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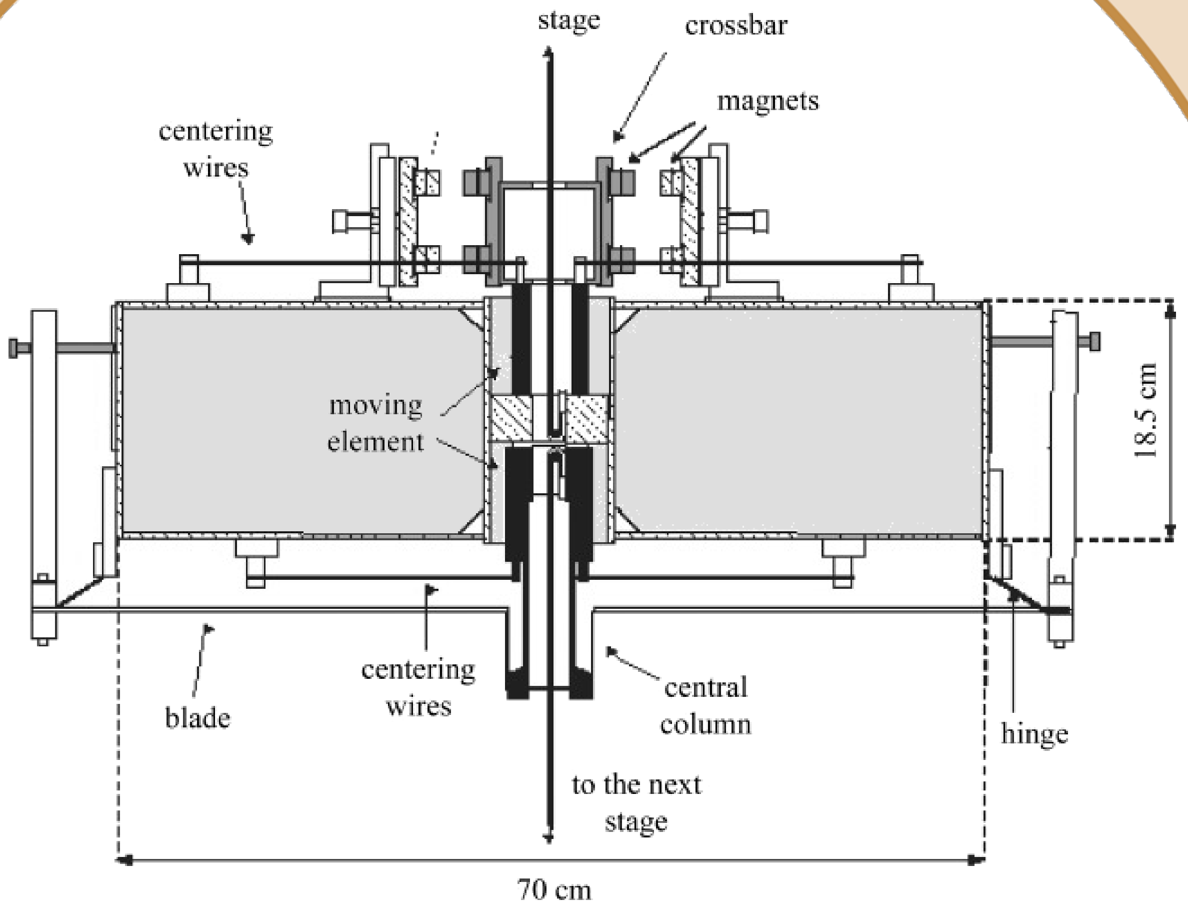
The attenuation of vertical seismic vibrations



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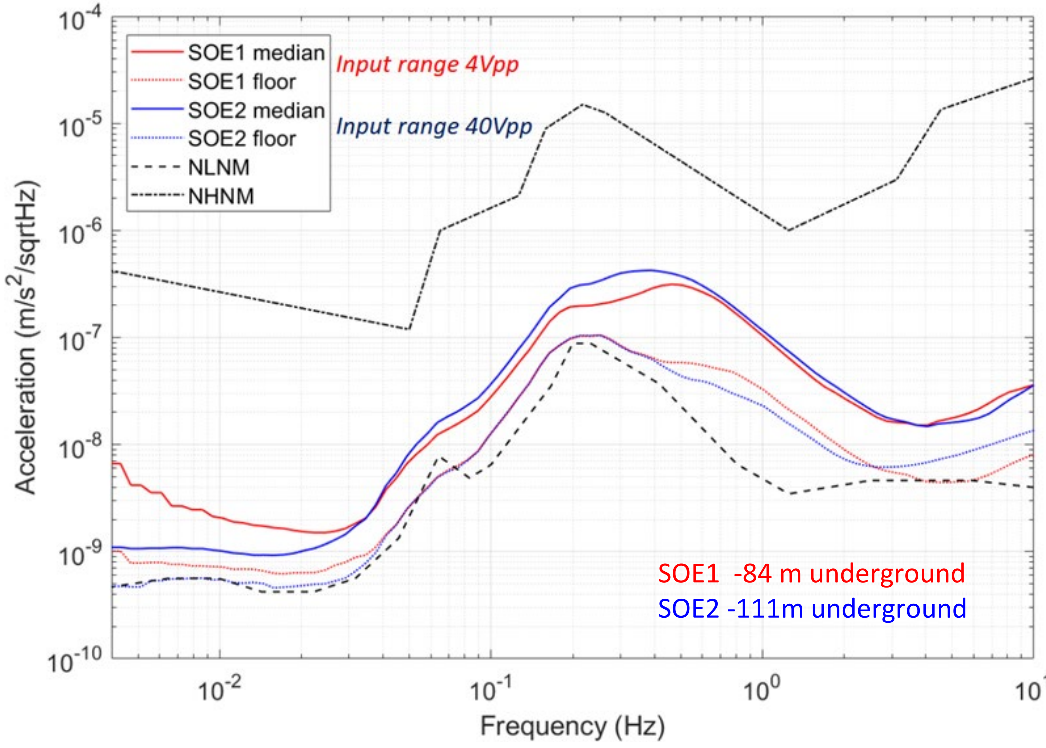
Seism never sleeps

Sos Enattos

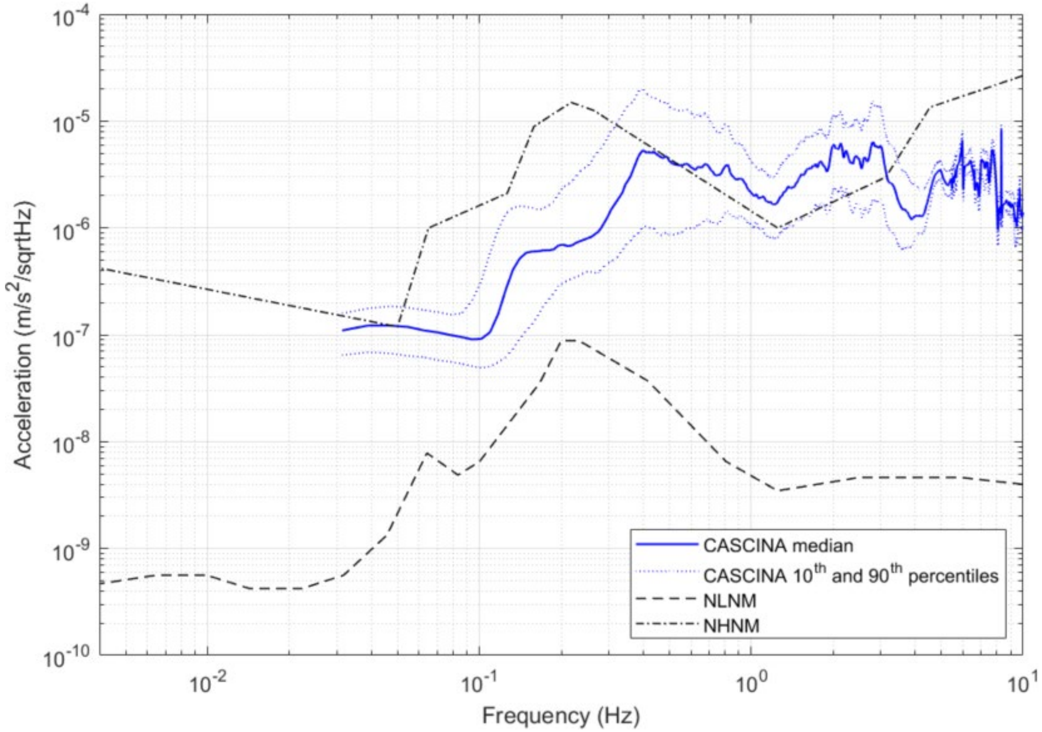
Vertical Acceleration ASD



L. Naticchioni – GWADW21 – May 17th – 21st 2021

Virgo

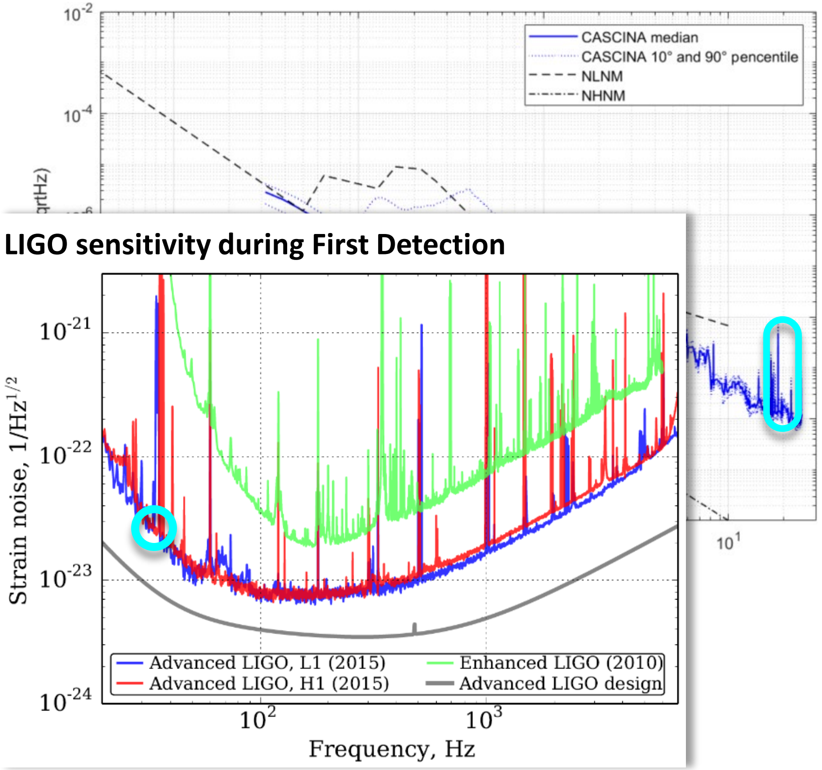
Horizontal Acceleration ASD



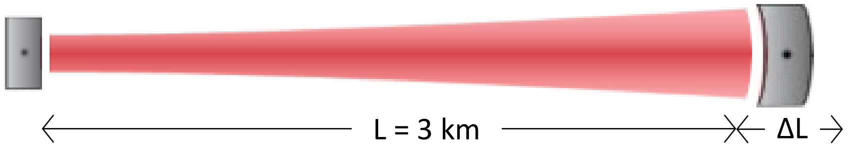
Courtesy of Irene Fiori (EGO-Cascina)

Vibrations must be rejected

Virgo Horizontal Displacement ASD



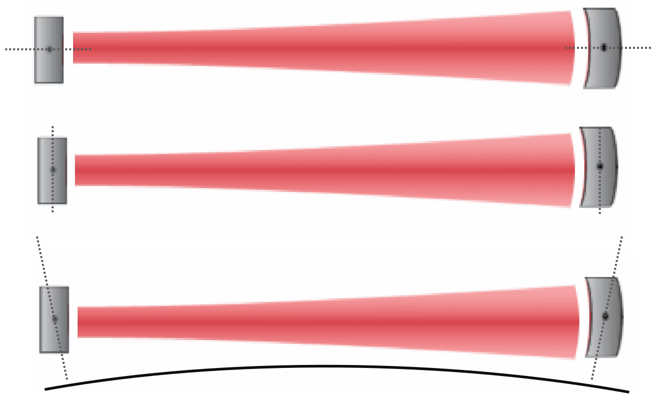
In Virgo, seismic displacements between $\mathbf{10^{-10}}$ and $\mathbf{10^{-8} m/\sqrt{Hz}}$ are recorded on ground **at 20 Hz**.

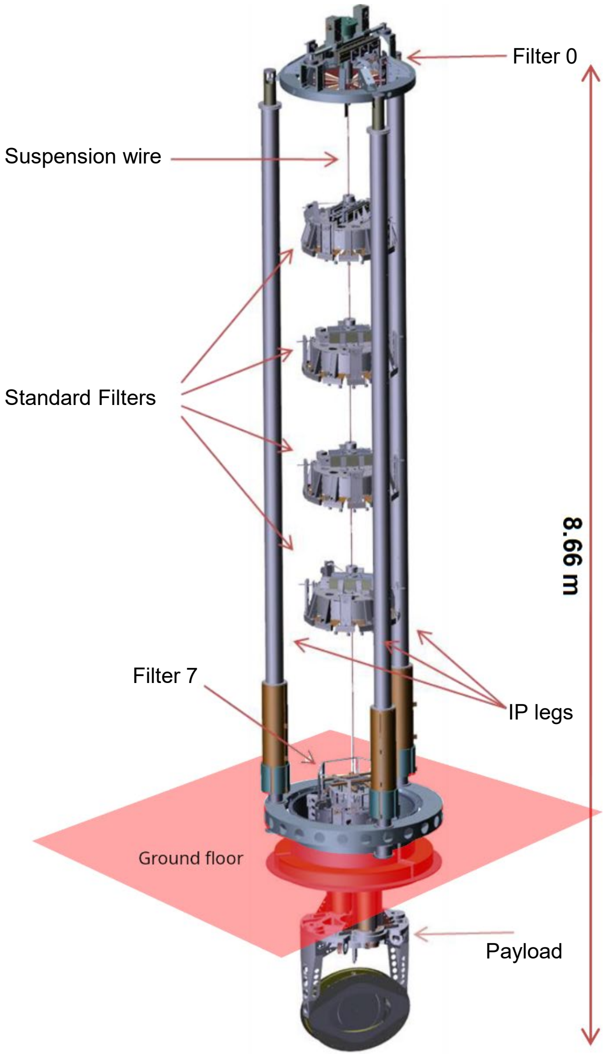


Without any filtering, this would mask strains of the $\mathbf{3\ km}$ arm like $h > \frac{10^{-10}\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}}{3{\cdot}10^{3}\,\mathrm{m}} = 3 \cdot 10^{-14}\frac{1}{\sqrt{\mathrm{Hz}}}$.

Horizontal vibrations act
- on the length of the optical paths;
- on the centering and alignment of the mirrors.

Vertical vibrations act
- on the centering and alignment of the mirrors;
- (slightly) on the length of the optical paths. This comes from the **convergence of the vertical directions** at a km-scale distance.





The Superattenuator solution

Each cavity mirror is currently suspended in Advanced Virgo from a Superattenuator (SA).

The AdVirgo SA is a **passive** mechanical filter whose main components are

- an **Inverted Pendulun** (IP), made of three identical legs and a top ring;
- a top vertical oscillator called **Filter 0**;
- a chain of four **Standard Filters**, suspended to each other;
- a special final filter (**Filter 7**), suspended from the previous and connected to the Payload.

The **Payload** components are

- a stiff reference cage rigidly connected to Filter 7;
- a Marionette, suspended from Filter 7 and controlled from the cage;
- the Mirror, suspended from the Marionette and controlled from the cage.

See Valerio Boschi – SUSP Training session, https://tds.virgo-gw.eu/?r=16165

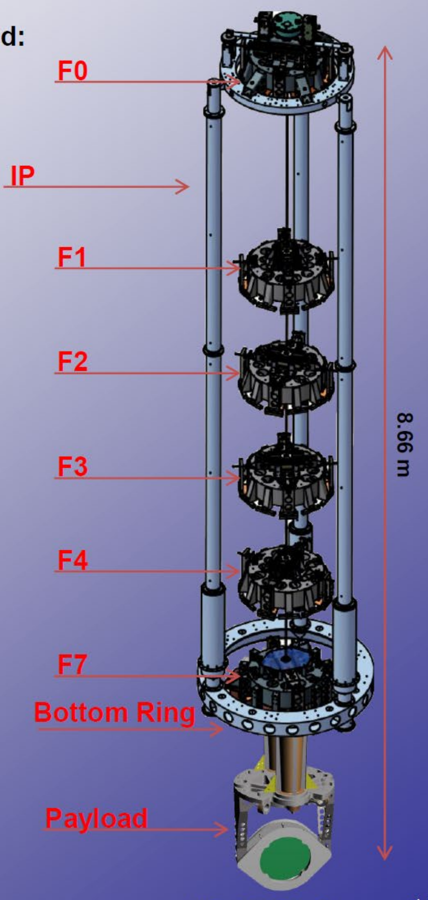
The Superattenuator solution

Not so passive, indeed

Introduction
Control system setup

On long superattenuators (BS, NI, NE, WI, WE, PR, SR) are installed:

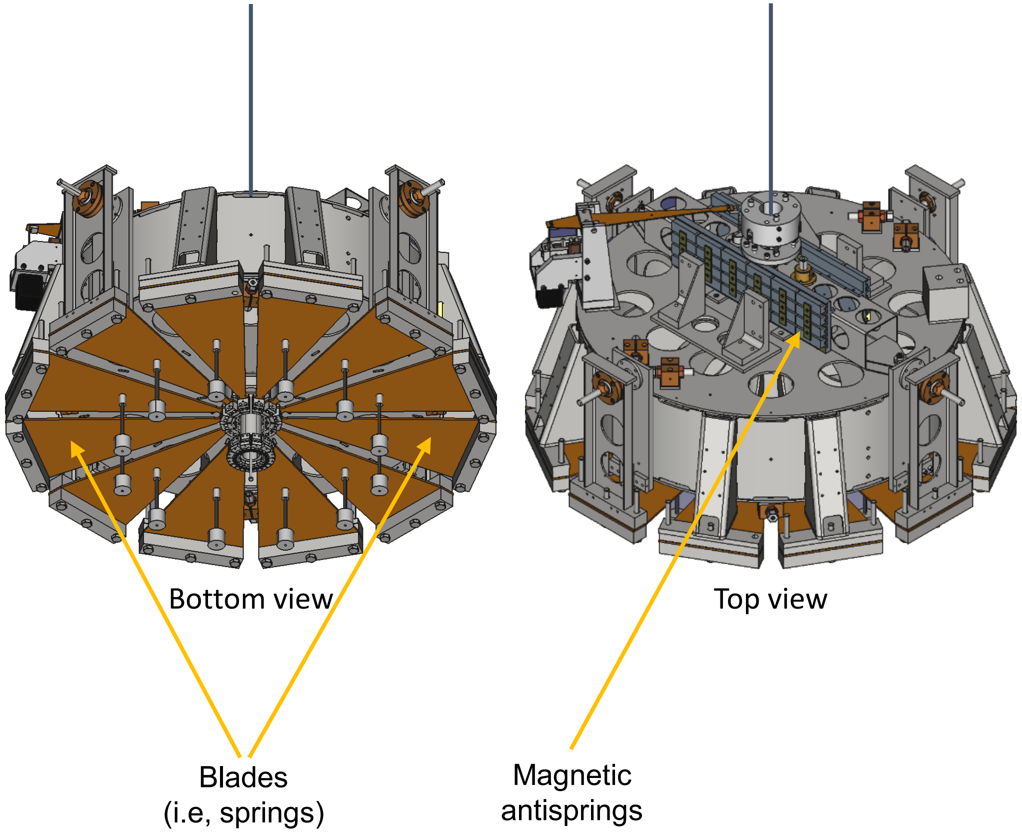
- **18 LVDTs** of 3 different types
  - 9 Vertical LVDTs (F0 – F7 Crossbar, Bottom Ring)
  - 3 F0 Horizontal LVDT
  - 6 F7 LVDTs
- **5 Accelerometers** of 2 different types installed on F0:
  - 3 Horizontal Accs
  - 2 Vertical Accs
- **23 Coils** of 4 different types
  - 5 F0 Coils
  - 6 F7 Coils
  - 8 Marionette coils
  - 4 Mirror coils
- **3 Piezos** on bottom ring (Not uset yet)
- **21 Motors**
  - 1 Top screw F0 vertical motor
  - 3 F0 trolley motors
  - 6 Fishing rod motors
  - 2 Marionette motors
  - 4 F7 motors
  - 5 Accelerometer motors



Training Session Cascina, 11-11-24 VIR-0XYZA-24 14

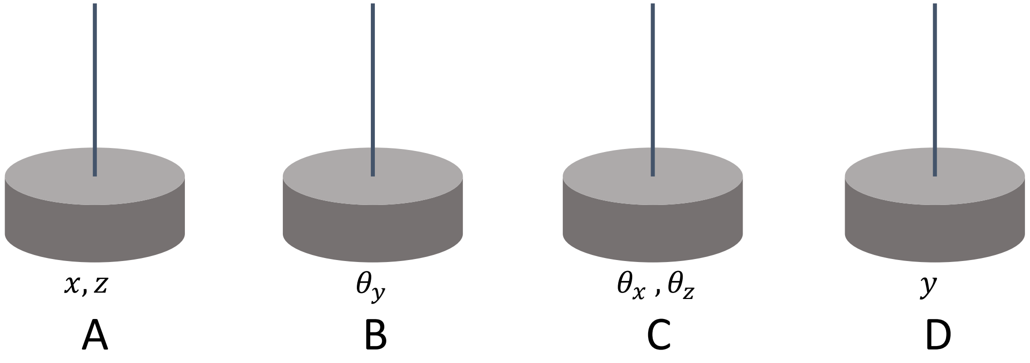
See Valerio Boschi – SUSP Training session, https://tds.virgo-gw.eu/?r=16165

Standard Filter



The AdV Superattenuator has four suspended Standard Filters

Each Standard Filter is a physical pendulum with several degrees of freedom.
Each DOF oscillates with its own frequency.



Oscillations A, B, C spontaneously occur depending on gravity, inertia of the massive body, wire properties.

Motion D require a **dedicated vertical oscillator**.

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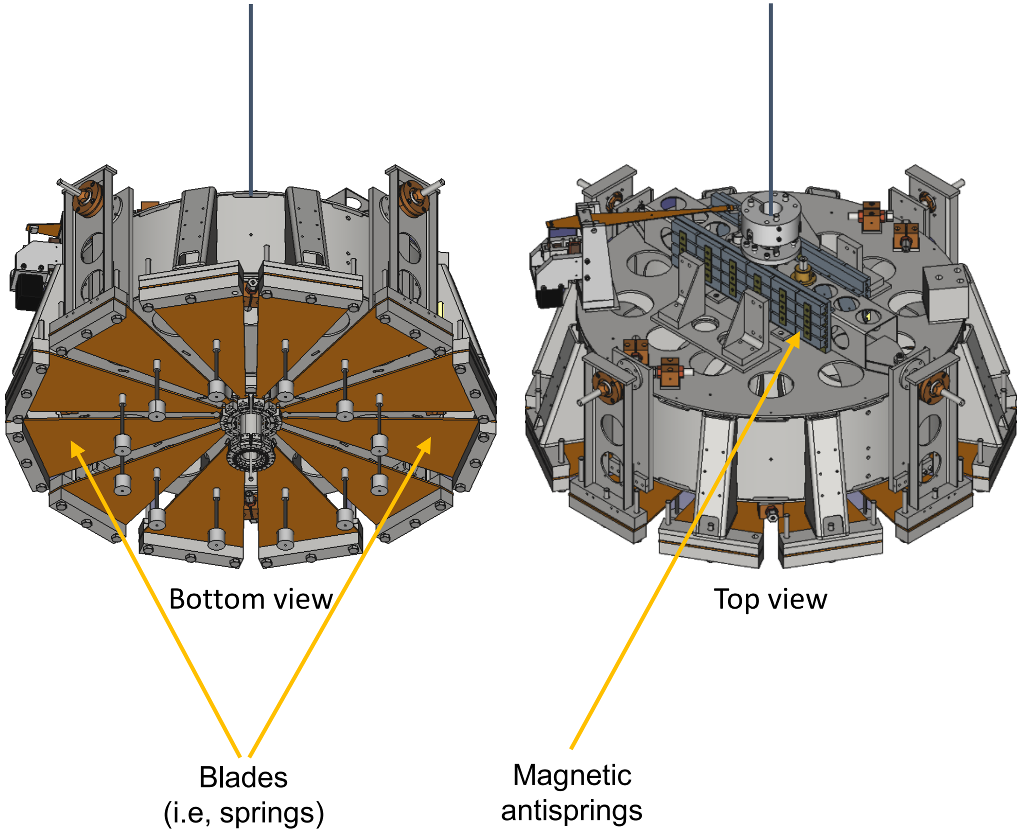
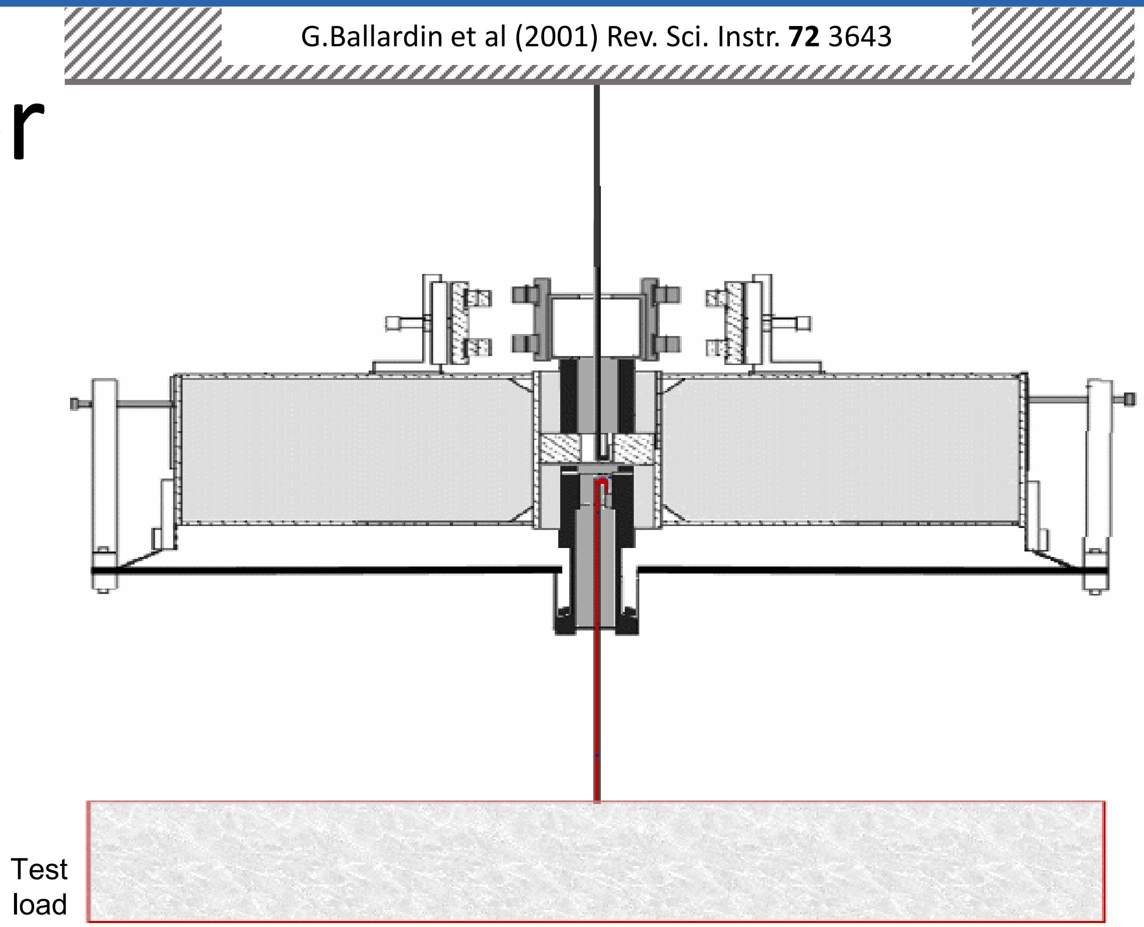
G.Ballardin et al (2001) Rev. Sci. Instr. **72** 3643

Built-in vertical oscillator





Vertical oscillators are set in
Filter 0, **Standard Filters**, **Filter 7**

A toy model of the oscillating filter

A mechanical filter can be understood in terms of **oscillators**.
Let's create a toy filter using an elastic oscillator as the building brick.

A metal spring is actuated on the left side by a piston whose position is $x_0(t)$.
At the right end of the spring a block can move over the spring axis.

$$k = k_0 \cdot (1 + i\,\phi)$$

The spring has a **loss angle ϕ**. This accounts for anelastic relaxations in the spring material.

$$m\,\ddot{x} = -k \cdot (x - x_0)$$

$$-m\omega^2 \tilde{x} = -k \cdot (\tilde{x} - \tilde{x}_0)$$

$$(k - m\omega^2)\tilde{x} = k\,\tilde{x}_0$$

$$\tilde{x} = -\frac{\omega_0^2 \cdot (1 + i\,\phi)}{\omega^2 - \omega_0^2 \cdot (1 + i\,\phi)}\tilde{x}_0$$

$$f_T(\omega) = -\frac{\omega_0^2 \cdot (1 + i\,\phi)}{\omega^2 - \omega_0^2 \cdot (1 + i\,\phi)}$$

A toy model of the oscillating filter

Transfer function

$$f_T(\omega) = \frac{-\omega_0^2 \cdot (1 + i\,\phi)}{\omega^2 - \omega_0^2 \cdot (1 + i\,\phi)}$$

$$k = k_0 \cdot (1 + i\,\phi)$$

$$Q \cong \phi^{-1}$$

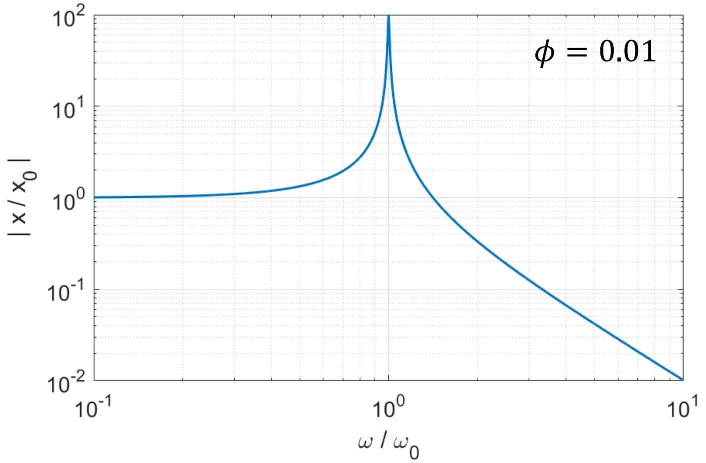
Peak value $\quad |f_T(\omega_0)| = \left|\frac{1+i\,\phi}{i\phi}\right| \cong \phi^{-1}$

Let's work the **curve width** out: $\left|f_T(\omega_{1/2})\right| = \frac{\phi^{-1}}{2}$ (half of the peak value)

$$\Rightarrow \left|\frac{{\omega_{1/2}}^2}{\omega_0^2 \cdot (1 + i\,\phi)} - 1\right| = 2\phi \quad \Rightarrow \left|\frac{{\omega_{1/2}}^2}{\omega_0^2}(1 - i\,\phi) - 1\right| = 2\phi$$

$$\Rightarrow \text{ with } \quad \omega_{1/2} = \omega_0 \cdot (1 + \delta), \qquad \left|\frac{\omega_0^2(1+2\delta)}{\omega_0^2}(1 - i\,\phi) - 1\right| = 2\phi$$

$$\Rightarrow |2\delta - i\,\phi| = 2\phi \qquad \Rightarrow 2\delta = \frac{\sqrt{3}}{2}\phi \qquad \Rightarrow FWHM = 0.866\,\phi\omega_0$$



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A toy model of the oscillating filter

$$k = k_0 \cdot (1 + i\,\phi)$$

Let's complicate it a bit.
Two identical oscillators are connected in cascade.

$$\begin{cases} -m\omega^2 \tilde{x}_1 = -k \cdot (\tilde{x}_1 - \tilde{x}_0) + k \cdot (\tilde{x}_2 - \tilde{x}_1) \\ -m\omega^2 \tilde{x}_2 = -k \cdot (\tilde{x}_2 - \tilde{x}_1) \end{cases}$$

$$\Rightarrow \begin{cases} (2k - m\omega^2)\tilde{x}_1 - k\tilde{x}_2 = k\tilde{x}_0 \\ -k\tilde{x}_1 + (k - m\omega^2)\tilde{x}_2 = 0 \end{cases} \Rightarrow \left[\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} - \frac{m\omega^2}{k} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{x}_0 \\ 0 \end{pmatrix}$$

$$\Rightarrow M \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{x}_0 \\ 0 \end{pmatrix}, \text{ with } M = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} - \frac{m\omega^2}{k} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \Rightarrow \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = M^{-1} \begin{pmatrix} \tilde{x}_0 \\ 0 \end{pmatrix}$$

A toy model of the oscillating filter

Even more complicated.
Many identical oscillators connected to each other.

$$\begin{cases} -m\omega^2\tilde{x}_1 = -k\cdot(\tilde{x}_1 - \tilde{x}_0) + k\cdot(\tilde{x}_2 - \tilde{x}_1) \\ -m\omega^2\tilde{x}_2 = -k\cdot(\tilde{x}_2 - \tilde{x}_1) + k\cdot(\tilde{x}_3 - \tilde{x}_2) \\ \dots \\ -m\omega^2\tilde{x}_n = -k\cdot(\tilde{x}_n - \tilde{x}_{n-1}) \end{cases}$$

$$\Rightarrow M = \begin{pmatrix} 2 & -1 & & & 0 \\ -1 & 2 & -1 & & \\ & -1 & \ddots & -1 & \\ & & -1 & 2 & -1 \\ 0 & & & -1 & 1 \end{pmatrix} - \frac{m\omega^2}{k}\begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ 0 & & & & 1 \end{pmatrix} = \begin{pmatrix} 2-\lambda & -1 & & & 0 \\ -1 & 2-\lambda & -1 & & \\ & -1 & \ddots & -1 & \\ & & -1 & 2-\lambda & -1 \\ 0 & & & -1 & 1-\lambda \end{pmatrix}$$

, with $\lambda = \frac{m\omega^2}{k}$

$$\Rightarrow \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_n \end{pmatrix} = M^{-1}\begin{pmatrix} \tilde{x}_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$\Rightarrow$ displacement of the **last block**: $\tilde{x}_n = (M^{-1})_{n,1}\,\tilde{x}_0$

$$(M^{-1})_{n,1} = (-1)^{n+1}\frac{\mathrm{Det}(M_{1,n})}{\mathrm{Det}(M)} = (-1)^{n+1}\frac{(-1)^{n-1}}{\mathrm{Det}(M)} = \frac{1}{\mathrm{Det}(M)}$$

$$\Rightarrow \boxed{f_T(\omega) = \frac{1}{\mathrm{Det}(M)}}$$

A toy model of the oscillating filter

The transfer function from **input displacement** x_0 to **output displacement** x_n:

$$f_T(\omega) = \frac{1}{\mathrm{Det}\begin{pmatrix} 2-\lambda & -1 & & & 0 \\ -1 & 2-\lambda & -1 & & \\ & -1 & \ddots & -1 & \\ & & -1 & 2-\lambda & -1 \\ 0 & & & -1 & 1-\lambda \end{pmatrix}}, \quad \text{with } \lambda = \frac{m\omega^2}{k}$$

$$f_T(\omega) = \frac{1}{(-1)^n\lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_0} = \frac{1}{(-1)^n(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_n)}$$

$$f_T(\omega) = \frac{[-\omega_0^2(1+i\phi)]^n}{[\omega^2-\lambda_1\omega_0^2(1+i\phi)]\;[\omega^2-\lambda_2\omega_0^2(1+i\phi)]\;\ldots\;[\omega^2-\lambda_n\omega_0^2(1+i\phi)]}$$

$\lambda_1, \lambda_2, \ldots \lambda_n$ are real numbers laying in the range (0,4).

For $\omega \gg 2\omega_0$,

$$f_T(\omega) = [-(1+i\phi)]^n \left(\frac{\omega_0}{\omega}\right)^{2n}$$

(Low pass filter)

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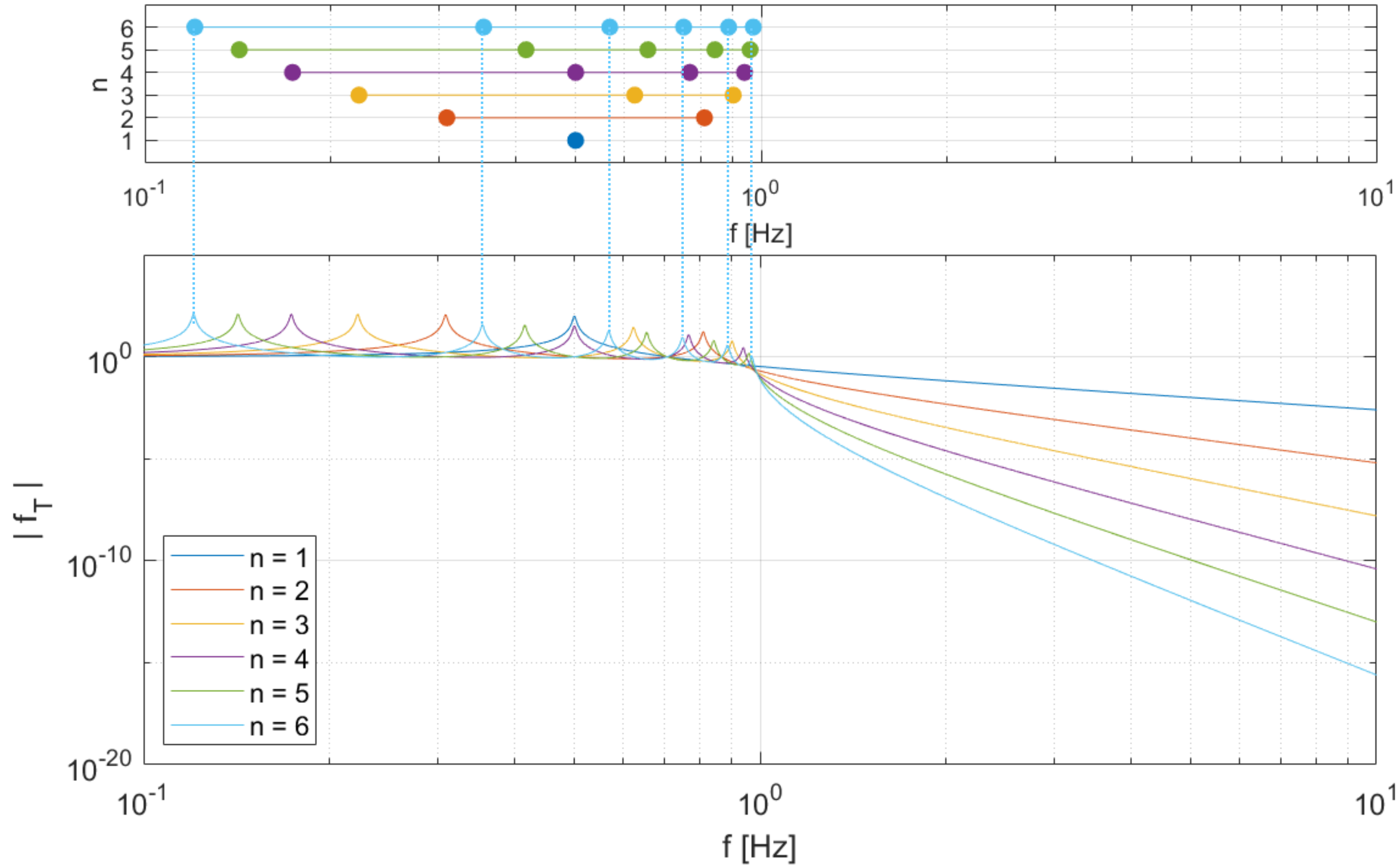
A toy model of the oscillating filter

The transfer function from **input displacement x_0** to **output displacement x_n**

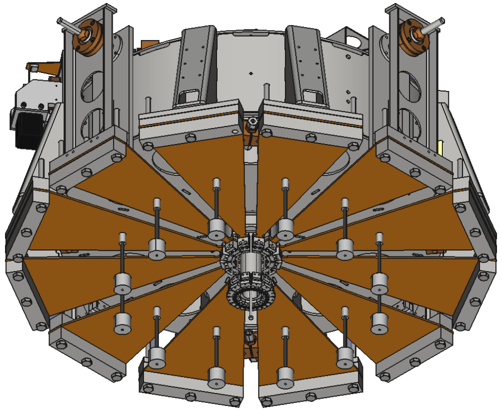


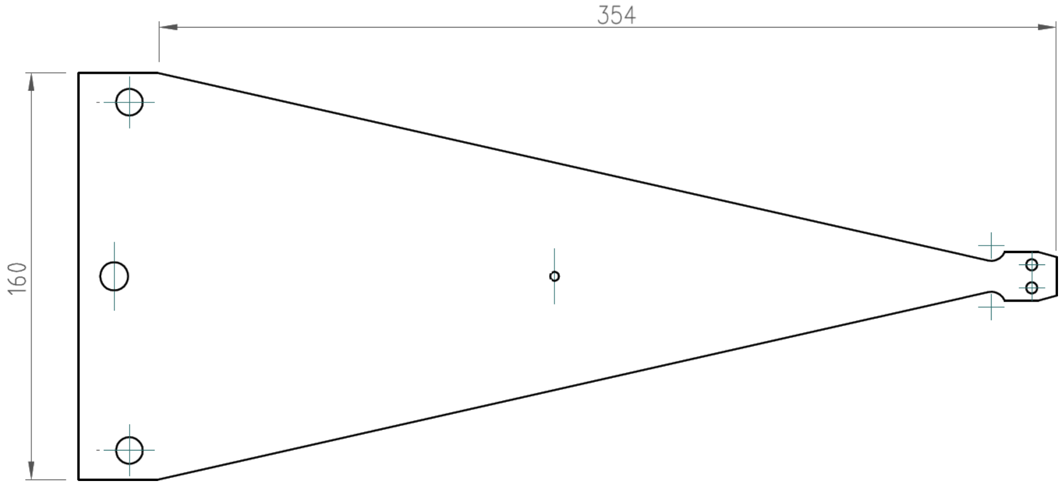
$$k = k_0 \cdot (1 + i\,\phi)$$

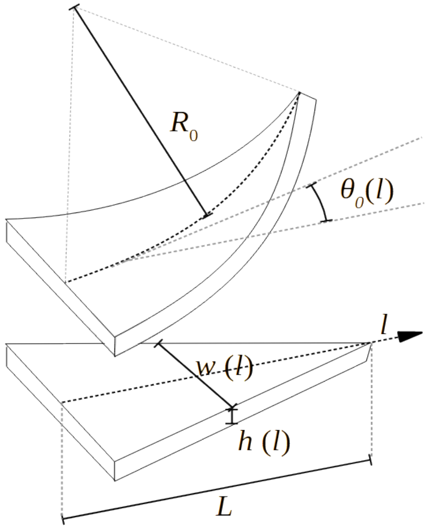
$$f_0 = 0.5 \text{ Hz}$$
$$\phi = 0.01$$
Growing n

Low n start filtering at lower frequencies.
High n have better rejections.
Above $2f_0$ all configurations filter anyway.

Elastic blades







SA blades are **curved at rest** and designed to be flattened by a load suspended from their tip.

They are made in **maraging steel** (maraging 250), a low carbon Fe alloy
Ni (18%), Co (8%), Mo (5%),
Ti (0.5%), C (≤0.03%) + Fe

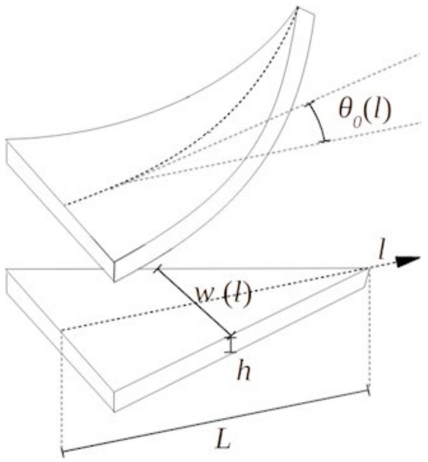
Relevant parameters

$E = 187$ GPa (at 20°C)
$UTS = 1.85$ GPa
$\phi = 3 \cdot 10^{-5}$

S.Braccini et al 2000 *Meas. Sci. Technol.* **11** 467

G.Cella, Personal communication - Write me for a copy

Blade mechanics



$$U = \frac{1}{2}\int_0^L E\,\frac{w(l)h^3}{12}\left(\frac{d\theta}{dl} - \frac{d\theta_0}{dl}\right)^2 dl + F\int_0^L \sin\theta\, dl$$

valid for a **very thin blade** with **any cut profile $w(l)$** and **any rest bending $\theta_0(l)$**

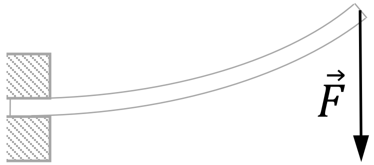
By minimizing this energy with respect to the possible profiles $\theta(l)$ we get a **condition** on the rest bending $\theta_0(l)$ for the blade **to lay flat** under the load $\bar{F}$

$$\frac{d\theta_0}{dl} = \frac{12\bar{F}}{Eh^3}\frac{(L-l)}{w(l)}$$

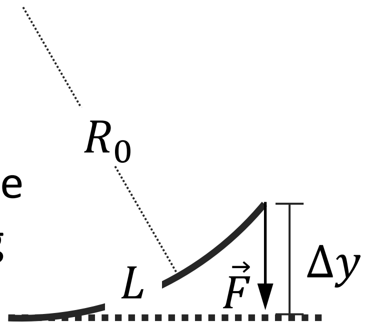
We discover that a **triangular profile** $w(l) = w_0\frac{(L-l)}{L}$ have a constant $\frac{d\theta_0}{dl}$.

This means an arc of circle profile with radius

$$R_0 = E\frac{w_0h^3}{12L\bar{F}}$$



The **spring constant** k can be retrieved from the flattening effect of the load

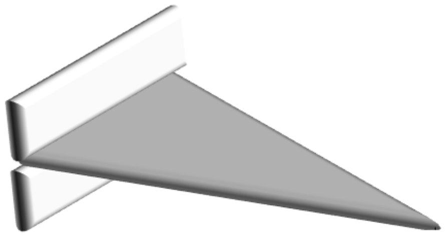


$$\Delta y = R_0\left(1 - \cos\frac{L}{R_0}\right) \cong R_0\frac{1}{2}\left(\frac{L}{R_0}\right)^2 = \frac{L^2}{2R_0}$$

$$k = \bar{F}/\Delta y \quad \Rightarrow \quad k = E\frac{w_0h^3}{12LR_0}\frac{2R_0}{L^2}$$

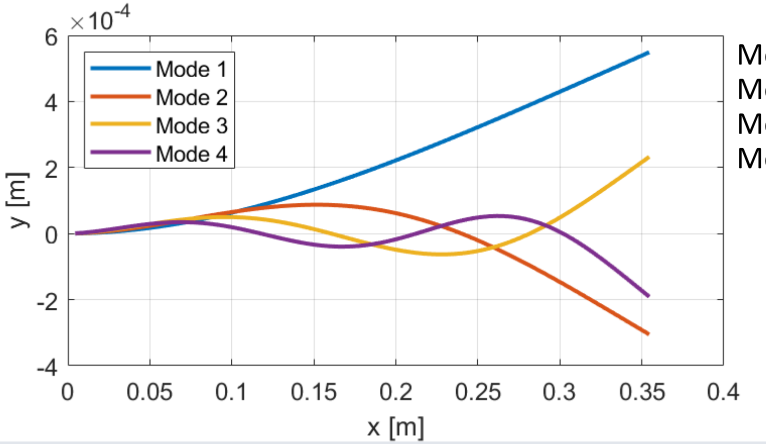
$$k = E\frac{w_0h^3}{6L^3}$$

Internal modes

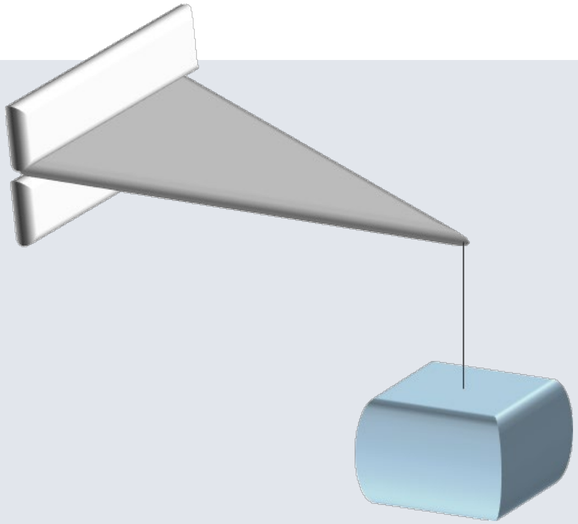


Flat unloaded triangular blade

$E = 187$ GPa
$\rho = 8000$ kg/m^3
$L = 354$ mm
$w_0 = 110$ mm
$h = 3.5$ mm
$R_0 = \infty$
$F = 0$

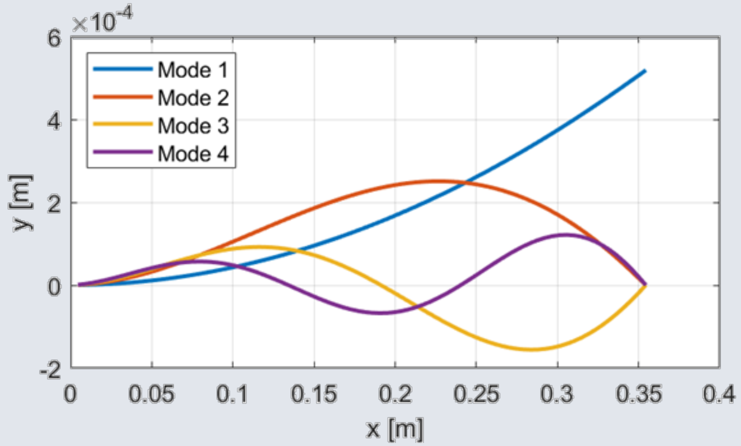


Mode 1 42 Hz
Mode 2 185 Hz
Mode 3 450 Hz
Mode 4 832 Hz



Pre-bent triangular blade
with adjusted (i.e. flattening) load

$E = 187$ GPa
$\rho = 8000$ kg/m^3
$L = 354$ mm
$w_0 = 110$ mm
$h = 3.5$ mm
$R_0 = 450$ mm
$F = 46.7 \text{ kg} \cdot g = 458$ N



Mode 1 1.35 Hz
Mode 2 101 Hz
Mode 3 311 Hz
Mode 4 642 Hz

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Triangular blade summary

Preset curvature radius for a load $\bar{F}$

$$R_0 = E\,\frac{w_0 h^3}{12\,L\,\bar{F}}$$

Single blade spring constant

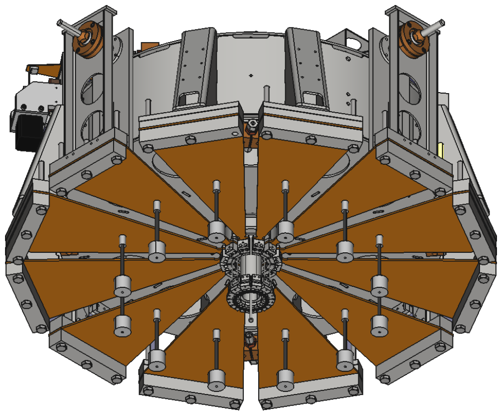
$$k = E\,\frac{w_0 h^3}{6L^3}$$

Single blade resonant frequency (1st)

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

Surface stress of the flattened blade

$$s = \frac{Eh}{2R_0}$$



Filter summary

$$\bar{F}_{tot} = \sum_i \bar{F}_i$$

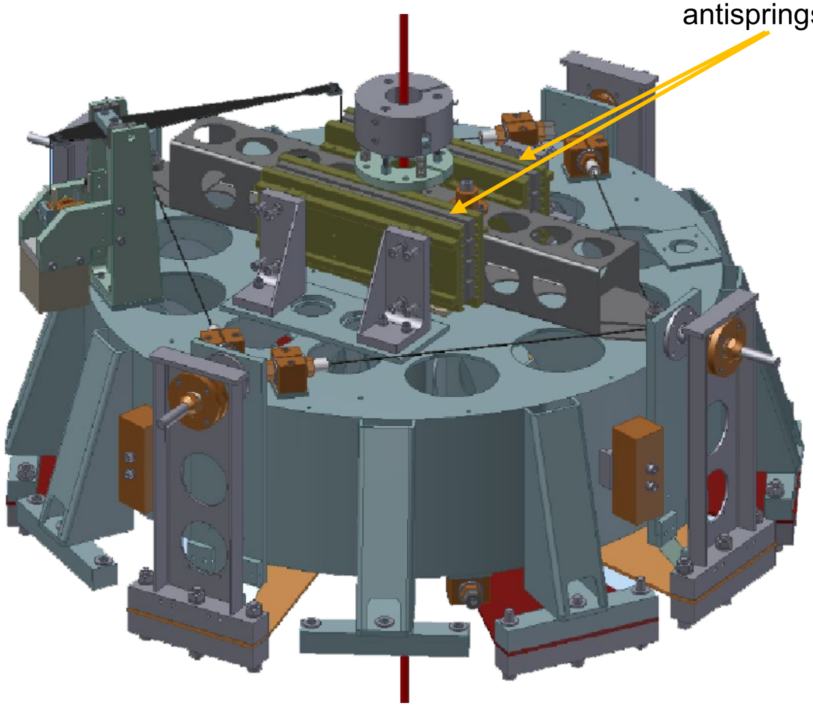
$$k_{tot} = \sum_i k_i$$

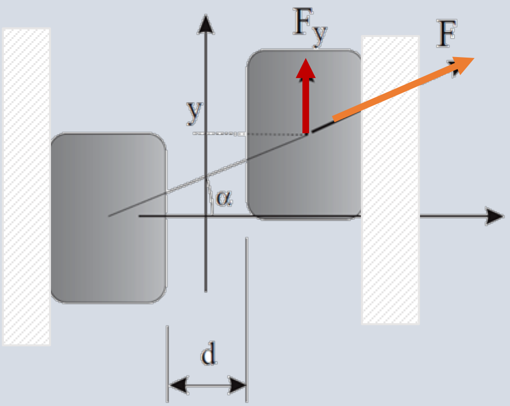
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_{tot}}{M_{tot}}}$$

$$s_i < 0.5\,UTS \;\; \forall i$$

Blades work **in symmetric couples** of identical specimens.
Couples can differ from each other.
If the **total load** of the filter is the **sum of the individual flattening loads** all blades work flat.

Magnetic Anti-Springs





MAS working principle

$$F_y = -(k_{as}y + a_{as}y^3)$$

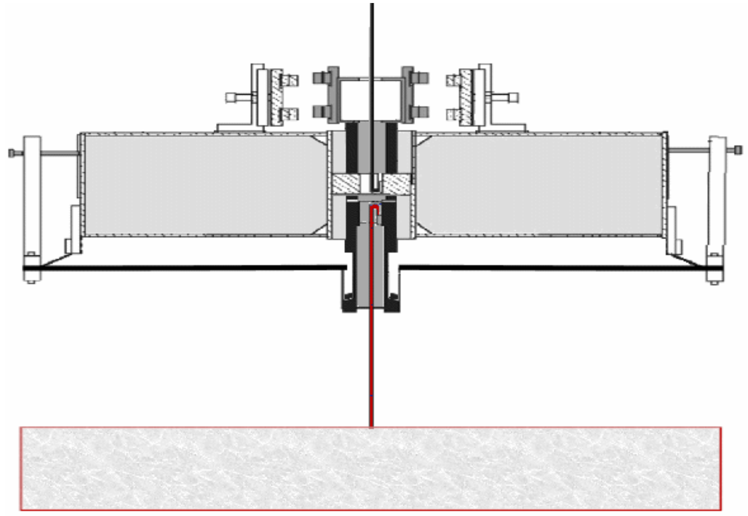
with

$$k_{as} < 0$$
$$a_{as} > 0$$

(unavoidable nonlinearity)

G. Curci et al, Effects of non-linearities…, https://tds.virgo-gw.eu/?r=7616

Two forces are exerted on the central column:



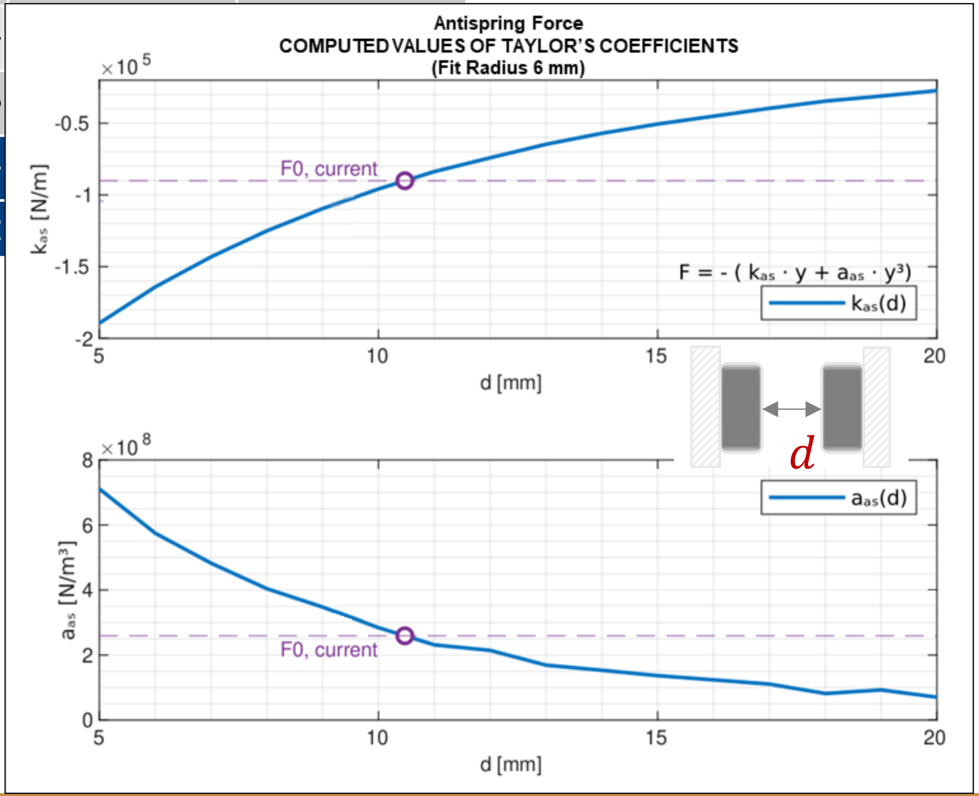
$$F_{tot} = -(k_{el} + k_{as})\, y + O_3(y)$$

stable as long as it is $k_{el} + k_{as} > 0$

Sum of ks

Current configuration		Filter 0	Filter 1	Filter 2	Filter 3	Filter 4	Filter 7
M	Suspended mass [kg]	1057	884	719	579	461	146
k_{el}	Elastic stiffness [N/m]	93863	78496	63840	51404		
k_{as}	Antispring stiffness [N/m]	- 90108	- 75356	- 61286	- 49348		
a_{as}	Nonlinearity [10^8 N/m^3]	2.6	2.3	1.8	1.4		
d	Antispring tuning [mm]	10.5	8.5	10.5	12		

Some figures in Virgo



Filter 0
Magnetic Anti-Spring simulation

For **Filter 0**,

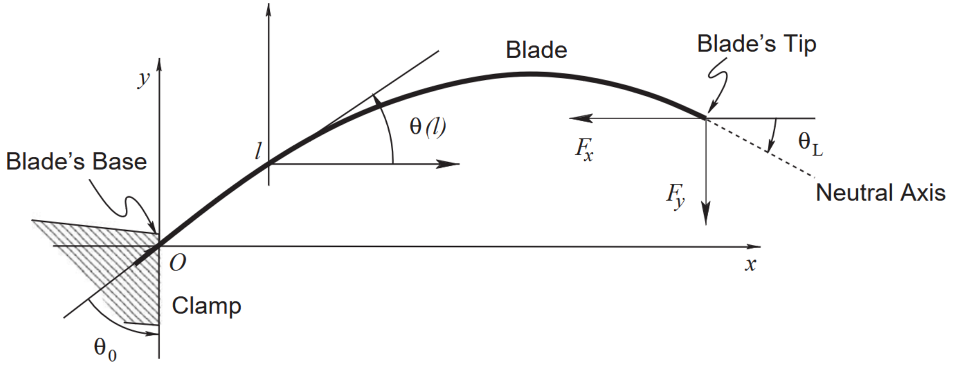
$$k = (93863 - 90108)\,\frac{\mathrm{N}}{\mathrm{m}} = 3755\,\frac{\mathrm{N}}{\mathrm{m}}$$

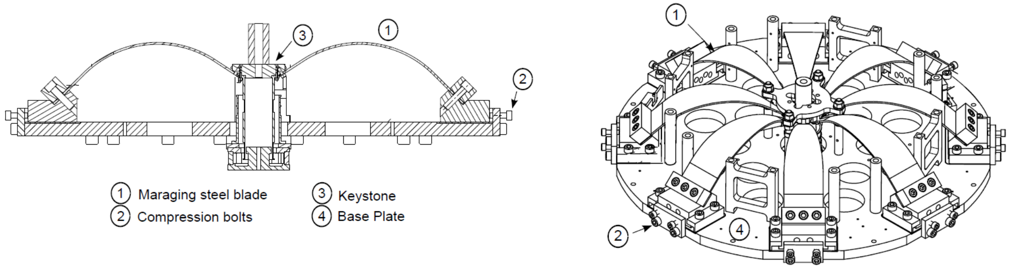
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{3755\frac{N}{m}}{1057\ \mathrm{kg}}} = 0.3\ \mathrm{Hz}$$

Without MAS

$$f_{el} = \frac{1}{2\pi}\sqrt{\frac{93863\,\frac{N}{m}}{1057\ \mathrm{kg}}} = 1.5\ \mathrm{Hz}$$

The Geometric Anti-Spring (i.e. a tunable blade)





(a) (b)

G.Cella et al. 2005 Nucl. Instr. and Meth. A **540** 502
M.R. Blom et al. 2015 Physics Procedia **61** 641
(non-exhaustive list!)

Energy of a compressed blade

$$U = \frac{1}{2}\int_0^L E\,\frac{w(l)h^3}{12}\left(\frac{d\theta}{dl}\right)^2 dl - F_y\int_0^L \sin\theta\, dl - F_x\int_0^L \cos\theta\, dl$$

Let's compare this to slide 15 (*Blade mechanics*)

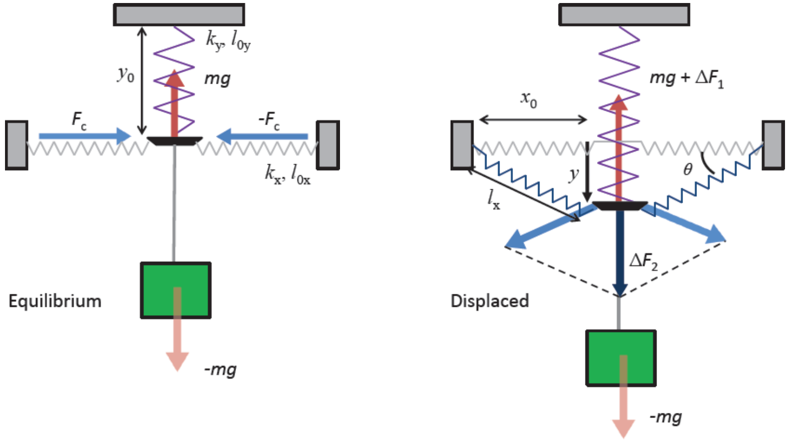
Similarities:

- Metal blade
- Constrained base angle
- Vertical load
- Constant and small thickness

Differences:

- No pre-curvature
- Different base angle ($\theta_O > 0$ instead of $\theta_O = 0$)
- Constrained tip angle ($\theta_L < 0$)
- Horizontal compression (Constrained length)

The Geometric Anti-Spring



(a) (b)

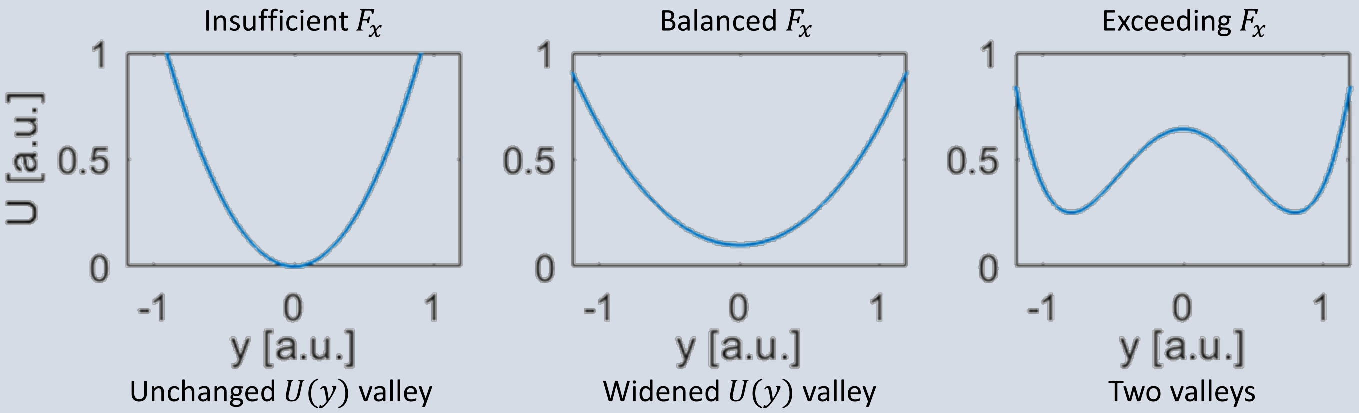
Intuitive model (not for real life design)

(a) A vertical spring is in equilibrium with the weight of a load. Two horizontal **counteracting compressed springs** are in an **unstable equilibrium**.

(b) When the load leaves the equilibrium position, the vertical spring exerts a recall, while the compressed springs expel the load.

Note: if **the repelling modulus exceeds the recalling one**, the compressed springs win and expand themselves until a stable equilibrium point. The system is **bistable**.

Less intuitive (and still insufficient) **model**



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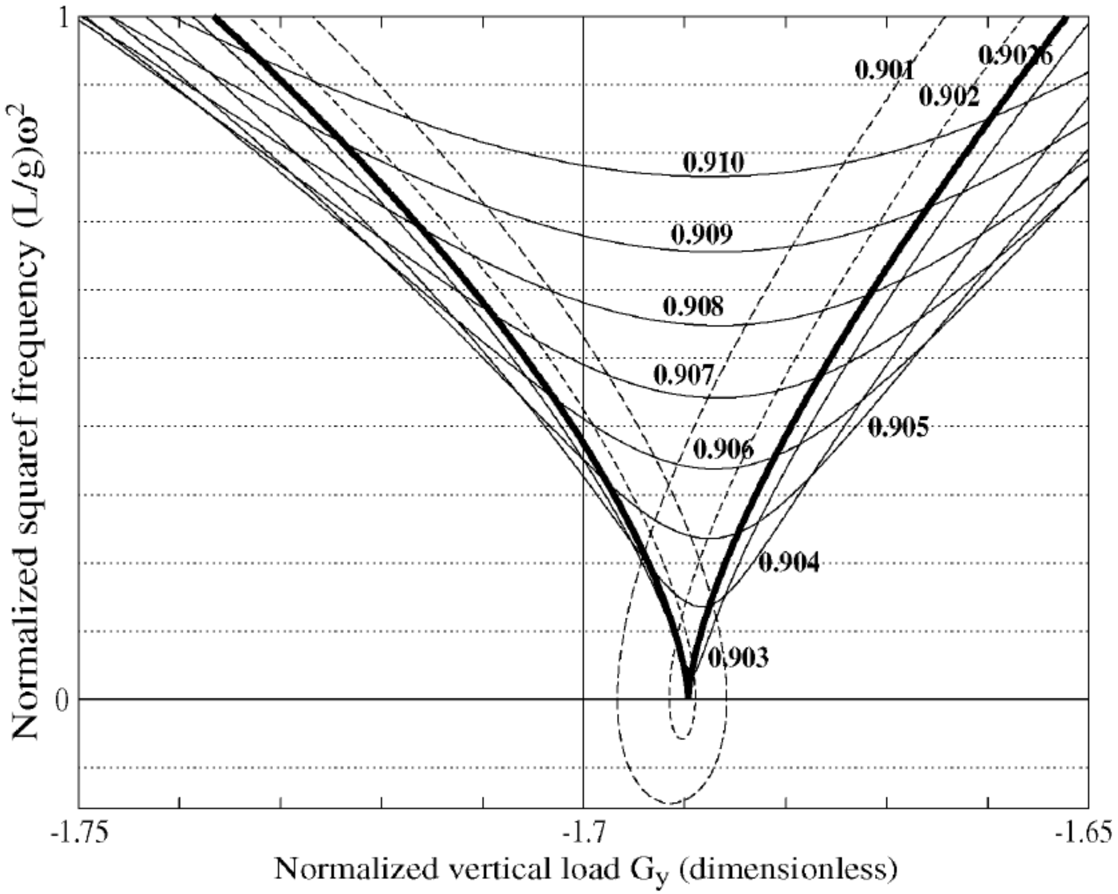
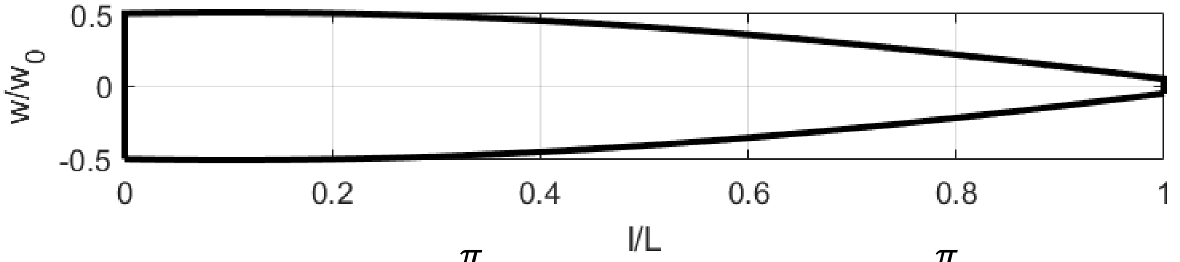
GAS how-to



G.Cella et al. 2005 Nucl. Instr. and Meth. A **540** 502

Computed general solution for a preset blade shape (TAMA)



with base angle $\theta_O = \frac{\pi}{4}$ and tip angle $\theta_L = -\frac{\pi}{6}$.

Dimensionless variables

$$G_y = 12\frac{L^2}{Ew_0h^3}F_y \text{ (load)}$$

$$x = X/L \text{ (horizontal constrain)}$$

Legend

Curves in full lines belong to stable states ($x > 0.9026$), dashed to bistable ($x < 0.9026$)

Stable states have minimum frequency for $G_y \cong 1.69$.

Some (numerical) outcomes

Simulated GAS

TAMA shape

$L = 354$ mm

$w_0 = 110$ mm

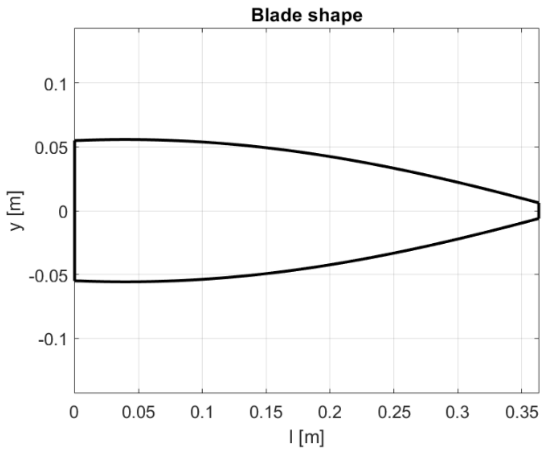
$h = 2.74$ mm

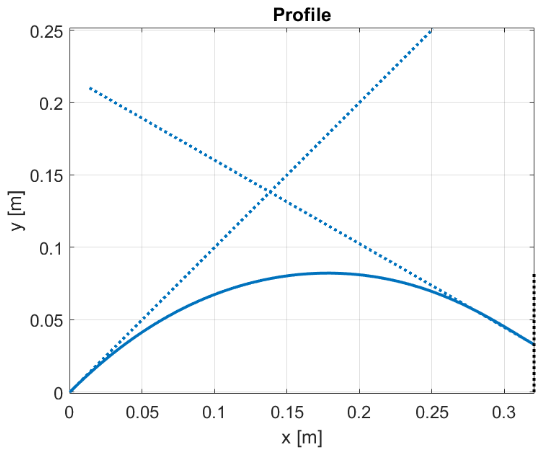
$E = 187$ GPa

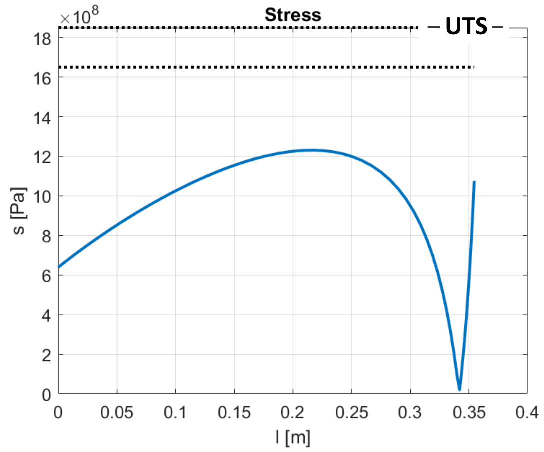
$UTS = 1.85$ GPa

$M = 48.4$ kg

$X = 0.9043\ L$







$\Rightarrow$ $f = 0.35$ Hz

$s = 0.66\ UTS$

That's all from my side
Let's go filter now

Same blade without GAS constrains

$\Rightarrow\ f = 1.20$ Hz

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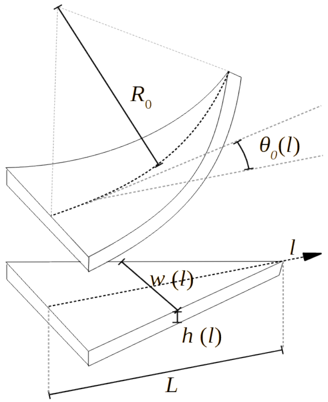
Problem #1

Set up the maraging 250 blades of a Virgo-like lowest filter.

$L = 354$ mm
$w_0 = 110$ mm
$h = 3.5$ mm
6 blades

$M = 290$ kg



Find
- correct rest **curvature $\boldsymbol{R_0}$**,
- **frequency** (assume blade mass << M),
- **stress**.

Compare stress with **UTS**.

Look around for necessary equations and data!

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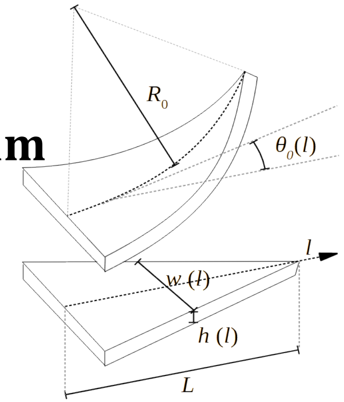
Problem #2

Choose between two possible blade bases w_0 and set thickness h to get the same f as in Problem #1. Which base is the best solution?

$L = 354 \text{ mm}$
$w_0 = \mathbf{55\ mm}$ or $\mathbf{110\ mm}$
6 blades

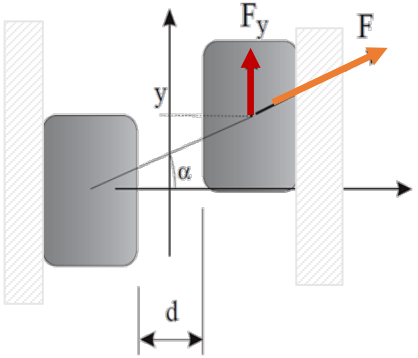
$M = 290 \text{ kg}$

[$f = 1.318 \text{ Hz}$]



Target
- find correct **curvature** $\boldsymbol{R_0}$ for both bases,
- find correct **thickness** $\boldsymbol{h}$ for both bases,
- find **stress** for both bases,
- choose the **best solution**.

Problem #3



Add ferrite Magnetic Anti-Springs to the same Vigo filter as in Problems #1 and 2.
Tune the MAS to the design frequency f, by choosing the correct distance d.

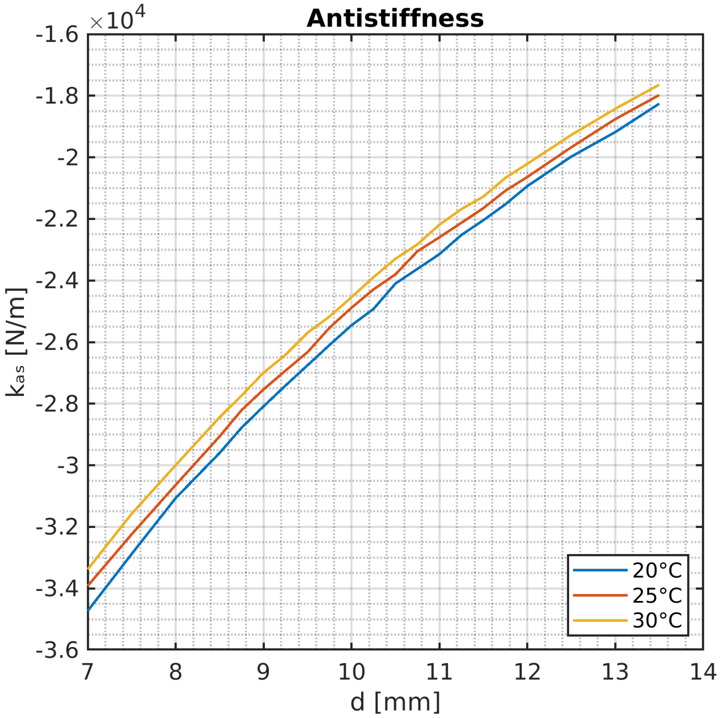
$M = 290$ kg

$f_{in} = 1.5$ Hz * (before MAS)

* Filter frequency is usually higher than the pure blade frequency

$f = \mathbf{0.50\ Hz}$

Negative stiffness of the installed MAS at some different temperatures



Find

- the correct **distance** $\boldsymbol{d}$.

Estimate the **frequency variation** with 5°C temperature increase.

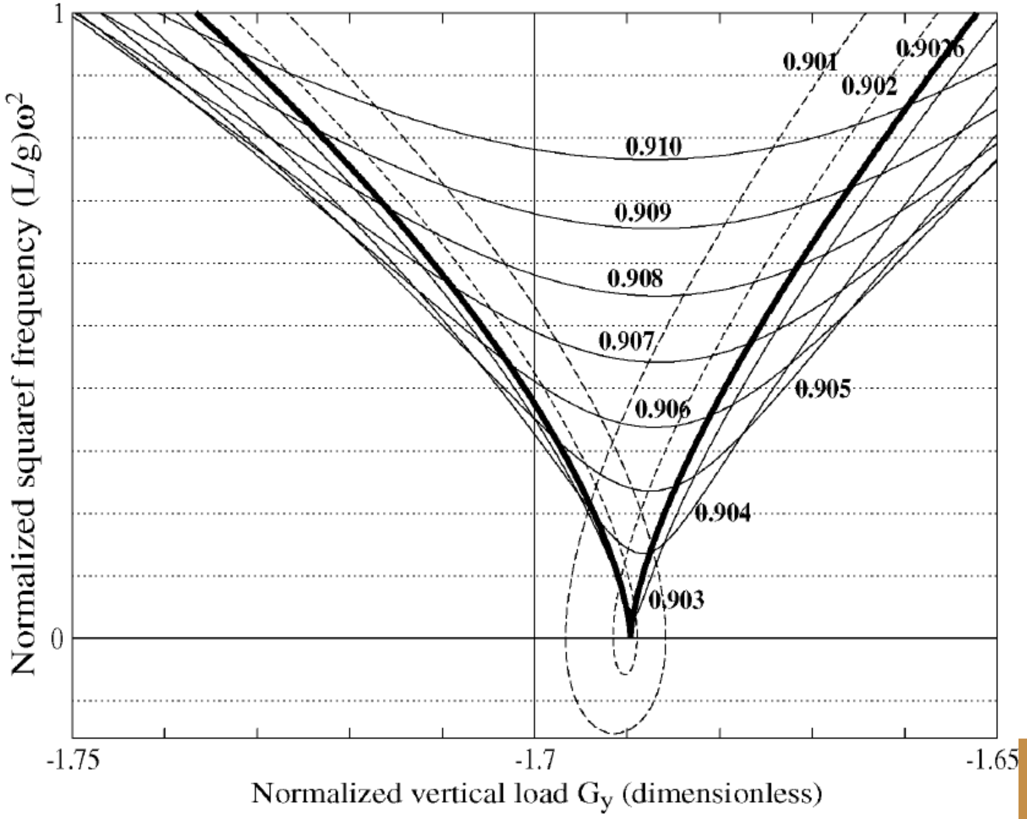
Problem #4

Let's switch to Geometric Anti-Springs.
Provide the same filter of the previous problems with GAS vertical oscillators instead of blades+MAS.
Tune the filter to the design frequency f.

TAMA shape
$L = 354$ mm
$w_0 = 110$ mm
6 blades

$M = 290$ kg

$f = \mathbf{0.50\ Hz}$



Find
- the correct **thickness** $\boldsymbol{h}$ that matches the optimum condition $G_y = -1.69$.
- the correct **horizontal compression factor** $\boldsymbol{x}$ that tunes the filter.

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Solutions

Problem #1	$R_0 = 0.438$ m, $f = 1.318$ Hz, $s = 0.404\ UTS$
Problem #2	$R_0 = 0.438$ m both, 110 mm base: same h and s as in Problem #1 55 mm base: $h = 4.4$ mm, $s = 0.508\ UTS$ (discarded!)
Problem #3	$d = 11.1$ mm, $\Delta f = +0.04$ Hz
Problem #4	$h = 2.74$ mm, $x = 0.906$