Thermal noise in the GW Detectors: Hands on

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Paola Puppo, INFN *Roma*

- The thermal noise is present in every macroscopic system at the thermodinamic equilibrium with the environment.

- The internal energy of the system is shared among all its degrees of freedom or equivalently among all the natural vibrational modes, with an average energy $k_b T$ (equipartition theorem).

- The motion of oscillating systems like pendula, springs, elastic bodies is always affected by the thermal noise.

- Its manifestation is in the random fluctuations of the macroscopic observable characterizing the system, limiting its sensitivity.

Thermal noise of the Test Masses and their suspension in the interferometer

- The thermal noise intensity is strictly related to the dissipative processes present in system

- The Levin* equation enhances this aspect giving also a method for evaluating the thermal noise of a system.

*(*) PRD, 57, 2, 1998 Y. Levin, '**Internal thermal noise in the LIGO test masses: A direct approach**'*

Interferometer readout

- The TM displacement is read with the light having a gaussian beam profile with the power:

$$P(\vec{r}) = \frac{2P_o}{\pi\sigma^2} e^{\frac{-2r^2}{\sigma^2}}$$

So the mirror face displacement at the ITF output is given by the weighted amplitude:

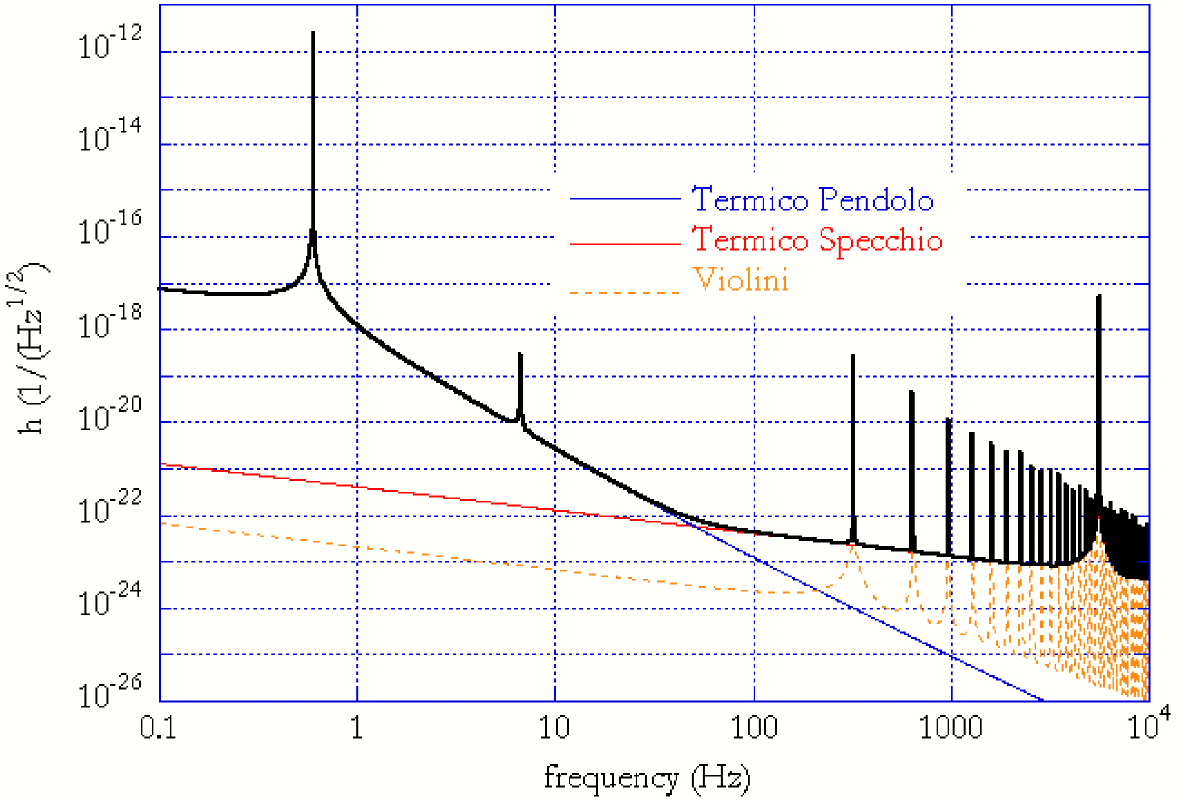
$$\mathrm{x(t)} = \int_S \; P(\vec{r})\, u(\vec{r}) d^2 r$$

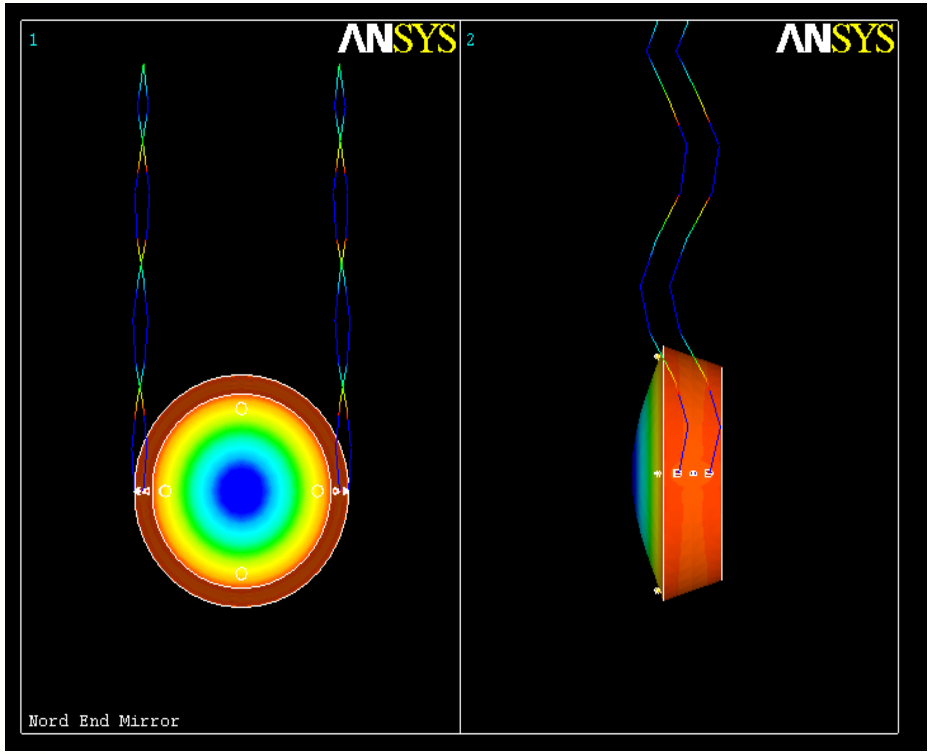
The motion of the TM and its suspension can be seen as the result of a collection of harmonic oscillators each having a different frequency and mass depending on the readout coordinate.

Thermal noise in the sensitivity curve

$$h_{equiv}(\omega) = 2\ \frac{\delta\ \ L}{L} = 2\ \frac{\sqrt{X^2_{tot}}}{L}$$

$$X^2_{tot} = X^2_{pend} + X^2_{mirror} + X^2_{viol}$$

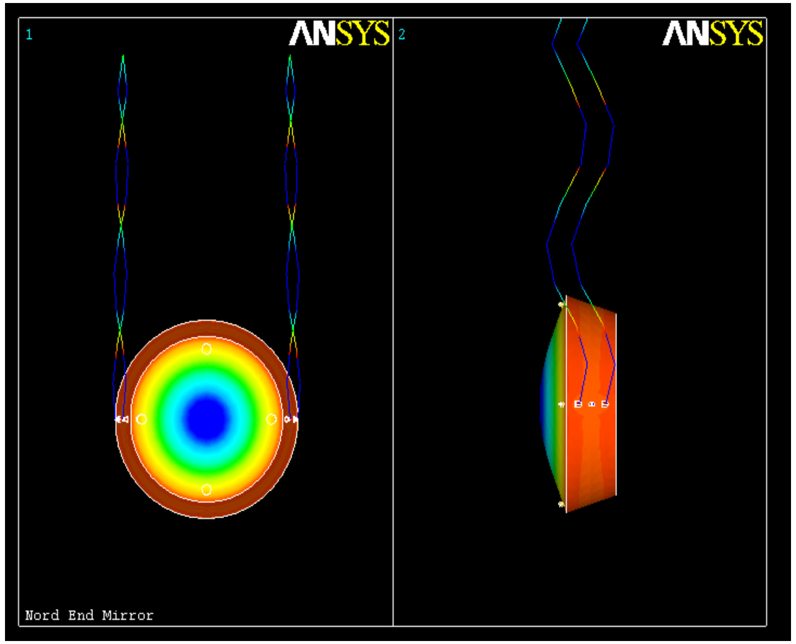




How can we estimate the TN

Test masses (modal expansion)

- Equipartition Energy Theorem:
  - Thermal Energy $k_b T$ same for each mode;
  - Mainly visible those modes coupled with gaussian beam (look at x_n^{eq});



$$X^2_{therm} = \frac{4k_bT}{\omega}\sum_n \frac{\omega_n^2\varphi_n}{m_n}\frac{1}{\left((\omega_n^2-\omega^2)^2+(\omega_n^2\varphi_n)^2\right)}$$

$$\frac{1}{2}m_m\omega_m^2 x_n^{eq\,2} = E_n = \frac{1}{2}k_bT$$

$$x_n^{eq} = \int_{surface} w_{n,z}(\vec{r})\,P(\vec{r})\,dS$$

Equivalent coordinate of the mode n

$$m_n = \frac{\int_{volume}\rho|\vec{w}_n(\vec{r})|^2 dV}{\left|\int_{surface} w_{n,z}(\vec{r})\,P(\vec{r})\,dS\right|^2}$$

Equivalent Mass of the mode n

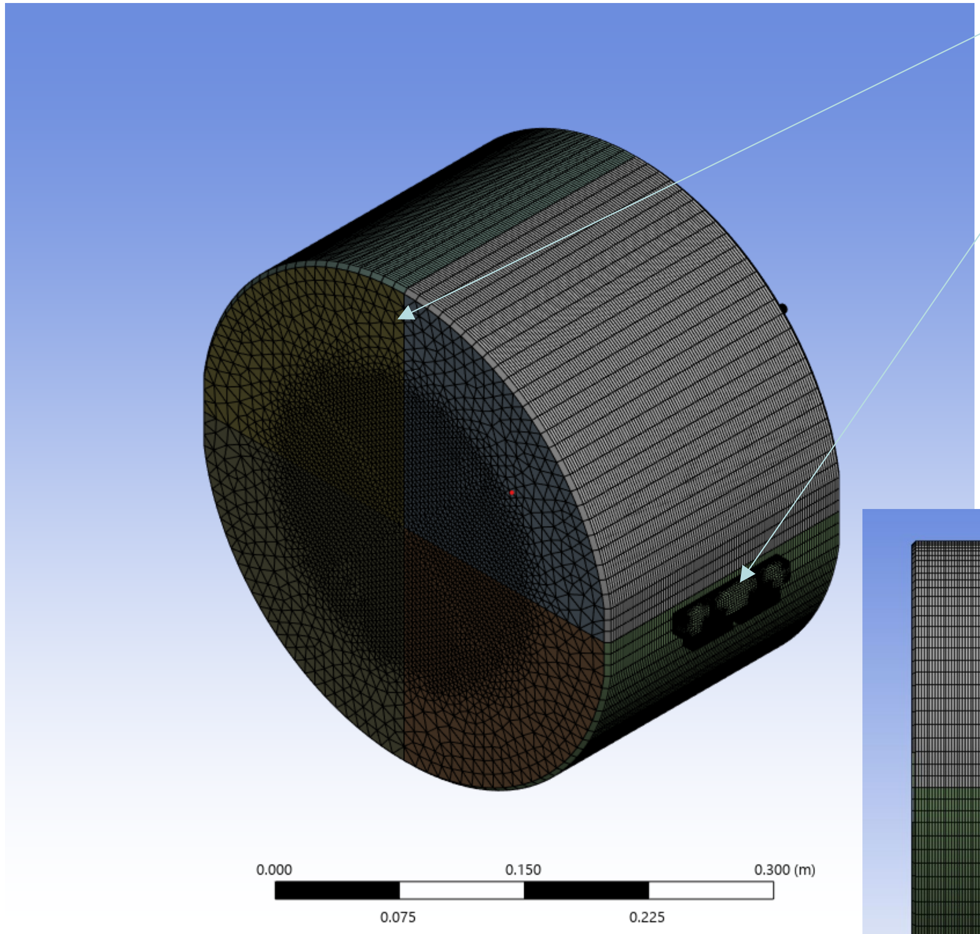
φ_m loss of mode n

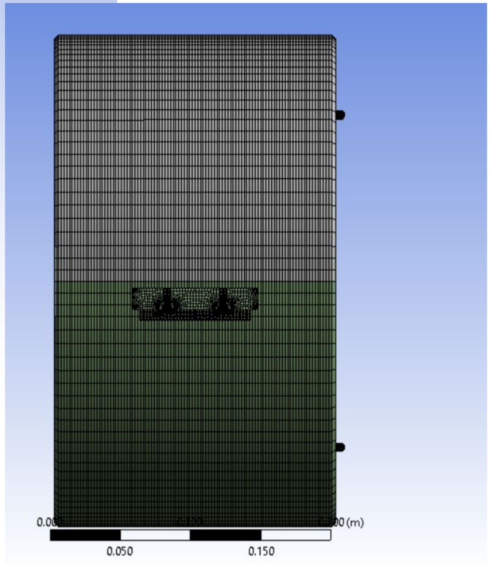
Estimation of the modes and effective masses

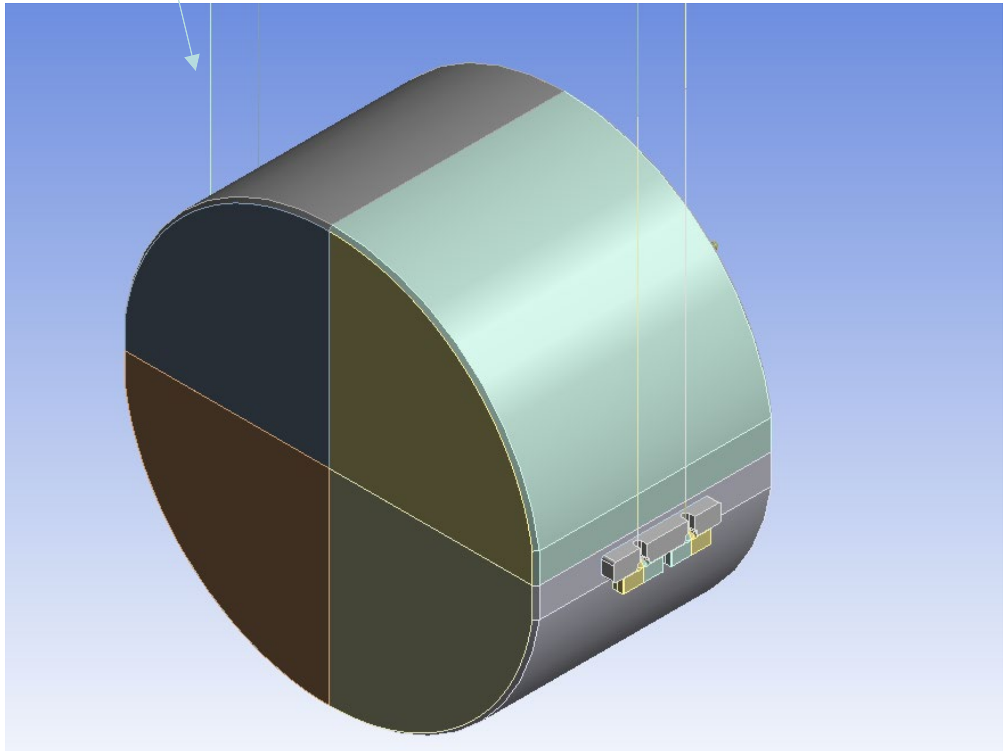
FEM

Dissipation Sources: Coating, HCBs (Ears and anchors)
Magnets

Wires for pendulum







The equivalent parameters

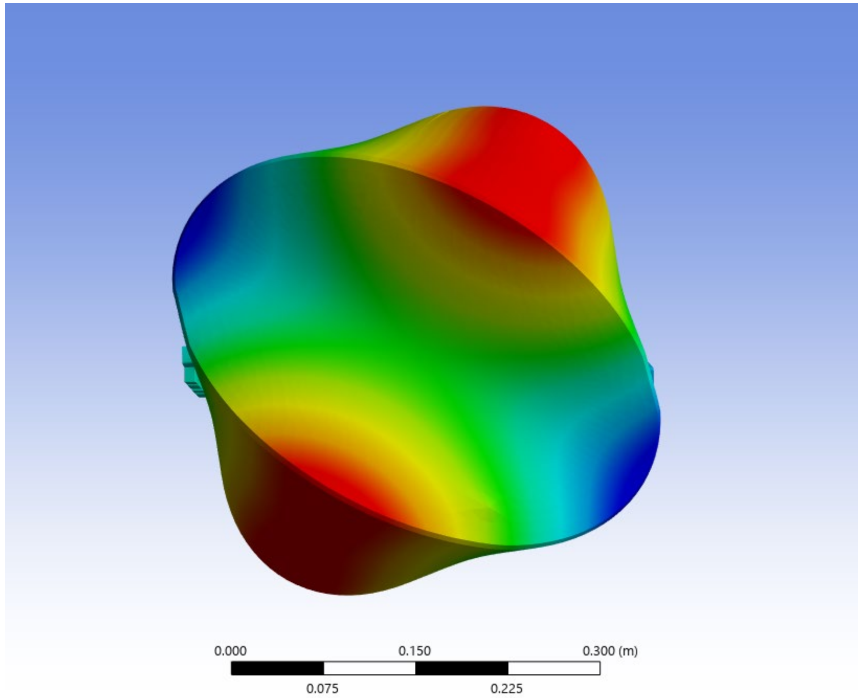
Pendulum (theory):

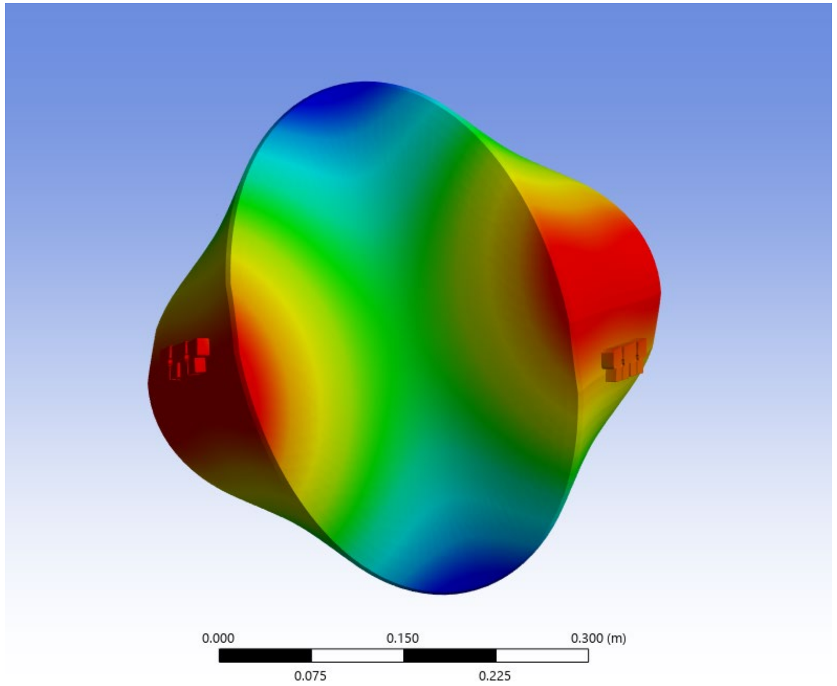
$M_{eq}=M_{mirror}$

$x_{eq}=x_{mirror}$

TM :

The asymmetric modes





Equivalent masses are infinite→not visible

But…

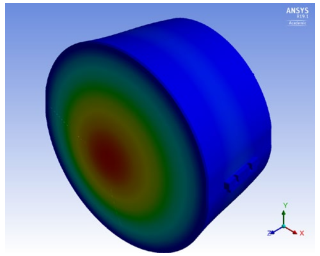
- Issues:

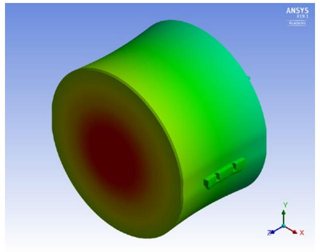
Beam centering

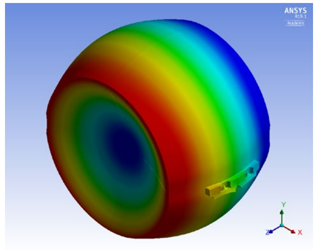
Shape defects, mounting defect

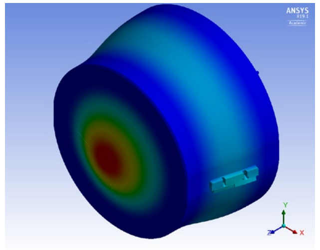
Hovewer…

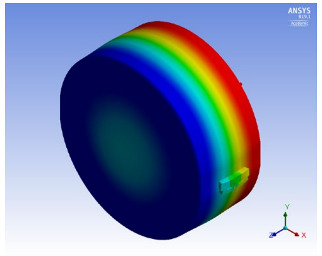
Their TN does not matter









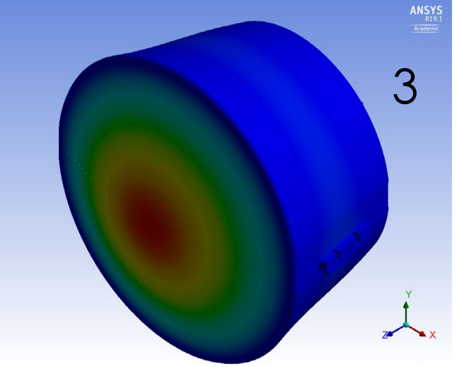


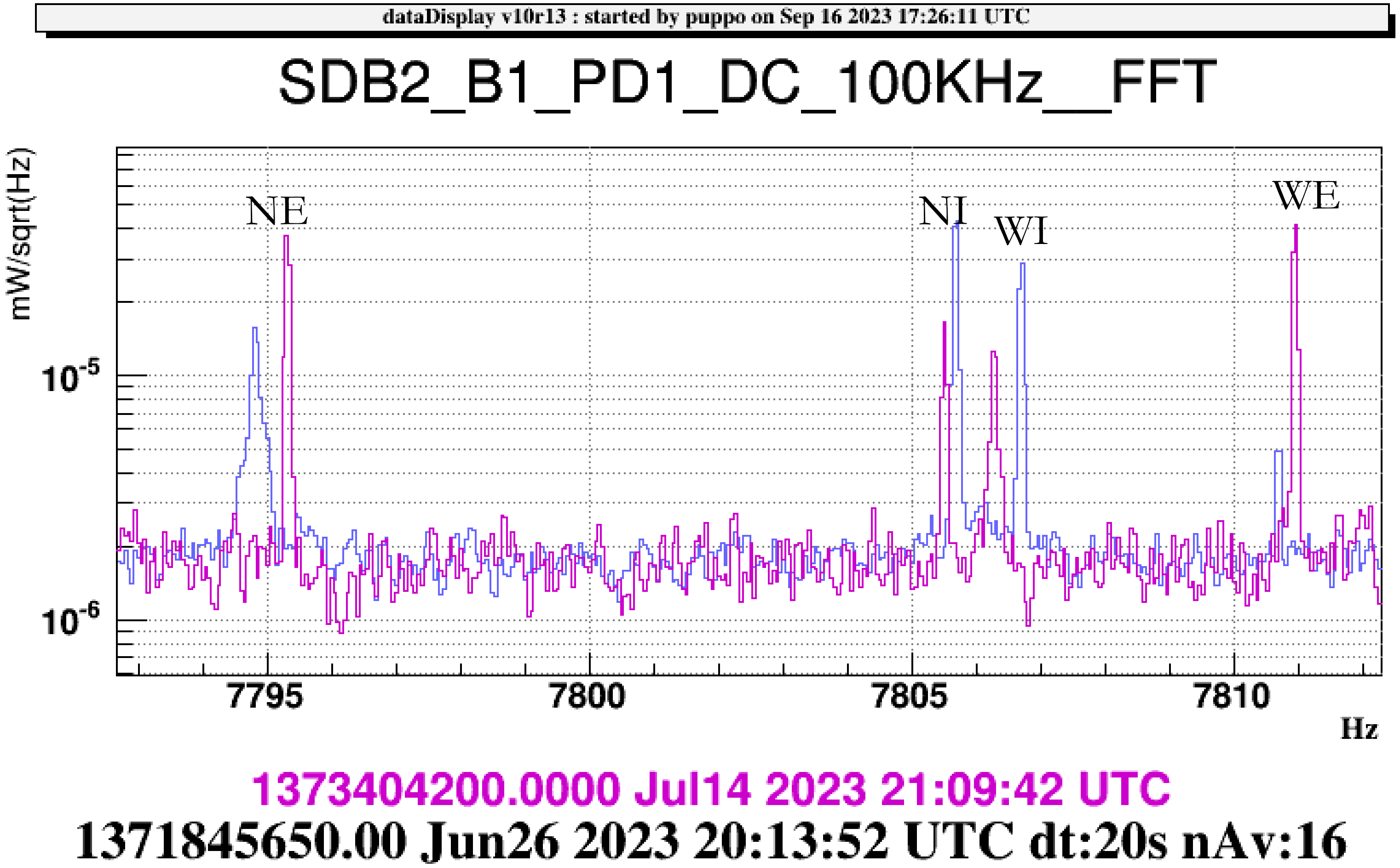
AdV TM
Modal Shapes
Symmetric Modes along optical axys

N	Name	Freq from FEM	Equivalent Mass	
3	Drum	7805 Hz	25	kg
13	Longitudinal	10100 Hz	78.2	kg
20	Barrel	12900 Hz	17.8	kg
33	DrumII	14835 Hz	9.2	kg
34	Longitudinal II	15008 Hz	54.8	kg

$$\langle x_n^{eq} \rangle = \sqrt{\frac{k_b T}{m_m \omega_m^2}}$$

A look on the output noise curve


ANSYS
3


dataDisplay v10r13 : started by puppo on Sep 16 2023 17:26:11 UTC
SDB2_B1_PD1_DC_100KHz__FFT
mW/sqrt(Hz)
NE
NI
WI
WE
10^{-5}
10^{-6}
7795
7800
7805
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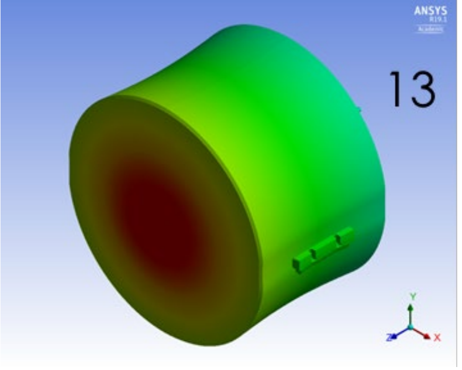
Hz
1373404200.0000 Jul14 2023 21:09:42 UTC
1371845650.00 Jun26 2023 20:13:52 UTC dt:20s nAv:16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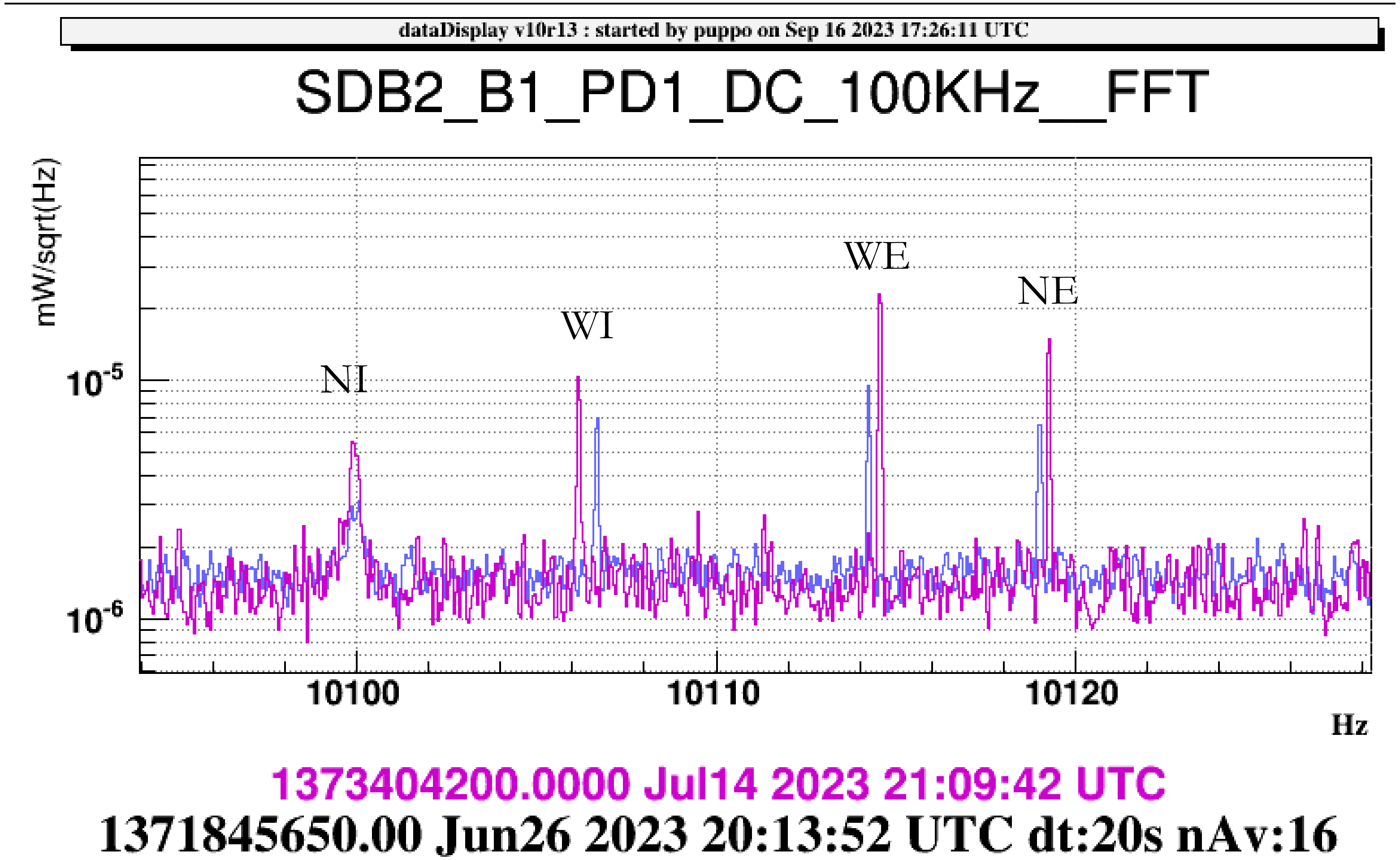
dataDisplay v10r13 : started by puppo on Sep 16 2023 17:26:11 UTC
SDB2_B1_PD1_DC_100KHz__FFT
mW/sqrt(Hz)
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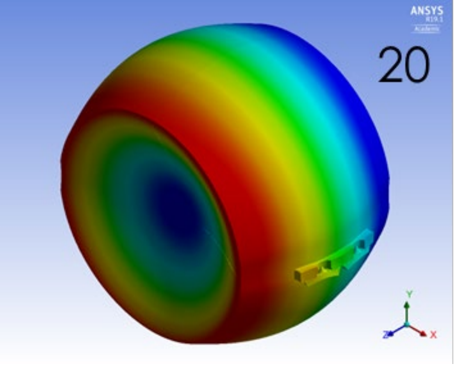
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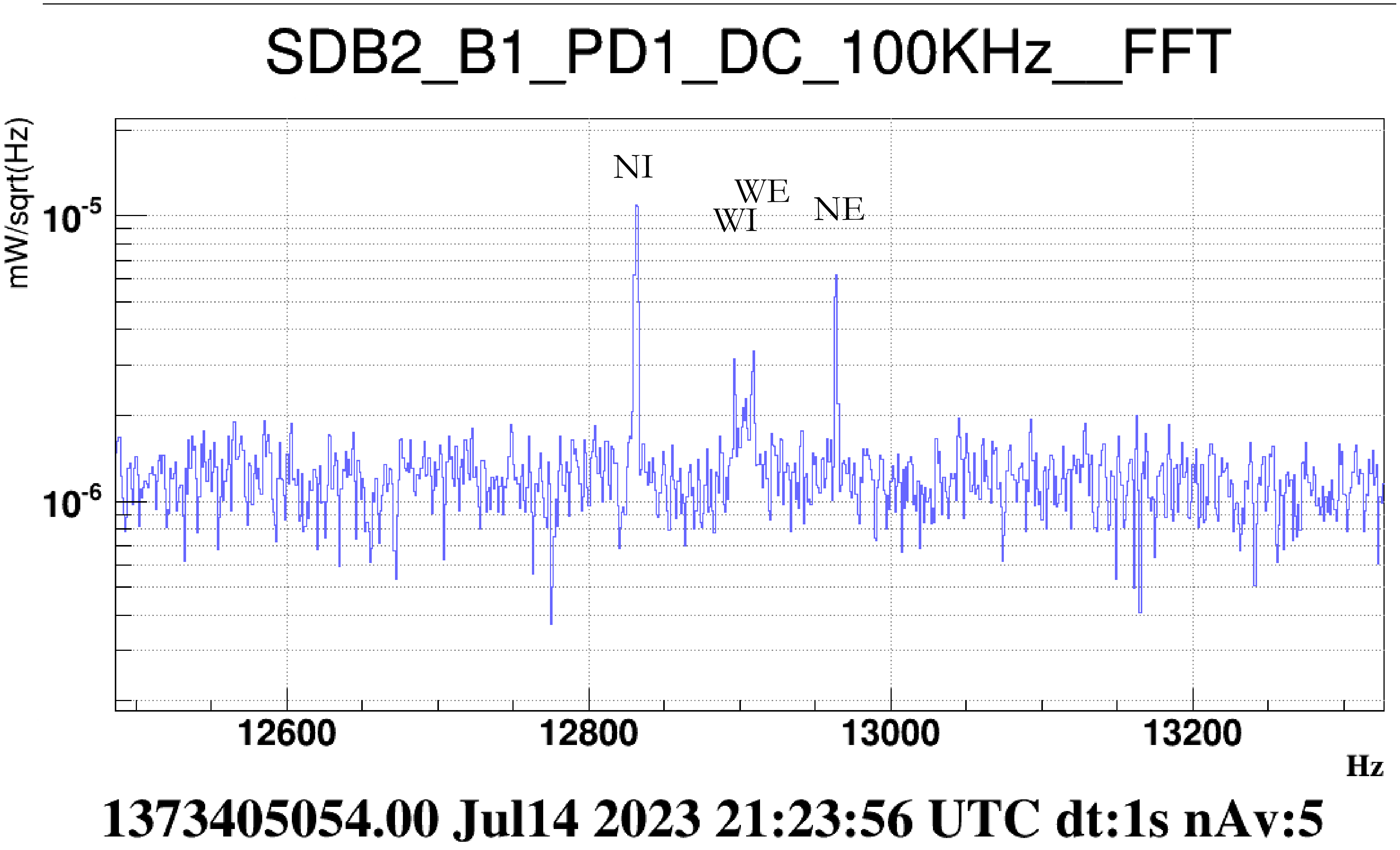
SDB2_B1_PD1_DC_100KHz__FFT
mW/sqrt(Hz)
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10^{-6}
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12800
13000
13200
Hz
1373405054.00 Jul14 2023 21:23:56 UTC dt:1s nAv:5

Levin Equation

To work out the thermal noise at a particular frequency f, one should mentally apply pressure oscillating at this frequency to the observed surface of the test mass. The spatial variation of this pressure should mimic that of the light beam intensity (for example, in the case of a Gaussian beam this oscillating pressure has a Gaussian profile of the same width as the beam). The thermal noise is then given by

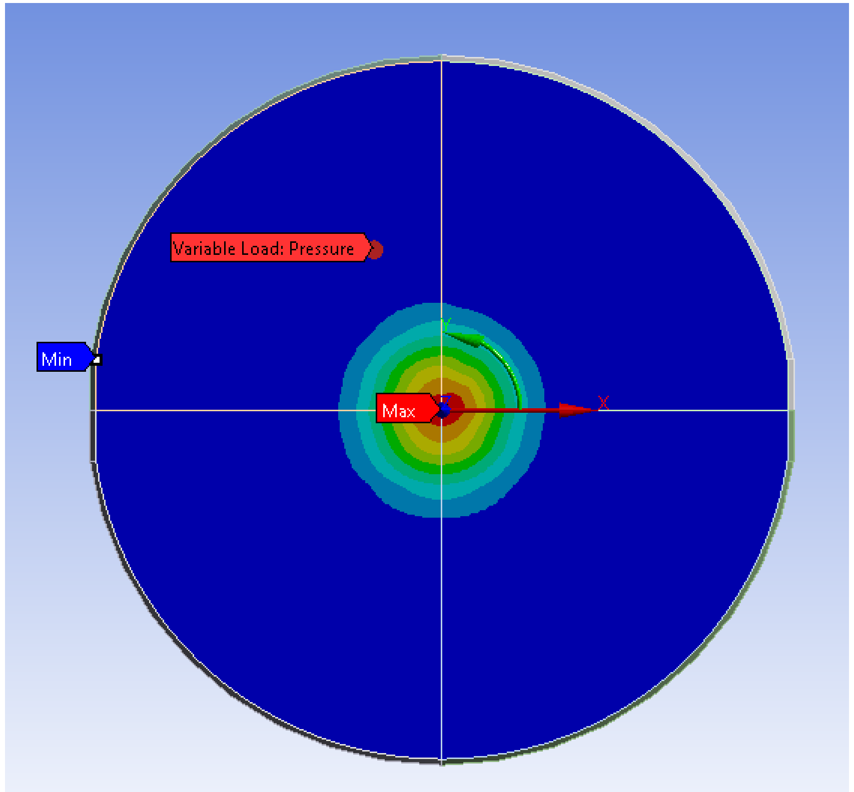
$$S_x(f) = \frac{2k_B T}{\pi^2 f^2} \frac{W_{\text{diss}}}{F_0^2}, \tag{1}$$

$$W_{diss} = 2\pi f \, \phi \, E_{strain}$$

How to calculate the strain energies

So applying on the mirror face an oscillatory pressure

$$P(\vec{r},\omega) = \frac{2F_o}{\pi\sigma^2} e^{\frac{-2r^2}{\sigma^2}} cos(\omega t)$$



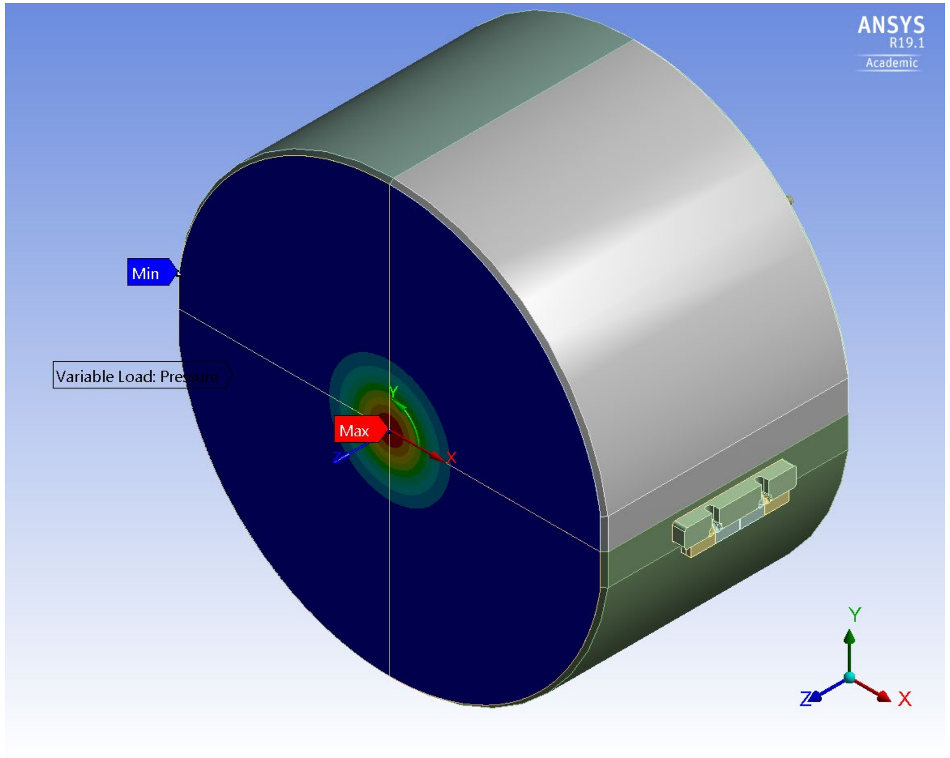
and a frequency ω we have

$$W_{diss} = \omega\, \phi\, E_{strain}$$

E_{strain} is the energy of elastic deformation at a moment when the test mass is maximally contracted or extended under the action of the oscillatory pressure.

Suspension Thermal
(Levin Formula)

$$S_X^{FEM}(\omega) = \frac{4\,k_b T}{\omega\, F_o^2} 2\left(\phi_{wires} E_{wires}(\omega) + \phi_{layers} E_{layers}(\omega) + \phi_{Mario} E_{Mario}(\omega) + \phi_{\substack{Silica\\parts}} E_{\substack{Silica\\parts}}(\omega) + \phi_{\substack{Mario\\Cable}}\, E_{\substack{Mario\\Cable}}(\omega) \ldots\right)$$



Strain energies $E_i(\omega)$ from the FEM applying a unitary gaussian force on the suspended mirror face, spanning from 10 to 1000Hz

Gaussian waist similar to the interferometer beams(for AdV – O3).

In the FEM:
IN: 4.87 cm
END: 5.8 cm

How can I estimate the losses?

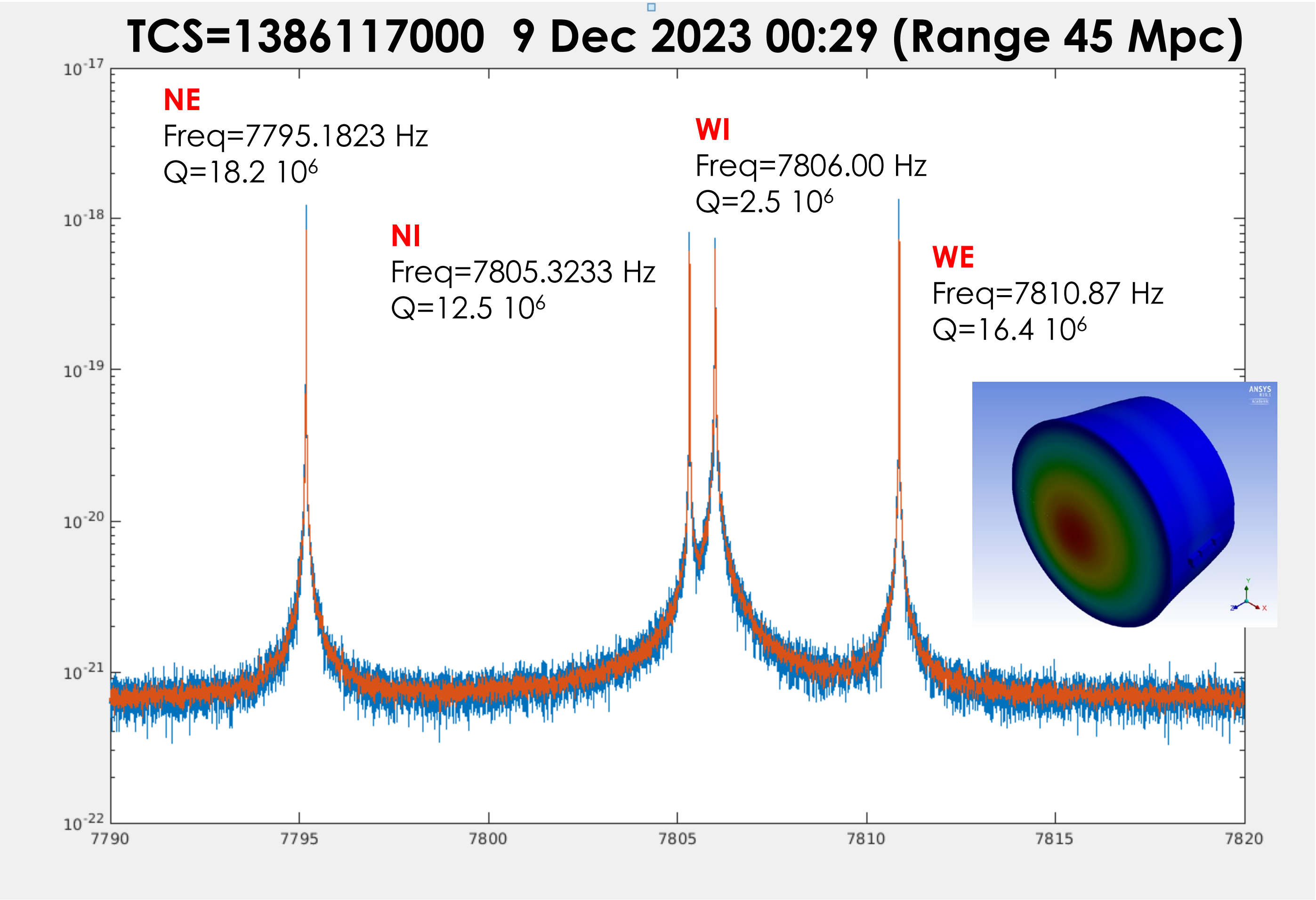
$$X_i^2 = \frac{4k_bT}{\omega}\frac{\omega_i^2\varphi_i}{M_i}\frac{1}{\left(\left(\omega_i^2-\omega^2\right)^2+\left(\omega_i^2\varphi_i\right)^2\right)} \quad =2$$

$$\Delta\omega_i = \omega_i\varphi$$

Lorentian widths (HWA)

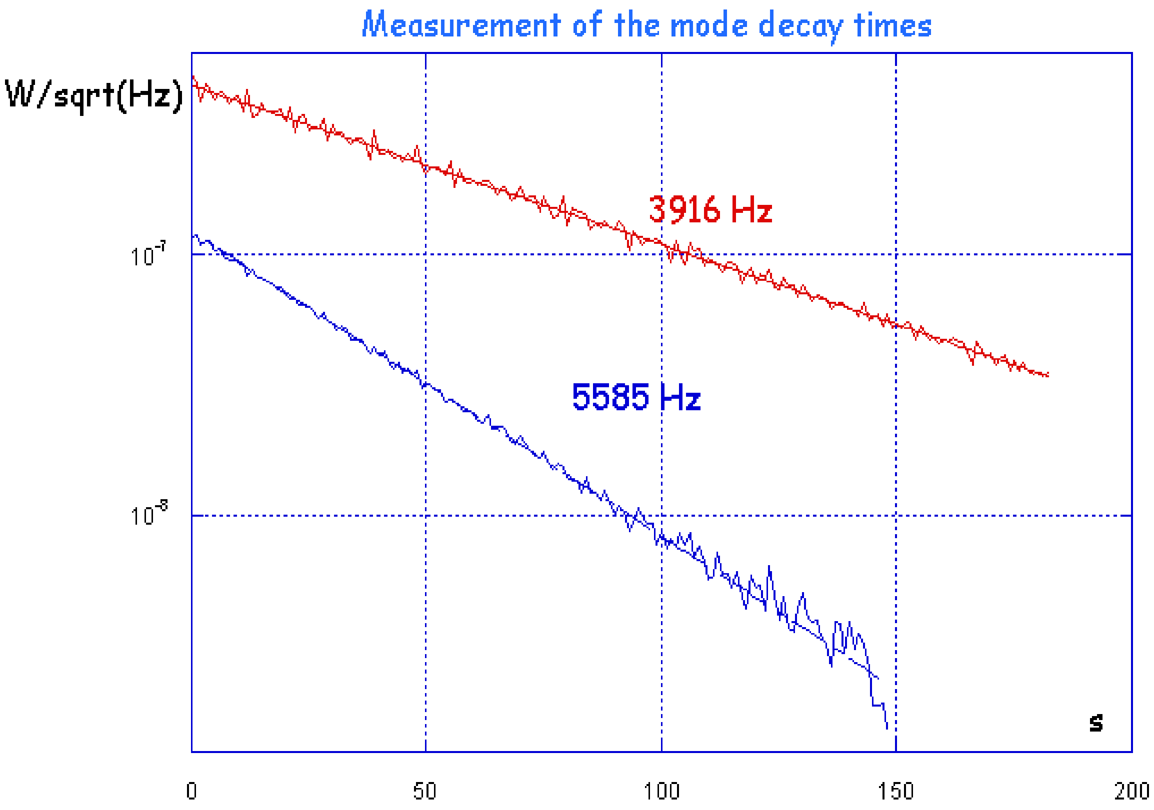
$$Q_i = \frac{\omega_i}{\Delta\omega_i} = \varphi_i^{-1}$$

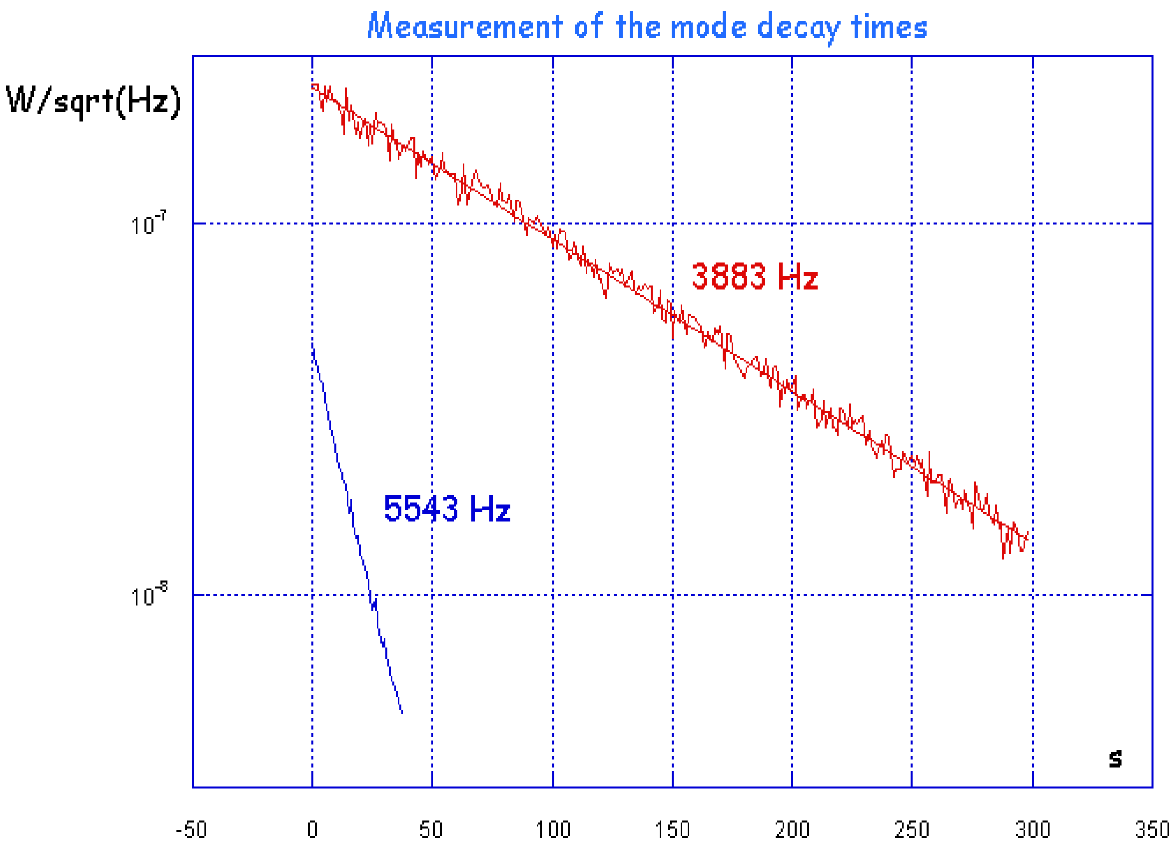
$$\langle x_n^{eq}\rangle = \sqrt{\frac{k_bT}{m_m\omega_m^2}}$$



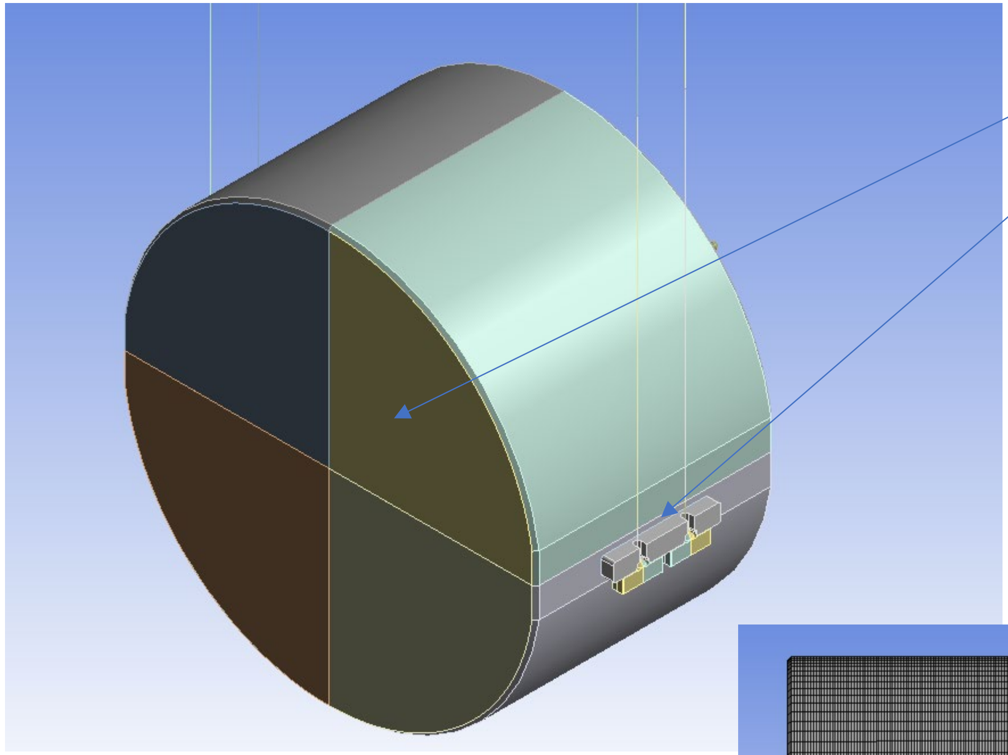
Ring downs

$$x(t) = x_o \exp\left(-\frac{t}{\tau}\right) \qquad Q = \frac{\omega_o \tau}{2}$$





FEM Model (for nonhomogeneous losses)



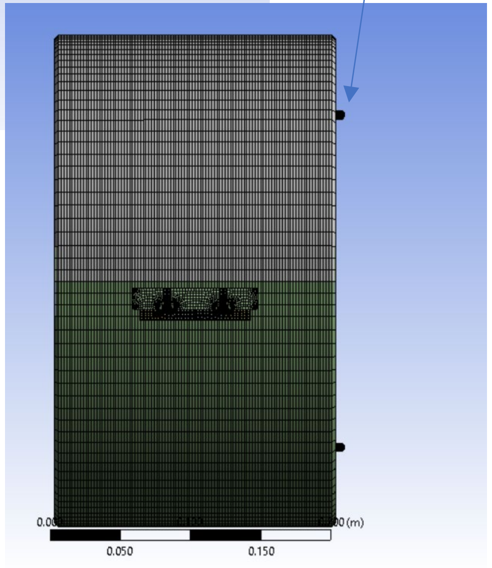
Dissipation Sources: Coating, HCBs (Ears and anchors) magnets

Calculation of the modal strain energies for Q calculations

$$\phi_{tot} = \phi_{mirror} + \phi_{coat} + \phi_{bondings}$$

$$\phi_i = \phi_{oi} \frac{E_i}{E_{tot}}$$

E: Strain Energy



$$Q_i = 1/\emptyset_i$$

Calculation of the Mirror Thermal Noise with Levin Method

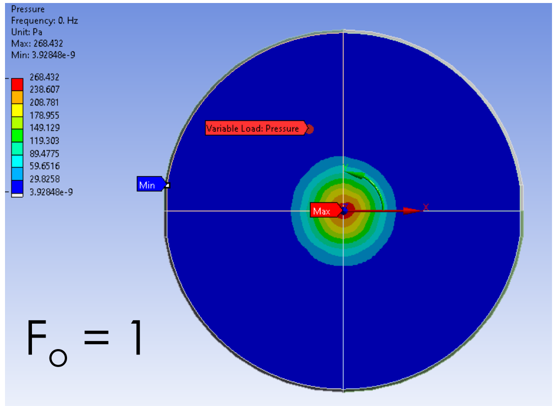
$$S_{mir} = \frac{8 k_b T}{\omega F_o^2} \left(\phi_{oi} E_i \right)$$

$i = 1, \ldots, n$

1: COATING

2: HCBs

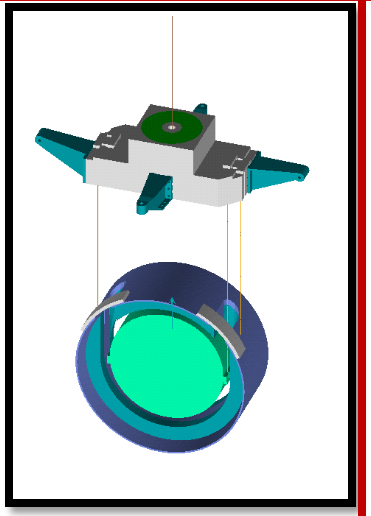
3: MAGNETS

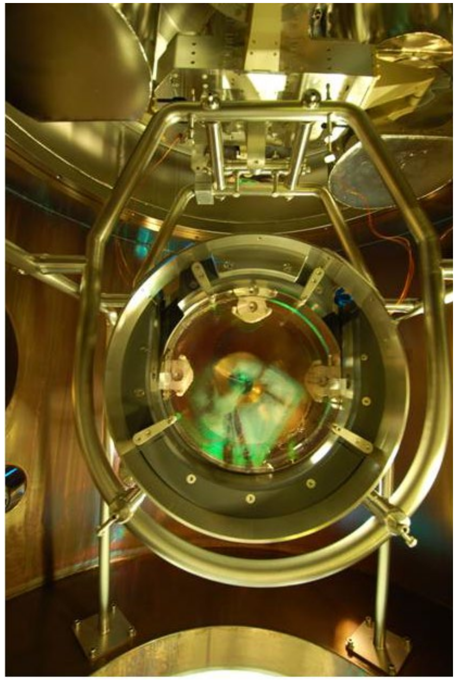


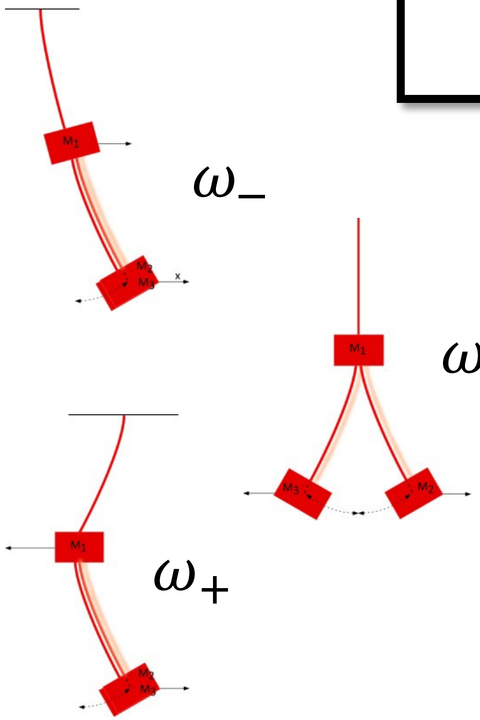
$F_o = 1$

Thermal noise computation: payload analytical models in Gwinc

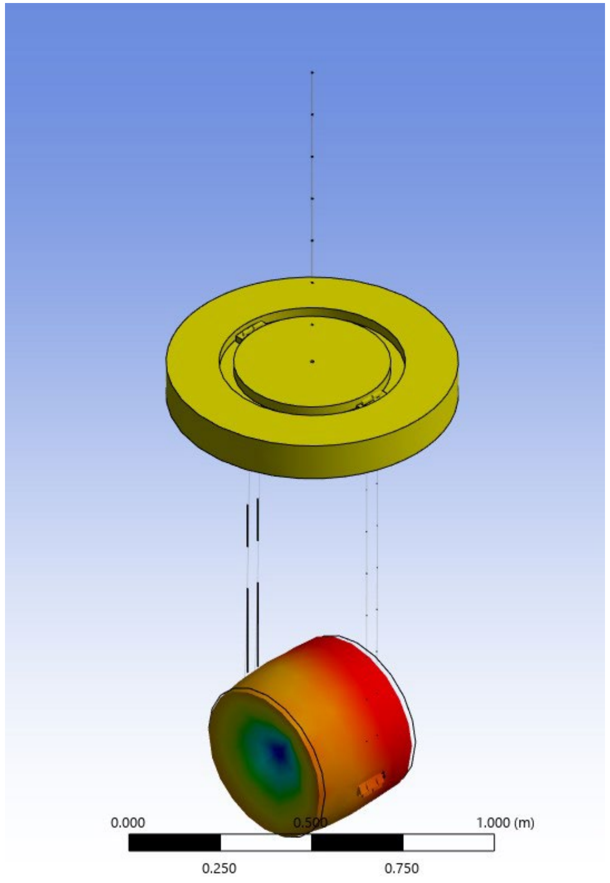
OldVirgo/Kagra Payload
Triple mode system

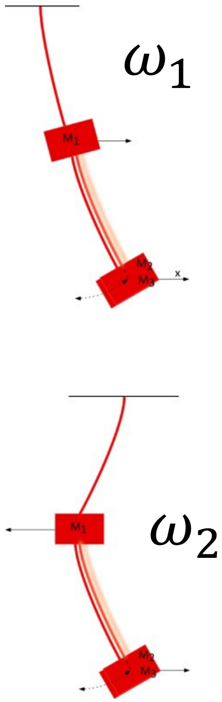






ET-AdV Payload
Double mode system

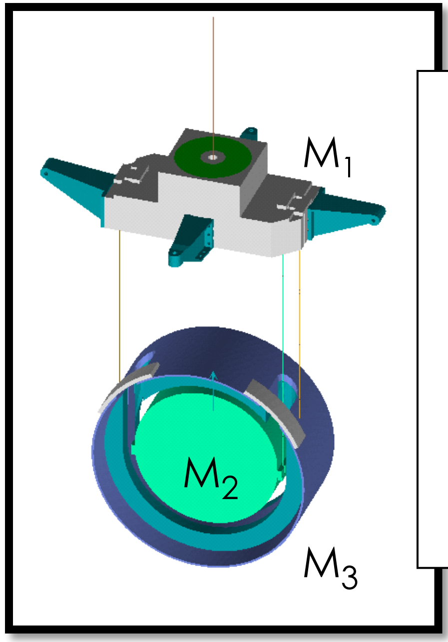


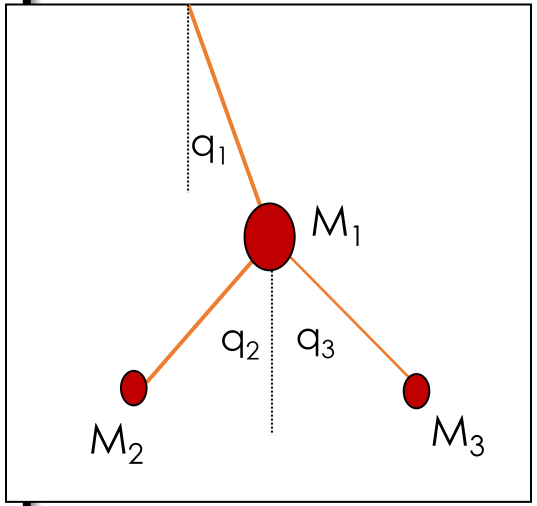


Double Pendulum (theory in Virgo Note VIR-015C-09)

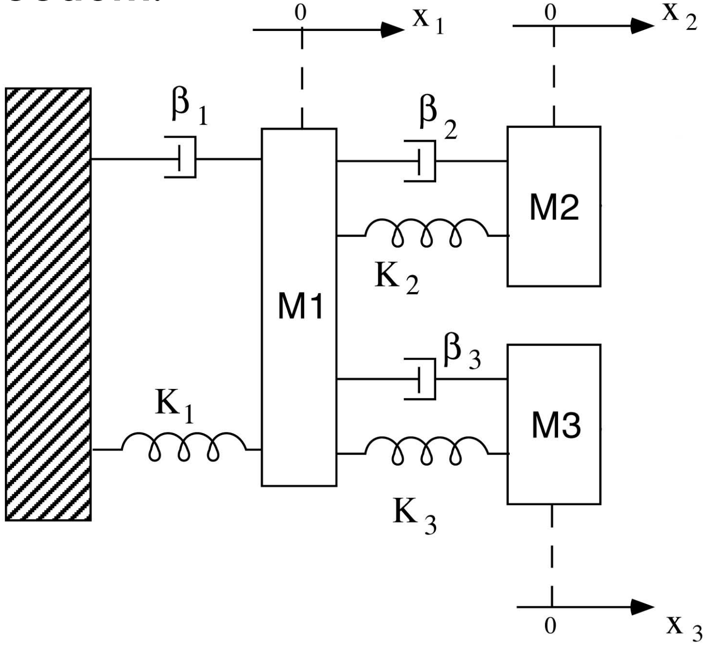
The mirror last stage suspension as a branched system

A Virgo-like last stage suspension is a cascade of three pendula. To the first pendulum (the marionette) the mirror and the recoil mass are hung as branches.





Equivalent to a branched combination of three harmonic oscillators [*]. This is true for horizontal and vertical degrees of freedom.



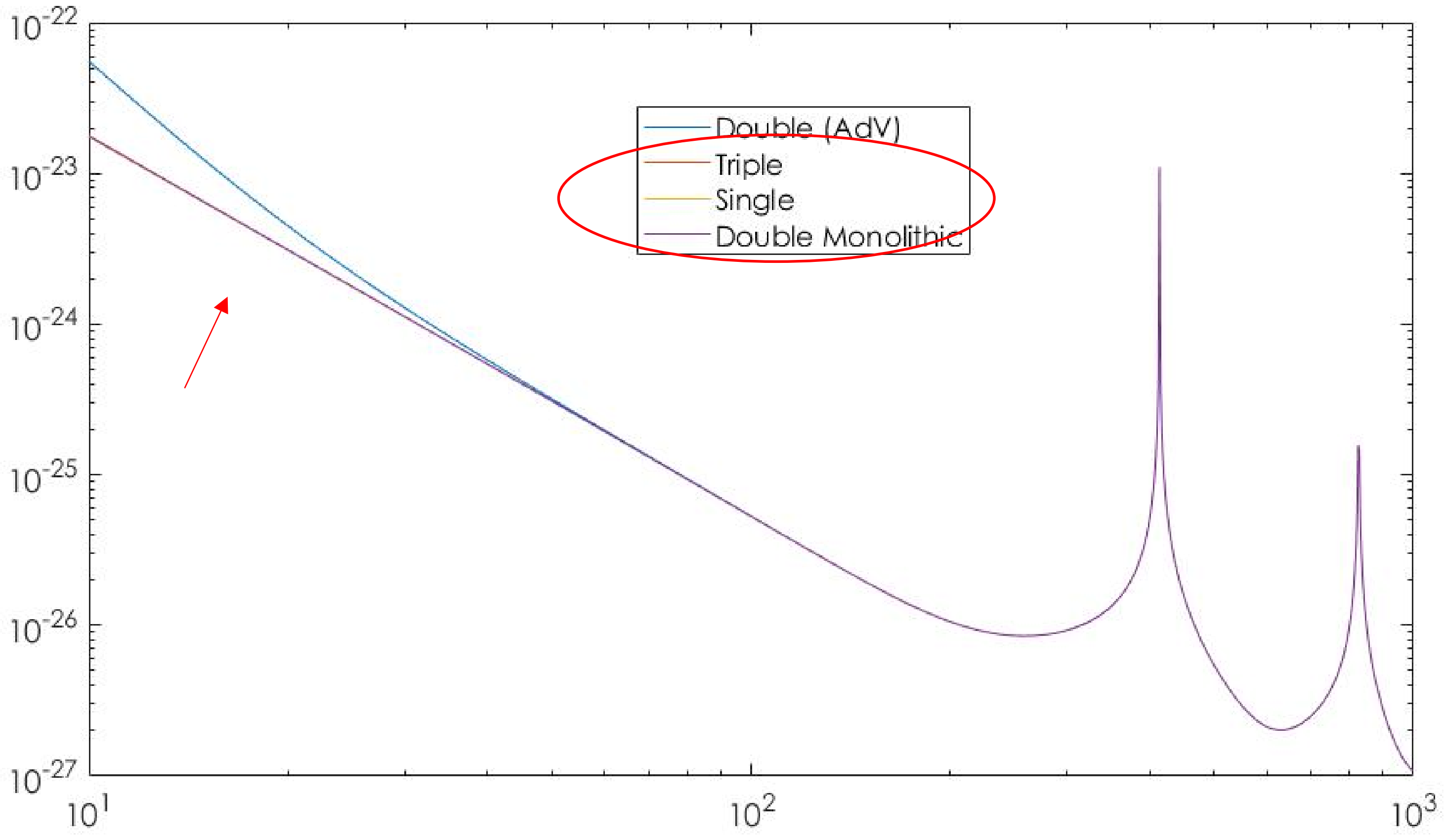
M_1marionette
M_2mirror
M_3recoil mass

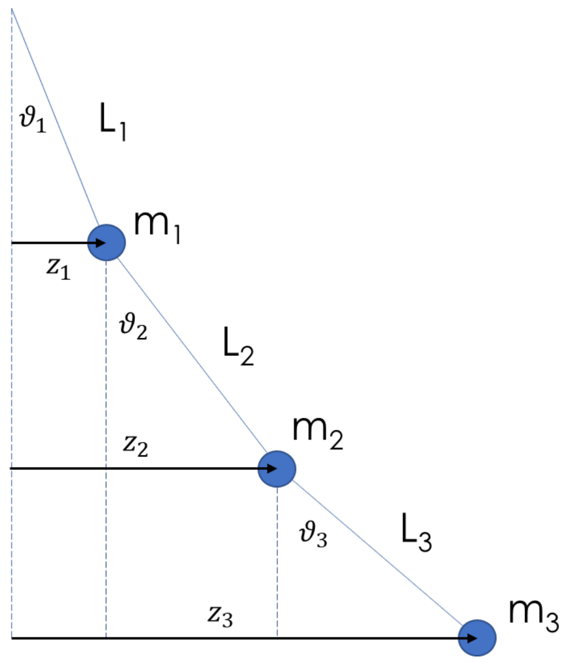
[*] Bernardini A., Majorana E. ,Puppo P. ,Rapagnani P. , Ricci F., Testi G. "Suspension last stages for the mirrors of the Virgo interferometric gravitational wave antenna." *Rev. Sci. Instr.* 70, no. 8 (1999): 3463.

Suspension thermal of the Triple pendulum

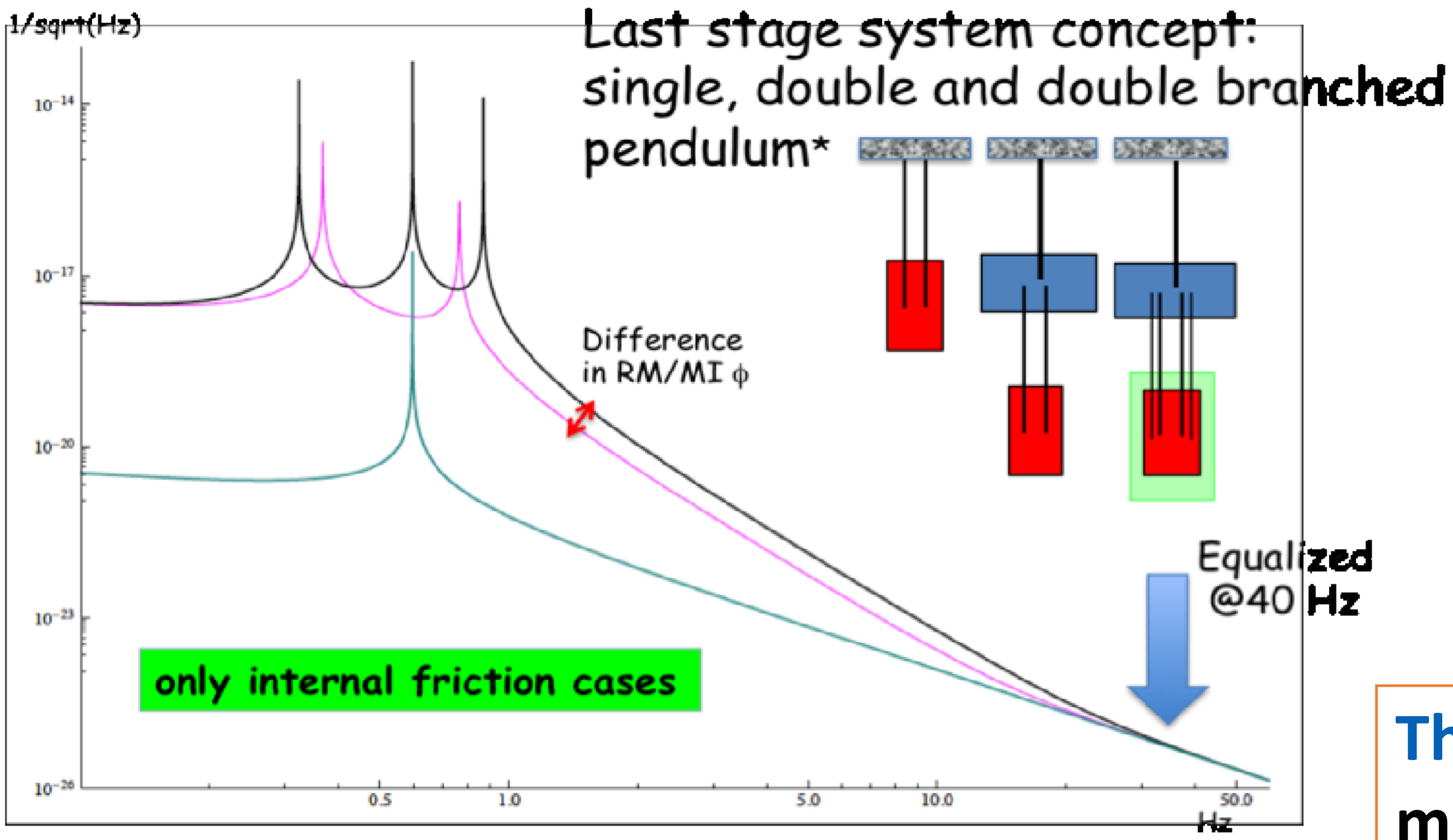
L_1=1125mm Maraging (Qmario=50000)
L_2=700 mm Silica
L_3=700 mm Silica (opt)

m_1=185 kg
m_2=105 kg
m_3=105 kg





From the Payload Scheme, we build a Dynamical Model with masses, momenta of inertia and control strategy (including locking of the ITF)



*reasonable values are assigned to describe double and simple pendulum parameters assumed here.

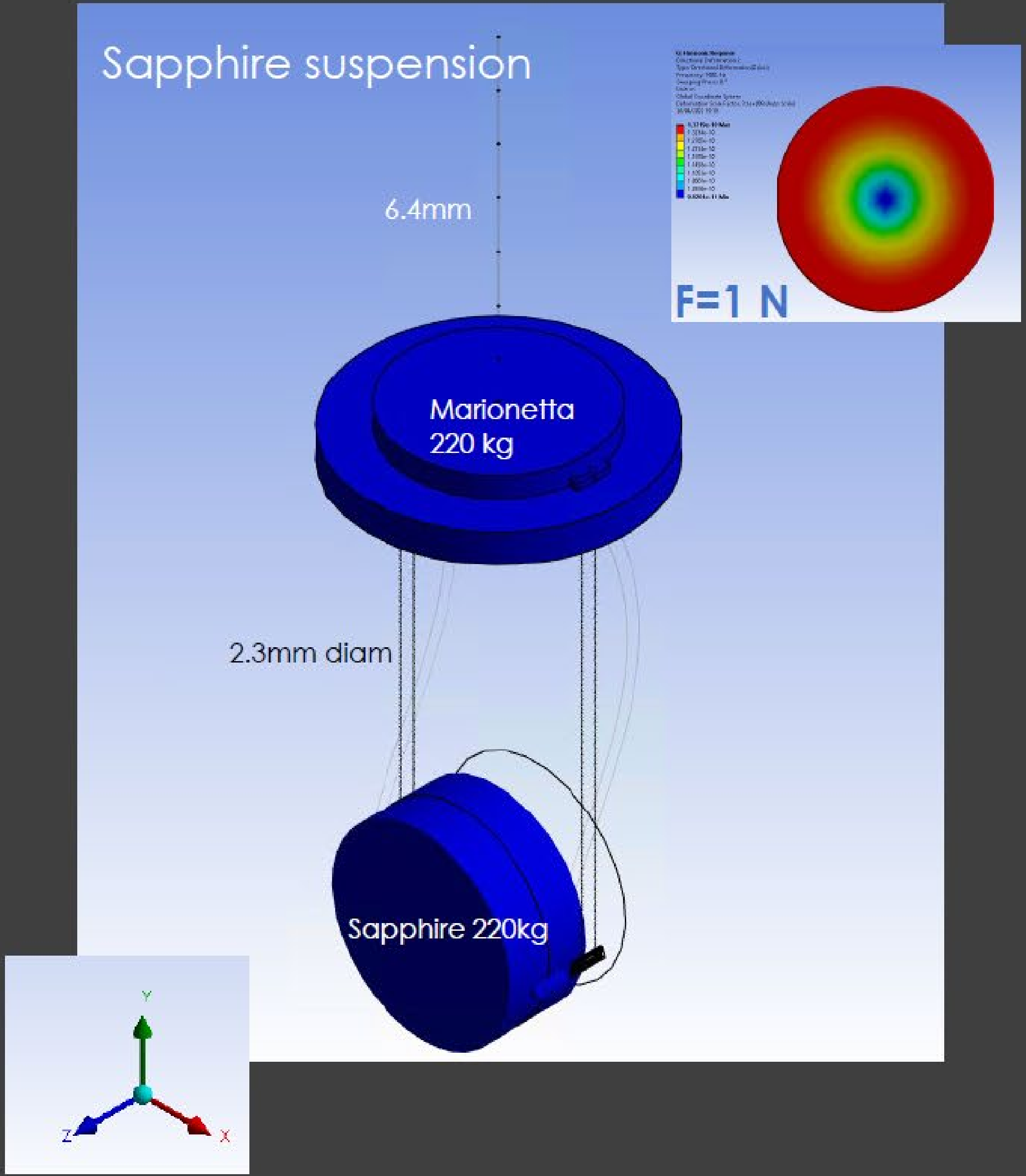
Thermal Noise check: **Pendulum modes of the payload must not spoil sensitivity**

STN in Gwinc cfr with FEM

let’s have a look on the example

Our example Parameters

	Marionetta	Mirror
Mass (kg)	220	220
Wire Diam (mm)	6.5	2.3
Wire Length (m)	1.0	1.2
Wire Material	Sapphire	Sapphire
Structural Loss Angle	$3\ 10^{-9}$	$3\ 10^{-9}$
Temperature (K)	20	20



Suspension Thermal with FEM

- $S^{FEM}_{Bulks}(\omega) = \frac{4\,k_b T}{\omega\, F_o^2} 2\, \sum_i \phi_i E_i T_i \qquad (T_{mario}\quad T_{mirror}\ .....)$
- $S^{FEM}_{Wires}(\omega) = \frac{4\,k_b T}{\omega\, F_o^2} 2\, \sum_i \phi(T_i, y_i) E_i(y_i) T_i$

  $\phi(T_i, y_i) = \phi_{thermo}\,(T_i, y_i) + \phi_{material} + \phi_{extra}$

$E_i(y_i)$ *from Mechanical*
$T_i(y_i)$ *from Thermal Steady*

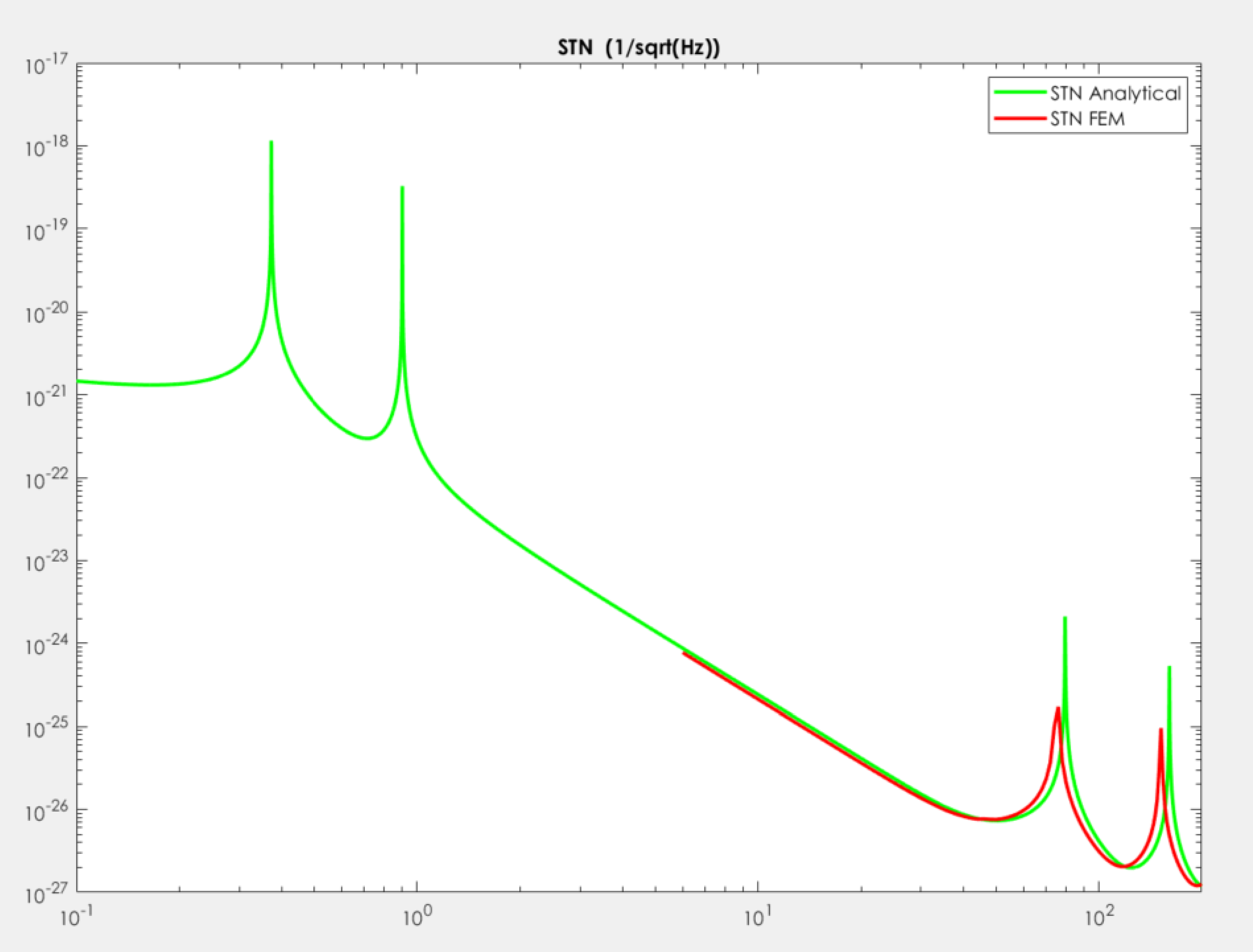
Files

- StrainModes.txt
- StrainModes_2.txt
- Strains_cavo1.txt
- Strains_cavo2.txt
- Strains_cavo3.txt
- Strains_cavo4.txt
- Strains_Mario.txt

Input files from FEM

- SuspET_Sapphire.m — Configuration File
- suspR_ET.m
- suspRmodal.m
- suspViol.m
- ThermalNoiseCalculation.m — Matlab Script File

Result


STN (1/sqrt(Hz))
STN Analytical
STN FEM
10^-17
10^-18
10^-19
10^-20
10^-21
10^-22
10^-23
10^-24
10^-25
10^-26
10^-27
10^-1
10^0
10^1
10^2

References

- *PRD, 57, 2, 1998 Y.* Levin, *'**Internal thermal noise in the LIGO test masses: A direct approach**'*

- VIR-0074B-12 (**Virgo+ Thermal Noise Study**)

- *The thermal noise of the Virgo+ and Virgo*
Advanced Last Stage Suspension (The PPP effect), Virgo Note VIR-015C-09, 2009.