

Probing double charge-exchange excitations with nuclear reactions

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Summary. — Collisional heavy-ion double charge exchange (DCE) reactions, which stem from second-order nucleon–nucleon interactions, provide a powerful probe into the nature of two-body transition densities (2BTD) involved in nuclear excitation processes. In this study, such reactions are analyzed within a second-order distorted wave approximation, wherein the fundamental interaction is formulated through an s -channel two-body operator.

This formalism allows to describe complementary DCE transitions in both the projectile and the target nuclei, enabling a consistent extraction of the corresponding 2BTDs. As an illustrative example, we present theoretical results for the reaction $^{18}\text{O} + ^{76}\text{Se} \rightarrow ^{18}\text{Ne} + ^{76}\text{Ge}$, at a laboratory beam energy of $T_{lab} = 270$ MeV.

1. – Introduction

Second-order processes play a pivotal role in testing our understanding of fundamental mechanisms in physics and in validating the hierarchical structure of theoretical models describing physical phenomena. Notable examples include double-gamma decay in quantum electrodynamics [1], and double-beta decay (DBD) - both with and without neutrino emission - within the electroweak interaction framework [2, 3, 4]. These processes serve as valuable probes of non-hadronic interactions.

In the hadronic sector, early investigations, aimed primarily at nuclear spectroscopy, employed pion-induced double charge-exchange (DCE) reactions to study second-order spin-isospin excitations in nuclei [5, 6]. However, the findings were inconclusive due to various experimental and theoretical limitations.

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A renewed opportunity for advancing studies in the nuclear interaction domain has emerged with recent developments in heavy-ion induced DCE reactions [7, 8, 9]. These reactions, described by transitions of the type $A(N, Z) + a(N', Z') \rightarrow B(N \pm 2, Z \mp 2) + b(N' \mp 2, Z' \pm 2)$, enable a complementary approach to investigating second-order spin-isospin nuclear excitations in the reaction partners.

Heavy-ion DCE processes generally proceed via two distinct mechanisms: (i) soft sequential nucleon or nucleon-pair transfers, reflecting underlying single-particle and mean-field properties, and (ii) hard, collisional charge-exchange interactions mediated by nucleon-nucleon (NN) scattering. The latter mechanism populates final nuclear states characterized by two-particle-two-hole (2p2h) configurations, analogous to those involved in two-neutrino and neutrinoless double-beta decay transitions [10, 9, 11]. Thus, collisional DCE reactions offer a direct means to probe isovector nuclear responses via operators similar to those governing DBD processes.

In this contribution, we focus on the scarcely explored spin-isospin two-body excitations and related transition densities (2BTD). Collisional DCE reactions involving isovector NN interactions present distinct advantages for such studies. Firstly, the final nuclear products are unambiguously characterized by suitable changes in their charge, clearly signaling the involvement of two-body nuclear processes. Secondly, the interaction dynamics are well-constrained by empirical data from NN scattering and single charge-exchange (SCE) reactions with light and heavy ions [12]. From a theoretical standpoint, nuclear DCE transitions can be interpreted as being mediated by dynamically generated effective two-body isotensor interactions of rank-2, arising transiently during the brief nuclear contact time.

2. – The DSCE cross section in s-channel representation

Heavy-ion DCE reactions typically occur under peripheral, grazing collision conditions. These processes are well characterized within the framework of direct reaction (DR) theory. The reaction begins in the entrance channel $\alpha = a + A$ and proceeds to the final DCE channel $\beta = b + B$ via a sequence of intermediate single charge exchange (SCE) states, labeled by $\gamma = c + C$; we denote by \mathbf{k}_λ the corresponding invariant three-momentum of the channel considered.

The double sequential charge exchange (DSCE) mechanism proceeds through two successive interactions mediated by the isovector ($T = 1$) nucleon-nucleon transition matrix, \mathcal{T}_{NN} - commonly modeled via virtual π - and ρ - meson exchanges. Consequently, the DSCE amplitude arises at second order in \mathcal{T}_{NN} . Within second order Distorted Wave (DW) theory, we adopt the approximation of averaging the intermediate channel propagator over an energy interval around its pole (that corresponds to energy conservation in the intermediate channel), leading to the logarithmic function $L_\gamma^{(+)}(\omega_\alpha)$ (*mean-energy approximation*), with ω_λ denoting the invariant energy. Under this assumption and employing the s-channel representation of the reaction dynamics, the complete DSCE transition matrix element (TME), summed over the spin $S_{1,2}$ of the two SCE steps, can be written as

$$M_{\beta\alpha}^{(2)}(\mathbf{k}_\beta, \mathbf{k}_\alpha) \approx L_\gamma^{(+)} \sum_{S_1, S_2} \sum_{S, M} (-1)^{\Delta_S} \sum_{c, C} \int d^3q F_{SM}^{(BCA)}(\mathbf{q}) F_{S-M}^{(bca)}(\mathbf{q}) \tilde{V}_{S_1 S_2}^{DSCE}(q) D_{\alpha\beta}(\mathbf{q}), \quad (1)$$

where we have considered, for simplicity, only rank-0 central interactions (see below), and $\Delta_S = S_1 + S_2 + S - M$, with $|S_1 - S_2| \leq S \leq S_1 + S_2$. In the above equation, we have

introduced the DCE form factors (or 2BTDs), e.g. $F_{SM}^{(BCA)}(\mathbf{q})$, as functions of the total momentum transfer \mathbf{q} . These quantities are integrated in the coordinate $\mathbf{v} = (\mathbf{p}_1 - \mathbf{p}_2)$ (being $\mathbf{p}_{1,2}$ the momentum transfer of the two successive steps), over a finite volume $V_v = \frac{4\pi\bar{v}^3}{3}$ with radius \bar{v} ("average approximation"):

$$(2) \quad F_{SM}^{(BCA)}(\mathbf{q}) \equiv \frac{(2\pi)^3}{V_v} \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} [F_{S_2}^{(BC)}(\mathbf{r}) \otimes F_{S_1}^{(CA)}(\mathbf{r})]_{S,M},$$

where $F_S^{(XY)}(\mathbf{r})$ denotes SCE form factors. The strengths of the components of the effective rank-2 iso-tensor interaction are determined by the vertex functionals:

$$(3) \quad \tilde{V}_{S_1 S_2}^{DSCE}(q) \equiv \int 4\pi r^2 dr j_0(qr) V_{S_2 T}(r) V_{S_1 T}(r),$$

where $V_{ST}(\mathbf{r})$ are the vertex functionals of the central isovector interactions, having spin-scalar (S=0) and spin-vector (S=1) character. Finally, in Eq.(1), $D_{\alpha\beta}(\mathbf{q})$ denotes the distortion coefficient, accounting for Initial State (ISI) and Final State Interactions (FSI), as usually appearing in DW theory. One can note that Eq. (1) exhibits a TME structure very similar to that of a single-step transition, except with an effective four-body transition form factor

$$(4) \quad \mathcal{F}^{S_1, S_2}(\mathbf{q}) = \sum_{S,M} (-1)^{S-M} \sum_C F_{SM}^{(BCA)}(\mathbf{q}) \sum_c F_{S-M}^{(bca)}(\mathbf{q})$$

which is composed of the product of a pair of two-body DCE projectile and target form factors. Uncertainties arising from the cut-off radius \bar{v} used in the "average approximation" can be controlled by comparing with the results of the standard t-channel representation [10, 13], taking for instance the global form factor at $\mathbf{q} \approx 0$ as a reference. It is also interesting to observe that the 2BTD of Eq. (2) are closely related to the nuclear form factors entering into DBD-Nuclear Matrix Elements (NME) [14, 15].

3. – Results for DSCE reactions

The relevant features of DSCE reactions are investigated and compared, adopting both *t*- and *s*-channel representations, in the context of the $^{76}\text{Se}(^{18}\text{O}, ^{18}\text{Ne}_{gs})^{76}\text{Ge}_{gs}$ process, currently being explored at LNS Catania [16].

Our analysis is restricted to $0^+ \rightarrow 0^+$ ground-state transitions in both the projectile and target nuclei. To account for initial- and final-state interaction (ISI and IFI) effects, responsible for a reduction of DCE cross sections due to absorption, we employ optical model potentials as prescribed in [9], following the methodology developed in [11]. The SCE interaction is modeled via free-space NN isovector *T*-matrices, using the Love-Franey representation with updated low-energy parameters [17, 9]. The NN interaction includes spin-scalar, spin-vector, and rank-2 tensor components [17].

Nuclear ground and excited states are treated within a self-consistent Hartree Fock + BCS approach extended via the Quasiparticle Random Phase Approximation (QRPA), using the computational framework developed by the Milano group [18], which is optimized for charge-exchange modes. For the particle-hole channel, we adopt three Skyrme parameterizations - SAMI [19], SKX [20], and SLy4 [21] - characterized by increasing

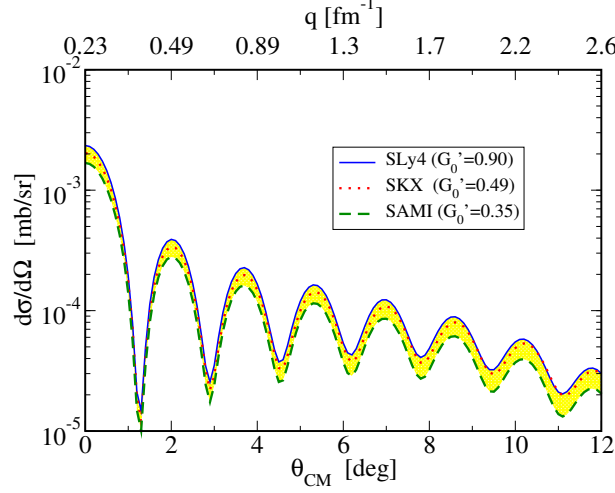


Fig. 1. – Angular distributions of the DSCE cross section computed in the t -channel representation, adopting three different Skyrme interaction [23]. The shaded yellow band represents the spread of the results. The corresponding momentum transfer is shown on the upper axis.

values of the spin-isospin Landau-Migdal parameter $G'_0 = 0.35, 0.49$, and 0.90 , respectively. This parameter significantly influences the spin-isospin response and hence the DSCE observables. Pairing correlations are incorporated via a density-dependent contact interaction in the particle-particle channel, yielding proton and neutron pairing gaps $\Delta_{p,n} \approx 1$ MeV in ^{76}Ge and a neutron gap $\Delta_n \approx 1$ MeV in ^{18}O . QRPA calculations include multipoles up to $J^\pi = 7^\pm$, in sufficiently large configuration spaces, ensuring that Ikeda-type sum rules [22] are fulfilled within 1%. As is standard in DBD studies, the two-step DSCE matrix elements are approximated by a coherent sum over QRPA-derived SCE matrix elements connecting (initial and final) ground states to intermediate excitations. All intermediate nuclear states up to 50 MeV of excitation energy are included, as transitions beyond this energy are found to contribute negligibly to the DSCE strength [23].

3.1. Differential Cross Section. – Employing the ingredients indicated above, the DSCE differential cross section is first computed within the standard t -channel representation. Figure 1 represents the results obtained with the three Skyrme interactions considered in our study. All cross sections are normalized using a common scaling factor, chosen to reproduce full DW calculations [13], with $|L_\gamma^{(+)}(\omega_\alpha)| \approx 1/300 \text{ MeV}^{-1}$. The effect of varying the choice of Skyrme interaction is encapsulated in the (yellow) shaded band, showing that larger values of G'_0 systematically enhance the DSCE cross section. This result highlights the sensitivity of the process to the spin-isospin properties of the nuclear interaction [23]. Then, following the formalism outlined in section 2, the DSCE differential cross sections is computed in the s -channel representation and compared to the t -channel results, in the case of the SLy4 interaction, see Figure 2. At small angles ($\theta < 5^\circ$), the s -channel approach yields results that overestimate the t -channel by about 35% (this discrepancy persists across different Skyrme parametrizations). However, once normalized at $\theta = 0^\circ$ (by suitably tuning the \bar{v} radius), both approaches remain in good agreement within the small-angle (namely small momentum transfer) regime,

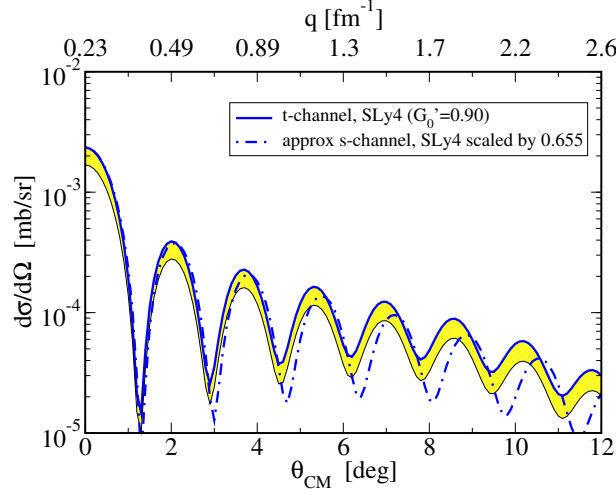


Fig. 2. – Angular distributions of the DSCE cross section computed using the “average approximation” in the s -channel representation (dashed curve) compared to t -channel results (solid curves), adopting the SLy4 Skyrme interaction [23]. The shaded yellow band represents the variation in t -channel results across different Skyrme parametrizations, see Fig.1. The corresponding momentum transfer is shown on the upper axis.

$k_{\alpha\beta} < 200$ MeV/ c (i.e., $k_{\alpha\beta} < 1$ fm $^{-1}$). It is interesting to note that s - and t -channel results agree reasonably well within the spread due to different Skyrme parametrizations. This result supports the suitability of the s -channel “average approximation”, by which the connection between the reaction cross section and (projectile and target) DCE 2BTDs is highlighted. At larger scattering angles, deviations emerge, resulting in phase differences in the angular distributions. This residual discrepancy is primarily attributed to the approximations introduced in the s -channel formalism. In particular, the factorized treatment tends to amplify contributions from high-multipolarity components in the intermediate states - especially for the heavier partner - due to the decoupling of projectile and target dynamics. This suggests that the s -channel scheme is more reliable in asymmetric systems, where one nucleus dominates the transition strength. In such cases, both t - and s -channel approaches are effectively governed by the same structure features of the heavier nucleus, leading to improved agreement between the methods.

4. – Conclusions

DCE reactions induced by heavy ions are effectively described within a second-order Distorted Wave framework, where the process is treated as a two-step mechanism. Building on the s -channel formalism developed in [10, 23], we incorporate a consistent extension that enables the application of practical approximations, such as the so-called “average approximation”. These assumptions allow for simplified, yet rigorous, formulations of the transition matrix element in terms of 2BTDs for both projectile and target nuclei, and of the effective two-step NN interaction potential.

A particularly interesting outcome of this approach is the formal factorization of the DCE TME - hence also the cross section - into distinct DCE NMEs for the projectile and target systems. Within the regime of small momentum transfers, specifically for

$k_{\alpha\beta} < 1 \text{ fm}^{-1}$, the corresponding differential cross sections, derived under the “average approximation”, show good agreement with results from benchmark t -channel based calculations. Notably, the accuracy of the approximation is expected to improve further in reactions involving heavier nuclear targets. This results holds significant promise, potentially providing a novel pathway for extracting information on NMEs akin to those involved in DBD, provided that all the reaction mechanisms contributing to the DCE cross section are under control and incorporated in a coherent manner [24].

Several improvements to the formalism are under development. These include refined treatments of nuclear structure effects, such as the incorporation of nuclear deformation and the adoption of models that more accurately reproduce experimental spectra. Comparative studies across different nuclear structure approaches such as QRPA, Shell Model, and Interacting Boson Model (IBM) are also being pursued to assess model dependencies and enhance predictive power. Moreover, the current studies, limited to $0^+ \rightarrow 0^+$ transitions, need to be extended to encompass a broader class of nuclear excitations. Finally, a key objective moving forward is the coherent inclusion of all relevant reaction mechanisms contributing to the overall DCE amplitude, as soon as their theoretical formulations become available.

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