat measure on the C's. Indeed

$$\overline{\langle\psi_i|\psi_j\rangle} = \int dC \,\langle\psi_a|C^*_{ia}C_{jb}|\psi_b\rangle = \delta_{ij}$$

One can now compute:

$$\overline{\langle\psi_i|\psi_j\rangle} = \int dC\,\langle\psi_a|C^*_{ia}C_{jb}|\psi_b\rangle = \delta_{ij}$$

$$\overline{\langle\psi_i|\psi_j\rangle\langle\psi_k|\psi_l\rangle} = \int dC\,\langle\psi_a|C^*_{ia}C_{jb}|\psi_b\rangle\langle\psi_c|C^*_{kc}C_{ld}|\psi_d\rangle$$
$$= \delta_{ij}\delta_{kl} + e^{-S}(\delta_{il}\delta_{jk} - \delta_{ij}\delta_{jk}\delta_{kl})$$

In particular

$$\overline{|\langle\psi_i|\psi_j\rangle|^2} = \delta_{ij} + e^{-S}(1-\delta_{ij})$$

This is sometimes written as (R is unit random matrix)

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + e^{-S/2}R_{ij}$$

So the gravitational prediction (using as input that black hole entropy counts states) is:

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + e^{-S/2}R_{ij}$$

If one did not want to make the a priori assumption that black hole entropy counts microstates, one can also derive this equation by directly computing

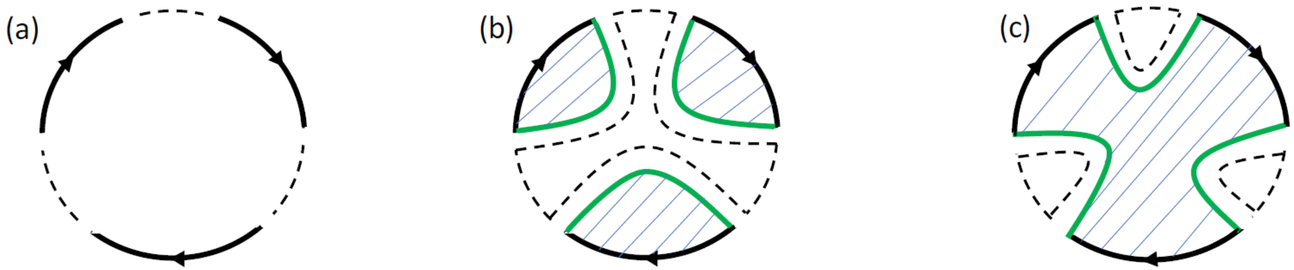
$$\overline{|\langle\psi_i|\psi_j\rangle|^2} = \delta_{ij} + e^{-S}(1-\delta_{ij})$$

using replica wormholes. Either way,

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + e^{-S/2}R_{ij}$$

is the key diagnostic to establish that one is probing a finite dimensional Hilbert space of dimension $\sim e^S$.

Cartoon how the correction arises from “replica wormholes

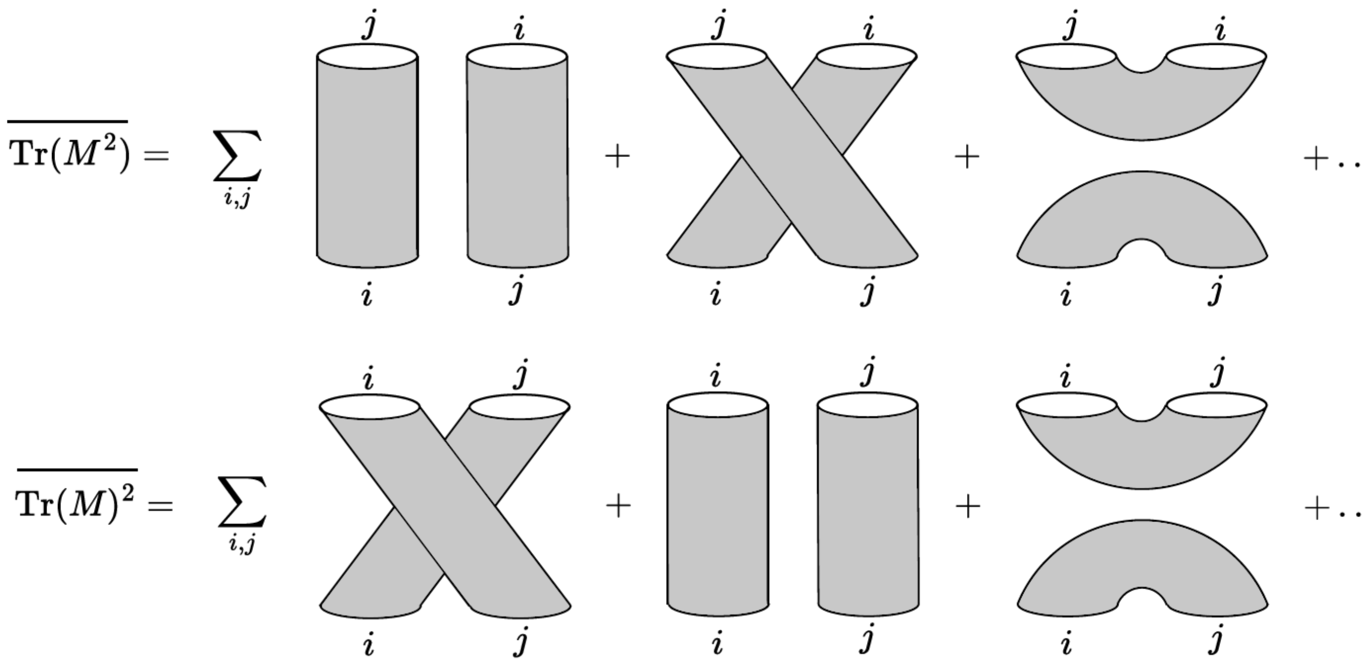


picture from Liu, Vardhan ‘20

Versions of this statistical picture appears in many papers, for example

Goel, Lam, Turiaci, Verlinde ‘18
Penington, Shenker, Stanford, Yang ’20
Pollack, Rozali, Sully, Wakeham ‘20
Liu, Vardhan ’20
Freivogel, Nikolakopoulou, Rotundo ’21
Chadra, Hartman ’22
Bah, Chen, Maldacena ’22
Balasubramanian, Lawrence, Magan, Sasieta ’22
JdB, Liska, Post, Sasieta ‘23
Climent, Emparan, Magan, Sasieta, Vilar Lopez ’24
Iliesiu, Levine, Lin, Maxfield, Mezei ‘24

One can apply these ideas to de Sitter (Harlow, Usatyuk, Zhao ‘25; Abdala, Antonini, Iliesiu, Levine ‘25)



Picture from Harlow, Usatyuk, Zhao, arXiv:2501.02359

Conclusion: de Sitter is an average over pure states

$$\int d\alpha \mu[\alpha] |\psi_\alpha\rangle\langle\psi_\alpha|$$

and it is hard to say more without adding additional structure. If one adds eternal observers with internal degrees of freedom then we end up with the previous situation

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + e^{-S/2} R_{ij}$$

Summary

This statistical physics interpretation of semi-classical gravitational physics is useful and puts many results in a single conceptual framework.

Some issues

- Nothing here disagrees with AdS/CFT - how is factorization restored in the full UV theory?
- What are the rules for including off-shell contributions?
- What is the statistical interpretation of the various wormholes in the literature?
- What to make of topological gravitational theories which are UV complete by themselves (like 2d and 3d gravity)?
- Did not describe another approach for CFT's by approximately imposing CFT axioms (Belin, JdB, Jafferis, Nayak, Sonner '23; Jafferis, Rozenberg, Wong '24)